Robot game winning probability

$$P(W_H) = (1 - P(s_1))(1 - P(s_2))P(W_H|B_1) + (1 - P(s_1))P(s_2)P(W_H|B_2)$$
$$+ P(s_1)(1 - P(s_2))P(W_H|B_3) + P(s_1)P(s_2)P(W_H|B_4)$$

$$= P(B_1) * P(W_H|B_1) + P(B_2) * P(W_H|B_2)$$
$$+ P(B_3) * P(W_H|B_3) + P(B_4) * P(W_H|B_4).$$

Since we redo the game when both robots fail,

$$P(W_H|B_1) = P(W_H)$$
$$P(W_H|B_1) = P(B_1) * P(W_H|B_1) + P(B_2) * P(W_H|B_2) + P(B_3) * P(W_H|B_3) + P(B_4) * P(W_H|B_4$$

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Let

$$r = (1 - P(s_1))(1 - P(s_2))$$
$$= P(B_1)$$

$$u_1 = P(s_1)P(1 - s_2) + P(s_1)P(s_2)P(B_4)$$
$$= P(B_2)P(W_H|B_2) + P(B_3)P(W_H|B_3) + P(B_4)P(W_H|B_4)$$

$$P(W_H|B_1) = P(B_1) * P(W_H|B_1) + P(B_2) * P(W_H|B_2) + P(B_3) * P(W_H|B_3) + P(B_4) * P(W_H|B_4$$
$$= r(P(W_H|B_1)) + u_1$$
$$= u_1 + r(u_1 + u(P(W_H|B_1)))$$
$$= u_1 + r(u_1 + r(u_1 + r(u_1 + r \ldots)))$$
$$= u_1 + ru_1 + r^2(u_1 + r(u_1 + r \ldots))$$
$$= u_1 + ru_1 + r^2u_1 + r^3u_1 + r^4u_1 \ldots.$$

The sum of a geometric series $y = u_1(r)^x$ is

$s_\infty = \frac{u_1}{1-r}$,

which I got from the IB Math formula booklet. Since $r < 1$, we know that this series converges and we can use this formula to find a definite answer to this infinite series. Plugging in actual values in place of stand in variables:

$$\begin{aligned}
s_\infty &= \frac{u_1}{1-r} \\
&= \frac{P(s_1)P(1-s_2)+P(s_1)P(s_2)P(B_4)}{P(B_1)} \\
&= \frac{P(s_1)P(1-s_2)+P(s_1)P(s_2)P(B_4)}{1-(1-P(s_2)-P(s_1)+P(s_1)P(s_2))} \\
&= \frac{P(s_1)P(1-s_2)+P(s_1)P(s_2)P(B_4)}{P(s_1)+P(s_2)-P(s_1)P(s_2)}
\end{aligned}$$

$$P(W_H|B_1) = \frac{(1-s_A)(s_B)(B_2)+(s_A)(1-s_B)(B_3)+(s_A)(s_B)(B_4)}{s_A+s_B-(s_A)(s_B)}$$

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$P(B_1) * P(W_H|B_1)$