University of Zurich[UZH]

Institute of Psychology – Cognitive Psychology

Improving inference about cognitive processes using mixture models.

Gidon Frischkorn & Vencislav Popov

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Workshop Agenda

I. What are Mixture Models and why can they be useful?

II. Specifying mixture models in brms
   a) Data formatting
   b) Setting up mixture families
   c) Understanding & Identifying parameters of mixture families
   d) Fitting & Summarizing results of mixture models

III. bmm – Easy implementation of mixture models for visual working memory tasks

--- Coffee Break ---

IV. Work with (your own) data

V. Outlook: Specifying custom mixture models for accuracy

What will you (not) learn today?

☺

- How to specify simple mixture models in brms
- How to use the *bmm* package to fit existing mixture models for visual working memory tasks
  - Two-parameter (Zhang & Luck, 2008)
  - Three-parameter (Bays et al., 2009)
  - Different flavors of the Interference Measurement model (Oberauer & Lin, 2017)
- Interpret & Summarize results of mixture models estimated using brms

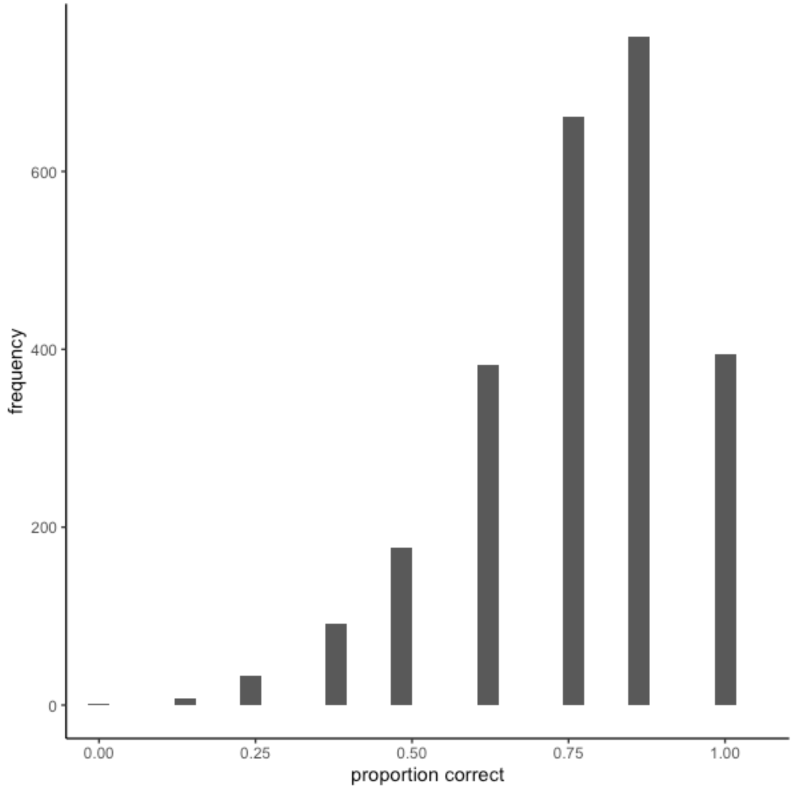
☹

- How to specify complicated mixture models or develop entirely new models
- How to fit mixture models for groups of subjects that differ in their behavior

What are Mixture Models and why can they be useful?

Assumptions in (standard) data analysis:

1. DV stems from a single distribution (oftentimes a normal distribution)
2. predict parameters (usually the mean) from these distributions by independent variables



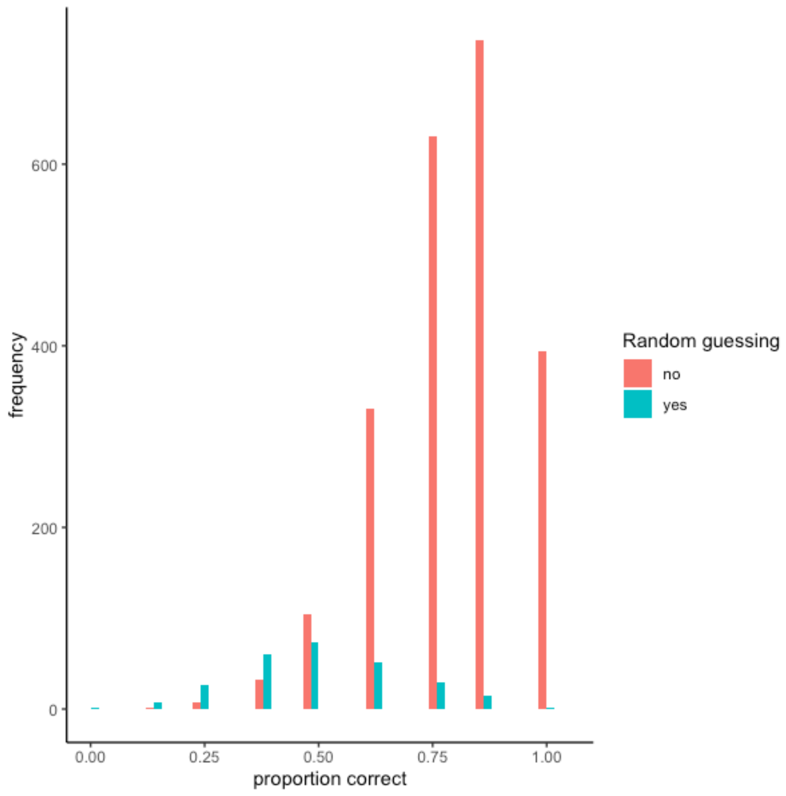
What are Mixture Models and why can they be useful?

Assumptions in (standard) data analysis:

1. DV stems from a single distribution (oftentimes a normal distribution)
2. predict parameters (usually the mean) from these distributions by independent variables

Problem:

→ Sometime data do stem from multiple different distributions



What are Mixture Models and why can they be useful?

Mixture Models…

…specify a set of distributions that data can stem from

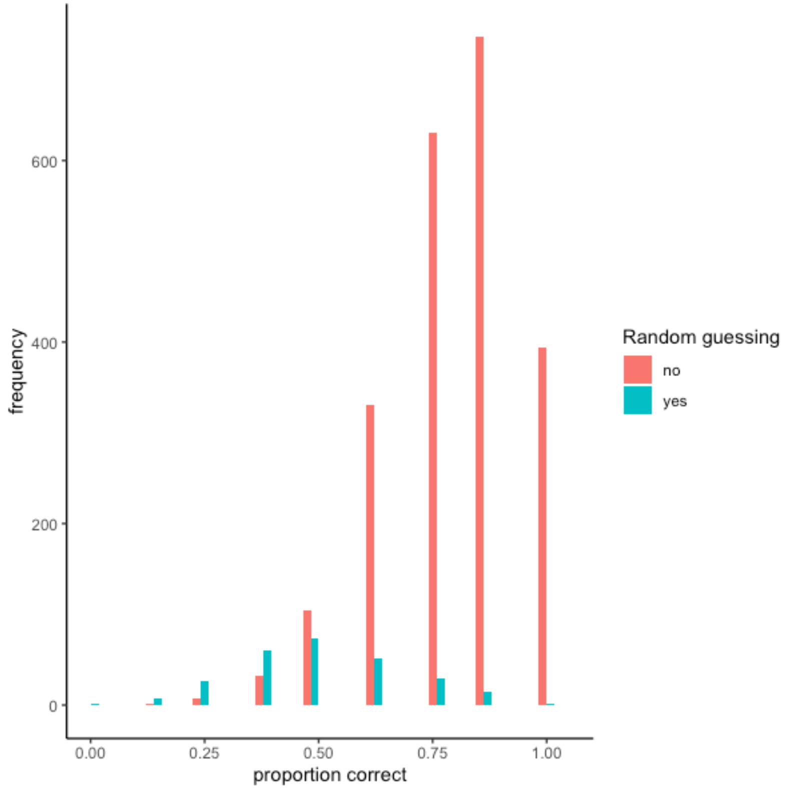
…allow to estimate what proportion of data stems from each distribution

…enable to predict parameters of the different distributions

Such mixtures can occur…

…within a participant (different cognitive states or sources of signal)

…between participants



What are Mixture Models and why can they be useful?

Application to visual working memory

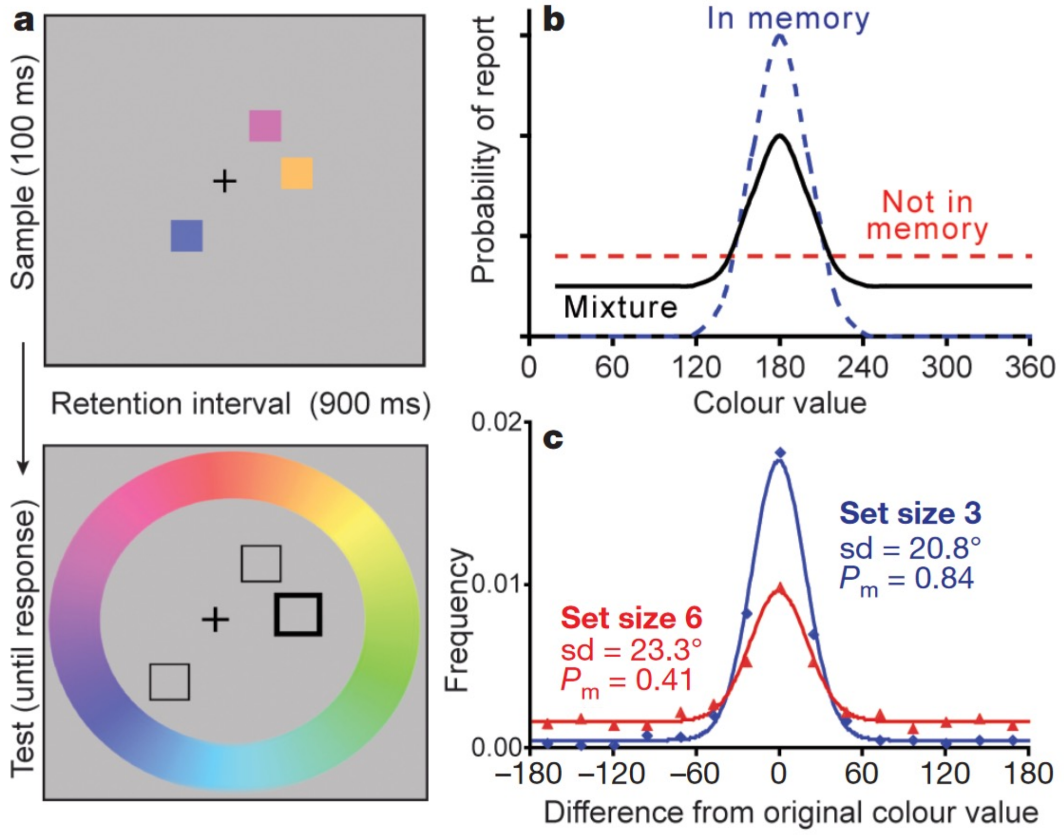
→ Theories separate different states we can be at during retrieval

a) encoded the item in memory → retrieval with the precision of memory representations

b) not encoded the item in memory → random guessing

Performance indicators:

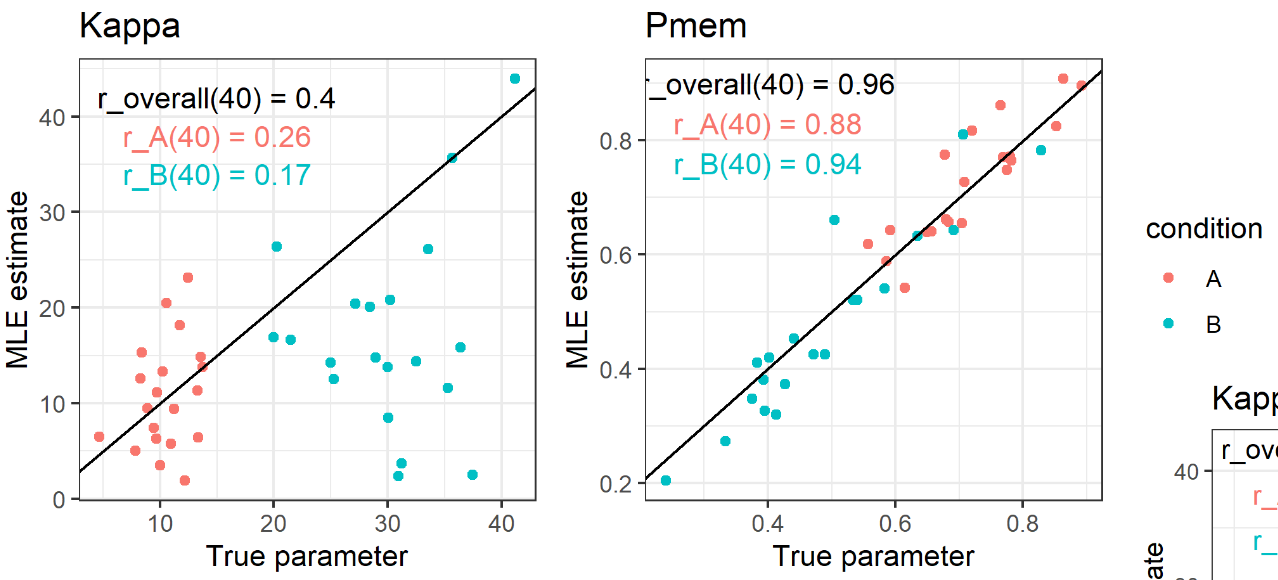
1. P_{mem} = probability of having an item in memory
2. Precision = deviations from correct item

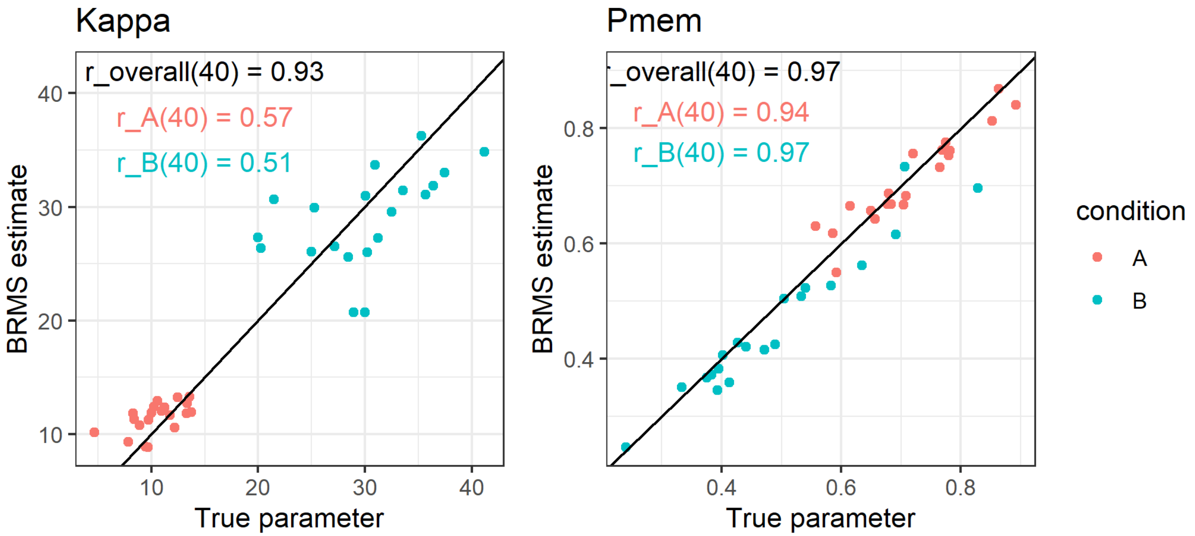


brms/bmm in R vs MemToolbox in Matlab

	brms/bmm	MemToolbox
Estimation	Bayesian	Bayesian and Maximum Likelihood
Fitting multiple conditions	Jointly (Linear model syntax)	Separately to each condition
Inference over multiple conditions	1-step procedure	2-step procedure
Allows continuous predictors	Yes	No
Can fix some parameters across conditions	Yes	No
Tasks	Continuous report, custom	Continuous report, Change detection
Included models “out-of-the-box”	2-parameter, 3-parameter, Interference measurement model	2-parameter, 3-parameter, Variable precision, Slot+averaging, Slots+resources
Can customize models	Yes	No

brms/bmm in R vs MemToolbox in Matlab





Specifying mixture models in brms

A (short) intro to brms (**B**ayesian **r**egression **m**odels using **S**tan)

I. interface to Stan → fit Bayesian generalized linear models

II. formula syntax similar to lme4 → provides familiar & simple regression analyses.

III. wide range of response distributions supported → fit wide range of data

IV. lots of further modeling options:
   - non-linear and smooth terms,
   - auto-correlation structures,
   - censored data,
   - missing value imputation,
   - and quite a few more (like mixture models ☺)

V. all parameters of response distribution can be predicted (means, standard deviations, etc.)

VI. flexible prior specifications → encourages users to apply prior distributions that reflect their beliefs.

VII. Model fit can easily be assessed → posterior predictive checks, cross-validation, and Bayes factors.

Specifying mixture models in brms

A (short) intro to brms (Bayesian regression models using Stan)

`brm` → main function to fit models using brms

required arguments:

1. formula → specifies the regression model to estimate
2. data → data set that contains all variables (**important:** match variable names to names in formula!)

defaults:

3. family = gaussian() → which probability distribution does the DV stem from
4. priors
5. sampler settings (number of warmup & post-warmup sample, number of MCMC chains, etc.)

```
brm(formula = DV ~ 1 + IV + (1 + IV | ID),
    data = myData,
    # optional arguments
    family = gaussian(),
    prior = myPriors,
    iter = 4000, warmup = 1500,
    nChains = 4)
```

Specifying mixture models in brms
Setting up mixture families

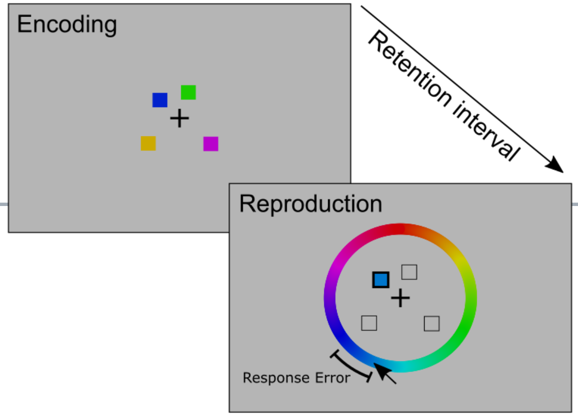
brms allows to specify mixtures of any of the supported data distributions ☺

Steps when setting up mixture models in *brms*:

1. Specify the mixture family
2. Specify model formula to predict parameters
3. Set priors to identify the different mixture components
4. Estimate the model (and have some patience!)
5. Evaluate results

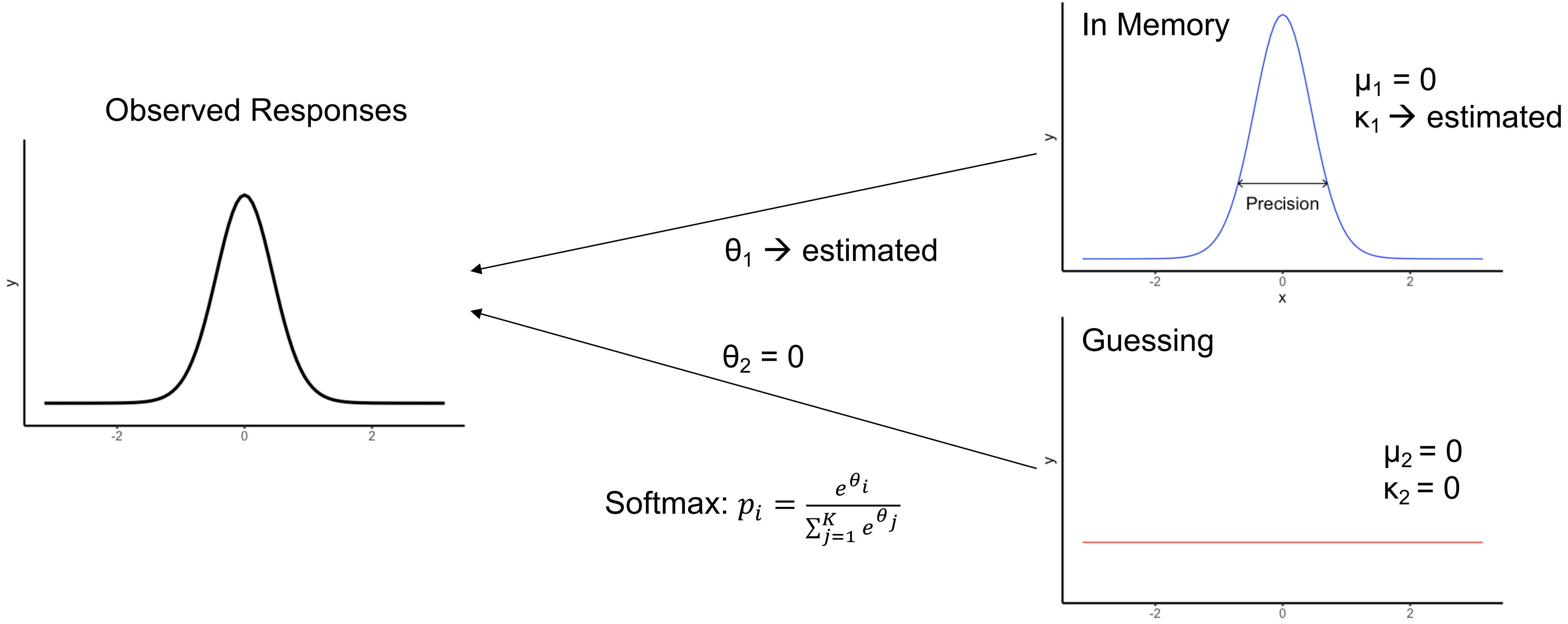
Option
If you want to follow along the next steps directly in R, open the R script „**2pMM_Zhang&Luck2008_brms.R**“.

This script implements all steps that we address now, using only brms syntax. Please note that there are some additional things implemented in the script, that are not included in the here.



Specifying mixture models in brms

Understanding & Identifying parameters of mixture families



Softmax: $p_i = \frac{e^{\theta_i}}{\sum_{j=1}^{K} e^{\theta_j}}$

Specifying mixture models in brms

Setting up mixture families

Step 1: Specifying mixture families in brms

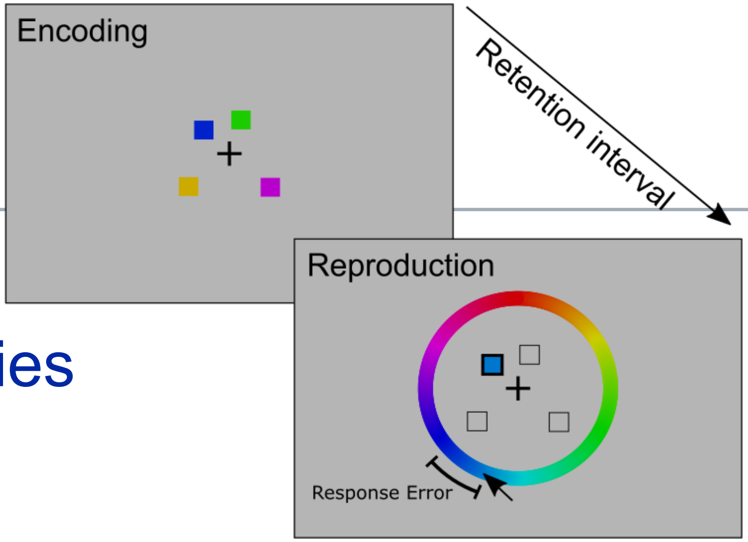
→ `mixture()` function allows setting up mixtures of any set of distributions implemented in *brms*

Example: Two-Parameter Mixture model

```
mixFamily_ZL <- mixture(von_mises, von_mises)
```

Parameters for Mixture Families:

- parameters of each mixture distribution :
  1) vonMises_1: mu1 & kappa1
  2) vonMises_2: mu2 & kappa2
- mixing proportions
  - Each mixture distribution gets a mixing proportion → theta
  - mixing proportions are converted to probabilities → Softmax
  - One mixing proportion needs to be fixed for scaling



Specifying mixture models in brms

Understanding & Identifying parameters of mixture families

Step 2: Specifying the model formula

```
formula_ZL <- brmsformula(ResponseError ~ 1,
                          kappa1 ~ 1 + setsize + (1 + setsize | ID),
                          mu2 ~ 1,
                          kappa2 ~ 1,
                          theta1 ~ 1 + setsize + (1 + setsize | ID))
```

Formula elements:

- Declaring the dependent variable → Response Error
- Prediciting estimated parameters
- Setting intercepts for constrained parameters → fixed via priors

Specifying mixture models in brms

Understanding & Identifying parameters of mixture families

Step 3: setting priors to identify distributions

```
priors_ZL <-
    prior(constant(0), class = Intercept, dpar = „mu1") +
    prior(constant(0), class = Intercept, dpar = „mu2") +
    prior(constant(log(0.0001)), class = Intercept, dpar = „kappa2")
```

Constraints in the Two-Parameter Mixture Model:

1. memory distribution is centered around zero → mu1 = 0
2. center the guessing distribution around zero → mu2 = 0
3. guessing distribution is flat → kappa2 ≈ 0 (brms uses a log-link function)
4. theta2 is internally fixed to zero by brms

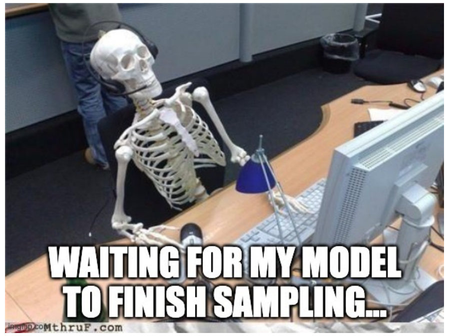
Specifying mixture models in brms

Fitting & Summarizing results of mixture models

Step 4: estimating the with the *brm* function

1. provide the specified formula,
2. names of formula variables = data variable names
3. Use mixture family as data distribution family
4. submit the defined priors to identify your mixture distributions

→ **Model estimation takes some time… be patient.**



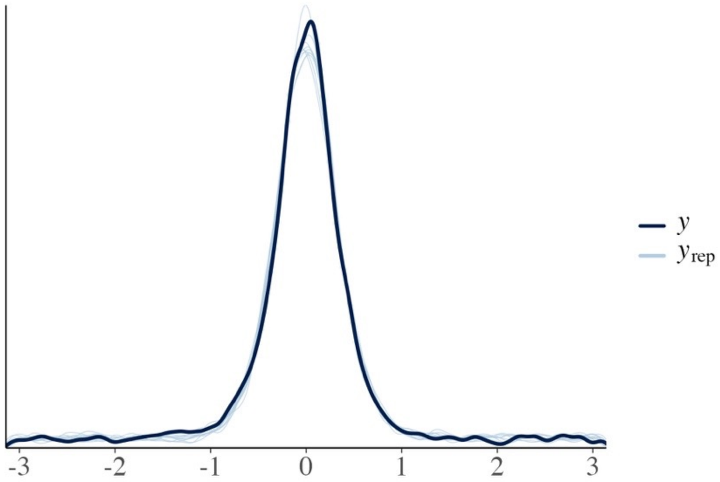

```
fit_ZLmodel <- brm(
        formula = formula_ZL,
        data = myData,
        # now required arguments
        family = mixFamily_ZL,
        prior = priors_ZL)
```

Specifying mixture models in brms

Fitting & Summarizing results of mixture models

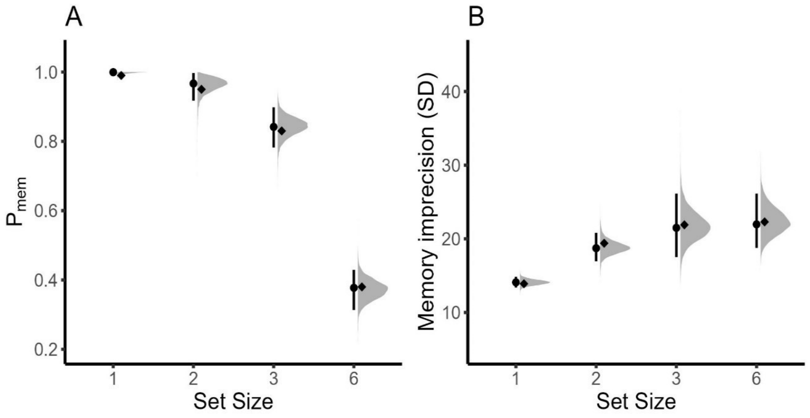
Step 5: Evaluating model results

1. `pp_check(fit_ZLmodel)` → posterior predictive plot to visually evaluate model fit
2. `summary(fit_ZLmodel)` → overview of the estimated parameters
3. `fixef(fit_ZLmodel)` & `ranef(fit_ZLmodel)` → get fixed and random effect estimates
4. `tidybayes & gpplot2` package → processing posterior draws; useful to plot model results




```
Group-Level Effects:
~subID (Number of levels: 8)
                      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sd(kappa1_setsize1)       0.07      0.06     0.00     0.21 1.00     3844     3490
sd(kappa1_setsize2)       0.21      0.13     0.02     0.51 1.00     2188     2385
sd(kappa1_setsize3)       0.48      0.20     0.20     0.97 1.00     2359     3564
sd(kappa1_setsize6)       0.19      0.16     0.01     0.58 1.00     3848     3370
sd(theta1_setsize1)       1.77      1.54     0.08     5.65 1.00     2326     2845
sd(theta1_setsize2)       1.49      0.70     0.57     3.26 1.00     2908     4425
sd(theta1_setsize3)       0.46      0.27     0.05     1.08 1.00     2633     2557
sd(theta1_setsize6)       0.14      0.12     0.00     0.43 1.00     3549     3506

Population-Level Effects:
                 Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
mu1_Intercept        0.00      0.00     0.00     0.00   NA       NA       NA
mu2_Intercept        0.00      0.00     0.00     0.00   NA       NA       NA
kappa2_Intercept  -100.00      0.00  -100.00  -100.00   NA       NA       NA
kappa1_setsize1      2.81      0.05     2.70     2.91 1.00     6449     6160
kappa1_setsize2      2.23      0.10     2.01     2.42 1.00     3597     4064
kappa1_setsize3      1.94      0.20     1.55     2.34 1.00     2156     2387
kappa1_setsize6      1.88      0.17     1.55     2.21 1.00     6705     6526
theta1_setsize1      6.46      1.61     4.39    10.78 1.00     2411     1564
theta1_setsize2      3.37      0.68     2.21     4.92 1.00     2532     4005
theta1_setsize3      1.69      0.22     1.26     2.14 1.00     3679     4257
theta1_setsize6     -0.53      0.13    -0.78    -0.28 1.00     5835     4647
```



Specifying mixture models in brms

Fitting & Summarizing results of mixture models

Step 5: Evaluating model results

Important: take care of link function and transformations when interpreting model parameters

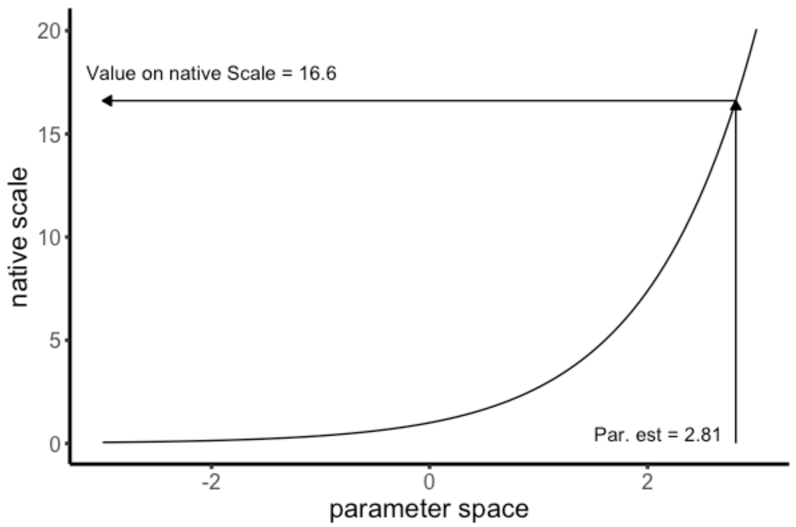
→ for computational efficiency and ideal sampling brms transforms parameters with bounded parameter spaces (e.g., precision > 0, 0 > probabilities < 1)

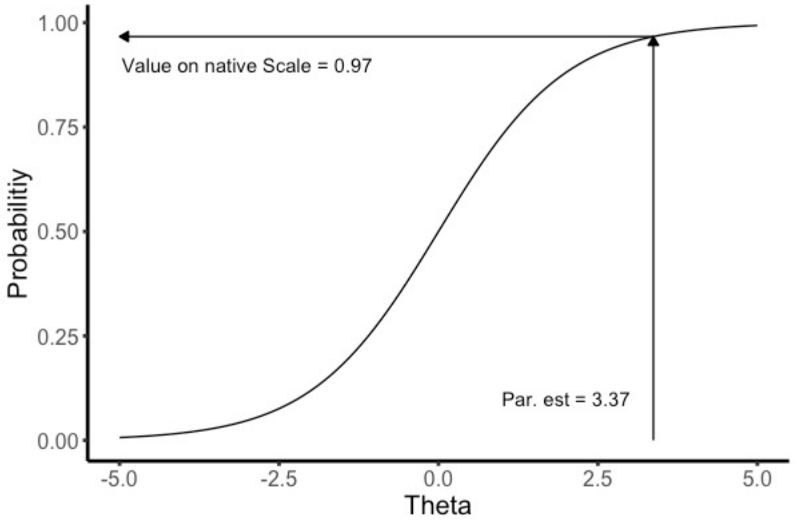
Kappa → log link

$$\kappa_{native} = e^{\kappa_{par}};\ \kappa_{par} = \log(\kappa_{native})$$

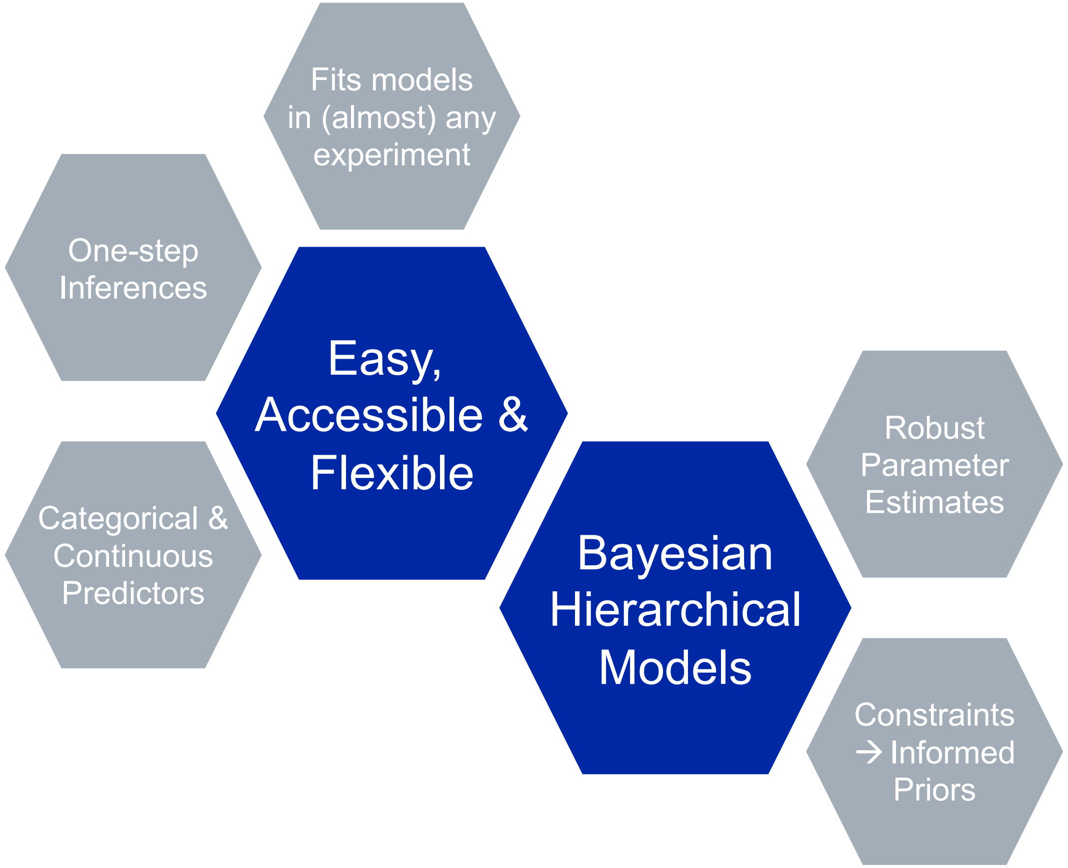
Theta → Softmax (for two mixtures = logit)

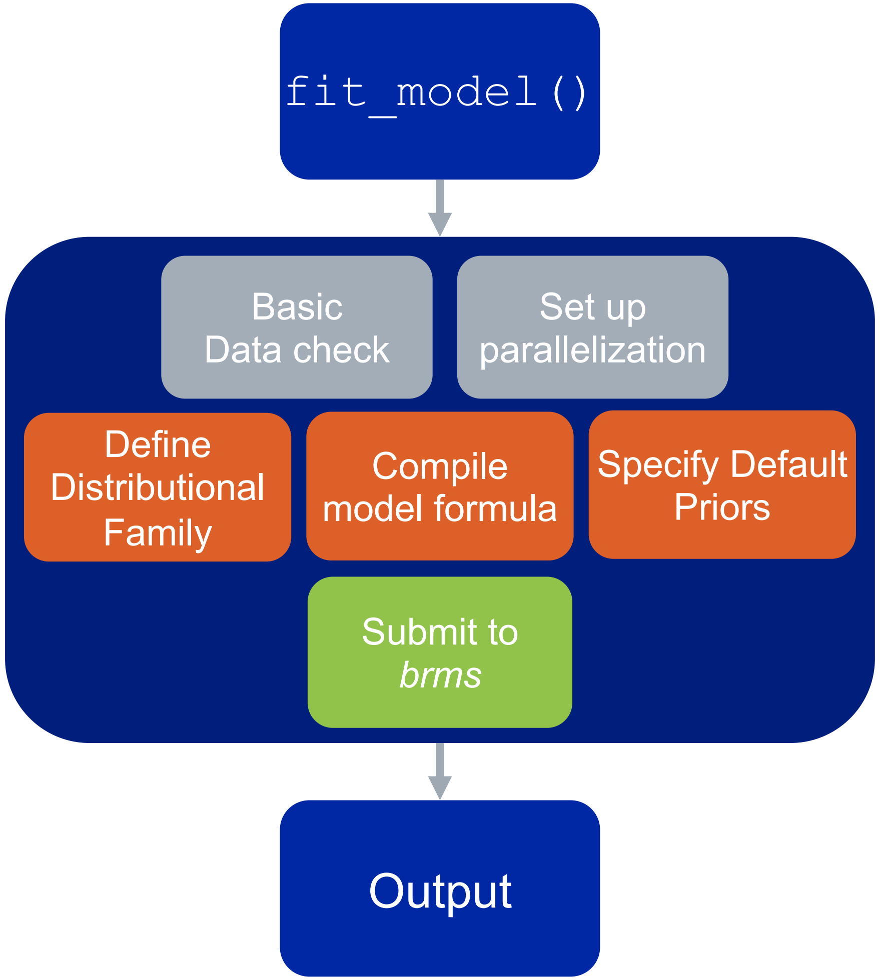
$$p_i = \frac{e^{\theta_i}}{\sum_{j=1}^{K} e^{\theta_j}};\ p_{Mem} = \frac{e^{\theta_{mem}}}{e^0 + e^{\theta_{mem}}} = \frac{e^{\theta_{mem}}}{1 + e^{\theta_{mem}}}$$





bmm: Easy implementations of mixture models for VWM tasks





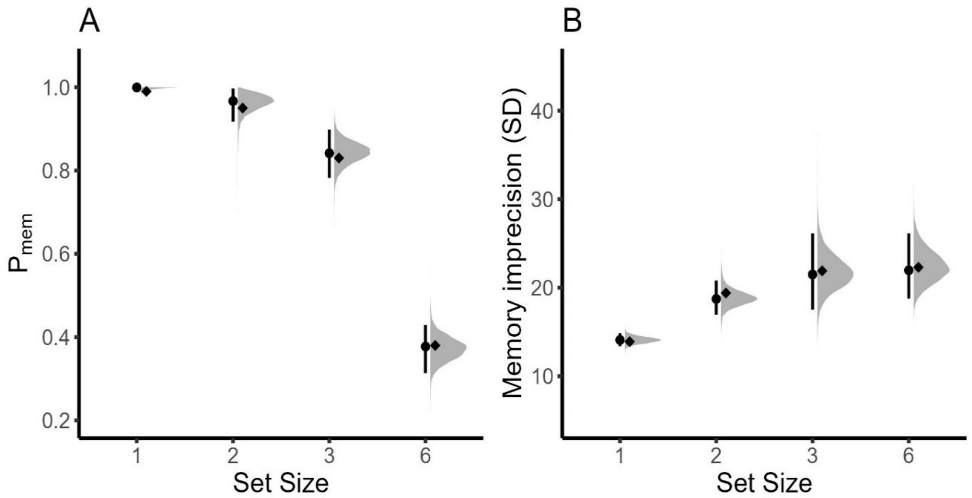
bmm: Easy implementations of mixture models for VWM tasks
Two-Parameter Mixture Model

1) Specify formula

Dependent Variable

Predictors of Parameters

```
ZL_mixFormula_bmm <- bf(RespErr ~ 1,
                        kappa ~ 0 + setsize + (0 + setsize || subID),
                        thetat ~ 0 + setsize + (0 + setsize || subID))
```




```
fit_ZL2009_bmm <- bmm::fit_model(formula = ZL_mixFormula,
                                 data = data_ZL2008,
                                 model_type = '2p',
                                 warmup=1000, iter=2000, parallel=TRUE)
```

Formula & Data

2) Fit Model

Select Model Type

Additional Arguments

bmm: Easy implementations of mixture models for VWM tasks
Three-Parameter Mixture Model

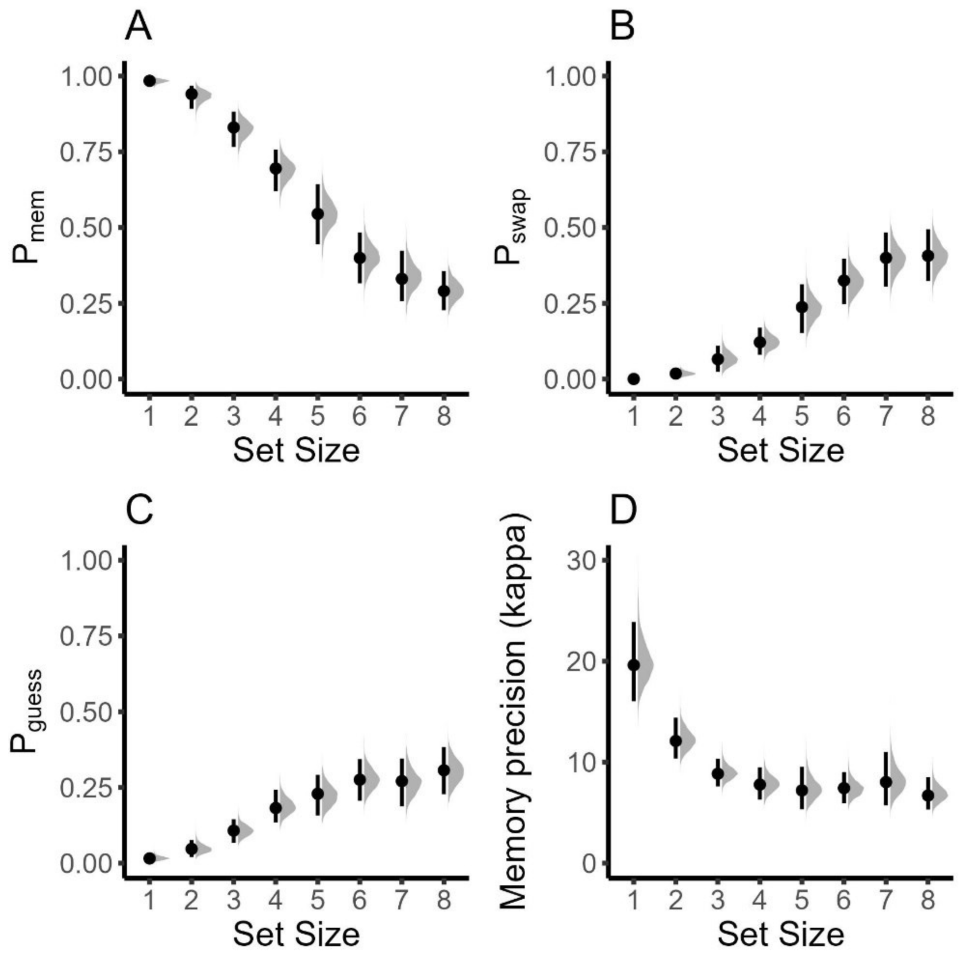
1) Specify formula

```
ff <- bf(devRad ~ 1,
         kappa ~ 0 + SetSize + (0 + SetSize || ID),
         thetat ~ 0 + SetSize + (0 + SetSize || ID),
         thetant ~ 0 + SetSize + (0 + SetSize || ID))
```

2) Fit Model

Additional Arguments

```
fit_3pMM <- bmm::fit_model(
    formula = ff,
    data = df_OberauerLin2017_E1,
    model_type = '3p',
    non_targets = paste0('Item',2:8,'_Col_rad'),
    setsize = "SetSize")
```



The Bayesian Measurement Model (*bmm*) packages

Interference Measurement Model

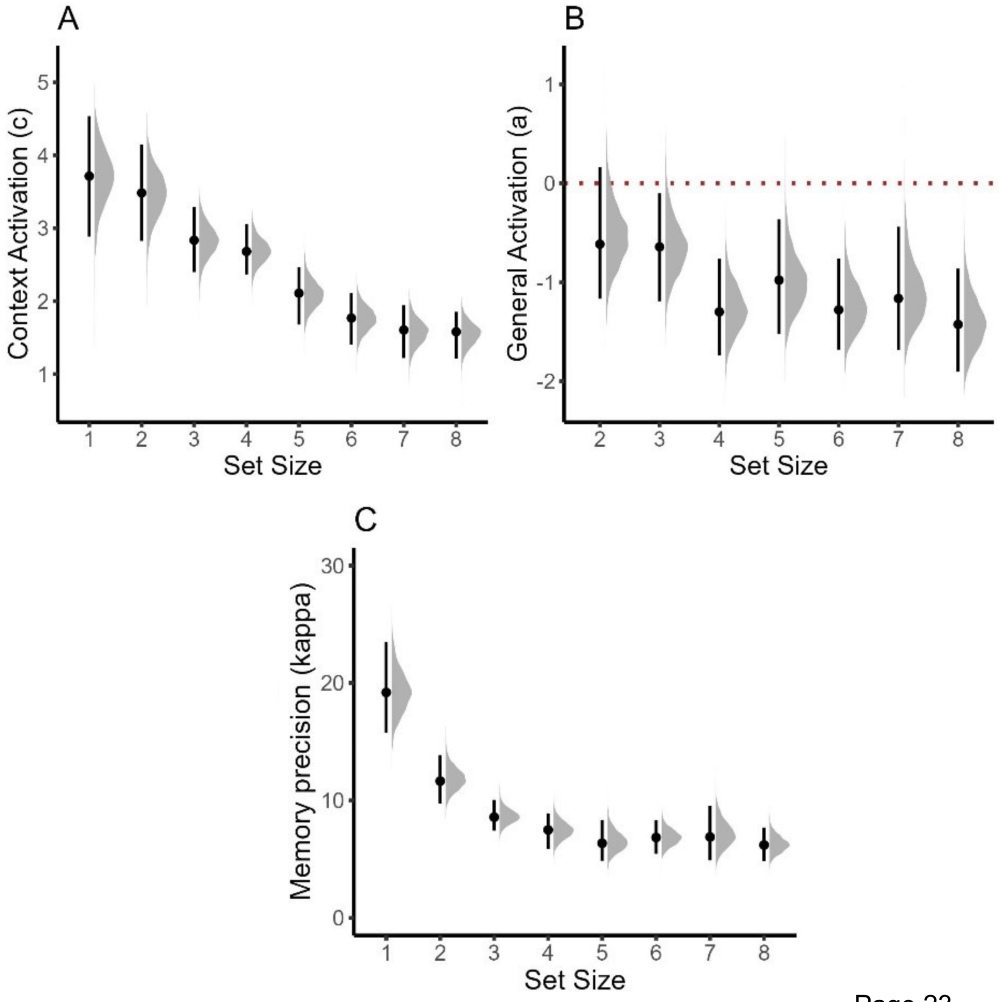
1) Specify formula

```
ff <- bf(devRad ~ 1,
         kappa ~ 0 + SetSize + (0 + SetSize || ID),
         c ~ 0 + SetSize + (0 + SetSize || ID),
         a ~ 0 + SetSize + (0 + SetSize || ID),
         s ~ 0 + SetSize + (0 + SetSize || ID),
)
```

2) Fit Model

Additional Arguments

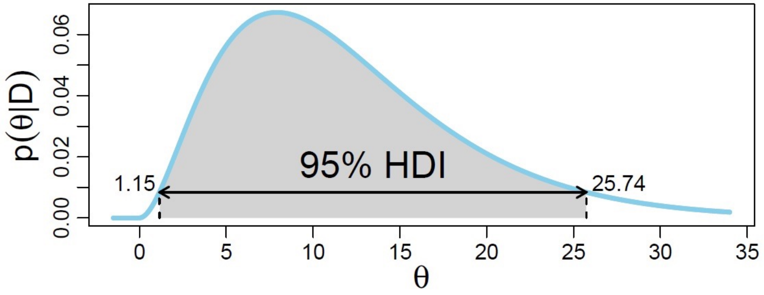
```
fit_IMMfull_mixMod <- fit_model(
    formula = ff,
    data = df_OberauerLin2017_E1,
    model_type = 'IMMfull',
    non_targets = paste0('Item',2:8,'_Col_rad'),
    spaPos = paste0('Item',2:8,'_Pos_rad'),
    setsize = "SetSize")
```



Testing Hypothesis with Bayesian models

1. **Evaluating 95% Highest Density Intervals**

   → Do my posterior estimates, include a specific value (e.g., 0) in their posterior distribution

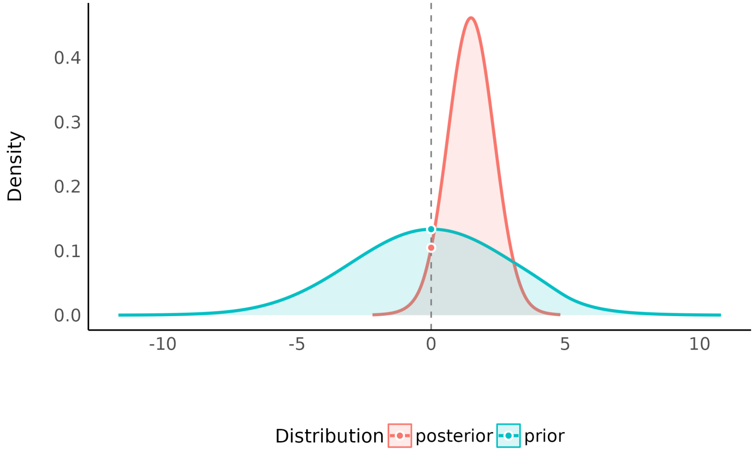


2. **Computing Bayes Factors for parameters (Savage-Dickey Ratio)**

   → Given the data, has my prior belief credibly changed

3. **Computing Bayes Factors for competing models (bridgesampling)**

   → Under which model are the observed data more probable?



Time for a coffee break!

Preprint introducing the *bmm* package:

Frischkorn, G. T., & Popov, V. (2023). *A tutorial for estimating mixture models for visual working memory tasks in brms: Introducing the Bayesian Measurement Modeling (bmm) package for R*. PsyArXiv. https://doi.org/10.31234/osf.io/umt57

Work with (your own) data

You have your own data

1. Prepare your data
   a) Transform to long format → each trial in a row
   b) Calculate response error in radians
   c) Calculate non-target locations relative to the target values
2. Specify a model you want to fit
3. Test if the modelling is sampling
   – Use a low number of samples to avoid lengthy wait times (iter = 500)
4. See if you can extract and plot results

You have no own data

1. Choose one of the data sets shared in the GitHub repository (For continous reproduction tasks, simulated binomial data, etc.)
2. Try to understand the different variables
3. Specify a model you want to fit
4. Test if the model is sampling
   – Use a low number of samples to avoid lengthy wait times (iter = 500)
5. Extract and plot results

Specifying custom mixture models for accuracy and reaction time data

A logic similar to VWM mixture models can be applied to accuracy data

- Lapses of attention → guessing performance for some trials
- perform with a certain level of ability (i.e., proportion correct)

```
mix_binomial <- mixture(binomial, binomial)
# fix probability to 0.50 via priors
priors_binomial <- prior(
   constant(0), class = Intercept, dpar = "mu2"
   )
```