Eigenfrequency study of a marimba bar

Goal

Estimate the harmonicity introduced by the expedients used by musical instrument makers for increasing the level of harmonicity of the sound produced by the marimba bars.

Description

The marimba bar has a lateral section characterized by a tapering at the center and the edges of the bottom surface. In the following you can find the Matlab code that models the bottom surface of the bar (given by the z variable), whereas upper surface is flat, i.e. z = 0.
Code for generating the profile of the bar:

```
%% Script for generating the thickness profile of a marimba bar as a function of
the horizontal position
% Fabio Antonacci, 2021 @ Musical Acoustics, A.A. 2021/2022
dx = 0.01; % Step of the horizontal axis
x = 0:dx:10-dx; % Horizontal axis
N = length(x);
z = zeros(size(x)); % Variable for storing the thickness of the bar as a
function of the horizontal position
a = 1.23; % To be modified in the range 0.1-->0.9, step 0.1
b = 1;
for n = 1:floor(N/8)-1
z(n) = a-2.5-a*cos((x(n)-1.25)*2*pi*b/5);
end
for n = floor(N/8):floor(N/4) % Constant thickness for the first fourth of the
length of the bar
    z(n) = -2.5;
end
for n = floor(N/4)+1:3*floor(N/4) % Sinusoidal thickness profile in the central
part of the bar
    z(n) = a-2.5+a*cos((x(n)-5)*2*pi*b/5);
end
for n = 6*floor(N/8)+1:7*floor(N/8) % Constant thickness for the last fourth of
the length of the bar
    z(n) = -2.5;
end
for n = 7*floor(N/8)+1:N
z(n) = a-2.5-a*cos(-(x(n)-8.75)*2*pi*b/5);
end

plot(x,z)
axis equal
```

Question 1

Generate in Comsol the mesh that reproduces the marimba bar. For doing so, generate the lateral section inspired by the Matlab code above, and then use the extrude operator to generate the full mesh (width: 2.5 cm). Consider the “a” variable as a parameter modeling the tapering, ranging in the interval 0.1 → 1.2 cm.

Question 2

Compute the eigenfrequencies of the first five modes of the bar as a function of the "a" parameter in the code above.

Question 3

Compute the inharmonicity of the system as a function of the "a" parameter. The inharmonicity is computed through the function

$$I = \sum_{n=2}^{N} \left| \frac{f_n}{f_{n-1}} - m_n \right|$$

Where f_n is the eigenfrequency of the mode n, N is an integer number in the range 2→5 and

$$m_n = \arg\min_m |f_n - m f_{n-1}|, \; m \in \mathbb{N}^+$$

Plot the I as a function of the parameter a for all the values of N as in the range identified above.

Important information

Deadline: upload the report by January 17, 2022.

- Answer concisely;
- Describe – concisely - the procedure used to obtain the results: if an error is present, I cannot identify the reason – numerical or conceptual - if the procedure is not described: in grading I will be forced to use the worst-case option.
- **All students must upload the report.**

In the PDF file and possibly in the filename, specify the name, surname and ID of all the students participating to the HW, if more than one student worked on it.