Musical Acoustics
Homework 1

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Introduction

We want to build, using COMSOL, a 3D model of a church bell and study its behaviour in terms of modeshapes and then perform a frequency domain and a time domain studies as well. Finally, we want to confront the results with an axisymmetrical model of the same bell.

1 3D model

1.1 Geometry

To build the model, we first used the sketch tool of COMSOL to design the section of one half of the bell as a 2D component (component 1), as a combination of quadratic Bézier curves and polygonal lines. We then used this 2D component, importing it the plane geometry of a work plane inside a 3D component (component 2). The *revolve* function is then used to rotate this object around the z axis, thus obtaining the 3D model for the church bell.
We then added the material of the bell (bronze, with $E = 98.6e9\, GPa, \rho = 8750\, Kg/m^3, \nu = 0.34$) and the physics we want to use (*solid mechanics*).

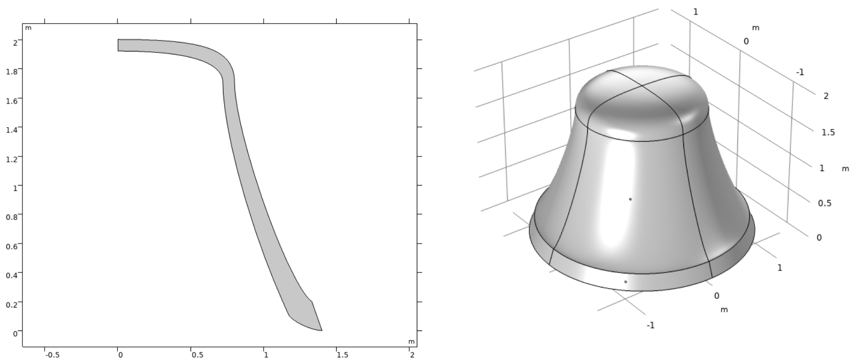


Figure 1: Section and 3D model of the bell.

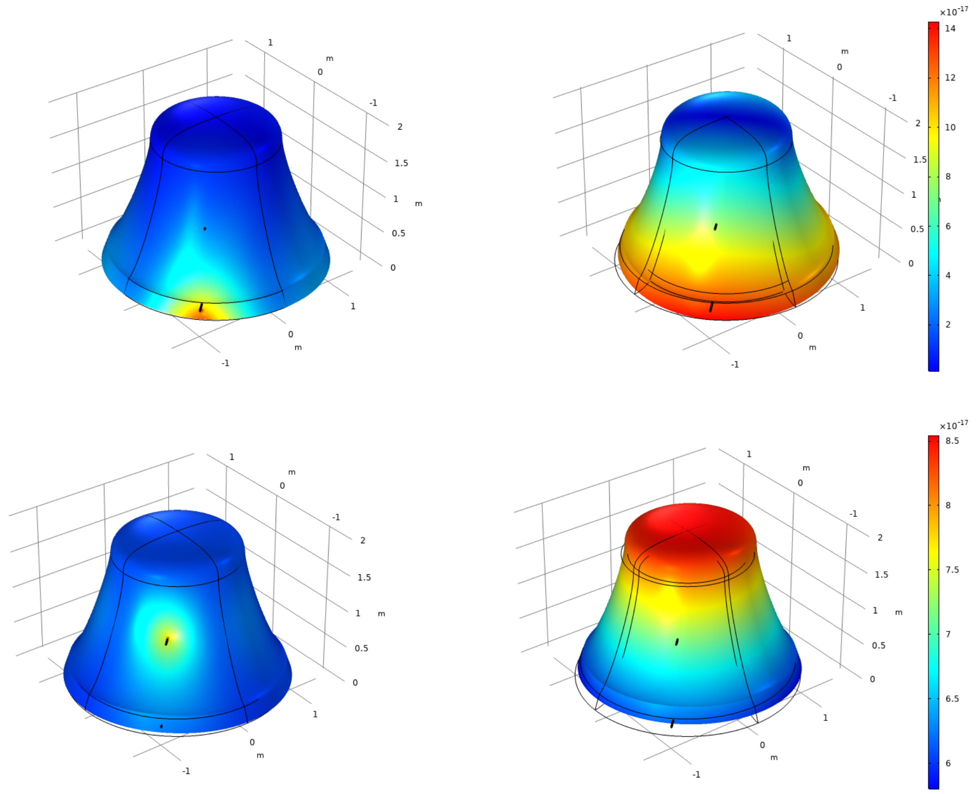
1.2 Eigenfrequency study

The first study we performed was an eigenfrequency study, in order to obtain the modeshapes of the bell. In particular, we computed the first 21 natural frequencies of our model.
Looking at the results of the study, we can observe that, among the derived frequencies, the first ones have very low values. These frequencies don't correspond to actual modeshapes of the model, but to rigid movement of the body, which is allowed since we didn't impose any boundary condition.
These six frequencies correspond to one degree of freedom of a tridimensional system and are supposed to be zero, but because of the approximation in the finite element solution, they assume very low values, some even complex.
Following these six frequencies, we find the ones corresponding to the actual modeshapes.
In particular, we can observe that almost all of these come in couple of values very close to each other (when not identical). This is due to the fact that, because of the symmetry under axial rotation, every mode incurs in degeneracy, except for mode 6, which is axisymmetrical.
The following table contains the resulting eigenfrequencies.

Eigenfrequencies [Hz]				
2.27e-5i	5.23e-5i	1.75e-5	5.71e-5	8.45e-5
1.10e-4	53.44	53.46	141.92	141.92
196.18	196.18	235.95	235.98	250.60
250.63	314.95	371.94	372.06	318.31
381.33				

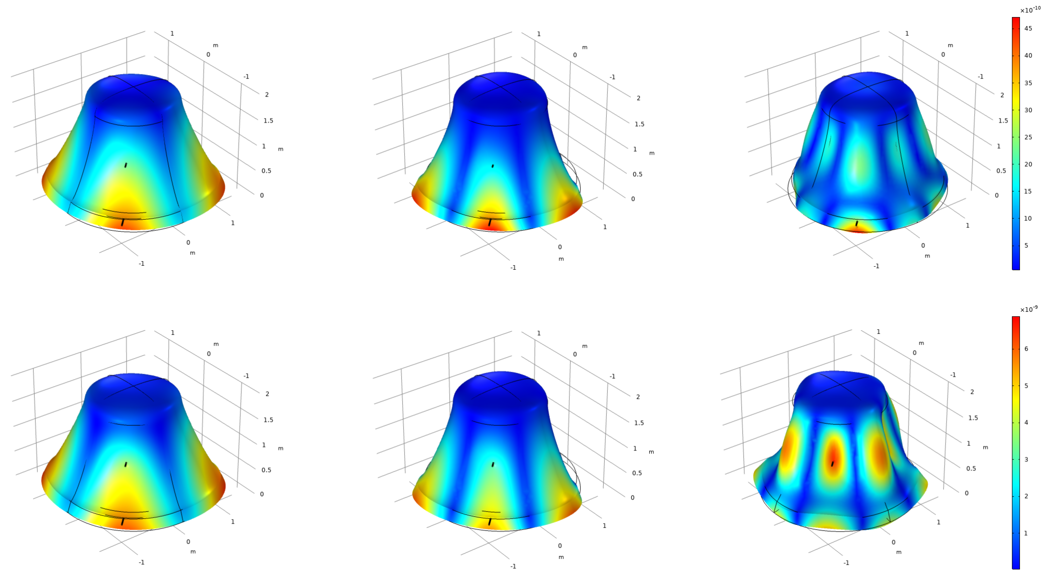
1.3 Time domain study

To perform a time domain study, we first added, in the definition section of the 3D component, a gaussian pulse ($\mu = 0.002, \sigma = 0.0001$), which we used to model an impulsive force applied as boundary load, going from time $t = 0\,s$ to $t = 0.5\,s$, with an increase of one millisecond per step.
We implemented two different boundary loads to study the differences in the response when the force is applied on the edge or on the side of the bell. In the "Time dependent" node of the study we named "Time response" it is possible to chose the correct force to apply ("Edge gaussian" or "Side gaussian").
The following plots show, for each position in which the force acts, two snapshots of the displacement, one taken in the early stages of the time response, and one around $t = 0.3s$. The first row refers to the force acting on the edge, the second on the side of the bell.



1.4 Frequency domain study

Similarly to the time domain study, to perform a frequency domain one we applied a force as boundary load, and in the "Frequency domain" node of the study named "Frequency response" it is possible to chose between an application of the force on the side or on the edge ("Harmonic edge" or "Harmonic side").
This time, the force is a harmonic one, with frequency ranging from 0 Hz to 400 Hz, with an increase of 10 Hz per step.
Below, we show the plots for a few frequencies of the force. In particular, we chose frequencies equal or very close to the eigenfrequency of three of the modes of vibration.
Like for the time domain study, the first row of images refers to the force acting on the edge of the bell, while the second one refers to the force acting on the side. We can see that the mode shapes are intelligible, although the displacement is slightly different between the results in the two configurations, as it can be seen clearly form the third pair of plots, corresponding to a higher frequency of the force.



2 Axisymmetric model

The second exercise demanded to build a model of the same bell as before using an axisymmetrical approach.

2.1 Geometry

We started by the *Model Wizard* of COMSOL, choosing the *2D-axisimmetric* model and applying *Solid Mechanics* as physics and *Eigenfrequency* between the general studies.
Inside the planar workplace for the geometry, we designed half of an axial section of the bell with the use of quadratic Bézier curves and polygonal lines, in respect of the same geometry used in the first exercise. Once the shape was concluded we added the material (again, bronze, with $E = 98.6e9\, GPa, \rho = 8750\, Kg/m^3, \nu = 0.34$).

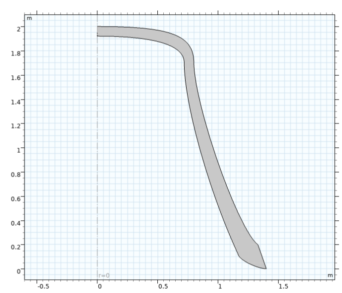


Figure 2: Half-section of the bell used for the axysimmetric approach.

2.2 Eigenfrequency study

In order to compute the study, we decided to apply a user-controlled mesh: we built it with triangular elements of normal size, but we applied the condition of having a boundary distribution of 150 elements on both the inner and the outer sides of the half-section geometry. The result is a really fine mesh with well distributed elements all along the section and not concentrated on the vertices, the total number of elements is 1850 with respect to the 2150 we would have had if we chose a physics-controlled mesh with extremely fine distribution.
Then we computed the study considering no boundary conditions for a total of 21 natural frequencies, which should correspond to the same mode shapes we found in the previous exercise.

Eigenfrequencies [Hz]				
3.36e-4	312.84	394.30	510.83	636.73
726.73	779.14	938.22	1079.2	1302.9
1435.9	1608.0	1774.6	1969.7	2252.3
2557.1	2664.6	2944.4	3255.7	3436.6
3629.9				

The obtained results show us that the first configuration is a rigid translation of the body, just as the first six modes of the 3D model; we have only one of these modes because a planar figure has only one degree of freedom which can maintain the symmetry with respect to the axis of rotation of the figure. After that we have 20 actual mode shapes.

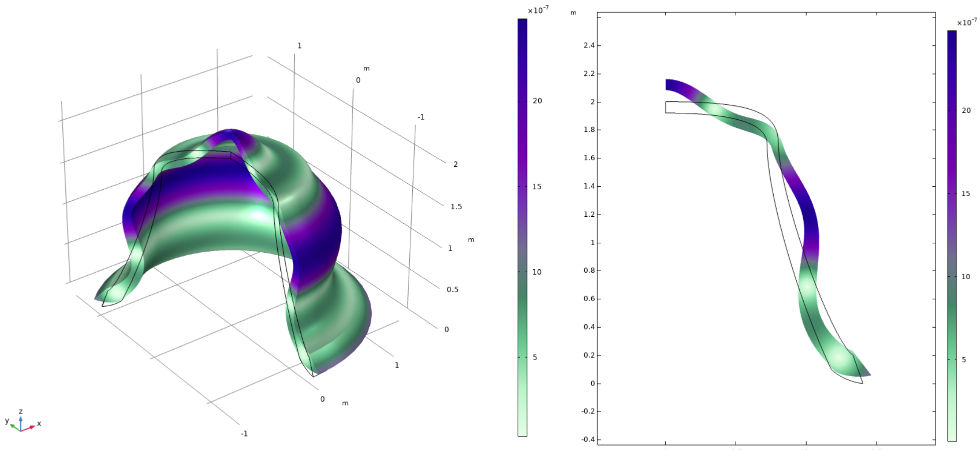


Figure 3: Mode shape of the bell at $f = 636.73\,Hz$.

2.3 Circumferential mode extension

The simple eigenfrequency study can't show us the circumferential modes, so we have to add this feature to *Solid Mechanics* options in order to plot them. Inside the first 21 natural frequencies we obtain the following new eigenfrequencies:

Eigenfrequencies [Hz]				
i4.82e-5	2.41e-4	312.84	394.30	491.07
510.83	636.73	726.73	779.14	938.22
946.55	1079.2	1302.9	1418.5	1435.9
1608.0	1774.6	1840.2	1969.7	2190.3
2252.3				

2.4 Comparison

The axisymmetric approach only shows 3D plots built as every physical quantity doesn't vary along the tangential direction. In this way we do not obtain the same results as in the previous exercise because, given two different sections of the bell, the displacements along these sections are, in general, not the same, so we cannot treat the bell as an axisymmetric body from an acoustic point of view.