Musical Acoustics
Homework laboratory II: Electric analogs

Marin Pasin Davide 10799610
De Bortoli Gian Marco 10805035

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Introduction

In this homework, we are first asked to model and simulate a Helmholtz resonator through Simscape.
The parameters for the model are the following:

- Volume of the resonator: $V_0 = 0,1\ m^3$
- Length of the neck of the resonator: $l = 10\ cm$
- Section of the neck: $S = 100\ m^2$
- Air speed: $c = 343\ m/s$
- Density of air: $\rho = 1.2\ kg/m^3$

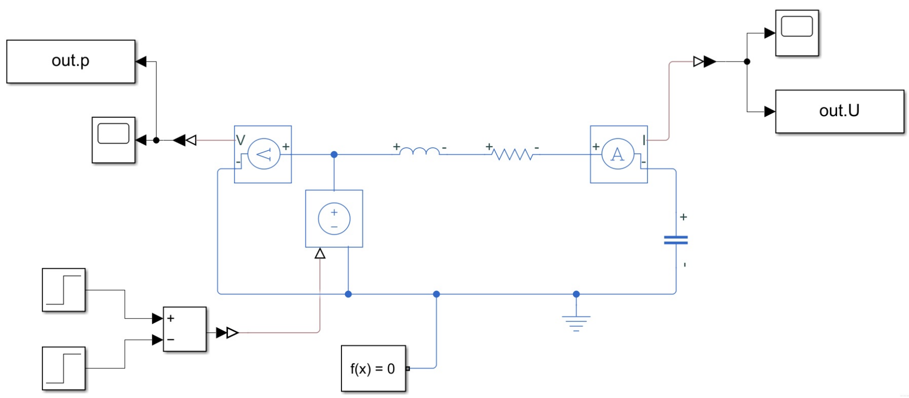
We then want to use this model of the Helmholtz resonator to build a more complex resonating system, connecting it to replicas of itself to build an Helmholtz resonator tree.

1 Single Helmholtz resonator

In order to build the model of the resonator in Simscape, we first need to derive the electric equivalent of each acoustic element of the system.
In particular, we can identify three components:

- a condenser, equivalent to the air volume, whose capacity C is calculated as:
$$C = \frac{V_0}{\rho c^2} \tag{1}$$
- an inductor, equivalent to the mass of the air in the neck of the resonator, whose inductance is:
$$L = \frac{\rho l_{tot}}{S} \tag{2}$$
where l_{tot} is the total length of the neck, keeping into account the end correction, given by $\Delta l = 2 \times \frac{8}{3\pi}\, a$, where $a = \sqrt{S/\pi}$ is the radius of the neck.
- a resistance that models the losses in the short tube, whose value is given by:
$$R = \frac{\rho c}{S} \tag{3}$$

We are interested in studying the frequency response (admittance) $Y(\omega)$ of the system, defined as the ratio between the acoustic flow $U(\omega)$ and the pressure $p(\omega)$, both measured at the entrance of the neck of the resonator.
$$Y(\omega) = \frac{U(\omega)}{p(\omega)} \tag{4}$$
In the simulation with the equivalent electric circuit, this corresponds to measuring the input voltage, equivalent to the pressure, and the current flowing through the circuit, equivalent to the acoustic flow. As an input, a Dirac's impulse as been implemented as the difference of two step functions, in order to excite every frequency.
The following picture shows the resulting circuit used for the simulation.



We also implemented the transfer function inside Matlab by considering the single acoustic elements' impedances, summing them together (since they are in series) obtaining the overall impedance and then inverting the latter to obtain the admittance:

$$Z(\omega) = j\omega L + R + \frac{1}{j\omega C} \tag{5}$$

$$Y(\omega) = Z(\omega)^{-1} \tag{6}$$

The plot in figure 1 shows the comparison between the transfer function resulting from the simulation and the transfer function computed analytically. Observing the plot, we can find the resonance

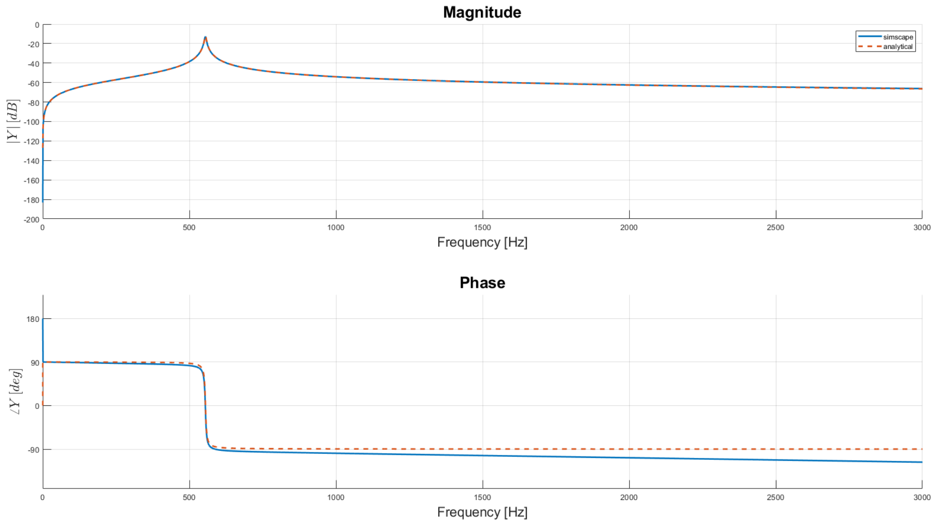


Figure 1: Admittance comparison between a single Helmholtz resonator built inside Simscape and the same one computer analytically.

frequency of the system using the *findpeaks()* function of Matlab: for the admittance obtained with the Simscape circuit it is located at $f_{res} = 555.05\ Hz$, while for the admittance obtained analytically inside Matlab it is located at $f_{res} = 554.91\ Hz$. We must consider that the resulting values of this method are influenced by the resolution given to the frequency axis inside the script.
We can now compute, analytically, the resonance frequency of the Helmholtz resonator, and confront it with the one resulting from the simulation of the equivalent circuit. To do so, we can use the formula for the resonance frequency of a Helmholtz resonator, always accounting for the end correction of the neck:

$$f_{res} = \frac{c}{2\pi}\sqrt{\frac{S}{V_0 l_{tot}}} = 554.53Hz \tag{7}$$

As we can see, the results of the simulation deviate from the theoretical result of about 0.1%, meaning that, while the lumped element approximation is valid, the equivalent circuit is a good way to simulate the system.

2 Helmholtz resonator tree

We now want to connect replicas of the Helmholtz resonator analyzed in the previous section in order to form a tree, characterized by two values:

- N, indicating the height of the tree;
- K, indicating the number of leaves for each branch.

We aim at observing how the admittance changes when changing the values of N and K, at first measuring the response on one of the outermost leaves.
The implementation of this tree inside the Matlab script follows the following considerations:

- the outermost leaves are simple replicas of the single Helmholtz resonator and, therefore, they have an impedance given by:

$$Z_0(\omega) = j\omega L + R + \frac{1}{j\omega C} \tag{8}$$

- the previous layer with respect to the outer leaves is given by the same electrical circuit, but with the outer leaves placed in parallel to the condenser, so that each leaf on this layer has the following impedance:

$$Z_1(\omega) = j\omega L + R + \frac{1}{j\omega C + \sum_{n=1}^{N} \frac{1}{Z_0(\omega)}} \tag{9}$$

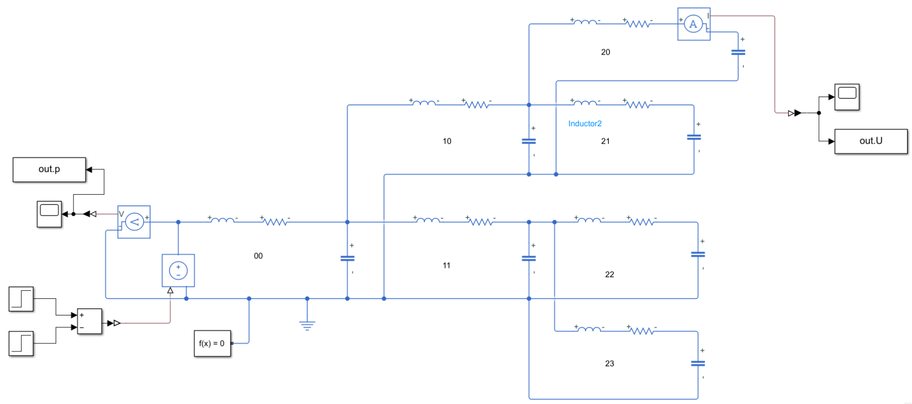
- then, going up another layer toward the root, with the same reasoning we obtain:

$$Z_2(\omega) = j\omega L + R + \frac{1}{j\omega C + \sum_{n=1}^{N} \frac{1}{Z_1(\omega)}} \tag{10}$$

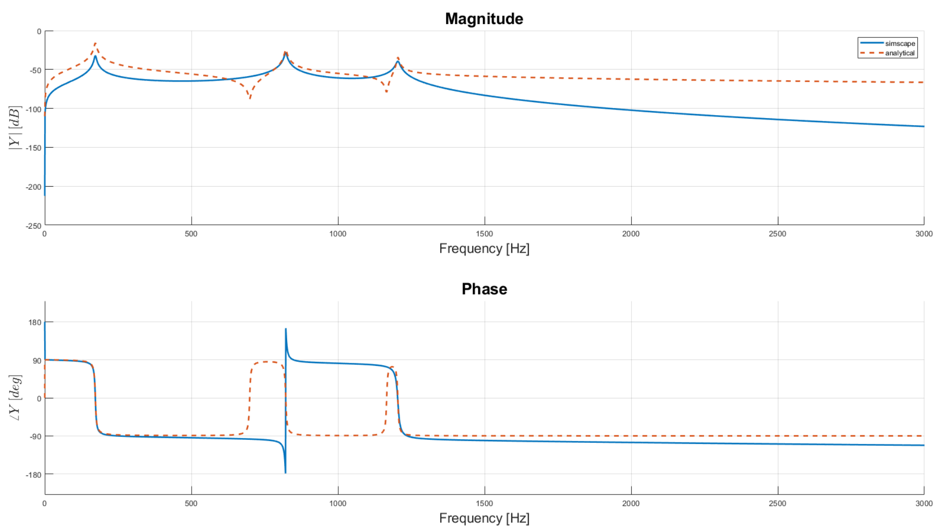
Going on like this, the K-*th* layer from the outermost leaves has an input impedance given by:

$$Z_K(\omega) = j\omega L + R + \frac{1}{j\omega C + \sum_{n=1}^{N} \frac{1}{Z_{K-1}(\omega)}} \tag{11}$$

Let's consider $K = 3$ and $N = 2$, the resulting circuit is the following:



In the following figure we show the comparison between the admittance diagram obtained by the Simscape simulation and the admittance diagram obtained with the Matlab script.

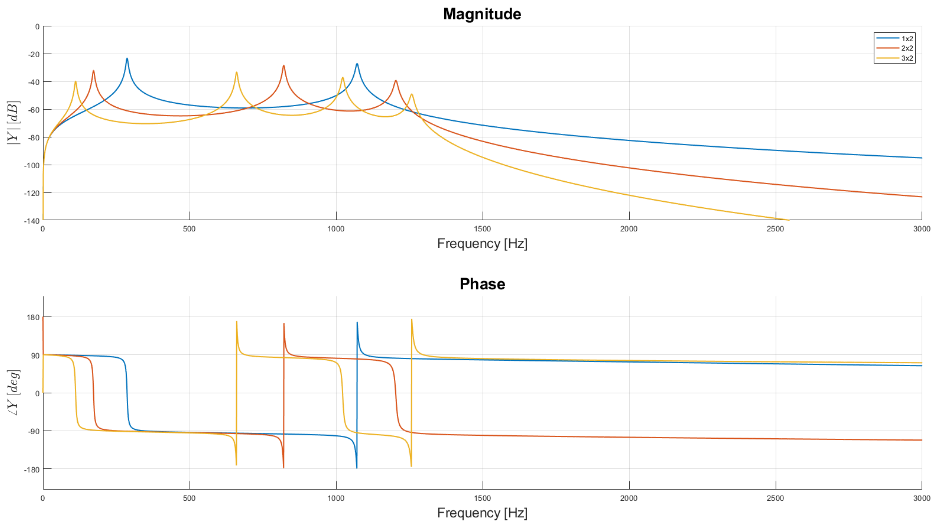


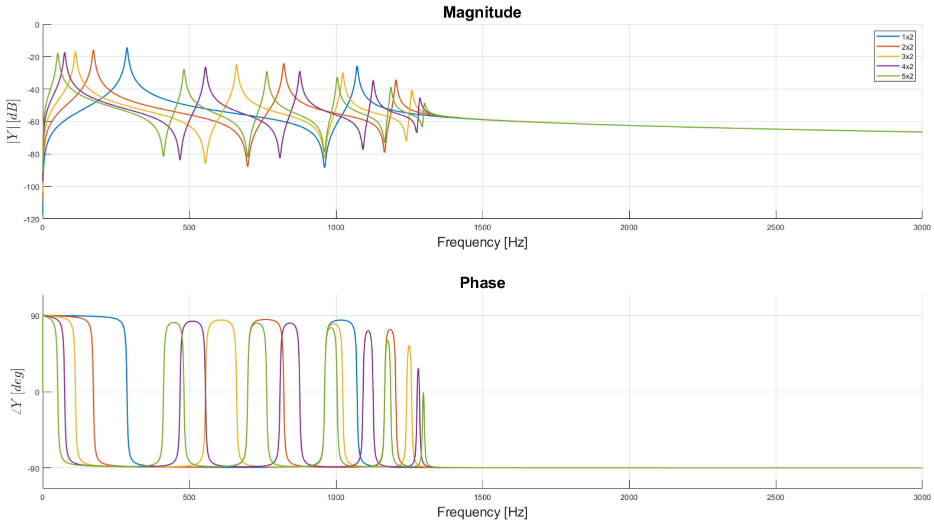
From the comparison we can notice that:

- the resonance frequencies are the same;
- the amplitude at resonance slightly changes, with particular emphasis on the one at lower frequency;
- the Simscape simulation does not show any antiresonance.

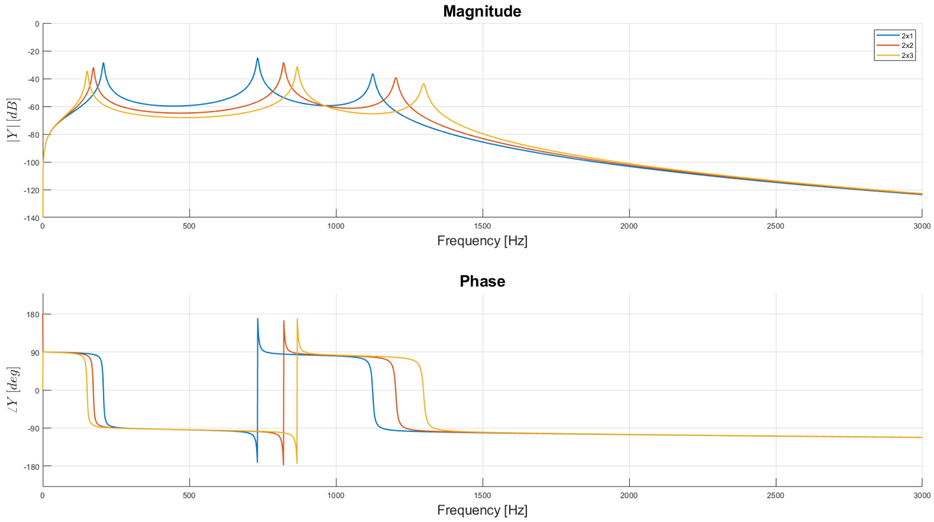
2.1 Changing values of N and K

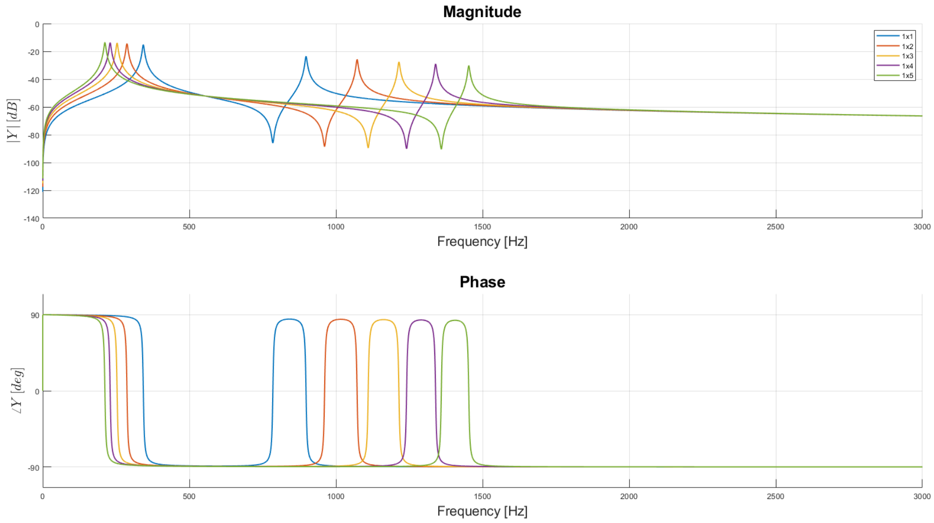
By changing the value of K and keeping fixed N we obtain the following diagrams, where the first one is obtained with the use of Simscape while the second one is obtained analytically with a Matlab script:





As we can see, for different values of K we obtain different numbers of resonances. More specifically, the total number of resonances corresponds to the total number of levels of the tree, which is $K+1$. By changing, instead, the value of N and keeping fixed K we obtain the followings, again the first one is obtained with the use of Simscape while the second one is obtained analytically with a Matlab script:





This time we can see that the influence of the number N is on the frequency value of the resonances, and not on their number. by increasing N we increment the distance between the resonances. The very first resonance lean towards lower frequencies while all the others tends to increasing frequencies.

2.2 Evaluation of the admittance in different points of the tree

Finally, we want to observe what changes in the admittance of the system if we evaluate it in points of the tree different than the outermost leaves. We can do so by moving the current sensor of the equivalent circuit in positions corresponding to different necks on the Helmholtz resonator tree.

By evaluating the frequency response in different leaves of different trees, we noticed that the results change between evaluations made in different levels of a tree.

In particular, the number and the positions of the resonance remain the same but, while in the evaluation in the outermost leaves no antiresonance appears, moving up the tree of one layer we can see the number of antiresonances increases by one each time.

The first antiresonance to appear is the one occurring between the highest frequency resonance and the second-to-highest one, and moving up the tree, antiresonances occurring at lower frequencies start appearing. We can also notice that the frequency of an antiresonance occurring between two resonances increases when moving up the levels of the tree.

Below, we show an example for a balanced (2×2) tree, with the admittance evaluated at each of the three levels. The plot presents 3 resonances, and the number of antiresonances increases form none to two. The one between the second and third resonances appears at a lower frequency when the evaluation is made on the second level, and at a lower one when evaluated at the first level of the tree.