Musical Acoustics
Homework 7: Eigenfrequency study of a marimba bar

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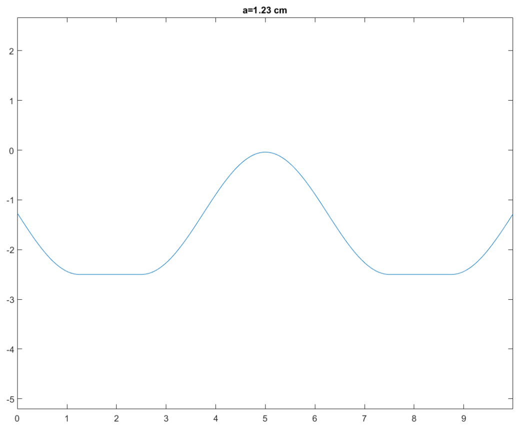
1 Introduction

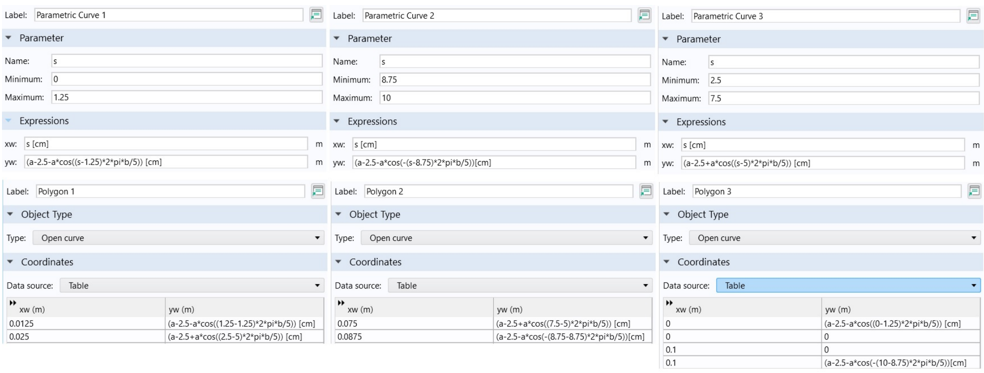
The aim of this homework is to study the eigenfrequency of a marimba bar, and to estimate the harmonicity introduced by the expedients used by musical instrument makers for increasing the level of harmonicity of the sound produced by the marimba bars.
The marimba bar has a lateral section characterized by a tapering at the center and the edges of the bottom surface, while the top surface is flat. We are provided with a portion of Matlab code that models the bottom surface of the bar, given by the variable z, which is equal to 0 for the top surface.
The tapering is defined as a function of the variable a, whose value is initially defined inside the code as $a = 1.23cm$, resulting in the shape displayed below:



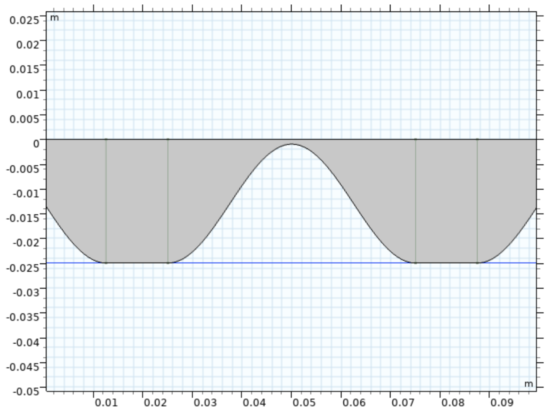
During the study we will change the value of the parameter a to see the resulting effect.

2 COMSOL geometry

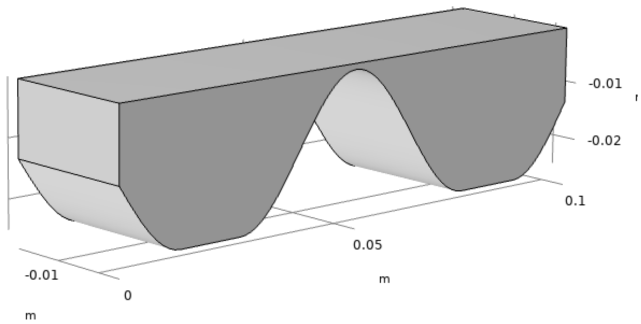
We want to perform our eigenfrequency study using a COMSOL simulation. To do so, we first need to define the geometry of the marimba bar.
We start from a 2D geometry, composed of a mixture of polygonal lines and parametric curves, defined as function of the variable a. The resulting curve goes the through a *union* operation followed by a conversion to solid.



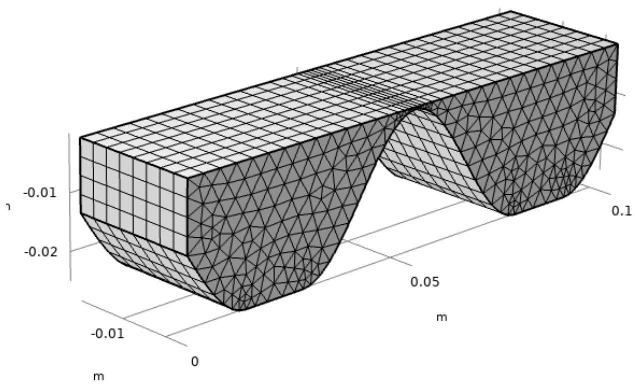
The resulting 2D geometry for $a = 1.23cm$ is the following:



To go from the 2D geometry to the 3D one, we perform an *extrude* operation of the 2D work plane, giving the bar a width of 2.5 cm. We also perform a rotation around the x-axis just to visualize the bar in the correct position. The resulting 3D geometry is the following:



The parameter a is defined as a global parameter, whose value will be changed with a parameter sweep during the eigenfrequency study described in the next section.
For the mesh, we chose a *free triangular* with extra fine size for the lateral faces, and performed a swept mesh for the rest of the element.



3 Eigenfrequency study

Before performing the study, we need to add a couple more elements to our component: a *Physics* node and a *Material* node
We applied a *Solid mechanics - Linear elastic solid* physics to our component. For the boundary conditions, we chose to consider all of the boundaries as free, with initial conditions of displacement and velocity equal to 0.
As for the material, marimba bar are generally made of rosewood. In particular, we chose to utilize the following values [1]:

- **Density**: $\delta = 1080\ kg/m^3$
- **Young's modulus**: $E = 20.3\ Gpa$
- **Poisson ratio**: $\nu = 0.30$

we can finally set up our study: we add a *Study* node, choosing an eigenfrequency one. We look for the first 5 eigenfrequencies of the marimba bar, but since we chose a free boundary condition we know the first six results will correspond to the rotations and the rigid motions on the three axis of the free body, so we chose the number of computed frequency accordingly.
We then add a parametric sweep, to change the value of a in a range going from 0.1 to 1.2 cm, with a step of 0.1.
After the computation, the resulting first 5 eigenfrequencies for each value of a are the following:

	$f_1\ [Hz]$	f_2	f_3	f_4	f_5
$a = 0.1$	8901.05	9318.30	11972.40	20732.77	20745.88
$a = 0.1$	8342.23	9231.87	11513.96	20476.47	20582.35
$a = 0.1$	7696.52	9103.21	10965.31	20071.90	20244.58
$a = 0.1$	6975.96	8930.37	10324.41	19501.49	19617.16
$a = 0.1$	6195.40	8711.04	9591.17	18751.60	18923.80
$a = 0.1$	5369.06	8440.57	8765.81	17809.20	18167.51
$a = 0.1$	4513.03	7850.36	8111.72	16665.44	17339.93
$a = 0.1$	3639.63	6843.63	7710.12	15300.40	16413.12
$a = 0.1$	2765.33	5745.14	7213.06	13684.36	15340.66
$a = 0.1$	1898.89	4540.55	6570.58	11724.01	14021.72
$a = 0.1$	1062.82	3202.48	5676.92	9212.19	12254.23
$a = 0.1$	296.97	1578.15	4151.59	5297.02	9281.766

As Shown in the table, the frequencies become lower with the increase of the parameter a.

4 Inharmonicity

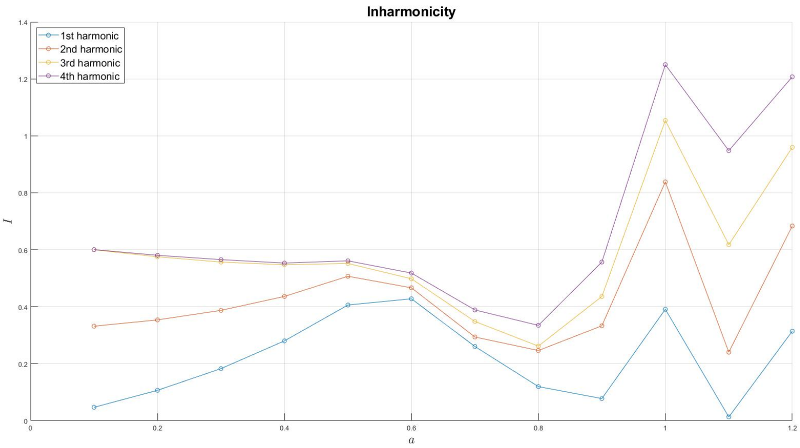
Finally, we want to compute the e inharmonicity of the system as a function of the parameter a. The inharmonicity is computed through the following function:

$$I = \sum_{n=2}^{N} \left| \frac{f_n}{f_{n-1}} - m_n \right| \tag{1}$$

Where f_n is the eigenfrequency of the mode n, N is an integer number ranging from 2 to 5, indicating the number of harmonics to consider, and

$$m_n = \underset{m}{\operatorname{argmin}} \left| f_n - m f_{n-1} \right|, \quad m \in N^+ \tag{2}$$

The following plot shows the value of I as a function of a, for all the values of N in the range previously defined:



As we can see from figure, at the value $a = 0.8$ the overall inharmonicity is minimized. This means that, even though we can still remove material from the bar, we won't further improve its harmonicity.

References

[1] I. Bork and A. Chaigne, "Comparison between modal analysis and finite-element modelling of a marimba bar", The Journal of the Acoustical Society of America, vol. 105, no. 2, pp. 1125, 1999.