POLITECNICO MILANO 1863	051483 Musical Acoustics Module 1: Modeling of musical instruments Academic Year 2021/2022	Homework Laboratory II Electric Analogs

Assignment: Helmholtz Resonator Tree (3.5 points)

Generate a hierarchical structure of Helmholtz resonators to model a complex resonant system.

Ex. 1: Model the response of a single Helmholtz resonator. (*1.5 points)*

Parameters

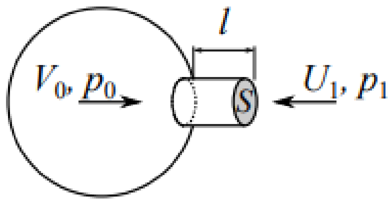
$V_0 = 0{,}1\ m^3$

$l = 10\ cm$

$S = 100\ m^2$

$\mathrm{c} = 343\ {}^{m}/_{s}$

$\rho = 1{,}2\ {}^{Kg}/_{m^3}$



(a) Set a simulation in Simscape and plot the frequency response (*1.0 points)*

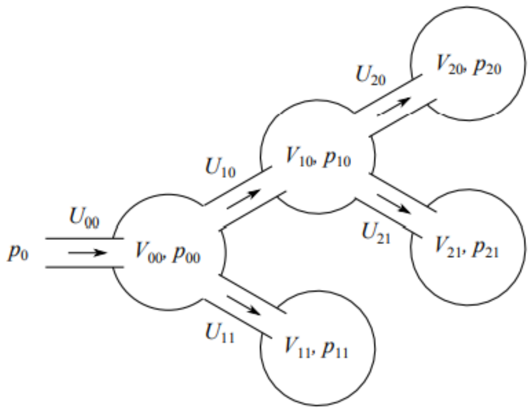
$$H(\omega) = \frac{U_1(\omega)}{p_1(\omega)}$$

(b) Compute the natural frequency of the resonator analytically and verify that it matches the results of the simulation (*0.5 points)*

Ex.2: Combine more resonators in a tree and analyze the response obtained. (*2 points)*

(a) Use the RLC circuit defined in Ex.1 and connect its replicas to build a NxK tree (*1.0 points)*

N - height of the tree
K - branch division
(= how many leaves for each branch)
example: tree in figure has N=2 and K=2



Use the same parameters for each component and analyze the frequency response using as output the current in one of the leaves ($U_{n,k}$) and pressure p_0 as input

(b) Try with different N and K and highlight what these parameters control in the final response (*0.5 points)*

(c) What happens if you change the location in which you evaluate the frequency response inside the tree hierarchy? Elaborate on that, showing some examples (*0.5 points)*

HINTS:

- model the pipe resistance with a frequency-independent resistance

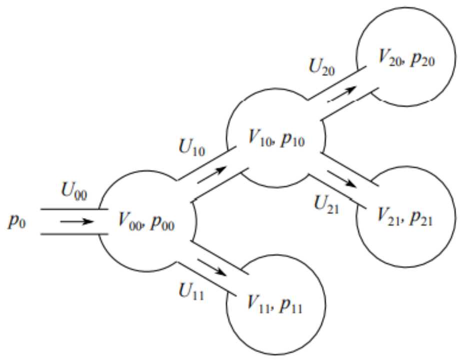
$$R = \frac{\rho \cdot c}{S}$$

- look at the Helmholtz Resonator Dimmer seen in class as starting point
- write the impedances in the network to evaluate the resonant frequency

Please provide the answers in a report as a PDF file by 3/12/2021 and upload it using the WeBeep platform in the "Assignment HL2" delivery folder. One file for each student must be uploaded. If more than one student participated to the assignment, write on the cover page of the assignment the name, surname and ID of the participating students. Also please provide the MATLAB files related to each of the exercises: remember that the platform allows to upload files up to 50 MB.

APPENDIX: HELMHOLTZ RESONATOR TREE

The Helmholtz resonator tree yields an intuitive way of creating physically-inspired resonating structures for synthesis. In this type of structure several resonators influence each other, and their parameters can be modified using acoustical parameters, which are intuitive even for non-technical people, although the exact resonance frequencies are unknown. Additionally, it provides the possibility of exciting the system at different points, resulting in timbre variations of the same musical instrument. This approach differs from the implementation of cascaded second order filters, since the resonators interact with each other, changing their resonance frequencies.



The acoustic impedance of a Helmholtz resonator tree can be derived in an iterative way. The acoustic impedance of a single resonator is given by

$$Z_1(\omega) = \frac{(\mathrm{j}\omega)^2 LC + \mathrm{j}\omega RC + 1}{\mathrm{j}\omega C}, \tag{5}$$

where ω is the frequency in rad/s and $\mathrm{j} = \sqrt{-1}$ is the imaginary unit. The impedance of a Helmholtz resonator tree of height two and with N leaves can be determined as

$$Z_2(\omega) = \mathrm{j}\omega L + R + \frac{1}{\mathrm{j}\omega C + \sum_{n=1}^{N} \frac{1}{Z_{1,n}}}, \tag{6}$$

where $Z_{1,n}(\omega)$ is the impedance of the n^{th} leaf, given by Eq. 5. The same procedure performed for a tree of height two can be generalized for a tree of height K, where the impedance of this tree $Z_K(\omega)$ is calculated based on the impedances of the subtrees $Z_{K-1,n}(\omega)$ connected to it, as illustrated in Fig. 2 (c). This procedure results in the general form of the Helmholtz resonator tree impedance

$$Z_K(\omega) = \frac{-\omega^2 LC + \mathrm{j}\omega RC + 1}{\mathrm{j}\omega C + \sum_{n=1}^{N} \frac{1}{Z_{K-1,n}}}. \tag{7}$$

Finally, the pressure-to-volume flow transfer function, can be obtained with the impedance $Z_K(\omega)$ as

$$H_K(\omega) = \frac{u(\omega)}{p(\omega)} = Z_K^{-1}(\omega). \tag{8}$$

This results in a complex resonating system where the resonances are given by the poles of $H_K(\omega)$ in Eq. 8.