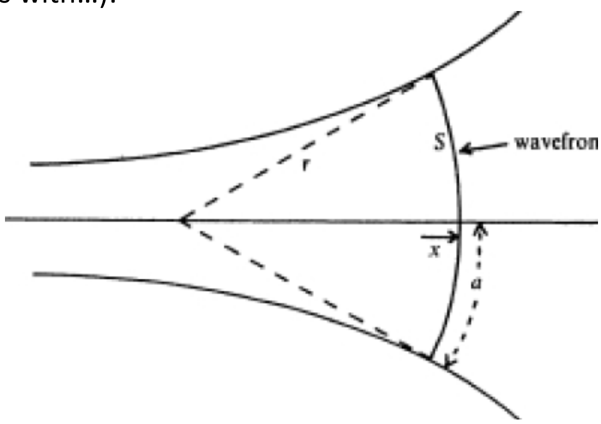
## Part 1: design of the exponential section

It is given an exponential horn. The equivalent radius (see diagram below) is $a = a_0 e^{mx}$, with $a_0$ = 0.01 m and $m$=4 m$^{-1}$. The length of the horn is L=0.4 m. We intend to approximate the exponential horn as a concatenation of short sections of conical horns. In particular, we assume that all the sections are of the same length δ.

Recall that in a Salmon horn the wavefronts can be approximated as planar (and therefore the equivalent radius coincides with…).




We aim at computing, in Matlab, the input impedance of the concatenation and compare it with the analytical expression of the impedance of the exponential horn in the frequency range [0,2kHz]. In a first stage, we neglect the impedance radiation at the mouth of the horn. We measure the similarity between the impedances $Z_1(\omega)$ and $Z_2(\omega)$, which are the input impedance of the approximated model and the analytical input impedance, respectively, through two metrics:

- $e_1 = \frac{1}{(\omega_{max}-\omega_{min})}\int_{\omega_{min}}^{\omega_{max}} |Z_1(\omega) - Z_2(\omega)|^2 d\omega$, which evaluates the mean squared error between the two impedances;
- $e_2 = \sum_1^5 \min |\arg max_\omega \operatorname{Re}(Z_1(\omega)) - \arg max_\omega \operatorname{Re}(Z_2(\omega))|$, which evaluates the difference in frequency between the first five maxima of the two impedances.

**a) Evaluate the error $e_1$ as a function of the length δ of the conical sections and plot it in Matlab.**

**b) Evaluate the error $e_2$ as a function of the length δ of the conical sections and plot it in Matlab.**

**c) (Extra: 0.5 additional points!) If one has to choose a different sampling strategy with respect to the uniform sampling on the *x* axis, which strategy could be chosen? Make some experiments and provide some conclusions.**

In the rest of the assignment, use the model that guarantees the minimum value of the error function $e_2$.

In the approximated model the adaptation to the external air is modeled through a further section whose impedance is $Z_L(\omega) = Z_{L0}(\omega)\frac{S_p}{S_s}$, where $Z_{L0}(\omega)$ is the impedance of an unflanged cylindrical pipe of radius *a*, given by $Z_{L0}(\omega) = 0.25\frac{\omega^2\rho}{\pi c} + 0.61j\frac{\rho\omega}{\pi a}$, $S_p$ is the cross-sectional area of the cylinder, and $S_s$ the spherical wave front area at the open end of the cone. It can be approximated by $S_s = \frac{2S_p}{1+\cos\theta}$, where θ is the flaring angle of the last conical section.

- **d) Compute the impedance of the approximated model when the radiation load is kept into account.**

## Part 2: design of the compound horn

Consider now a compound horn, composed by a cylindrical pipe, whose length is 0.6m, followed by the exponential horn. For the exponential horn use the impedance obtained before, including the radiation load.

- **e) Compute the impedance of the compound horn and list in a table the frequencies of the first ten maxima of the impedance.**

## Upload instructions

- Upload the report on WeBeep by Nov. 18.
- If two students collaborated to the same report, all the students must upload the report on WeBeep.
- On the front page of the report specify the name, surname and ID of all the students participating to the HW.

## Tip…

This assignment is a (small) research challenge and not all the answers are found in the course material. Some degrees of freedom to students: different solutions to the same problem can be found. Use an engineering approach!