WORLDQUANT UNIVERSITY

Group Work Project

MScFE 630

Computational Finance

Revision Date: 06/29/2021

Submission Requirements

Each Group Work Project submission must include both source code and a report. Use the following instructions for each submission:

1. Source Code Submission Requirements
    - Provide appropriate source code that encapsulates each task
    - The coding language must be Python
    - Use Jupyter Notebook to develop and execute your code
    - In your source code, properly label all functions and methods, and incorporate detailed comments
    - Submit your source code **both** as a .zip file **and** as an attachment to your report

2. Report Submission Requirements:
    - Use the " **MScFE 630 CF REPORT TEMPLATE**" document provided in the online course room to develop your report. In case you do not have access to MS Office, you can upload the document on a Google Drive and use it (Google Docs)

Note: The PDF file with your report must be uploaded **separately** from the zipped folder that includes your other files. This allows Turnitin to generate the similarity report.

Submission 1: Price a European Up-and-out Call Option

The goal of Submission 1 is to price a European up-and-out call option held with a risky counterparty. This is a type of call option whose payoff is reduced to 0 if the share price becomes too high over its lifetime. Note that this limits the final payoff of the option, and as a consequence it becomes cheaper than a vanilla call option.

The payoff of this option at maturity T is as follows:

$$v(S_T) = (S_T - K)^+ \quad \text{given} \quad \max_{t \epsilon [0,T]} S - t < L$$

where K is the strike of the option, L is the barrier level, and S_t is the share price at time t.

Observe that the payoff of the option is dependent on the value of the share price between the inception of the option and maturity. This means that the option payoff is dependent on the history of the share price, and not just on its terminal value. As a result, you will need to simulate entire share price paths to estimate the price of this option.

You may make the assumptions of the Black-Scholes-Merton model (i.e. assume that both the stock and counterparty firm values follow Geometric Brownian Motion [GBM] with constant drift and volatilities, and default only occurs at maturity). Use the following parameters:

- Option maturity is one year
- The option is struck at-the-money
- The up-and-out barrier for the option is $150
- The current share price is $100
- The risk-free continuously compounded interest rate is 8%
- The volatility for the underlying share is 30%
- The volatility for the counterparty's firm value is 25%
- The counterparty's debt, due in one year, is $175
- The correlation between the counterparty and the stock is constant at 0.2
- The recovery rate with the counterparty is 25%.

For this submission, complete the following tasks:

General: 4 points

1. What are the advantages and disadvantages of purchasing the up-and-out barrier call option, compared to the plain vanilla European call option? **(2 pts)**
2. Would you expect to find this option on an exchange, or Over-The-Counter? **(1 pt.)**
3. Is there a closed-form, analytical solution for pricing an up-and-out barrier call option? **(1 pt.)**

Pricing: 32 points

4. Price a European call option with the information provided. **(2 pts)**
5. Price a European up-and-out barrier call option: Simulate paths for the underlying share and for the counterparty's firm value using sample sizes of 1000, 2000, …, 50000. Do monthly simulations for the lifetime of the option. **(10 pts)**
6. Price a European up-and-in barrier call option. Hint: Use the 2 other option prices. **(2 pts)**
7. Repeat Question 5 (Price up-and-out barrier call) 6 times, keeping all the data the same, but using a new strike level in each case: a) 85, b) 90, c) 95, d) 105, e) 110, f) 115. Produce a table of 7 rows that shows the strike, and the option price. **(2 pts)**
8. Determine Monte Carlo estimates of both the default-free value of the option and the Credit Valuation Adjustment (CVA). You can take an initial firm value like of $200 for your calculations. **(8 pts)**
9. Calculate the Monte Carlo estimates for the price of the option incorporating counterparty risk, given by the default-free price less the CVA. **(8 pts)**

Discussing: 9 points

10. Write a 1-page non-technical document that explains the difference in the default-free value of the option, and the Credit Valuation Adjustment. **(9 pts)**

Submission 2: Price a Vanilla European Call Option

The goal of Submission 2 is to price a vanilla European call option, except we will now allow the volatility term to vary. For this submission, complete the following tasks:

Pricing a vanilla European call option: 32 points

1. Using a simple Fourier pricing technique (using $N = 100$ intervals, and using an effective upper bound of integration of 30), price a vanilla call option assuming that the underlying share follows the Heston model dynamics. Use the parameter values from the previous section (GWP #1), as well as the following parameter values:
   - $v_0 = 0.06$
   - $\kappa = 9$
   - $\theta = 0.06$
   - $\rho = -0.4$

   (8 pts)

2. We will now simulate a share price path. Assume that $\sigma(t_i, t_{i+1}) = \sigma(S_{ti})^{\gamma-1}$, where $\sigma = 0.3$ and $\gamma = 0.75$. Using the formula below, simulate paths for the underlying share using sample sizes of 1000, 2000, …, 50000. Perform monthly simulations for a period of a year. We can simulate the next step in a share price path using the following formula:

   $$S_{t_{i+1}} = S_{t_i} e^{\left(r - \frac{\sigma^2(t_i, t_{i+1})}{2}\right)(t_{i+1} - t_i) + \sigma(t_i, t_{i+1})\sqrt{t_{i+1} - t_i}Z},$$

   where S_{ti} is the share price at time t_i, $\sigma(t_i, t_{i+1})$ is the volatility for the period $[t_i, t_{i+1}]$, r is the risk-free interest rate, and $Z \sim N(0,1)$.

   Note that we are attempting to run simulations using the CEV model. However, while the CEV model assumes that volatility is a continuous function of time and share price, we are making a simplifying assumption that volatility is constant over each simulation period. **(8 pts)**

3. Augment your code in part 2 to calculate Monte Carlo estimates, as well as the standard deviations for these estimates, for the price of a vanilla call option (with the same strike term as in Submission 1). **(8 pts)**

4. Plot the Monte Carlo estimates generated in part 3 with respect to sample size, as well as three standard deviation error bounds around these estimates. **(8 pts)**

Volatility Smile: 7 points

5. Graph the volatility smile **(2 pts)**
    - Visit https://finance.yahoo.com/quote/FB/options/
    - Find the strike closest to the current price of Facebook stock (take whatever maturity/ies you'd like). Call this K. Find its volatility. Call this sigma
    - Then find the 3 closest strikes below K. Find their implied volatilities from the website.
    - Then find the 3 closest strikes above K. Find their implied volatilities from the website.
    - Graph the strikes along the x-axis, and the implied volatilities along the y-axis.
    - Label this graph the "Facebook Option Volatility Smile".
6. Write a function to compute the implied volatility (Hint: Think bisection algorithm as one of the choices). Use this function to validate at least 2 of the implied volatilities from the website. Note: Feel free to use the 1-month T-Bill as the risk-free rate. **(2 pts)**
7. The volatility skewness is defined as the change in implied volatility (delta y) divided by the change in strike level (delta x). Using the strikes below K, calculate the volatility skew for Facebook. **(1 pt.)**
8. In Black Scholes, does the volatility depend on the strike level? Why or why not? **(1 pt.)**
9. How does the Heston Model better estimate the volatility smile? Specifically, what is different in Heston's model from Black Scholes that allows more estimate option pricing? **(1 pt.)**

Discussing: 6 points

10. Write a 1-page non-technical document that explains how the Heston model better prices the volatility smile than the Black-Scholes model. <u>Be sure to be non-technical – no formulas, no equations, no code snippets, no names of code routines</u>. **(6 pts)**

Submission 3: Simulate Asset Price Evolutions and Reprice Risky up-and-out Call Option

The goal of Submission 3 is to reprice the risky up-and-out call option from Submission 1, but now implementing a non-constant interest rate and local volatility. With the exception of the interest rates and volatilities, you may make the same assumptions as in Submission 1:

- Option maturity is one year
- The option is struck at-the-money
- The up-and-out barrier for the option is \$150
- The current share price is \$100
- The current firm value for the counterparty is \$200
- The counterparty's debt, due in one year, is \$175
- The correlation between the counterparty and the stock is constant at 0.2
- The recovery rate with the counterparty is 25%.

The local volatility functions for both the stock and the counterparty have the same form as in part 2, namely $\sigma(t_i, t_{i+1}) = \sigma(S_{ti})^{\gamma-1}$. For the stock $\sigma_S(t_i, t_{i+1}) = 0.3(S_{ti})^{\gamma-1}$, and for the counterparty, $\sigma_V(t_i, t_{i+1}) = 0.3(V_{ti})^{\gamma-1}$, where $\gamma = 0.75$.

We can simulate the next step in a share price path using the following formula:

$$S_{t_{i+1}} = S_{t_i} e^{\left(r - \frac{\sigma^2(t_i, t_{i+1})}{2}\right)(t_{i+1} - t_i) + \sigma(t_i, t_{i+1})\sqrt{t_{i+1} - t_i} Z},$$

where S_{ti} is the share price at time t_i, $\sigma(t_i, t_{i+1})$ is the volatility for the period $[t_i, t_{i+1}]$, r_{ti} is the risk-free interest rate, and $Z \sim N(0,1)$. The counterparty firm values can be simulated similarly.

You observe the following zero-coupon bond prices (per \$100 nominal) in the market:

Maturity	Price
1 month	\$99.38
2 months	\$98.76
3 months	\$98.15
4 months	\$97.54
5 months	\$96.94
6 months	\$96.34
7 months	\$95.74
8 months	\$95.16
9 months	\$94.57
10 months	\$93.99
11 months	\$93.42
12 months	\$92.85

You are required to use a LIBOR forward rate model to simulate interest rates. The initial values for the LIBOR forward rates need to be calibrated to the market forward rates which can be deduced through the market zero-coupon bond prices given above. This continuously compounded interest rate for $[t_i, t_{i+1}]$ at time t_i, is then given by the solution to:

$$e^{r_{ti}(t_{i+1}-t_i)} = 1 + \mathrm{L}(t_i, t_{i+1})(t_{i+1} - t_i),$$

Where $\mathrm{L}(t_i, t_{i+1})$ is the LIBOR forward rate which applies from t_i to t_{i+1}, at time t_i. Note that these LIBOR rates are updated as you run through the simulation, and so your continuously compounded rates should be as well.

For this submission, complete the following tasks:

1. Using a sample size of 100000, jointly simulate LIBOR forward rates, stock paths, and counterparty firm values. You should simulate the values monthly, and should have LIBOR forward rates applying over one month, starting one month apart, up to maturity. You may assume that the counterparty firm and stock values are uncorrelated with LIBOR forward rates. **(32 pts)**
2. Calculate the one-year discount factor which applies for each simulation, and use this to find first the value of the option for the jointly simulated stock and firm paths with no default risk, and then the value of the option with counterparty default risk. (Hint: you may want to use the reshape and ravel attributes of numpy arrays to ensure your dimensions match correctly). **(32 pts)**
3. If you bought the option (rather than sold it), how does your own credit risk affect the value of the option? **(2 pts)**
4. Suppose interest rates increased 25 basis points. What is the new value of your option? (**4 pts)**

Presentation: 20 points

5. Write a 5 (or more) page <u>technical report</u> <u>INCLUDING GRAPHS</u> that illustrate and explain the differences: (20 pts)
    a. between vanilla options and barrier options (Hint: focus on payoffs and uses: this part can be non-technical) **(2 pts)**
    b. between pricing out-of-the-money calls and at-the-money calls (Hint: focus on smile) **(2 pts)**
    c. between default-free option pricing and CVA option pricing. **(4 pts)**
    d. between analytical-pricing methods and Monte Carlo simulation methods **(4 pts)**
    e. among Black-Scholes volatility, Heston volatility, and local volatility. **(8 pts)**