WORLDQUANT UNIVERSITY

Group Work Project

MScFE 620

Discrete-time Stochastic Processes

Revision Date: 05/18/2021

Overview

This document describes the requirements for the three Group Work Project assignments which must be submitted at the end of week 3, 5, and 7 respectively. Within a week of each submission, your group will receive feedback from the WQU Instructional Team, enabling you to use the feedback to revise your assignment ahead of the second and third submissions. You will use the Group Work Forum to communicate with your peers throughout the course.

Please make use of the **LIRN Library** located on the left pane of your screen as the primary resource for your research.

Your research should favor authoritative, scholarly sources, and you must cite all sources where relevant. The task is not to reproduce the research of others, but instead to develop your own systematic narrative that addresses the research topic and is informed by the research of others. Not only are you required to cite accurate and relevant facts, but you must also present your own clear logic when linking and contextualizing these facts.

Visit the **Student Resource Center (SRC)** where you can find resources on **how to conduct research**, how to use different sources of information, how to **cite references to** a**void plagiarism**, and how to use the **MLA citation style.**

Note: All Group Work Project assignments must be submitted via **Turnitin**, the anti-plagiarism software.

Submission 1: Measure Theory Concepts put in Practice

Use Python or R to perform the following:

1. Build a binomial tree for stock price evolution. Specifically, set the upper movement of the price (u) to an expression that will depend on the number on your group.

   $$u = (1.10 + Group\ Number/100),$$

   Group 1, u = 1.11. Group 2 = 1.12. Group 3 = 1.13, …, Group 50 = 1.60

   You can define d = 1/u.

   a. Calculate and show the new Binomial tree for N=6.
   b. What are the terminal values of each path? Define the appropriate filtrations for each of these values.

2. Finally, recalculate the tree for N=4,000.
   a. Plot the terminal prices produced by the model (You may use a histogram for this).
   b. Can you identify what type of statistical distribution do these prices resemble to?
   c. If we defined $Return = Ln(\frac{Price_t}{Price_{t-1}})$, this is, using a log-function from previous prices, which statistical distribution would Returns follow? Indicate the appropriate Probability Density Function for it.

3. Market completeness
   a. How many fundamental securities are there in the market? (Hint: a derivative, such as an option, is NOT a fundamental security. Derivatives are derived from fundamental securities).
   b. At any given node, how many states of the world are there in the binomial tree?
   c. Define market completeness using parts a and b.
   d. Suppose the underlying stock price jumped. By jumps, we mean that it moves by a factor larger than u (or smaller than d) from 1 node to the next. Would that market still be complete? Why or why not?

Submission Requirements

Use the "**MScFE_620_DTSP_REPORT_TEMPLATE**" published in the online course room to develop a PDF report with a 5-page maximum length.

In addition to the report, you must submit the source code.

Submission 2: Pricing Options under Binomial Models

Use Python or R to perform the following:

1. Write code to price a European Call option:
   a. The underlying stock that is currently trading at $95. The option has a strike price of $105 and 1 year maturity. Use the Binomial model with the parameters r=0 and 3 steps in the pricing process. Additionally, set the upper movement of the price (u) to an expression that will depend on the number on your group.

$$u = (1.10 + Group\ Number/100),$$

   Group 1, u = 1.11. Group 2 = 1.12. Group 3 = 1.13, …, Group 50 = 1.60

   You can define d = 1/u.

   b. Using the information from (a), show the value of the derivative, $H(\omega)$, for each of the paths.

2. Write code to price a European Put Option:
   a. Consider the same parameters as in section (a) in part (1) above but now with N=2. What is the price of the option?

   b. Construct a Table (alike the ones you have in the notes) that includes, for each price path and each t when it corresponds, the information on stock price evolution, $X_t(\omega)$, the value of the option, $V_t^H(\omega)$, the payoff of the option, $H(\omega)$, and the hedging strategy, φ_t^H.

3. Market Completeness Revisited
   a. Form a matrix with 2 rows and 2 columns. Each row contains a state of the world (up and down). Each column contains a security (stock and bond).

   b. Pick a specific node. Write down the values for the A matrix.

   c. From that node, the stock price may go up or down. Write down the b matrix (which is a column matrix). The first value in b contains the option value if the stock price went up. The second value in b contains the option value if the stock price went down.

   d. The no-arbitrage equation can be written as Ax=b. How do you solve this equation for x?

   e. Using matrix algebra, solve for x.

   f. Show that your solution matches that from the binomial tree.

4. Put Call Parity
    a. When a stock option expires, there are 3 states of the world. 1) Final stock price > Strike 2) Final Stock price = Strike 3) Final Stock price < Strike. Consider a call and put with the same underlying, strike, European exercise style, and expiration. Assume a constant risk-free rate. For each state of the world, show the value of the following portfolios:
        i. Long 1 call and short 1 put
        ii. Long 1 stock and borrow K (strike) dollars at the risk free rate for T (option maturity) years
    b. Using your answers from the previous parts, check that the call price, the put price, the stock price, and the borrowed funds satisfy put-call parity. If not, explain why it may not match more precisely. (Hint: it may not match even if you did everything correctly!)

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Submission 3: Pricing American Options

In this submission we are going to work with American Options. Write R or Python code for the following:

1. First, consider the case of an American Call option with underlying stock trading at $95, Strike price of $90 and 1 year maturity. Use the Binomial model with the parameters r=0 and 5 steps in the pricing process. Additionally, set the upper movement of the price (u) to an expression that will depend on the number on your group:

$$u = (1.10 + Group\ Number/100),$$

Group 1, u = 1.11. Group 2 = 1.12. Group 3 = 1.13, …, Group 50 = 1.60

You can define d = 1/u.

   a. In this setting, find the value of the derivative at each node.

   b. Is there any point time where we, as buyers of the option, benefit from early exercise?

   c. Explain whether your previous answer is (or is not) always the case.

2. Consider the same parameters and setting as in (1), but now for pricing an American Put Option.

   a. Find the value of the derivative at each node.

   b. Is there any point time where we, as buyers of the option, benefit from early exercise?

   c. Explain whether your previous answer is (or is not) always the case.

3. Finally, modify your code accordingly in order to price an exotic option with European payoff. In this case we are going to price an Up-and-Out (UAO) European Call Option. The payoff at maturity (T) of such option is as follows:

$$v(S_T) = (S_T - K)^+ \quad given \quad max_{t\epsilon[0,T]} S_t < L$$

where L is the barrier level such that, if trespassed, the value of the option becomes 0. Notice that the value of this option is path-dependent, not just on terminal value. The maturity of the option is 1 year, and is currently struck at-the-money (ATM). The up-and-out barrier for the option is $130, while the current underlying stock price is $100. Consider the Binomial model under the same u, d, and r parameters from part (1) in this submission and 5 steps in the pricing process.

   a. Find the value of this option.

   b. Which option is more expensive: the European call, or the UAO European call?

   c. What is the advantage of the up-and-out European call option?

   d. There is another exotic option called an Up-and-In (UAI) European Call option. Without running a binomial tree, what is the price of UAI with the same strike and maturity as the European call, and the same strike, maturity, and barrier as the UAO. Assume the current stock price is below the barrier. (Hint: Think of a parity using the European call, UAI, and UAO).

4. Completeness Revisited
   a. Using the binomial tree, if we price an American call or put option, is the market complete?
   b. Using the binomial tree, if we price a European UAO call option, is the market complete?
   c. Describe market completeness in mathematical terms using the rank of the A matrix.
   d. Describe market completeness in non-technical terms, in at most 1 sentence, or even 1 word! Be concise, but specific.

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