Submission Number: 1

Group Number: 4

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Statement of integrity: By typing the names of all group members in the text box below, you confirm that the assignment submitted is original work produced by the group (*excluding any non-contributing members identified with an "X" above*).

Artem Kornev, Ishaan Narula, Nqobile Nyathi

Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed.

N/A

** Note, you may be required to provide proof of your outreach to non-contributing members upon request.*

Answer 1

Note: Please review this answer in conjunction with the file WQU_MScFE_620_Project1_vF.ipynb

Part (a)

The binomial tree has been constructed taking $S_0 = 100$ and $u = 1.14$

Function:

```
def asset_px_binomial_tree(N, u, d, S0):
    tree_holder = np.zeros([N+1, N+1])
    freq = np.zeros(N+1)

    u_increment = u**(1/N)
    d_increment=1/u_increment

    # populate the tree:
    for i in range(N+1):
        for j in range(i+1):
            tree_holder[j,i]=S0*(d_increment**j)*(u_increment**(i-j))
        freq[i]= Decimal(ncr(N, i))

    return (tree_holder, freq)
```

Output:

```
Binomial tree for an asset price:
 [[100 102 104 106 109 111 114]
 [  0  97 100 102 104 106 109]
 [  0   0  95  97 100 102 104]
 [  0   0   0  93  95  97  99]
 [  0   0   0   0  91  93  95]
 [  0   0   0   0   0  89  91]
 [  0   0   0   0   0   0  87]]
```

Part (b)

The terminal values are given as follows. Frequency here refers to the number of paths which lead to a given terminal price.

Function:

```
def terminal_px_extractor(binomial_tree):
  size = binomial_tree.shape[0]
  terminal_px_holder=[]

  for i in range(size):
    temp = binomial_tree[i,-1]
    terminal_px_holder.append(temp)

  return terminal_px_holder
```

Output:

	Terminal Value	Frequency
0	114.000000	1.0
1	109.128093	6.0
2	104.464393	15.0
3	100.000000	20.0
4	95.726398	15.0
5	91.635432	6.0
6	87.719298	1.0

The filtrations are given by:

F0 = $\{\emptyset, \Omega\}$

F1 = {{U},{D}}

F2 = $\{\{U^2D^0\},\{U^1D^1\},\{U^0D^2\}\}$

F3 = $\{\{U^3D^0\},\{U^2D^1\},\{U^1D^2\},\{U^0D^3\}\}$

F4 = $\{\{U^4D^0\},\{U^3D^1\},\{U^2D^2\},\{U^1D^3\},\{U^0D^4\}\}$

F5 = $\{\{U^5D^0\},\{U^4D^1\},\{U^3D^2\}, \{U^2D^3\},\{U^1D^4\}, \{U^0D^5\}\}$

F6 = $\{\{U^6D^0\},\{U^5D^1\},\{U^4D^2\},\{U^3D^3\},\{U^2D^4\},\{U^1D^5\},\{U^0D^6\}\}$

Answer 2

Note: Please review this answer in conjunction with the file WQU_MScFE_620_Project1_vF.ipynb

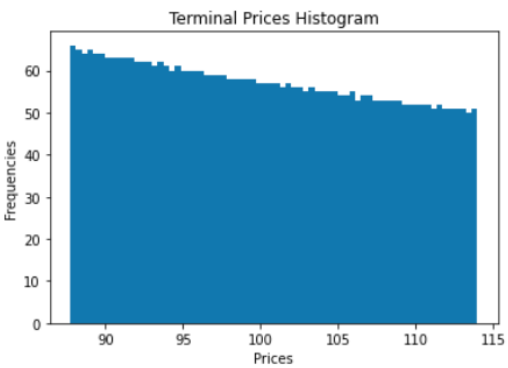
Part (a)

Code:

```
task_2, freq_task2 = asset_px_binomial_tree(4000, u_total, d_total, S_0)
task_2_TV = terminal_px_extractor(task_2)
```

```
plt.hist(task_2_TV, bins=70)
plt.title('Terminal Prices Histogram')
plt.xlabel('Prices')
plt.ylabel('Frequencies')
```

Output:



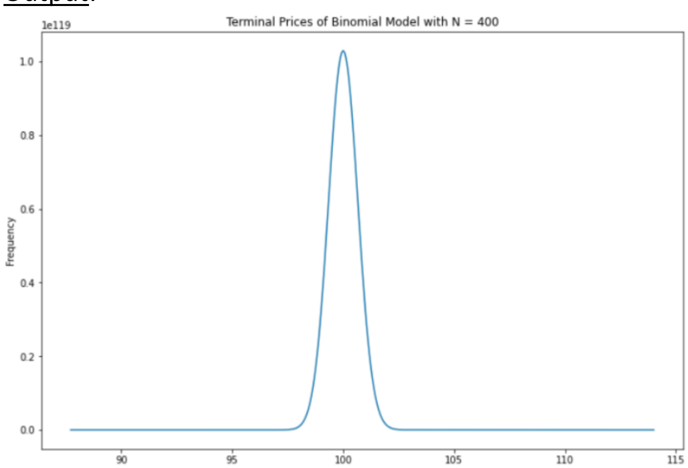
Part (b)

After running the code to produce terminal prices for a binomial tree with 400 periods (plot's y-axis was not scaling appropriately for N = 4000), we noticed a lognormal distribution, as shown below.

Code:

```
fig, ax = plt.subplots(figsize = (12,8))
ax.plot(terminal_val_task2['Terminal Value'].values, terminal_val_task2['Frequency'].values, label = 'Terminal Values'
ax.set_xlabel('Terminal Values')
ax.set_ylabel('Frequency')
ax.set_title('Terminal Prices of Binomial Model with N = 400');
```

Output:



Based on the above observation, we can conclude that the terminal prices for a tree with 4000 periods would also follow a lognormal distribution.

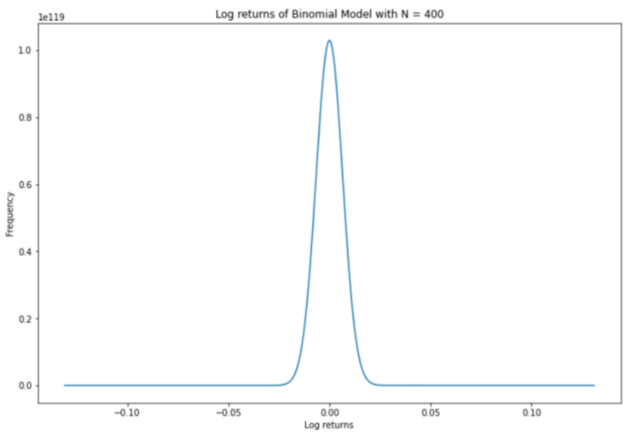
Part (c)

We now create a data frame of log returns for terminal prices wherein these are calculated as $Return_t = Ln\left(\frac{Price_t}{Price_0}\right)$. We plot this and observe that the log returns follow a normal distribution, which makes sense since the prices follow a lognormal distribution.

Code:
```
terminal_val_task2['Log Returns'] = np.log(terminal_val_task2['Terminal Value']/100)

fig, ax = plt.subplots(figsize = (12,8))
plt.plot(terminal_val_task2['Log Returns'].values, terminal_val_task2['Frequency'].values, label = 'Log returns')
ax.set_xlabel('Log returns')
ax.set_ylabel('Frequency')
ax.set_title('Log returns of Binomial Model with N = 400');
```

Output:



Answer 3

Part (a)

There are 4 fundamental financial instruments in a market, viz. equity, debt, bank account (cash) and hybrid securities.

Equity Securities

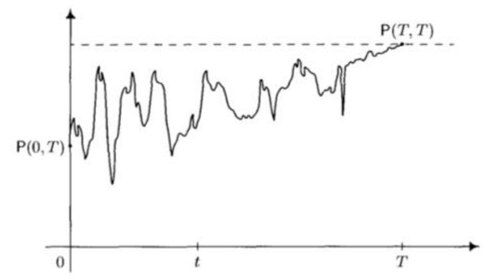
- Ownership claims on the net assets of corporations (i.e. a residual claim) and exposure to its operating performance
- Companies not contractually obligated to repay the amount received from shareholders nor pay any dividends
- Investors of equity securities typically seek a total return in the form of capital/ price appreciation and/ or dividend incomes
- Equity securities are typically of 2 types: common stock and preferred stock
    - *Common Stock*: offer share in operating performance, come with voting rights and a claim on net assets in the event of bankruptcy and liquidation. May have embedded optionality (calls, puts)
    - *Preferred Stock*: is senior to common stock in terms of dividend payments and claim over net assets in the event of liquidation. Have characteristics of both debt and equity

- Equity securities can be private or publicly traded. The former are typically issued to institutional investors via non-public offerings. The latter may be traded on stock exchanges or OTC markets

Debt/ Fixed Income Securities/ Bonds

- Promissory notes issued by corporations, banks, financial institutions or other establishments (issuers) to borrow capital from investors

- Bondholders get paid a fixed (sometimes floating) interest at regular intervals (independent of company performance), the principal is repaid at a specified time and debt investors have a senior claim over the company's net assets



- Although debt is less risky than equity, it is not entirely risk-free (credit risk, interest rate risk, spread risk etc.)

- A standard bond's value at any given time can be characterised as conditional stochastic process (shown above) $P(t,T)$ since its face value $P(T,T)$ is fixed

- There is a large variety of bond categories depending on differences in their features (maturity: money market securities, capital market securities, coupon: floaters, fixed-rate bonds, currency: dual-currency bonds, currency option bonds, etc.)

Bank Accounts

- We can regard it as a security of bond kind that reduces in effect the bank's obligation to pay certain interest on the sum put into one's account

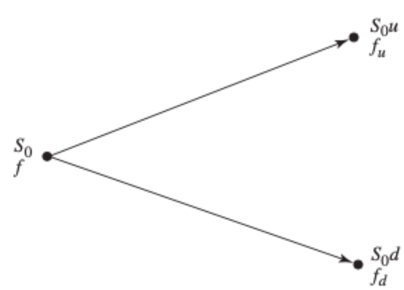
Hybrid Securities

- Typically characterise bonds with which is attached rights to securities which could be equities or sometimes bonds of the issuer

- This generic heading encompasses a seemingly endless array of finance instruments, including convertible bonds, mandatory convertibles, reverse convertibles, preferred shares, bonds with equity warrants etc.

Part (b)

At any given node of a binomial tree, there are 2 states of the world, viz. the 'up' state wherein the asset price at the node increases by a factor of u, and the 'down' state wherein the price decreases by the factor d $(= \frac{1}{u})$.



The underlying assumption is that the asset price follows a random walk, and at each time step, it has a certain probability of moving in either of the above 2 states.

Part (c)

Context

Consider a market with $d+1$ assets whose prices are discrete-time stochastic processes. So, we have

$(B,S) = \left(B, S^1, S^2, \dots, S^d\right)$ where

B = risk-free bank account
$S^i = \{S_t^i : t = 0,1,2,\dots,T\}$

We, therefore, mathematically represent a market with the tuple $((\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P}), (B, S))$

B has a predictable and strictly positive price process. So,
$B_t = B_{t-1}.(1 + r_t)$ where $t = 0,1,2,\dots,T$

Expressing all S^i in terms of B, the numeraire, we get discounted prices $(1, X) = (1, X^1, X^2, \dots, X^d)$ where

$$X^i = \frac{S^i}{B}, \quad i = 1,\dots,d$$

Lastly, an Equivalent Martingale Measure (EMM) $\mathbb{P}^*$ for X is one for which $\mathbb{P}^*$ is equivalent to $\mathbb{P}$, i.e. $\mathbb{P}(A) = 0 \Leftrightarrow \mathbb{P}^*(A) = 0 \forall A \in \mathcal{F}$ and X is an $(\mathbb{F}, \mathbb{P}^*)$-martingale.

Market Completeness Definition
Now, consider a contingent claim H which is basically an $\mathcal{F}^T$-measurable random variable.

Given that the market $((\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P}), (B, S))$ is arbitrage-free (i.e. there are no trading strategies requiring a starting capital of 0 and producing a positive profit with a positive probability), it is said to be complete if every contingent claim is attainable, i.e. its payoffs can be replicated by a strategy φ so that

$$V_T(\varphi) = H$$

More concretely, an arbitrage-free market is complete if any one of the following equivalent conditions holds:

- The set of Equivalent Martingale Measures P has only one element
- Every $(\mathbb{F}, \mathbb{P})$-martingale M with $M_0 = 0$ can be written as a martingale transform w.r.t X, i.e. X has a Predictable Representation Property (PRP)

The above is referred to as the Fundamental Theorem of Asset Pricing II (FTAP II).

Using the PRP, an expression for the replicating strategy for each contingent claim H in a complete market can be derived. This is given below:

$$\varphi_t^M = \frac{\mathbb{E}^*(H \mid \mathcal{F}_t) - \mathbb{E}^*(H \mid \mathcal{F}_{t-1})}{X_t - X_{t-1}}$$

Part (d)
A complete market is a market with an equilibrium price for every asset in every possible state of the world and it can also be defined as one in which a derivative product can be artificially made from more basic instruments, such as cash and the underlying asset.

In the case study above, the market will not be complete but would rather be incomplete because there exists an arbitrage opportunity in the market due to that "larger than expected jump". The concept of an incomplete market can further be described as simply a market where we are unable to construct portfolios to achieve a desired payoff function of a derivative. In such a market, a derivative is priced using the risk preferences of investors rather than by theory.