2.2 Regge trajectories and anomalous dimensions

Let us list some families of local operators that will be important in this work. The lowest twist families are "double-twist" operators built from two scalars ϕ^i. These can be grouped into $O(N)$ singlets, antisymmetric tensors, and symmetric tensors as follows:

$$\begin{aligned}
&[(\phi^i\phi^j)_s]_J \sim \phi^i\partial_{\mu_1}\cdots\partial_{\mu_J}\phi^i + \cdots\,, && \tau = d-2+O(1/N)\,,\\
&[(\phi^i\phi^j)_{\mathrm{sym}}]_J \sim \phi^{(i}\partial_{\mu_1}\cdots\partial_{\mu_J}\phi^{j)} + \cdots\,, && \tau = d-2+O(1/N)\,,\\
&[(\phi^i\phi^j)_{\mathrm{asym}}]_J \sim \phi^{[i}\partial_{\mu_1}\cdots\partial_{\mu_J}\phi^{j]} + \cdots\,, && \tau = d-2+O(1/N)\,.
\end{aligned} \tag{2.5}$$

Here "..." represents combinations of derivatives needed to ensure that the operator is primary. By analogy with QCD, we will sometimes refer to these leading twist operators as DGLAP-type [2, 69, 70]. In the limit $N \to \infty$, these operators become higher-spin conserved currents with scaling dimensions $\Delta = J + d - 2$. At finite N, the higher-spin symmetries are slightly broken and these operators acquire anomalous dimensions, except for the spin-2 singlet current, which is the stress tensor $[(\phi^i\phi^i)_s]_2 = T_{\mu\nu}$. The leading anomalous dimensions are [71, 72]

$$\begin{aligned}
\gamma_{\mathrm{sym},J} = \gamma_{\mathrm{asym},J} &= \frac{1}{N}\frac{16(J-1)(d+J-2)\sin\left(\frac{\pi d}{2}\right)\Gamma(d-2)}{\pi(d+2J-4)(d+2J-2)\Gamma\left(\frac{d}{2}-2\right)\Gamma\left(\frac{d}{2}+1\right)}\,,\\
\gamma_{s,J} = \gamma_{\mathrm{(a)sym},J} &- \frac{1}{N}\frac{8\sin\left(\frac{\pi d}{2}\right)\Gamma(d-2)\Gamma(d+1)\Gamma(J+1)}{\pi(d-1)(d+2J-4)(d+2J-2)\Gamma\left(\frac{d}{2}-2\right)\Gamma\left(\frac{d}{2}+1\right)\Gamma(d+J-3)}\,.
\end{aligned} \tag{2.6}$$

Some example higher-twist families are

$$\begin{aligned}
&[\sigma\phi^i]_J \sim \sigma\partial_{\mu_1}\cdots\partial_{\mu_J}\phi^i\,, && \tau = \tfrac{d+2}{2}+O(1/N)\,,\\
&[\sigma(\phi^i\phi^j)_\rho]_J \sim \sigma(\phi^i\partial_{\mu_1}\cdots\partial_{\mu_J}\phi^i)_\rho\,, && \tau = d+O(1/N)\,,\\
&[\sigma\sigma]_J \sim \sigma\partial_{\mu_1}\cdots\partial_{\mu_J}\sigma\,, && \tau = 4+O(1/N)\,.
\end{aligned} \tag{2.7}$$

Here and below, we use the notation that ρ stands for an $O(N)$ representation that can be either "s", "sym", or "asym". The equation of motion implies that $\Box\phi \sim \sigma\phi$. Thus, $[\sigma(\phi^i\phi^j)_\rho]_J$ plays the role of a subleading double-twist family for ϕ. Each additional insertion of σ or ϕ generates additional subleading trajectories. Thus, even in the infinite-N limit, the theory contains a rich set of families that become dense in τ at large τ (for generic d).

2.3 A first look at the Chew-Frautschi plot

A Regge trajectory is a family of light-ray operators depending continuously on spin J. Nonvanishing light-ray operators with nonnegative integer J become light-transforms of local operators $\mathbf{L}[\mathcal{O}]$ (or their shadow transforms). We can visualize Regge trajectories using a Chew-Frautschi plot. The coordinates of the Chew-Frautschi plot are conventionally chosen to be $\Delta - \frac{d}{2}$ and J, where $(\Delta, J) = (1-J_L, 1-\Delta_L)$, where (Δ_L, J_L) are the quantum numbers