Homework 6

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Problem 1.

Part (a)

True, a groundtrack plots the motion of a satellite relative to the Earth. In a prograde orbit below geosynchronous a satellite is travelling eastward faster than the Earth's rotation so its groundtrack has eastward motion.

Part (b)

True. After J_2 resonance and third-body perturbations, solar radiation pressure has the largest effect in GEO.

Part (c)

True, because gravity can be expressed as a potential.

Part (d)

False, looking at the equation for apsidal rotation:

$$\dot{\omega} = \frac{3}{4} J_2 \frac{n R_E^2}{(1-e^2)^2 a^2} \cos i \left(5 \cos^2 i - 1\right) \tag{1}$$

When $i = 63.4^\circ$ (Molniya orbit), there is no apsidal rotation.

Problem 2.

Part (a)

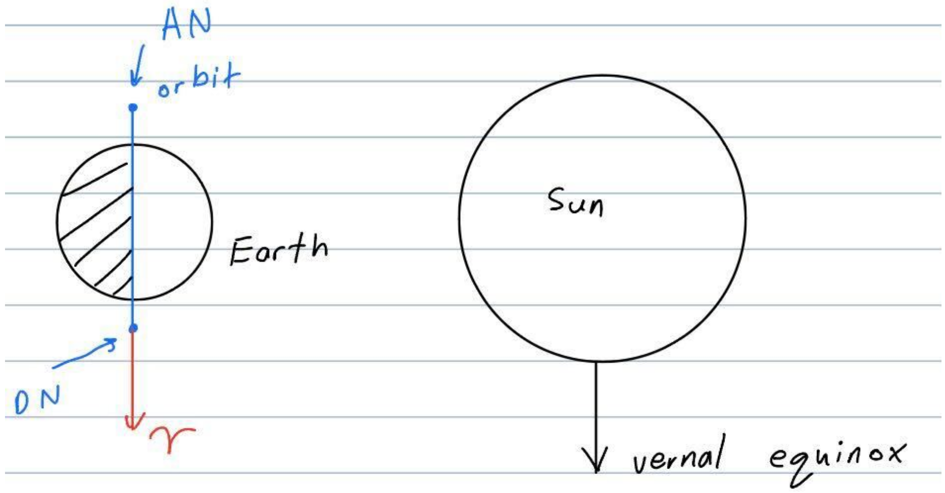

Figure 1: Top-Down View of Orbit

Part (b)

These design criteria call for a Dawn-Dusk Sun-Synchronous Orbit using J2 effects.
First, consider the precession rate due to J_2 and set it so that the satellite is aligned with the dawn-dusk line, which rotates 360 degrees every year.

$$\frac{d\Omega}{dt} = \frac{360^\circ}{365.25d} = \frac{3}{2} J_2 \left(\frac{R_E}{a(1-e^2)}\right)^2 \cos i \tag{2}$$

Solving for inclination yields:

$$i = \cos^{-1}\left(\frac{360^\circ}{365.25d} \times \left(\frac{2}{3}\right) \times \left(\frac{1}{nJ_2}\right) \times \left(\frac{a(1-e^2)}{R_E}\right)^2\right) \tag{3}$$

Next, consider the repeat orbits constraint with 60 Orbits in 40 days.

$$\frac{2\pi\sqrt{\frac{a^3}{\mu}}}{T_{\text{solar day}}} = \frac{m}{n} \tag{4}$$

Where:

- m = number of days (4 days)
- n = number of orbits (60 orbits)

Rearrange to solve for a:

$$a = \left(\mu \times \left(\frac{m \times T_{\text{solar day}}}{2 \times \pi \times n}\right)^2\right)^{1/3} \tag{5}$$

Here, use $T_{\text{solar day}}$ to account for J_2, specifically the precession of Ω, while ignoring perturbations in the argument of perigee ($\dot{\omega}$) and mean anomaly ($\dot{M}$) since they are negligible.
Next, consider the two orbits that lead to a symmetric ground track: circular or eccentric with an eccentricity vector aligned with the lines of nodes. For the eccentric case, there will be apsidal precession due to J_2, which will cause the ground track to change over time and not be symmetric. This effect can be avoided if $i = 63.4°$ (Molniya orbit), but this inclination is not sufficient enough for a dawn-dusk sun-syncrhonous orbit. Thus, the orbit must be circular ($e = 0$), which means that the argument of periapsis (ω) is undefined.
Based on the right ascension definition at epoch, the RAAN (Ω) is opposite to the vernal equinox:

$$\Omega = 180° \tag{6}$$

Calculating the final orbital parameters yields:

$$\begin{aligned}
\mathbf{a} &= \mathbf{6945.0} \text{ km} \\
\mathbf{e} &= \mathbf{0} \\
\mathbf{i} &= \mathbf{97.663°} \\
\boldsymbol{\Omega} &= \mathbf{180°} \\
\omega &= \text{undefined}
\end{aligned}$$

Part (c)

To find the minimum swath needed to capture the entire equator at sunrise, determine the number of orbits in one solar day and divide Earth's circumference by that.

$$1 \text{ solar day} = 24 \text{ hrs } 4 \text{ minutes} \tag{7}$$

Number of orbits:

$$\text{number of orbits} = \frac{1 \text{ solar day}}{T_{\text{satellite}}} \tag{8}$$

Minimum swath width:

$$\text{Minimum swath} = \frac{\text{Earth's circumference}}{\text{number of orbits}} = \frac{2\pi R_E}{\text{number of orbits}} \tag{9}$$

Evaluating this, yields a minimum swath of: **2671.610 km**.

Problem 3.

For an areo-stationary orbit, want circular, equatorial: $e = 0$, $i = 0$. And want the final rate of change of mean longitude equal to Mars' rotation rate:

$$n_{\text{goal}} = \omega_{\text{Mars}} \tag{10}$$

First, find a_0, n_0 for a spherical Mars with no J2 effects:

$$a_0 = \left(\frac{\mu_{\text{Mars}}}{n_0^2}\right)^{1/3}, \quad n_0 = \frac{\mu_{\text{Mars}}}{a_0^3} \tag{11}$$

Evaluating this, the initial semi-major axis is **20463.280 km**.
Now considering J2 effects, the initial rate of change of mean longitude is given by:

$$\dot{l} = \dot{\omega} + \dot{\Omega} + \dot{M} \tag{12}$$

where:

$$\dot{\omega} = \frac{3}{4} J_2 \left(\frac{R_{\text{Mars}}}{a}\right)^2 (4)n \tag{13}$$

$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{R_{\text{Mars}}}{a}\right)^2 n \tag{14}$$

$$\dot{M} = n + \frac{3}{4} J_2 \left(\frac{R_{\text{Mars}}}{a}\right)^2 (2)n \tag{15}$$

Thus, the initial mean longitude rate or initial mean motion is:

$$n_{\text{initial}} = \dot{l}_{\text{initial}} = n_0 \left(1 + 3J_2 \left(\frac{R\text{Mars}}{a}\right)^2\right) \tag{16}$$

To achieve n_{goal}, adjust $n_i nitial$ as:

$$n_{\text{goal}} = n_{\text{initial}} + \Delta n \quad \Rightarrow \quad \Delta n = n_{\text{goal}} - n_{\text{initial}} \tag{17}$$

$$\Delta n = n_{\text{goal}} - n_0 \left(1 + 3J_2 \left(\frac{R_{\text{Mars}}}{a}\right)^2\right) \tag{18}$$

This adjustment can then be propagated into an appropriate adjustment in the semi-major axis by:

$$a = \left(\frac{\mu_{\text{Mars}}}{n^2}\right)^{1/3} \tag{19}$$

$$\frac{da}{dn} = \frac{-2}{3} \frac{\mu_{\text{Mars}}^{1/3}}{n^{5/3}} = \frac{-2a}{n} \tag{20}$$

$$\Delta a = \frac{da}{dn} \Delta n \tag{21}$$

$$a_{\text{initial}} = a_0 = \left(\frac{\mu_{\text{Mars}}}{\omega_{\text{Mars}}^2}\right)^{1/3} \tag{22}$$

$$a_{\text{goal}} = a_{\text{initial}} + \Delta a \tag{23}$$

Evaluating this, the final semi-major axis is **20465.496 km**.

Appendix 1

```
%% Space Mechanics Homework 6
% Tycho Bogdanowitsch
clc; clear;
% Constants
R_earth = 6378; % km
DU_earth = R_earth; % km
AU = 1.496e8; % km
mu_earth = 3.986e5; % km^3/s^2
mu_mars = 4.283e4; % km^3/s^2
mu_sun = 1.327e11; % km^3/s^2
R_sun = 6.963e5; % km

%% Problem 2
disp('Problem 2')
% Part b
e = 0;
RAAN = 180;
d = 86400; % seconds in a day
J_2 = 0.001082;
num_orbits = 60; % orbits
num_days = 4; % days

T_sidereal_day = 86164.1; % seconds

T_solar_day = 86400; % seconds

a = (mu_earth*(num_days*T_solar_day/(2*pi*num_orbits))^2)^(1/3)
   ;

T = 2*pi*sqrt(a^3/mu_earth);

n = 2*pi/T;

year_sec = 365.25*24*3600; % days --> seconds
dRAANdt = deg2rad(360)/year_sec;

i = acosd(dRAANdt*(-2/3)*(1/(n*J_2))*((a*(1-e^2)/R_earth))^2);

e_test = 0.9885;
i_test = 63.4;
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```
dRAANdt_test = (3/2)*n*J_2*(R_earth/(a*(1-e_test^2))^2*cosd(
   i_test));

fprintf('a: %.3f km\n', a);
fprintf('e: %.3f \n', e);
fprintf('i: %.3f degrees\n', i);
fprintf('RAAN: %.3f degrees\n', RAAN);
fprintf('omega: undefined\n');

% Part c
solar_day = 86400; % seconds
orbits_per_day = (solar_day)/(T);
min_swath = 2*pi*R_earth/orbits_per_day;

fprintf('Min swath: %.3f km\n', min_swath);

%% Problem 3
disp('Problem 3')
J_2_mars = 1.9643e-3;
R_mars = 3397.2; % km
mu_mars = 4.305e4; % km^3/s^2
omega_mars = 7.088e-5; % rad/s
e = 0;
i = 0;

a_initial = (mu_mars/omega_mars^2)^(1/3);

n_0 = omega_mars;

n_goal = omega_mars;

n_initial = n_0*(1+3*J_2_mars*(R_mars/a_initial)^2);

dn = n_goal-n_initial;

dadn = -(2/3)*(a_initial)/(n_goal);

da = dadn*dn;

a_goal = a_initial + da;

fprintf('Initial altitude: %.3f km\n', a_initial);
fprintf('Delta altitude: %.3f km\n', da);
fprintf('Goal altitude: %.3f km\n', a_goal);
```