AA279A, Winter 2025
Homework 3

Due: Wednesday, February 5th, 2025 @ 3:00 PM PST

Notes:

Submission Instructions

Please submit your solutions as a PDF file on Gradescope (there is a link to Gradescope on Canvas). We require your document to be typeset using LaTeX, Microsoft Word, or another word processor. We have provided a LaTeX template on Canvas that can be used to typeset your solutions. For problems that require programming, please include in your submission a copy of your code (with comments) and any figures that you are asked to plot. Include your code as text in your PDF, please do not submit extra files.

Topics

Lecture 5 : Orbital relationships, Kepler's equation, time relationships, orbital elements, coordinate systems.

Problem 1: Travel Time

Many years in the future, a crew of Martian colonists boards a SpaceX Starship rocket and lifts off from the surface of Mars to return home. The Starship eventually enters an elliptical trajectory around Mars with eccentricity, $e = 0.89$, and radius of apoapsis, $r_a = 3800$ km. Assume that the true anomaly at epoch is $\nu = 110°$. What is the time remaining (in minutes) before the Starship reaches apoapsis?

Problem 2: My First Simulink Model

Halley's Comet is the only known comet that is clearly visible to the naked eye from the Earth. Its orbit around the Sun has a semi-major axis, $a = 17.834$ AU and an eccentricity, $e = 0.967$. Develop an orbital simulation in MATLAB and Simulink to investigate the orbit's evolution with time. Recall that 1 AU $\approx 1.496 \times 10^8$ km is the semi-major axis of the Earth's orbit around the Sun.

(i) What is the orbital period, T, and the mean motion, n, for Halley's Comet? Provide your answers in units of Earth years and rad/s, respectively.

(ii) Write a MATLAB function which accepts inputs of mean anomaly M (in radians), eccentricity e, and tolerance ϵ (in radians), then returns eccentric anomaly E (in radians) as its output. Calculate E numerically using the Newton-Raphson method as discussed in lecture.

(iii) Use Simulink to simulate the comet's position on its orbit in the perifocal coordinate system. Run your simulation from $t = 0$ TU to $t = T$ TU. Use the MATLAB function written in part (ii) in Simulink with a tolerance value of 10^{-10} rad. Submit a screenshot of your Simulink block diagram and include any relevant code.

(iv) Plot the orbit in MATLAB. Label axes and include units. Be sure to use a time step that results in a smooth-looking plot. On the same figure, plot circular tick marks on the orbital trace at every time interval of $0.04T$ TU. *Hint: You can obtain the position of the comet in MATLAB from the position data output from Simulink using indexing:* `https://goo.gl/Csrka4`

(v) Comment on the relationship between the spacing of the tick marks near the periapsis versus those near the apoapsis.

Please remember to submit your Simulink block diagram, the MATLAB code, and the figure.

Problem 3: Kessler Syndrome

There are an estimated 670,000 or so small orbital debris (between one and 10 centimeters in diameter) and about 29,000 larger bits (larger than 10 centimeters) in the vicinity of the Earth right now. Therefore, the probability of a piece of orbital debris striking a satellite is not negligible. Consider one such piece of orbital debris for which the following data is available at epoch t_0:

$$\vec{r_0} = 8\vec{i} \text{ DU}$$
$$\vec{v_0} = -\frac{1}{\sqrt{8}}\vec{i} + \frac{1}{\sqrt{8}}\vec{j} \text{ DU/TU}$$

Here $\vec{r}$ is the position of the orbital debris relative to the center of the Earth, and $\vec{v}$ is its inertial velocity. The Earth-Centered Inertial basis unit vectors are $\vec{i}$, $\vec{j}$, and $\vec{k}$ where $\vec{i}$ and $\vec{j}$ identify the equatorial plane. Assume the Earth is spherical with radius $R_E = 1$ DU $= 6378$ km.

(i) What kind of orbit is the debris in? State all of the following that apply: circular, elliptical, parabolic, hyperbolic, equatorial, polar

(ii) Imagine that a satellite orbits the Earth in a circular, equatorial, prograde orbit at an altitude of 19134 km. Is it possible for the orbital debris to collide with this satellite at some time in the *future*? To check for a potential collision scenario, you should determine whether the position of the debris meets the following condition at any time $t \geq t_0 : ||\vec{r}|| \leq R_E + 19134$ km.

(iii) Draw a sketch or make a plot of the orbits of the debris and the satellite.

(iv) If a collision were to happen, what would be the velocity vector of the orbital debris at the collision point *relative to the satellite*? What is the true anomaly of the orbital debris at the given initial position and velocity?

Problem 4. ECI2OE

The Envisat (Environmental Satellite) is an Earth-observing satellite built by the European Space Agency (ESA). After 10 years of service, contact with Envisat was lost in 2012 and given its orbit and area-to-mass ratio, it will take 150 years before it gradually gets pulled into the Earth's atmosphere. Its position and velocity in the Earth-Centered Inertial (ECI) frame on February 3, 2024 at 22:08:04 UTC were:

$$\vec{r}_{IJK} = \begin{bmatrix} 3105.4128 & -880.8531 & 6368.9408 \end{bmatrix} \text{km}$$
$$\vec{v}_{IJK} = \begin{bmatrix} -6.7276 & -0.5612 & 3.2023 \end{bmatrix} \text{km/s}$$

Write a function in MATLAB that accepts position and velocity in ECI frame as inputs and outputs the six Keplerian orbital elements $(a, e, i, \Omega, \omega, \nu)$. Submit your code as well as the Keplerian elements for Envisat. **Please provide all angles** (i, Ω, ω, ν) **in degrees.**