AA279A, Winter 2025
Homework 2

Due: Wednesday, January 29th, 2025 @ 3:00 PM PST

Notes:

Submission Instructions

Please submit your solutions as a PDF file on Gradescope (there is a link to Gradescope on Canvas). We require your document to be typeset using LaTeX, Microsoft Word, or another word processor. We have provided a LaTeX template on Canvas that can be used to typeset your solutions. For problems that require programming, please include in your submission a copy of your code (with comments) and any figures that you are asked to plot. Include your code as text in your PDF, please do not submit extra files.

Topics

Lecture 3 and 4: Keplerian Orbital Elements, Orbit Prediction

Problem 1: True or False

Mark each of the following statements as either TRUE or FALSE, then provide a brief explanation for your answer. Note that if a statement is not always TRUE, then it should be considered FALSE generally.

(i) On an elliptical non-circular orbit, the flight path angle is 90° only at periapsis and apoapsis.

(ii) For elliptical orbits, mean anomaly increases linearly with time during each orbit.

(iii) For elliptical orbits, true anomaly increases at varying rates and is measured from the center of the ellipse.

(iv) One orbital period can be measured as the time it takes for the mean anomaly to vary by 2π or 360°.

(v) Given the eccentricity of an orbit, the eccentric and true anomaly can be related geometrically.

(vi) Specific mechanical energy can be completely determined by the perigee distance.

Problem 2: Visualizing Eccentricity

For a perigee distance, $r_P = 3.0$DU and the 4 different eccentricity values below, use MATLAB to plot the shape of the orbit described by the conic trajectory equation. Let the true anomaly, ν, of your plotted points vary from 0 to 360 degrees in 1 degree increments. Sketch in asymptotes for any hyperbolic trajectories and do not plot invalid points. Also annotate p, a, and r_P and give their values. If not possible say why.
Your MATLAB figures should have titles, and their axes should be labeled (with units). You should find MATLAB's axis equal, hold on, and hold off functions useful. Provide your printed MATLAB code and plots.

(i) e = 0.7

(ii) e = 0

(iii) e = 1.6

(iv) e = 1.0

(v) All four plots above on the same axes for comparison (no annotation necessary).

Problem 3: Locating the Target

An imaging satellite has just been injected into an orbit around the Earth that is highly eccentric and inclined. The satellite operator wishes to know its current position along the trajectory, but only has the following details:

$$\vec{r} = -8050\hat{i} + 1900\hat{j} + 2600\hat{k} \text{ km}$$
$$\vec{v} = -7.0\hat{i} - 2.8\hat{j} - 4.0\hat{k} \text{ km/s}$$

They also know that the satellite already passed its periapsis point, but has not yet reached apoapsis. Determine the following properties of the orbit:

(i) Eccentricity, e

(ii) Current True Anomaly, ν, **in degrees**

(iii) Current Flight Path Angle, ϕ, **in degrees**

(iv) Eccentric Anomaly, E, **in degrees**

(v) Mean Anomaly, M, **in degrees**

Problem 4: OE2ECI

The Hubble Space Telescope was launched aboard the space shuttle in 1990 and has since traveled 3 billion miles along a circular low Earth orbit. As of January 28, 2024 at 13:38pm UTC (Coordinated Universal Time, the primary global time standard), its orbit could be described using the following Keplerian orbital elements:

$$\begin{array}{cc} a = 6897\text{km} & \Omega = 205.7537^\circ \\ e = 0.0002422 & \omega = 318.2020^\circ \\ i = 28.4666^\circ & \nu = 41.8386^\circ \end{array}$$

Write a MATLAB function to compute the position and velocity of the Hubble Space Telescope in the ECI (Earth-Centered Inertial) reference frame. Your function's input should be $[a, e, i, \Omega, \omega, \nu]$ and its output should be $[r_{ECI}, v_{ECI}]$. Use your new function to compute the r_{ECI} and v_{ECI} of the Hubble Space Telescope on January 28, 2024 using the aforementioned Keplerian elements.