AA 279A, Winter 2025
Homework 6

Due: Wednesday, March 5, 2025 @ 3:00 PM PST

Notes:

Submission Instructions

Please submit your solutions as a PDF file on Gradescope (there is a link to Gradescope on Canvas). We require your document to be typeset using LaTeX, Microsoft Word, or another word processor. We have provided a LaTeX template on Canvas that can be used to typeset your solutions. For problems that require programming, please include in your submission a copy of your code (with comments) and any figures that you are asked to plot. Include your code as text in your PDF, please do not submit extra files.

Topics

Lecture 9 + 10 + 11 : Gauss variational equations. Impulsive maneuvers in near-circular orbits. J2 effects.

Problem 1. True or False

Mark each of the following statements as either TRUE or FALSE, then provide a brief explanation for your answer. Note that if a statement is not always TRUE, then it should be considered FALSE generally.

(a) Eastward motion on a groundtrack indicates an object is in a prograde orbit about the Earth.

(b) Consider a spacecraft with a ballistic coefficient of $0.01\text{m}^2/\text{kg}$. Neglecting orbital resonances, the largest perturbation to geostationary Keplerian orbit is solar radiation pressure (SRP).

(c) All gravity forces are considered conservative.

(d) The perigee of an elliptical orbits will always drift due to the J2 perturbation.

Problem 2. Full Earth Coverage

The company Planet (formerly, Planet Labs) is using a constellation of satellites to image the entire Earth each day (see this MIT Technology Review article featuring Prof. D'Amico's comments). Planet recruits you to design the orbit of their new Earth-imaging satellite to map the entire world. You are provided with the following requirements/details:

1. Assume the satellite is in Keplerian motion subject to secular J2 effects (ignore short-term oscillations).
2. The satellite's groundtrack should be symmetric with respect to the latitudes and longitudes.
3. The satellite should have a dawn-dusk orbit, i.e. the local time at ascending node should always be 18:00.

4. The right ascension of the Sun should be 90° at epoch. In other words, the line from the Earth to the Sun is 90° counter-clockwise from the vernal equinox.

5. The satellite's groundtrack should repeat itself in 4 days. Moreover, it should have completed 60 orbits in those 4 days. *Hint:* This is referring to a repeat orbit, i.e. an orbit where an integral number of satellite periods must be equal to an integral number of days.

Now answer the following questions:

(a) Sketch a top-down view of the orbit, the Earth, the direction of the vernal equinox, and the Sun.

(b) What are the mean orbital elements a, e, i, Ω, ω of the satellite's orbit that meet Planet's requirements?

(c) Now consider that the camera on the satellite is always pointing in Nadir direction (negative radial direction). What is the minimum width of the swath of the camera images so that they capture the entire equator at sunrise? In other words, what is the separation between the successive ascending nodes along the equator? Your answer should be in kilometers.

Problem 3. Areo-Stationary Orbit

Consider an areo-stationary satellite in circular-equatorial orbit about an oblate Mars (J_2 only). An areo-stationary orbit is the Mars equivalent of a geo-stationary orbit about the Earth. We would like the ground-track to occupy the same position (geocentric longitude and latitude) on Mars surface during the mission lifetime. Use Mars' physical properties as follows: $\mathrm{J}_2 = 1.964 \times 10^{-3}$, $R_{\mathrm{Mars}} = 3397.2$ [km], $\mu_{\mathrm{Mars}} = 4.305 \times 10^4 [\mathrm{km}^3/\mathrm{s}^2]$, and $\omega_{\mathrm{Mars}} = 7.088 \times 10^{-5}$ [rad/s]. What is the radius or semi-major axis to be selected for the areo-stationary satellites orbit considering the combined secular J_2 effects on the mean longitude?

Hints:

- Recall the definition of mean longitude $l = \omega + \Omega + M$ for circular-equatorial orbits.
- Consider first computing the semi-major axis for spherical mass distribution. Afterward, seek a small correction to mean motion to compensate J2 effects. The following partial derivative will turn useful to find the desired first order correction of semi-major axis: $\mathrm{d}a/\mathrm{d}n$.