Homework 2

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AA 279A - Space Mechanics

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Problem 1.

Part (a)

False: The flight path angle is 0 degrees at the periapsis and apoapsis.

Part (b)

True: For elliptical orbits, the mean anomaly is defined as the mean motion, n, multiplied by time elapsed plus an initial mean anomaly.

Part (c)

False: For elliptical orbits, the true anomaly increases at varying rates, but is measured form the central body at one of the foci.

Part (d)

True: By definition one orbital period can be measured as the time it takes for the mean anomaly to vary by 460 degrees since mean motion is defined as $\frac{2\pi}{T}$

Part (e)

True: Given the eccentricity of an orbit, the eccentric and true anomaly can be related geometrically by the following equation: $tan(\frac{E}{2}) = \sqrt{\frac{1-e}{1+e}} tan(\frac{\nu}{2})$

Part (f)

False: Specific mechanical energy can be completely determined by the semi-major axis, which can be found with the perigee distance and the eccentricity.

Problem 2.

For all cases, the periapsis radius is given as:

$$r_p = 3.0 \times DU_{\text{earth}} \quad (\text{km})$$

Additionally, the Cartesian coordinates are calculated as follows with ν as the true anomaly:

$$x = r\cos\nu, \quad y = r\sin\nu$$

Part (a)

For an elliptical orbit ($e = 0.7$), the semi-major axis is given by:

$$a = \frac{r_p}{1-e}$$

The semi-parameter is calculated as:

$$p = a(1-e^2)$$

The radial distance as a function of true anomaly ν is:

$$r = \frac{p}{1+e\cos\nu}$$

From these equations, the elliptical orbit can be plotted with the parameters shown in Figure 1.

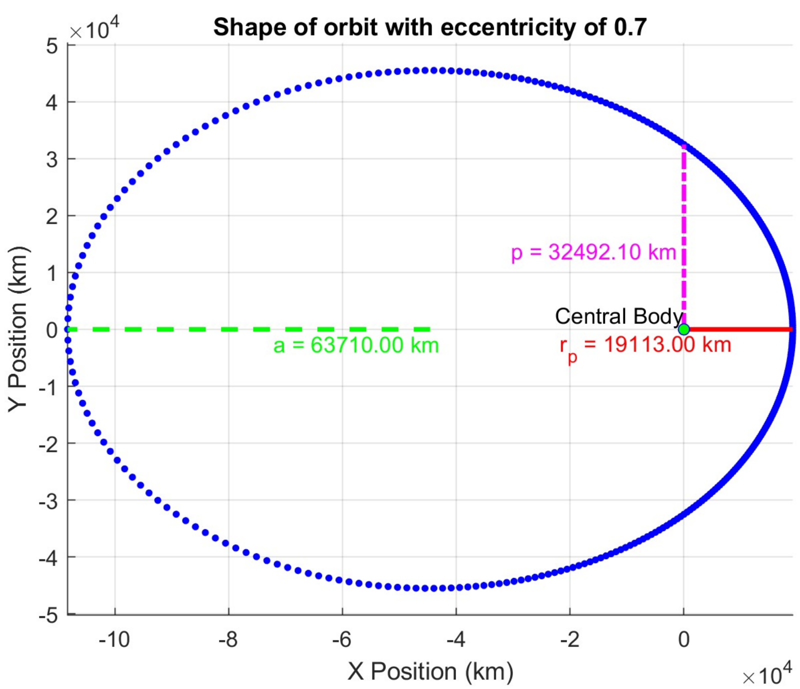

Figure 1: Elliptical Orbit

Part (b)

For a circular orbit ($e = 0.0$), the eccentricity is zero, which means that:

$$a = r_p$$

Since $e = 0$, the semi-parameter is now just:

$$p = a$$

Similarly, the radial distance remains constant:

$$r = p$$

From these equations, the circular orbit can be plotted with the parameters shown in Figure 2.

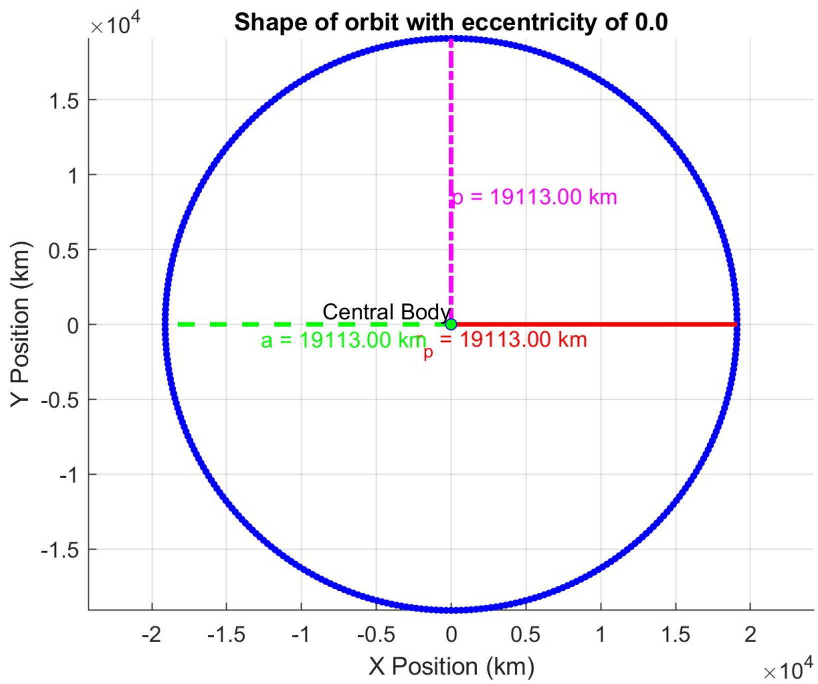

Figure 2: Circular Orbit

Part (c)

For a hyperbolic trajectory $e = 1.6$, the semi-major axis is given by:

$$a = \frac{r_p}{1 - e}$$

and the semi-parameter is:

$$p = a(1 - e^2)$$

The true anomaly range is restricted by:

$$\delta = 2\sin^{-1}\left(\frac{1}{e}\right)$$

so the valid range is:

$$90^\circ + \frac{\delta}{2} < \nu < 360^\circ - \left(90^\circ + \frac{\delta}{2}\right)$$

The asymptotes are defined with a slope:

$$\text{slope} = \sqrt{e^2 - 1}$$

leading to asymptote equations:

$$y_\pm = \pm(\text{slope} \cdot x - \text{slope} \cdot (r_p - a))$$

From these equations, the hyperbolic orbit can be plotted with the parameters shown in Figure 3.

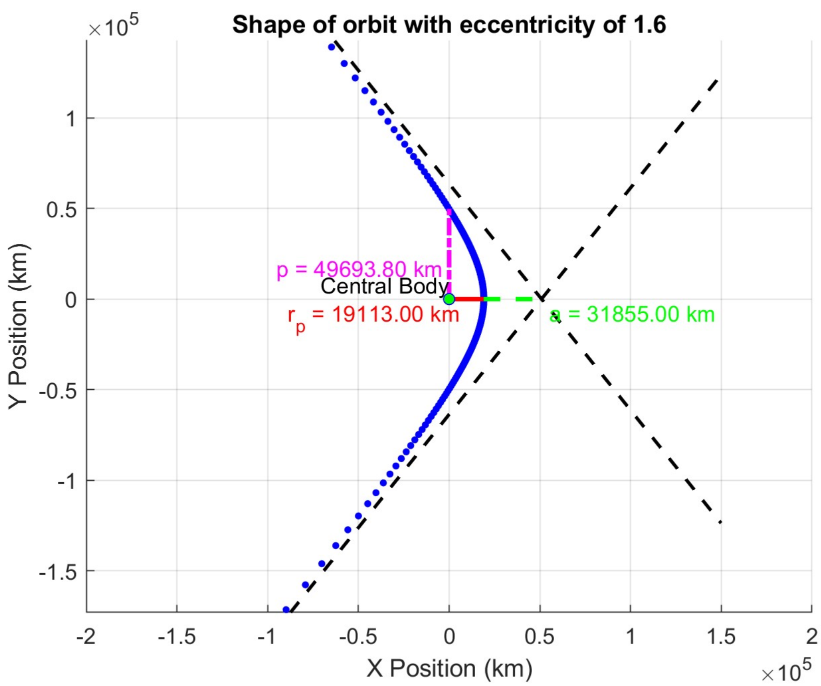

Figure 3: Hyperbolic Orbit

Part (d)

For a parabolic orbit ($e = 1.0$), the semi-major axis approaches infinity ($a \to \infty$), so we use the semi-parameter instead, which is given by:

$$p = 2r_p$$

The radial distance as a function of true anomaly is:

$$r = \frac{p}{1 + e\cos\nu}$$

From these equations, the parabolic orbit can be plotted with the parameters shown in Figure 4.

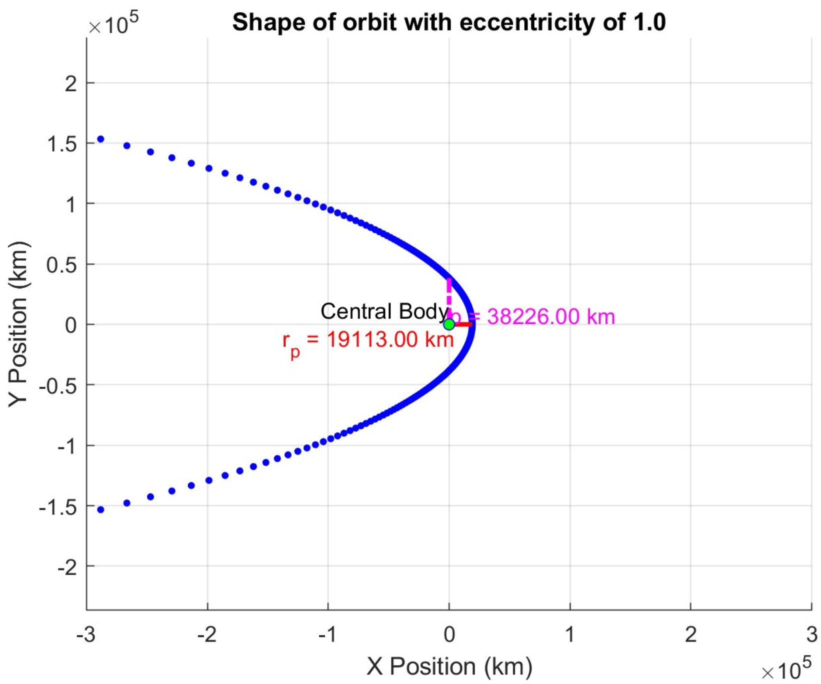

Figure 4: Parabolic Orbit

Part (e)

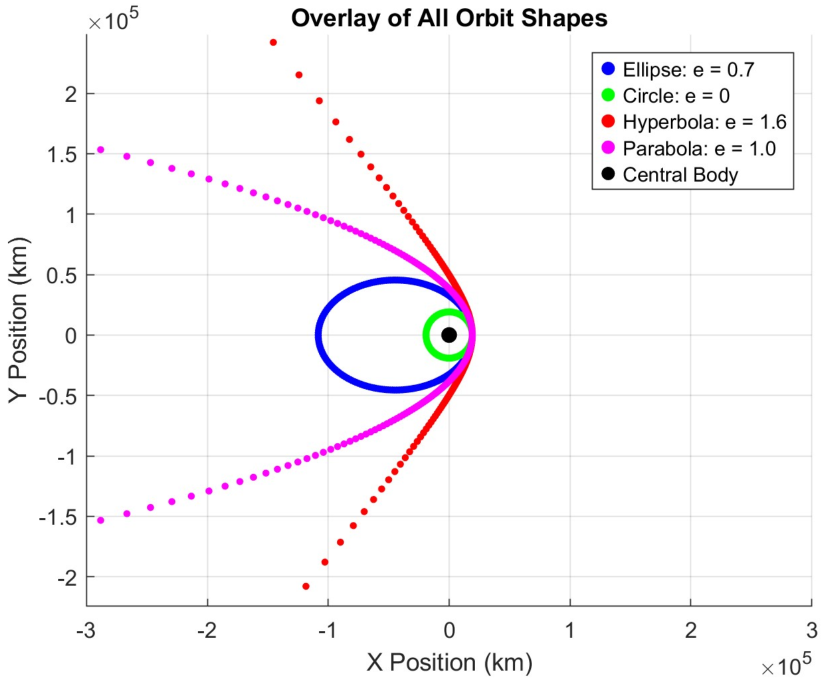

Figure 5: All plots on the same axes

Problem 3.

Part (a)

The initial position and velocity vectors in the Earth-Centered Inertial (ECI) frame are:

$$\mathbf{r}_{ECI} = \begin{bmatrix} -8050 & 1900 & 2600 \end{bmatrix} \quad \text{(km)}$$

$$\mathbf{v}_{ECI} = \begin{bmatrix} -7 & -2.8 & -4 \end{bmatrix} \quad \text{(km/s)}$$

The magnitudes of these vectors are:

$$r = \|\mathbf{r}_{ECI}\|, \quad v = \|\mathbf{v}_{ECI}\|$$

The specific mechanical energy is calculated as:

$$\epsilon = \frac{v^2}{2} - \frac{\mu_{\text{earth}}}{r}$$

where μ_{earth} is the standard gravitational parameter of Earth.
The specific angular momentum vector is:

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

with magnitude:

$$h = \|\mathbf{h}\|$$

The eccentricity is computed as:

$$e = \sqrt{1 + \frac{2h^2\epsilon}{\mu_{\text{earth}}^2}}$$

Evaluating this with the given values yields an eccentricity of: $\mathbf{e = 0.735}$.

Part (b)

The semi-parameter is given by:

$$p = \frac{h^2}{\mu_{\text{earth}}}$$

The current true anomaly is calculated using:

$$\nu = \cos^{-1}\left(\frac{1}{e}\left(\frac{p}{r} - 1\right)\right)$$

converted to degrees:

$$\nu_{\text{deg}} = \frac{\nu \cdot 180}{\pi}$$

Evaluating this with the given values yields a current true anomaly of: $\nu = \mathbf{81.650}°$.

Part (c)

The flight path angle is determined by:

$$\gamma = \cos^{-1}\left(\frac{h}{rv}\right)$$

converted to degrees:

$$\gamma_{\text{deg}} = \frac{\gamma \cdot 180}{\pi}$$

Evaluating this with the given values yields a current flight path angle of: $\gamma = \mathbf{33.304}°$.

Part (d)

The semi-major axis is:

$$a = -\frac{\mu_{\text{earth}}}{2\epsilon}$$

The eccentric anomaly is calculated using:

$$E = \cos^{-1}\left(\frac{1}{e}\left(1 - \frac{r}{a}\right)\right)$$

converted to degrees:

$$E_{\text{deg}} = \frac{E \cdot 180}{\pi}$$

Evaluating this with the given values yields an eccentric anomaly of: $\mathbf{E} = \mathbf{37.323}°$.

Part (e)

The mean anomaly is determined through Kepler's Equation:

$$M = E - e\sin E$$

converted to degrees:

$$M_{\text{deg}} = \frac{M \cdot 180}{\pi}$$

Evaluating this with the given values yields a mean anomaly of: $\mathbf{M} = \mathbf{11.794}°$.

Problem 4.

The orbital elements for the Hubble Space Telescope are given as:

$$a = 6897 \quad \text{km} \quad \text{(semi-major axis)}$$

$$e = 0.0002422 \quad \text{(eccentricity)}$$

$$i = 28.4666^\circ \quad \text{(inclination)}$$

$$\Omega = 205.7537^\circ \quad \text{(Right Ascension of the Ascending Node, RAAN)}$$

$$\omega = 318.2020^\circ \quad \text{(Argument of Periapsis)}$$

$$\nu = 41.8386^\circ \quad \text{(True Anomaly)}$$

First, the radial distance is computed using:

$$r = \frac{a(1-e^2)}{1+e\cos\nu}$$

In the Perifocal Coordinate System (PQW), the position vector is:

$$\mathbf{r}_{PQW} = \begin{bmatrix} r\cos\nu \\ r\sin\nu \\ 0 \end{bmatrix}$$

Next, the mean motion is given by:

$$n = \sqrt{\frac{\mu_{\text{earth}}}{a^3}}$$

where $\mu_{\text{earth}} = 3.986 \times 10^5 \text{ km}^3/\text{s}^2$.
Next, the eccentric anomaly is computed using:

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\nu}{2}\right)$$

Then, the velocity vector in the PQW frame is computed as:

$$\mathbf{v}_{PQW} = \frac{an}{1-e\cos E}\begin{bmatrix} -\sin E \\ \sqrt{1-e^2}\cos E \\ 0 \end{bmatrix}$$

The transformation matrix from PQW to ECI coordinates is calculated by applying three successive rotations:

$$R_{\text{PQW}\to\text{ECI}} = R_z(-\Omega)R_x(-i)R_z(-\omega)$$

Expanding the matrices step by step:

$$R_z(-\Omega) = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x(-i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

$$R_z(-\omega) = \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying these matrices together:

$$R_{\text{PQW}\to\text{IJK}} = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Performing the matrix multiplication step-by-step:
First two matrices:

$$\begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega\cos i & \sin\Omega\sin i \\ -\sin\Omega & \cos\Omega\cos i & \cos\Omega\sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

Now multiplying with $R_z(-\omega)$:

$$\begin{bmatrix} \cos\Omega & \sin\Omega\cos i & \sin\Omega\sin i \\ -\sin\Omega & \cos\Omega\cos i & \cos\Omega\sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The resulting matrix is:

$$R_{\text{PQW}\to\text{ECI}} = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\cos i\sin\omega & \cos\Omega\sin\omega + \sin\Omega\cos i\cos\omega & \sin\Omega\sin i \\ -\sin\Omega\cos\omega - \cos\Omega\cos i\sin\omega & -\sin\Omega\sin\omega + \cos\Omega\cos i\cos\omega & \cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & \cos i \end{bmatrix}$$

The final position and velocity vectors in the ECI frame are obtained using:

$$\mathbf{r}_{ECI} = R_{PQW\to ECI}\mathbf{r}_{PQW}$$

$$\mathbf{v}_{ECI} = R_{PQW\to ECI}\mathbf{v}_{PQW}$$

Evaluating these with the given values, yields:

$$\mathbf{r}_{ECI} = \begin{bmatrix} -6209.9 & -3000.1 & 2.3 \end{bmatrix} \quad \text{(km)}$$

$$\mathbf{v}_{ECI} = \begin{bmatrix} -2.9081 & -6.0185 & 3.6242 \end{bmatrix} \quad \text{(km/s)}$$

Appendix 1

```
%% Space Mechanics Homework 2
% Tycho Bogdanowitsch
clc; clear;
% Constants
R_earth = 6371; % km
DU_earth = R_earth; % km
mu_earth = 3.986e5; % km^3/s^2

%% Problem 2
close all;
r_p = 3.0*DU_earth; % km

% Part i (ellipse)
nu = 0:1:360; % degrees, true anomaly
e = 0.7;

a = r_p/(1-e);
p = a*(1-e^2);
r_ECI = p ./(1+e*cosd(nu));
x_ellipse = r_ECI.*cosd(nu);
y_ellipse = r_ECI.*sind(nu);

figure(1)
hold on
scatter(x_ellipse,y_ellipse,10,'blue','filled');

plot([0, r_p], [0, 0], 'r-', 'LineWidth', 2);
text(r_p/2, 0.1, sprintf('r_p = %.2f km ', r_p), '
   HorizontalAlignment', 'right', 'VerticalAlignment', 'top','
   FontSize', 10, 'Color', 'r');

plot([-a*e, -(a+a*e)], [0, 0], 'g--', 'LineWidth', 2);
text(-(a+a*e)/1.5, -0.1, sprintf('a = %.2f km ', a), '
   HorizontalAlignment', 'left', 'VerticalAlignment', 'top', '
   FontSize', 10, 'Color', 'g');

plot([0, 0], [0, p], 'm-.', 'LineWidth', 2);
text(0.1, p/2, sprintf('p = %.2f km ', p), 'HorizontalAlignment
   ', 'right', 'VerticalAlignment', 'top','FontSize', 10, '
   Color', 'm');

plot(0, 0, 'bo', 'MarkerSize', 5, 'MarkerFaceColor', 'g');
```

```
text(0, 0, 'Central Body', 'HorizontalAlignment', 'right', '
   VerticalAlignment', 'bottom', 'FontSize', 10);
title('Shape of orbit with eccentricity of 0.7')
xlabel('X Position (km)')
ylabel('Y Position (km)')
grid on
axis equal
hold off

% Part ii (circle)
nu = 0:1:360; % degrees, true anomaly
e = 0;

a = r_p/(1-e);
p = a*(1-e^2);
r_ECI = p ./(1+e*cosd(nu));
x_circle = r_ECI.*cosd(nu);
y_circle = r_ECI.*sind(nu);

figure(2)
hold on
scatter(x_circle,y_circle,10,'blue','filled');

plot([0, r_p], [0, 0], 'r-', 'LineWidth', 2);
text(r_p/2, 0.1, sprintf('r_p = %.2f km ', r_p), '
   HorizontalAlignment', 'right', 'VerticalAlignment', 'top','
   FontSize', 10, 'Color', 'r');

plot([-a*e, -(a+a*e)], [0, 0], 'g--', 'LineWidth', 2);
text(-(a+a*e)/1.5, -0.1, sprintf('a = %.2f km ', a), '
   HorizontalAlignment', 'left', 'VerticalAlignment', 'top', '
   FontSize', 10, 'Color', 'g');

plot([0, 0], [0, p], 'm-.', 'LineWidth', 2);
text(0.1, p/2, sprintf('p = %.2f km ', p), 'HorizontalAlignment
   ', 'left', 'VerticalAlignment', 'top','FontSize', 10, 'Color
   ', 'm');

plot(0, 0, 'bo', 'MarkerSize', 5, 'MarkerFaceColor', 'g');
text(0, 0, 'Central Body', 'HorizontalAlignment', 'right', '
   VerticalAlignment', 'bottom', 'FontSize', 10);
title('Shape of orbit with eccentricity of 0.0')
xlabel('X Position (km)')
ylabel('Y Position (km)')
grid on
```

```
axis equal
hold off

% Part iii (hyperbola)
e = 1.6;

delta = rad2deg(2*asin(1/e));
valid_high = 90 + delta/2;
valid_low = 360 - 90 - delta/2;

nu_1 = 0:1:valid_high-1;
nu_2 = valid_low+1:1:360;
nu = [nu_1 nu_2]; % degrees, true anomaly

a = r_p/(1-e);
p = a*(1-e^2);
r_ECI = p ./(1+e*cosd(nu));
x_hyperbola = r_ECI.*cosd(nu);
y_hyperbola = r_ECI.*sind(nu);

x_asymptote = linspace(-1.5e5, 1.5e5, 1000);
slope_asymptote = sqrt(e^2 - 1);

y_asymptote_positive = slope_asymptote * x_asymptote - (
   slope_asymptote*(r_p-a));
y_asymptote_negative = -slope_asymptote * x_asymptote + (
   slope_asymptote*(r_p-a));

figure(3)
hold on
scatter(x_hyperbola,y_hyperbola,10,'blue','filled');

plot(x_asymptote, y_asymptote_positive, 'k--', 'LineWidth',
   1.5, 'DisplayName', 'Asymptote (+)');
plot(x_asymptote, y_asymptote_negative, 'k--', 'LineWidth',
   1.5, 'DisplayName', 'Asymptote (-)');

plot([0, r_p], [0, 0], 'r-', 'LineWidth', 2);
text(r_p/2, 0.1, sprintf('r_p = %.2f km ', r_p), '
   HorizontalAlignment', 'right', 'VerticalAlignment', 'top','
   FontSize', 10, 'Color', 'r');

plot([r_p,r_p+(-a)], [0, 0], 'g--', 'LineWidth', 2);
text(-(a+a*e)/1.5, -0.1, sprintf('a = %.2f km ', -a), '
   HorizontalAlignment', 'left', 'VerticalAlignment', 'top', '
```

```
    FontSize', 10, 'Color', 'g');

plot([0, 0], [0, p], 'm-.', 'LineWidth', 2);
text(0.1, p/2, sprintf('p = %.2f km ', p), 'HorizontalAlignment
    ', 'right', 'VerticalAlignment', 'top','FontSize', 10, '
    Color', 'm');

plot(0, 0, 'bo', 'MarkerSize', 5, 'MarkerFaceColor', 'g');
text(0, 0, 'Central Body', 'HorizontalAlignment', 'right', '
    VerticalAlignment', 'bottom', 'FontSize', 10);
title('Shape of orbit with eccentricity of 1.6')
xlabel('X Position (km)')
ylabel('Y Position (km)')
axis equal
xlim([-2e5 2e5])
grid on

hold off

% Part iv (parabola)
nu_1 = 0:1:179;
nu_2 = 181:1:360;
nu = [nu_1 nu_2]; % degrees, true anomaly
nu = 0:1:360;
e = 1;

a = r_p/(1-e);
p = 2*r_p;
r_ECI = p ./(1+e*cosd(nu));
x_parabola = r_ECI.*cosd(nu);
y_parabola = r_ECI.*sind(nu);

figure(4)
hold on
scatter(x_parabola,y_parabola,10,'blue','filled');

plot([0, r_p], [0, 0], 'r-', 'LineWidth', 2);
text(r_p/2, 0.1, sprintf('r_p = %.2f km ', r_p), '
    HorizontalAlignment', 'right', 'VerticalAlignment', 'top','
    FontSize', 10, 'Color', 'r');

%plot([-a*e, -(a+a*e)], [0, 0], 'g--', 'LineWidth', 2);
%text(-(a+a*e)/1.5, -0.1, sprintf('a = %.2f km ', a), '
    HorizontalAlignment', 'left', 'VerticalAlignment', 'top', '
    FontSize', 10, 'Color', 'g');
```

```
plot([0, 0], [0, p], 'm-.', 'LineWidth', 2);
text(0.1, p/2, sprintf('p = %.2f km ', p), 'HorizontalAlignment
    ', 'left', 'VerticalAlignment', 'top','FontSize', 10, 'Color
    ', 'm');

plot(0, 0, 'bo', 'MarkerSize', 5, 'MarkerFaceColor', 'g');
text(0, 0, 'Central Body', 'HorizontalAlignment', 'right', '
    VerticalAlignment', 'bottom', 'FontSize', 10);
title('Shape of orbit with eccentricity of 1.0')
xlabel('X Position (km)')
ylabel('Y Position (km)')
xlim([-3e5 3e5])
grid on
axis equal
hold off

figure(5);
hold on;
scatter(x_ellipse, y_ellipse, 10, 'b', 'filled', 'DisplayName',
     'Ellipse: e = 0.7');
scatter(x_circle, y_circle, 10, 'g', 'filled', 'DisplayName', '
    Circle: e = 0');
scatter(x_hyperbola, y_hyperbola, 10, 'r', 'filled', '
    DisplayName', 'Hyperbola: e = 1.6');
scatter(x_parabola, y_parabola, 10, 'm', 'filled', 'DisplayName
    ', 'Parabola: e = 1.0');
scatter(0, 0, 50, 'k', 'filled', 'DisplayName', 'Central Body')
    ;
xlabel('X Position (km)');
ylabel('Y Position (km)');
title('Overlay of All Orbit Shapes');
legend show;
axis equal;
xlim([-3e5 3e5])
grid on;
hold off;

%% Problem 3
r_ECI = [-8050 1900 2600]; % km
v_ECi = [-7 -2.8 -4]; % km/s
r_mag = norm(r_ECI); % km
v_mag = norm(v_ECi); % km/s

% Part i
```

```
epsilon = (v_mag^2/2) - (mu_earth/r_mag); % km^2/s^2
h = cross(r_ECI,v_ECi); % km^2/s
h_mag = norm(h); % km^2/s
e = sqrt(1 + (2*h_mag^2*epsilon/mu_earth^2));

fprintf('The Eccentricity is: %.3f \n', e);

% Part ii
p = h_mag^2/mu_earth; % km
nu = rad2deg(acos((1/e)*((p/r_mag) - 1))); % deg

fprintf('The True Anomaly is: %.3f deg \n', nu);

% Part iii
fpa = rad2deg(acos(h_mag/(r_mag*v_mag))); % deg

fprintf('The Flight Path Angle is: %.3f deg \n', fpa);

% Part iv
a = -mu_earth/(2*epsilon); % km
E = rad2deg(acos((1/e)*(1-(r_mag/a)))); % deg

fprintf('The Eccentric Anomaly is: %.3f deg \n', E);

% Part v
E_rad = deg2rad(E);
M = rad2deg(E_rad - e*sin(E_rad)); % deg

fprintf('The Mean Anomaly is: %.3f deg \n', M);

%% Problem 4
clear;
a = 6897; % km
e = 0.0002422;
i = 28.4666; % deg
RAAN = 205.7537; % deg
omega = 318.2020; % deg
nu = 41.8386; % deg

[r_result,v_result] = OE2ECI(a,e,i,RAAN,omega,nu);

disp('Hubble Position Vector (km):')
disp(r_result)
disp('Hubble Velocity Vector (km/s):')
disp(v_result)
```

```
function [r_ECI,v_ECI] = OE2ECI(a,e,i,RAAN,omega,nu)
    mu_earth = 3.986e5; % km^3/s^2
    i = deg2rad(i);
    RAAN = deg2rad(RAAN);
    omega = deg2rad(omega);
    nu = deg2rad(nu);
    %u = omega + nu;
    r = a*(1-e^2)/(1+e*cos(nu));
    r_PQW = [r * cos(nu); r * sin(nu); 0];
    %r_IJK = r * [cos(u)*cos(RAAN)+sin(u)*cos(i)*sin(RAAN);
    %     cos(u)*sin(RAAN)+sin(u)*cos(i)*cos(RAAN);
    %     sin(u)*sin(i)];

    n = sqrt(mu_earth/a^3);
    E = 2*atan2(sqrt((1-e)/(1+e))*tan(nu/2),1);
    R_PQR_IJK = [cos(RAAN)*cos(omega)-sin(RAAN)*cos(i)*sin(
       omega),...
        -cos(RAAN)*sin(omega)-sin(RAAN)*cos(i)*cos(omega),...
        sin(RAAN)*sin(i);
        sin(RAAN)*cos(omega)+cos(RAAN)*cos(i)*sin(omega),...
        -sin(RAAN)*sin(omega)+cos(RAAN)*cos(i)*cos(omega),...
        -cos(RAAN)*sin(i);
        sin(i)*sin(omega), sin(i)*cos(omega), cos(i)];
    v_PQW = (a*n/(1-e*cos(E)))*[-sin(E); sqrt(1-e^2)*cos(E);
       0];
    %v_IJK = R_PQR_IJK*v_PQW;

    r_ECI = R_PQR_IJK*r_PQW;
    v_ECI = R_PQR_IJK*v_PQW;

end
```