Homework 1

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Problem 1.

Part (a)

True: The assumption that the mass of the orbiting body is much less than the mass of the central body allows for the usage of μ which only depends on the central body.

Part (b)

False: Geostationary orbits have an eccentricity of 0 as they are a special type of geosynchronous orbit that appears stationary over a fixed point on Earth.

Part (c)

False: The third body must be very far from the two primary bodies for it to be neglected, which is because this leads to the position vectors to that third body align more the farther away the third body is.

Part (d)

True: Angular momentum is defined by the following equation: $h = \sqrt{\mu p}$. The semi-parameter determines the orbit shape, which determines angular momentum.

Part (e)

True: The flight path angle approaches +/- 90 deg far away from the central body because the velocity vector becomes aligned with the radial direction.

Part (f)

False: Gravity is a conservative force because it can be represented as a potential field and work done along a closed path is zero.

Problem 2.

Part (a)

The distance from the Earth's center to the satellite is:

$$r_{\text{satellite-Earth}} = R_{\text{Earth}} + h,$$

where R_{Earth} is the Earth's radius and h is the satellite's altitude.
As discussed in lecture, the acceleration of the satellite due to the Earth and third bodies relative to the Earth is given by:

$$\ddot{\vec{r}}_{12} = -G(m_1 + m_2)\frac{\vec{r}_{12}}{r_{12}^3} - \sum_{k=3}^{N} Gm_k \left(\frac{\vec{r}_{k2}}{r_{k2}^3} - \frac{\vec{r}_{k1}}{r_{k1}^3}\right)$$

where $j = 2$ represents the satellite, $j = 1$ represents the Earth, and $r_{12} = r_{\text{satellite-Earth}}$. As defined by the problem statement, $r_{k2} = a_j - a_e - r_{\text{satellite-Earth}}$, where a_j and a_e are the semi-major axes of the perturbing body and Earth with respect to the Sun, respectively. Also, $r_{k1} = a_j - a_e$, which represents the acceleration on Earth from the third body.

By ignoring mass of the satellite,m_2, (since it is much less massive than Earth) and considering only the magnitudes (since directions are not accounted for in this problem), the acceleration simplifies to:

$$\ddot{\vec{r}}_{12} = \frac{Gm_1}{r_{12}^2} + \sum_{k=3}^{N} Gm_k \left(\frac{1}{r_{k2}^2} - \frac{1}{r_{k1}^2}\right)$$

Here, the term $\frac{Gm_1}{r_{12}^2}$ represents the gravitational acceleration due to Earth. And the summation term represents the perturbative effects of other celestial bodies.

Thus, Earth's gravitational acceleration on the satellite can be found using:

$$a_{\text{Earth}} = \frac{G \cdot m_{\text{Earth}}}{r_{\text{satellite}}^2}$$

The perturbation acceleration due to the Sun can be found using:

$$a_{\text{Sun}} = G \cdot m_{\text{Sun}} \left(\frac{1}{(r_{\text{Sun-Earth}} - r_{\text{satellite}})^2} - \frac{1}{r_{\text{Sun-Earth}}^2}\right)$$

And, the perturbation acceleration due to the Moon can be found using:

$$a_{\text{Moon}} = G \cdot m_{\text{Moon}} \left(\frac{1}{(r_{\text{Moon-Earth}} - r_{\text{satellite}})^2} - \frac{1}{r_{\text{Moon-Earth}}^2}\right)$$

For all other third bodies, the perturbation acceleration can be calculated as:

$$a_{\text{perturb}} = \left| G \cdot m_{\text{body}} \left(\frac{1}{(r_{\text{body}} - r_{\text{Sun-Earth}} - r_{\text{satellite}})^2} - \frac{1}{(r_{\text{body}} - r_{\text{Sun-Earth}})^2} \right) \right|$$

Evaluating all of these, we find the perturbation accelerations in km/s^2 to be:

Body	**LEO (600 km)**	**MEO (12,000 km)**	**GEO (35,786 km)**
Earth	0.0081943	0.0011799	0.00022406
Sun	5.5279×10^{-10}	1.457×10^{-9}	3.3442×10^{-9}
Moon	1.2408×10^{-9}	3.4249×10^{-9}	8.7058×10^{-9}
Mercury	4.0378×10^{-16}	1.0639×10^{-15}	2.4405×10^{-15}
Venus	6.1606×10^{-14}	1.6229×10^{-13}	3.7209×10^{-13}
Mars	1.2684×10^{-15}	3.3434×10^{-15}	7.6759×10^{-15}
Jupiter	7.1159×10^{-15}	1.8753×10^{-14}	4.3037×10^{-14}
Saturn	2.5331×10^{-16}	6.6757×10^{-16}	1.532×10^{-15}
Uranus	4.0059×10^{-18}	1.0557×10^{-17}	2.4226×10^{-17}
Neptune	1.1545×10^{-18}	3.0425×10^{-18}	6.9818×10^{-18}

Part (b)

The total perturbation acceleration at each altitude is calculated as:

$$a_{\text{total}} = a_{\text{Sun}} + a_{\text{Moon}} + \sum_i a_{\text{Planet},i}$$

where the summation term does not include Earth.

Evaluating this, we find the total perturbation acceleration at LEO and GEO to be $\mathbf{1.794 \times 10^{-9}}$ **km/s^2** and $\mathbf{1.205 \times 10^{-8}}$ **km/s^2**, respectively. This means that the total perturbation acceleration is **6.718** times larger at GEO than at LEO. This is because the satellite is closer to the perturbing third body in GEO than it is in LEO.

Problem 3.

Part (a)

First, we can calculate the apogee distance ($r_{\text{NS,apogee}}$) and semi-major axis (a_{NS}) of the New Shepard's trajectory:

$$r_{\text{NS,apogee}} = R_{\text{Earth}} + h_{\text{NS,apogee}}$$
$$a_{\text{NS}} = \frac{r_{\text{NS,apogee}}}{1 + e_{\text{NS}}}$$

where:

$$h_{\text{NS,apogee}} = 100\,\text{km}$$
$$e_{\text{NS}} = 0.87$$

Next, we can calculate the velocity of New Shepard at apogee ($v_{\text{NS,apogee}}$) using the vis-viva equation:

$$v_{\text{NS,apogee}} = \sqrt{\mu_{\text{Earth}} \left(\frac{2}{r_{\text{NS,apogee}}} - \frac{1}{a_{\text{NS}}} \right)}$$

Evaluating this, we find that the velocity of New Shepard at its apogee is **2.830 km/s**.

For comparison, the velocity required for a circular orbit at this altitude ($v_{\text{NS,circular}}$) is given by:

$$v_{\text{NS,circular}} = \sqrt{\frac{\mu_{\text{Earth}}}{r_{\text{NS,apogee}}}}$$

Evaluating this, we find that at 100 km, New Shepard must have a velocity of at least **7.848 km/s** for a circular orbit. Thus, the New Shepard velocity is not sufficient for a circular orbit around the Earth at this altitude.

Part (b)

The Falcon 9 velocity is given in km/hr. First, we can convert it to km/s:

$$v_{\text{F9}} = \frac{9100}{3600}\,\text{km/s}$$

Evaluating this, we find that the Falcon 9 velocity is **2.528 km/s**, which is also not sufficient for a circular orbit around the Earth at 100 km.

Part (c)

We can calculate the specific mechanical energy (ϵ) using the formula:

$$\epsilon = \frac{v^2}{2} - \frac{\mu_{\text{Earth}}}{r}$$

For New Shepard:

$$\epsilon_{\text{NS}} = \frac{v_{\text{NS,apogee}}^2}{2} - \frac{\mu_{\text{Earth}}}{r_{\text{NS,apogee}}}$$

For Falcon 9:

$$\epsilon_{\text{F9}} = \frac{v_{\text{F9}}^2}{2} - \frac{\mu_{\text{Earth}}}{r_{\text{F9}}}$$

Evaluating both of these yields that the New Shepard specific mechanical energy is **-57.59 km²/s²** and the Falcon 9 specific mechanical energy is **-58.40 km²/s²**. Thus, the New Shepard specific mechanical energy is greater.

Part (d)

We can calculate the escape velocity as:

$$v_{\text{escape}} = \sqrt{\frac{2\mu_{\text{Earth}}}{r}}$$

For an altitude of 100 km:

$$v_{0,100} = \sqrt{\frac{2\mu_{\text{Earth}}}{R_{\text{Earth}} + 100}}$$

For an altitude of 1000 km:

$$v_{0,1000} = \sqrt{\frac{2\mu_{\text{Earth}}}{R_{\text{Earth}} + 1000}}$$

Evaluating these, we find that the escape velocity at an altitude of 100 km is **11.099 km/s** and that at an altitude of 1000 km, it is **10.400 km/s**. The escape velocity decreases as the altitude increases because the gravitational influence of Earth reduces at greater distances.

Problem 4.

Part (a)

The orbital eccentricity (e) can be calculated using the formula:

$$e = \frac{r_a - r_p}{r_a + r_p}$$

For Earth:

$$e_{\text{Earth}} = \frac{r_{a,\text{Earth}} - r_{p,\text{Earth}}}{r_{a,\text{Earth}} + r_{p,\text{Earth}}}$$

For comet 67P:

$$e_{\text{67P}} = \frac{r_{a,\text{67P}} - r_{p,\text{67P}}}{r_{a,\text{67P}} + r_{p,\text{67P}}}$$

Evaluating these, we find that the eccentricity of Earth's orbit is **0.017** and that the eccentricity of 67P's orbit is **0.659**.

Part (b)

The semi-major axis (a) can calculated as:

$$a = \frac{r_a}{1 + e}$$

For Earth:

$$a_{\text{Earth}} = \frac{r_{a,\text{Earth}}}{1 + e_{\text{Earth}}}$$

For comet 67P:

$$a_{\text{67P}} = \frac{r_{a,\text{67P}}}{1 + e_{\text{67P}}}$$

The maximum and minimum velocities along the orbit are calculated using the vis-viva equation:

$$v_{\max} = \sqrt{\mu_{\text{Sun}} \left(\frac{2}{r_p} - \frac{1}{a} \right)}$$

$$v_{\min} = \sqrt{\mu_{\text{Sun}} \left(\frac{2}{r_a} - \frac{1}{a} \right)}$$

Evaluating these for Earth, we find that the minimum and maximum inertial velocities are **29.290 km/s** and **30.285 km/s** respectively. Evaluating these for 67P, we find that the minimum and maximum inertial velocities are **7.709 km/s** and **37.564 km/s** respectively.

Part (c)

The orbital period (T) is given by:

$$T = 2\pi\sqrt{\frac{a^3}{\mu_{\text{Sun}}}}$$

To convert the period from seconds to mean solar days:

$$T_{\text{days}} = \frac{T}{86400}$$

Evaluating these leads to an orbital period of **365.274 mean solar days** for Earth and **1958.168 mean solar days** for 67P.

Appendix 1

```
%% Space Mechanics Homework 1
% Tycho Bogdanowitsch
clc; clear;
%% Constants
G = 6.67e-20; % km^3/kg/s^2
AU = 149597870.7; % km
m_sun = 1.99e30; % kg
m_moon = 7.35e22; % kg
m_planets = [0.33e24,4.87e24,0.642e24,...
    1898e24,568e24,86.8e24,102e24]; % kg
m_earth = 5.97e24; % kg
r_sun_earth = 1*AU; % km
r_moon_earth = 3.84e5; % km
r_sun_planets = AU*[0.39, 0.72, 1.52, 5.2, 9.54, 19.19, 30.06];
    % km

mu_earth = 3.986e5; % km^3/s^2
mu_sun = 1.327e11; % km^3/s^2
R_earth = 6371; % km

%% Problem 2
disp('')
disp('Problem 2')
% Part 1
altitudes = [600,12000,35786]; % km

r_satellite_earth = R_earth + altitudes; % km

calc_accel = @(m_body, r_body) ...
    abs(G * m_body * ((1 ./(r_body - r_sun_earth -
       r_satellite_earth).^2) - ...
    (1 ./(r_body - r_sun_earth).^2)));

acceleration_earth = G * m_earth ./ (r_satellite_earth).^2;
acceleration_sun = G * m_sun * ((1 ./(r_sun_earth -
   r_satellite_earth).^2) - ...
    (1 ./(r_sun_earth).^2));
acceleration_moon = G * m_moon * ((1 ./(r_moon_earth -
   r_satellite_earth).^2) - ...
    (1 ./(r_moon_earth).^2));
acceleration_planets = zeros(length(m_planets),length(altitudes
   ));
```

```
for i = 1:length(m_planets)
    acceleration_planets(i,:) = calc_accel(m_planets(i),
        r_sun_planets(i));
end

perturbations = [acceleration_earth;acceleration_sun;
   acceleration_moon;acceleration_planets];
names = [{'Earth'},{'Sun'},{'Moon'},{'Mercury'},{'Venus'},{'
   Mars'},...
    {'Jupiter'},{'Saturn'},{'Uranus'},{'Neptune'}];
perturbations_table = array2table(perturbations,...
    'VariableNames',{'LEO (600 km)','MEO (12,000 km)','GEO
        (35,786 km)'},...
    'RowNames',names);

disp('Perturbation Accelerations (km/s^2):');
disp(perturbations_table)

% Part 2
total_accel_LEO = acceleration_sun(1) + acceleration_moon(1)...
    + sum(acceleration_planets(:,1));
total_accel_GEO = acceleration_sun(3) + acceleration_moon(3)...
    + sum(acceleration_planets(:,3));

fprintf(['Total perturbation acceleration at 600 km altitude (
   LEO): ' ...
    '%.3e km/s^2\n'], total_accel_LEO);
fprintf(['Total perturbation acceleration at 35,786 km altitude
    (GEO): ' ...
    '%.3e km/s^2\n'], total_accel_GEO);
fprintf(['The total perturbation acceleration at GEO is %.3f
   times larger ' ...
    'than at LEO\n'], total_accel_GEO/total_accel_LEO);

%% Problem 3
disp('')
disp('Problem 3')
% Part 1
h_NS_apogee = 100; % km
e_NS = 0.87;

r_NS_apogee = R_earth + h_NS_apogee;
a_NS = r_NS_apogee / (1 + e_NS);
```

```
v_NS_apogee = sqrt(mu_earth * (2 / r_NS_apogee - 1 / a_NS));

v_NS_circular = sqrt(mu_earth / r_NS_apogee);

fprintf('New Shepard Velocity at Apogee: %.3f km/s\n',
   v_NS_apogee);
fprintf('Required Circular Orbit Velocity: %.3f km/s\n',
   v_NS_circular);
fprintf(['The New Shepard velocity is not sufficient for a
   circular ' ...
    'orbit around the Earth at an altitude of 100km\n']);

% Part 2
h_F9 = 100; % km
v_F9 = 9100; % km/hr

r_F9 = R_earth + h_F9;
v_F9_kms = v_F9/3600; % km/s

fprintf('Falcon 9 Velocity at 100km Altitude: %.3f km/s\n',
   v_F9_kms);
fprintf(['The Falcon 9 velocity is not sufficient for a
   circular ' ...
    'orbit around the Earth at an altitude of 100km\n']);

% Part 3
epsilon_NS = ((v_NS_apogee)^2/2)-(mu_earth/r_NS_apogee); % km
   ^2/s^2
epsilon_F9 = ((v_F9_kms)^2/2)-(mu_earth/r_F9); % km^2/s^2

fprintf('New Shepard specific mechanical energy: %.3e km^2/s^2\
   n', epsilon_NS);
fprintf('Falcon 9 specific mechanical energy: %.3e km^2/s^2\n',
    epsilon_F9);
fprintf('The New Shepard specific mechanical energy is greater\
   n')

% Part 4
r_0_100 = R_earth + 100; % km
r_0_1000 = R_earth + 1000; % km

v_0_100 = sqrt(2*mu_earth/r_0_100); % km/s
v_0_1000 = sqrt(2*mu_earth/r_0_1000); % km/s
```

```
fprintf('Escape velocity at an altitude of 100km: %.3f km/s\n',
    v_0_100);
fprintf('Escape velocity at an altitude of 1000km: %.3f km/s\n'
   , v_0_1000);
fprintf(['As altitude increases, escape velocity decreases
   because a higher ' ...
    'altitude reduces Earths gravitational influence \n']);

%% Problem 4
disp('')
disp('Problem 4')
r_a_earth = 1.0167*AU; % km
r_p_earth = 0.9833*AU; % km

r_a_67P = 5.0829*AU; % km
r_p_67P = 1.0432*AU; % km

% Part 1
e_earth = (r_a_earth - r_p_earth) / (r_a_earth + r_p_earth);
e_67P = (r_a_67P - r_p_67P) / (r_a_67P + r_p_67P);
fprintf('Eccentricity of Earth orbit: %.3f \n', e_earth);
fprintf('Eccentricity of 67P orbit: %.3f \n', e_67P);

% Part 2
a_earth = r_a_earth / (1+e_earth); % km
a_67P = r_a_67P / (1+e_67P); % km

v_earth_max = sqrt(mu_sun * (2 / r_p_earth - 1 / a_earth)); %
   km/s
v_earth_min = sqrt(mu_sun * (2 / r_a_earth - 1 / a_earth)); %
   km/s
v_67P_max = sqrt(mu_sun * (2 / r_p_67P - 1 / a_67P)); % km/s
v_67P_min = sqrt(mu_sun * (2 / r_a_67P - 1 / a_67P)); % km/s
fprintf('Maximum inertial velocity of the Earth: %.3f km/s\n',
   v_earth_max);
fprintf('Minimum inertial velocity of the Earth: %.3f km/s\n',
   v_earth_min);
fprintf('Maximum inertial velocity of 67P: %.3f km/s\n',
   v_67P_max);
fprintf('Minimum inertial velocity of 67P: %.3f km/s\n',
   v_67P_min);

% Part 3
T_earth = 2*pi*sqrt(a_earth^3/mu_sun)/86400; % sec --> mean
   solar days
```

```
T_67P = 2*pi*sqrt(a_67P^3/mu_sun)/86400; % sec --> mean solar
   days
fprintf('Orbital period of the Earth: %.3f mean solar days\n',
   T_earth);
fprintf('Orbital period of 67P: %.3f mean solar days\n', T_67P)
   ;
```