AA279A, Winter 2025
Homework 4

Due: Wednesday, February 12th, 2025 @ 3:00 PM PST

Notes:

Submission Instructions

Please submit your solutions as a PDF file on Gradescope (there is a link to Gradescope on Canvas). We require your document to be typeset using LaTeX, Microsoft Word, or another word processor. We have provided a LaTeX template on Canvas that can be used to typeset your solutions. For problems that require programming, please include in your submission a copy of your code (with comments) and any figures that you are asked to plot. Include your code as text in your PDF, please do not submit extra files.

Topics

Lectures 6 and 7: Time and Reference Systems. Remote Sensing and Ground Tracks.

Problem 1. True or False

Mark each of the following statements as either TRUE or FALSE, then provide a brief explanation for your answer. Note that if a statement is not always TRUE, then it should be considered FALSE generally.

(a) Given a set of orbital elements associated with an elliptical, inclined orbit, the mapping to ECI position and velocity is unique.

(b) In circular, equatorial orbits, the inclination of the orbit is undefined.

(c) Imagine a line extending from the center of the Earth through the Prime Meridian. Over the course of one solar day, this line subtends a larger angle than it does over the course of one sidereal day.

(d) If a satellite is in a geostationary orbit, its groundtrack will always look like a single point on the equator.

(e) The inclination of an orbit can be determined from the maximum latitude of the orbit's groundtrack.

(f) It is not possible to deduce the semi-major axis of a prograde orbit from its groundtrack.

Problem 2. Calculating Time

A satellite operator is planning an orbital maneuver for one of its satellites at epoch: February 1, 2024 12:00h UT1.

(a) Write a function in MATLAB to input Universal Time (UT1) in calendar form (e.g., in [MM,DD,YYYY] format) and output UT1 expressed in Modified Julian Days (MJD). Use the formula developed in Montenbruck, A.1.1, page 321 to convert from calendar format to MJD. When implementing Eq. A.6, use the following formula in MATLAB:

$$MJD = 365y - 679004 + \text{floor}(B) + \text{floor}(30.6001 * (m + 1)) + D$$

What is the Universal Time (UT1) at epoch expressed in MJD? See Satellite Orbits (Page 320) to validate your calendar date to MJD function.

Note: In in MJD tables such as those in the textbook, MJD is provided at day 0.0 of each month. To obtain the MJD of a given date and time, you must add the day and fractions of day to the tabulated value for the respective month and year. Example: MJD(2000 Jan. 1, 12h) = 51543 + 1.5 = 51544.5.

(b) Write a function in MATLAB to input UT1 time in MJD format and output Greenwich Mean Sidereal Time (GMST) expressed in radians. What is the GMST at epoch **expressed in radians?** Report the number by wrapping it to 2π.

Hint: look into the wrapTo2Pi function in MATLAB.

(c) Write a function in MATLAB to input GMST expressed in radians and output the rotation matrix from Celestial Reference Frame (CRF) to Terrestrial Reference Frame (TRF). Neglect nutation, precession, polar motion, and equation of equinoxes. What is the rotation matrix from CRF to TRF at epoch considering only the Earth's rotation about its axis?

Problem 3. International Space Simulation

With more than an acre of solar arrays to power itself, the International Space Station (ISS) is also the next brightest object in the night sky after the Moon. Its Keplerian orbital elements at $t_0 = 30.0$ days were:

$$\begin{aligned} a &= 6796.3 \text{ km} \\ \Omega &= 257.7630^\circ \\ e &= 0.00023 \\ \omega &= 195.9983^\circ \\ i &= 51.6412^\circ \\ M &= 240.3224^\circ \end{aligned}$$

(a) Which of the following properties are true for the ISS's orbit?

- Retrograde, prograde
- Circular, elliptical
- Equatorial, polar, inclined

(b) Use Simulink to simulate the orbit of the ISS in perifocal coordinates. Start at epoch $t_1 = 100.0$ days and run for exactly 1 day until $t_2 = 101.0$ days. You may re-use the Simulink model and functions created in the previous problem set.

(c) Extend your Simulink model so that it computes the trajectory of the orbit in the Earth-Centered Inertial (ECI) frame over the same time frame. In MATLAB, plot this trajectory from the point of view of an observer fixed in inertial space. Remember to label the axes on all your figures with units. Show an Earth-sized sphere for reference in your figure. See the code in Figure 1 for hints.

(d) Further extend your Simulink model to compute the trajectory of the ISS orbit as it would be seen by an observer on the Earth (ie. Earth-Centered Earth-Fixed frame). Assume that t_0 corresponds to February 1, 2024 at 00:00h UT1 and plot the orbit in ECEF coordinates over the time period from t_1 to t_2. As in part (c), include an Earth-sized sphere in your figure.

Note: You may neglect nutation, precession, polar motion, and equation of equinoxes. You only need to consider the rotation of the Earth when computing ECEF coordinates.

Remember to submit your Simulink block diagram, the MATLAB code, and the figure.

```
[xE, yE, zE] = ellipsoid(0, 0, 0, rE, rE, rE, 20);

figure
surface(xE, yE, zE, 'FaceColor', 'blue', 'EdgeColor', 'black');
axis equal;
view(3);
grid on;
```

Figure 1: Sample code for generating a plot of an Earth-sized sphere

Problem 4. Super Bowl Superstar Rendezvous

Taylor Swift has to get to Las Vegas in time for the Super Bowl, so she decides to travel by spacecraft. The spacecraft must pass directly over Allegiant Stadium ($\phi = 36.0909°, \lambda = -115.1833°$) in order to drop Taylor off. Through your simulation software, you find that the spacecraft reaches the following position in the ECEF frame:

$$\vec{r}_{ECEF} = \begin{bmatrix} -2195.7 \\ -4669.6 \\ 3761.5 \end{bmatrix} \text{km}$$

(a) Develop a function in MATLAB which will convert the provided ECEF position vector into a set of Geocentric (spherical Earth) coordinates. Is the spacecraft passing over the stadium? If not, how far away is it (in km) and in what direction?

(b) Using the approach described in class, create another function to determine the Geodetic (elliptical Earth) coordinates of the spacecraft. When accounting for the oblateness of the Earth, is the spacecraft passing over the stadium? If not, how far away is it (in km) and in what direction?

Note: If the spacecraft flies within +/- 0.001°, it can be considered a direct flyover. If the spacecraft does not fly over the stadium, you can approximate the distance using a circular arc length assuming a constant radius for the Earth. Take the Earth radius as 6378.1 km.

Feel free to validate your answer to part b) using MATLAB's built-in `geoc2geod` function, although you must write and submit your own function as well. Note that MATLAB's `ecef2lla` function **cannot** be used to validate your answer for part a), as it will produce a different result.