AA279A, Winter 2025
Homework 1

Due: Wednesday, January 22, 2025 @ 3:00 PM PST

Notes:

Submission Instructions

Please submit your solutions as a PDF file on Gradescope (there is a link to Gradescope on Canvas). We require your document to be typeset using LaTeX, Microsoft Word, or another word processor. We have provided a LaTeX template on Canvas that can be used to typeset your solutions. For problems that require programming, please include in your submission a copy of your code (with comments) and any figures that you are asked to plot. Include your code as text in your PDF, please do not submit extra files.

Topics

Lectures 1, 2, and 3: N-body problem, 2-body problem, orbital motion, conic sections, Keplerian orbital elements

Problem 1: True/False

For each of the following, determine if the statement is true or false. Provide a brief description explaining why. *Hint: If the statement is not **always** true or is only partially true, it is false.*

(i) The restricted 2-body problem assumes the mass of the orbiting body is much less than the mass of the central body.

(ii) Geostationary orbits can be represented as a conic section with high eccentricity.

(iii) The influence of third bodies on the motion of a spacecraft can generally be neglected if the third body is very close to the two primary bodies.

(iv) Angular momentum and semi-parameter completely determine each other.

(v) On a hyperbolic orbit, there is no apoapsis, and the flight path angle approaches $+90$ deg or -90 deg far away from the central body.

(vi) The gravity force field is non-conservative, i.e. its work done along a closed path is non-zero.

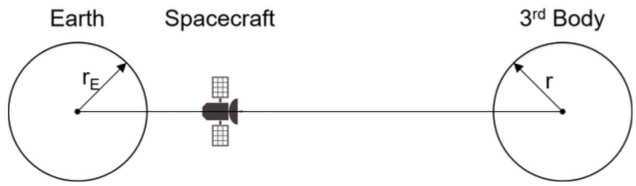

Figure 1: Collinear alignment of Earth, spacecraft, and 3rd body

Problem 2: N-Body Problem

A satellite operator claims that the combined perturbation to the orbit of their satellites relative to the Earth due to the gravitational pull of the Sun, the Moon, and the planets of the solar system is $\approx 10^{-9}\frac{km}{s^2}$. Verify the operator's claim as follows:

(i) Compute the individual perturbation accelerations due to the Sun, the Moon, and the eight planets of the solar system, including Earth, at orbital altitudes (see Figure 2) of 600 km (LEO), 12,000 km (MEO), and 35,786 km (GEO). Please list these accelerations, in units of $\frac{km}{s^2}$, in table form.

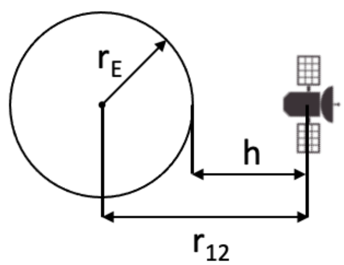

Figure 2: Orbital altitude, h, is defined as the difference between orbit radius, r_{12}, and the equatorial radius of the central body, r_E

For simplicity, you may assume that the distance between the Earth and the perturbing body is equal to the difference in the semi-major axes (relative to the Sun) of the orbits of the Earth and the perturbing planet. i.e.,

$$\vec{r}_{Earth-3rdBody} = \vec{r}_{Sun-3rdBody} - \vec{r}_{Sun-Earth} \tag{1}$$

You may also assume that the centers of mass of the Earth, the spacecraft, and the 3rd body are collinear, with the spacecraft in the middle (see Figure 1).

Hint: Feel free to use MATLAB to perform these computations. If you choose to do so, please don't forget to include your code in your submission.

(ii) Calculate the **total** perturbation acceleration (in $\frac{km}{s^2}$) due to the Sun, the Moon, and the planets of the solar system, excluding Earth, for a spacecraft at an altitude of 600 km. Compare this with the total perturbation acceleration for a geostationary orbit with an altitude of 35,786 km.

Hint: The total perturbation acceleration at a given altitude can be calculated by summing the magnitudes of the individual perturbation accelerations from the various bodies at that altitude.

Problem 3. Orbital Velocities

In recent years, both Blue Origin and SpaceX have successfully completed vertical take-offs and landings of their rockets (the New Shepard and the Falcon 9, respectively). Through this problem, we will compare these achievements from the perspective of orbital mechanics.

(i) Treat the trajectory of the New Shepard as an ellipse with an eccentricity, e, of 0.87 and its apogee at an altitude of 100 km. What is the velocity (in km/s) of the New Shepard at its apogee? Is this velocity sufficient for a circular orbit around the Earth at this altitude?

(ii) Treat the trajectory of the Falcon 9 as another ellipse. Assume the rocket travels at 9100 km/hr at an altitude of 100 km. Is this velocity sufficient for a circular orbit around the Earth at this altitude?

(iii) Compute the specific mechanical energy, ε, for both the Falcon 9 and New Shepard. Which is greater? Provide your answers in $\mathrm{km}^2/\mathrm{s}^2$.

(iv) What should be the velocity (relative to Earth) of a rocket at an altitude of 100 km if it were to be on an escape trajectory? What about a rocket at an altitude of 1000 km? Can you explain the effect of altitude on escape velocity?

Problem 4. Orbit Parameters

In 2014, the European Space Agency's Rosetta became the first spacecraft to orbit a comet. The Rosetta spacecraft was launched from the Guiana Space Centre in 2004 and performed three flybys of the Earth and one flyby of Mars on its way to the 67P/ ChuryumovGerasimenko (67P) comet.

Let us compare the vastly different orbits of the Earth and 67P around the Sun. The Earth's aphelion and perihelion distances are 1.0167AU and 0.9833AU, respectively. The aphelion and perihelion distances of 67P are 5.0829AU and 1.0432AU, respectively.

Note: 1 AU $= 149,597,870.7$ km

(i) Find the eccentricities of the orbits of the Earth and 67P.

(ii) What are the maximum and minimum inertial velocities (relative to the Sun) of the Earth and 67P? Provide your answers in km/s.

(iii) What are the orbital periods of the Earth and 67P? Provide your answers in mean solar days.