AA279A, Winter 2025
Homework 5

Due: Wednesday, February 26, 2025 @ 3:00 PM PST

Notes:

Submission Instructions

Please submit your solutions as a PDF file on Gradescope (there is a link to Gradescope on Canvas). We require your document to be typeset using LaTeX, Microsoft Word, or another word processor. We have provided a LaTeX template on Canvas that can be used to typeset your solutions. For problems that require programming, please include in your submission a copy of your code (with comments) and any figures that you are asked to plot. Include your code as text in your PDF, please do not submit extra files.

Topics

Lectures 7, 8, and 9: Remote Sensing and Ground Tracks. Orbit Design. Gauss' Variational Equations.

Problem 1. The Spy Orbit

During the Cold War, Molniya (Russian for lightning) Orbits were a popular choice for spy satellites for the Soviet Union and the USA. Some of these satellites are still operational as communications satellites. The position and velocity of one such satellite, Molniya 3-31, as expressed in the Earth-Centered Inertial frame on February 1, 2023 00:00h UTC (epoch t_0) were as follows:

$$\vec{r}_{IJK} = \begin{bmatrix} 3727.9 & -826.3 & -7693.2 \end{bmatrix} \text{km}$$
$$\vec{v}_{IJK} = \begin{bmatrix} 4.0256 & 7.8002 & -0.8610 \end{bmatrix} \text{km/s}$$

(a) Unlike in Problem Set 4 where the epoch was provided in the UT1 time standard, we must now account for the fact that our epoch is expressed in UTC time. Referring to the IERS Bulletin (`https://www.iers.org/IERS/EN/Publications/Bulletins/bulletins.html`), what is the value for dUT1 on February 1, 2023? Report your answer in milliseconds. *Hint: IERS Bulletin A gives precise values for dUT1*

(b) Use Simulink to simulate the orbit of Molniya 3-31 for exactly 24 hours starting from time t_0. Plot the orbit as it would appear to an Earth-fixed observer (ie. in the ECEF frame), along with tick marks at every hour. Include the 3D Earth sphere in your figure and don't forget to convert Simulink's clock output from UTC to UT1 when computing the Greenwich Mean Sidereal Time (GMST).

(c) Expand your Simulink model so that it generates the Geodetic coordinates of the orbit over time. Plot the satellite groundtrack and hourly tick marks over the 24 hour period on an Earth map that includes the outlines of the continents. Make sure you use an ellipsoidal Earth model when computing latitude and longitude.

(d) Above which continents does the Molniya 3-31 spend the most amount of time during the simulation? Why does this orbit represent good use of the satellite time on orbit?

Problem 2. Topocentric Coordinates

The Iridium constellation consists of 66 working satellites that provides voice and date coverage to satellite phones across the world. In 2017, 10 new Iridium satellites were placed into orbit by a SpaceX Falcon 9 rocket to maintain the constellation. One of the Iridium satellites, Iridium-100 has the following orbital elements at epoch, $t_0 = 59987.6458$ MJD :

$$a = 7155.81 \text{ km}$$
$$\Omega = 230.875^\circ$$
$$e = 0.0001971$$
$$\omega = 93.069^\circ$$
$$i = 86.403^\circ$$
$$M = 267.074^\circ$$

(a) In Simulink, compute the position of the Iridium-100 relative to an observer on the roof of Stanford's Durand building ($\phi = 37.426622^\circ, \lambda = -122.173355^\circ$). The position vector should be expressed in the East-North-Up (ENU) coordinates. Run your simulation for two hours from epoch using an appropriate time step. What are the ENU coordinates (in kilometers) of the satellite at the end of your simulation? You may assume that the latitude and longitude of the ground station are given in geocentric coordinates.

(b) Plot the following quantities versus the simulation time (in minutes)

(i) The distance between the satellite and the observer (in km)

(ii) The time rate of change of the range (in km/s)

(iii) Azimuth (in degrees)

(iv) Elevation (in degrees)

Only plot these for the interval when the satellite is visible to the observer. Refer to the Lecture 8 notes.

Hint: *The rate of change of range can be computed numerically in MATLAB by differentiating the range, i.e., the distance between the ground station and the satellite, with respect to simulation time using the* diff *function.*

(c) How long after t_0 is the Iridium-100 closest to the observer? Express your answer in minutes.

(d) How long is the Iridium-100 visible to the observer? Your answer should be in minutes.

(e) What is the maximum elevation of the Iridium-100? Your answer should be in degrees.

(f) What are the maximum and minimum distances to the Iridium-100 from the observer? Your answer should be in kilometers.

(g) Suppose the observer wants a camera to track the path of the Iridium-100 across the sky. What is the expected maximum angular velocity for the stepper motor rotating the camera? In other words, what is the maximum value of the rate of change of the azimuth and elevation angles, respectively? You can use the diff function in MATLAB to differentiate the azimuth and elevation angles with respect to simulation time. Both your answers should be in degrees per second.

Remember to submit your MATLAB code and Simulink block diagram.

Problem 3. Impulsive Reasoning

A satellite-based maritime tracking company wants to fly a satellite in a sun-synchronous, near-circular low Earth orbit with the following mean Keplerian elements:

$$a = 6720 \text{ [km]},\ e = 0.0002100,\ \omega = 270°,\ \Omega = 90°$$

(a) Consider that the orbit is sun-synchronous with a local solar time at the ascending node passages of 18:00 (dawn-dusk orbit). What should be the inclination, i, of the orbit? Consider Earth-oblateness effects.

(b) The satellite operators need to increase the orbit's semi-major axis by $\Delta a = a^+ - a^- = 200$ [m] to compensate for the accumulated drag effects. Consider the Gauss Variational Equations for near-circular orbits including impulsive maneuvers. Neglecting any effects on the other orbital elements, what is the size and the direction of the required ΔV? At which argument of latitude would you place the maneuver? Why? Do you expect this maneuver to change other orbital elements?

(c) For part b), again considering the Gauss Variational Equations, would it be possible to plan two along-track maneuvers Δv_{T1} at u_1 and Δv_{T2} at u_2 separated by half an orbital revolution (i.e. $u_2 - u_1 = 180°$) instead of one single impulse to obtain the desired Δa without a net change of eccentricity vector after the pair of maneuvers? Why? What is the resulting size and location of the maneuvers?

(d) The satellite operators need to increase the orbit's inclination by $a\Delta i = a(i^+ + i^-) = 200$ [m] to compensate for the accumulated third body effects. Consider the Gauss Variational Equations for near-circular orbits including impulsive maneuvers. Neglecting any effects on the other orbital elements, what is the size and the direction of the required delta-v? At which argument of latitude would you place the maneuver? Why? Do you expect this maneuver to change other orbital elements?