Control moment gyroscope (AE 353 Project 1)

Import modules and configure the notebook.

In [1]:
```
# This module is part of the python standard library
import time

# These modules are part of other existing libraries
import numpy as np
import matplotlib.pyplot as plt
import sympy as sym
from scipy import linalg

# Given pybullet script (it is an interface to the pybullet simulator)
import ae353_cmg

import importlib
importlib.reload(ae353_cmg)
```

Out[1]:
```
<module 'ae353_cmg' from 'c:\\Users\\anshu\\Desktop\\Class Materials\\AE 353 Git\\ae353-sp21\\projects\\01_cmg\\01_code\\ae353_cmg.py'>
```

In this notebook, the following steps will be taken to generate all the results for the project.

- Linearize the system
- Find the eigenvalues of F for different values of K
- Pick value of K that gives negative eigenvalues for F
- Run the simulation in pybullet with the calculated K gains
- Plot the results

Linearizing the System

In [2]:
```
#All the required symbols
q1,q2,v1,v2,v3,tau2,tau3 = sym.symbols('q1,q2,v1,v2,v3,tau2,tau3')
#The provided equations of motion
f = sym.Matrix([[v1],[-(5*(200*tau3*sym.sin(q2)+sym.sin(2*q2)*v1*v2+2*sym.cos(q2)*v2*v3
# Make f and executable function
f_num = sym.lambdify((q1,q2,v1,v2,v3,tau2,tau3), f)
```

In [3]:
```
# See F
f
```

Out[3]:

$$\begin{bmatrix} v_1 \\ -\frac{1000\tau_3 \sin(q_2) + 5v_1v_2 \sin(2q_2) + 10v_2v_3 \cos(q_2)}{10\sin^2(q_2) - 511} \\ v_2 \\ 90.9090909090909\tau_2 - 0.909090909090909v_1v_3 \cos(q_2) \\ -\frac{51100\tau_3 + 511v_1v_2 \cos(q_2) + 5v_2v_3 \sin(2q_2)}{10\sin^2(q_2) - 511} \end{bmatrix}$$

Equilibrium values

- q1 is not in any of the equations, so it can be anything, let's make it about 45 degrees (desired angle)
- v3 can also be anything, but since we're starting off at 100 rpm, I set it as that.

In [4]:
```
q1_e = 45*np.pi/180
q2_e = 0
v1_e = 0
v2_e = 0
v3_e = 10.472 #100 rpm, starting v3
tau2_e = 0
tau3_e = 0
```

In [5]:
```
#checking if the equilibrium points work:
f_eq = f_num(q1_e,q2_e,v1_e,v2_e,v3_e,tau2_e,tau3_e)
f_eq #evaluates to 0! Nice!
```

Out[5]:
```
array([[0.],
       [0.],
       [0.],
       [0.],
       [0.]])
```

Now we are ready to linearize the system.

In [6]:
```
#Now we find the Jacobians
f_jacob_x = f.jacobian([q1,q2,v1,v2,v3])
f_jacob_u = f.jacobian([tau2,tau3])

#And then we find functions for A and B using this jacobian
A_num = sym.lambdify((q1,q2,v1,v2,v3,tau2,tau3),f_jacob_x)
B_num = sym.lambdify((q1,q2,v1,v2,v3,tau2,tau3),f_jacob_u)

#Finally, we find the linearized state space model by evaluating A_num and B_num at the
A = A_num(q1_e,q2_e,v1_e,v2_e,v3_e,tau2_e,tau3_e).astype(float)
B = B_num(q1_e,q2_e,v1_e,v2_e,v3_e,tau2_e,tau3_e).astype(float)
```

With all these steps, we have linearized our equations of motions about the chosen equilibrium point to create a state space model.

In [7]:
```
# See A
A
```

Out[7]:
```
array([[ 0.        ,  0.        ,  1.        ,  0.        ,  0.        ],
       [ 0.        ,  0.        , -0.        ,  0.20493151, -0.        ],
       [ 0.        ,  0.        ,  0.        ,  1.        ,  0.        ],
       [ 0.        ,  0.        , -9.52      ,  0.        , -0.        ],
       [ 0.        ,  0.        , -0.        ,  0.        , -0.        ]])
```

In [8]:
```
# See B
B
```

Out[8]:
```
array([[  0.        ,   0.        ],
       [  0.        ,   0.        ],
       [  0.        ,   0.        ],
       [ 90.90909091,   0.        ],
       [  0.        , 100.        ]])
```

Finding Stable Controller Gains

We still have not covered a method to efficiently calculate working K gains. Therefore, here we resort to using random numbers. There are some values in K that are set to 0 as a design choice based on the system. More information on that in the report.

In [9]:
```
# The next section is commented out because I already found the K gains I wanted. But t
# Credit: Alan Hong on a campuswire post.
'''i = 0
while i == 0:
    K = np.array([[np.random.rand(),np.random.rand(),np.random.rand(),np.random.rand(),
    F  = A - B@K
    s = linalg.eigvals(F)
    if (s.real < 0).all() and (s.imag == 0).all():
        i += 1'''
# Good K gains
K = np.array([[0.8085914584901315, 0.1356123563018472, 0.748872791294181, 0.25935575979
F  = A - B@K
s = linalg.eigvals(F)
print(K.tolist())
s
```

```
[[0.8085914584901315, 0.1356123563018472, 0.748872791294181, 0.2593557597971461, 0.0],
[0.0, 0.0, 0.0, 0.0, 0.9166361309236439]]
```

Out[9]:
```
array([-1.96998847e+01+0.j, -2.10670987e+00+0.j, -1.77120183e+00+0.j,
       -3.13077743e-16+0.j, -9.16636131e+01+0.j])
```

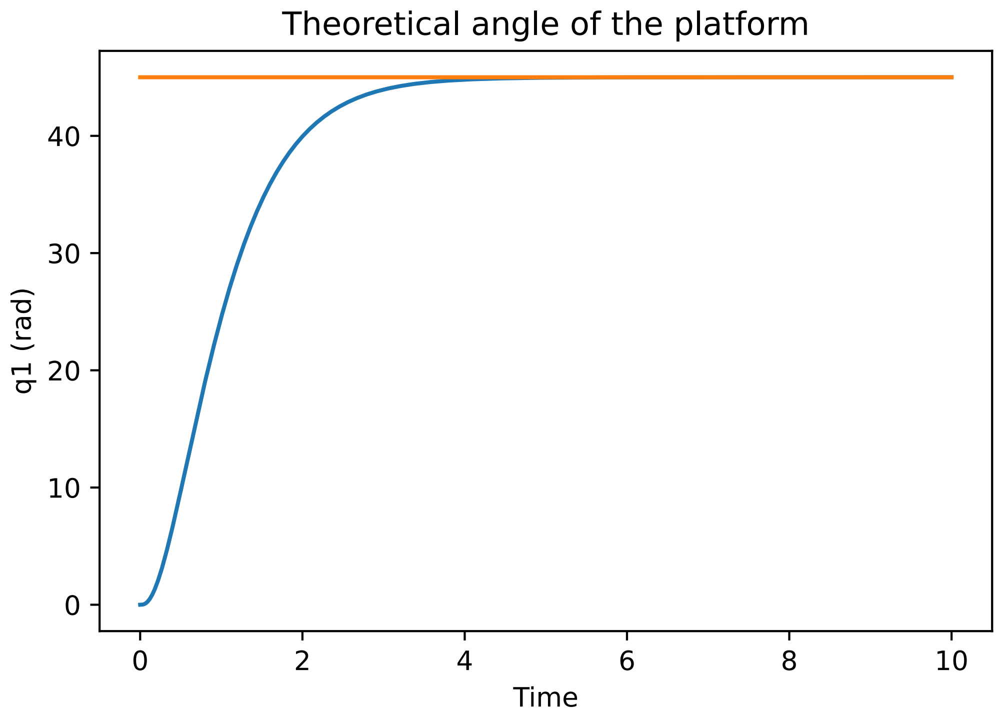
Plotting the Response

We can now calculate x(t) by diagonalizing F and then using the x = V *e^(St)* Vinv *x(0).

In [10]:
```
x0 = sym.Matrix([[-45*np.pi/180],[0],[0],[0],[10.472-50]])
F_sym = sym.simplify(sym.Matrix(F))
t = sym.Symbol('t')
s,V = linalg.eig(F)
Vinv = linalg.inv(V)
S = np.diag(s)
S_sym = sym.simplify(sym.Matrix(S))
V_sym = sym.simplify(sym.Matrix(V))
Vinv_sym = sym.simplify(sym.Matrix(Vinv))
x = V_sym @ sym.exp(S_sym*t) @ Vinv_sym @ x0
x = sym.simplify(x)
x_num = sym.lambdify(t, x)
```

In [11]:
```
q = []
for i in np.linspace(0,10,1001):
    q.append(x_num(i)[0])
q = np.array(q)
plt.plot(np.linspace(0,10,1001),(q+45*np.pi/180)*180/np.pi)
plt.plot(np.linspace(0,10,1001),(45*np.pi/180*np.ones(1001))*180/np.pi)
plt.xlabel("Time")
plt.ylabel("q1 (rad)")
plt.title("Theoretical angle of the platform")
```

Out[11]:
```
Text(0.5, 1.0, 'Theoretical angle of the platform')
```



Running the Simulation

Now we are ready to run the actual simulation. Most of this is taken from CMGDemo.ipynb. The major changes are in the controller definition.

Creating an instance of the robot simulator

There are three optional parameters:

- `damping` is the coefficient of viscouse friction at each joint (the default value is `0.`)
- `dt` is the length of each time step in the simulator (the default value is `0.001`)
- `display` is a flag that says whether or not to open the GUI (the default value is `True`)

The `display` parameter, in particular, is likely to be of use to you. If you set `display=True`, then you will run the simulator in real-time with a GUI. If you set `display=False`, then you will run the simulator as fast as possible (faster than real-time) without a GUI. This is convenient when all you want is the data.

NOTE: it is still necessary to reset the kernel (see the "Kernel" menu above) before creating a new instance of the robot simulator, even when running without a GUI - if you don't, then you'll notice that simulation slows way down.

In [12]:
```
robot = ae353_cmg.RobotSimulator(damping=0., dt=0.001, display=True)
```

Some useful notes from the EOM notebook.

Joint angles:

- q1 is the angle of the "outer" link. This is what we want to control and get to a particular angle The goal of this project is going to be to get this angle to 45 degrees.

- q2 is the angle of the "inner" link

- q3 is the angle of the wheel

The same order applies to the velocities.

Define and create an instance of the robot controller.

In [13]:
```
class RobotController:
    def __init__(self, dt=0.001, q1_des = 45*np.pi/180):
        self.dt = dt #Timestep property of the RobotController class

        #Desired angles and velocities and the gain matrix
        self.K = np.array([[0.8085914584901315, 0.1356123563018472, 0.748872791294181,
        self.q1_des = q1_des
        self.q2_des = 0
        self.v1_des = 0
        self.v2_des = 0
        self.v3_des = 50 #rad/s

    def run(self, q_1, v_1, q_2, v_2, q_3, v_3):
        #state
        x = np.array([[q_1 - self.q1_des],[v_1 - self.v1_des],[q_2 - self.q2_des], [v_2

        #input
        u = - self.K @ x

        tau_2 = u[0,0] # <-- torque applied to gimbal.
        tau_3 = u[1,0] # <-- torque applied to rotor.
        #tau_3 = 0.
        return tau_2, tau_3

# Test case with 45 degrees
controller = RobotController(dt=robot.dt,q1_des = 45*np.pi/180)

# Gimbal Lock position
# controller = RobotController(dt=robot.dt,q1_des = 209.5861*np.pi/180)

#Test case with 180 degrees
# controller = RobotController(dt=robot.dt,q1_des = np.pi)
```

Run the simulation. It is a loop. At each iteration, we:

- get sensor measurements
- choose actuator commands
- go forward one time step

We also log data so that we can plot it later, if we want.

In [14]:
```
# Restore the simulation to its initial state
# Random initial state
# robot.set_state(np.random.randn(3), np.random.randn(3))
# Default initial state
robot.reset(rotor_rpm=100.)
```

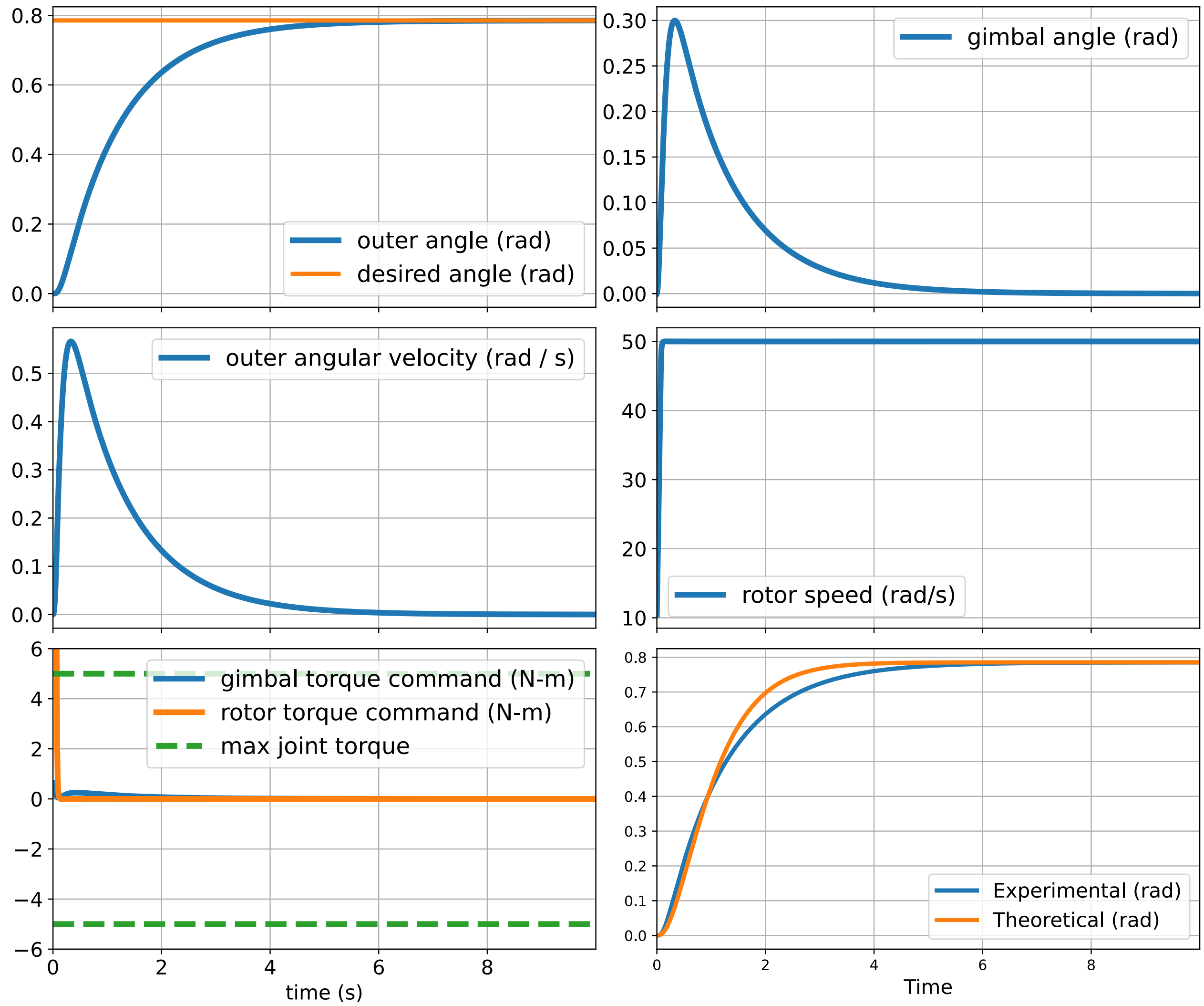
```
# Choose how long we want to run the simulation, and
# compute the corresponding number of time steps
run_time = 10.
num_steps = int(run_time/robot.dt)

# Create a dictionary in which to store results
data = {
    't': np.empty(num_steps, dtype=float),
    'q_1': np.empty(num_steps, dtype=float),
    'v_1': np.empty(num_steps, dtype=float),
    'q_2': np.empty(num_steps, dtype=float),
    'v_2': np.empty(num_steps, dtype=float),
    'q_3': np.empty(num_steps, dtype=float),
    'v_3': np.empty(num_steps, dtype=float),
    'tau_2': np.empty(num_steps, dtype=float),
    'tau_3': np.empty(num_steps, dtype=float),
}

# Run the simulation loop
start_time = time.time()
for step in range(num_steps):
    # Get the current time
    t = robot.dt * step

    # Get the sensor measurements
    q_1, v_1, q_2, v_2, q_3, v_3 = robot.get_sensor_measurements()

    # Choose the actuator command (by running the controller)
    tau_2, tau_3 = controller.run(q_1, v_1, q_2, v_2, q_3, v_3)

    # Log the data from this time step
    data['t'][step] = t
    data['q_1'][step] = q_1
    data['v_1'][step] = v_1
    data['q_2'][step] = q_2
    data['v_2'][step] = v_2
    data['q_3'][step] = q_3
    data['v_3'][step] = v_3
    data['tau_2'][step] = tau_2
    data['tau_3'][step] = tau_3

    # Send the actuator commands to robot and go forward one time
    # step (this is where the actual simulation happens)
    robot.set_actuator_commands(tau_2, tau_3)
    robot.step(t=(start_time + (robot.dt * (step + 1))))
```

Plotting the Results

In [15]:

```
# Create a figure with three subplots
fig, ax = plt.subplots(3, 2, figsize=(12, 10), sharex=True)

# Plot angle of outer gimbal on first subplot
ax[0,0].plot(data['t'], data['q_1'], label='outer angle (rad)', linewidth=4)
ax[0,0].plot(data['t'], np.ones_like(data['t']) * controller.q1_des, label='desired ang
ax[0,0].grid()
ax[0,0].legend(fontsize=16)
ax[0,0].tick_params(labelsize=14)

# Plot angular velocity of outer gimbal on second subplot
```

```
ax[1,0].plot(data['t'], data['v_1'], label='outer angular velocity (rad / s)', linewidt
ax[1,0].grid()
ax[1,0].legend(fontsize=16)
ax[1,0].tick_params(labelsize=14)

# Plot torques on third subplot
ax[2,0].plot(data['t'], data['tau_2'], label='gimbal torque command (N-m)', linewidth=4
ax[2,0].plot(data['t'], data['tau_3'], label='rotor torque command (N-m)', linewidth=4)
ax[2,0].plot(data['t'], np.ones_like(data['t']) * robot.tau_max, '--', label='max joint
ax[2,0].plot(data['t'], -np.ones_like(data['t']) * robot.tau_max, '--', linewidth=4, co
ax[2,0].grid()
ax[2,0].legend(fontsize=16)
ax[2,0].tick_params(labelsize=14)
ax[2,0].set_ylim(-1.2 * robot.tau_max, 1.2 * robot.tau_max)

# Plot gimbal angle on fourth subplot
ax[0,1].plot(data['t'], data['q_2'], label='gimbal angle (rad)', linewidth=4)
ax[0,1].grid()
ax[0,1].legend(fontsize=16)
ax[0,1].tick_params(labelsize=14)

# Plot rotor speed to see the effectiveness
ax[1,1].plot(data['t'], data['v_3'], label='rotor speed (rad/s)', linewidth=4)
ax[1,1].grid()
ax[1,1].legend(fontsize=16)
ax[1,1].tick_params(labelsize=14)

# Plot angle of actual vs theoretical output
ax[2,1].plot(data['t'], data['q_1'], label='Experimental (rad)', linewidth=3)
ax[2,1].plot(np.linspace(0,10,1001),(q+45*np.pi/180), label='Theoretical (rad)', linewi
ax[2,1].grid()
ax[2,1].set_xlabel("Time", fontsize=14)
ax[2,1].legend(fontsize=14)
ax[2,1].tick_params(labelsize=10)

ax[2,0].set_xlabel('time (s)', fontsize=14)
ax[2,0].set_xlim([data['t'][0], data['t'][-1]])
ax[2,1].set_xlim([data['t'][0], data['t'][-1]])

# Make the arrangement of subplots look nice
fig.tight_layout()
```



outer angle (rad)
desired angle (rad)
gimbal angle (rad)
outer angular velocity (rad / s)
rotor speed (rad/s)
gimbal torque command (N-m)
rotor torque command (N-m)
max joint torque
Experimental (rad)
Theoretical (rad)
time (s)
Time