AE353 (Spring ZOZI)

Day 18. Acker

T. Bretl. Controllability

Controllable Canonical Form (CCF)

$$A = \begin{bmatrix} I - a_1 & \cdots & -a_n \end{bmatrix} \quad B = \begin{bmatrix} I & I & I \\ I & I & I \\ I & I & I \end{bmatrix}$$

Facts

$$det(sI - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n$$

$$A - BK = \begin{bmatrix} I - a_1 - k_1 & \cdots & -a_n - k_n \end{bmatrix}$$

$$det(sI - (A - BK)) = s^n + (a_1 + k_1) s^{n-1} + \cdots + (a_{n-1} + k_{n-1}) s + (a_n + k_n)$$

Consequence

if you want
$$s^n + \Gamma_1 s^{n-1} + \cdots + \Gamma_{n-1} s + \Gamma_n$$
then
$$k_1 = \Gamma_1 - a_1 + \cdots + \Gamma_{n-1} s + \Gamma_n$$

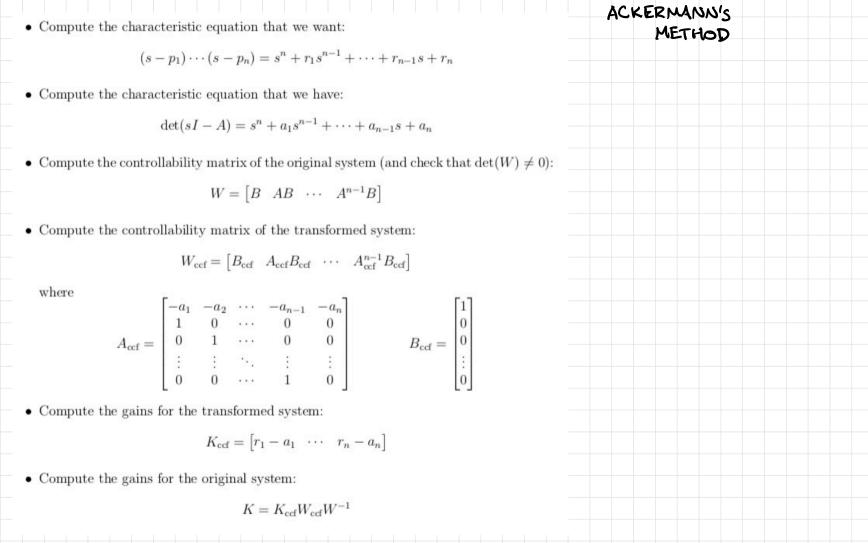
$$k_n = \Gamma_n - a_n$$

If we could put a system in CCF... $\dot{x} = A_{x} + B_{y}$ $\dot{x} = V_{z}$ $\dot{x} = V_{z}$ $\dot{z} = A_{ccf} z + B_{ccf} u$

Then ... easy to find
$$u = -K_{CCF} = Z$$

 $\dot{x} = Ax + Bu$ $\dot{z} = AVz + Bu$ $\dot{z} = VAVz + VBu$

solve for V (that's what we need to find K How? given Kax) ACCF = VAV BCCF = VB Bccf = AUFBUF = AUF BUF = Accf Bccf = [BOOF ACCEBUF AUFBOUF -- ACCE BOOF] = VI[WCCF is invertible - works as long as



States X) The system x = Ax+ Bu is controllable if A"-1B] W = [B AB ··· has full rank. in python this means "np. linalg. matrix_rank (w)" is the same as "n" if there is only one input, W is square, and so "full rank" and "invertible" mean the same thing - so, for a system with only one input, you can simply check if det(w) 70