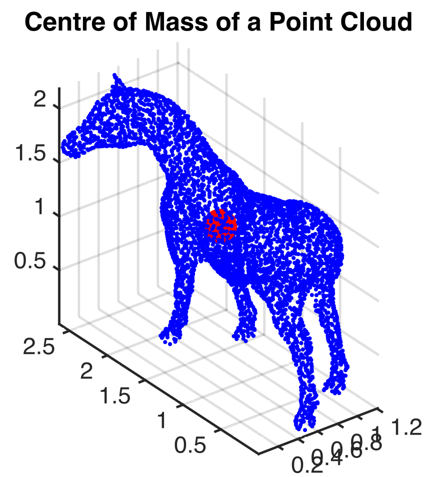
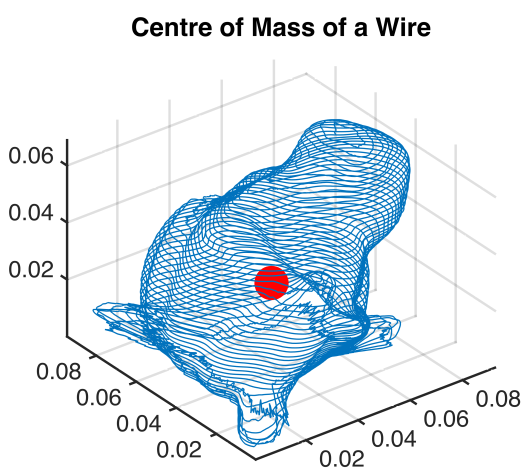
UBC MECH 222: MATLAB Computer Lab 2

Centre of Mass




```
>> load('point_cloud_A.mat')
>> [x_cm,y_cm,z_cm,m_tot] = cm_points(x,y,z,m,'y');
```




```
>> load('wire_nodes_C.mat')
>> [x_cm,y_cm,z_cm,m_tot] = cm_wire(x,y,z,rho,'y');
```

Instructions

1. The centre of mass of a collection of N points with coordinates $\{(x_i, y_i, z_i)\}_{i=1}^N$ (each with mass m_i) has coordinates $(x_{\text{cm}}, y_{\text{cm}}, z_{\text{cm}})$ where

$$x_{\text{cm}} = \frac{\sum_{i=1}^N x_i m_i}{m_{\text{tot}}} \quad y_{\text{cm}} = \frac{\sum_{i=1}^N y_i m_i}{m_{\text{tot}}} \quad z_{\text{cm}} = \frac{\sum_{i=1}^N z_i m_i}{m_{\text{tot}}} \quad m_{\text{tot}} = \sum_{i=1}^N m_i$$

   Write a function called `cm_points` which takes 5 input parameters `x`, `y`, `z`, `m` and `print`:

```
function [x_cm,y_cm,z_cm,m_tot] = cm_points(x,y,z,m,print)
% Compute the centre of mass [x_cm,y_cm,z_cm] and
% the total mass m_tot of the point cloud
% with coordinates given by x, y, and z, and mass m.
```

   The vectors `x`, `y`, `z`, and `m` have the same length N and represent the 3D coordinates of a cloud of points where the point at position (x_i, y_i, z_i) has mass m_i. The parameter `print` is a string. The function performs the following tasks:

   - ☐ Compute the coordinates of the centre of mass of the point cloud and the total mass. The function should only use array operations and **not** `for` loops.
   - ☐ If `print == 'y'`, then use the `plot3` command to plot the point cloud as blue dots and the centre of mass as a larger red dot in the same figure.
   - ☐ Write comments at the beginning of your function to describe its purpose, inputs, outputs and **include your name and student number**.
   - ☐ Test your function with many different inputs. For example, load each of the workspace files (such as `point_cloud_A.mat`) and evaluate your function with the variables provided.

   When you have completed each item above and are satisfied with your function, submit your M-file (called `cm_points.m`) to Connect.

2. Write a function called `cm_wire` which takes 5 input parameters `x`, `y`, `z`, `rho` and `print`:

```
function [x_cm,y_cm,z_cm,m_tot] = cm_wire(x,y,z,rho,print)
% Compute the centre of mass [x_cm,y_cm,z_cm] and
% the total mass m_tot of the wire
% with coordinates given by x, y, and z, and density rho.
```

   The vectors `x`, `y`, `z`, and `rho` have the same length N and represent the 3D coordinates of a wire where the wire has density ρ_i at position (x_i, y_i, z_i). The parameter `print` is a string. The function performs the following tasks:

   - ☐ Compute the coordinates of the centre of mass of the wire and the total mass (see Hints below). The function should only use array operations and **not** `for` loops.

- ☐ If `print == 'y'`, then use the `plot3` command to plot the wire and the centre of mass as a larger red dot in the same figure.
- ☐ Write comments at the beginning of your function to describe its purpose, inputs, outputs and **include your name and student number**.
- ☐ Test your function with many different inputs. For example, load each of the workspace files (such as `wire_nodes_A.mat`) and evaluate your function with the variables provided.

When you have completed each item above and are satisfied with your function, submit your M-file (called `cm_wire.m`) to Connect.

Hints

For the function `cm_wire`, we are approximating a continuous wire by a discrete set of points $\{(x_i, y_i, z_i)\}_{i=1}^N$ where the density of the wire at point (x_i, y_i, z_i) is ρ_i and varies linearly from point to point. A good way to compute the centre of mass of the wire is:

1. Treat the wire as a collection of $N-1$ segments: straight lines connecting each point $\mathbf{r}_i = (x_i, y_i, z_i)$ with the next $\mathbf{r}_{i+1} = (x_{i+1}, y_{i+1}, z_{i+1})$.
2. Calculate the mass of each segment: the density of a segment of length L is given by the linear function
$$\rho(s) = \rho_0 + \frac{s}{L}(\rho_1 - \rho_0) \ , \ 0 \leq s \leq L$$
where $\rho(0) = \rho_0$ and $\rho(L) = \rho_1$ are the densities at each end of the wire therefore
$$\begin{aligned} m = \int_0^L \rho(s)\, ds &= \int_0^L (\rho_0 + \frac{s}{L}(\rho_1 - \rho_0))\, ds \\ &= \rho_0 s + \frac{s^2}{2L}(\rho_1 - \rho_0)\Big|_0^L \\ &= \rho_0 L + \frac{L}{2}(\rho_1 - \rho_0) \\ &= \frac{L(\rho_0 + \rho_1)}{2} \end{aligned}$$
3. Calculate the centre of mass of each segment:
$$\begin{aligned} s_{\text{cm}} = \frac{1}{m}\int_0^L s\rho(s)\, ds &= \frac{1}{m}\int_0^L s\left(\rho_0 + \frac{s}{L}(\rho_1 - \rho_0)\right) ds \\ &= \frac{1}{m}\left(\frac{s^2}{2}\rho_0 + \frac{s^3}{3L}(\rho_1 - \rho_0)\right)\Big|_0^L \\ &= \frac{1}{m}\left(\frac{L^2}{2}\rho_0 + \frac{L^2}{3}(\rho_1 - \rho_0)\right) \\ &= \frac{L^2}{6m}(\rho_0 + 2\rho_1) \\ &= \frac{L}{3}\frac{\rho_0 + 2\rho_1}{\rho_0 + \rho_1} \end{aligned}$$

therefore the centre of mass of the segment of length $L_i = ||\mathbf{r}_{i+1} - \mathbf{r}_i||$ connecting the points $\mathbf{r}_i = (x_i, y_i, z_i)$ and $\mathbf{r}_{i+1} = (x_{i+1}, y_{i+1}, z_{i+1})$ is located at

$$\mathbf{r}_{i,\mathrm{cm}} = \mathbf{r}_i + \frac{s_{\mathrm{cm}}}{L_i}(\mathbf{r}_{i+1} - \mathbf{r}_i) = \mathbf{r}_i + \frac{1}{3}\frac{\rho_i + 2\rho_{i+1}}{\rho_i + \rho_{i+1}}(\mathbf{r}_{i+1} - \mathbf{r}_i)$$

4. Find the total mass of the wire by summing the masses of all the line segments.

5. Find the centre of mass of the wire by using the previous function `cm_points` (with `print` set to `'n'`) with the set of centres of mass of all the line segments.