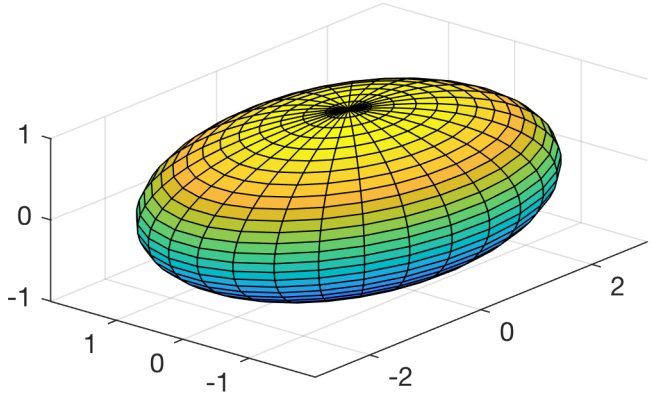
UBC MECH 222: MATLAB Computer Lab 3

Ellipsoids: Surface area and 3D plotting in spherical coordinates




```
>> A = ellipsoid_area(3,2,1)

A =

    48.8821
```

Instructions

An ellipsoid is a quadratic surface defined by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

for positive parameters a, b and c. Write a function called `ellipsoid_area` which takes 3 input parameters a, b and c such that the function performs the following tasks:

- ☐ Plot the ellipsoid for the given parameters a, b and c using spherical coordinates (do **not** use the MATLAB function `ellipsoid`)
- ☐ Compute an approximation for the surface area of the ellipsoid (using `integral2` or `trapz`)

When you have completed each item above and are satisfied with your function, submit your M-file (called `ellipsoid_area.m`) to Connect.

Hints

1. The following code plots the unit sphere using spherical coordinates:

```
N = 30;
theta = linspace(0,2*pi,N);
phi = linspace(0,pi,N);
[THETA,PHI] = meshgrid(theta,phi);
X = sin(PHI) .* cos(THETA);
Y = sin(PHI) .* sin(THETA);
Z = cos(PHI);
```

   Modify this code to plot an ellipsoid with parameters a, b and c.

2. The general formula for the surface area of a surface $z = f(x, y)$ over a region R is

$$A = \iint_R \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dA$$

   Thanks to symmetry, the surface area of an ellipsoid is 8 times the area of the ellipsoid in the first octant $x, y, z \geq 0$ where the ellipsoid is given by the formula

$$z = c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$

   over the region $0 \leq x \leq a$ and $0 \leq y \leq b\sqrt{1 - x^2/a^2}$. Therefore, after some calculations and some algebra, the surface area of an ellipsoid is

$$A = 8 \int_0^a \int_0^{b\sqrt{1-\frac{x^2}{a^2}}} \sqrt{\frac{1 - \left(1 - \frac{c^2}{a^2}\right)\frac{x^2}{a^2} - \left(1 - \frac{c^2}{b^2}\right)\frac{y^2}{b^2}}{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}}\, dy\, dx$$

3. MATLAB has several functions for numerically approximating integrals. The most convenient function for calculating double integrals is `integral2`. For instructions on how to compute integrals numerically, check out the documentation:

`https://www.mathworks.com/help/matlab/numerical-integration-and-differentiation.html`