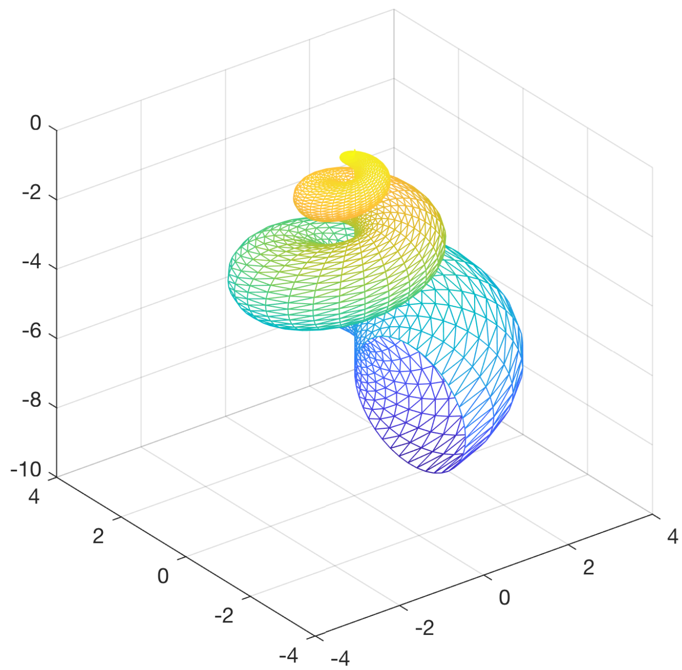
UBC MECH 222: MATLAB Computer Lab 5

Triangulated parametric surfaces




```
[T,X,Y,Z] = triangulate(@conch,[0,1],[0,pi],[80,40]);
A = tri_surface_area(T,X,Y,Z);

function p = conch(u,v)
    x = 2*(1-exp(u)).*sin(6*pi*u).*cos(v).^2;
    y = 2.*(1-exp(u)).*cos(6*pi*u).*cos(v).^2;
    z = 1 - exp(2*u) - sin(2*v) + exp(u).*sin(2*v);
    p = [x,y,z];
end
```

Instructions

Write a function called `tri_surface_area` which takes 4 input parameters:

- ☐ `T` is a matrix of vertex indices of a triangulation
- ☐ `X` is the vector of x coordinates of the vertices
- ☐ `Y` is the vector of y coordinates of the vertices
- ☐ `Z` is the vector of z coordinates of the vertices

The function performs the following tasks:

- ☐ Use `trimesh` to plot the triangulated surface defined by `T`, `X`, `Y` and `Z`.
- ☐ Compute the surface area of the surface

Write comments at the beginning of your function to describe its purpose, inputs, outputs and **include your name and student number**. When you are satisfied with your function, submit your M-file (called `tri_surface_area.m`) to Connect.

Hints

1. A *triangulation* of a surface is a net of triangles which covers the surface. Triangulations are often used in finite element methods in engineering and physics applications. For more information, see:

   `https://en.wikipedia.org/wiki/Surface_triangulation`
   `https://en.wikipedia.org/wiki/Finite_element_method`

2. A triangulation is essentially a collection of triangles whose vertices are on the surface. We record the points by vectors X, Y and Z of x, y and z coordinates. Each triangle is a list of 3 points which we list as *indices*. For example, a vector $[5, 2, 3]$ is the triangle where the 3 vertices are given by the points (x_5, y_5, z_5), (x_2, y_2, z_2) and (x_3, y_3, z_3) (where for example x_5 is the fifth entry in X). Therefore we record the data of a triangulation by a matrix T of indices and vectors X, Y and Z of coordinators. For example, consider the tetrahedron with vertices at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ (and label them 1, 2, 3 and 4 respectively). Each face is already a triangle and so a triangulation is given by:

$$T = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix} X = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} Y = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} Z = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

   Again, the first row of T is $[1, 2, 4]$ which designates the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$ and $(0, 0, 1)$, and so on.

3. The function `triangulate` (see `triangulate.m` posted with these instructions) is provided for you to create a triangulation from a parameterization

$$\mathbf{r}(u, v) = (f(u, v), g(u, v), h(u, v))$$

For example, a parameterization of the torus (with radius R and inside radius r) is given by

$$\begin{aligned} x(\theta, \varphi) &= (R + r\cos\theta)\cos\varphi \\ y(\theta, \varphi) &= (R + r\cos\theta)\sin\varphi \\ z(\theta, \varphi) &= r\sin\theta \end{aligned}$$

where $0 \leq \theta \leq 2\pi$ and $0 \leq \varphi \leq 2\pi$ and the surface area of the torus is $4\pi^2 Rr$. For more information, see:

`https://en.wikipedia.org/wiki/Torus`

4. The area of a triangle with vertices A, B and C is

$$\text{Area} = \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$$

and the surface area of a triangulation is the sum of the areas of all the triangles in the triangulation.