

GIGAYASA

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Contents

1	Car	rier Frequency Offset Estimation and Correction in 5G Networks	1
	1.1	What is Carrier Frequency Offset?	1
	1.2	CFO Estimation Techniques	1
	1.3	Results	5
	1.4	Useful links	7
	1.5	Exercise	7
2	Ref	erences	g



1 | Carrier Frequency Offset Estimation and Correction in 5G Networks

The air interface of 5G NR is based on orthogonal frequency division multiplexing (OFDM), which divides the wideband carrier into small orthogonal subcarriers. Unlike the wideband signal, these subcarriers experience flat fading, resulting in significantly reduced receiver complexity and improved spectral efficiency. However, OFDM is very sensitive to time and frequency synchronization errors. These errors can arise in the frequency or time domains due to carrier frequency offset (CFO) and symbol time offset (STO), respectively. This experiment will focus on the carrier frequency offset and its impact on the system, as well as algorithms to estimate and correct the CFO using the synchronization signal block (SSB).

1.1 | What is Carrier Frequency Offset?

The carrier frequency offset is attributed to a mismatch in the local oscillator frequency of the transmitter and receiver. This offset causes a shift in the phase of the received symbols, as shown in Figure 1.1, which leads to incorrect decoding and degradation of performance. When a UE is moving, CFO estimation becomes even more difficult.

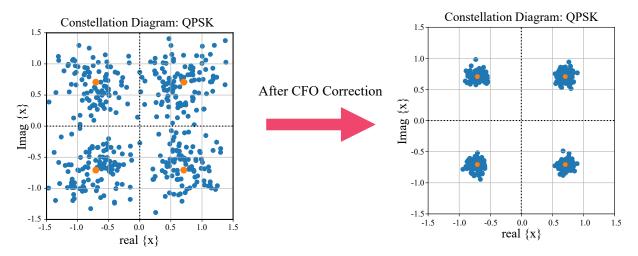


Figure 1.1: Carrier Frequency Offset (CFO) in the SSB/PBCH in 5G Networks.

The primary objective of the UE is to decode the data transmitted by the BS. However, initially, the UE doesn't know the OFDM symbol/frame boundary, which is accomplished using the process of time synchronization detailed in Chapter-??. Once the transmitter and receiver are time synchronized, the receiver estimates the CFO and compensates its effect from the received samples. Inaccuracies in CFO correction result in shifts in subcarrier frequencies, compromising the orthogonality among the sub-carriers and leading to inter-carrier interference (ICI) as shown in Fig-1.2. These errors can accumulate and significantly deteriorate the performance of SSB detection. Hence, it becomes important to estimate and correct CFO accurately at the UE. In the next section, we will discuss different techniques that have been widely used to estimate CFO.

1.2 | CFO Estimation Techniques

The most commonly used methods to estimate CFO are cyclic prefix (CP)-based CFO estimation and pilot-based CFO estimation. CFO estimation can be performed in the time domain or frequency domain. In the frequency domain, only the integer part of CFO can be estimated, while the fractional CFO can only be estimated in the time domain. Fractional CFO results in phase rotation across time samples, which can be estimated using either a pilot-based scheme or a CP-based scheme.

1.2.1 | System Model

For the estimation and correction of CFO, we assume that the wireless channel between the transmitter and receiver undergoes flat fading, modelled as follows,

$$y[n] = h.x[n] + w[n],$$
 (1.1)

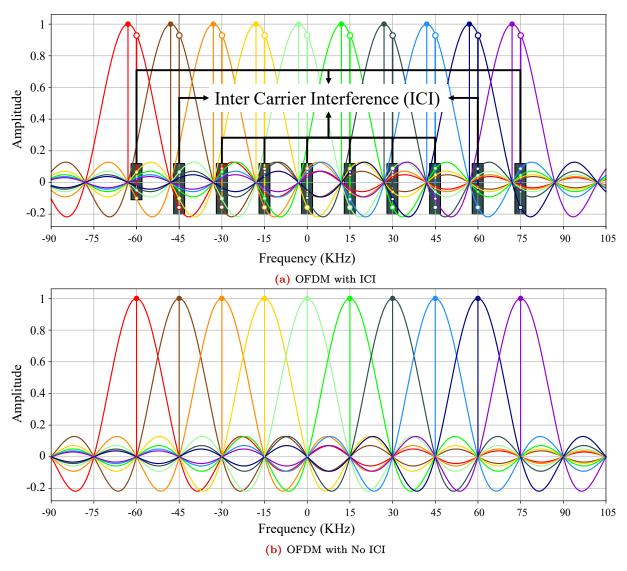


Figure 1.2: Inter carrier interference (ICI) in OFDM system

where y[n], x[n], w[n] and h are received signal, transmitted signal, noise and flat fading channel between transmitter and receiver respectively at time instant n. This system model proposed in equation-1.1 can be modified to capture the effect of CFO at the receiver as follows,

$$y[n] = h.x[n]e^{j2\pi\epsilon n} + w[n]. \tag{1.2}$$

where ϵ denotes the fractional part of the CFO resulting in phase rotation of transmitted symbols.

1.2.2 | CP Based CFO Estimation

The cyclic prefix (CP) is employed in an OFDM system, where the last $L_{\rm CP}$ samples of the OFDM symbol are appended to the front of it, as shown in Fig-1.3. The CP helps the OFDM system mitigate the effects of intersymbol interference (ISI) caused by multipath propagation. The large number of multi-path components result in a higher delay spread. To prevent ISI, the length of $L_{\rm CP}$ must be larger than the delay spread of the channel.

The length of one OFDM symbol is $N_{\rm FFT} + L_{\rm CP}$, as shown in Fig-1.3, where $N_{\rm FFT}$ is the size of the FFT, and $L_{\rm CP}$ is the length of the cyclic prefix (CP) used in OFDM modulation. In a OFDM symbol, the first and the last $L_{\rm CP}$ sample are same,

$$x[n] = x[n + N_{\text{FFT}}], n = 0, 1, 2, \dots, L_{\text{CP}} - 1.$$

This property of OFDM can be utilized for CFO estimation at the receiver. It can be observed from the equation 1.1, the first $L_{\rm CP}$ received samples of the OFDM symbols can be written as,



$$y[n] = hx[n]e^{j2\pi\epsilon n} + w[n], n = 0, 1, 2, \dots, L_{CP} - 1.$$
(1.3)

the last $L_{\rm CP}$ received samples are given as,

$$y[n + N_{\text{FFT}}] = hx[n + N_{\text{FFT}}]e^{j2\pi\epsilon(n + N_{\text{FFT}})} + w[n + N_{\text{FFT}}], n = 0, 1, 2, \dots, L_{\text{CP}} - 1.$$

= $hx[n]e^{j2\pi\epsilon(n + N_{\text{FFT}})} + w[n + N_{\text{FFT}}]$

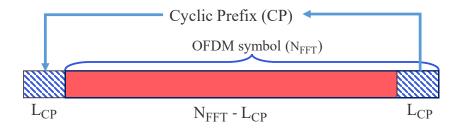


Figure 1.3: Time domain OFDM symbol with information and CP.

The differential phase of the last and first $L_{\rm CP}$ samples can be computed as follows,

$$\phi(n) = \frac{y[n + N_{\text{FFT}}]}{y[n]}, n = 0, 1, 2, \dots, L_{\text{CP}} - 1.$$

$$= e^{j2\pi\epsilon N_{\text{FFT}}} + \frac{w[n + N_{\text{FFT}}]}{w[n]}$$
(1.4)

The differential phase of $\phi(n)$ can be used to estimate the CFO at time instant n as follows,

$$\hat{\epsilon}(n) = \frac{\arg\{\phi(n)\}}{2\pi N_{\text{FFT}}}.$$
(1.5)

To reduce the effect of noise, the ML estimation is performed, The ML estimate of CFO ($\hat{\epsilon}$) is computed as,

$$\hat{\epsilon} = \frac{1}{L_{\text{CP}}} \sum_{n=1}^{L_{\text{CP}}} \hat{\epsilon}(n), \tag{1.6}$$

Algorithm: The CP based CFO Estimation Method

1. Collect the first $L_{\rm CP}$ samples from OFDM symbol

$$\{y[n]\}_{n=1}^{L_{\text{CP}}} = \{hx[n]e^{j2\pi\epsilon N_{\text{FFT}}}\}_{n=1}^{L_{\text{CP}}}.$$

2. Collect the last $L_{\rm CP}$ samples from OFDM symbol.

$$\{y[n+N_{\rm FFT}]\}_{n=1}^{L_{\rm CP}} = \{hx[n+N_{\rm FFT}]e^{j2\pi\epsilon N_{\rm FFT}}\}_{n=1}^{L_{\rm CP}}.$$

3. Compute the differential phase,

$$\phi[n] = \left\{ \frac{y[n+N_{\rm FFT}]}{y[n]} \right\}_{n=0}^{n=L_{\rm CP}-1} = e^{j2\pi\epsilon N_{\rm FFT}}.$$

4. Compute the CFO estimate for time instant-n,

$$\hat{\epsilon}[n] = \frac{arg\{\phi[n]\}}{2\pi N_{\text{FFT}}}.$$



5. Compute the ML estimate of CFO,

$$\hat{\epsilon} = \frac{1}{L_{\mathrm{CP}}} \sum_{n=1}^{L_{\mathrm{CP}}} \hat{\epsilon}[n],$$

6. Perform CFO correction,

$$\hat{y}[n] = y[n]e^{-j2\pi\hat{\epsilon}n},$$

The accuracy of CFO estimation using the CP-based technique depends on the length of the CP. When the length of the CP is shorter, the estimate of CFO tends to be inaccurate, leading to poor CFO correction. This can be significantly improved using a pilot-based scheme, which provides a larger number of observations available for CFO estimation.

1.2.3 | Pilot Based CFO Estimation

As discussed above, in CP based CFO estimation the first and last $L_{\rm CP}$ samples is considered. However, in pilot based CFO estimation all $N_{\rm FFT} + L_{\rm CP}$ samples of OFDM symbol are considered. The $N_{\rm FFT} + L_{\rm CP}$ received samples of OFDM symbol is given by

$$y[n] = hx[n]e^{j2\pi\epsilon n} + w[n], n = 1, 2, \dots, N_{\text{FFT}} + L_{\text{CP}}$$
 (1.7)

The channel tap estimate for time index n is estimated as follows,

$$y[n + N_{\text{FFT}}] = hx[n + N_{\text{FFT}}]e^{j2\pi\epsilon(n + N_{\text{FFT}})} + w[n + N_{\text{FFT}}], n = 1, 2, \dots, L_{\text{CP}}$$

$$= hx[n]e^{j2\pi\epsilon(n + N_{\text{FFT}})} + w[n + N_{\text{FFT}}]$$
(1.8)

$$h[n] = \frac{y[n]}{x[n]}$$

$$= he^{j2\pi\epsilon n} + \frac{w[n]}{x[n]}$$

$$= he^{j2\pi\epsilon n} + \bar{w}[n],$$
(1.10)

where $n = 1, 2, \dots, N_{\text{FFT}} + L_{\text{CP}}$. The differential phase of channel taps can be estimated as,

$$\phi[n] = \frac{h[n+1]}{h[n]} = e^{j2\pi\epsilon n} + \tilde{w}[n]$$
(1.11)

 $\overline{w}[n]$ denotes the noise component. The equation 6.15 assumes that the channel is changing smoothly across time. The CFO for time sample n can be estimate as,

$$\hat{\epsilon}(n) = \frac{\angle \phi[n]}{2\pi} \tag{1.12}$$

In order to mitigate the effect of noise, the mean of all $N_{\rm FFT} + L_{\rm CP} - 1$ samples are taken. The resulting maximum likelihood CFO estimation is given as,

$$\hat{\epsilon} = \frac{1}{N_{\text{FFT}} + L_{\text{CP}} - 1} \sum_{n=1}^{N_{\text{FFT}} + L_{\text{CP}} - 1} \hat{\epsilon}[n]$$
(1.13)

Algorithm: The Pilot based CFO Estimation Method

- **1.** Extract all the $N_{\text{FFT}} + L_{\text{CP}}$ received samples of OFDM symbol: $\{y[n]\}$
- 2. $x[n] = \text{Extract all the } N_{\text{FFT}} + L_{\text{CP}}$ transmitted samples of OFDM symbol.
- **3.** Estimate the channel taps $h[n] = \frac{y[n]}{x[n]}$



4. Compute differential phase for time instant n,

$$\phi[n] = \frac{h[n+1]}{h[n]} = e^{j2\pi\epsilon n} + \tilde{w}[n]$$
(1.14)

5. CFO estimate at time instant n,

$$\hat{\epsilon}[n] = \frac{\angle \phi[n]}{2\pi} \tag{1.15}$$

6. Maximum Likelihood (ML) estimation of CFO is given by,

$$\hat{\epsilon} = \frac{1}{N_{\text{FFT}} + L_{\text{CP}} - 1} \sum_{n=1}^{N_{\text{FFT}} + L_{\text{CP}} - 1} \hat{\epsilon}[n]$$
 (1.16)

In this subsection, we discussed the CP-based and pilot-based CFO estimation techniques in detail. In the upcoming results section, we will analyze the performance of both CFO estimation techniques and compare them.

1.3 | Results

The hardware emulations are conducted using the simulation parameters specified in table-1.1. The evaluations are performed under various link conditions, including line of sight (LoS) or non-line of sight (NLoS), and at different distances between the transmitter and receiver.

Table 1.1: Simulation parameters and evaluation methodology

Parameter	Value
Carrier frequency (f_c)	$1000~\mathrm{MHz}$
Bandwidth (B)	5 MHz
FFT size $(N_{\rm FFT})$	1024
Subcarrier spacing (Δf)	15 KHz
Transmitter-Receiver separation	10 cm, 1 m, 5m, 10m
Physical Channel	PBCH
Channel Estimation	PBCH-DMRS
Channel Estimation/Interpolation Method	Least Squares with linear interpolation
Symbol Equalization Method	Zero forcing
Time Synchronization	PSS based time correlation
CFO Estimation	CP-based

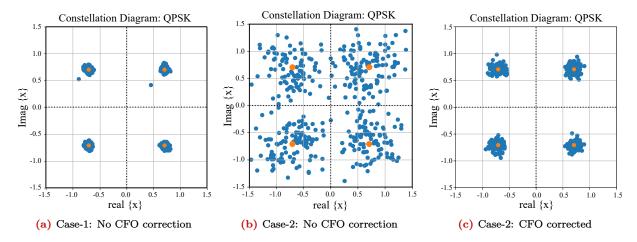


Figure 1.4: Constellation diagrams of received QPSK symbols for case-1 and case-2. | Case-1: Transmitter and receiver are on same SDR | Case-2: Transmitter and Receiver are on two different SDRs.



Observation 1: When the same SDR is used as both the transmitter (BS) and the receiver (UE), no CFO is observed because both the transmitter and receiver use the same local oscillator.

It can be clearly observed from Figure 1.4a that when the same SDR is employed for transmission and reception, there is no effect of CFO. When the same SDR is used at both the transmitter and receiver, the frequency of the local oscillator and the clocks used by the digital-to-analog converter and analog-to-digital converter are exactly the same. This results in nearly perfect time and frequency synchronization between the transmitter and receiver, resulting in very clean constellation of the received symbols.

Observation 2: When two different SDRs using two different clock sources are used for decoding data, the carrier frequency offset (CFO) becomes non-negligible.

It is practically very difficult to manufacture identical devices or components, such as clock frequencies and local oscillators. This results in mismatches in the clock rate and carrier frequency of the clock source and local oscillator between the transmitter and the receiver. These hardware impairments can severely corrupt the quality of the received symbols, as shown in Fig-1.4b. The effect of this carrier frequency offset (CFO) is low when the pilot density is high, which is the case for PBCH-SSB. However, CFO can significantly degrade the quality of the constellation for PDSCH as shown in Fig-??, where the density of pilots across time is low. This impact can be even more severe for higher constellation orders.

Observation 3: Quality of the constellation of the received equalized symbols improves drastically when a CFO correction algorithms is applied.

The effect of the CFO can be reduced using CFO estimation algorithm as shown in Fig[1.4c.

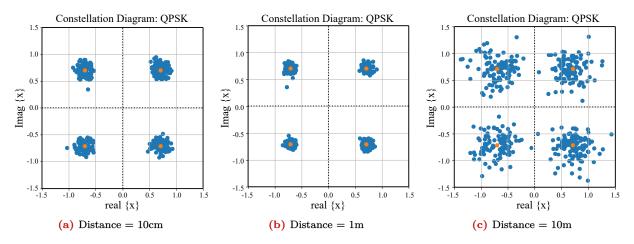


Figure 1.5: Quality of QPSK constellation after CFO correction for different Tx-Rx (LoS link) separation.

Observation 4: The performance of the CFO estimation algorithms changes significantly with channel conditions (LoS/NLoS links) and the transmitter and receiver separation.

CFO algorithms are designed with certain assumptions, such as CP-based CFO estimation algorithms is optimal in flat-faded channel conditions. The performance of these CFO algorithms changes with channel conditions, as illustrated in Fig-1.5 and Fig-1.6. As the separation between the transmitter and receiver increases, the signal-to-noise ratio (SNR) degrades. Additionally, in general, for non-line-of-sight (NLoS) links, the frequency selectivity in the channel increases with distance. Both of these factors contribute to the degradation in performance of the employed CFO estimation algorithm.

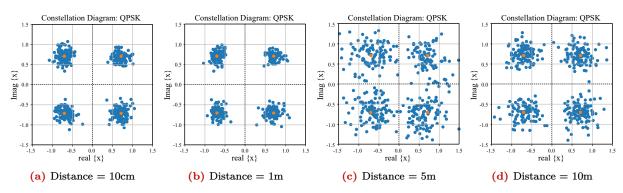


Figure 1.6: Quality of QPSK constellation after CFO correction for different Tx-Rx(NLoS link) separation.



An important metric that determines the quality of equalized symbol constellation is Error Vector Magnitude (EVM). EVM computation measures the spread of received symbols with respect to transmitted constellation symbols. The lower the EVM, the better the performance of the CFO estimation method. The estimated CFO and EVM performance of the CFO estimation method for different channel conditions are listed in table-1.2.

Error Vector Magnitude (EVM): The EVM computes the euclidean distance between each received samples with nearest symbol(in the transmitted constellation). The EVM gives the measure of how well the CFO algorithm works. Ideally, the EVM must be 0(typically when using single SDR as transmitter and receiver), indicating the all received lies on the point as transmitted samples. The procedure to compute the EVM is detailed below:

- Compute the distance of all received symbols from constellation points of modulation.
- For each received symbol consider the minimum distance.
- Compute mean of the minimum distances of all received symbols from previous step to get EVM.

Tx-Rx Separation (m)	Link state (LoS/NLoS)	Estimated CFO (ϵ)	EVM
0	Same SDR (LoS)	0.013	0.045
0.1	Different SDR (LoS)	-1.233	0.129
1	Different SDR (LoS)	-1.244	0.143
10	Different SDR (LoS)	-1.1841	0.0789
50	Different SDR (LoS)	-0.697	0.605
0.1	Different SDR (NLoS)	-1.132	0.0424
1	Different SDR (NLoS)	-1.254	0.117
10	Different SDR (NLoS)	-1.180	0.2509
50	Different SDR (NLoS)	-1.105	0.627

Table 1.2: CFO Estimation Performance for different channel conditions

Table 1.2 shows the observation table, students must fill the values of estimated CFO and EVM for each distance between SDRs. It can be noted from results that as distance between the two SDRs increases the EVM increases. The above experiment was performed for 0 dB transmitter gain and 50 dB receiver gain.

1.4 Useful links

- Carrier frequency synchronization in the downlink
- CFO Estimation and Correction in 5G Toolkit.
- Video tutorial of this chapter.

1.5 | Exercise

- 1. For the diagram given in Fig-1.7, compute the EVM,
- 2. Estimate the CFO using the OFDM symbols based on CP based CFO estimation.
 - The received samples and the transmitted samples can downloaded from the link.
 - $N_{\rm FFT} = 1024$
 - $\Delta f = 15 \text{ kHz}$
 - $L_{\rm CP} = 72$
 - Assumption: Channel is flat-faded.

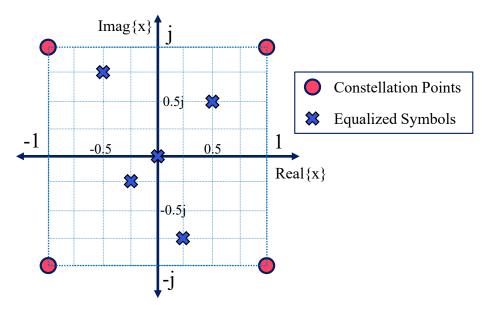


Figure 1.7: Transmitted vs received Symbols

- **3.** Estimate the CFO using the OFDM symbols based on Pilot based CFO estimation.
 - Assume the same data as in previous example.



2 | References