BPDecoderPlus: Circuit-Level Quantum Error Correction with Belief Propagation and Tropical Tensor Networks

Si-Yuan Chen*, Meng-Kun Liu*, Shen Yang* and Jin-Guo Liu*

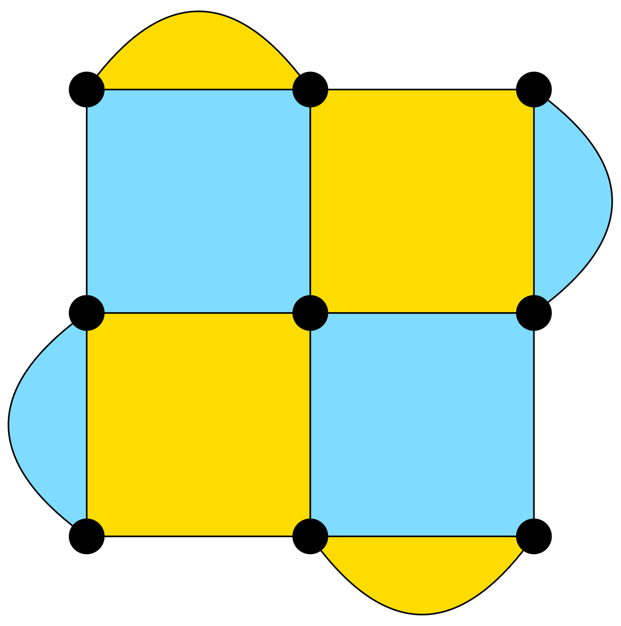
Abstract

Quantum error correction (QEC) is essential for fault-tolerant quantum computing. We present **BPDecoderPlus**, a Python package implementing two complementary approaches for decoding surface codes under circuit-level noise:

1. **BP+OSD Decoder**: Belief propagation with ordered statistics decoding post-processing, achieving near-optimal performance on quantum LDPC codes.
2. **Tropical Tensor Networks**: Exact Most Probable Explanation (MPE) computation via tropical semiring contraction, providing optimal solutions for moderate-size instances.

Our implementation correctly resolves the circuit-level error threshold at $\approx 0.7\%$ for rotated surface codes, validating against established literature. The package features GPU acceleration via PyTorch, comprehensive CLI tools, and integration with Google's Stim simulator.

Rotated Surface Code



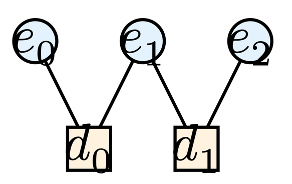
Detection events (not raw syndromes):
- Measurement errors flip syndrome values
- Event = XOR of consecutive measurements
- Localizes errors in space-time

Detector Error Model (DEM):
- Error probabilities per fault
- Detector-error associations
- Observable flip annotations

Detection Events → **Decoder** → Observable Flip Prediction

BP+OSD Decoder

Belief Propagation iteratively passes messages on a factor graph to compute marginal probabilities. For QEC, the factor graph is derived from the parity check matrix H.



Factor Graph

Message passing:

$$\mu_{v\to f} = \prod_{f'\in N(v)\setminus f} \mu_{f'\to v}, \mu_{f\to v} = \sum_{\boldsymbol{x}:x_v=0,1} \psi_f(\boldsymbol{x}) \prod_{v'\in N(f)\setminus v} \mu_{v'\to f}$$

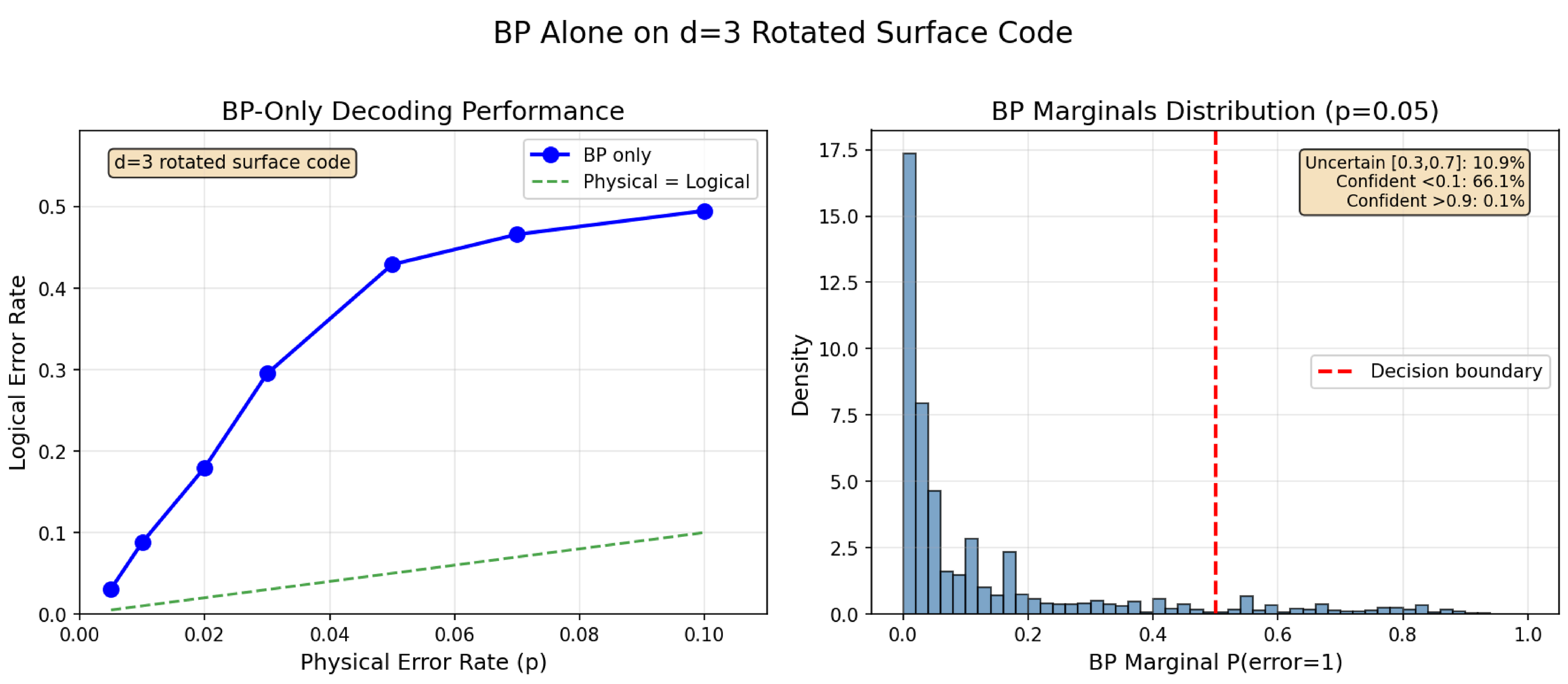


Figure 1: BP alone fails due to degeneracy: the decoder outputs an invalid solution that does not satisfy the syndrome.

Ordered Statistics Decoding (OSD) post-processes BP output:
1. Sort variables by BP reliability
2. Fix most reliable bits using Gaussian elimination
3. Exhaustively search remaining bits (OSD-w searches w bits)

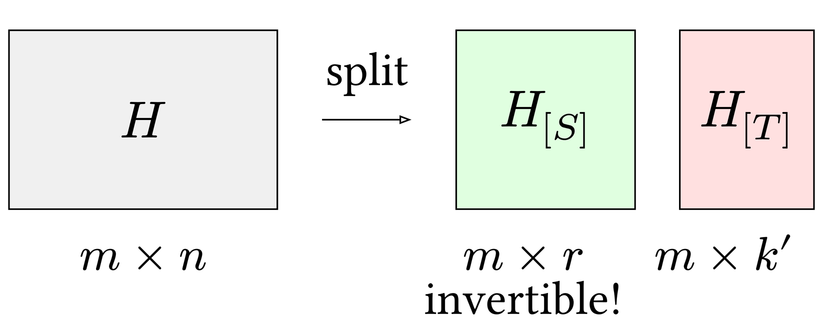
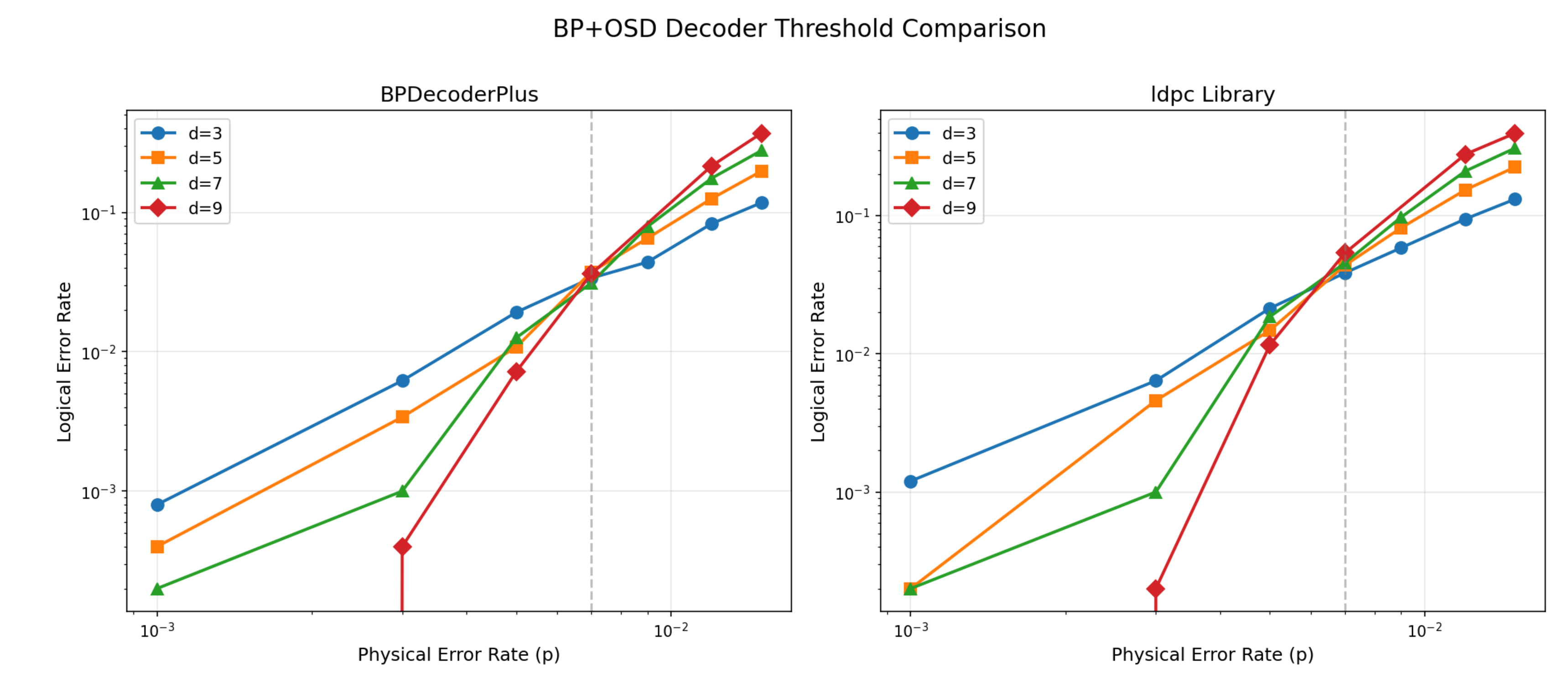


Figure 2: Splitting H into basis and remainder parts

BP+OSD Threshold Results



Noise Model	BP Only	BP+OSD	Optimal
Code capacity	N/A	$\approx 9.9\%$	10.3%
Circuit-level	N/A	$\approx 0.7\%$	$\approx 1\%$

Configuration:
- BP: 60 iterations, min-sum
- Damping: 0.2, OSD order: 10

Validation:
- Matches ldpc library [1]
- Curves cross at threshold

Tropical Tensor Networks for MPE

The **Most Probable Explanation** (MPE) problem finds the most likely error pattern given observations. We solve this exactly using tropical tensor networks.

Tropical Semiring: $(\mathbb{R} \cup \{-\infty\}, \max, +)$
- Addition → max operation
- Multiplication → standard addition

Standard tensor contraction:

$$C_{ik} = \sum_j A_{ij} \cdot B_{jk}$$

Tropical contraction:

$$C_{ik} = \max_j (A_{ij} + B_{jk})$$

For probabilistic graphical models in log-space:

$$\log P(\boldsymbol{x}) = \sum_f \log \psi_f(\boldsymbol{x}_f)$$

Tropical contraction computes $\max_{\boldsymbol{x}} \log P(\boldsymbol{x})$ exactly!

Implementation highlights:
- Uses `omeco` for optimal contraction ordering
- PyTorch backend with GPU support
- Backtracking recovers the optimal assignment
- Complexity: $O(2^{\text{treewidth}})$

Software Architecture

Stim Circuit → DEM → Factor Graph → BP/Tropical → Prediction

Key features:
- **Stim integration**: Generate noisy circuits for rotated surface codes
- **DEM parsing**: Two-stage processing (separator splitting + hyperedge merging)
- **PyTorch backend**: GPU-accelerated batch inference
- **CLI tools**: `generate-noisy-circuits` for dataset creation
- **Modular design**: BP and Tropical modules are independent

Developed with **vibe coding**: human-AI collaboration using Claude Code accelerated development from concept to working threshold plots.

github.com/TensorBFS/BPDecoderPlus

References

[1] O. Higgott and N. P. Breuckmann, "Improved decoding of circuit noise and fragile boundaries of tailored surface codes," *Physical Review X*, vol. 13, no. 3, p. 31007, 2023.

TensorBFS/BPDecoderPlus jinguoliu@hkust-gz.edu.cn