Getting Started with BPDecoderPlus

BP+OSD Decoder for Quantum Error Correction

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Outline

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Outline

What Are We Decoding?

Quantum error correction protects logical qubits by encoding them in many physical qubits.

Physical errors → Syndrome measurements → **Decoder** → Logical error prediction

The decoder's job: Given syndrome measurements, infer what errors occurred and whether they flipped the logical qubit.

Key Challenge

- Degeneracy: identical syndromes.
- Must predict the **logical effect**, not the exact physical errors

Goal

Minimize the **logical error rate** - the probability of incorrect logical predictions

Circuit-Level vs Code-Capacity Noise

Model	Description	Realism
Code-capacity	Errors only on data qubits, perfect measurements	Simplified
Circuit-level	Errors on all operations, noisy measurements	**Realistic**

BPDecoderPlus uses **circuit-level noise** - the realistic model where measurement operations themselves can fail.

Code-Capacity

- Single round of perfect measurements
- Error model: p on each data qubit
- Useful for theoretical analysis

Circuit-Level

- Multiple rounds of noisy measurements
- Errors on gates, idles, measurements
- Required for real hardware

Detection Events

We don't directly use raw syndrome bits. Instead, we use **detection events**:

$$\text{Detection event} = \text{syndrome}[\text{round } t] \oplus \text{syndrome}[\text{round } t-1]$$

A detection event fires (value = 1) when the syndrome **changes** between rounds.

Why Detection Events?

- Raw syndromes are noisy (measurement errors)
- Detection events **cancel** measurement errors that persist across rounds
- Stim's detector error model (DEM) is defined in terms of detection events

Example Timeline

Round	$t-1$	t	$t+1$
Syndrome s	0	1	1
Detection d	—	1	0

Detection $d_t = s_t \oplus s_{t-1}$

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Pipeline Steps

Step	Input	Output	Purpose
1. Generate Circuit	Parameters (d, r, p)	`.stim` file	Define noisy quantum operations
2. Extract DEM	`.stim` circuit	`.dem` file	Map errors → detections
3. Build H Matrix	`.dem` file	H, priors, obs_flip	Decoder input format
4. Sample Syndromes	`.stim` circuit	`.npz` file	Training/test data
5. Decode	H + syndromes	Predictions	Infer logical errors

The DEM: Detector Error Model

The DEM is the crucial link between physical errors and observable detection events.

```
error(0.01) D0 D5 L0
```

This entry means: *"There's a 1% probability of an error that triggers detectors 0 and 5, and flips the logical observable."*

DEM Specifies

- **What errors** can occur (each line is one error mechanism)
- **Which detectors** fire (D0, D1, etc.)
- **Logical effect**
- **Probability** (the number in parentheses)

Generated By

- Stim's circuit analysis (Gidney, 2021)
- Automatic error propagation
- Decomposition of correlated errors

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Factor Graph from H Matrix

The parity check matrix H defines a **factor graph** (Tanner graph) (Kschischang et al., 2001):

- **Variable nodes** (circles): Error mechanisms (columns of H)
- **Check nodes** (squares): Detectors (rows of H)
- **Edges**: $H[i, j] = 1$ connects detector i to error j

	e_1	e_2	e_3	e_4
D_0	1	1	0	1
D_1	0	1	1	1

D_0 checks: $e_1 \oplus e_2 \oplus e_4$
D_1 checks: $e_2 \oplus e_3 \oplus e_4$

Example: $H = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$

- 4 error variables, 2 detectors
- e_2 and e_4 connected to both checks

Message Passing Intuition

BP iteratively passes "beliefs" between nodes (Pearl, 1988):

$$e \xrightarrow{\mu_{e\to D}} D \qquad D \xrightarrow{\mu_{D\to e}} e$$

Step 1
Variable → Check

"Here's my current probability of being an error"

Step 2
Check → Variable

"Given what others told me, here's what you should be to satisfy the parity"

Step 3
Repeat

Until convergence or max iterations reached

After convergence, each variable has a **marginal probability** of being an error.

The Degeneracy Problem

BP works perfectly on trees, but quantum codes have loops!

On loopy graphs, BP can:

- Fail to converge
- Converge to wrong probabilities
- Output invalid solutions ($H \cdot e \neq$ syndrome)

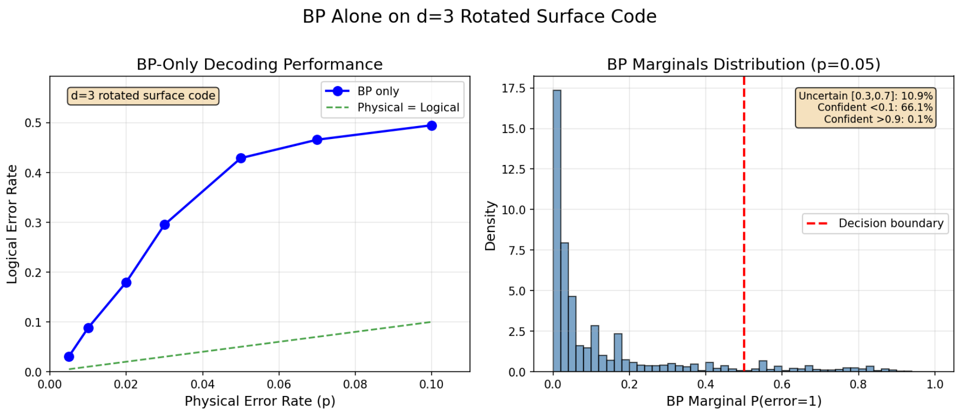
The Degeneracy Problem

Most Critically

BP outputs **probabilities**, but rounding them doesn't guarantee a valid solution.

Why Quantum Codes Are Hard

- Classical LDPC: Each error has unique syndrome signature
- **Quantum surface codes**: Multiple error patterns produce the **same syndrome** (degeneracy)
- BP gets "confused" by equivalent solutions



BP's marginals produce invalid error pattern

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The Key Insight

OSD (Ordered Statistics Decoding) (Fossorier & Lin, 1995) forces a **unique, valid solution** by treating decoding as a system of linear equations:

$$\boxed{H \cdot e = s \pmod 2}$$

Given syndrome s, find error vector e that satisfies this constraint.

BP Provides

- Soft information (probabilities)
- **Unreliable** hard decisions
- No validity guarantee

OSD Guarantees

- **Valid** solutions ($H \cdot e = s$)
- Uses BP's confidence to guide search
- Polynomial time complexity

OSD-0 Algorithm in 3 Steps

Step	**Operation**	**Result**
1. Sort	Order columns by $\lvert \text{LLR} \rvert$ descending	High confidence columns first
2. Row Reduce	Gaussian elimination on H	$[I \mid P]$ form with pivots
3. Solve	Back-substitution with s	Valid error vector e

Result: A valid codeword that respects BP's confident decisions.

OSD-W: Search for Better Solutions

OSD-0 fixes non-pivot bits to BP's decision. **OSD-W** searches over 2^W combinations of the W least confident non-pivot bits.

BP Marginals $\rightarrow$ Sort by Confidence $\rightarrow$ Gaussian Elimination $\rightarrow$ $\boxed{\text{OSD-0 / OSD-W}}$ $\rightarrow$ Best Solution

Method	**Candidates**	**Trade-off**
OSD-0	1	Fast, single solution
OSD-W	2^W	Search W least confident bits
OSD-CS	$1 + k + \binom{W}{2}$	Efficient: weight-0,1,2 patterns

OSD-W: Search for Better Solutions

OSD Order Trade-off

- **OSD-0**: Fast, single solution
- **OSD-10**: $2^{\{10\}}$ candidates, better accuracy
- **OSD-20**: Near-optimal, slower

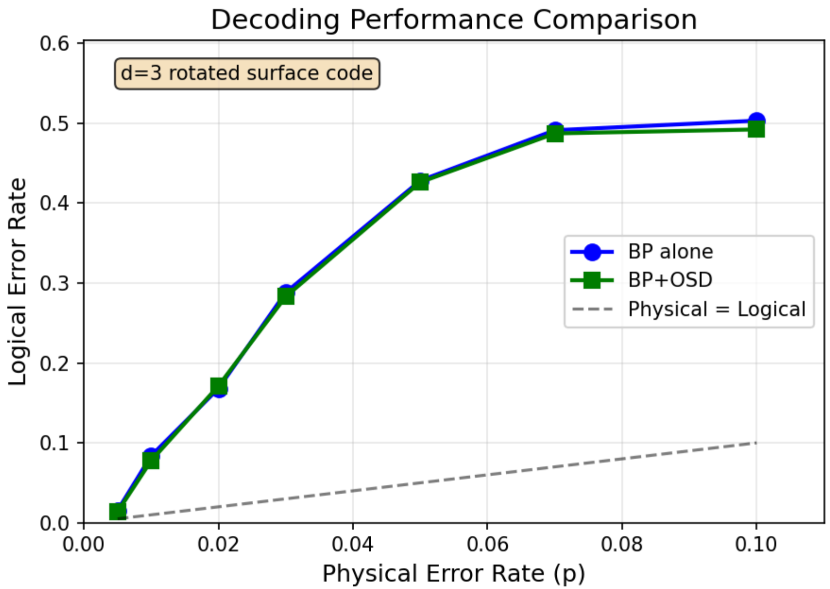
Selection Criterion

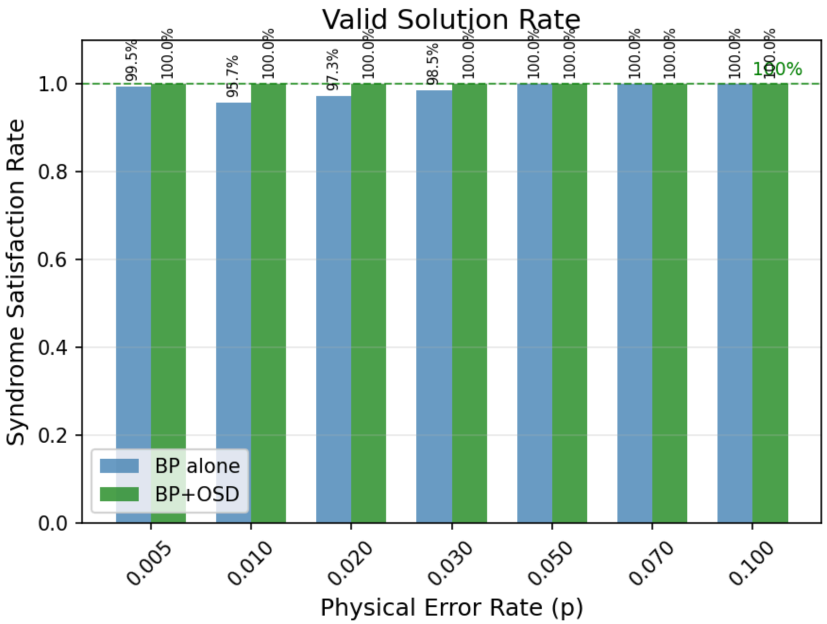
Pick the candidate with lowest **soft-weighted cost**:

$$\mathrm{cost}(e) = \sum_i |\mathrm{LLR}_i| \cdot e_i$$

The Fix in Action

BP+OSD Solves the Hard-Decision Failure





The same syndrome, now decoded correctly with OSD post-processing.

OSD guarantees $H \cdot e = s$

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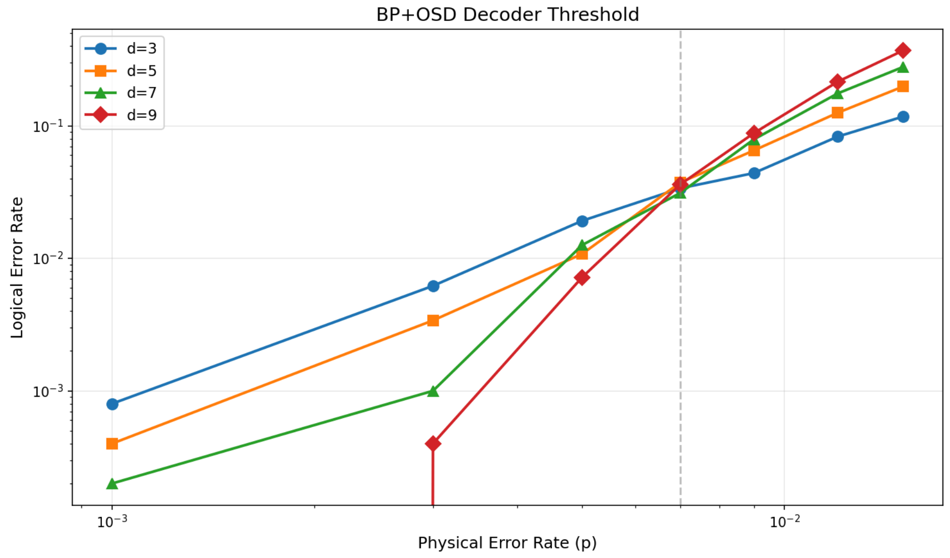
Threshold Results

The **threshold** is the physical error rate below which larger codes perform better.

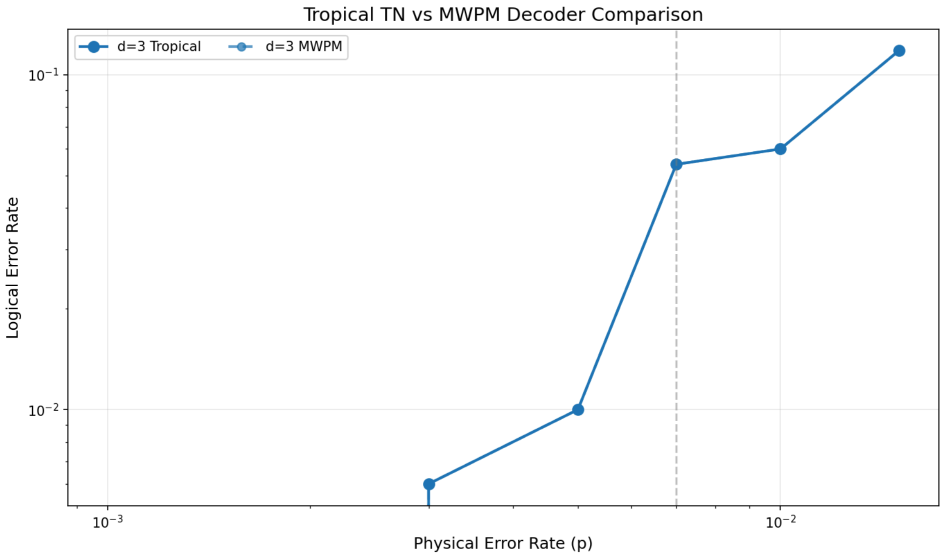


Decoder	Threshold	Notes
BP (damped)	N/A	Fast, limited by loops
BP+OSD	$\sim 0.7\%$	Near-optimal
MWPM	$\sim 0.7\%$	Gold standard (Higgott & Gidney, 2025)

BP+OSD achieves **near-optimal threshold** with good computational efficiency.

Tropical Results

The **threshold** for tropical tensor network decoder require more memory. Further approximate contraction method?



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Tropical is too resource consuming!

Summary

BPDecoderPlus: Fast and accurate decoding for quantum error correction

Key Takeaways

1. **Decoding** = syndrome $\longrightarrow$ error prediction
2. **DEM** maps physical errors to detections
3. **BP** provides soft information but may fail
4. **OSD** guarantees valid solutions
5. **BP+OSD** achieves near-optimal threshold

Next Steps

- Try: `uv run python examples/minimal_example.py`
- CLI: `uv run bpdecoder --help`
- Docs: `docs/usage_guide.md`
- Math: `docs/mathematical_description.md`

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Summary

GitHub: `GiggleLiu/BPDecoderPlus`

Bibliography

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