Statement of Research

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I am a Post-Doctoral fellow in Mikhail Lukin's group at the department of physics at Harvard University with an interest in understanding the connection between computing and physics. The majority of my current research is about understanding the computational hardness and its relation to quantum physics, which I believe is the key to understanding the nature of computation. In the following, I will explain my works in the past two years mainly from two aspects, one is understanding the computational hardness from the solution space properties, and another is embedding computational-hard problems in a physical system.

1 Solution space properties of hard combinatorial optimization problems

1.1 Current Work

In the past two years, I have been most dedicated to creating a unified framework to solve the *solution space properties* [1] of a class of hard combinatorial optimization problems. Here, the solution space property refers to a class of quantities that not only include the maximum or minimum set size, but also include the number of sets at a given size, enumeration of all sets at a given size, and direct sampling of such sets when they are too large to be fit into memory; the class of problem includes but is not limited to the independent set problem, the maximum cut problem, the vertex coloring problem, the maximal clique problem, the dominating set problem, and the satisfiability problem, among others. The framework I and my collaborators created is called generic tensor network, where the word "generic" comes from generic programming in computer science. While the relationship between a counting problem and a tensor network is well known, these works did not catch much attention of computational scientists due to their limited use cases. We take a different view of a tensor network and show how different solution space properties can be computed with the same program by elevating the tensor element algebra to commutative semiring algebras. We show the real algebra is related to the counting of all solutions, the (extended) tropical algebra is related to the largest solution size(s), the polynomial algebra is related to the graph polynomials, the truncated polynomial algebra is related to the degeneracy of solutions with largest or smallest several sizes, the bit string algebra is related to finding one best solution, and the set algebra is related to solution enumeration.

I created an open-source package GraphTensorNetworks (will be public on Github soon) that might benefit people in the field of computational complexity and statistic physics. During the development of this package, I also contributed a lot to the open-source community. I and Chris Elrod wrote the package TropicalGEMM for fast tropical matrix multiplication, which has a very close to the theoretical optimal speed, i.e. half the speed of floating-point number; I rewrote the tensor permutation in Julia base and CUDA to speed up the high-rank tensor manipulation; I wrote the package OMEinsumContractionOrders for the state of the art `einsum` contraction order optimization.

This project is highly motivated by another experimental project that I participated in, where we use variational quantum algorithms to solve the maximum independent set (MIS) problem on a diagonal-coupled unit-disk grid graph (DUGG) by embedding a problem instance into a Rydberg atom array Hamiltonian [2]. In this experiment, it is crucially important to explain why our quantum algorithm works better than classical simulated annealing in some graph instances but not in others. The results found by using the generic tensor network deepened our understanding of the MIS experiment. By checking the degeneracies of independent set solutions at different sizes, we find that the degeneracy ratio can be a good indicator of the classical hardness of a problem instance. By inspecting the configuration space connectivity, we find the absence of the overlap-gap-properties [3] in most of the target problems. Here, the overlap gap property is an important feature of a solution space geometry, the presence of which indicates a no-go for a local-search-based algorithm. These solution space properties that helped understand quantum and classical algorithms are not feasible in the previous frameworks that mainly targeted on best solutions. Our method fills the gap between finding one solution for a hard problem and understanding a hard problem.

1.2 Future Work

It would like to generalize the idea of generic programming to other algorithms using the inclusion-exclusion principle or subset convolution and see what new properties can be computed. To this end, it is worth mentioning dynamic programming. Dynamic programming is closely related to a tropical tensor network, for example, the Viterbi algorithm for finding the most probable configuration in a hidden Markov model can be interpreted as a matrix product state featured with tropical algebra, and the tropical tensor network in the main text is probably equivalent to dynamic programming in finding the best solution. However, while having broader applications, dynamic programming does not have a clear algebraic interpretation required by computing solution space properties beyond finding the best solution.

2 Solving the maximum independent set problem with Rydberg atom arrays

2.1 Current Work

I and my collaborators proposed a scheme to reduce the MIS problem on a general graph to one on a DUGG and showed this reduction is very likely to be optimal [4]. The proposed method only introduce an overhead that is linear to the pathwidth of the source graph, where the pathwidth is a graph characteristic that is upper bounded by the number of vertices. This result may have impacts on both fields of quantum computing and computational complexity.

In the quantum computing field, it is a crucial step toward solving the MIS problem on a general graph with quantum algorithms implemented on Rydberg atom arrays that have highly constrained two-dimensional geometry. In the previous MIS experiment paper, we studied the MIS problem on a DUGG. We showed although DUGGs have highly constrained topology, finding their MISs is nevertheless NP-complete. However the reduction in that paper from an MIS problem on a general graph to an MIS problem on DUGG has an overhead of n^8. If one wants to solve an MIS problem on a general graph with 100 vertices, which can be solved in a classical computer in milliseconds, the required number of qubits is $\sim 10^{16}$, which is a number not feasible in the near term. Our new mapping scheme resolved these concerns.

In the computational complexity field, it can bridge some important results about the MIS problem. For example, a lemma of our reduction is: the MIS size of a unit-disk graph is hard to approximate with an error smaller than $\sqrt{n}$ because the MIS size of a general graph is hard to approximate within $n^{1-\epsilon}$ [5].

Figure 1 is an example of mapping the Petersen graph to a DUGG using the proposed mapping scheme. The paper for this work as well as an open source software will be released soon.

2.2 Future Work

DUGGs have many properties that make them look simple, like the absence of overlap gap property in a random instance, the treewidth that scales square root of the size, and can be constant-approximated in polynomial time, unlike the general MIS problem. We also have a preliminary simulated annealing result that indicates an average DUGG can be solved in polynomial time, which is too good to be true. One possible explanation is: the mapped

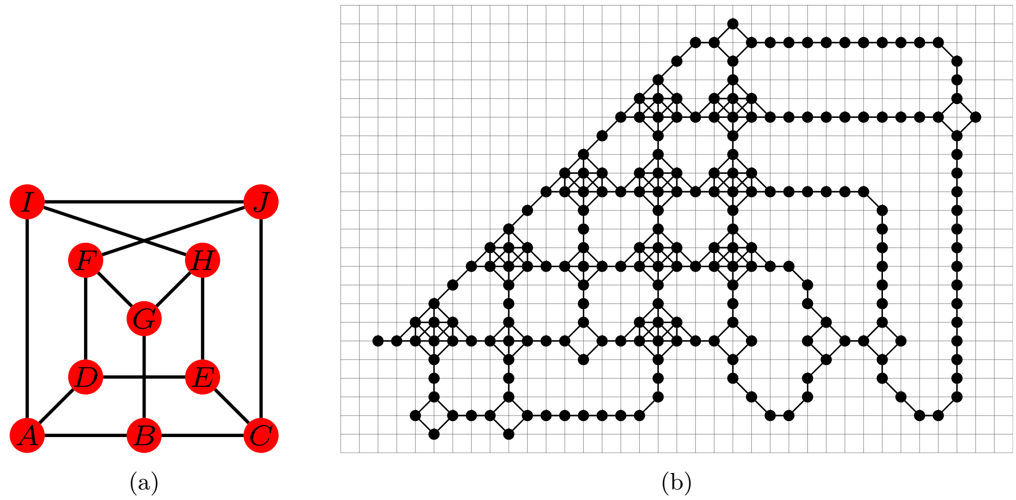

Figure 1: (a) The Petersen graph and (b) its diagonal coupled unit disk grid graph embedding.

graphs live in the hardest corner of the whole set of DUGGs. I want to study and understand DUGGs more to explain these observations seemingly inconsistent with our previous understanding. I also want to study the quantum dynamics of the mapped graph under annealing and deepen my understanding of the performance of quantum annealing on this may be the hardest portion of DUGGs.

3 Summary

To summarize, at this point in my career, my primary interests are computational-hard problems and near-term quantum algorithms. In the future, I want to understand the nature of computational hardness more from the aspects of quantum thermal dynamics and the integrability of a quantum system. Except for these research interests, I want to devote half of my time to the help build up the open-source scientific software ecosystem. I believe programming is a proper way to formalize some part of human knowledge, and open-source software development is an integrable and collaborative practice in this direction.

References

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