

$$L = \| \hat{A} \cdot \vec{x} - \vec{b} \|_2$$

$$= \sqrt{(A \cdot x - b) \cdot (A \cdot x - b)}$$

$$\vec{c} = \hat{A}^T \cdot \vec{x}$$

$$\vec{d} = \vec{c} - \vec{b}$$

$$e = \vec{d}^T \cdot \vec{d}$$

$$L = \sqrt{e}$$

Diagram showing the flow from  $\vec{c}$  to  $\vec{d}$  to  $e$  to  $L$ .

$$\vec{x} = \hat{A}^T \cdot \vec{c}$$

$$\hat{A} = \vec{c} \cdot \vec{x}^T$$

$$\frac{\partial L}{\partial e} = \frac{1}{2\sqrt{e}} \cdot \frac{\partial L}{\partial e}$$

$$\frac{\partial L}{\partial \hat{A}} = \frac{\partial L}{\partial \vec{c}} \cdot \frac{\partial \vec{c}}{\partial \hat{A}} = \frac{\partial L}{\partial \vec{c}} \cdot \vec{x}^T$$

$$\frac{\partial L}{\partial \vec{d}} = \frac{\partial L}{\partial e} \cdot \frac{\partial e}{\partial \vec{d}} = \frac{\partial L}{\partial e} \cdot \vec{d}$$

$$\frac{\partial L}{\partial \vec{x}} = \frac{\partial L}{\partial \vec{c}} \cdot \frac{\partial \vec{c}}{\partial \vec{x}} = A^T \cdot \frac{\partial L}{\partial \vec{c}}$$

$$\frac{\partial L}{\partial \vec{b}} = \frac{\partial L}{\partial \vec{d}} \cdot \frac{\partial \vec{d}}{\partial \vec{b}} = \frac{\partial L}{\partial \vec{d}} \cdot (-1) = -\frac{\partial L}{\partial \vec{d}}$$

$$\frac{\partial L}{\partial \vec{c}} = \frac{\partial L}{\partial \vec{d}} \cdot \frac{\partial \vec{d}}{\partial \vec{c}} = \frac{\partial L}{\partial \vec{d}} \cdot \vec{1}$$