
SOLVING THE MAXIMUM INDEPENDANT SET PROBLEM WITH TROPICAL TENSOR NETWORKS

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ABSTRACT

Solving the maximum independent set size problem: the maximum independent set size, the degeneracy, the optimal configuration and the equivalence between different graphs.

1 TOOLS

OMEinsum a package for einsum

<https://github.com/under-Peter/OMEinsum.jl>

TropicalGEMM a package for efficient tropical matrix multiplication (compatible with OMEinsum)

<https://github.com/TensorBFS/TropicalGEMM.jl>

TropicalNumbers a package providing tropical number types and tropical algebra, one o the dependency of TropicalGEMM

<https://github.com/TensorBFS/TropicalNumbers.jl>

2 COMPUTING DEGENERACY

3 UTILIZING THE SPARSITY

Tensor network compression is an important tool to utilize sparsity.

We contract the tensors in a subregion $R \subseteq G$ of a graph G , and obtain a resulting tensor A of rank $|C|$, where C is the set of vertex tensors at the cut. The maximum independent set size in this region with boundary configuration $\sigma \in \{0, 1\} \otimes |C|$ is A_σ . We say an entry A_{σ_a} is “better” than A_{σ_b} if

$$(\sigma_a \wedge \sigma_b = \sigma_a) \wedge (A_{\sigma_a} \geq A_{\sigma_b}), \quad (1)$$

where \wedge is a bitwise and operations. The first term means that whenever a bit in σ_a has boolean value 1, the corresponding bit in σ_b is also 1. While the second term means the maximum independent set size with boundary configuration fixed to σ_a is not less than that fixed to σ_b . The word “better” means the best solution with boundary configuration σ_a is never worse than that with σ_b . When Eq. 1 holds, It is easy to see that if $\sigma_b \cup \overline{\sigma_b}$ is one of the solutions for maximum independent sets in G , $\sigma_a \cup \overline{\sigma_b}$ is also a solution.

3.1 THE EQUIVALENCE BETWEEN BRANCHING AND COMPRESSION

We are going the verify the Lemmas used in branching algorithm in book (Fomin & Kaski, 2013).

Rule 1. *If a vertex v is in an independent set I , then none of its neighbors can be in I . On the other hand, if I is a maximum (and thus maximal) independent set, and thus if v is not in I then at least one of its neighbors is in I .*

Rule 2. *Let $G = (V, E)$ be a graph, let v and w be adjacent vertices of G such that $N[v] \subseteq N[w]$. Then*

$$\alpha(G) = \alpha(G \setminus w). \quad (2)$$

Rule 3. Let $G = (V, E)$ be a graph and let v be a vertex of G . If no maximum independent set of G contains v then every maximum independent set of G contains at least two vertices of $N(v)$.

Rule 4. Let $G = (V, E)$ be a graph and v a vertex of G . Then

$$\alpha(G) = \max(1 + \alpha(G \setminus N[v]), \alpha(G \setminus (M(v) \cup \{v\}))). \quad (3)$$

Rule 5. Let $G = (V, E)$ be a graph and v be a vertex of G such that $N[v]$ is a clique. Then

$$\alpha(G) = 1 + \alpha(G \setminus N[v]). \quad (4)$$

Rule 6. Let G be a graph, let S be a separator of G and let $I(S)$ be the set of all subsets of S being an independent set of G . Then

$$\alpha(G) = \max_{A \in I(S)} |A| + \alpha(G \setminus (S \cup N[A])). \quad (5)$$

Rule 7. Let $G = (V, E)$ be a disconnected graph and $C \subseteq V$ a connected component of G . Then

$$\alpha(G) = \alpha(G[C]) + \alpha(G \setminus C). \quad (6)$$

if $|V| = 0$ then return 0 if $\exists v \in V$ with $d(v) \leq 1$ then return $1 + \max_{u \in N[v]} \alpha(G \setminus N[v])$ if $\exists v \in V$ with $d(v) = 2$ then let u_1 and u_2 be the neighbors of v and return $\max(\alpha(G \setminus N[v]), \alpha(G \setminus (N[v] \cup \{u_1, u_2\})))$ else return $\alpha(G)$ (7)

REFERENCES

Fedor V Fomin and Petteri Kaski. Exact exponential algorithms. *Communications of the ACM*, 56(3):80–88, 2013.