SOLVING THE MAXIMUM INDEPENDANT SET PROBLEM WITH TROPICAL TENSOR NETWORKS

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ABSTRACT

Solving the maximum independent set size problem: the maximum independent set size, the degeneracy, the optimal configuration and the equivalence between different graphs.

1 TOOLS

OMEinsum a package for einsum
https://github.com/under-Peter/OMEinsum.jl

TropicalGEMM a package for efficient tropical matrix multiplication (compatible with OMEinsum)
https://github.com/TensorBFS/TropicalGEMM.jl

TropicalNumbers a package providing tropical number types and tropical algebra, one o the dependency of TropicalGEMM
https://github.com/TensorBFS/TropicalNumbers.jl

2 COMPUTING DEGENERACY

3 UTILIZING THE SPARSITY

Tensor network compression is an important tool to utilize sparsity.

3.1 THE EQUIVALENCE BETWEEN BRANCHING AND COMPRESSION

We contract the tensors in a subregion $R \subseteq G$ of a graph G, and obtain a resulting tensor A of rank $|C|$, where C is the set of vertice tensors at the cut. The maximum independant set size in this region with boundary configuration $\sigma \in \{0,1\} \otimes |C|$ is A_σ. We say an entry A_{σ_a} is "better" than A_{σ_b} if

$$(\sigma_a \wedge \sigma_b = \sigma_a) \wedge (A_{\sigma_a} \geq A_{\sigma_b}), \tag{1}$$

where $\wedge$ is a bitwise and operations. The first term means that whenever a bit in σ_a has boolean value 1, the corresponding bit in σ_b is also 1. While the second term means the maximum independant set size with boundary configuration fixed to σ_a is not less than that fixed to σ_b. The word "better" means the best solution with boundary configuration σ_a is never worse than that with σ_b. When Eq. 1 holds, It is easy to see that if $\sigma_b \cup \overline{\sigma_b}$ is one of the solutions for maximum independant sets in G, $\sigma_a \cup \overline{\sigma_b}$ is also a solution.

4 VERTIFYING A WIDGET

5

REFERENCES