Numerical Methods for Bulk Entanglement Spectrum

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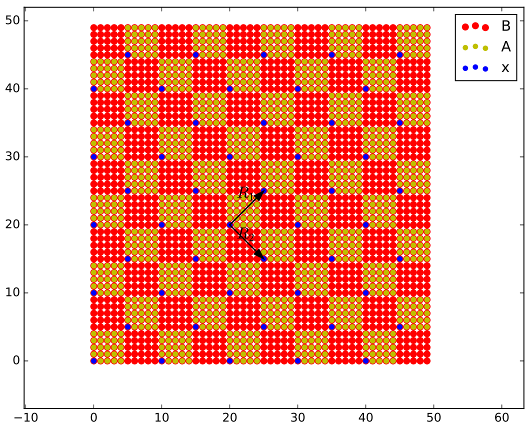

FIG. 1. The lattice configuration, the lattice sites in A are colored in black while those in B in red. $\mathbf{x}$ in blue is used to label the position of blocks(inter-block label). The block spacing is $L \times L$ and the size of A-block is $S \times S$.

I. DEDUCTION OF TRACING PROCEDURE

This note is an improved deduction of the main result in Ref.[1]. The target is to get the bilinear expectation value $\langle c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma'}\rangle$ for region A in Fig.1, with $\mathbf{k}$ the k-vector for block position $\mathbf{x}$ and σ is the inna-block index (it is different with the k-vector $\tilde{\mathbf{k}}$ which is for site position $\tilde{\mathbf{x}}$).

The first step is to express $c_{\mathbf{k},\sigma}$ in terms of $c_{\tilde{\mathbf{k}}}$. Here, the inna-block label σ is also the relative site position with respect to $\mathbf{x}$(the number of possible values of σ is $S \times S$). In other words, $\mathbf{x} + \sigma$ is the global position of a site($\tilde{\mathbf{x}}$).

With N_0 the number of sites and $N(= N_0/L^2)$ the number of blocks, we have

$$c_{\mathbf{k},\sigma} = \frac{1}{\sqrt{N}} \sum_{\mathbf{x}} e^{-i\mathbf{k}\mathbf{x}} c_{\mathbf{x}\sigma} \tag{1}$$

$$= \frac{1}{\sqrt{NN_0}} \sum_{\mathbf{x}} e^{-i\mathbf{k}\mathbf{x}} \left(\sum_{\tilde{\mathbf{k}}} e^{i\tilde{\mathbf{k}}(\sigma+\mathbf{x})} c_{\tilde{\mathbf{k}}} \right) \tag{2}$$

$$= \frac{1}{\sqrt{NN_0}} \sum_{\tilde{\mathbf{k}}} \left(\sum_{\mathbf{x}} e^{i(\tilde{\mathbf{k}}-\mathbf{k})\mathbf{x}} \right) e^{i\tilde{\mathbf{k}}\sigma} c_{\tilde{\mathbf{k}}} \tag{3}$$

Up to now, the deduction is straight forward, then we are going to to evaluate the equation in the parentheses.

Soon we notice that only those $\tilde{\mathbf{k}}$s 'equivalent' to $\mathbf{k}$ will contribute to the above equation, that is

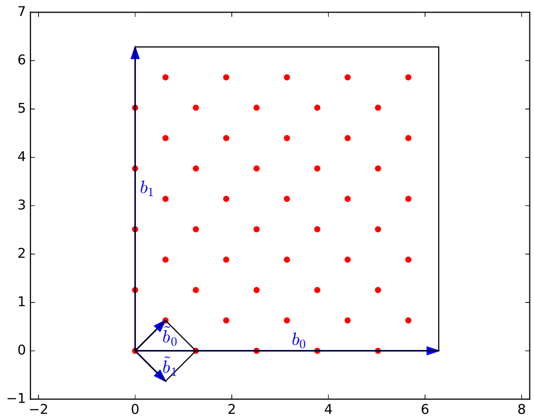

FIG. 2. The first Brillouin zones for site space(with unit vector $\tilde{\mathbf{b}}$) and block space(with unit vector $\mathbf{b}$), momentum equivalent to Γ with respect to $\mathbf{b}$ are colored in red, the collection of these points is Γ^*.

$$c_{\mathbf{k},\sigma} = \sqrt{\frac{N}{N_0}} \sum_{\tilde{\mathbf{k}}} \delta(\tilde{\mathbf{k}} - \mathbf{k} - n_1\mathbf{b}_1 - n_2\mathbf{b}_2) e^{i\tilde{\mathbf{k}}\sigma} c_{\tilde{\mathbf{k}}} \tag{4}$$

with n_1, n_2 arbituary integers.

To find out these equivalent points, take a glance of the first Bruillouin zones as shown in Fig.2,

The number of members in Γ^* is $2L \times L$ and they are $n_1\mathbf{b}_1 + n_2\mathbf{b}_2$. Thus we have,

$$c_{\mathbf{k},\sigma} = \sqrt{\frac{N}{N_0}} \sum_{\tilde{\mathbf{k}}-\mathbf{k}\in\Gamma^*} e^{i\tilde{\mathbf{k}}\sigma} c_{\tilde{\mathbf{k}}} \tag{5}$$

Then, it is straight forward to express a bilinear operator after the above tracing procedure,

$$\langle c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma'}\rangle = \frac{N}{N_0} \sum_{\tilde{\mathbf{k}}-\mathbf{k}\in\Gamma^*} e^{i\tilde{\mathbf{k}}(\sigma'-\sigma)} \langle c^\dagger_{\tilde{\mathbf{k}}} c_{\tilde{\mathbf{k}}}\rangle \tag{6}$$

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[1] T. H. Hsieh and L. Fu, Phys. Rev. Lett. **113**, 106801 (2014).