University of Nottingham
UK | CHINA | MALAYSIA

Foundation Algebra for Physical Sciences and Engineering (CELEN036)

Homework 5

Note: for questions 1 to 3, $p(x)$ is the polynomial (dividend), $s(x)$ is the divisor, $q(x)$ is the quotient, and r is the remainder.

1. Divide using long division. Write the result as $p(x) = s(x)q(x) + r$.

   a. $\dfrac{x^3 - 5x^2 - 4x + 23}{x - 2}$

   b. $(2x^3 + 5x^2 + 4x + 17) \div (x + 3)$

   c. $(x^3 - 8x^2 + 11x + 20) \div (x - 5)$

2. Divide using synthetic division. Write the result as $\dfrac{p(x)}{s(x)} = q(x) + \dfrac{r}{s(x)}$.

   a. $\dfrac{2x^2 - 5x - 3}{x - 3}$

   b. $(x^3 - 3x^2 - 14x - 8) \div (x + 2)$

   c. $\dfrac{x^3 - 5x^2 - 4x + 23}{x - 2}$

   d. $(2x^3 - 5x^2 - 11x - 17) \div (x - 4)$

3. Divide using synthetic division. Write the result as $p(x) = s(x)q(x) + r$.

   a. $(x^3 + 5x^2 + 7) \div (x + 1)$

   b. $(x^3 - 13x^2 - 12) \div (x - 4)$

   c. $\dfrac{3x^3 - 8x + 12}{x - 1}$

   d. $(n^3 + 27) \div (n + 3)$

   e. $(x^4 + 3x^3 - 16x - 8) \div (x - 2)$

4. Use the remainder theorem to show that the given number is a zero of the polynomial.

   a. $P(x) = x^3 + 2x^2 - 5x - 6.\ x = -3$

   b. $f(x) = x^3 - 7x + 6.\ x = 2$

   c. $h(x) = 9x^3 + 18x^2 - 4x - 8.\ x = \dfrac{2}{3}$

5. Use the factor theorem to determine if the expressions given are factors of the polynomial.

   $a.$ $f(x) = x^3 - 3x^2 - 13x + 15$
   $i.$ $(x + 3)$ $ii.$ $(x - 5)$

   $b.$ $h(x) = x^3 - 6x^2 + 3x + 10$
   : $i.$ $(x + 2)$ $ii.$ $(x - 5)$

   $c.$ $q(x) = -2x^3 - x^2 + 12x - 9$
   : $i.$ $(x + 3)$ $ii.$ $(2x - 3)$

6. If $p(x)$ is a polynomial with integer coefficients, and a leading coefficient of 1, then at least one of the integer zeros of $p(x)$ if they exist must be a factor of the constant term. Use this property to completely factorise the following polynomials.

   $a.$ $p(x) = x^3 - 3x^2 - 9x + 27$

   $b.$ $p(x) = x^3 - 6x^2 + 12x - 8$