

Homework 4 (Questions and Answers)

1. Use the addition and subtraction formulae to prove the following trig identities.

$$(i) \quad \frac{\cos(\alpha - \beta)}{\cos \alpha \sin \beta} = \tan \alpha + \cot \beta$$

$$(ii) \quad \frac{\sin(s + t)}{\cos s \cos t} = \tan s + \tan t$$

$$(iii) \quad \frac{\sin(x - y)}{\sin(x + y)} = \frac{\tan x - \tan y}{\tan x + \tan y}$$

$$(iv) \quad \frac{\sin(x + y)}{\cos(x - y)} = \frac{\cot x + \cot y}{1 + \cot x \cot y}$$

$$(v) \quad \frac{\sin(s - t)}{\sin t} + \frac{\cos(s - t)}{\cos t} = \frac{\sin s}{\sin t \cos t}$$

$$(vi) \quad \frac{\tan(\alpha + \beta) - \tan \beta}{1 + \tan(\alpha + \beta) \tan \beta} = \tan \alpha$$

2. Use factor formulae to factorise the following trigonometric expressions.

$$(i) \quad \sin 3A + \sin A$$

$$(ii) \quad \cos 5A + \cos 3A$$

$$(iii) \quad \sin 4A - \sin 2A$$

$$(iv) \quad \cos 7A - \cos A$$

$$(v) \quad \sin 3A - \sin 5A$$

$$(vi) \quad \cos A - \cos 5A$$

$$(vii) \quad \cos 70^\circ + \cos 50^\circ$$

$$(viii) \quad \sin 2A + 1$$

$$(ix) \quad 1 + \cos 4A$$

3. Prove the following identities.

$$(i) \quad \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} = \frac{\tan(A + B)}{\tan(A - B)}$$

$$(ii) \quad \frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \tan 5A$$

$$(iii) \quad \frac{\sin 3x + \sin 5x}{\sin 4x + \sin 6x} = \frac{\sin 4x}{\sin 5x}$$

$$(iv) \quad \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$$

$$(v) \quad \cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 12\theta = 4 \cos 3\theta \cos 4\theta \cos 5\theta$$

4. Solve the following equations giving the values of θ from 0° to 360° .

(i) $\cos 2\theta = \sin \theta$

(ii) $\sin 2\theta + \cos \theta = 0$

(iii) $4 - 5 \cos \theta = 2 \sin^2 \theta$

(iv) $\tan \theta \tan 2\theta = 2$

(v) $\sin 2\theta - 1 = \cos 2\theta$

(vi) $5 \cos \theta \sin 2\theta + 4 \sin^2 \theta = 4$

5. Transform the following equations into the suggested compound angle form ; $0 < \theta < \frac{\pi}{2}$.

(i.) $\sqrt{3} \cos x - \sin x \quad r \cos(x + \theta)$

(ii.) $\cos x + 3 \sin x \quad r \cos(x - \theta)$

(iii.) $4 \sin x - 3 \cos x \quad r \sin(x - \theta)$

(iv.) $\cos 2x - \sin 2x \quad r \cos(2x + \theta)$

Answers

2. (i) $2 \sin 2A \cos A$ (ii) $2 \cos 4A \cos A$ (iii) $2 \cos 3A \sin A$
- (iv) $-2 \sin 4A \sin 3A$ (v) $-2 \cos 4A \sin A$ (vi) $2 \sin 3A \sin 2A$
- (vii) $2 \cos 60^\circ \cos 10^\circ$ (viii) $2 \sin(A + 45^\circ) \cos(A - 45^\circ)$ (ix) $2 \cos^2 2A$
4. (i) $30^\circ, 150^\circ, 270^\circ$ (ii) $90^\circ, 210^\circ, 270^\circ, 330^\circ$ (iii) $60^\circ, 330^\circ,$
- (iv) $35.26^\circ, 144.74^\circ, 215.26^\circ, 324.74^\circ$ (v) $90^\circ, 270^\circ, 45^\circ, 225^\circ$
- (vi) $90^\circ, 270^\circ, 23.58^\circ, 156.42^\circ$
5. (i) $2 \cos\left(x + \frac{\pi}{6}\right)$ (ii) $\sqrt{10} \cos(x - 71.57^\circ)$ (iii) $5 \sin(x - 36.87^\circ)$
- (iv) $\sqrt{2} \cos\left(2x + \frac{\pi}{4}\right)$