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Foundation Calculus and Mathematical Techniques (CELEN037)

Answers to Worksheet #3

1.

(i) $-\dfrac{b}{a}$ (ii) -1 (iii) 1

(iv) -1 (v) $\dfrac{v_0 \sin\alpha - gt}{v_0 \cos\alpha}$ (vi) $-\dfrac{2t}{1-t^2}$

2.

(i) $1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$

(ii) $1 + x + x^2 + x^3 + x^4 + \cdots$

(iii) $1 - \dfrac{x}{2} - \dfrac{x^2}{8} - \dfrac{x^3}{16} + \cdots$

(iv) $1 + x - \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots$

(v) $x + \dfrac{x^3}{3} + \dfrac{2x^5}{15} + \dfrac{17x^7}{315} + \dfrac{62x^9}{2835} + \cdots$

(vi) $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$

$e^{-x} = 1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \cdots$

$$\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \cdots = \sum_1^\infty \frac{x^{2k-1}}{(2k-1)!}$$

(vii) $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$

$$\ln\left(\frac{1+x}{1-x}\right) = 2x + \frac{2x^3}{3} + \frac{2x^5}{5} - \frac{2x^7}{7} + \cdots = 2\sum_1^\infty \frac{x^{2k-1}}{(2k-1)}$$

3.

(i) Equation of tangent line $y + x + 2 = 0$ Equation of normal line $y = x$

(ii) Equation of tangent line $y - 2x + 3 = 0$ Equation of normal line $2y + x + 1 = 0$

(iii) Equation of tangent line $2y + 3x - 6 = 0$ Equation of normal line $3y - 2x - 9 = 0$

4.

(i) $x = 1.93457$

(ii) $x = 1.895494$

(iii) $x = 1.021690$

(iv) $x = -1.272020$

(v) $x = 1.1777035$

(vi) $x = 2.09455148$

(vii) $x = 2.09455148$

(viii) $x = 1.25992105$

5.

(i) $f'(x) = \sec^2 x > 0$ for all $x \in \mathbb{R}$

(ii) $f'(x) = -\sin x > 0$ for all $x \in (2k\pi - \frac{\pi}{2}, 2k\pi), k \in \mathbb{Z}$

(iii) $f'(x) = -2(e^{-2x}) < 0$ for all $x \in \mathbb{R}$

(iv) $f'(x) = 2x + 2 < 0$ for all $x < -1$

(v) In $(-\infty, -1.3)$ and $(1.3, \infty)$, f is increasing.
In $(-1.3, 1.3)$, f is decreasing.

(vi) In $(-\infty, -\frac{3\sqrt{73}+9}{16})$ and $(0, \frac{3\sqrt{73}-9}{16})$, f is decreasing.
In $(-\frac{3\sqrt{73}+9}{16}, 0)$ and $(\frac{3\sqrt{73}-9}{16}, \infty)$, f is increasing.