COMP3065
Computer Vision

Topic 4 – Image Stitching 1

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2025 Spring

Outline

- Geometric Transformation
  - Translation
  - Euclidean
  - Similarity
  - Affine
  - Projective
- Image Stitching
- Compute Transformation
- RANSAC

Image Stitching

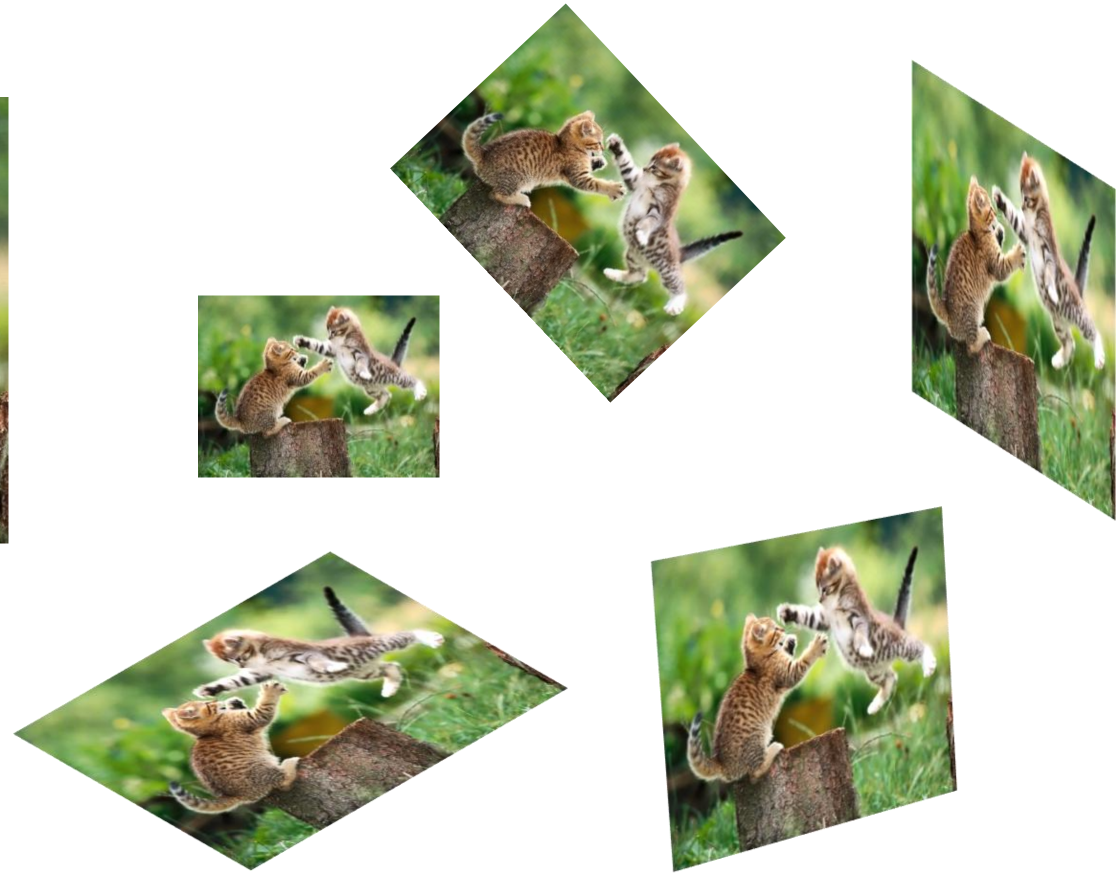
Combine two or more overlapping images to make one larger image

Slides credit: Shapino

Image Stitching: Geometric Transformation

We need to **transform** one image first in order to stitch the two images to together!

What Are Geometric Transformations?

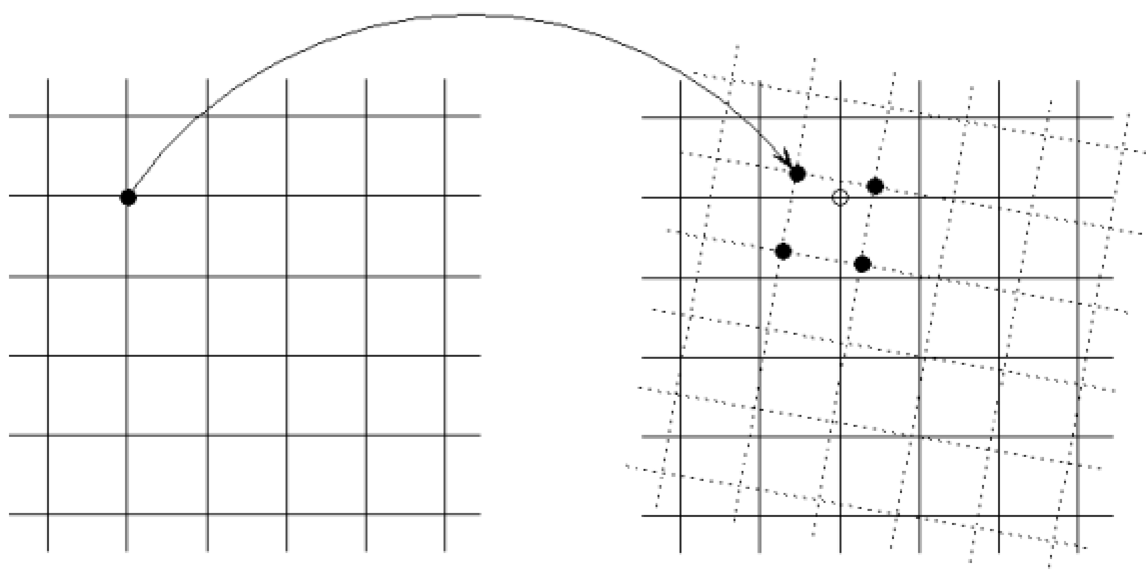


How do we do them?

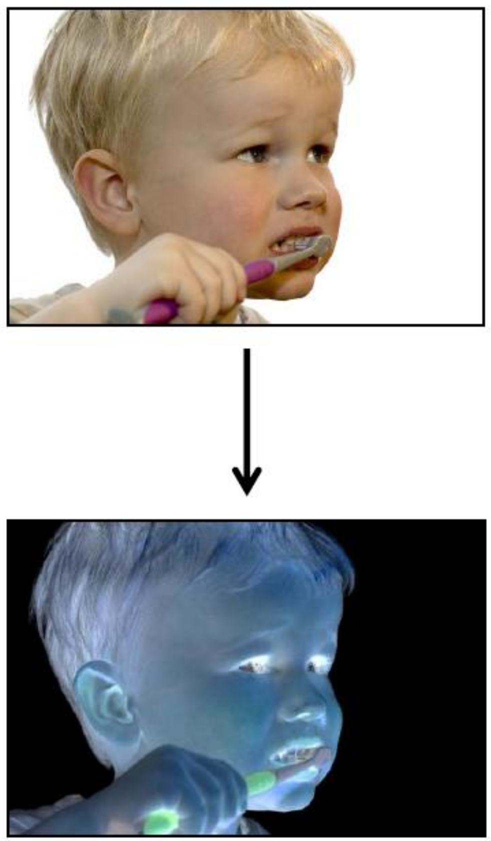
What Are Geometric Transformations?

- In a geometric transformation each point (x, y) of image A is mapped to a point (u, v) in a new coordinate system

$$u = f_1(x, y)$$
$$v = f_2(x, y)$$

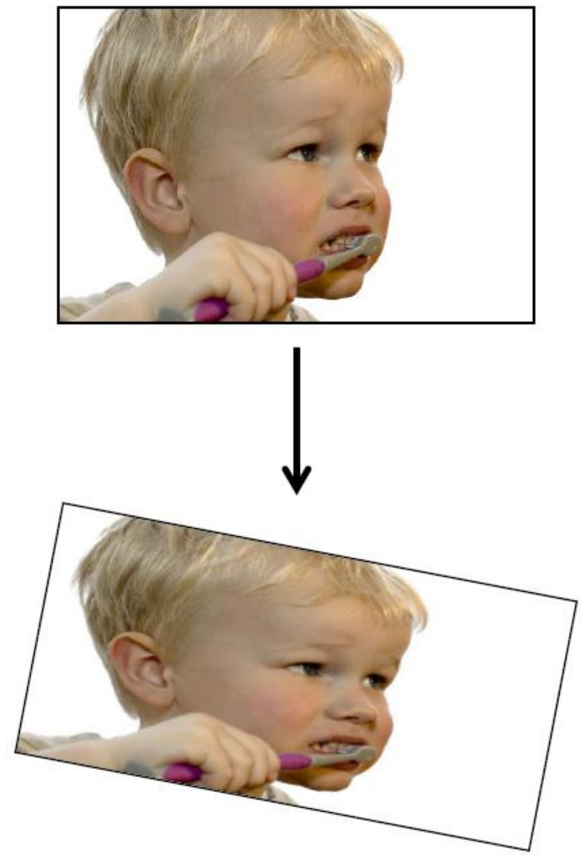


What Are Geometric Transformations?



changes pixel *values*

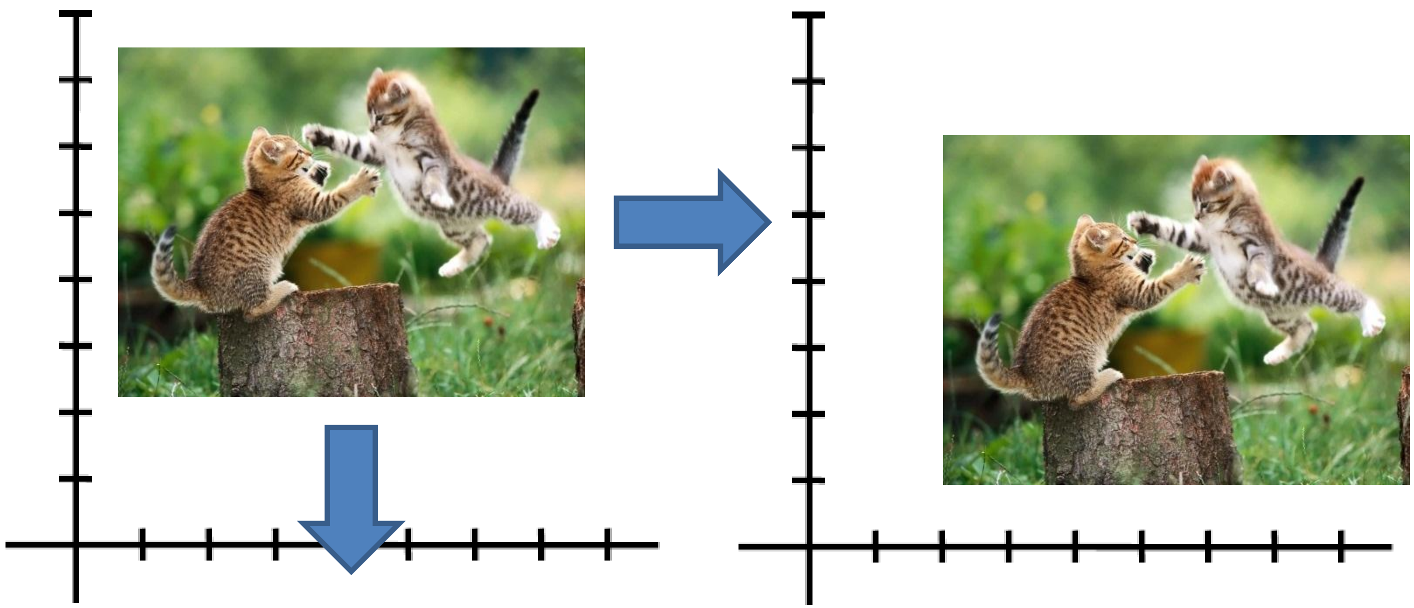
changes *range* of image function



changes pixel *locations*

changes *domain* of image function

Translation

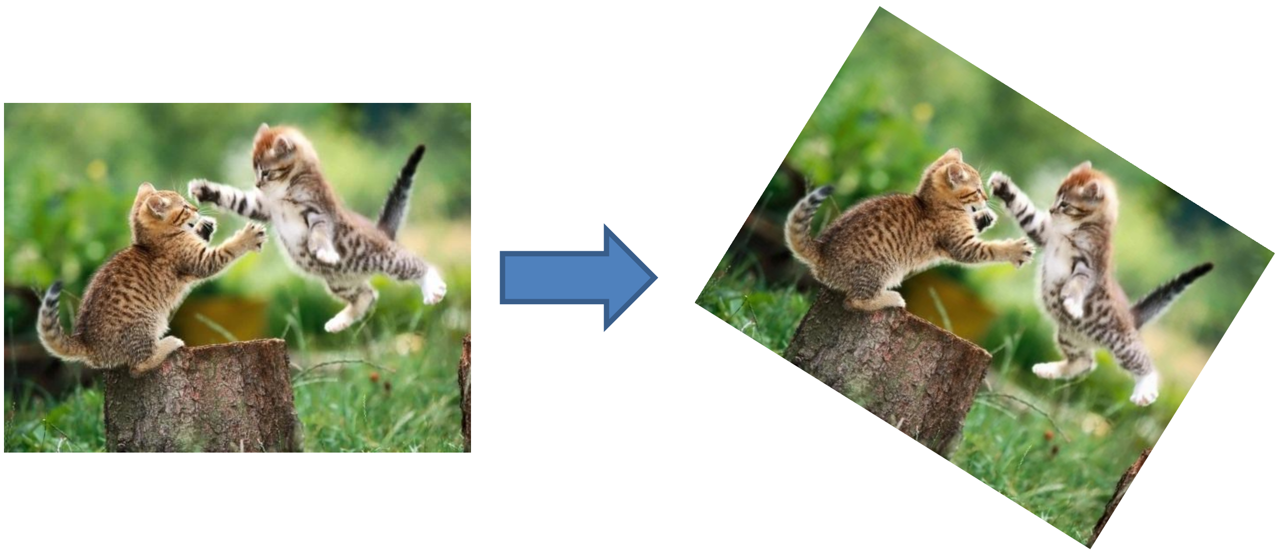


x and *y* are pixel location.

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

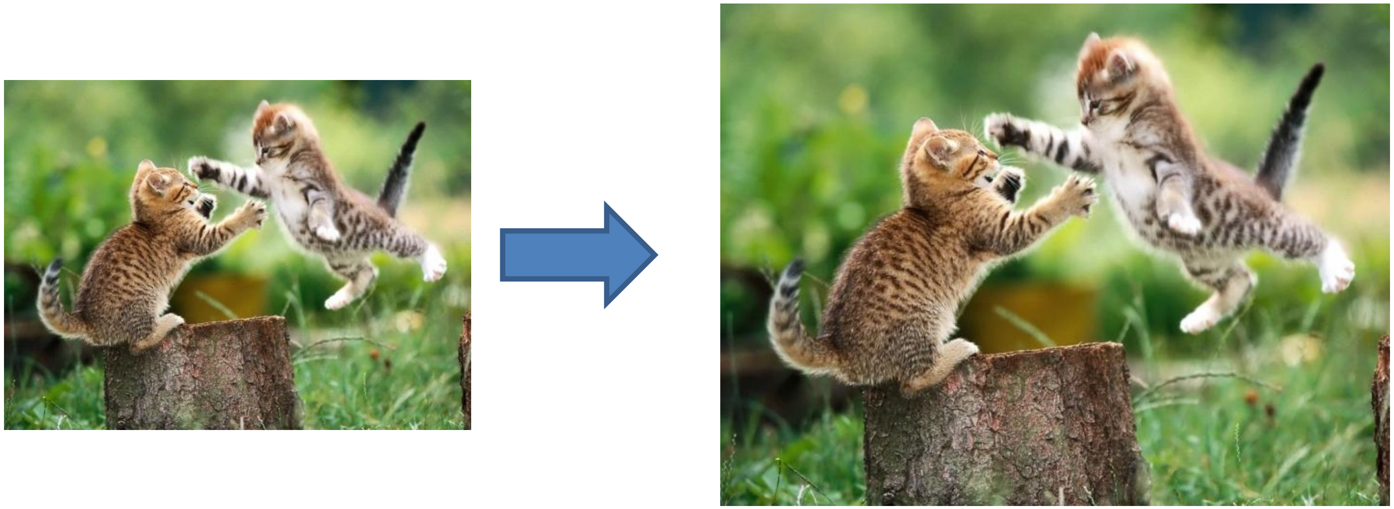
Ignore last row for now

Rotation



$$\begin{bmatrix} cos(\theta) & -sin(\theta) & t_x \\ sin(\theta) & cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

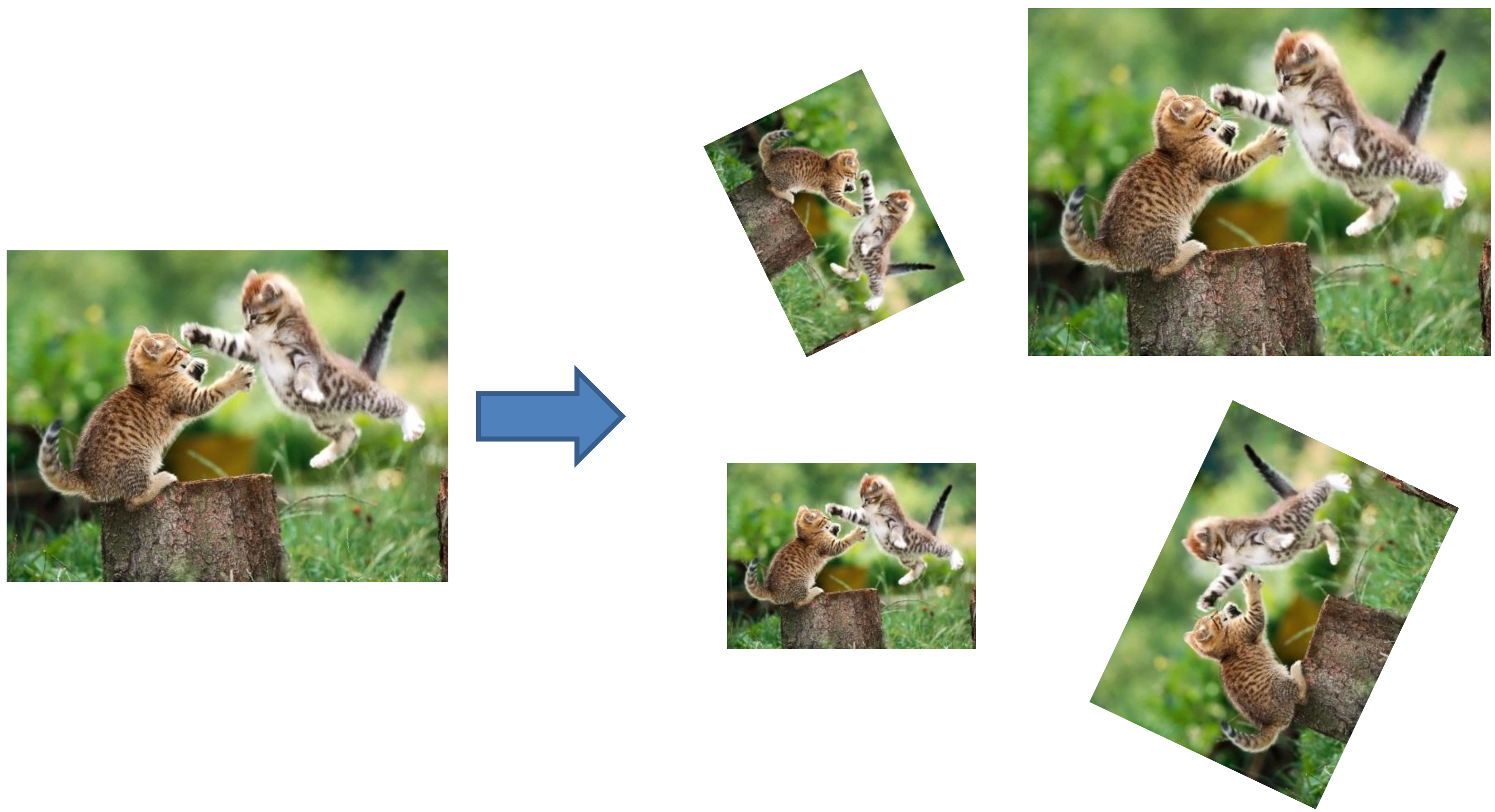
Scale



Each component is multiplied by a scalar

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Similarity



Euclidean transform = translation + rotation

Similarity transform = translation + rotation + scale

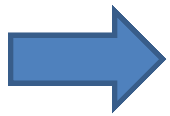
Similarity

- Any transform of the form:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ -b & a & d \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- 4 degrees of freedom (DOF)

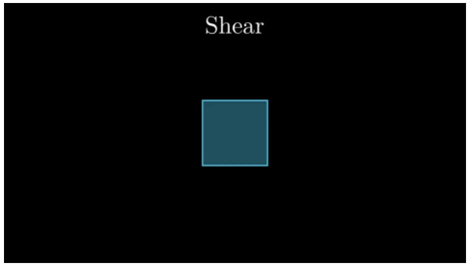
Aspect Ratio



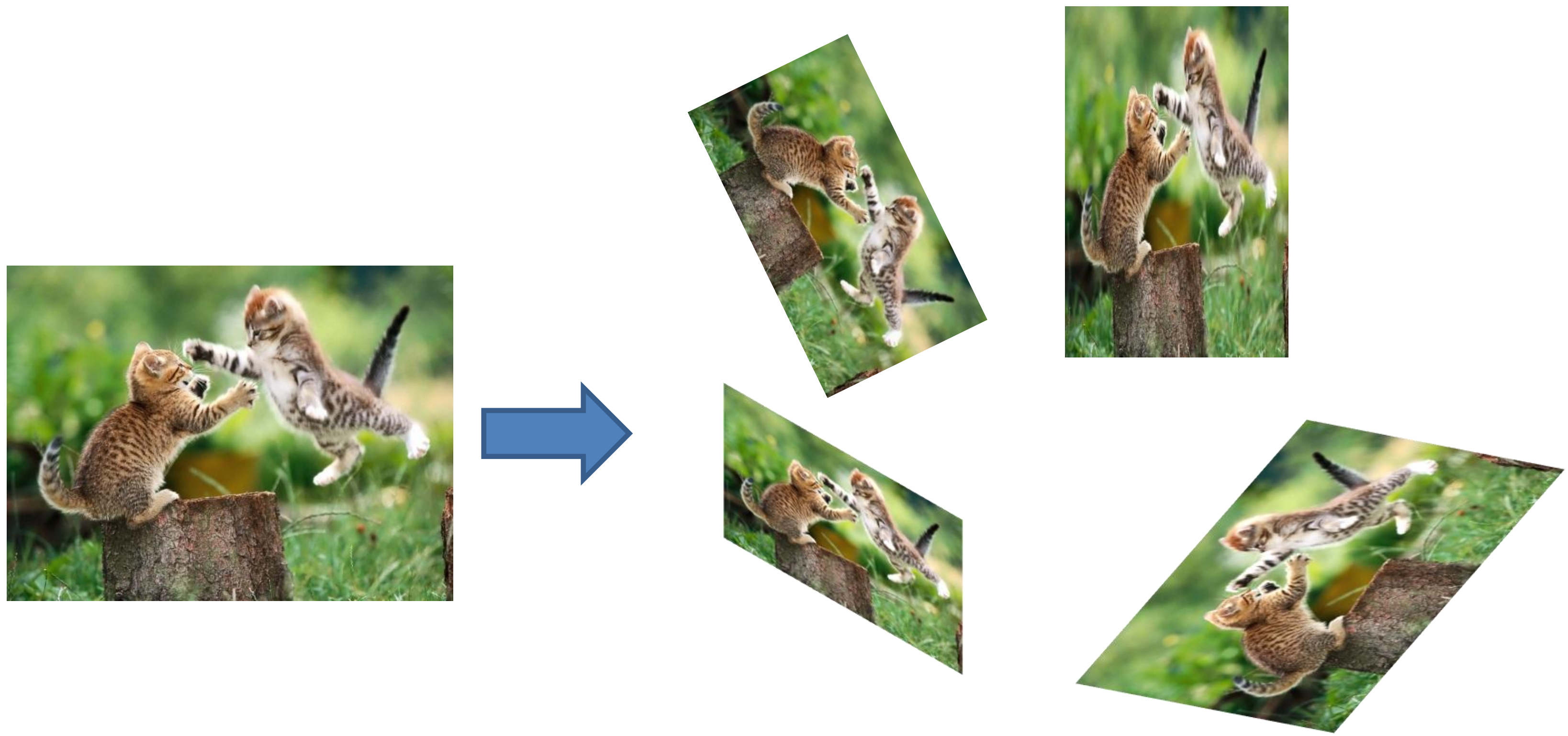
$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Shear

$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$



Affine



Affine transform = translation + rotation +
scale + aspect ratio + shear

Preserves: Parallelism

Affine

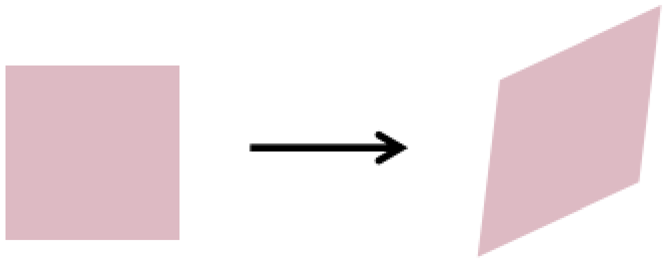
- Any transform of the form:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- ? degrees of freedom (DOF)

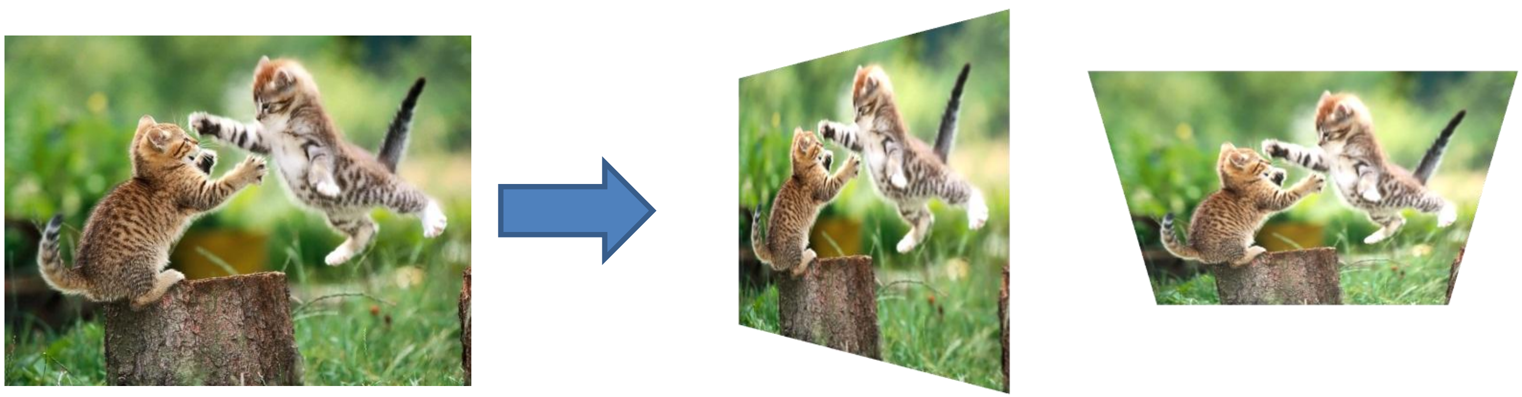
Properties of Affine

- Origin **does not** necessarily **map** to origin
- Lines **map** to lines
- Parallel lines **map** to parallel lines
- Ratios (lengths and areas) are **preserved**
- **Compositions** of affine transforms are also affine transforms



Anything Else?

- Are there any other planar transformation?



Homogeneous Coordinates

heterogeneous coordinates		homogeneous coordinates
$\begin{bmatrix} x \\ y \end{bmatrix}$	$\Rightarrow$	$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

add a 1 here

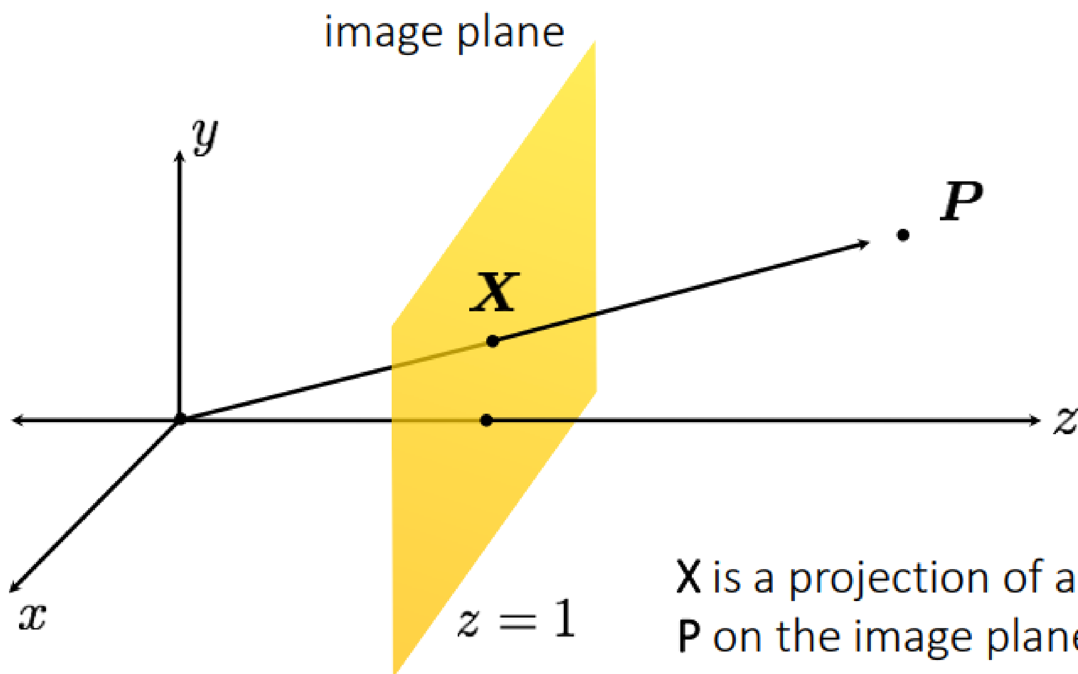
- Represent 2D point with a 3D vector

Projective Geometry

image point in pixel coordinates $\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\Updownarrow$$

image point in homogeneous coordinates $\boldsymbol{X} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$



X is a projection of a point **P** on the image plane

- Model an image as a plane in space, and project it onto any other image

Homogeneous Coordinates

homogeneous → heterogeneous

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow \begin{bmatrix} x/w \\ y/w \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

One extra step:

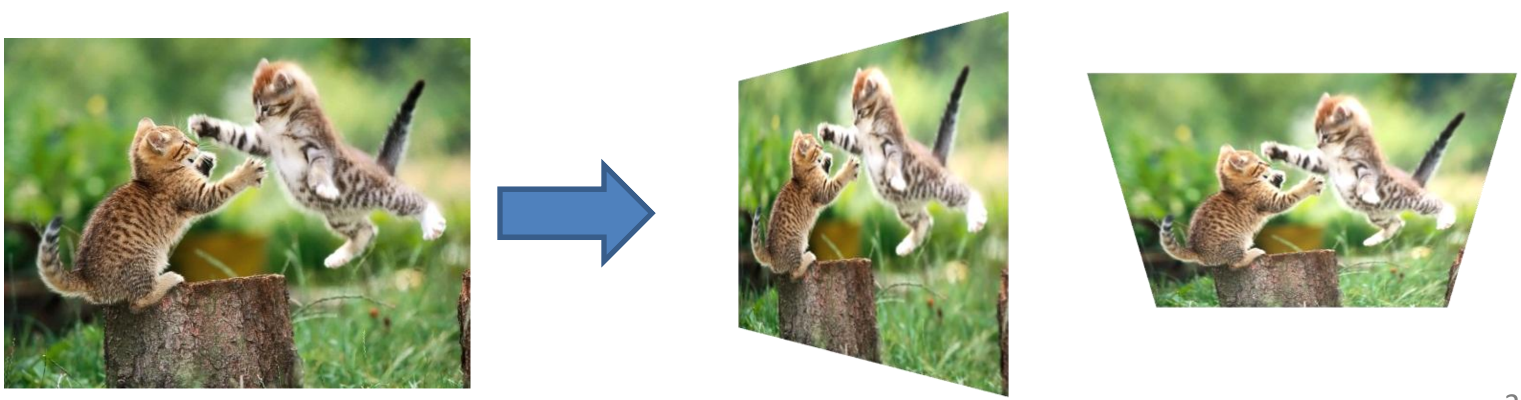
$$x' = u/w$$

$$y' = v/w$$

Projective Transformation

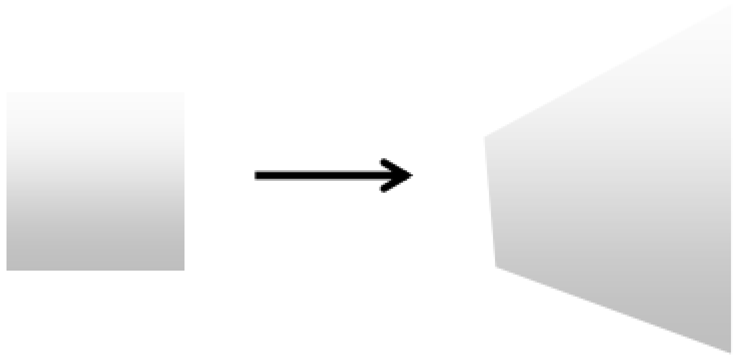
$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$x' = u/w$$
$$y' = v/w$$



Properties of Projective

- Origin **does not** necessarily **map** to origin
- Lines **map** to lines
- Parallel lines **do not** necessarily **map** to parallel lines
- Ratios (lengths and areas) are **not** necessarily **preserved**
- **Compositions** of projective transforms are also projective transforms



Classification of 2D transformations

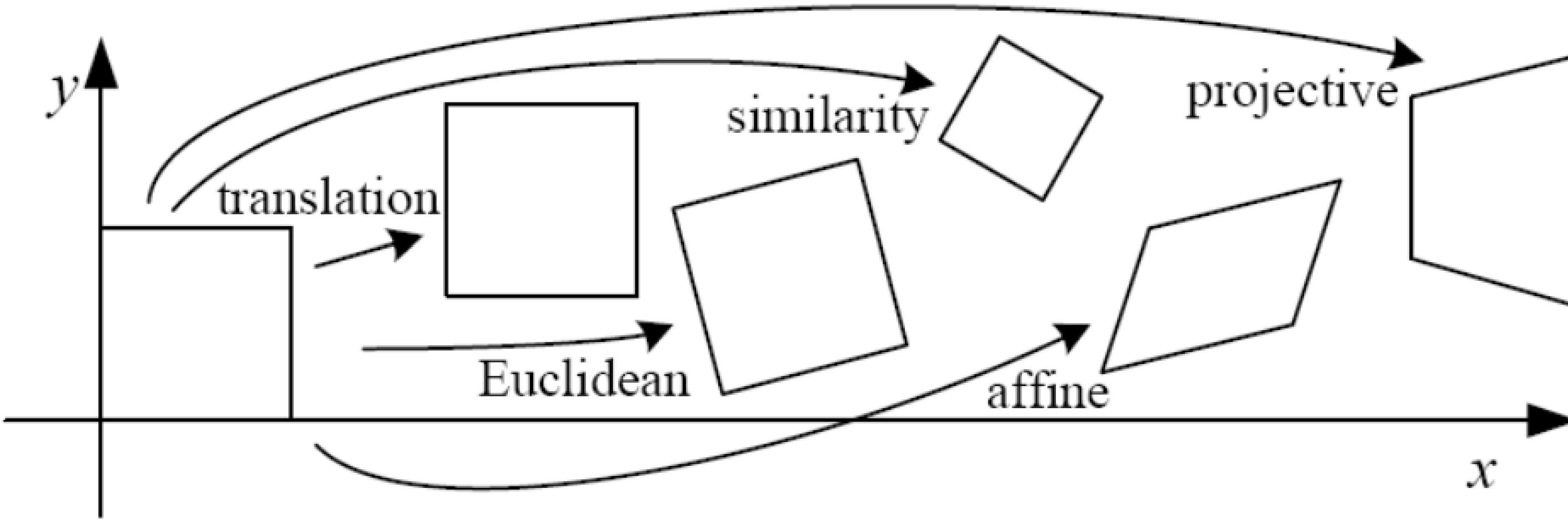


Image Stitching: The Idea

1. Take a sequence of images from the same position.
2. To stitch two images: compute transformation between second image and first.
3. Shift (warp) the second image to overlap with the first.
4. Blend the two together to create a mosaic.
5. If there are more images, repeat step 2 to 4.

The Idea

1. Take a sequence of images from the same position.
   - Rotate the camera about its optical center, (more accurate with camera mounted on a tripod).
   - No tripods? Hold the camera and turn the body without changing the position.

The Idea

2. Compute transformation between images.
    - Extract interest points
    - Find Matches
    - Compute transformation

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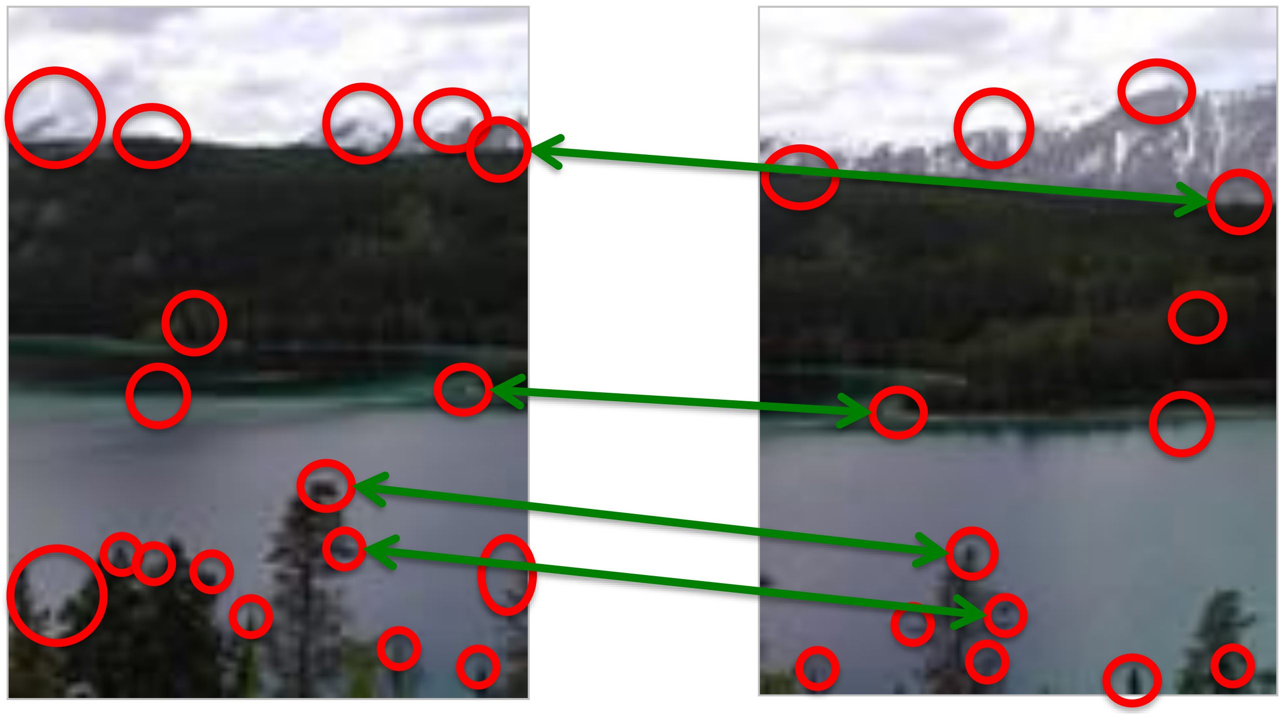

The Idea

3. Shift image to overlap.

The Idea

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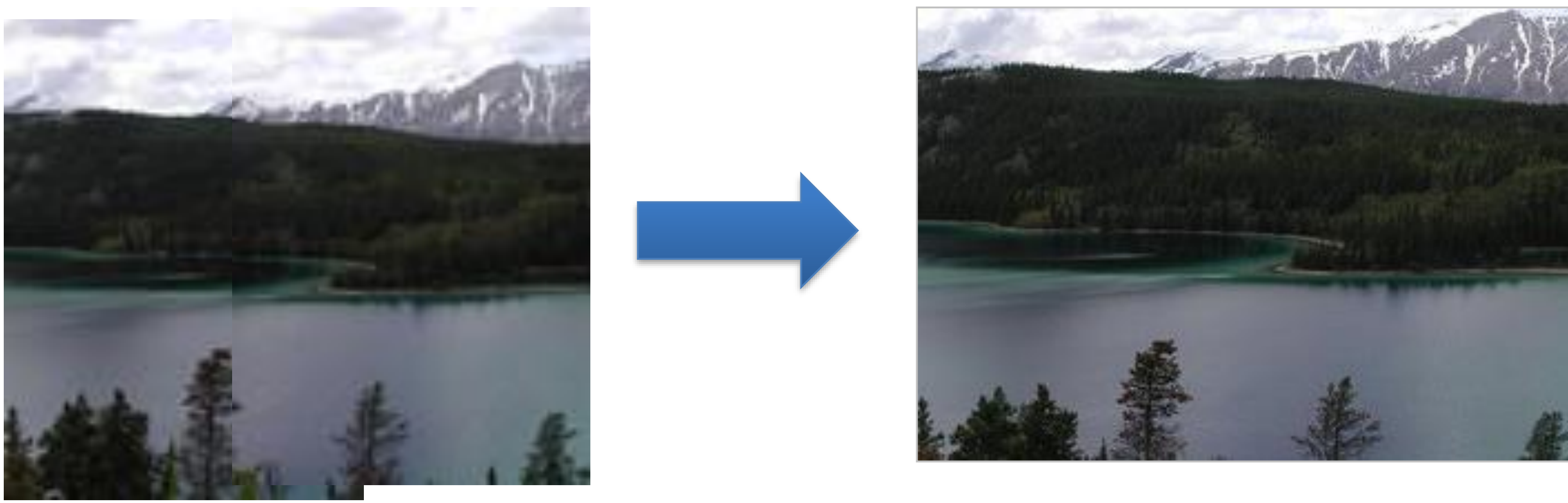
The Idea

4. Blend together to create mosaic.



The Idea

5. Repeat for all images.

The Idea

1. Take a sequence of images from the same position.
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The details …

Compute Transformation

- Extract feature points (e.g. SIFT)
- Find good matches (e.g. compare feature vectors)

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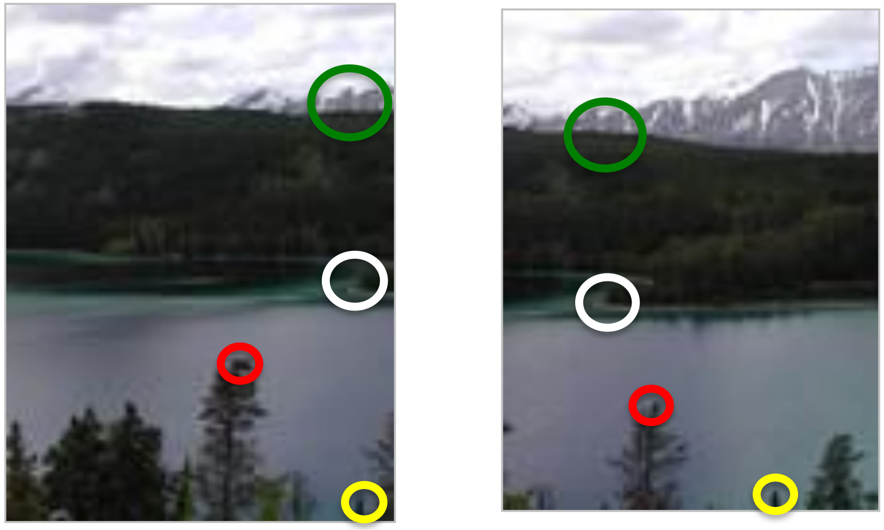
- Find good matches (e.g. compare feature vectors)
- **Compute Transformation**

Let’s assume we are given a set of good matching interest points



The Underline Theory: Image Reprojection

The mosaic has a natural interpretation in 3D.

- The images are reprojected onto a common plane.
- The mosaic is formed on this plane.

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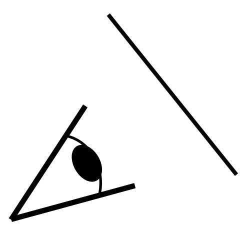

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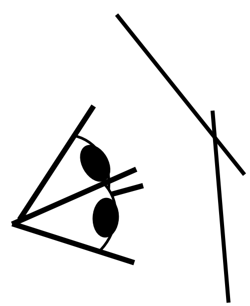

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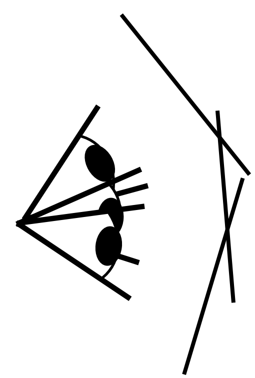

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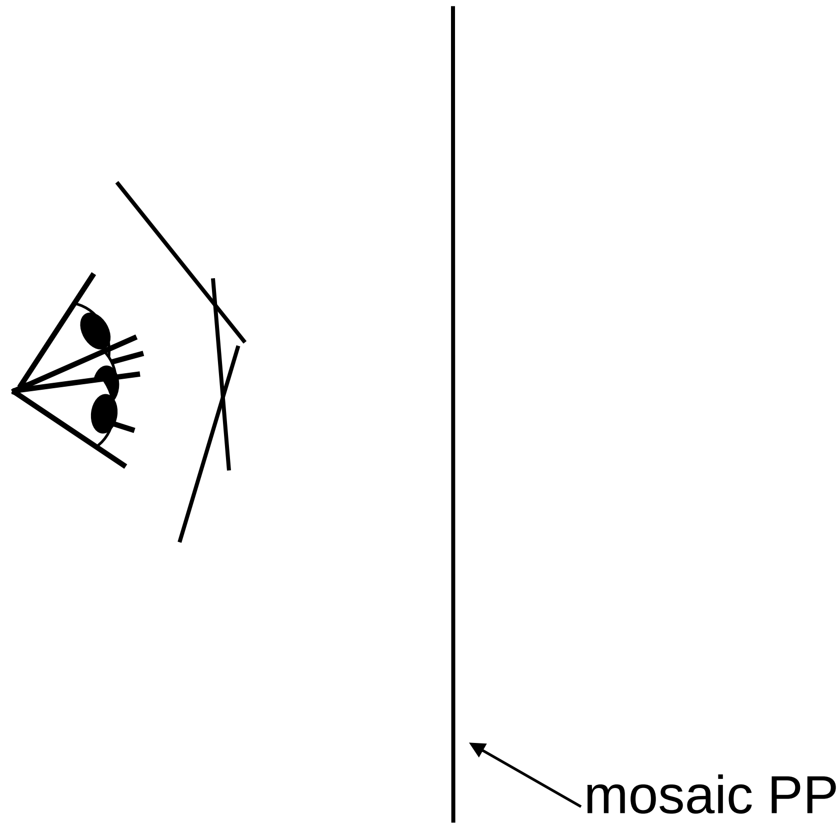

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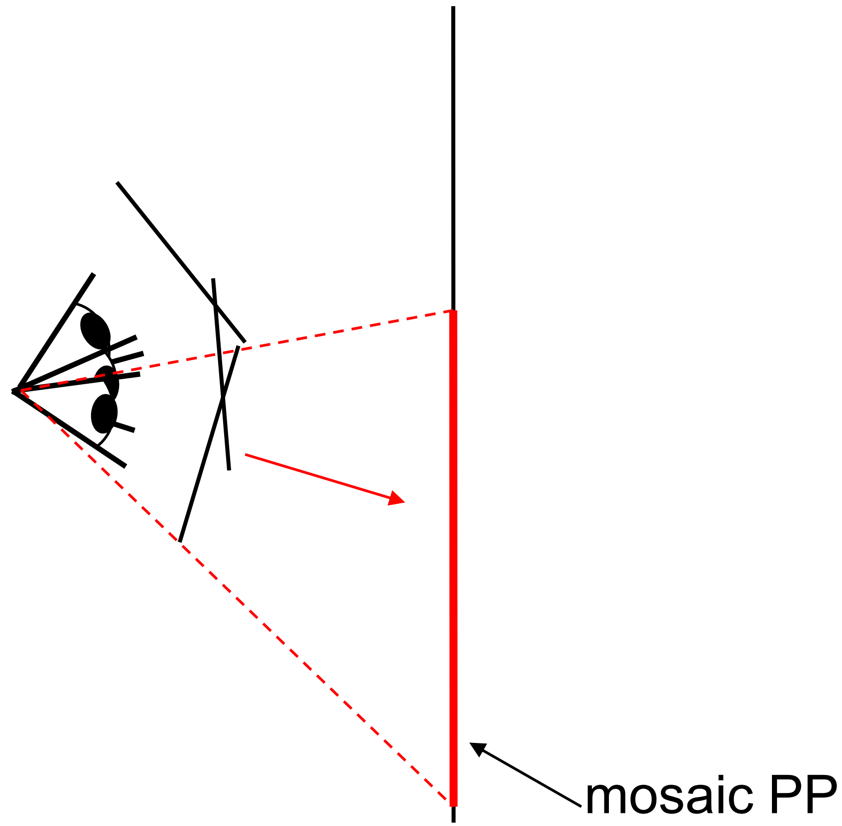

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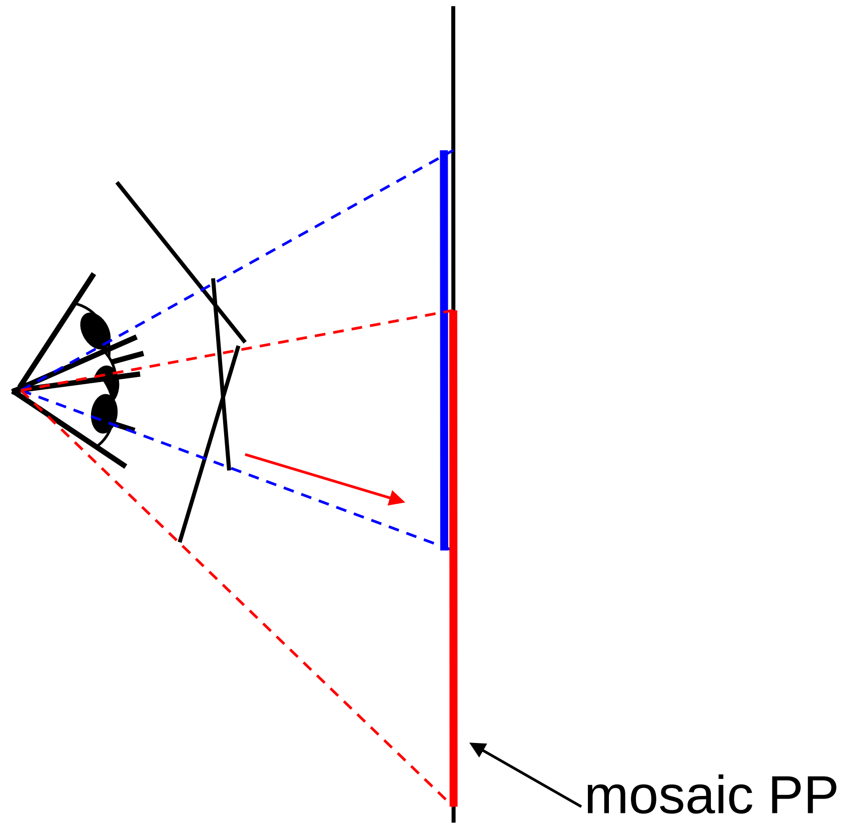

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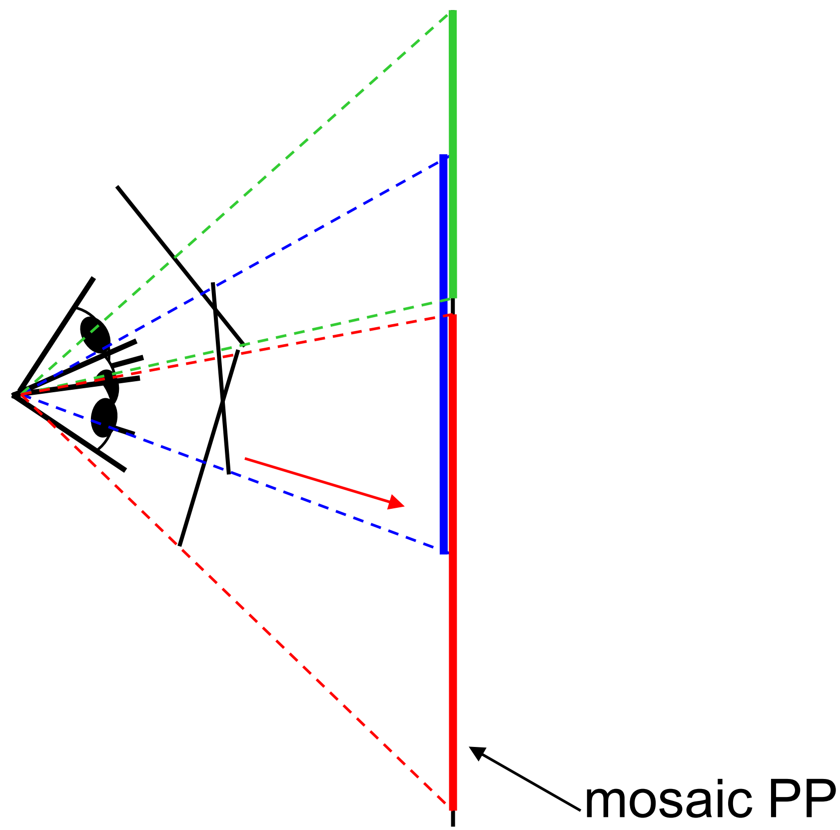
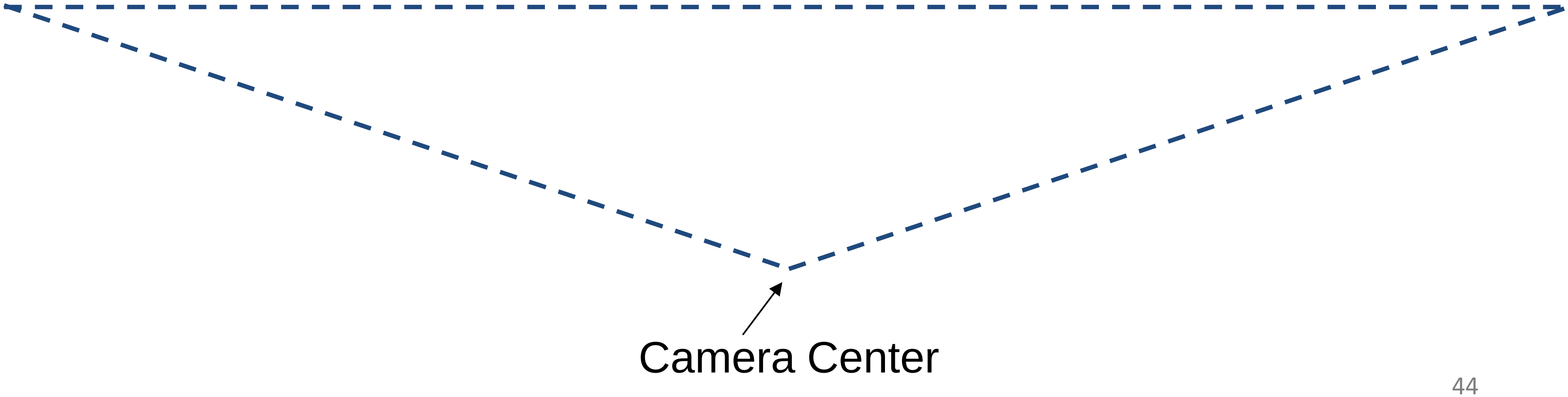


Example



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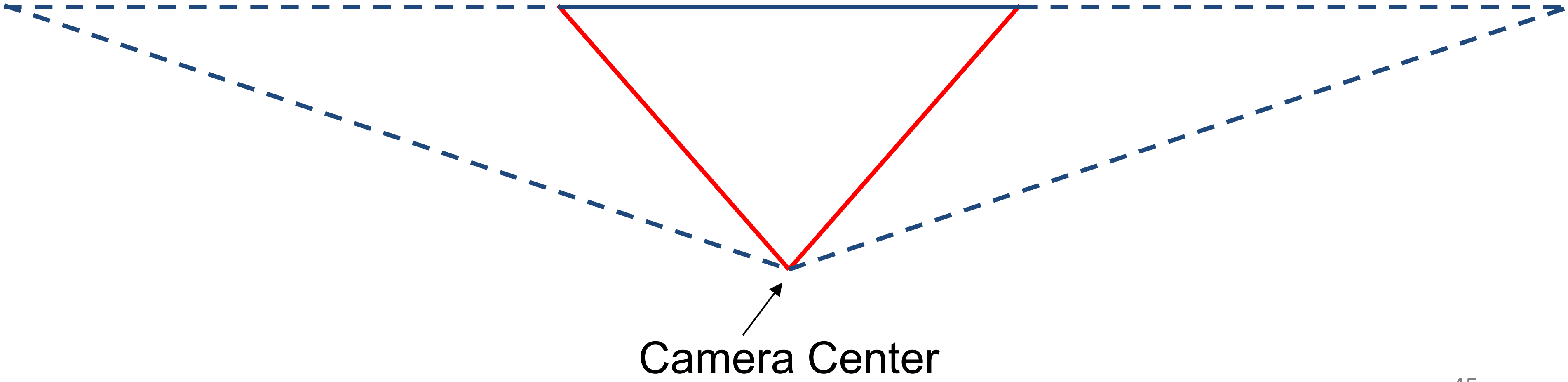

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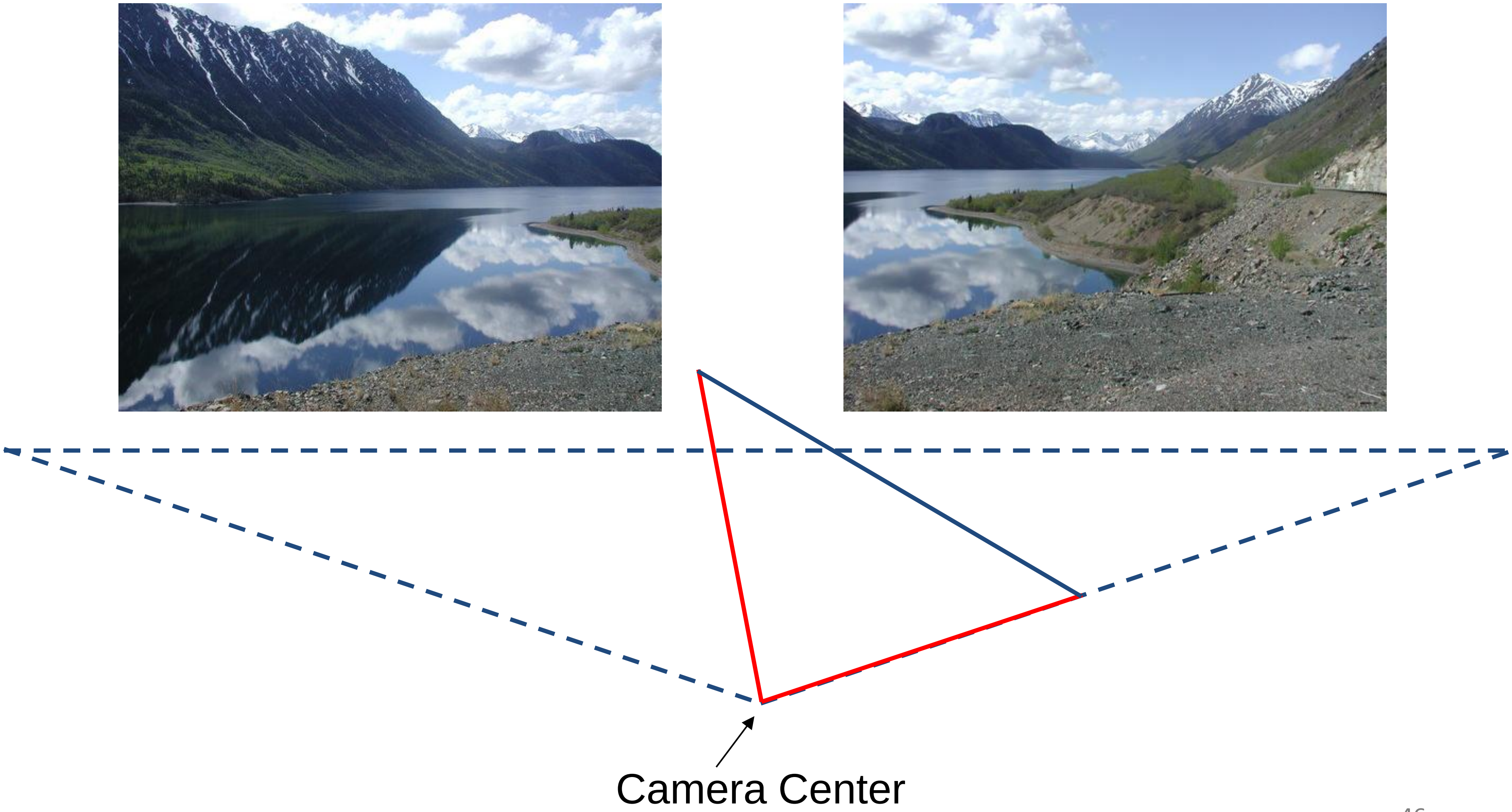

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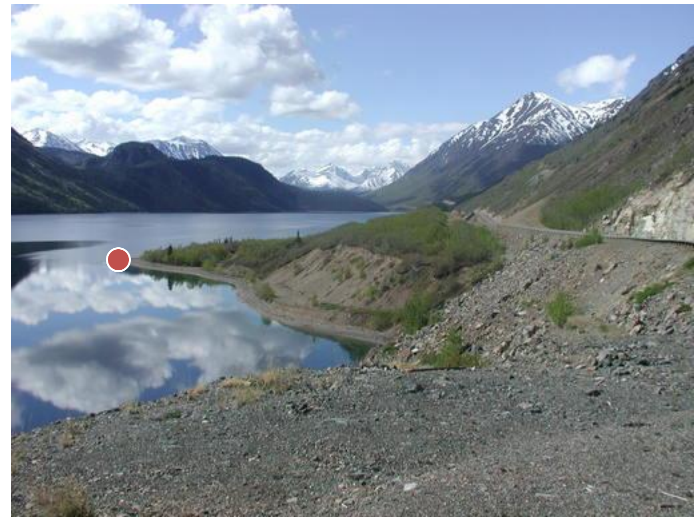
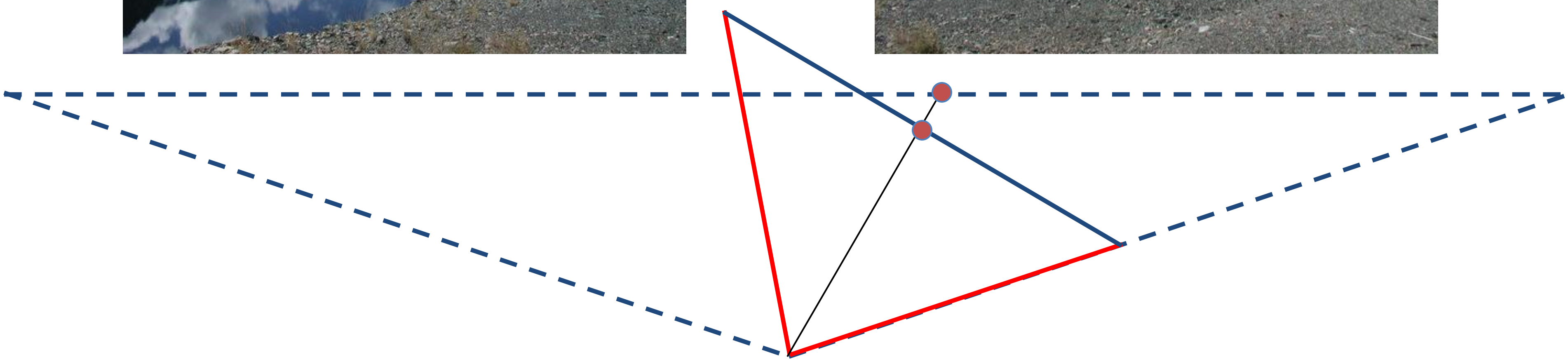
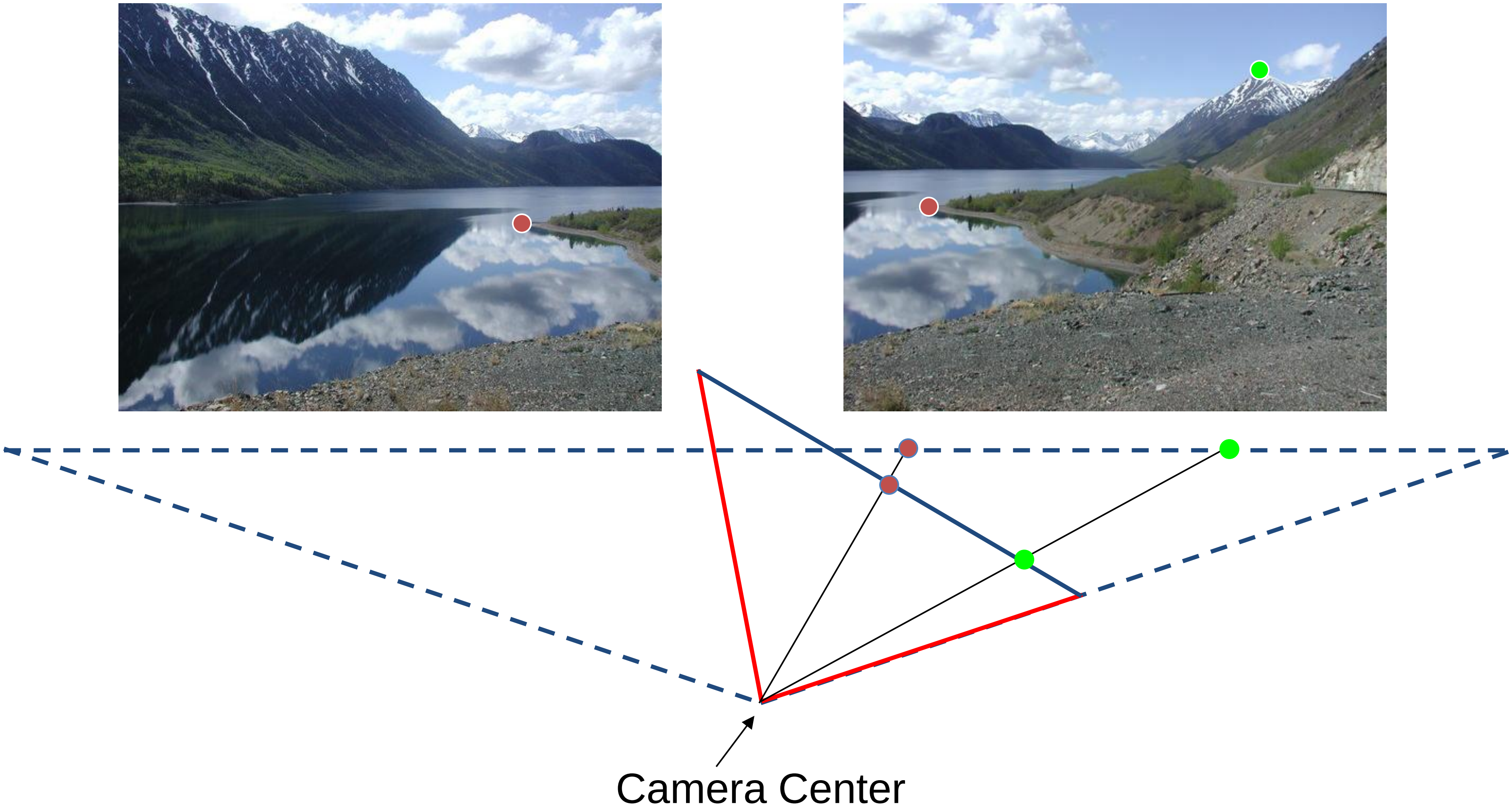


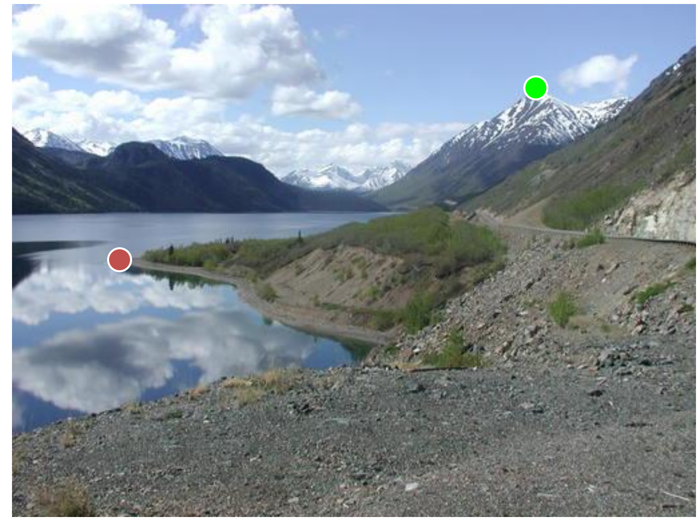


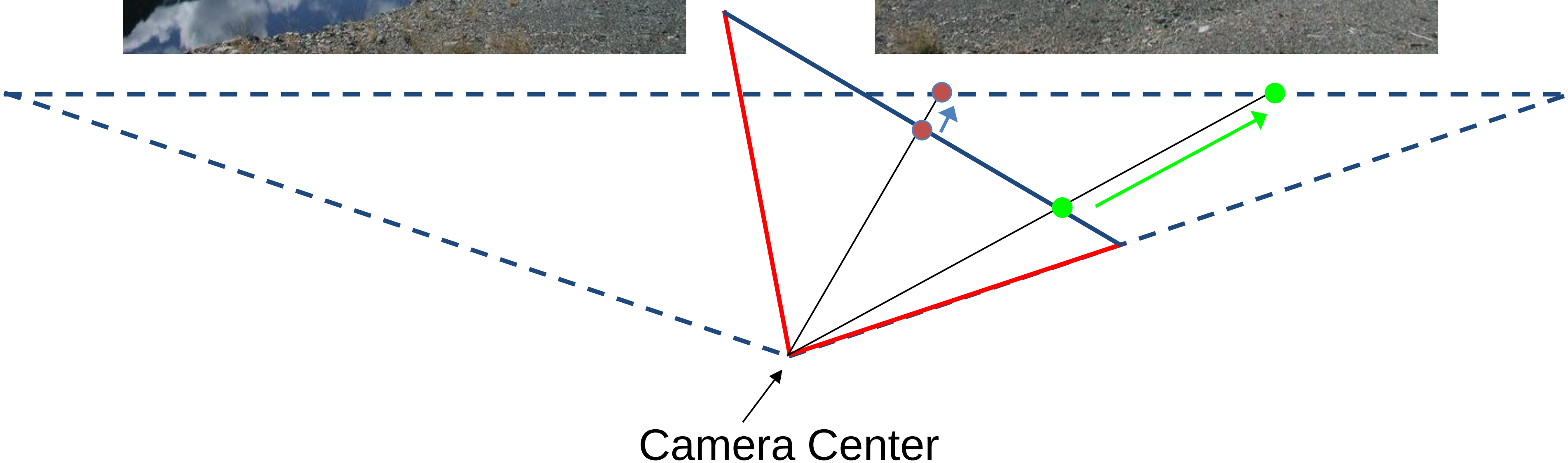
Camera Center

Example

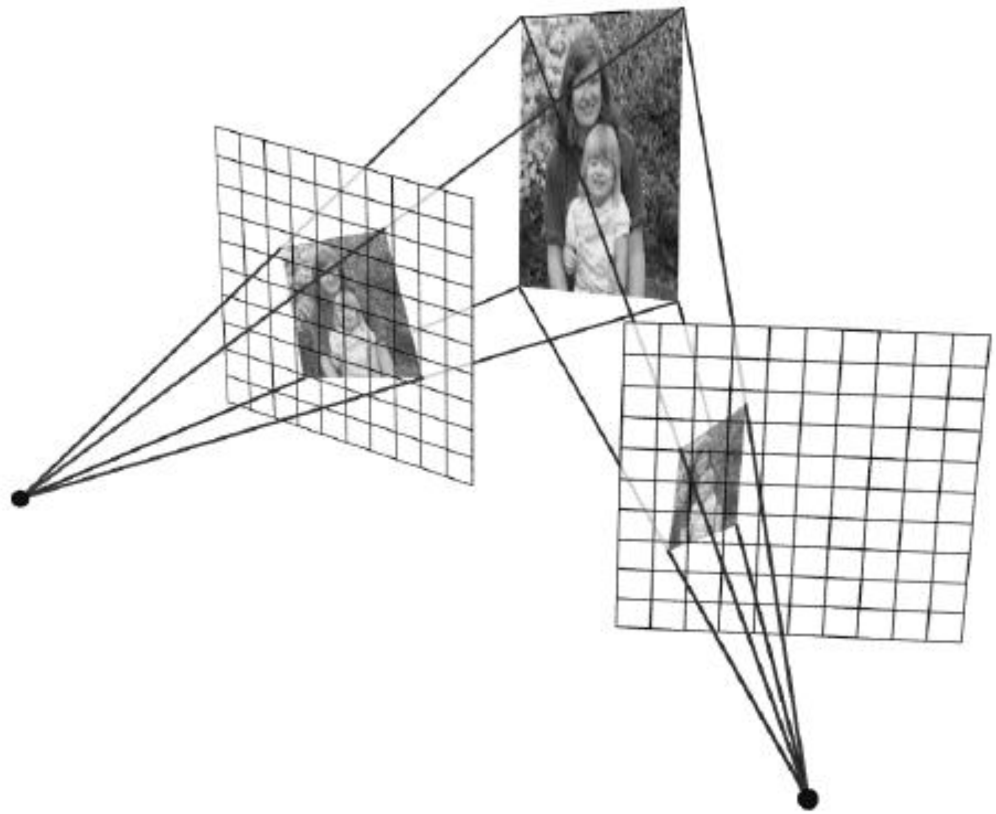


Example





The Underline Theory: Image Reprojection



- Although the theory says the process is a 3D reprojection.
- <u>We can also think of it as a **2D image warp** from one image to another.</u>

This is how we actually do it!

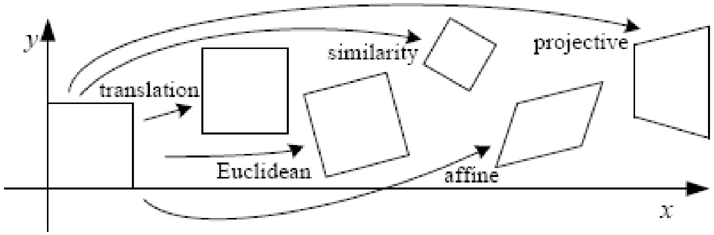
Compute Transformation: Motion Models

- What happens when we take two images with a camera and try to align them?

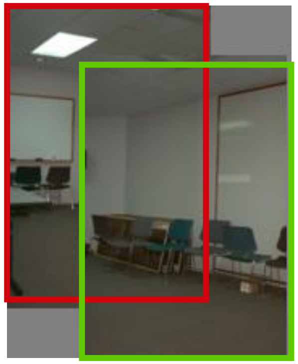
 

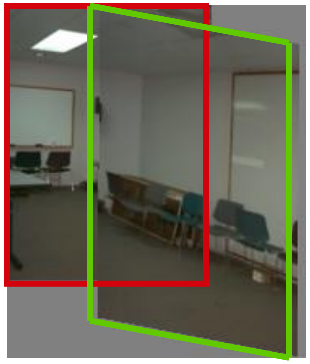
Translation? Rotation? Scale? Affine? Projective?

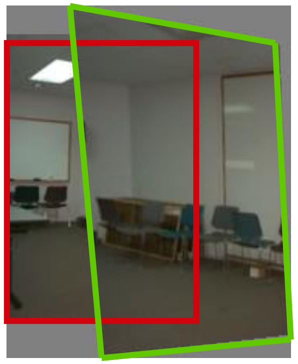
The Transformation: Motion Models



Translation | **Affine** | **Projective**







2 unknowns | **6 unknowns** | **8 unknowns**

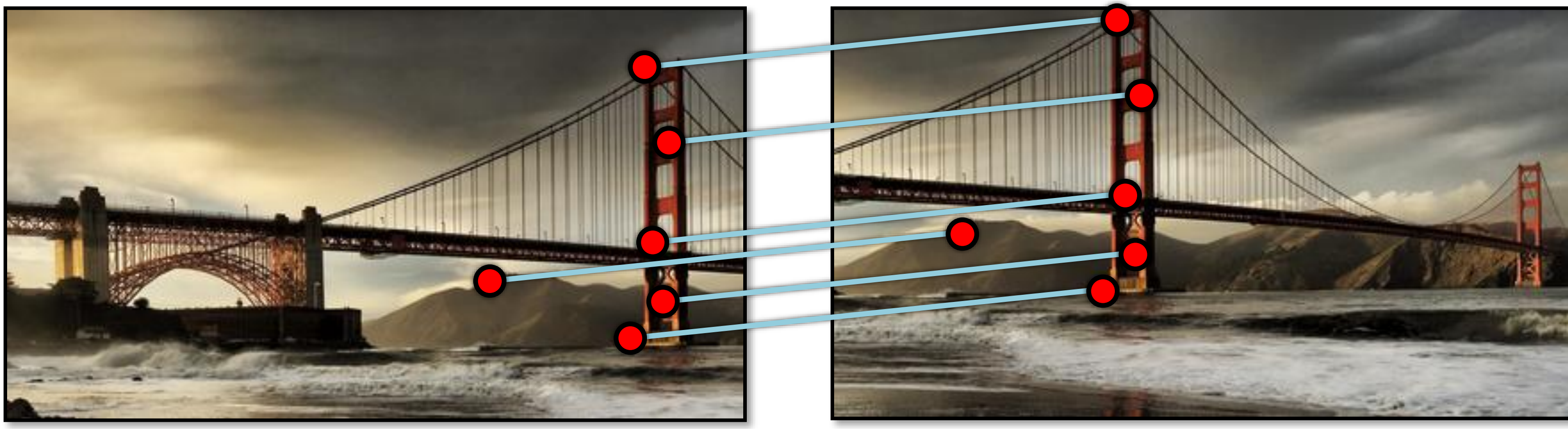
Finding the Transformation

- Translation = 2 degrees of freedom
- Similarity = 4 degrees of freedom
- Affine = 6 degrees of freedom
- Projective (Homography) = 8 degrees of freedom

- How many corresponding points do we need to solve?

Simple Case: Translations

Simple Case: Translations



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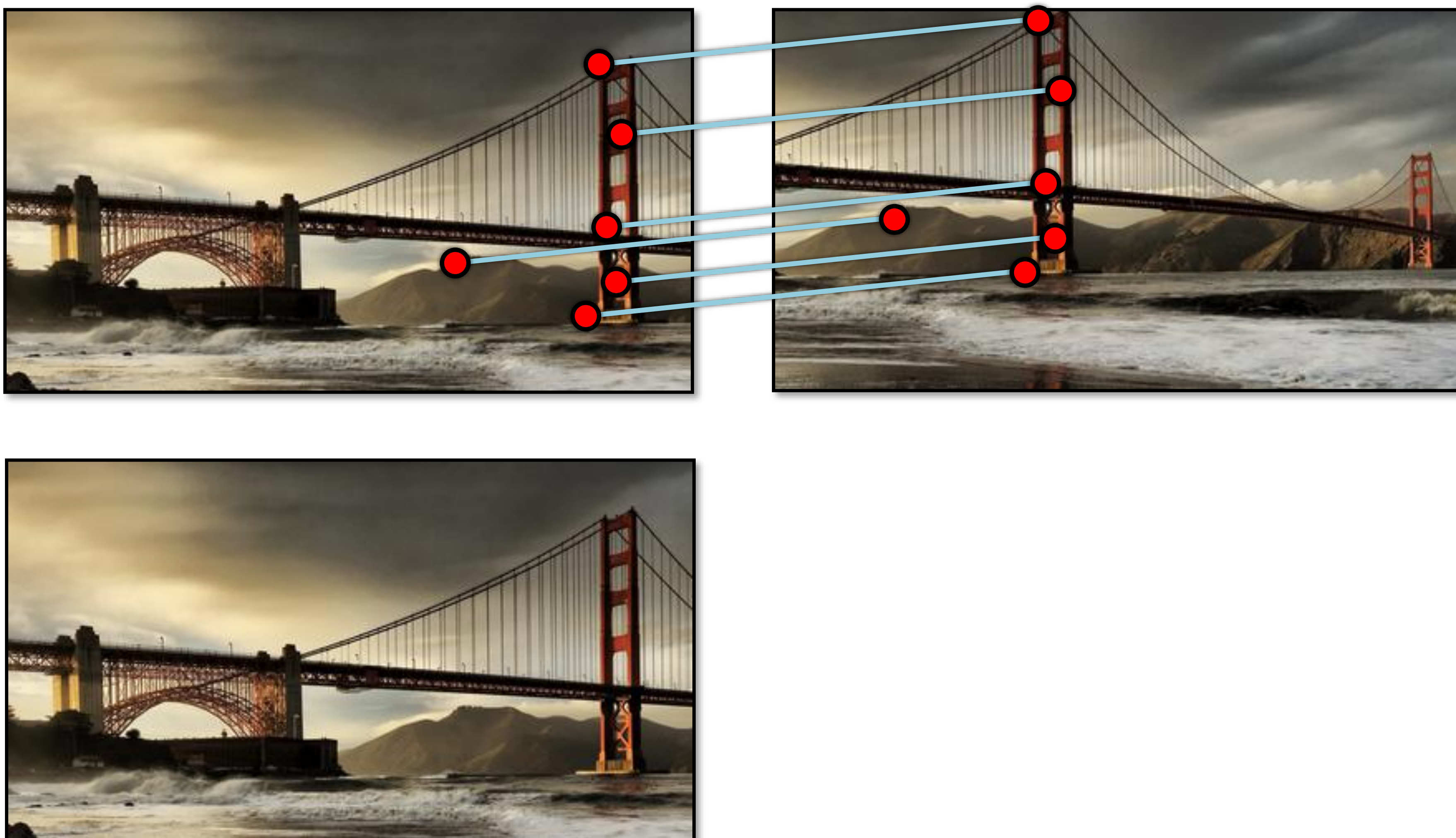

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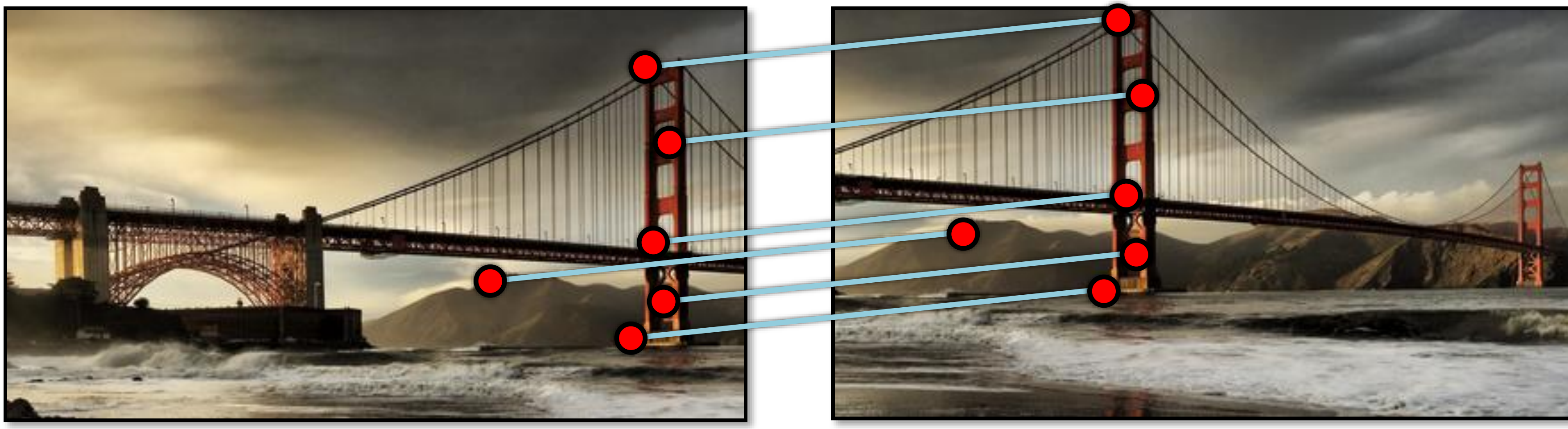

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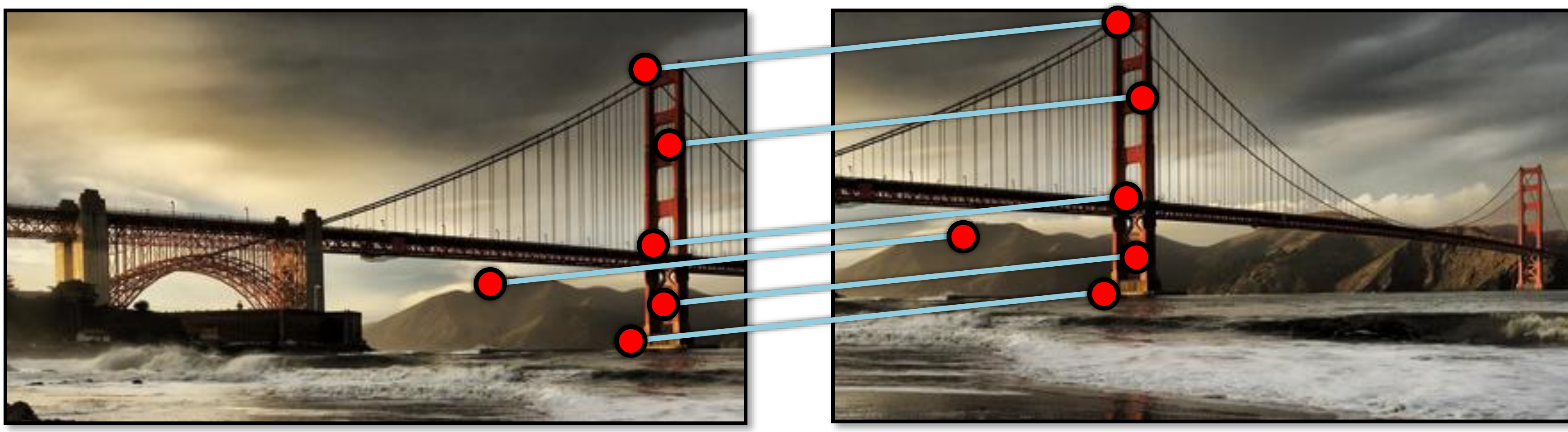
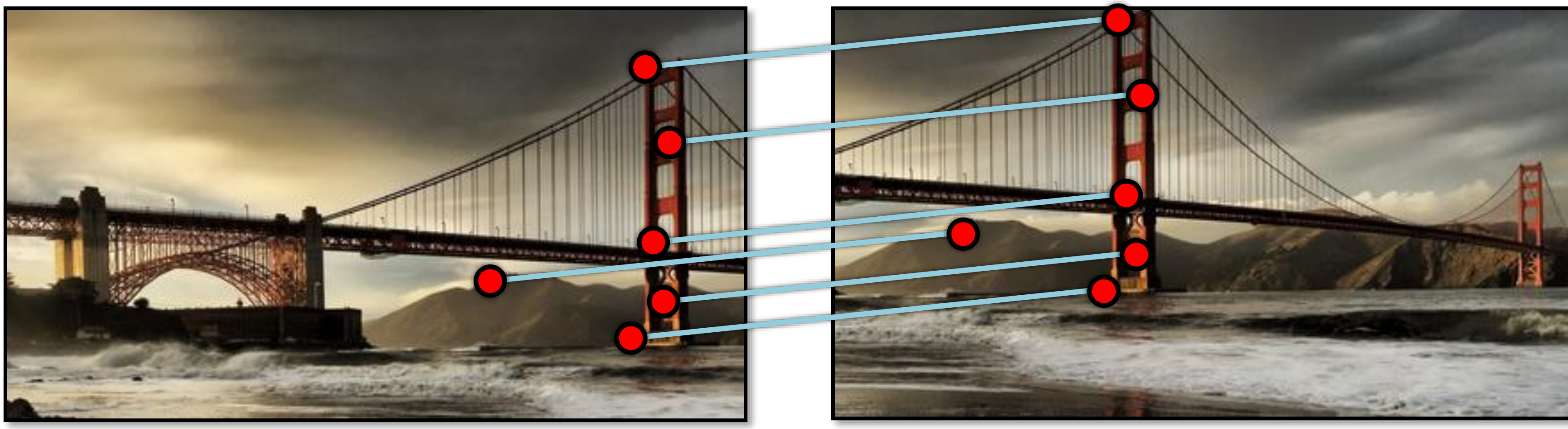


$(\mathbf{x}_t, \mathbf{y}_t)$

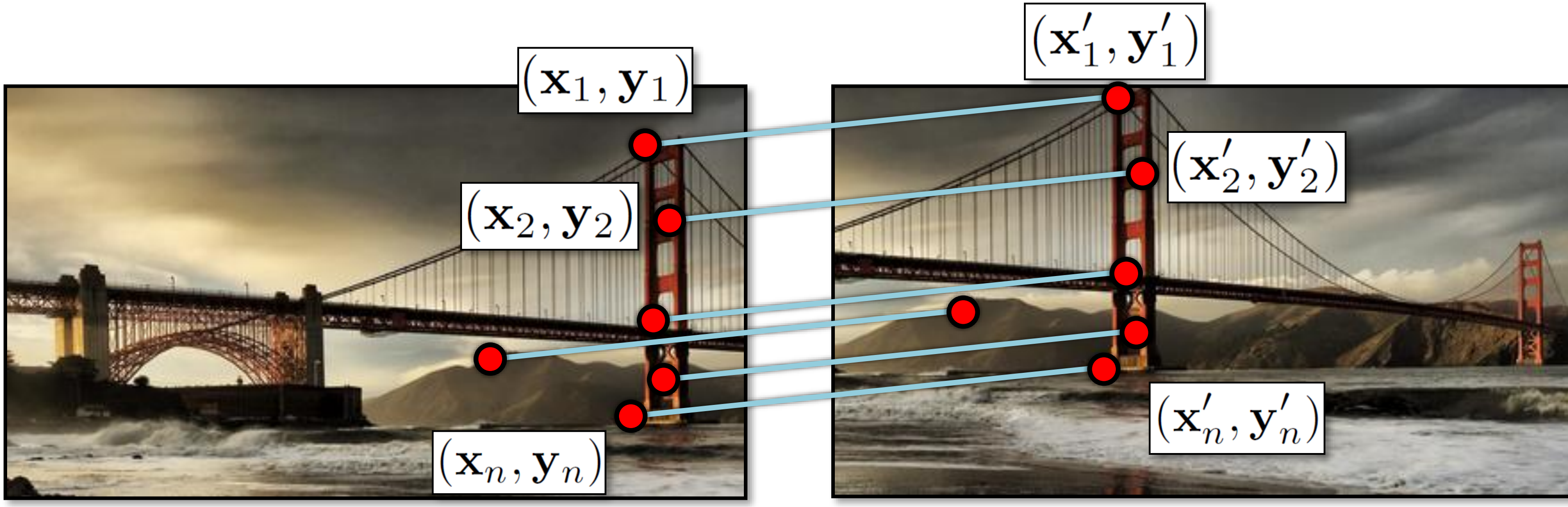
Simple Case: Translations



$(\mathbf{x}_t, \mathbf{y}_t)$

How do we solve for $(\mathbf{x}_t, \mathbf{y}_t)$?

Simple Case: Translations



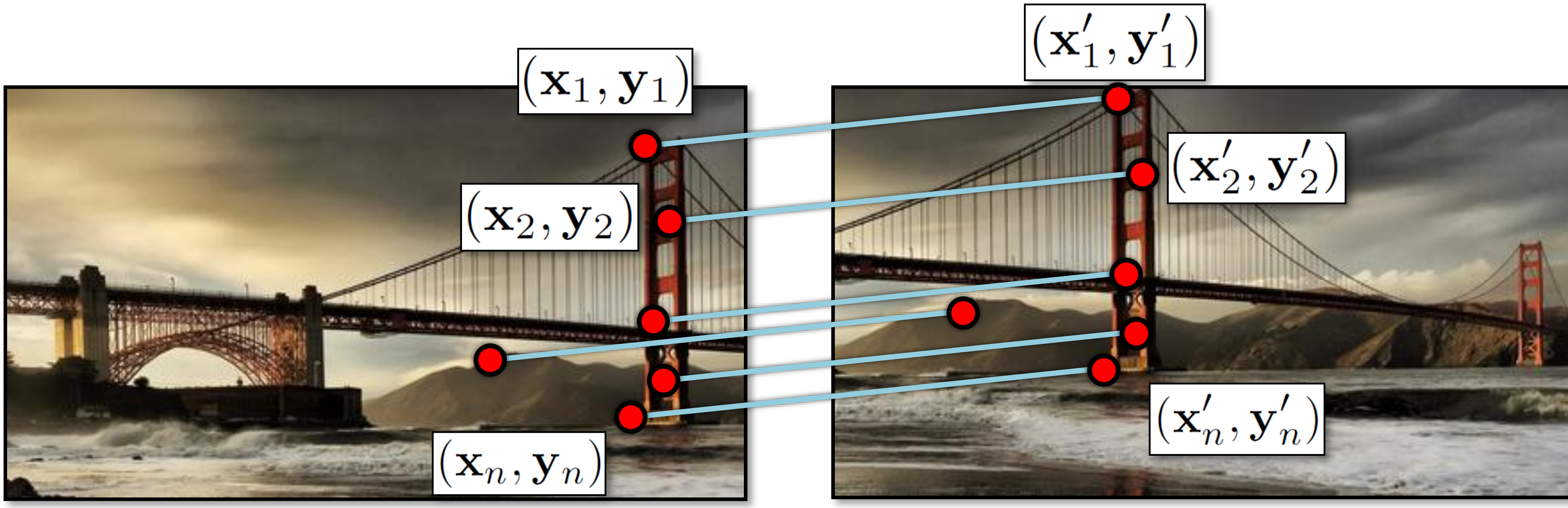
For each pair of matching points, we have

$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}'_i$$

$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}'_i$$

- System of linear equations
  - What are the knowns? Unknowns?
  - How many unknowns? How many equations (per match)?

Simple Case: Translations



For each pair of matching points, we have

$$\begin{aligned} \mathbf{x}_i + \mathbf{x_t} &= \mathbf{x}'_i \\ \mathbf{y}_i + \mathbf{y_t} &= \mathbf{y}'_i \end{aligned}$$

- The problem: more equations than unknowns.
  - "Overdetermined" system of equations.
  - **We find the *least square* solution.**

Least Squares Formulation

- For each point $(\mathbf{x}_i, \mathbf{y}_i)$

$$
\begin{aligned}
\mathbf{x}_i + \mathbf{x_t} &= \mathbf{x}_i' \\
\mathbf{y}_i + \mathbf{y_t} &= \mathbf{y}_i'
\end{aligned}
$$

- we define the *residuals* as

$$
\begin{aligned}
r_{\mathbf{x}_i}(\mathbf{x}_t) &= (\mathbf{x}_i + \mathbf{x}_t) - \mathbf{x}_i' \\
r_{\mathbf{y}_i}(\mathbf{y}_t) &= (\mathbf{y}_i + \mathbf{y}_t) - \mathbf{y}_i'
\end{aligned}
$$

Least Squares Formulation

- Goal: minimize sum of squared residuals.

$$C(\mathbf{x}_t, \mathbf{y}_t) = \sum_{i=1}^{n} \left(r_{\mathbf{x}_i}(\mathbf{x}_t)^2 + r_{\mathbf{y}_i}(\mathbf{y}_t)^2 \right)$$

- "Least squares" solution.

Solving for Translations

- Using least squares

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} x'_1 - x_1 \\ y'_1 - y_1 \\ x'_2 - x_2 \\ y'_2 - y_2 \\ \vdots \\ x'_n - x_n \\ y'_n - y_n \end{bmatrix}$$

$$\underset{2n \times 2}{\mathbf{A}} \quad \underset{2 \times 1}{\mathbf{t}} = \underset{2n \times 1}{\mathbf{b}}$$

Least Squares

$$\mathbf{A}\mathbf{t} = \mathbf{b}$$

- Find **t** that minimizes

$$||\mathbf{A}\mathbf{t} - \mathbf{b}||^2$$

- To solve, form the *normal equations*

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{t} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

$$\mathbf{t} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Least Squares: Example

- $(x_1,y_1) = (1,1)$, $(x_1',y_1') = (4.3,14.5)$, $(x_2,y_2) = (2,2)$, $(x_2',y_2') = (1.9,-3)$

```
A =

     1     0
     0     1
     1     0
     0     1
```

```
b =

    3.3000
   13.5000
   -0.1000
   -5.0000
```

- Least square solution is:

```
>> inv(A'*A)*A'*b

ans =

    1.6000
    4.2500
```

Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- How many unknowns?

- How many matches do we need?

Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} ax + by + c \\ dx + ey + f \\ 1 \end{bmatrix}$$

- How many unknowns?
  - 6 unknowns
- How many matches do we need?
  - At least 6 equations, so 3 matches.

Affine Transformations

- Residuals:

$$
\begin{aligned}
r_{x_i}(a,b,c,d,e,f) &= (ax_i + by_i + c) - x'_i \\
r_{y_i}(a,b,c,d,e,f) &= (dx_i + ey_i + f) - y'_i
\end{aligned}
$$

- Cost function:

$$
C(a,b,c,d,e,f) = \\
\sum_{i=1}^{n} \left(r_{x_i}(a,b,c,d,e,f)^2 + r_{y_i}(a,b,c,d,e,f)^2 \right)
$$

Affine Transformations

- Matrix form

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ & & & \vdots & & \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}$$

$$\underset{2n \times 6}{\mathbf{A}} \qquad \underset{6 \times 1}{\mathbf{t}} = \underset{2n \times 1}{\mathbf{b}}$$

Solving for Homographies

$$\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Why is this now a variable and not just 1?

- A homography is a projective object, in that it has no scale. It is represented by the above matrix, up to scale.
- One way of fixing the scale is to **set one of the coordinates to 1**, though that choice is arbitrary.
- But that's what most people do.

Solving for Homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x_i' = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$y_i' = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

Why the division?

$$x_i'(h_{20}x_i + h_{21}y_i + h_{22}) = h_{00}x_i + h_{01}y_i + h_{02}$$

$$y_i'(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

Solving for Homographies

$$
\begin{aligned}
x'_i(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\
y'_i(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{10}x_i + h_{11}y_i + h_{12}
\end{aligned}
$$

$$
\begin{bmatrix}
x_i & y_i & 1 & 0 & 0 & 0 & -x'_i x_i & -x'_i y_i & -x'_i \\
0 & 0 & 0 & x_i & y_i & 1 & -y'_i x_i & -y'_i y_i & -y'_i
\end{bmatrix}
\begin{bmatrix}
h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0
\end{bmatrix}
$$

This is just for one pair of points.

Direct Linear Transforms (n points)

$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 9}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix}}_{\substack{\mathbf{h} \\ 9}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\substack{\mathbf{0} \\ 2n}}$$

Defines a least squares problem: minimize $\|\mathbf{A}\mathbf{h} - \mathbf{0}\|^2$

- Since $\mathbf{h}$ is only defined up to scale, solve for **unit vector** $\hat{\mathbf{h}}$
- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

Direct Linear Transforms

- Why could we not solve for the homography in exactly the same way we did for the affine transform, ie.

$$\mathbf{t} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

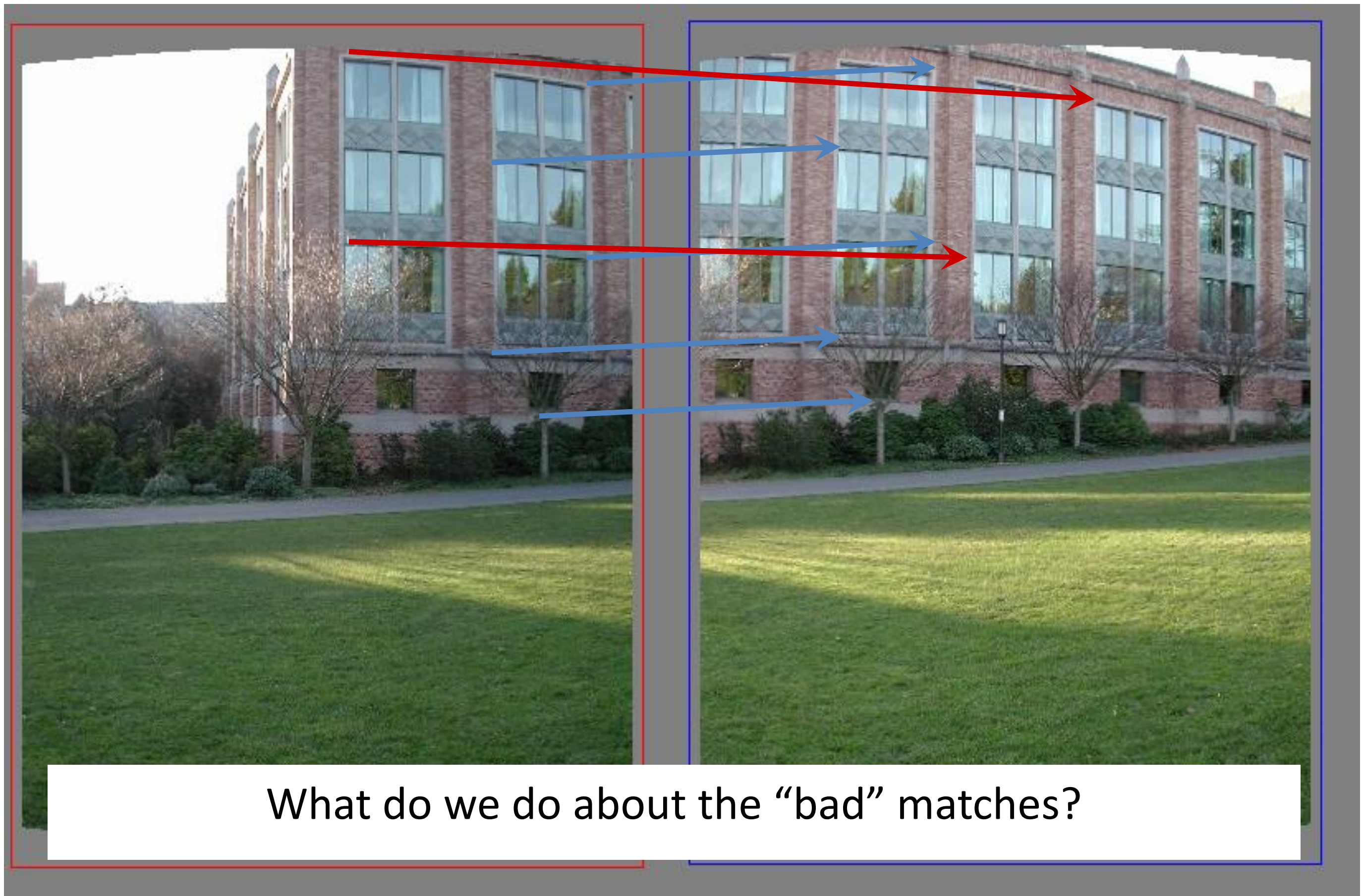
The Answer

- For an affine transform, we have equations of the form $Ax_i + b = y_i$, solvable by linear regression.
- For the homography, the equation is of the form

  $H\tilde{x}_i \sim \tilde{y}_i$ (homogeneous coordinates)

and the ~ means it holds only up to scale. The affine solution does not hold.

Matching features



What do we do about the “bad” matches?

RAndom SAmple Consensus

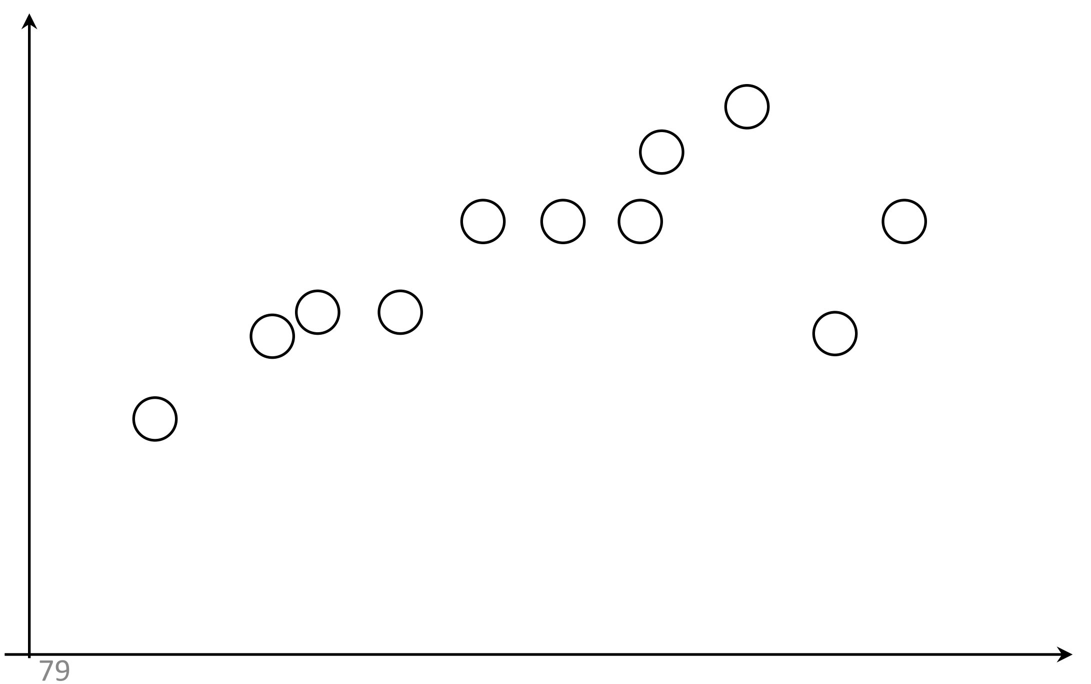
- RANSAC loop for estimating homography:

1. Select four feature pairs (at random)
2. Compute homography $\boldsymbol{H}$ (exact)
3. Compute inliers where $\|p_i', \boldsymbol{H}\, p_i\| < \varepsilon$

- Keep the largest set of inliers
- Re-compute least-squares $\boldsymbol{H}$ estimate using all of the inliers

Simple example: fit a line

- Rather than homography H (8 numbers) fit y=ax+b (2 numbers a, b) to 2D pairs



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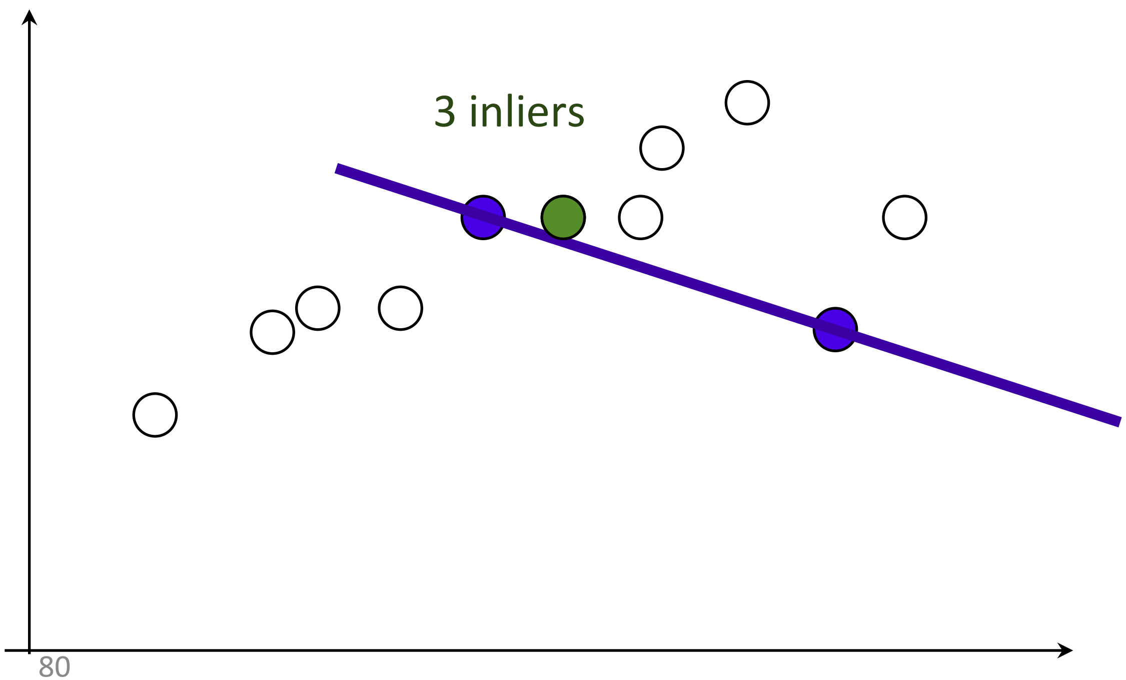
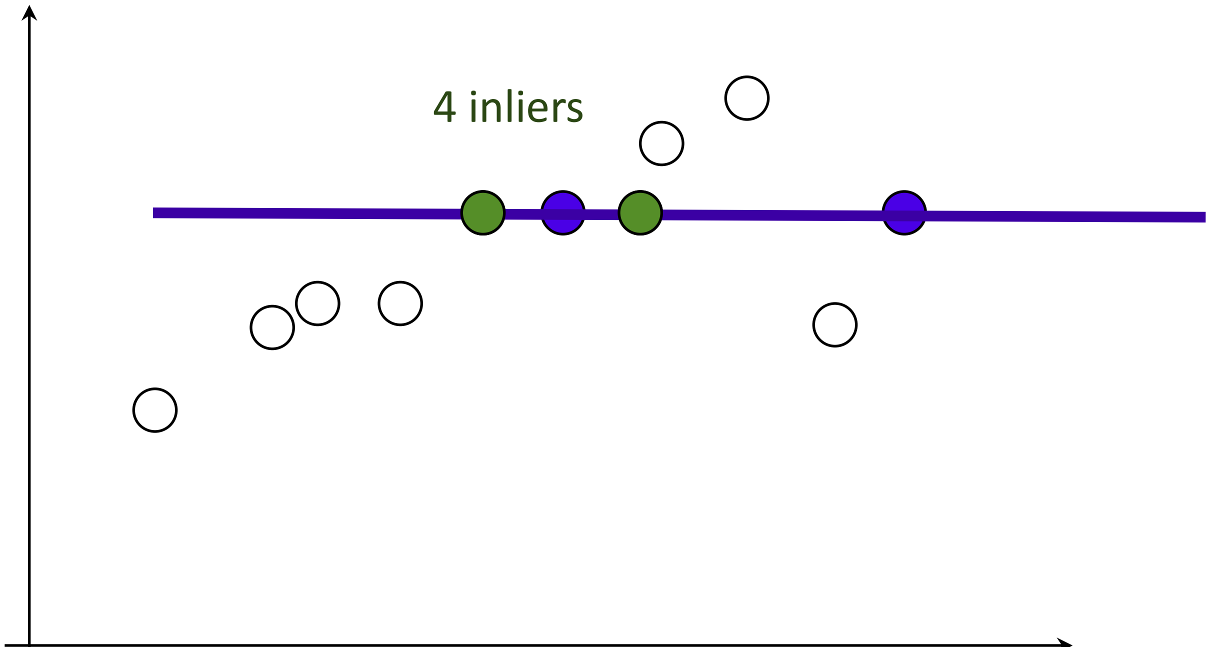
Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



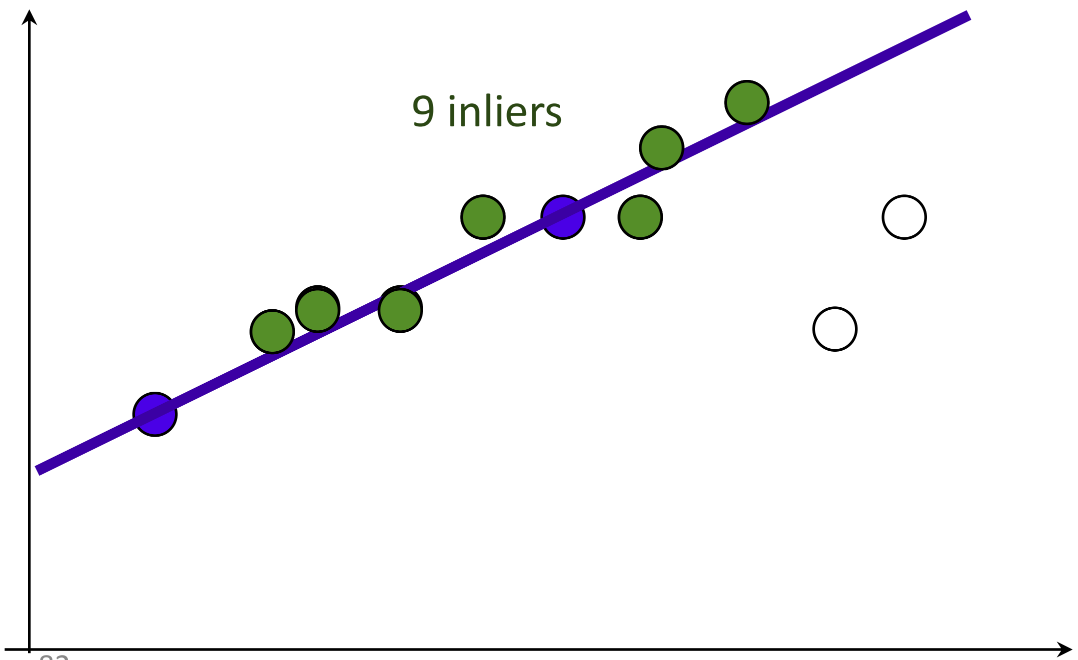
Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



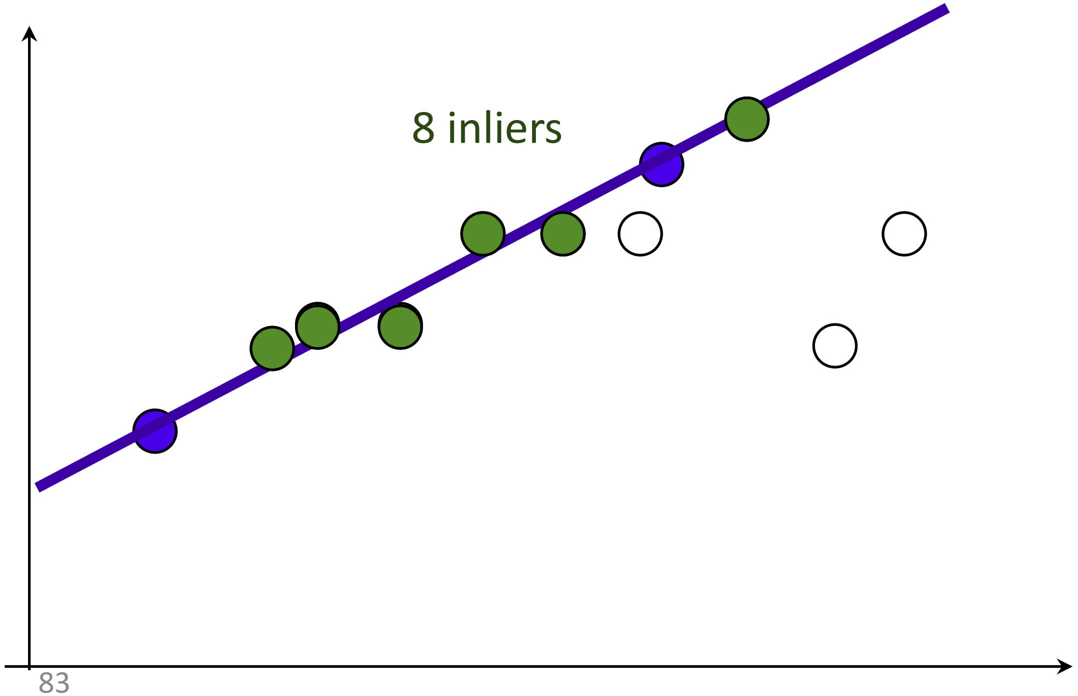
Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers



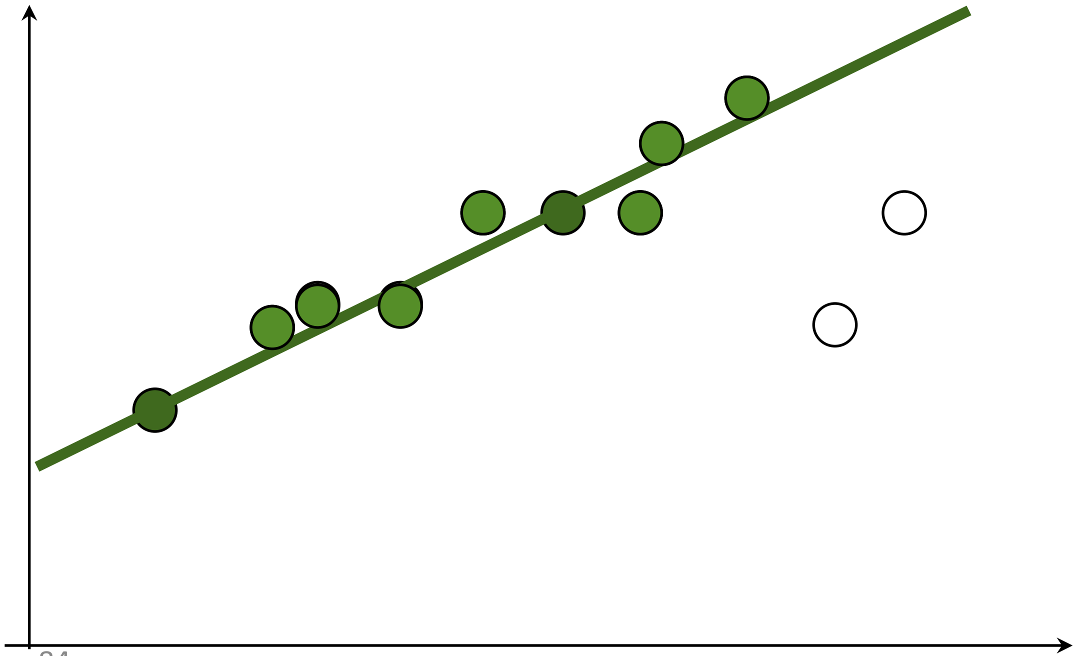
Simple example: fit a line

- Pick 2 points
- Fit line
- Count inliers

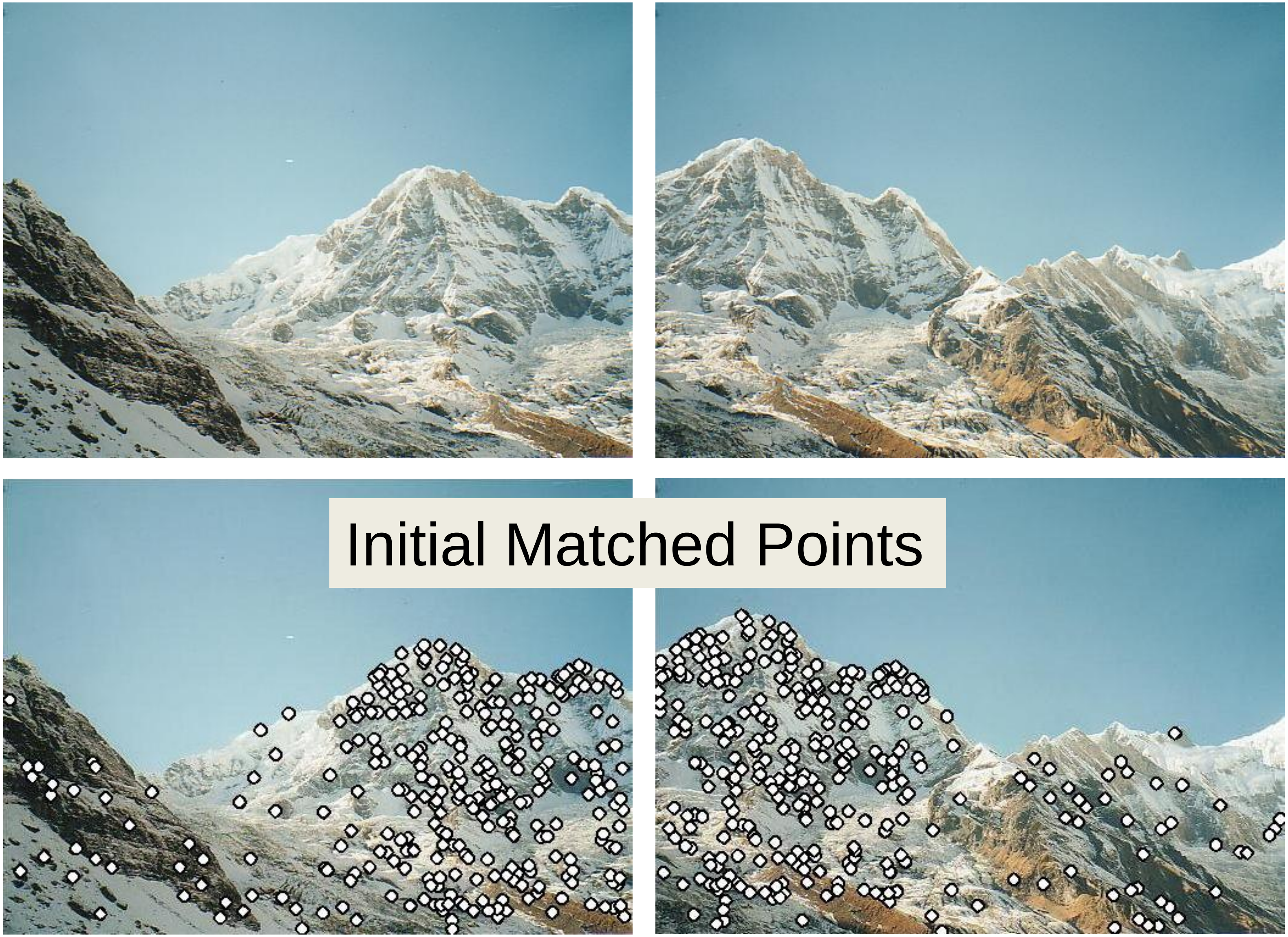


Simple example: fit a line

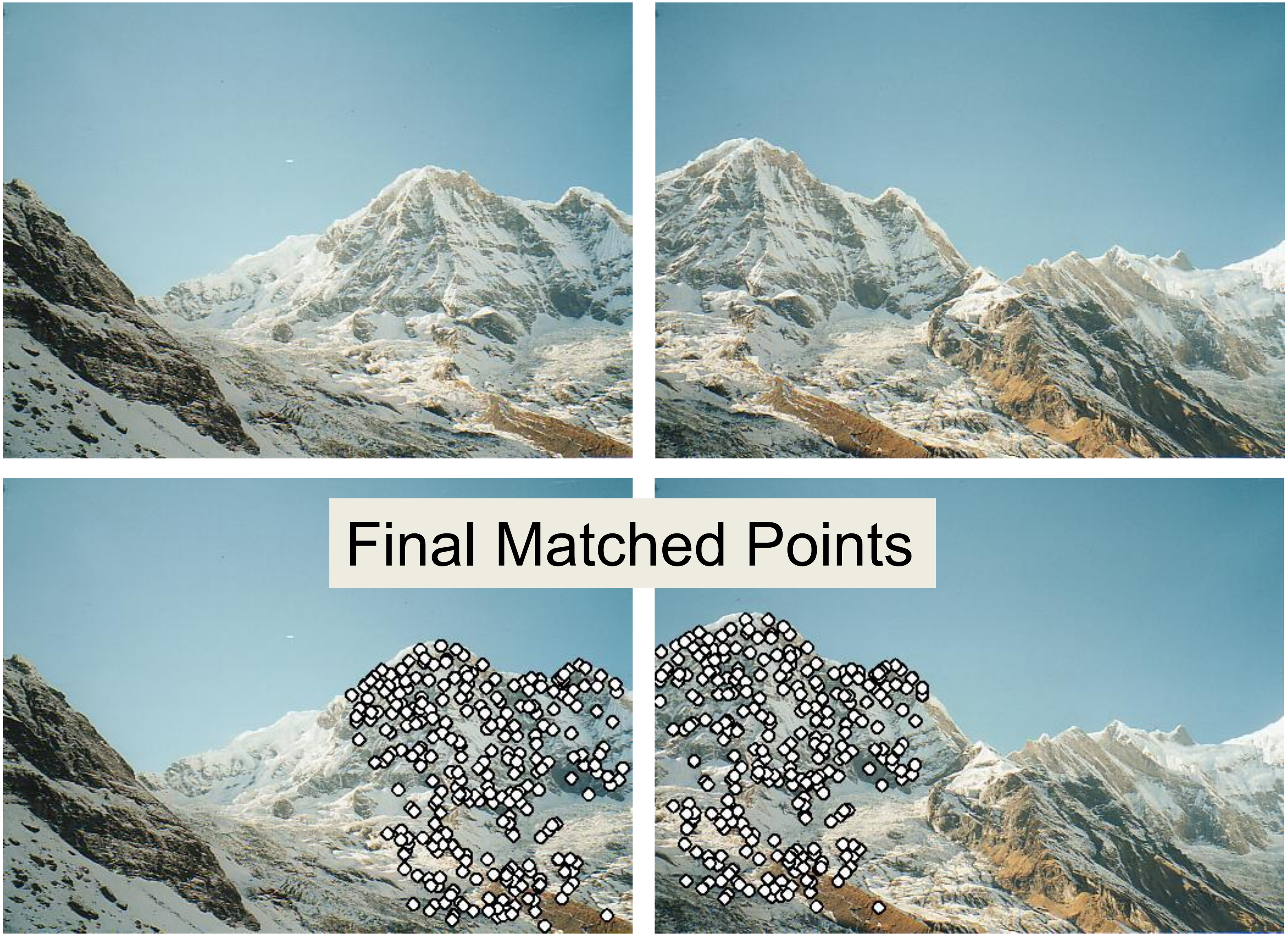
- Use biggest set of inliers
- Do least-square fit



RANSAC for Homography



RANSAC for Homography



Conclusion

- Geometric transformation
  - Translation, rotation, scale, affine, homography…
- The idea of image stitching
  - The overall process.
  - Finding transformation given an image pair.
  - Least square solution, etc.
  - RANSAC.