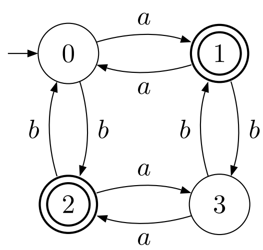
Answer to Exercise 2.4

$L = \{\epsilon, a, b, aa, ab, ba, bb, bc, aaa, aab, aba, abb, abc, baa, bab, bba, bbb, bbc, bca, bcb\}$

Answer to Exercise 3.1

1. DFA A



2.

w	$w \in L(A)$
ϵ	no
b	yes
$abaab$	yes
$bababbba$	no

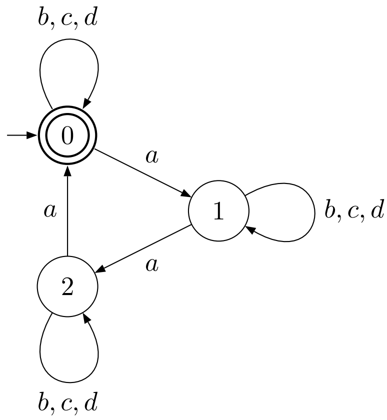
3.

$$\begin{array}{rcll}
\hat{\delta}_A(0, abba) & = & \hat{\delta}_A(\delta_A(0, a), bba) & \text{def. } \hat{\delta}_A \\
& = & \hat{\delta}_A(1, bba) & \text{because } \delta_A(0, a) = 1 \\
& = & \hat{\delta}_A(\delta_A(1, b), ba) & \text{def. } \hat{\delta}_A \\
& = & \hat{\delta}_A(3, ba) & \text{because } \delta_A(1, b) = 3 \\
& = & \hat{\delta}_A(\delta_A(3, b), a) & \text{def. } \hat{\delta}_A \\
& = & \hat{\delta}_A(1, a) & \text{because } \delta_A(3, b) = 1 \\
& = & \hat{\delta}_A(\delta_A(1, a), \epsilon) & \text{def. } \hat{\delta}_A \\
& = & \hat{\delta}_A(0, \epsilon) & \text{because } \delta_A(1, a) = 0 \\
& = & 0 & \text{def. } \hat{\delta}_A
\end{array}$$

4. $L(A)$ contains all words over $\{a, b\}$ in which the number of a's is even and the number of b's is odd, or vice versa. But that's the same as saying all the words over $\{a, b\}$ containing an odd number of symbols. Which in turn suggests there is a DFA with fewer states that accepts the same language. (Can you find it?)

Answer to Exercise 3.2

We need to count the number of a's modulo 3, i.e. we need to keep track of whether the remainder when we divide the total number of a's seen so far by 3 is 0, 1, or 2. Thus we need 3 states. They are named 0, 1, and 2 below, to indicate said remainder. When any symbol other than a is read, the machine does not change state as the number of a's seen remain unchanged. 0 should be the accepting state because a remainder of 0 indicates that the number of a's seen is a multiple of 3. Note that 0 is a multiple of 3. Thus the empty string is accepted, and the accepting state is thus also the initial state.



Answer to Exercise 3.5

1. (a) $\epsilon \in L(A)$
   (b) $aaa \in L(A)$
   (c) $bbc \in L(A)$
   (d) $cbc \notin L(A)$
   (e) $abcacb \in L(A)$
2. Starting from $S_A = \{q_0, q_1, q_3\}$, the start state of $D(A)$, we compute $\hat{\delta}_A(S_A, x)$ for each $x \in \Sigma_A$. Whenever we encounter a state $P \subseteq Q_A$ of $D(A)$ that has not been considered before, we add P to the table and proceed to tabulate $\hat{\delta}_A(P, x)$ for each $x \in \Sigma_A$. We repeat the process until no new states are encountered. Finally, we identify the initial state ($\rightarrow$ to the left of the state) and all accepting states ($*$ to the left of the state). Note that a DFA state is accepting iff it contains at least one accepting NFA state (as this means it is *possible* to reach at least one accepting state on a given word, which means that word is considered to be in the language of the NFA).

$\delta_{D(A)}$	a	b	c
$\rightarrow *$ $\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_3\} \cup \emptyset \cup \emptyset = \{q_0, q_1, q_3\}$	$\{q_0\} \cup \{q_1\} \cup \{q_4\} = \{q_0, q_1, q_4\}$	$\{q_0\} \cup \{q_2\} \cup \{q_3\} = \{q_0, q_2, q_3\}$
$*$ $\{q_0, q_1, q_4\}$	$\{q_0, q_1, q_3\} \cup \emptyset \cup \emptyset = \{q_0, q_1, q_3\}$	$\{q_0\} \cup \{q_1\} \cup \emptyset = \{q_0, q_1\}$	$\{q_0\} \cup \{q_2\} \cup \emptyset = \{q_0, q_2\}$
$*$ $\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_3\} \cup \emptyset \cup \emptyset = \{q_0, q_1, q_3\}$	$\{q_0\} \cup \emptyset \cup \{q_4\} = \{q_0, q_4\}$	$\{q_0\} \cup \emptyset \cup \{q_3\} = \{q_0, q_3\}$
$*$ $\{q_0, q_1\}$	$\{q_0, q_1, q_3\} \cup \emptyset = \{q_0, q_1, q_3\}$	$\{q_0\} \cup \{q_1\} = \{q_0, q_1\}$	$\{q_0\} \cup \{q_2\} = \{q_0, q_2\}$
$*$ $\{q_0, q_2\}$	$\{q_0, q_1, q_3\} \cup \emptyset = \{q_0, q_1, q_3\}$	$\{q_0\} \cup \emptyset = \{q_0\}$	$\{q_0\} \cup \emptyset = \{q_0\}$
$*$ $\{q_0, q_4\}$	$\{q_0, q_1, q_3\} \cup \emptyset = \{q_0, q_1, q_3\}$	$\{q_0\} \cup \emptyset = \{q_0\}$	$\{q_0\} \cup \emptyset = \{q_0\}$
$*$ $\{q_0, q_3\}$	$\{q_0, q_1, q_3\} \cup \emptyset = \{q_0, q_1, q_3\}$	$\{q_0\} \cup \{q_4\} = \{q_0, q_4\}$	$\{q_0\} \cup \{q_3\} = \{q_0, q_3\}$
$\{q_0\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$	$\{q_0\}$

(Note that we only needed to consider 8 states, a lot fewer than the $2^5 = 32$ possible states in this case. $32 - 8 = 24$ states are thus not reachable from the initial state.)
Giving simple names to the states resulting from the subset construction can facilitate drawing the transition diagram:

$\delta_{D(A)}$	a	b	c
$\rightarrow *$ $\{q_0, q_1, q_3\} = A$	$\{q_0, q_1, q_3\} = A$	$\{q_0, q_1, q_4\} = B$	$\{q_0, q_2, q_3\} = C$
$*$ $\{q_0, q_1, q_4\} = B$	$\{q_0, q_1, q_3\} = A$	$\{q_0, q_1\} = D$	$\{q_0, q_2\} = E$
$*$ $\{q_0, q_2, q_3\} = C$	$\{q_0, q_1, q_3\} = A$	$\{q_0, q_4\} = F$	$\{q_0, q_3\} = G$
$*$ $\{q_0, q_1\} = D$	$\{q_0, q_1, q_3\} = A$	$\{q_0, q_1\} = D$	$\{q_0, q_2\} = E$
$*$ $\{q_0, q_2\} = E$	$\{q_0, q_1, q_3\} = A$	$\{q_0\} = H$	$\{q_0\} = H$
$*$ $\{q_0, q_4\} = F$	$\{q_0, q_1, q_3\} = A$	$\{q_0\} = H$	$\{q_0\} = H$
$*$ $\{q_0, q_3\} = G$	$\{q_0, q_1, q_3\} = A$	$\{q_0, q_4\} = F$	$\{q_0, q_3\} = G$
$\{q_0\} = H$	$\{q_0, q_1, q_3\} = A$	$\{q_0\} = H$	$\{q_0\} = H$

3. We can now draw the transition diagram for $D(A)$: