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Foundation Calculus & Mathematical Techniques CELEN037

Weekly Worksheet-2 **Topics:** Quotient & Chain Rules, Implicit, Logarithmic, and Inverse diff.

Type 1: Chain Rule for differentiation $\boxed{\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dv} \cdot \dfrac{dv}{dw} \cdot \dfrac{dw}{dx}}$

1. Use the chain rule for derivatives to find $\dfrac{dy}{dx}$ where,

(i) $y = \cos(e^x)$	(ii) $y = \sin(\cos(\ln x))$	(iii) $y = \cos(\sin x)$
(iv) $y = \tan(\ln x)$	(v) $y = \ln(\sec x)$	(vi) $y = \sin(\cos(\ln x))$
(vii) $y = \sin(\ln(\cos x))$	$(viii)$ $y = \tan\left(\cos\left(\sqrt{x}\right)\right)$	(ix) $y = \ln(\sin(e^x))$

Type 2: (Fast-track) Chain Rule for differentiation

2. Use the fast-track chain rule for derivatives to find $\dfrac{dy}{dx}$ where,

(i) $y = \ln(\cos(e^x))$	(ii) $y = \sin(\cos(\ln x))$	(iii) $y = \sin(\ln(\cos x))$
(iv) $y = \tan\left(\cos\left(\sqrt{x}\right)\right)$	(v) $y = \ln(\sin(e^x))$	(vi) $y = \sqrt{\sin(e^{\cos x})}$

Type 3: Logarithmic differentiation

<u>The Method</u>:

- Take Logarithm on both sides.
- Apply rules of logarithms.
- Differentiate both sides w.r.t. x

3. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for the following functions:

(i) $y = (\tan x)^{\sin x}$	(ii) $y = (\cos x)^{\sin x}$	(iii) $y = (\sin x)^{\cos x}$
(iv) $y = (\cos x)^{x}$	(v) $y = (x)^{\cos x}$	(vi) $y = (\ln x)^{\tan x}$
(vii) $y = (x)^{x}$	$(viii)$ $y = \sqrt[x]{x} = x^{1/x}$	(ix) $y = \sin(x^x)$

(x) $y = \dfrac{\sqrt[3]{x} \cdot \tan^4 x}{\cos(e^x)}$

Type 4: Implicit Differentiation

4. Use the method of implicit differentiation to find $\dfrac{dy}{dx}$ for the following functions:

(i)	$x^3 + y^3 = 3xy$	(ii)	$\cos(x+y) = x^2 + y^2$
(iii)	$\cos(x\,y) = \sqrt{x+y}$	(iv)	$\sin(x\,y) = x^2 - y^2$
(v)	$\ln(x+y) = \ln(x\,y) + 1$	(vi)	$\tan(x\,y) + 2\,x\,y = \sqrt{x^2 - y^2}$
(vii)	$x^2 + y^2 + 2\,x^2\,y = 4\,x\,y^2$		Also find $\left.\dfrac{dy}{dx}\right\|_{(1,1)}$.
$(viii)$	$(1+x-y)^3 = (1-x+y)^2$		Also find $\left.\dfrac{dy}{dx}\right\|_{(0,0)}$.
(ix)	$x^3 + y^3 = x\,y^2 + x^2\,y$		Also find $\left.\dfrac{dy}{dx}\right\|_{(1,-1)}$.
(x)	$2\,x + x^2\,y^3 - 3\,x\,y = 5\,x\,y^2 - 8$		Also find $\left.\dfrac{dy}{dx}\right\|_{(2,1)}$.

Type 5: Derivatives of Inverse Functions

5. (i) Given $y = \operatorname{cosec}^{-1}(x) = \sin^{-1}\left(\dfrac{1}{x}\right)$; $|\,x\,| > 1$. Find $\dfrac{dy}{dx}$. **Hint**: $\sqrt{x^2} = |\,x\,|$.

(ii) Given $y = \sec^{-1}\left(\sqrt{1+x^2}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$; $x > 0$. Find $\dfrac{dy}{dx}$.

Hint: $\sqrt{x^2} = |\,x\,|$, $\dfrac{d}{dx}\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{-1}{2\,x^{3/2}}$

(iii) Given $y = \tan^{-1}\left(x + \sqrt{1+x^2}\right)$, find $\dfrac{dy}{dx}$. **Hint**: $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{1}{2\sqrt{x}}$.

(iv) Given $y = \tan^{-1}(\sec x + \tan x)$, find $\dfrac{dy}{dx}$.

(v) Given $y = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$; $|\,x\,| < 1$, show that $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$.

Hint: Use the quotient rule $\boxed{\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v \cdot \dfrac{du}{dx} - u \cdot \dfrac{dv}{dx}}{v^2}}$.

(vi) Given $y = \tan^{-1}\left(\dfrac{2x}{1+x^2}\right)$, find $\dfrac{dy}{dx}$.

(vii) Given $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$; $x \geq 0$, find $\dfrac{dy}{dx}$. **Hint**: $\sqrt{x^2} = |\,x\,|$.

$(viii)$ Find $\dfrac{d}{dx}\left[\sin^{-1}(\cos x) + \cos^{-1}(\sin x)\right]$ where

(**a**) $0 < x < \dfrac{\pi}{2}$ (**b**) $\dfrac{\pi}{2} < x < \pi$ **Hint** : $\sqrt{\sin^2 x} = |\,\sin x\,|$.