FOUNDATION SCIENCE A

SEMINAR 8: RESISTORS, KIRCHOFF'S LAW, AND MAGNETISM







LEARNING OUTCOMES

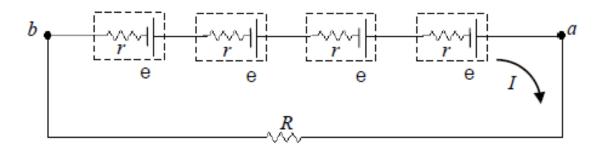
- To be able to use an expression for the respective resistors in series and in parallel.
- To solve for complex electrical circuit problems by using Kirchoff's Law.
- To understand how to charge and discharge a capacitor in an RC circuit.
- To investigate the relationship between magnetic force and fields.

Emf and Terminal Voltage

(1) Four 1.50 V cells are connected in series to a 12 Ω lightbulb. If the resulting current is 0.45 A, what is the internal resistance of each cell, assuming they are identical and neglecting the resistance of the wires?

Answer:

See the circuit diagram below. The current in the circuit is I. The voltage V_{ab} is given by Ohm's law to be $V_{ab} = IR$. That same voltage is the terminal voltage of the series EMF.



$$V_{ab} = (e - Ir) + (e - Ir) + (e - Ir) + (e - Ir) = 4(e - Ir)$$
 and $V_{ab} = IR$
 $4(e - Ir) = IR \rightarrow r = \frac{e - \frac{1}{4}IR}{I} = \frac{(1.5V) - \frac{1}{4}(0.45A)(12\Omega)}{0.45A} = 0.333\Omega \approx \boxed{0.3\Omega}$

Resistors in Series and Parallel

(2) Three I.70 $k\Omega$ resistors can be connected together in four different ways, making combinations of series and/or parallel circuits. What are these four ways, and what is the net resistance in each case?

Answer:

(i) The resistors can all be connected in series.

$$R_{eq} = R + R + R = 3(1.70 \text{k}\Omega) = 5.10 \text{k}\Omega$$

(ii) The resistors can all be connected in parallel.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow R_{\text{eq}} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3} = \frac{1.70 \,\text{k}\Omega}{3} = \boxed{567 \,\Omega}$$

Resistors in Series and Parallel

(2) Three 1.70 k Ω resistors can be connected together in four different ways, making combinations of series and/or parallel circuits. What are these four ways, and what is the net resistance in each case?

Answer:

(iii) Two resistors in series can be placed in parallel with the third.

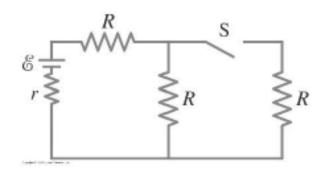
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R+R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \rightarrow R_{\text{eq}} = \frac{2R}{3} = \frac{2(1.70 \,\text{k}\Omega)}{3} = \boxed{1.13 \,\text{k}\Omega}$$

(iv) Two resistors in parallel can be placed in series with the third.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = R + \frac{R}{2} = \frac{3}{2}(1.70 \,\text{k}\Omega) = \boxed{2.55 \,\text{k}\Omega}$$

QUESTION 3

Three equal resistors (R) are connected to a battery as shown in the figure. Qualitatively, what happens to (a) the voltage drop across each of these resistors when the switch is opened?



Answer:

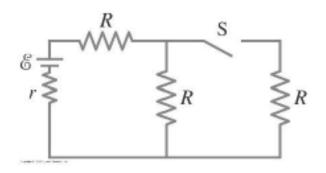
We label identical resistors from left to right as R_{left} , R_{middle} , and R_{right} .

When the switch is opened, the equivalent resistance of the circuit increases from $\frac{3}{2}R + r$ to 2R + r.

Thus the current delivered by the battery decreases, from $\frac{e}{\frac{3}{2}(R+r)}$ to $\frac{e}{2R+r}$. Note that this

is LESS than a 50% decrease.

(3) Three equal resistors (R) are connected to a battery as shown in the figure. Qualitatively, what happens to (a) the voltage drop across each of these resistors?

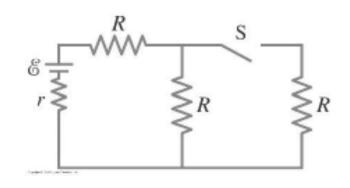


Answer:

(a) Because the current from the battery has decreased, the voltage drop across R_{left} will decrease, since it will have less current than before. The voltage drop across R_{right} decreases to 0, since no current is flowing in it. The voltage drop across R_{middle} will increase, because even though the total current has decreased, the current flowing through R_{middle} has increased since before the switch was opened, only half the total current was flowing through R_{middle} .

$$V_{
m left}$$
 decreases ; $V_{
m middle}$ increases ; $V_{
m right}$ goes to 0

3(b&c) Three equal resistors (*R*) are connected to a battery as shown in the figure below. Qualitatively, what happens to the current flow through each, and the terminal voltage of the battery, when the switch S is opened, after having been closed for a long time?



Answer:

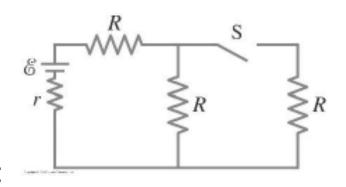
(b) By Ohm's law, the current is proportional to the voltage for a fixed resistance.

$$I_{\mathrm{left}}$$
 decreases ; I_{middle} increases ; I_{right} goes to 0

(c) Since the current from the battery has decreased, the voltage drop across *r* will decrease, and thus the terminal voltage increases.

QUESTION 3d&e:

Three equal resistors (R) are connected to a battery as shown in the figure below. Quantitatively, what happens to If the emf of the battery is 9.0 V, what is its terminal voltage when the switch is closed if the internal resistance r is 0.50 Ω and $R = 5.50 \Omega$? (e) What is the terminal voltage when the switch is open?



Answer:

(d) With the switch closed, the equivalent resistance is $\frac{3}{2}(R+r)$. Thus the current in the circuit is $I_{closed} = \frac{e}{\frac{3}{2}(R+r)}$, and the terminal voltage is given by the equation:

$$V_{\text{terminal closed}} = \mathbf{e} - I_{\text{closed}} r = \mathbf{e} - \frac{e}{\frac{3}{2}R + r} r = \mathbf{e} \left(1 - \frac{r}{\frac{3}{2}R + r} \right) = (9.0 \text{ V}) \left(1 - \frac{0.50 \Omega}{\frac{3}{2} (5.50 \Omega) + 0.50 \Omega} \right) = 8.486 \text{ V} \approx \boxed{8.5 \text{ V}}$$

(e) With the switch open, the equivalent resistance is 2R+r. Thus the current in the circuit is $I_{closed} = \frac{e}{2R+r}$, and again the terminal voltage is given by the equation below:

$$V_{\text{terminal closed}} = \mathbf{e} - I_{\text{closed}} r = \mathbf{e} - \frac{\mathbf{e}}{2R + r} r = \mathbf{e} \left(1 - \frac{r}{2R + r} \right) = (9.0 \, \text{V}) \left(1 - \frac{0.50 \, \Omega}{2 \left(5.50 \, \Omega \right) + 0.50 \, \Omega} \right) \\ = 8.609 \, \text{V} \approx \boxed{8.6 \, \text{V}}$$

QUESTION 4a:

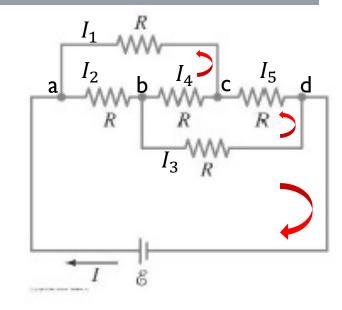
A network of five equal resistors R is connected to a battery \mathcal{E} as shown in the figure. Determine the current I that flows out of the battery.

Answer:

We label each of the currents as shown in the accompanying figure. Using Kirchhoff's junction rule and the first three junctions (a-c) we write equations relating the entering and exiting currents.

We use Kirchhoff's loop rule to write equations for loops abca, abcda, and bdcb.

We have six unknown currents and six equations.



$$I = I_1 + I_2 \tag{1}$$

$$I_2 = I_3 + I_4$$
 [2]

$$I_1 + I_4 = I_5 {3}$$

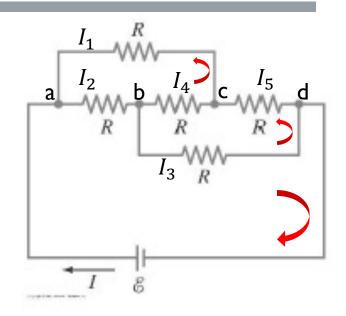
$$0 = -I_2 R - I_4 R + I_1 R$$
 [4]

$$0 = -I_2 R - I_3 R + e$$
 [5]

$$0 = -I_3 R + I_5 R + I_4 R$$
 [6]

QUESTION 4a:

A network of five equal resistors R is connected to a battery \mathcal{E} as shown in the figure. Determine the current I that flows out of the battery.



Answer:

We solve these equations by substitution. First, insert Eq. [3] into [6] to eliminate current I_5 . Next insert Eq. [2] into Eqs. [1], [4], and [5] to eliminate I_5 .

$$0 = -I_{3}R + (I_{1} + I_{4})R + I_{4}R \rightarrow 0 = -I_{3}R + I_{1}R + 2I_{4}R$$
 [6*]

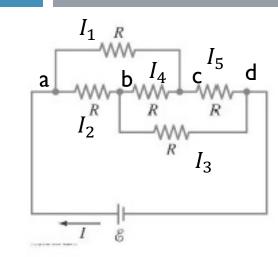
$$I = I_{1} + I_{3} + I_{4}$$
 [1*]

$$0 = -(I_{3} + I_{4})R - I_{4}R + I_{1}R \rightarrow 0 = -I_{3}R - 2I_{4}R + I_{1}R$$
 [4*]

$$0 = -(I_{3} + I_{4})R - I_{3}R + e \rightarrow 0 = -I_{4}R - 2I_{3}R + e$$
 [5*]

QUESTION 4a:

A network of five equal resistors R is connected to a battery \mathcal{E} as shown in the figure. Determine the current I that flows out of the battery.



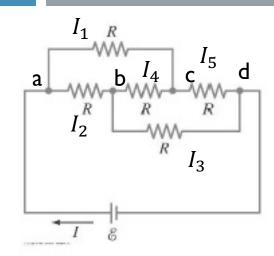
Answer:

Next we solve Eq. [4*] for I_4 and insert the result into Equations. [1*], [5*], and [6*].

$$\begin{split} 0 &= -I_3 R - 2I_4 R + I_1 R \to I_4 = \frac{1}{2}I_1 - \frac{1}{2}I_3 \\ I &= I_1 + I_3 + \frac{1}{2}I_1 - \frac{1}{2}I_3 \to I = \frac{3}{2}I_1 + \frac{1}{2}I_3 \\ 0 &= -I_3 R + I_1 R + 2\left(\frac{1}{2}I_1 - \frac{1}{2}I_3\right) R = -2I_3 R + 2I_1 R \to I_1 = I_3 \\ 0 &= -\left(\frac{1}{2}I_1 - \frac{1}{2}I_3\right) R - 2I_3 R + \Theta \to 0 = -\frac{1}{2}I_1 R - \frac{3}{2}I_3 R + \Theta \end{split}$$
 [5**]

QUESTION 4a:

A network of five equal resistors R is connected to a battery $\mathcal E$ as shown in the figure. Determine the current I that flows out of the battery.



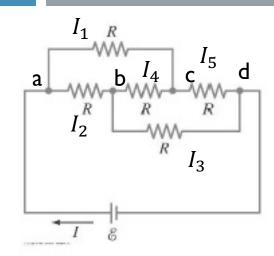
Answer:

Finally we substitute Eq. $[6^{**}]$ into Eq $[5^{**}]$ and solve for I_1 . We insert this result into Eq. $[1^{**}]$ to write an equation for the current through the battery in terms of the battery emf and resistance.

$$0 = -\frac{1}{2}I_1R - \frac{3}{2}I_1R + \mathbf{e} \to I_1 = \frac{\mathbf{e}}{2R} \; \; ; \; I = \frac{3}{2}I_1 + \frac{1}{2}I_1 = 2I_1 \to \boxed{I = \frac{\mathbf{e}}{R}}$$

QUESTION 4b:

Use the value determined for I to find the single resistor R_{eq} that is equivalent to the five-resistor network.



Answer:

We divide the battery emf by the current to determine the effective resistance.

$$R_{eq} = \frac{e}{I} = \frac{e}{e/R} = \boxed{R}$$

QUESTION 5:

Determine the current through each of the resistors in the figure.

Answer:

The circuit diagram has been labeled with six different currents. We apply the junction rule to junctions a, b, and c. We apply the loop rule to the three loops labeled in the diagram.

1)
$$I = I_1 + I_2$$
; 2) $I_1 = I_3 + I_5$; 3) $I_3 + I_4 = I$

4)
$$-I_1R_1 - I_5R_5 + I_2R_2 = 0$$
; 5) $-I_3R_3 + I_4R_4 + I_5R_5 = 0$

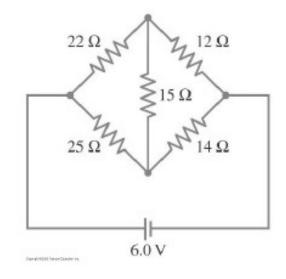
6)
$$e - I_2 R_2 - I_4 R_4 = 0$$

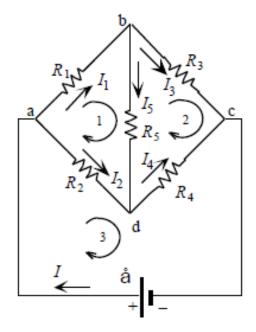
Eliminate *I* using equations 1) and 3).

1)
$$I_3 + I_4 = I_1 + I_2$$
; 2) $I_1 = I_3 + I_5$

4)
$$-I_1R_1 - I_5R_5 + I_2R_2 = 0$$
; 5) $-I_3R_3 + I_4R_4 + I_5R_5 = 0$

6)
$$e - I_2 R_2 - I_4 R_4 = 0$$





QUESTION 5:

Determine the current through each of the resistors in the figure.

Answer:

Eliminate I_2 using equation 2.

1)
$$I_3 + I_4 = I_3 + I_5 + I_2 \rightarrow I_4 = I_5 + I_2$$

4)
$$-(I_3 + I_5)R_1 - I_5R_5 + I_2R_2 = 0 \rightarrow -I_3R_1 - I_5(R_1 + R_5) + I_2R_2 = 0$$

$$5) -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

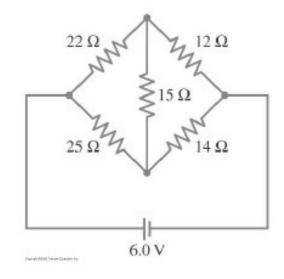
6)
$$e - I_2 R_2 - I_4 R_4 = 0$$

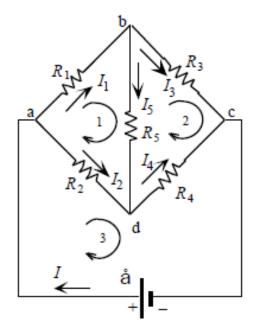
Eliminate I_4 using equation 1.

4)
$$-I_3R_1-I_5(R_1+R_5)+I_2R_2=0$$

$$5) -I_3R_3 + (I_5 + I_2)R_4 + I_5R_5 = 0 \rightarrow -I_3R_3 + I_5(R_4 + R_5) + I_2R_4 = 0$$

6)
$$e - I_2 R_2 - (I_5 + I_2) R_4 = 0 \rightarrow e - I_2 (R_2 + R_4) - I_5 R_4 = 0$$





QUESTION 5:

Determine the current through each of the resistors in the figure.

Answer:

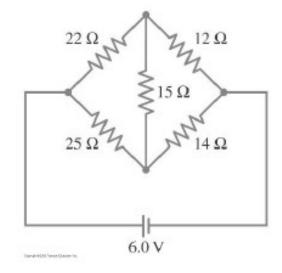
Eliminate
$$I_2$$
 using equation 4: $I_2 = \frac{1}{R_2} [I_3 R_1 + I_5 (R_1 + R_5)].$

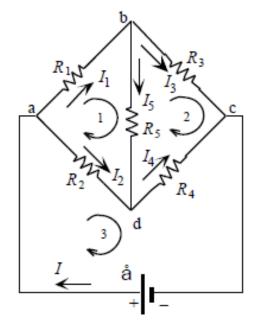
5)
$$-I_3R_3 + I_5(R_4 + R_5) + \frac{1}{R_2} [I_3R_1 + I_5(R_1 + R_5)]R_4 = 0 \rightarrow$$

 $I_3(R_1R_4 - R_2R_3) + I_5(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4) = 0$

6)
$$e - \frac{1}{R_2} [I_3 R_1 + I_5 (R_1 + R_5)] (R_2 + R_4) - I_5 R_4 = 0 \rightarrow$$

 $e R_2 - I_3 R_1 (R_2 + R_4) - I_5 (R_1 R_2 + R_1 R_4 + R_5 R_2 + R_5 R_4 + R_2 R_4) = 0$



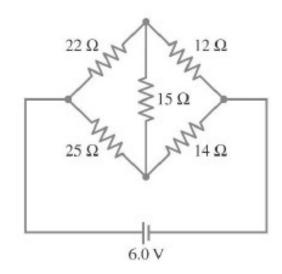


QUESTION 5:

Determine the current through each of the resistors in the figure.

Answer:

Eliminate
$$I_3$$
 using equation 5: $I_3 = -I_5 \frac{\left(R_2 R_4 + R_2 R_5 + R_1 R_4 + R_5 R_4\right)}{\left(R_1 R_4 - R_2 R_3\right)}$

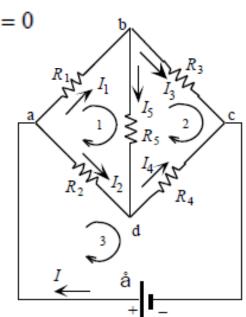


$$eR_{2} + \left[I_{5} \frac{\left(R_{2}R_{4} + R_{2}R_{5} + R_{1}R_{4} + R_{5}R_{4}\right)}{\left(R_{1}R_{4} - R_{2}R_{3}\right)}\right]R_{1}\left(R_{2} + R_{4}\right) - I_{5}\left(R_{1}R_{2} + R_{1}R_{4} + R_{5}R_{2} + R_{5}R_{4} + R_{2}R_{4}\right) = 0$$

$$e = -\frac{I_5}{R_2} \left\{ \left[\frac{\left(R_2 R_4 + R_2 R_5 + R_1 R_4 + R_5 R_4 \right)}{\left(R_1 R_4 - R_2 R_3 \right)} \right] R_1 \left(R_2 + R_4 \right) - \left(R_1 R_2 + R_1 R_4 + R_5 R_2 + R_5 R_4 + R_2 R_4 \right) \right\}$$

$$= -\frac{I_5}{25\Omega} \left\{ \begin{bmatrix} \frac{(25\Omega)(14\Omega) + (25\Omega)(15\Omega) + (22\Omega)(14\Omega) + (15\Omega)(14\Omega)}{(22\Omega)(14\Omega) - (25\Omega)(12\Omega)} \\ -[(22\Omega)(25\Omega) + (22)(14) + (15\Omega)(25\Omega) + (15\Omega)(14\Omega) + (25\Omega)(14\Omega) \end{bmatrix} \right\}$$

$$=-I_5(5261\Omega) \rightarrow I_5 = -\frac{6.0 \text{ V}}{5261\Omega} = -1.140 \text{ mA (upwards)}$$



QUESTION 5:

Determine the current through each of the resistors in the figure.

Answer:

$$I_{3} = -I_{5} \frac{\left(R_{2}R_{4} + R_{2}R_{5} + R_{1}|R_{4} + R_{5}R_{4}\right)}{\left(R_{1}R_{4} - R_{2}R_{3}\right)}$$

$$= -(-1.140 \,\mathrm{mA}) \frac{(25\Omega)(14\Omega) + (25\Omega)(15\Omega) + (22\Omega)(14\Omega) + (15\Omega)(14\Omega)}{(22\Omega)(14\Omega) - (25\Omega)(12\Omega)} = 0.1771 \,\mathrm{A}$$

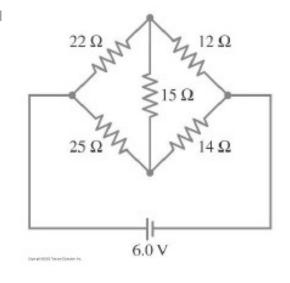
$$I_2 = \frac{1}{R_2} \left[I_3 R_1 + I_5 \left(R_1 + R_5 \right) \right] = \frac{1}{25\Omega} \left[(0.1771 \,\text{A}) (22\Omega) + (-0.00114 \,\text{A}) (37\Omega) \right] = 0.1542 \,\text{A}$$

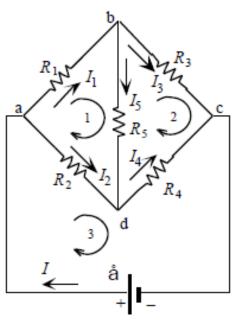
$$I_4 = I_5 + I_2 = -0.00114 \,\text{A} + 0.1542 \,\text{A} = 0.1531 \,\text{A}$$

$$I_1 = I_3 + I_5 = 0.1771 \,\mathrm{A} - 0.00114 \,\mathrm{A} = 0.1760 \,\mathrm{A}$$

We keep an extra significant figure to show the slight difference in the currents.

the currents.
$$I_{22\Omega} = 0.176 \text{A} \quad I_{25\Omega} = 0.154 \, \text{A} \quad I_{12\Omega} = 0.177 \, \text{A} \quad I_{14\Omega} = 0.153 \, \text{A} \quad I_{15\Omega} = 0.001 \, \text{A}, \text{ upwards}$$

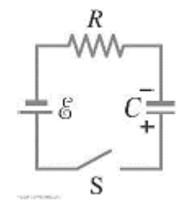




RC Circuits

QUESTION 6:

In the figure below, the total resistance is 15.0 k Ω , and the battery's emf is 24.0 V. If the time constant is measured to be 24.0 μ s, calculate (a) the total capacitance of the circuit and (b) the time it takes for the voltage across the resistor to reach 16.0 V after the switch is closed.



Answer:

(a) The product RC is equal to the time constant τ .

$$\tau = RC \rightarrow C = \frac{\tau}{R} = \frac{24.0 \times 10^{-6} \text{ s}}{15.0 \times 10^{3} \Omega} = \boxed{1.60 \times 10^{-9} \text{ F}}$$

(b) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_{C} = e \left(1 - e^{-t/\tau} \right) \rightarrow e^{-t/\tau} = \left(1 - \frac{V_{C}}{e} \right) \rightarrow -\frac{t}{\tau} = \ln \left(1 - \frac{V_{C}}{e} \right) \rightarrow t = -\tau \ln \left(1 - \frac{V_{C}}{e} \right) = -\left(24.0 \times 10^{-6} \text{s} \right) \ln \left(1 - \frac{8.0 \text{ V}}{24.0 \text{ V}} \right) = 9.73 \times 10^{-6} \text{s}$$

RC Circuits

QUESTION 7:

How long does it take for the energy stored in a capacitor in a series RC circuit (shown in the figure above) to reach 75% of its maximum value? Express your answer in terms of the time constant $\tau = RC$?

Answer:

Express the stored energy in terms of the charge on the capacitor, using the equation below. The charge on the capacitor is given by:

$$U = \frac{1}{2} \frac{Q^{2}}{C} = \frac{1}{2} \frac{\left[Ce \left(1 - e^{-t/\tau} \right) \right]^{2}}{C} = \frac{1}{2} Ce^{2} \left(1 - e^{-t/\tau} \right)^{2} = U_{\text{max}} \left(1 - e^{-t/\tau} \right)^{2} \; ;$$

$$U = 0.75 U_{\text{max}} \quad \rightarrow \quad U_{\text{max}} \left(1 - e^{-t/\tau} \right)^{2} = 0.75 U_{\text{max}} \quad \rightarrow \quad \left(1 - e^{-t/\tau} \right)^{2} = 0.75 \quad \rightarrow \quad t = -\tau \ln \left(1 - \sqrt{0.75} \right) = \boxed{2.01\tau}$$

Forces on Electric Current in Magnetic Fields

QUESTION 8:

Calculate the magnitude of the magnetic force on a 240 m length of wire stretched between two towers and carrying a 150 A current. The Earth's magnetic field of 5.0×10^{-5} T makes an angle of 68° with the wire.

Answer:

Use the equation below to calculate the force.

$$F = I | B \sin \theta = (150 \,\text{A})(240 \,\text{m})(5.0 \times 10^{-6} \,\text{T}) \sin 68^{\circ} = 1.7 \,\text{N}$$

Forces on Electric Current in Magnetic Fields

QUESTION 9:

Suppose a straight 1.00 mm diameter copper wire could just 'float' horizontally in air because of the force due to the Earth's magnetic field \vec{B} , which is horizontal, perpendicular to the wire, and of magnitude 5.0×10^{-5} T. What current would the wire carry? Does the answer seem feasible? Explain briefly.

Answer:

- The magnetic force must be equal in magnitude to the force of gravity on the wire.
- The maximum magnetic force is applicable since the wire is perpendicular to the magnetic field.
- The mass of the wire is the density of copper times the volume of the wire.

$$F_{\rm B} = mg \rightarrow I1B = \rho \pi \left(\frac{1}{2}d\right)^2 1g \rightarrow$$

$$I = \frac{\rho \pi d^2 g}{4B} = \frac{\left(8.9 \times 10^3 \text{ kg/m}^3\right) \pi \left(1.00 \times 10^{-3} \text{m}\right)^2 \left(9.80 \text{ m/s}^2\right)}{4 \left(5.0 \times 10^{-5} \text{T}\right)} = \boxed{1400 \text{ A}}$$

Forces on Charge Moving in Magnetic Fields

QUESTION 10:

An electron is projected vertically upward with a speed of $1.70 \times 10^6 \text{ m} \cdot \text{s}^{-1}$ into a uniform magnetic field of 0.480 T that is directed horizontally away from the observer. Describe the electron's path in this field.

Answer:

The magnetic force will cause **centripetal motion**, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

$$F_{\text{max}} = qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31} \text{kg})(1.70 \times 10^6 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.480 \text{ T})} = \boxed{2.02 \times 10^{-5} \text{m}}$$

Forces on Charge Moving in Magnetic Fields

QUESTION II:

A 3.40 g bullet moves with a speed of I55 m·s⁻¹ perpendicular to the Earth's magnetic field of 5.0×10^{-5} T. If the bullet possesses a net charge of 18.5×10^{-9} C, by what distance will it be deflected from its path due to the Earth's magnetic field after it has traveled I.00 km?

Answer:

The magnetic force produces an acceleration that is **perpendicular** to the original motion. If that perpendicular acceleration is small, it will produce a small deflection, and the original velocity can be assumed to always be perpendicular to the magnetic field. This leads to a constant perpendicular acceleration. The time that this (approximately) constant acceleration acts can be found from the original velocity v and the distance traveled l. The starting speed in the perpendicular direction will be zero.

$$F_{\perp} = ma_{\perp} = qvB \rightarrow a_{\perp} = \frac{qvB}{m} \qquad d_{\perp} = v_{0\perp}t + \frac{1}{2}a_{\perp}t^{2} = \frac{1}{2}\frac{qvB}{m}\left(\frac{1}{v}\right)^{2} = \frac{qB1^{2}}{2mv}$$

$$= \frac{\left(18.5 \times 10^{-9} \text{C}\right)\left(5.00 \times 10^{-5} \text{T}\right)\left(1.00 \times 10^{3} \text{m}\right)^{2}}{2\left(3.40 \times 10^{-3} \text{kg}\right)\left(155 \text{m/s}\right)} = 8.8 \times 10^{-7} \text{m}$$

Q&A? OFFICE HOURS:

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