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Foundation Calculus and Mathematical Techniques (CELEN037)

Answers to WorkSheet #6

1.

(i) $\dfrac{\sin^5 x}{5} - \dfrac{\sin^7 x}{7} + C$

(ii) $-\dfrac{\cos^8 x}{8} + \dfrac{\cos^{10} x}{10} + C$

(iii) $-\dfrac{\cos^3 x}{3} + \dfrac{\cos^5 x}{5} + C$

(iv) $\dfrac{\sin^3 x}{3} - \dfrac{2\sin^5 x}{5} + \dfrac{\sin^7 x}{7} + C$

2.

(i) $\ln\left(1+\sin^2 x\right) + C$

(ii) $-\ln\left(1+\cos^2 x\right) + C$

(iii) $\dfrac{\ln\left(1+x^2\right)}{2} + C$

(iv) $\dfrac{\ln\left|1+x^3\right|}{3} + C$

(v) $-\ln\left|\cos x\right| + C$

(vi) $\ln\left|\sin x\right| + C$

(vii) $-\ln\left|\sec x - \tan x\right| + C$

(viii) $\ln\left|\text{cosec } x - \cot x\right| + C$

(ix) $\ln\left|x\right| + x + C$

(x) $\ln\left(e^x + e^{-x}\right) + C$

(xi) $\ln\left(e^x + e^{-x}\right) + C$

(xii) $\ln\left|e^x - e^{-x}\right| + C$

3.

(ii) $\tan^{-1}(x+1) + C$

(ii) $\dfrac{1}{3}\tan^{-1}\left(\dfrac{x+1}{3}\right) + C$

(iii) $\dfrac{1}{\sqrt{5}}\tan^{-1}\left(\dfrac{x+2}{\sqrt{5}}\right) + C$

(iv) $\dfrac{1}{2\sqrt{13}}\ln\left|\dfrac{x+2+\sqrt{13}}{x+2-\sqrt{13}}\right| + C$

(v) $\dfrac{1}{6}\ln\left|\dfrac{x+2}{x-4}\right| + C$

(vi) $\ln\left|x+2+\sqrt{(x+2)^2-3^2}\right| + C$

(vii) $\ln\left|x+2+\sqrt{(x+2)^2-1}\right| + C$

(viii) $\ln\left|x+\dfrac{9}{2}+\sqrt{\left(x+\dfrac{9}{2}\right)^2-\dfrac{101}{4}}\right| + C$

(ix) $\sin^{-1}\left(\dfrac{x-2}{3}\right) + C$

(x) $\dfrac{1}{2}\ln\left(x+\dfrac{1}{2}+\sqrt{\left(x+\dfrac{1}{2}\right)^2+\dfrac{1}{2}}\right) + C$

4.

(i) $\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+C$

(ii) $\dfrac{1}{\sqrt{3}}\ln\left|\dfrac{\tan\left(\frac{x}{2}\right)+\sqrt{3}}{\tan\left(\frac{x}{2}\right)-\sqrt{3}}\right|+C$

(iii) $\dfrac{1}{\sqrt{5}}\ln\left|\dfrac{\tan\left(\frac{x}{2}\right)+\sqrt{5}}{\tan\left(\frac{x}{2}\right)-\sqrt{5}}\right|+C$

(iv) $\dfrac{1}{\sqrt{5}}\ln\left|\dfrac{\tan\left(\frac{x}{2}\right)-\frac{1}{\sqrt{5}}}{\tan\left(\frac{x}{2}\right)+\frac{1}{\sqrt{5}}}\right|+C$

(v) $\dfrac{1}{\sqrt{15}}\ln\left|\dfrac{\tan\left(\frac{x}{2}\right)+\sqrt{\frac{5}{3}}}{\tan\left(\frac{x}{2}\right)-\sqrt{\frac{5}{3}}}\right|+C$

(vi) $\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right)+C$

(vii) $\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2\tan\left(\frac{x}{2}\right)+1}{\sqrt{3}}\right)+C$

(viii) $-\dfrac{1}{\tan\left(\frac{x}{2}\right)+2}+C$

5.

(ii) $\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{\tan x}{\sqrt{3}}\right)+C$

(ii) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\tan x}{\sqrt{2}}\right)+C$

(iii) $\dfrac{1}{\sqrt{10}}\tan^{-1}\left(\dfrac{\tan x}{\sqrt{\frac{2}{5}}}\right)+C$

(iv) $\dfrac{1}{2}\ln\left|\dfrac{\tan x+1}{\tan x-1}\right|+C$

(v) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+C$

(vi) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{\tan x}{2}\right)+C$