Master Theorem

Additional Practice Questions

For each of the following questions: Identify which case of the Master Theorem each applies to (if any) and find the scaling behaviour. The base case for all questions is $T(1) = 1$.

Questions 1-10 can be found in the accompanying tutorial slides!

Q11. $\boldsymbol{T(n) = 3 \cdot T(n/3)}$

- $f(n) =$ return 0 hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_3 3$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^1) = \Theta(n)$

Q12. $\boldsymbol{T(n) = T(n-1) + n}$

- $a = 1; b =$ not defined
- M.T. not applicable – solve by induction.

Q13. $\boldsymbol{T(n) = 2 \cdot T(n/2) + 1}$

- $f(n) = 1$ hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_2 2$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^1) = \Theta(n)$

Q14. $\boldsymbol{T(n) = 2 \cdot T(n/2) + n^2}$

- $f(n) = n^2$ hence is $O(n^2)$
- $c = 2$ and $2 > \log_2 2$ hence case 3 applies.
- $T(n)$ is $\Theta(f(n)) = \Theta(n^2)$

Q15. $\boldsymbol{T(n) = 4 \cdot T(n/4)}$

- $f(n) =$ return 0 hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_4 4$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^1) = \Theta(n)$

Q16. $\boldsymbol{T(n) = 3 \cdot T(n/3) + 1}$

- $f(n) = 1$ hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_3 3$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^1) = \Theta(n)$

Q17. $\boldsymbol{T(n) = 3 \cdot T(n-1)}$

- $a = 3; b =$ not defined
- M.T. not applicable – solve by induction.

Q18. $\boldsymbol{T(n) = 2 \cdot T(n/2) + n \log n}$

- $f(n) = n \log n$ is $\Theta(n \log n)$
- $c = 1 = \log_b a$
- Hence M.T. case 2 with $k = 1$:
- $T(n)$ is $\Theta(n(\log n)^{1+1}) = \Theta(n(\log n)^2)$

Q19. $\boldsymbol{T(n) = 2 \cdot T(n/2) + 2n}$

- $f(n) = 2n$ is $\Theta(n)$
- $c = 1 = \log_b a$
- Hence M.T. case 2 with $k = 0$:
- $T(n)$ is $\Theta(n(\log n)^{0+1}) = \Theta(n \log n)$

Q20. $\boldsymbol{T(n) = 4 \cdot T(n/2) + 1}$

- $f(n) = 1$ hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_2 4$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^2)$

Q21. $\boldsymbol{T(n) = 2 \cdot T(n/4)}$

- $f(n) = 0$ hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_4 2$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^{0.5})$

Q22. $\boldsymbol{T(n) = n \cdot T(n-1)}$

- $b =$ not defined

- M.T. not applicable – solve by induction.

Q23. $\boldsymbol{T(n) = 4 \cdot T(n/2)}$

- $f(n) =$ return 0 hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_2 4$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^2)$

Q24. $\boldsymbol{T(n) = 2 \cdot T(n/4) + n^2}$

- $f(n) = n^2$ hence is $O(n^2)$
- $c = 2$ and $2 > \log_4 2$ hence case 3 applies.
- $T(n)$ is $\Theta(f(n)) = \Theta(n^2)$

Q25. $\boldsymbol{T(n) = 2 \cdot T(n/2) + n \log n + 3n}$

- $f(n) = n \log n + 3n$ is $\Theta(n \log n)$
- $c = 1 = \log_b a$
- Hence M.T. case 2 with $k = 1$:
- $T(n)$ is $\Theta(n(\log n)^{1+1}) = \Theta(n\,(\log n)^2)$

Q26. $\boldsymbol{T(n) = 2 \cdot T(n/2) + 2}$

- $f(n) = 2$ hence is $O(1) = O(n^0)$
- $c = 0$ and $0 < \log_2 2$ hence case 1 applies.
- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^1) = \Theta(n)$

Q27. $\boldsymbol{T(n) = 3 \cdot T(n/3) + n}$

- $f(n) = n$ is $\Theta(n)$
- $c = 1 = \log_b a$
- Hence M.T. case 2 with $k = 0$:
- $T(n)$ is $\Theta(n(\log n)^{0+1}) = \Theta(n \log n)$

Q28. $\boldsymbol{T(n) = 4 \cdot T(n/2) + 2n}$

- $f(n) = 2n$ hence is $O(n) = O(n^1)$
- $c = 1$ and $1 < \log_2 4$ hence case 1 applies.

- $T(n)$ is $\Theta\left(n^{\log_b a}\right) = \Theta(n^2)$

Q29. $\boldsymbol{T(n) = 3 \cdot T(n/3) + n^2}$

- $f(n) = n^2$ hence is $O(n^2)$
- $c = 2$ and $2 > \log_3 3$ hence case 3 applies.
- $T(n)$ is $\Theta(f(n)) = \Theta(n^2)$