University of Nottingham
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Foundation Algebra (CELEN036)

Problem Sheet 10 **Topics: Partial fractions, sequences**

Topic 1: Partial fractions – non-repeated linear factors

$$\boxed{\frac{1}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}}$$

1. Express the following as the sum of partial fractions:

(i) $\dfrac{-3}{(2x+1)(x-3)}$ (ii) $\dfrac{3x}{(x-1)(x+2)}$ (iii) $\dfrac{3x-2}{(x+1)(x-1)}$

(iv) $\dfrac{2x-1}{(x+2)(x-3)}$ (v) $\dfrac{2x+5}{(x-2)(x+1)}$ (vi) $\dfrac{7x+10}{(2x+3)(x+1)}$

(vii) $\dfrac{1}{x^2+5x+6}$ (viii) $\dfrac{2x+3}{x^2+3x+2}$ (ix) $\dfrac{9-4x}{3x^2-x-2}$

(x) $\dfrac{11x+1}{2x^2-x-1}$ (xi) $\dfrac{5x-1}{x^2-x-2}$ (xii) $\dfrac{7x+25}{x^2+7x+12}$

(xiii) $\dfrac{3}{x(x-1)(x+1)}$ (xiv) $\dfrac{6x+12}{x(x-2)(x+3)}$ (xv) $\dfrac{5x+1}{(x-1)(x-2)(x-3)}$

Topic 2: Partial fractions – non-repeated quadratic factor

$$\boxed{\frac{1}{(x+a)(x^2+b)} = \frac{A}{(x+a)} + \frac{Bx+C}{(x^2+b)}}$$

2. Express the following as the sum of partial fractions:

(i) $\dfrac{3x+1}{(x^2+x+10)(x-1)}$ (ii) $\dfrac{5x}{(x^2+x+1)(x-2)}$ (iii) $\dfrac{13}{(x^2+1)(2x+3)}$

(iv) $\dfrac{x}{(x^2-x+1)(3x-2)}$ (v) $\dfrac{3}{(x^2+x+1)(x-2)}$ (vi) $\dfrac{x^2-3x-7}{(x^2+x+2)(2x-1)}$

Topic 3: Partial fractions – repeated linear factors

$$\boxed{\frac{1}{(x+a)^2(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+b)}}$$

3. Express the following as the sum of partial fractions:

(i) $\dfrac{5}{(x+1)^2(x-1)}$ (ii) $\dfrac{3x}{(x+2)^2(x-1)}$ (iii) $\dfrac{x}{(x-3)^2(2x+1)}$

(iv) $\dfrac{x^2+1}{(x-1)^2(x+1)}$ (v) $\dfrac{3x+1}{(x-1)^2(x+2)}$ (vi) $\dfrac{3x+7}{(x+2)^2(x+3)}$

Topic 4: Finding terms of a sequence

4. Find the first five terms (i.e., for $n = 1, 2, 3, 4, 5$) of the following sequence:

   (i) $f(n) = 3n + 2$ (ii) $f(n) = \dfrac{n+2}{n(n+1)}$ (iii) $f(n) = \dfrac{1}{n(n+1)}$

   (iv) $f(n) = 1 + n^2$ (v) $f(n) = 1 + (-1)^n$ (vi) $f(n) = (-1)^n(n+1)$

5. Find the formula of the n^{th} term of the following sequences:

   (i) $3,\ 7,\ 11,\ 15,\ 19, \cdots$ (ii) $5,\ 2,\ -1,\ -4,\ -7, \cdots$ (iii) $1,\ 10,\ 19,\ 28,\ 37, \cdots$

   (iv) $2,\ 6,\ 18,\ 54, \cdots$ (v) $1,\ \dfrac{1}{2},\ \dfrac{1}{4},\ \dfrac{1}{8}, \cdots$ (vi) $4,\ -8,\ 16,\ -32,\ 64, \cdots$

   (vii) $-1,\ -\dfrac{1}{3},\ -\dfrac{1}{9},\ -\dfrac{1}{27}, \cdots$

Topic 5: Arithmetic progression

6. Find the 10^{th} term of the sequence: $3,\ 15,\ 27,\ 39, \cdots$

7. Find the 14^{th} term of the sequence: $-19,\ -16,\ -13,\ -10, \cdots$

8. Find the 12^{th} term of the sequence: $95,\ 80,\ 65,\ 50, \cdots$

9. Find the 16^{th} term of the sequence: $22.5,\ 20,\ 17.5,\ 15, \cdots$

10. Obtain the twelfth, the twenty-seventh, and the fortieth terms of the arithmetic sequence: $3,\ 7,\ 11,\ 15, \cdots$

11. The first term of an arithmetic sequence is 7 and its common difference is 5. If the n^{th} term $a_n = 462$, find n.

12. For an arithmetic sequence $152,\ 145,\ 138,\ 131, \cdots$, if the n^{th} term $a_n = 12$, find n.

Topic 6: Geometric progression

13. The first term of a geometric sequence is 2 and the common ratio is 3. Find the fourth term.

14. The first term of a geometric sequence is -3 and the third term is -12. Find the second term a_2.

15. The sixth term of a geometric sequence is 16 and the third term is 2. Find the first term a and the common ratio r.

16. For a geometric sequence $\dfrac{1}{3},\ \dfrac{1}{6},\ \dfrac{1}{12},\ \dfrac{1}{24},\ \cdots$, if the n^{th} term $a_n = \dfrac{1}{3072}$, find n.

17. Find the negative common ratio of a geometric progression whose first term is 8 and the fifth term is $\frac{1}{2}$.

18. The second term of a geometric progression with only positive terms is $\frac{1}{4}$ and the sum of the first four terms is $\frac{1}{16}$ of the sum of the next four terms. Find the first term and the common ratio of this sequence.

19. Given some geometric progression, find those three consecutive terms whose sum is 65 and whose product is 3375.

20. The sum and product of three consecutive terms in a geometric progression are 52 and 1728 respectively. Find these three terms.

Answers

1. (i) $\frac{6/7}{2x+1}+\frac{-3/7}{x-3}$ (ii) $\frac{1}{x-1}+\frac{2}{x+2}$ (iii) $\frac{5/2}{x+1}+\frac{1/2}{x-1}$

   (iv) $\frac{1}{x+2}+\frac{1}{x-3}$ (v) $\frac{3}{x-2}+\frac{-1}{x+1}$ (vi) $\frac{1}{2x+3}+\frac{3}{x+1}$

   (vii) $\frac{1}{x+2}+\frac{-1}{x+3}$ (viii) $\frac{1}{x+1}+\frac{1}{x+2}$ (ix) $\frac{1}{x-1}+\frac{-7}{3x+2}$

   (x) $\frac{4}{x-1}+\frac{3}{2x+1}$ (xi) $\frac{3}{x-2}+\frac{2}{x+1}$ (xii) $\frac{3}{x+4}+\frac{4}{x+3}$

   (xiii) $\frac{3/2}{x+1}+\frac{3/2}{x-1}+\frac{-3}{x}$ (xiv) $\frac{12/5}{x-2}+\frac{-2/5}{x+3}+\frac{-2}{x}$ (xv) $\frac{3}{x-1}+\frac{-11}{x-2}+\frac{8}{x-3}$

2. (i) $\frac{1/3}{x-1}+\frac{-x/3+7/3}{x^2+x+10}$ (ii) $\frac{10/7}{x-2}+\frac{-10x/7+5/7}{x^2+x+1}$ (iii) $\frac{4}{2x+3}+\frac{-2x+3}{x^2+1}$

   (iv) $\frac{6/7}{3x-2}+\frac{-2x/7+3/7}{x^2-x+1}$ (v) $\frac{3/7}{x-2}+\frac{-3x/7-9/7}{x^2+x+1}$ (vi) $\frac{-3}{2x-1}+\frac{2x+1}{x^2+x+2}$

3. (i) $\frac{-5/4}{x+1}+\frac{-5/2}{(x+1)^2}+\frac{5/4}{x-1}$ (ii) $\frac{-1/3}{x+2}+\frac{2}{(x+2)^2}+\frac{1/3}{x-1}$ (iii) $\frac{1/49}{x-3}+\frac{3/7}{(x-3)^2}+\frac{-2/49}{2x+1}$

   (iv) $\frac{1/2}{x-1}+\frac{1}{(x-1)^2}+\frac{1/2}{x+1}$ (v) $\frac{5/9}{x-1}+\frac{4/3}{(x-1)^2}+\frac{-5/9}{x+2}$ (vi) $\frac{2}{x+2}+\frac{1}{(x+2)^2}+\frac{-2}{x+3}$

4. (i) $5, 8, 11, 14, 17$ (ii) $\frac{3}{2}, \frac{2}{3}, \frac{5}{12}, \frac{3}{10}, \frac{7}{30}$ (iii) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}$

   (iv) $2, 5, 10, 17, 26$ (v) $0, 2, 0, 2, 0$ (vi) $-2, 3, -4, 5, -6$

5. (i) $4n - 1$ (ii) $8 - 3n$ (iii) $9n - 8$ (iv) $2 \cdot 3^{n-1}$

   (v) $\left(\frac{1}{2}\right)^{n-1}$ (vi) $(-2)^{n+1}$ (vii) $-\left(\frac{1}{3}\right)^{n-1}$

6. 111

7. 20

8. -70

9. -15

10. $47, \quad 107, \quad 159$

11. 92

12. 21

13. 54

14. ± 6

15. $a = \frac{1}{2}, \quad r = 2$

16. 11

17. $-\frac{1}{2}$

18. $a = \frac{1}{8}, \quad r = 2$

19. $45, 15, 5$ and $5, 15, 45$

20. $4, 12, 36$ and $36, 12, 4$