Foundation Algebra for Physical Sciences and Engineering (CELEN036)

Homework 7

1. Given that $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$,

and $n! = n(n-1)(n-2)!$ or $n! = n(n-1)(n-2)(n-3).....3 \times 2 \times 1$, where $n \in \mathbb{N}$.

Evaluate and simplify the following.

(a) $\binom{n}{n-1}$

(b) $\binom{n+1}{n-1}$

(c) $\dfrac{\binom{n}{k+1}}{\binom{n}{k}}$

(d) $\dfrac{\binom{n+1}{r}}{\binom{n}{r-1}}$

2. Use the formula for the binomial theorem to expand the following expressions.

(a) $(x-1)^5$

(b) $(x-3y)^4$

(c) $\left(\dfrac{1}{x} - 2y^2\right)^4$

(d) $\left(x^2 + \dfrac{2}{x}\right)^6$

(e) $\left(3x + \dfrac{1}{x^2}\right)^5$

(f) $(x-1)^7$

3. Find the coefficient of the term that contains the given power of x^n.

(a) $(x^3 - \dfrac{2}{x})^4; \quad x^4$

(b) $\left(2x - \dfrac{1}{3}\right)^{10}; \quad x^7$

(c) $(x^2+2)^{11}; \quad x^8$

(d) $\left(x^2 - \dfrac{2}{x}\right)^{10}; \quad x^8$

(e) $\left(x^3 - \dfrac{1}{x}\right)^{15}; \quad x^{25}$

(f) $\left(x^2 - \dfrac{3}{x}\right)^{12}; \quad x^9$

(g) $(x^2 - 2x + 1)^3; \quad x^4$

4.

(a) Use the generalised binomial theorem to find an approximate value for $(1.05)^{-\frac{1}{2}}$ correct to 4 decimal places, by expanding until the term with x^3.

(b) Given that $\beta = \pi\alpha v^{\frac{2}{3}}$, where α is a constant. If the error in calculating v is 1.25%, find the error in calculating β. Give your answer correct to 3 decimal places, and use the expansion from the generalised binomial theorem up to the term with x^3.