University of Nottingham
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Foundation Algebra for Physical Science & Engineering (CELEN036)

Homework 3

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1. Solve the following equations for angles in the interval $[0, 2\pi]$ or $[0, 360°]$.

   (i) $\cos\theta = \frac{1}{2}$ (ii) $\tan\theta = -\frac{1}{\sqrt{3}}$ (iii) $\sin\theta = \frac{\sqrt{2}}{2}$

   (iv) $\sec\theta = -3$ (v) $\cot\theta = 2$ (vi) $\operatorname{cosec}\theta = 4$

   (vii) $\cos\theta = 0.84$ (viii) $\operatorname{cosec}\theta = -2.5$ (ix) $\tan\theta = 0.75$

   (x) $\sqrt{3}\tan\theta = 2\sin\theta$ $\left(\textit{Hint: } \tan\theta \equiv \frac{\sin\theta}{\cos\theta}\right)$

   (xi) $2\sin\theta\cos\theta + \sin\theta = 0$

2. Solve the following equations for angles in the range $-180° \leq \theta \leq 180°$.

   (i) $\sec^2\theta + \tan^2\theta = 6$ (ii) $4\cos^2\theta + 5\sin\theta = 3$

   (iii) $\cot^2\theta = \operatorname{cosec}\theta$ (iv) $\tan\theta + \cot\theta = 2$

   (v) $\tan\theta + 3\cot\theta = 5\sec\theta$ (vi) $\sec\theta = 1 - 2\tan^2\theta$

3. If $\tan\theta = \frac{4}{3}$ and θ is acute, find the value of the following expressions.

   (i) $\sin 2\theta$ (ii) $\tan\frac{\theta}{2}$ (iii) $\cot 2\theta$

4. If $t \equiv \tan\frac{\theta}{2}$, express the following trig functions in terms of t.

   (i) $\frac{1-\cos\theta}{1+\cos\theta}$ (ii) $\frac{\sin\theta}{1-\cos\theta}$ (iii) $\cot\theta\cot\frac{\theta}{2}$

   (iv) $\frac{\cos^2\frac{\theta}{2}}{3\sin\theta + 4\cos\theta - 1}$ (v) $\frac{1-2\sin\theta}{2\cos\theta+1}$

5. Prove the following identities.

   (i) $\cot\theta + \tan\theta \equiv \sec\theta \operatorname{cosec}\theta$

   (ii) $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} \equiv \sin A + \cos A$

   (iii) $\tan^2\theta + \cot^2\theta \equiv \sec^2\theta + \operatorname{cosec}^2\theta - 2$

   (iv) $\dfrac{\sin A}{1+\cos A} \equiv \dfrac{1-\cos A}{\sin A}$

   (v) $(\sec^2\theta + \tan^2\theta)(\operatorname{cosec}^2\theta + \cot^2\theta) \equiv 1 + 2\sec^2\theta\operatorname{cosec}^2\theta$

   (vi) $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} \equiv \dfrac{2}{\sin A}$

   (vii) $\sec^2 A \equiv \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - \sin A}$

   (viii) $(1+\sin\theta+\cos\theta)^2 \equiv 2(1+\sin\theta)(1+\cos\theta)$

   (ix) $\dfrac{\tan^2 A + \cos^2 A}{\sin A + \sec A} \equiv \sec A - \sin A$

6. Find the exact value of each real number y if it exists.

   (i) $y = \sin^{-1} 0$ (ii) $y = \cos^{-1}(-1)$ (iii) $y = \tan^{-1} 1$

   (iv) $y = \arctan 0$ (v) $y = \arcsin\left(\dfrac{-\sqrt{3}}{2}\right)$ (vi) $y = \arccos\left(\dfrac{-\sqrt{3}}{2}\right)$

   (vii) $y = \sin^{-1}\sqrt{3}$ (viii) $y = \cot^{-1}(-1)$ (ix) $y = \operatorname{cosec}^{-1}(-2)$

   (x) $y = \operatorname{arcsec}\dfrac{2\sqrt{3}}{3}$ (xi) $y = \sec^{-1} 1$ (xii) $y = \operatorname{cosec}^{-1}\dfrac{\sqrt{2}}{2}$