Science A Physics

Lecture 2:

Kinematics

Aims of today's lecture

1. Kinematics
2. Instantaneous velocity
3. Finding position from velocity
4. Kinematics equations for constant acceleration
5. Instantaneous acceleration
6. Finding velocity from acceleration

1. Kinematics

Kinematics

- **Kinematics** is the name for the equations that we use to describe motion.
- As we have seen, the motion of an object is described by its position, velocity, and acceleration.
- In one dimension, these quantities are represented by $\boldsymbol{x}$ (or $\boldsymbol{s}$), $\boldsymbol{v_x}$, and $\boldsymbol{a_x}$.

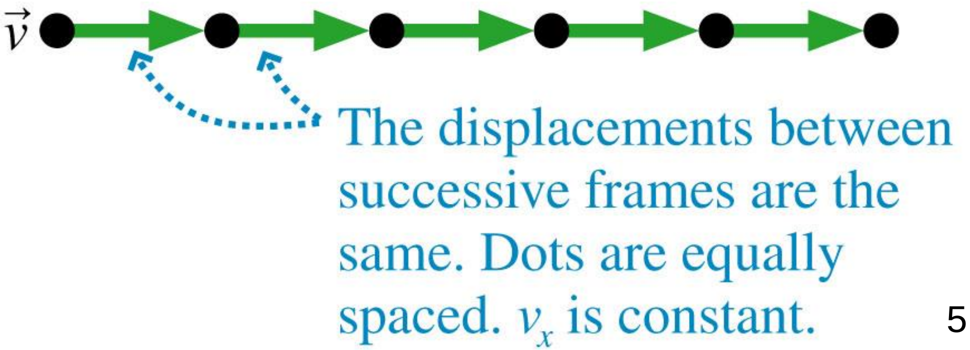
Kinematics

- If you drive your car at a perfectly steady 60 km/h, this means you change your position by 60 km for every time interval of 1 hour.

- **Uniform motion** is when equal displacements occur during any successive equal-time intervals.

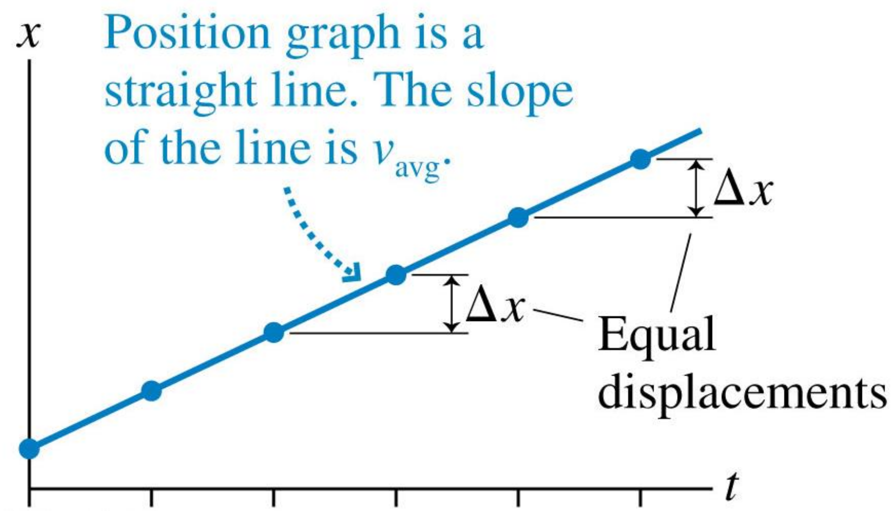
- Uniform motion is always along a straight line.

Riding steadily over level ground is a good example of uniform motion.



Kinematics

- An object's motion is uniform if and only if its position-versus-time graph is a straight line.

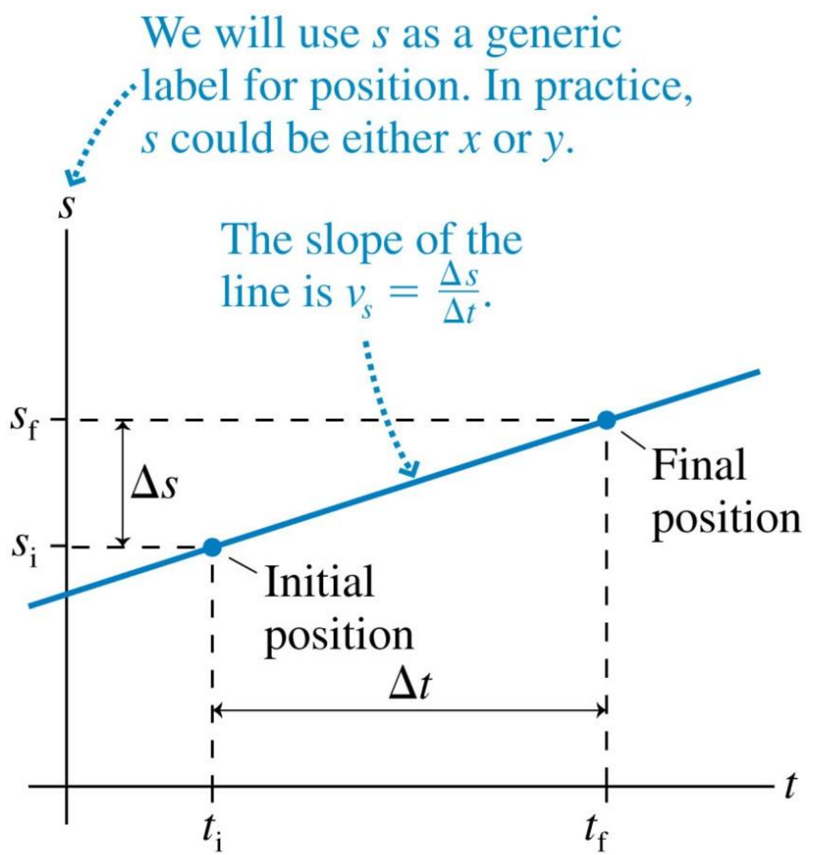


- The **average velocity** is the slope of the position-versus-time graph.

- The SI units of velocity are m/s.

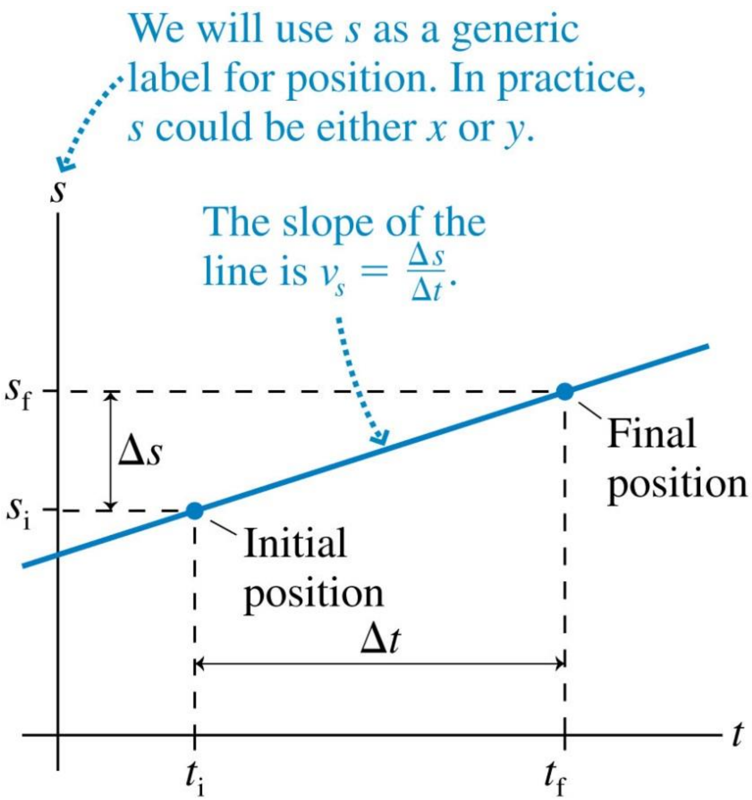
$$v_{avg} = \frac{\Delta x}{\Delta t} \text{ or } \frac{\Delta y}{\Delta t} = \quad \text{slope of the position-versus-time graph}$$

Kinematics



- Consider an object in uniform motion along the s-axis, as shown in the graph.

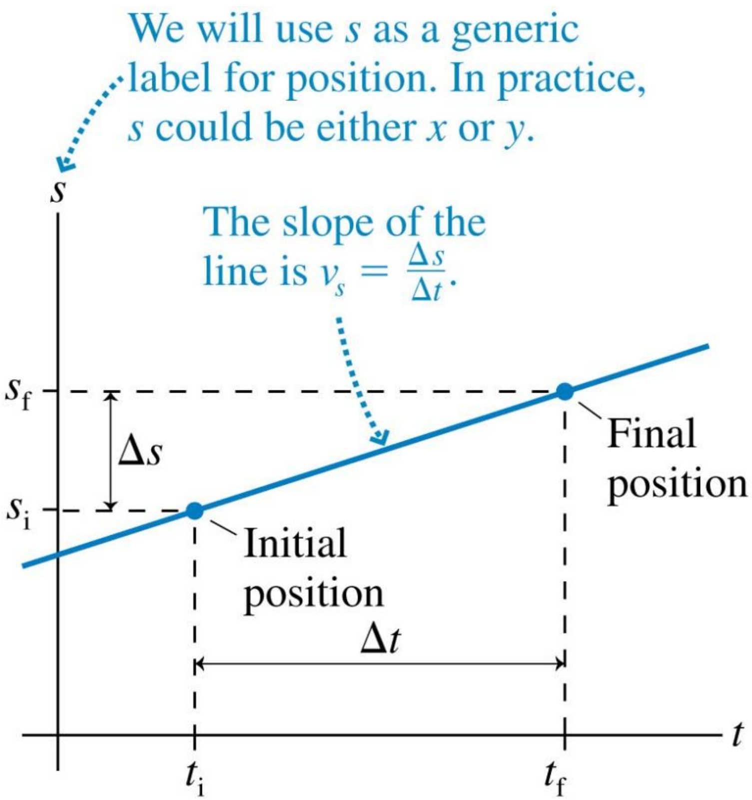
Kinematics



$$s_f = s_i + v_s \Delta t \qquad \text{(uniform motion)}$$

- The above equation is the first kinematic equation that we have derived; it describes the position of an object that has a constant velocity, as a function of time.

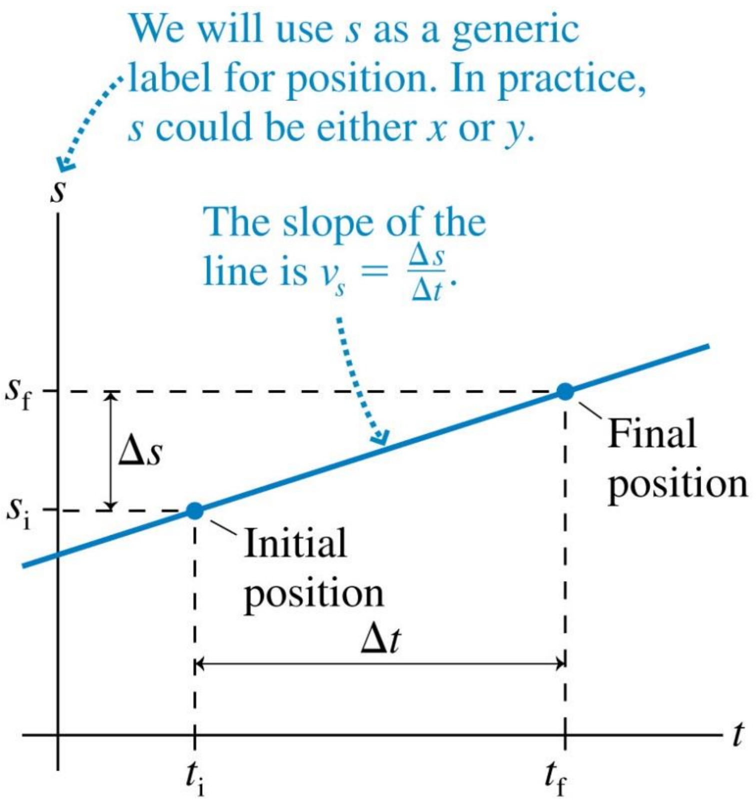
Kinematics



$$s_f = s_i + v_s \Delta t \qquad \text{(uniform motion)}$$

- The above position-versus-time graph is a straight line, and it shows us that the velocity of the object is the same at any particular instant in time; in other words, the object has uniform motion.
- But what if the velocity of the object is not constant; how do we find the velocity of the object at an instant in time?

Kinematics

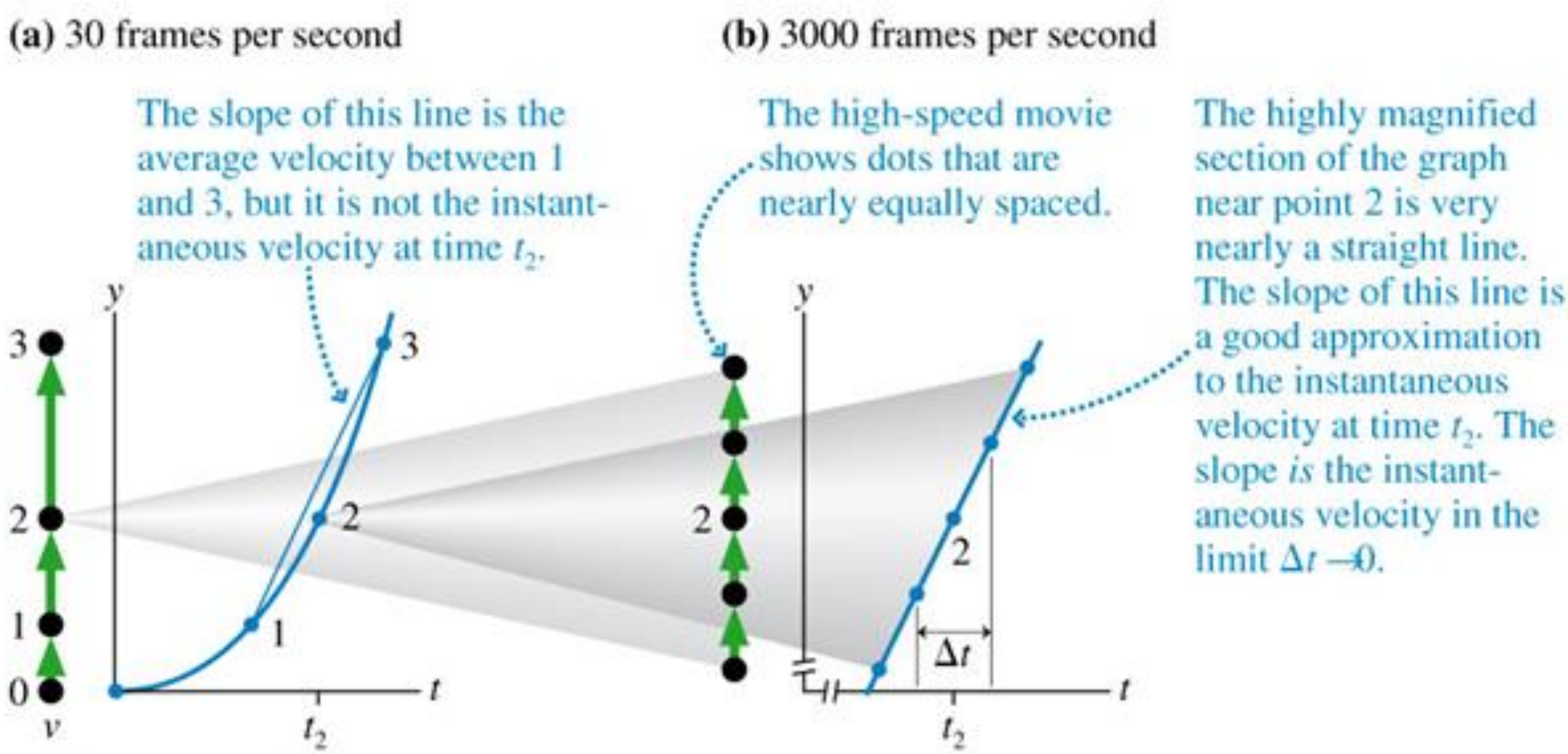


$$s_f = s_i + v_s \Delta t \qquad \text{(uniform motion)}$$

- We must consider a new idea to answer the aforementioned question, namely the idea of **instantaneous velocity**.
- When we consider instantaneous velocity, we first talk about average velocity in the context of an object whose motion is non-uniform.

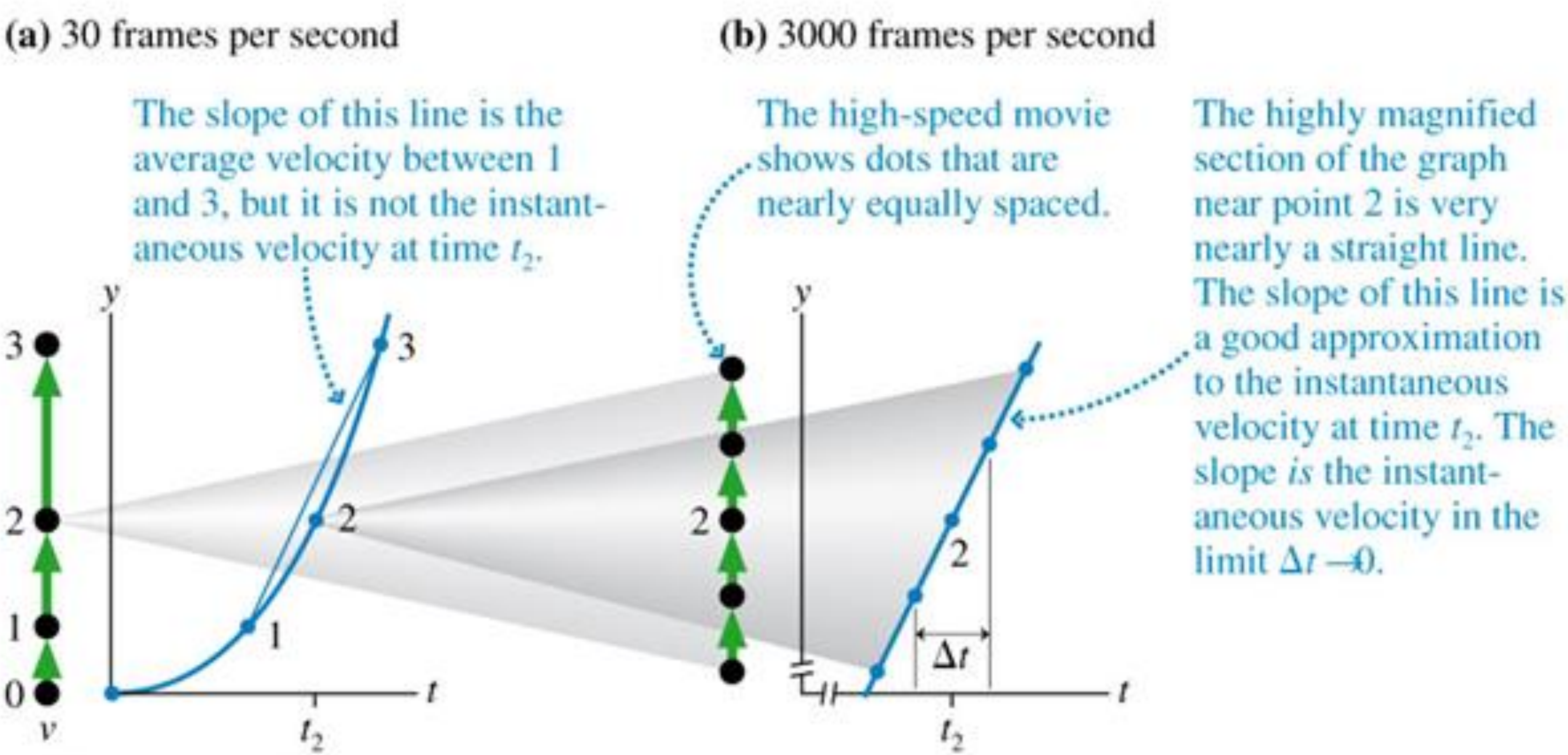
2. Instantaneous Velocity

Instantaneous Velocity



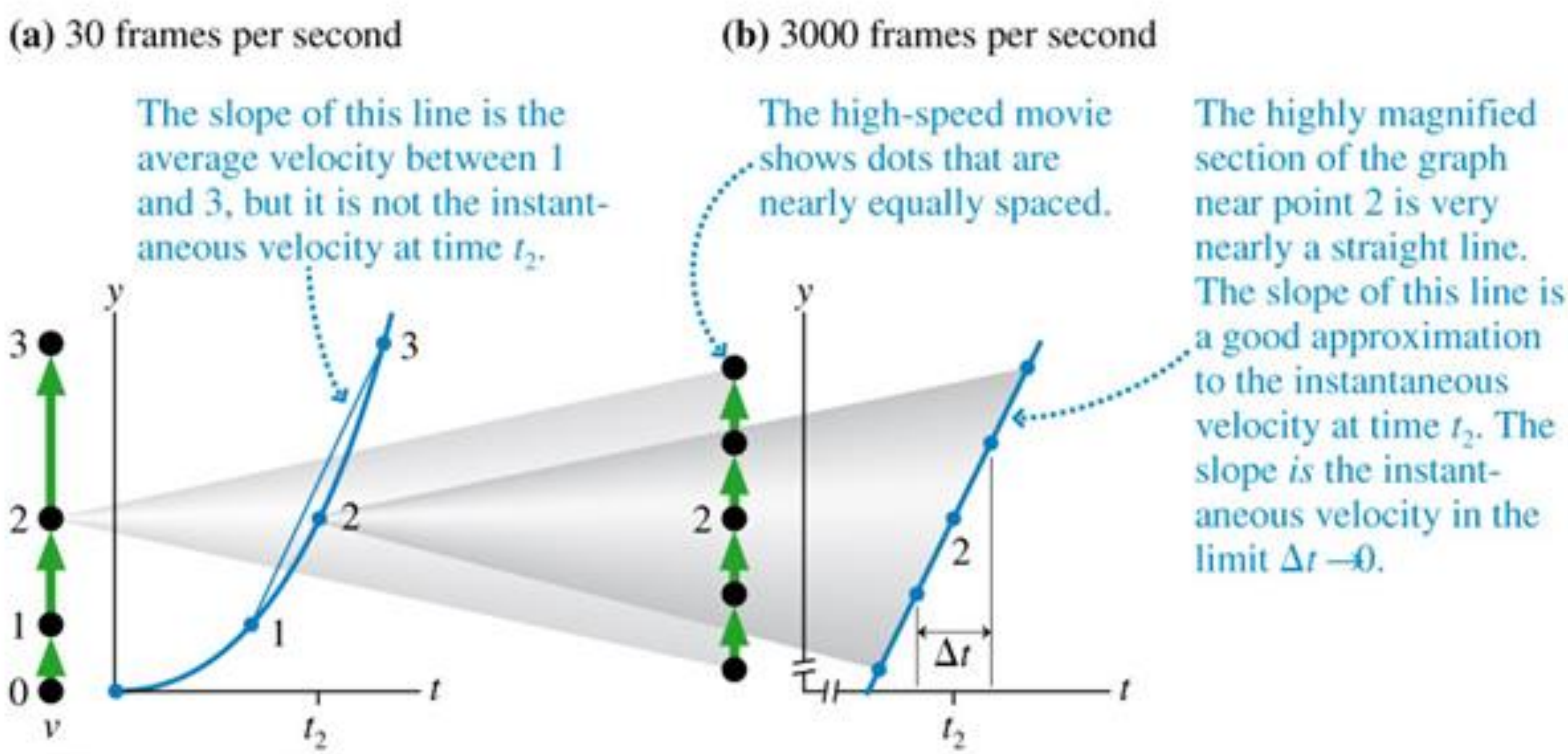
- We can determine the average speed v_{avg} between any two times separated by a time interval, Δt, by finding the slope of the straight-line connection between the two points.

Instantaneous Velocity



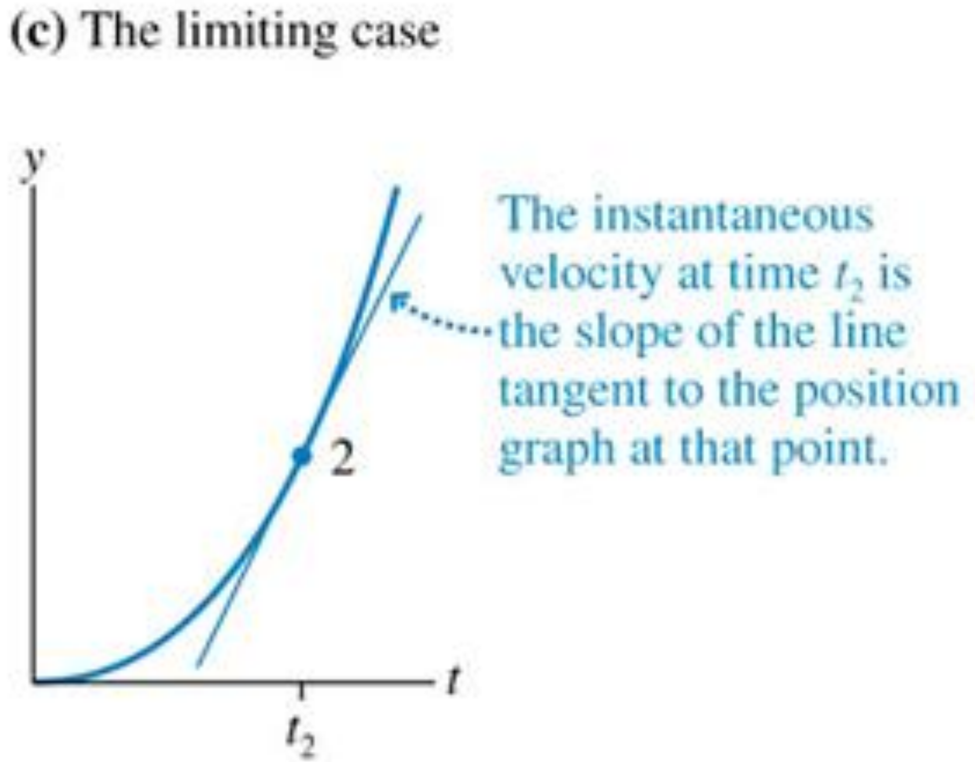
- The <u>**instantaneous velocity**</u> is the object's velocity at a single **instant** of time, t.

Instantaneous Velocity



- The average velocity $\boldsymbol{v_{avg}} = \Delta \boldsymbol{s}/\Delta \boldsymbol{t}$ becomes a better and better approximation to the instantaneous velocity as Δt gets smaller and smaller.

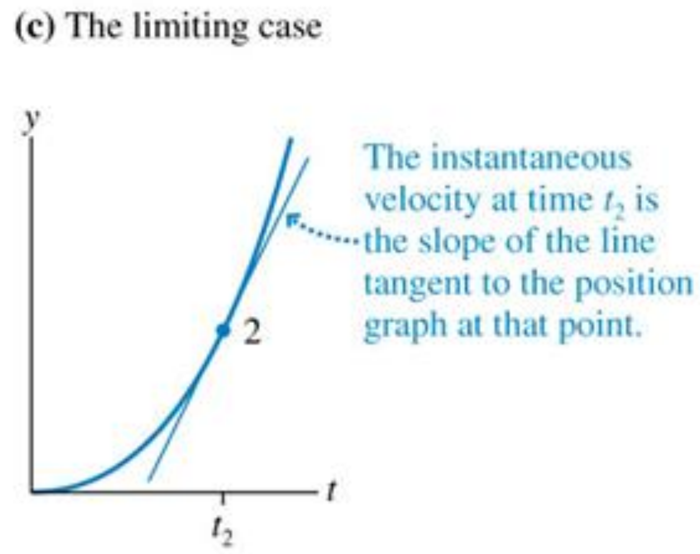
Instantaneous Velocity



$$v_s = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)}$$

NB: Calculus is not examined in this module.

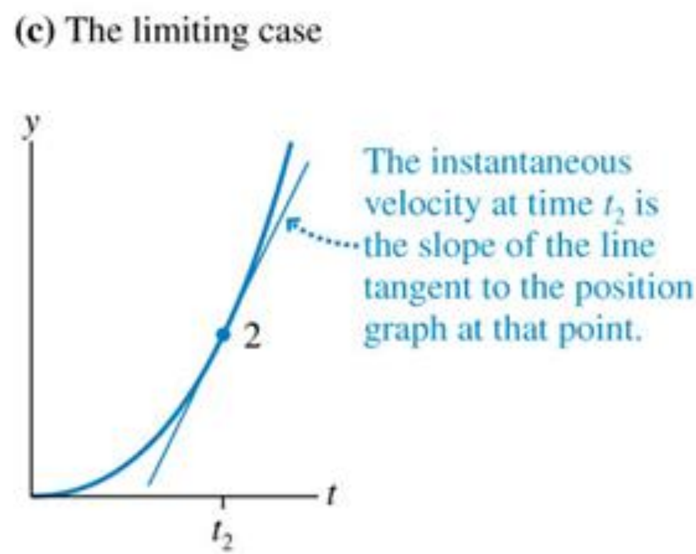
Instantaneous Velocity



$$v_s = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)}$$

- As Δt continues to get smaller, the average velocity $v_{avg} = \Delta s/\Delta t$ reaches a constant or **limiting** value.
- The instantaneous velocity at time t is the average velocity during a time interval Δt centred on t, as Δt approaches zero.

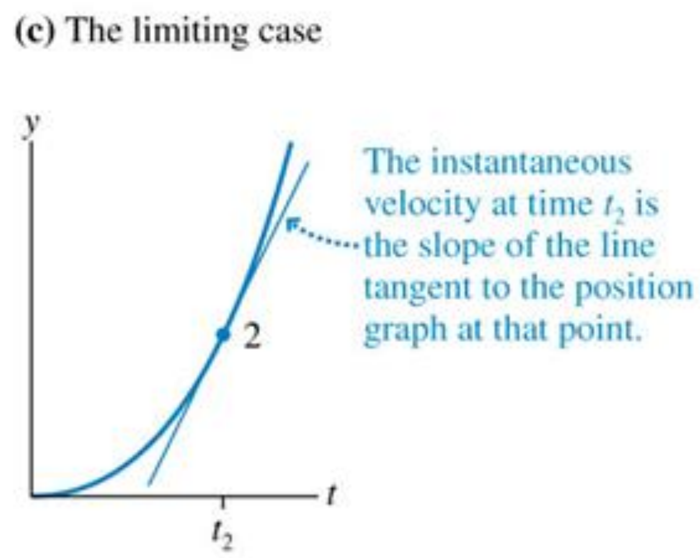
Instantaneous Velocity



$$v_s = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)}$$

- The instantaneous velocity at time t is the average velocity during a time interval Δt centred on t, as Δt approaches zero.
- In **calculus**, this is called **the derivative of s with respect to t**.

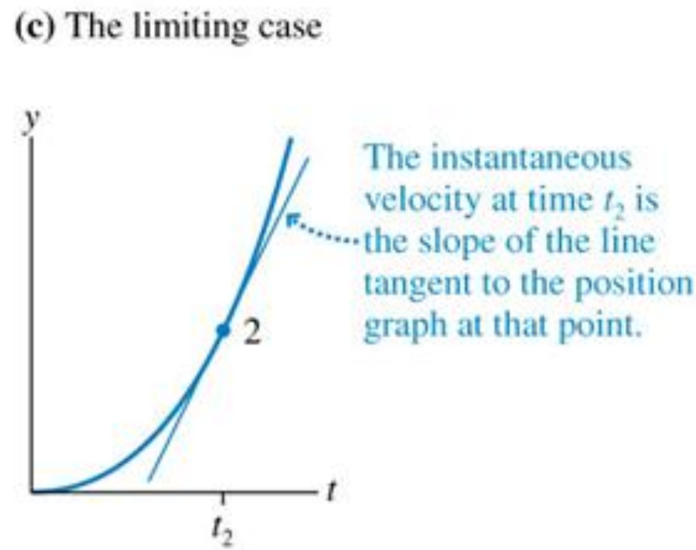
Instantaneous Velocity



$$v_s = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)}$$

- Graphically, $\Delta s/\Delta t$ is the slope of a straight line.
- In the limit $\Delta t \to 0$, the straight line is tangent to the curve.
- The instantaneous velocity at time t is the slope of the line that is tangent to the position-versus-time graph at time t.

Instantaneous Velocity



$$v_s = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)}$$

- The instantaneous velocity at time t is the slope of the line that is tangent to the position-versus-time graph at time t.

v_s = slope of the position-versus-time graph at time t

Some Calculus

- ds/dt is called *the* **derivative of s with respect to t.**

- ds/dt is the **slope** of the line that is tangent to the position-versus-time graph.

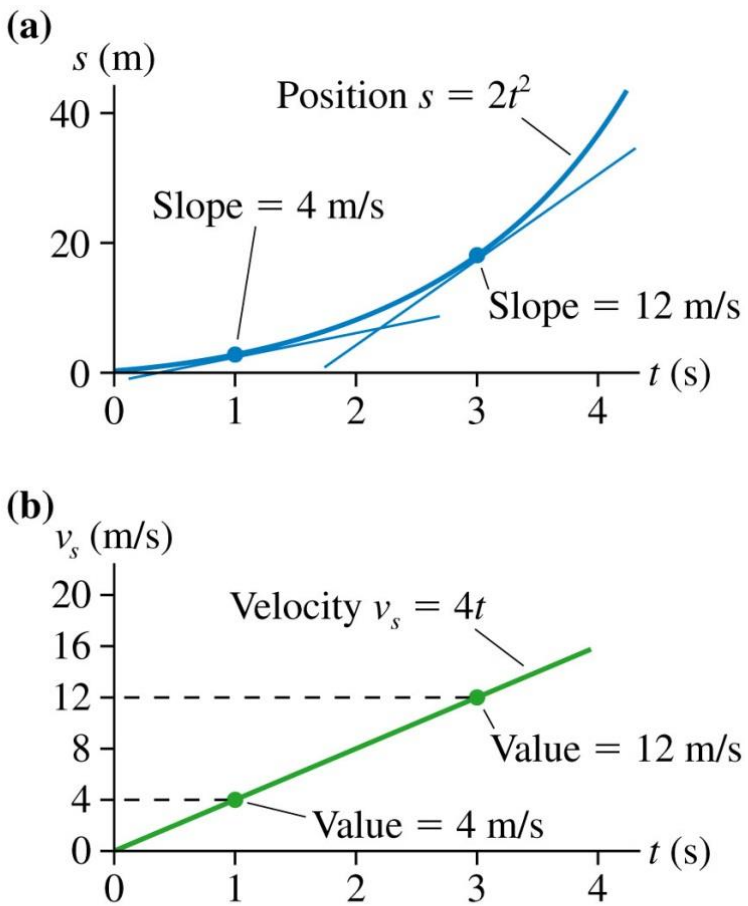
- Consider a function u that depends on time as $u = ct^n$, where c and n are constants:

$$\text{The derivative of } u = ct^n \text{ is } \frac{du}{dt} = nct^{n-1}$$

- The derivative of a constant is zero:

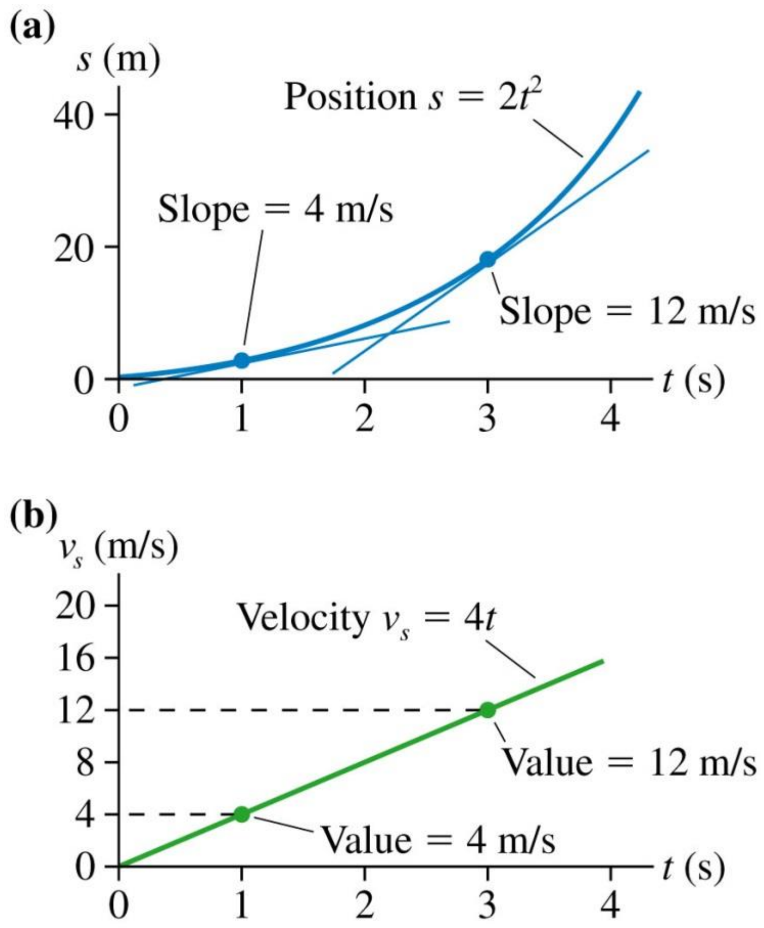
$$\frac{du}{dt} = 0 \text{ if } u = c = \text{constant}$$

Calculus in Physics



- Suppose the position of a particle as a function of time is $s = 2t^2$, where t is in s. What is the particle's velocity?

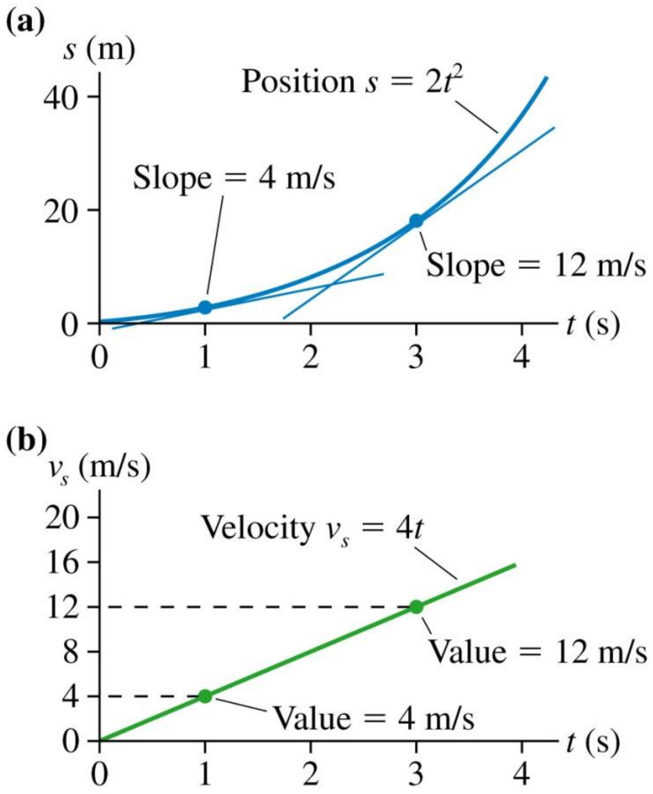
Calculus in Physics



- Velocity is the derivative of s with respect to t:

$$v_s = \frac{ds}{dt} = 2 \cdot 2t^{2-1} = 4t$$

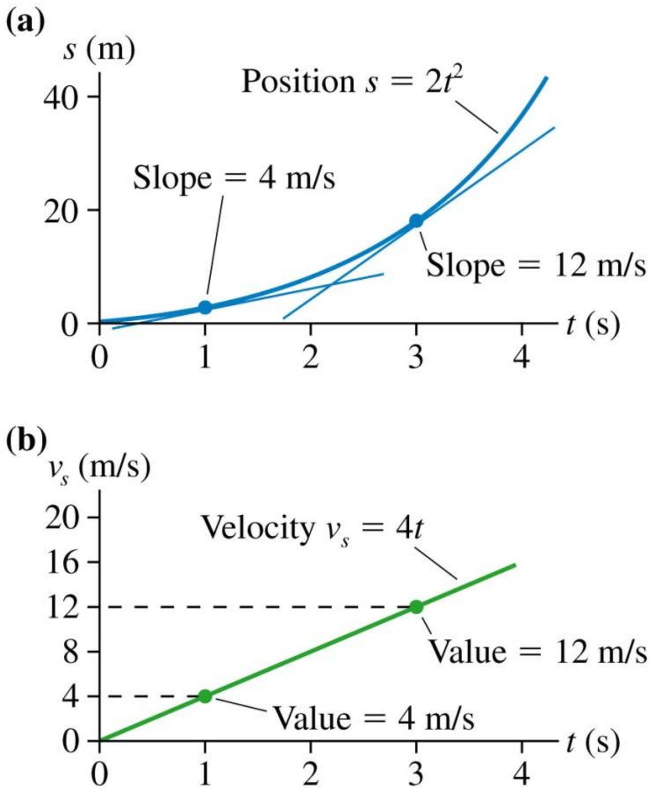
Calculus in Physics



- The value of the velocity graph at any instant of time is the **slope** of the position graph at that same time:

$$v_s = \frac{ds}{dt} = 2 \cdot 2t^{2-1} = 4t$$

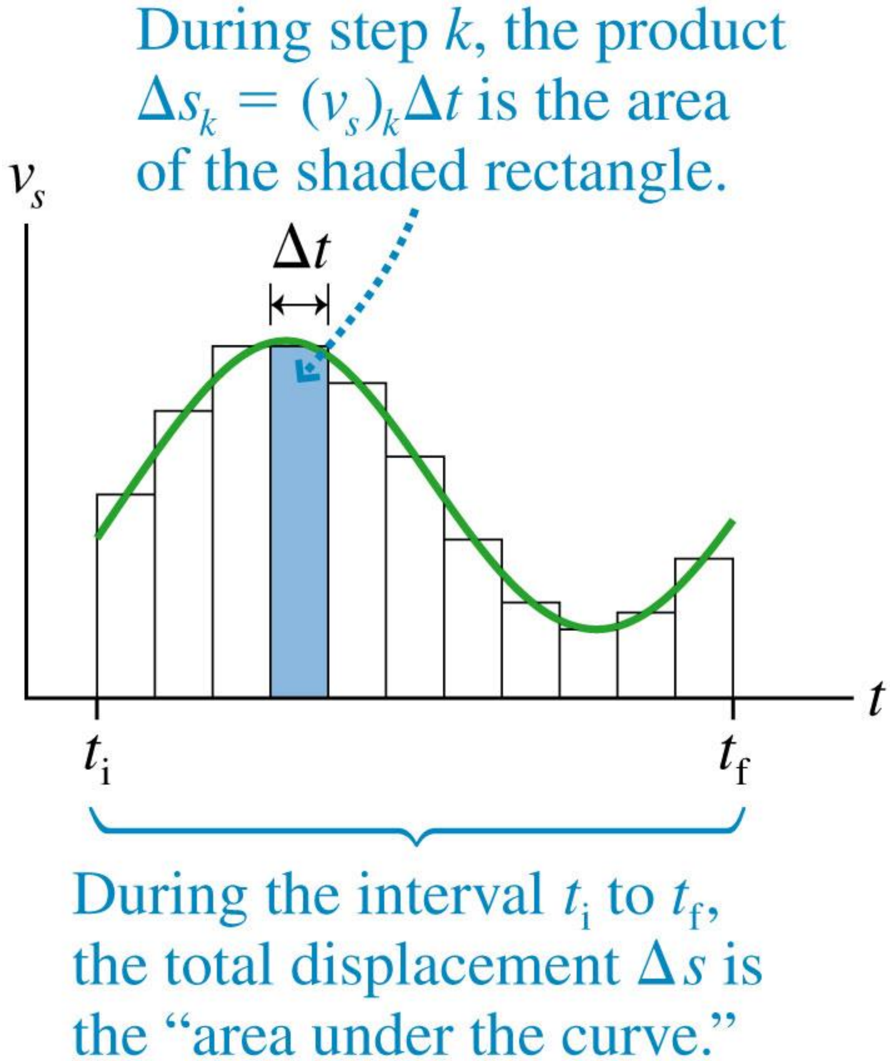
Calculus in Physics



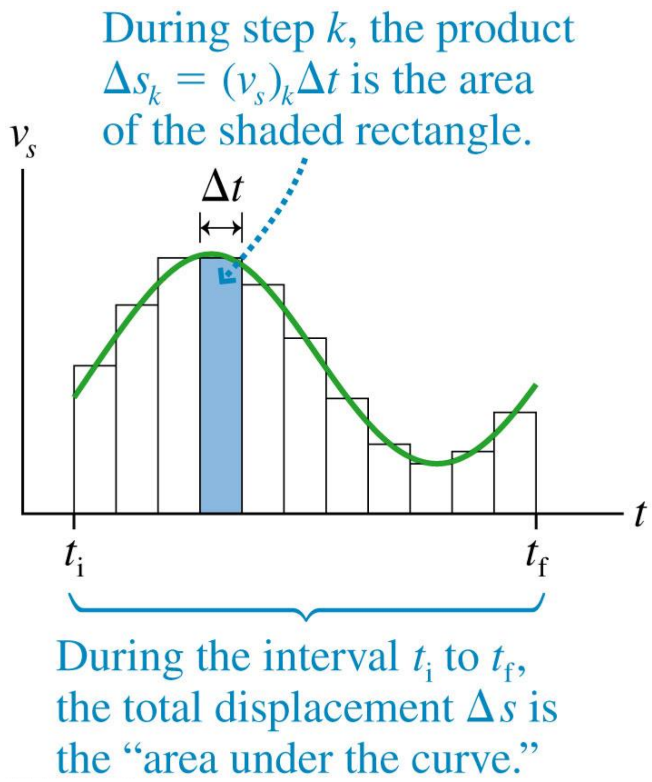
- What we are going to consider now, although very briefly, is the idea of calculating the area between the velocity graph and the x-axis over a certain time interval. Believe it or not, this area represents the positional change (displacement) that the object undergoes within this time interval.

3. Finding Position from Velocity

Finding Position from Velocity



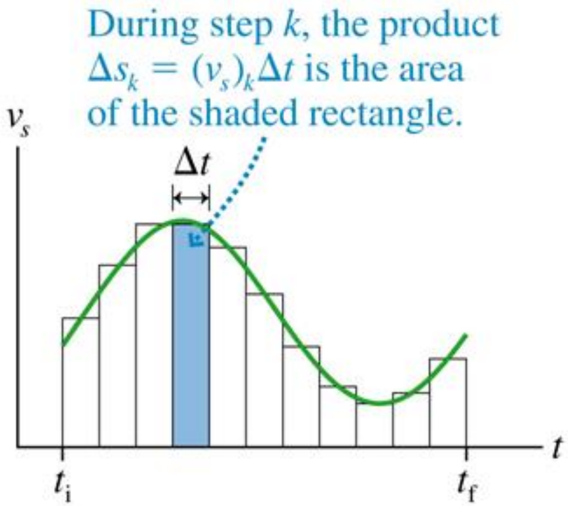
Finding Position from Velocity



- The **integral** may be interpreted graphically as the total area enclosed between the t-axis (over a specified interval) and the velocity curve.

$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f$$

Finding Position from Velocity

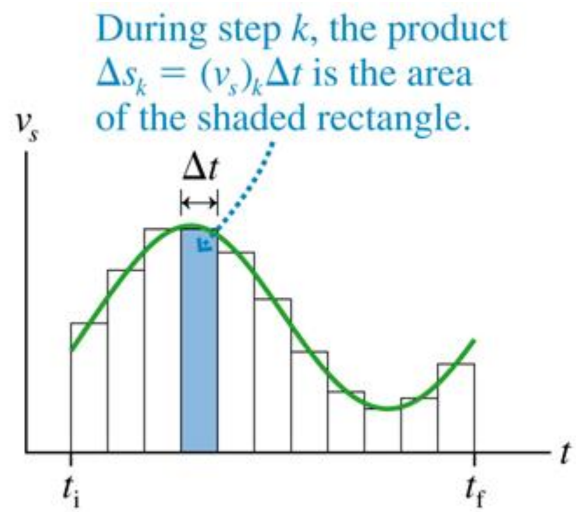


- Suppose we know an object's position to be s_i at an initial time t_i.
- Suppose we also know the velocity as a function of time between t_i and some later time t_f, then:

even if the velocity is not constant, we can divide the motion into N steps in which it is approximately constant, and calculate the final position as:

$$s_f = s_i + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s \, dt$$

Finding Position from Velocity

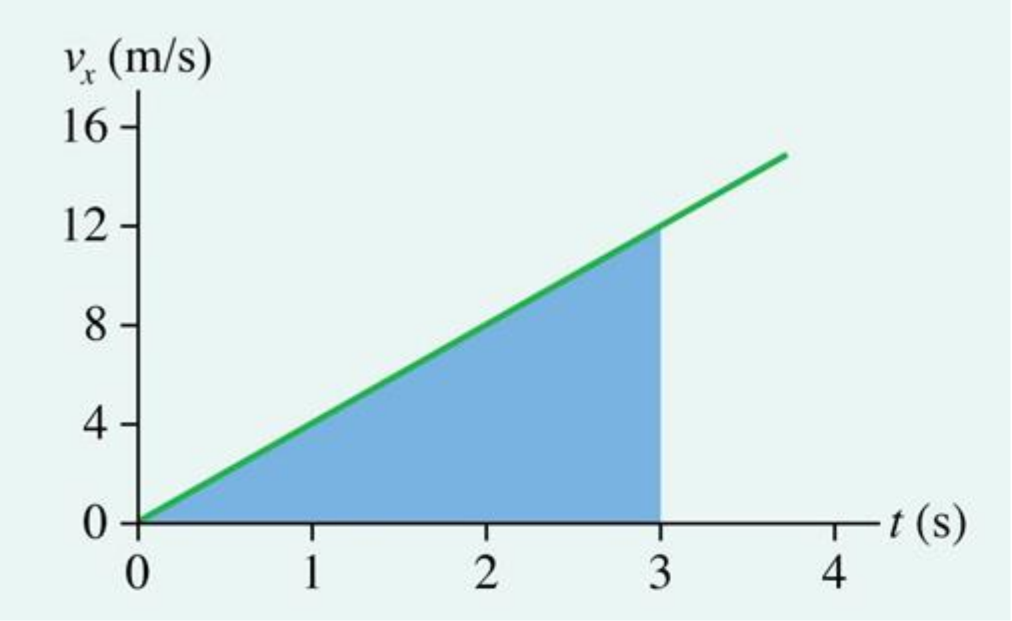


$$s_f = s_i + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s \, dt$$

- The **sigma symbol means the sum of an infinite number of rectangles**.
- The expression on the right is read, 'the integral of $v_s \, dt$ from t_i to t_f'.

Have a Think: Displacement during a Race

Q.1 The figure below shows the velocity-versus-time graph of a car racer. How far does the racer move (or how far is the racer displaced) during the first 3.0 s?



Calculus: Key Points

- Taking the derivative of a function is equivalent to generating a function (called the derivative function) which gives you a value for the slope of the tangent (at any particular point) on the graph of the original function.

- Evaluating an integral (also called an integrand function) is equivalent to finding the area between the graph of the integrand function and its horizontal axis, either over a specified interval (referred to as a definite integral) or over a non-specified interval (referred to as an indefinite integral)

N.B.

Do not worry if you do not fully understand the above two points yet. By the time you your mathematics module in Semester 2, these two key points will make more sense.

Previously: Acceleration

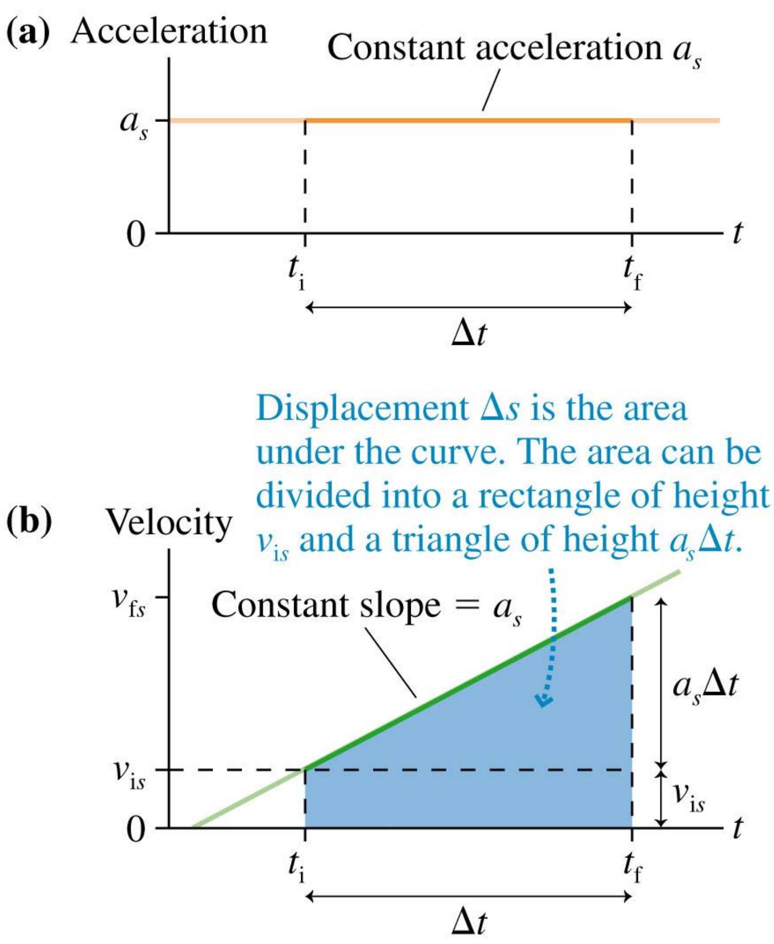
- The **average acceleration** during a time interval Δt is:

$$a_{avg} = \frac{\Delta v_s}{\Delta t} \qquad \text{(average acceleration)}$$

- Graphically, $\boldsymbol{a_{avg}}$ is the **slope** of a straight-line velocity-versus-time graph.
- If acceleration is constant, the acceleration a_s is the same as a_{avg}.
- Acceleration, like velocity, is a vector quantity, and has both magnitude and direction.
- Let's apply our understanding of integration so far to objects which have constant acceleration, or zero acceleration; doing so, will allow us to derive more kinematic equations of motion.

4. Kinematics Equations for Constant Acceleration

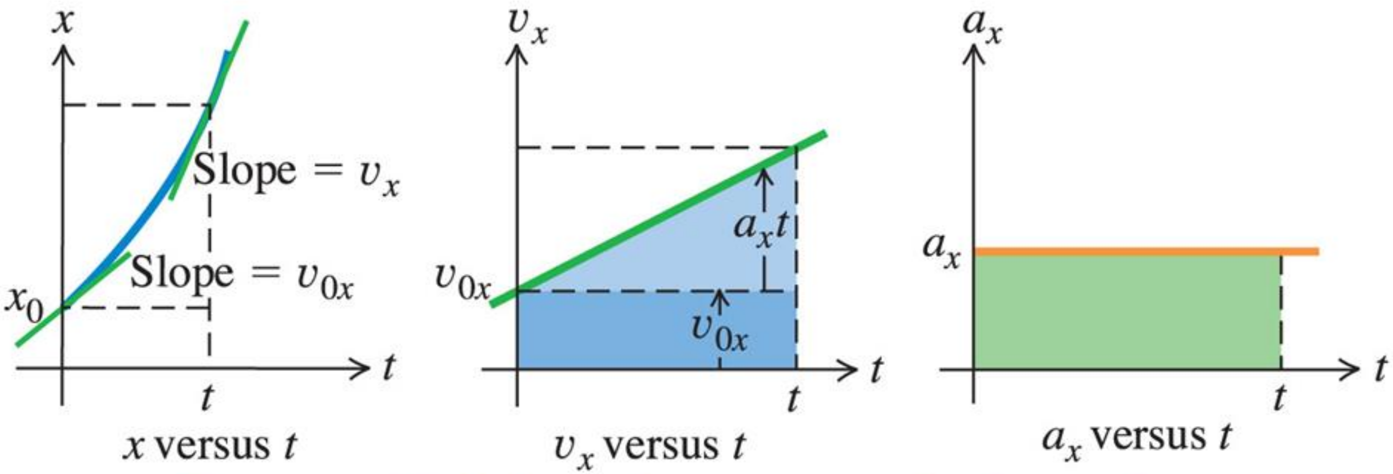
Kinematic Equations for Constant Acceleration



- Suppose we know an object's position to be s_i at an initial time t_i.
- It's constant acceleration a_s is shown in graph (a).
- The velocity-versus-time graph is shown in graph (b).
- The final position s_f is s_i plus the area under the curve of v_{is} between t_i and t_f:

$$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$$

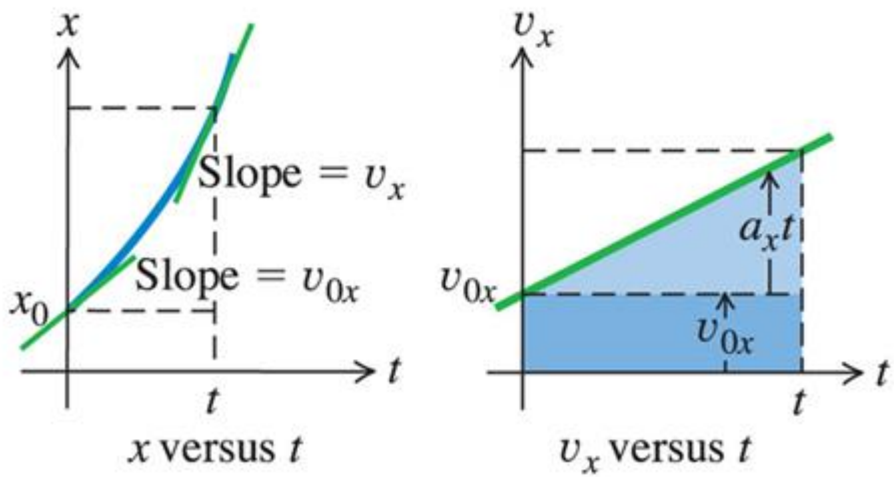
Kinematic Equations for Constant Acceleration



Three graphical views of constant-acceleration motion.

- The above graphs show the position, velocity and acceleration for an object that has constant acceleration, as a function of time.
- If we integrate the function for the acceleration graph, we get the velocity function, and if we integrate the velocity function, we get the position function.

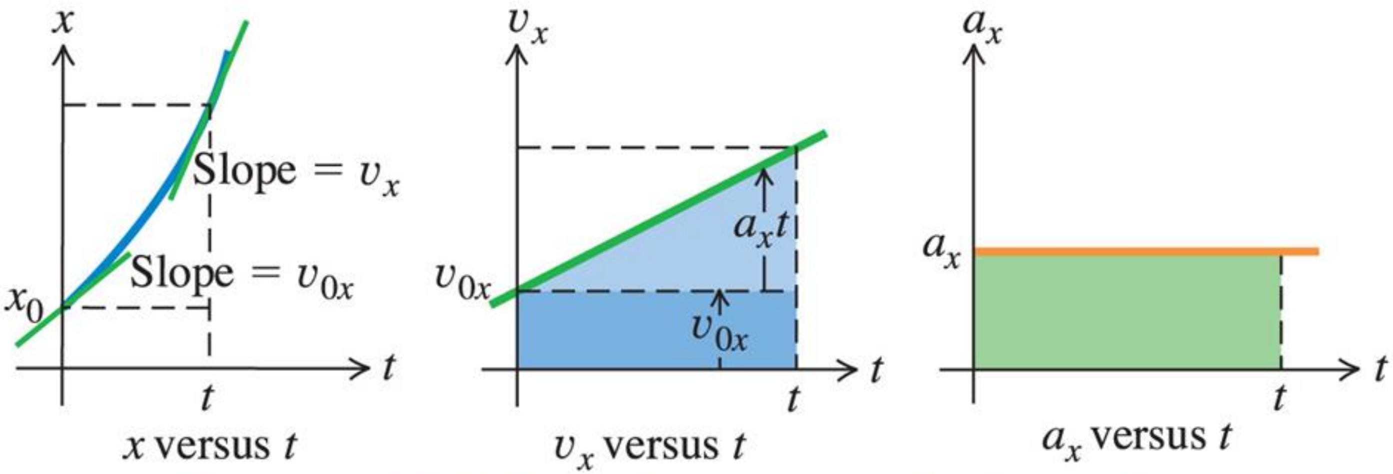
Kinematic Equations for Constant Acceleration



- Suppose we know an object's velocity to be v_{is} at an initial time t_i.
- We also know the object has a constant acceleration of a_s over the time interval $\Delta t = t_f - t_i$.
- We can then find the object's velocity at the later time t_f as:

$$v_{fs} = v_{is} + a_s \Delta t$$

Kinematic Equations for Constant Acceleration



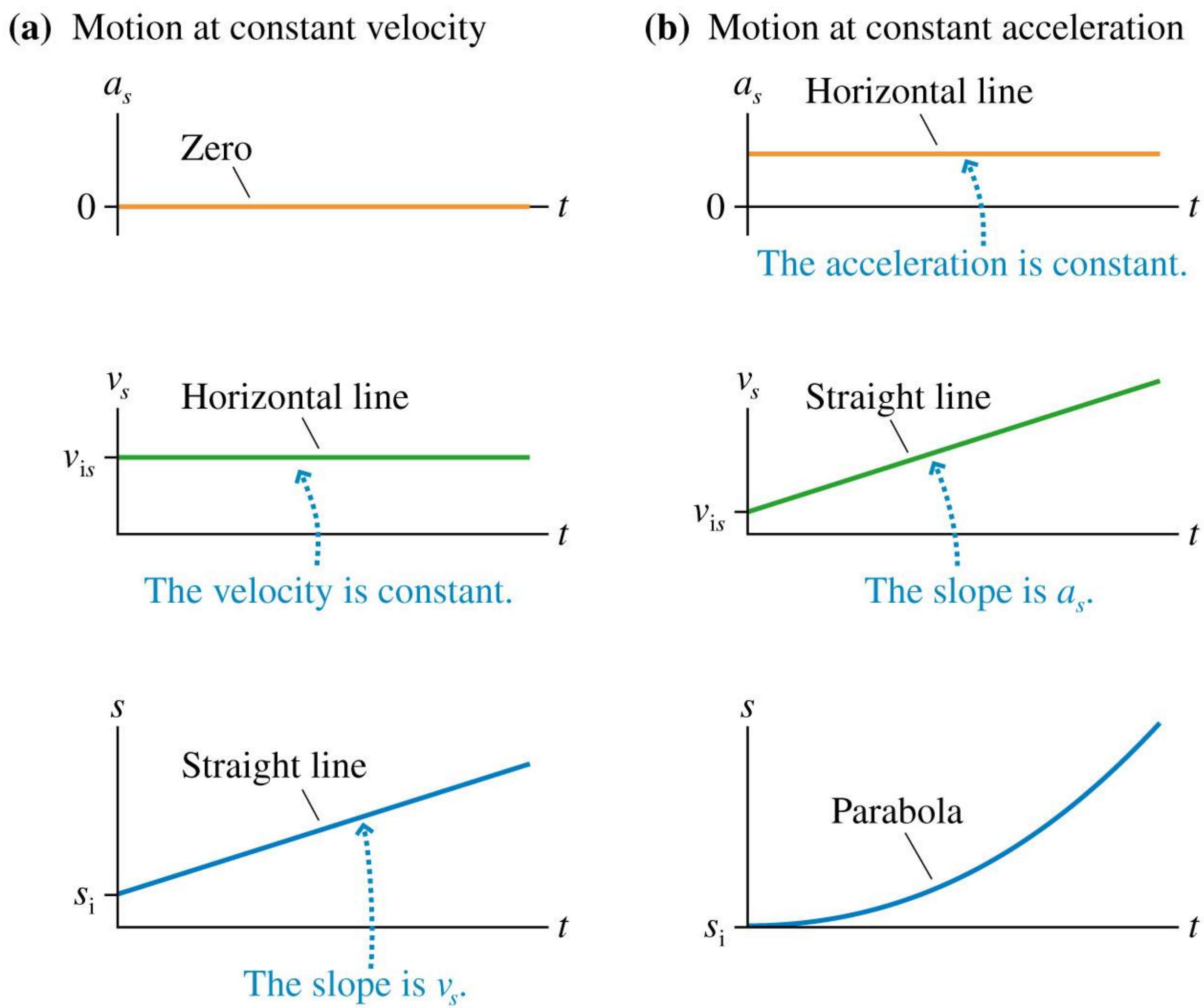
Three graphical views of constant-acceleration motion.

$$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$$

- It is left as an exercise for you to show that given the above equation, we can use it to derive another equation which also allows us to find the object's velocity at the final position s_f; the equation is as follows:

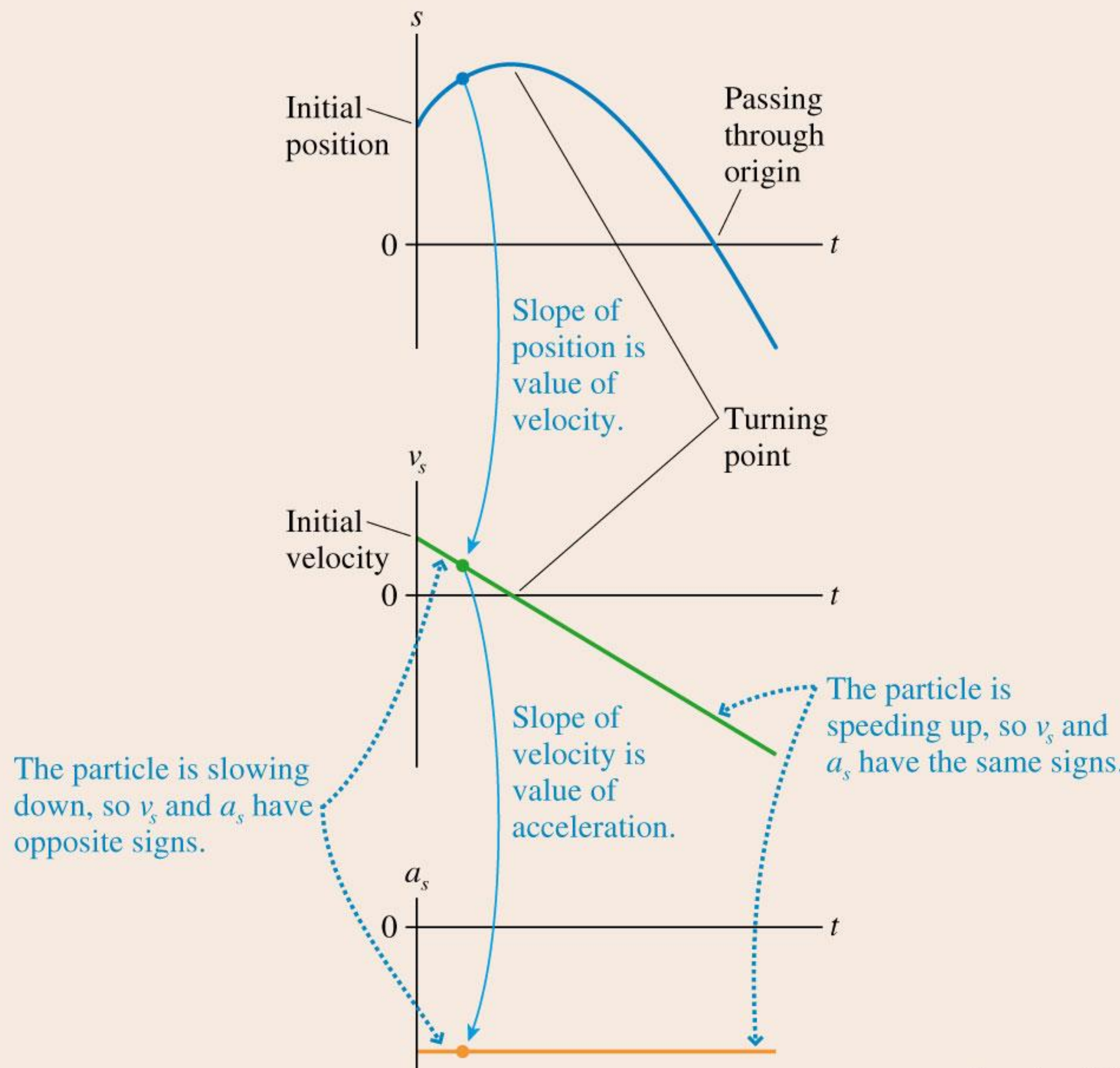
$$v_{fs}{}^2 = v_{is}{}^2 + 2a_s\Delta s$$

Motion with constant velocity and constant acceleration. These graphs assume $s_i = 0$, $v_{is} > 0$, and (for constant acceleration) $a_s > 0$.



TACTICS BOX 2.2

Interpreting graphical representations of motion



Exercises 15, 16, 22

Previously: Acceleration

- The **average acceleration** during a time interval Δt is:

$$a_{avg} = \frac{\Delta v_s}{\Delta t} \qquad \text{(average acceleration)}$$

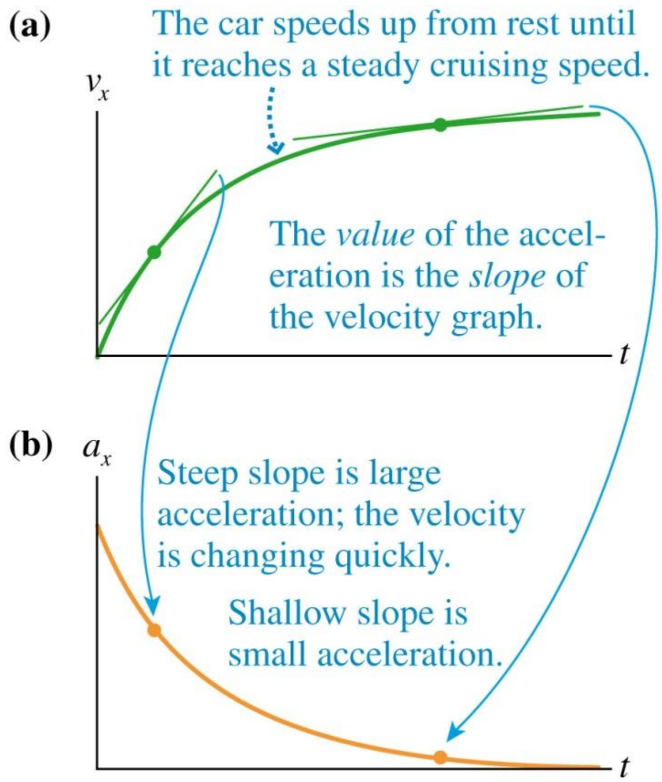
- Graphically, $\boldsymbol{a_{avg}}$ is the **slope** of a straight-line velocity-versus-time graph.
- If acceleration is constant, the acceleration a_s is the same as a_{avg}.
- Acceleration, like velocity, is a vector quantity and has both magnitude and direction.

N.B.

If the velocity-versus-time graph is not a straight-line graph, then we can talk about **instantaneous acceleration**.

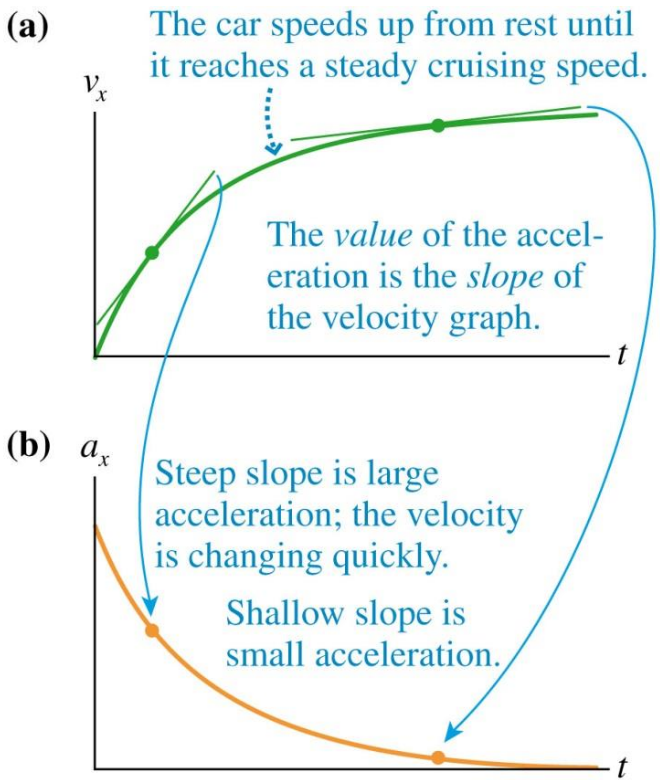
5. Instantaneous Acceleration

Instantaneous Acceleration



- Figure (a) shows a realistic velocity-versus-time graph for a car leaving a stop sign.
- The graph is not a straight line, so this is not motion with a constant acceleration.

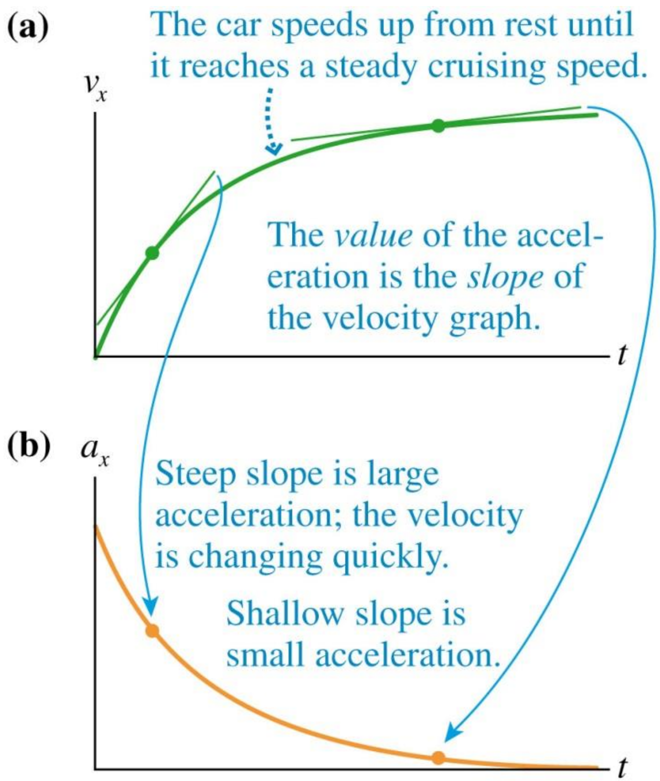
Instantaneous Acceleration



$$a_s = \frac{dv_s}{dt} = \text{slope of the velocity-versus-time graph at time } t$$

- Figure (b) shows the car's acceleration graph; the derivative of the velocity as a function of time, in other words.
- The **instantaneous acceleration** a_s is the slope of the line that is tangent to the velocity-versus-time curve at time t.

Instantaneous Acceleration



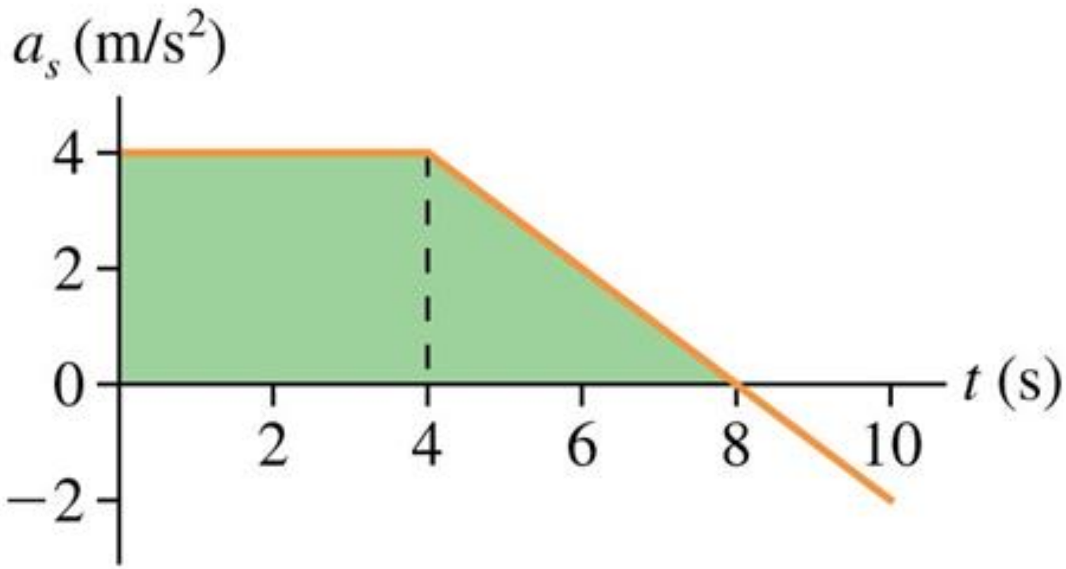
$$a_s = \frac{dv_s}{dt} = \text{slope of the velocity-versus-time graph at time } t$$

- Finally, let's see how we apply integration to the acceleration-versus-time graph for an object to find the velocity of that object at a certain time; we will look at a simple case.

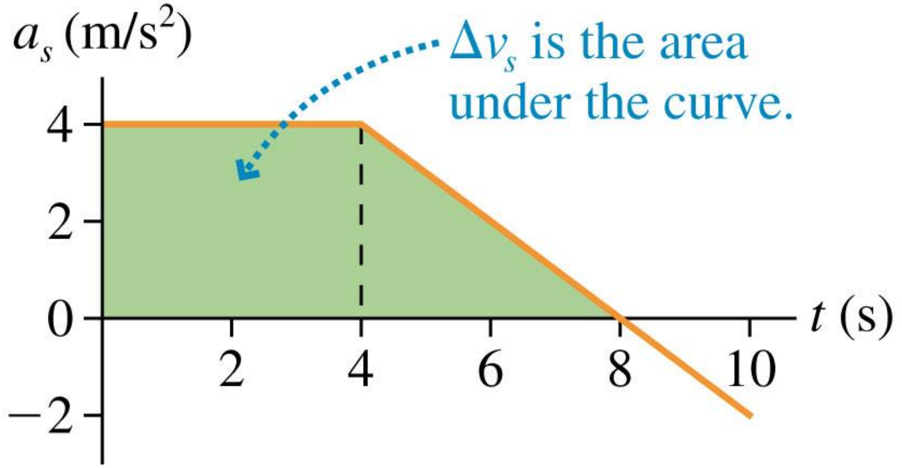
5. Finding Velocity from Acceleration

Finding Velocity from Acceleration

Q.2 The figure below shows the acceleration graph for a particle with an initial velocity of $10\ \mathrm{m/s}$. What is the particle's velocity at $t = 8\ s$?



Finding Velocity from Acceleration



MODEL:

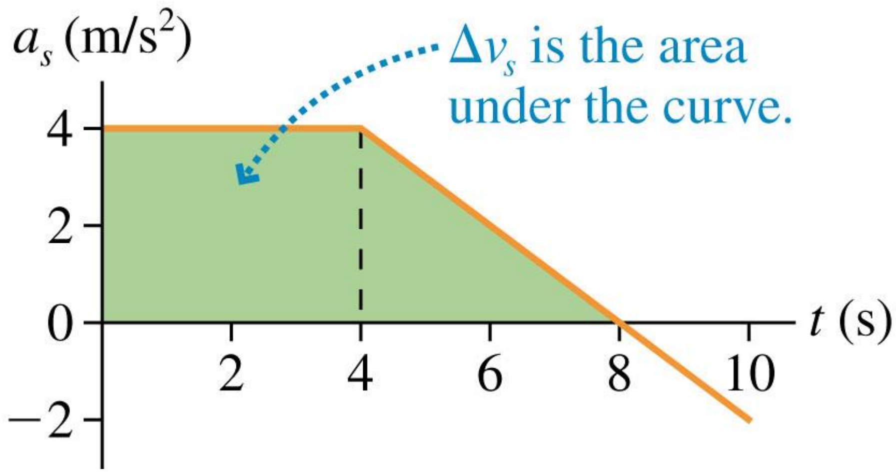
We're told this is the motion of a particle.

SOLVE:

The change in velocity is found as the area under the acceleration curve:

$v_f = v_i$ + area under the acceleration curve between t_i and t_f

Finding Velocity from Acceleration



SOLVE:

The area under the curve between $t_i = 0\text{ s}$ and $t_f = 8\text{ s}$ can be subdivided into a rectangle ($0\text{ s} \le t \le 4\text{ s}$)and a triangle ($4\text{ s} \le t \le 8\text{ s}$). These areas are easily calculated. Thus

v_s (at t = 8 s) = 10 m/s + (4 (m/s)/s)(4 s) + ½(4 (m/s)/s)(4 s)

= **34 m/s**

Calculus Once Again

- If we know an object's velocity to be v_{is} at an initial time t_i , and we also know the acceleration as a function of time between t_i and some later time t_f ,then:

  even if the acceleration is not constant, we can divide the motion into N steps of length Δt in which it is approximately constant.

- In the limit $\Delta t \to 0$, we can calculate the final velocity as:

$$v_{fs} = v_{is} + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (a_s)_k \Delta t = v_{is} + \int_{t_i}^{t_f} a_s dt$$

- The integral may be interpreted graphically as a_s, the area under the acceleration curve between t_i and t_f.

Summary of today's Lecture

1. Kinematics
2. Instantaneous velocity
3. Finding position from velocity
4. Kinematics equations for constant acceleration
5. Instantaneous acceleration
6. Finding velocity from acceleration

Lecture 2: Optional Reading

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- **Ch. 2.5**, Motion at constant acceleration; p.38-39.
- **Ch. 2.6**, Solving problems; p.40-43.
- **Ch. 2.8**, Variable acceleration; p.49-50.
- **Ch. 2.9**, Graphical analysis and numerical integration; p.50-52
- **Ch. 3.1-3.4**, Vectors (various topics); p.66-72.

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.