Science A Physics

Lectures 4-6:

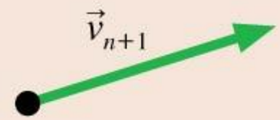
Additional Problems: Free-body Diagrams, Energy, and Momentum

The Acceleration Vector

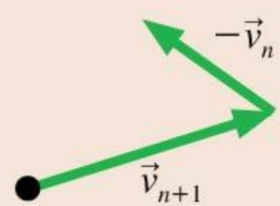
TACTICS BOX 4.1 **Finding the acceleration vector**

To find the acceleration between velocity $\vec{v}_n$ and velocity $\vec{v}_{n+1}$:

❶ Draw the velocity vector $\vec{v}_{n+1}$.

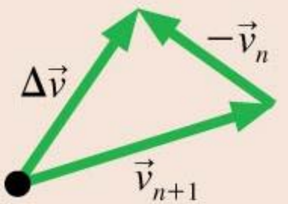


❷ Draw $-\vec{v}_n$ at the tip of $\vec{v}_{n+1}$.

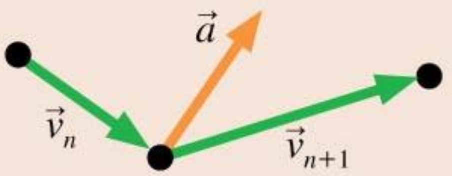


The Acceleration Vector

③ Draw $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$
$= \vec{v}_{n+1} + (-\vec{v}_n)$
This is the direction of $\vec{a}$.



④ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration between $\vec{v}_n$ and $\vec{v}_{n+1}$.



Exercises 1–4

Through the Valley

Q.1 A ball rolls down a long hill, through the valley, and back up the other side. Draw a complete motion diagram of the ball, showing velocity and acceleration vectors.

PROBLEM-SOLVING STRATEGY 10.1

Conservation of mechanical energy

MP®

MODEL Choose a system that is isolated and has no friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + U_f = K_i + U_i$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 8

TACTICS BOX 10.1 Interpreting an energy diagram

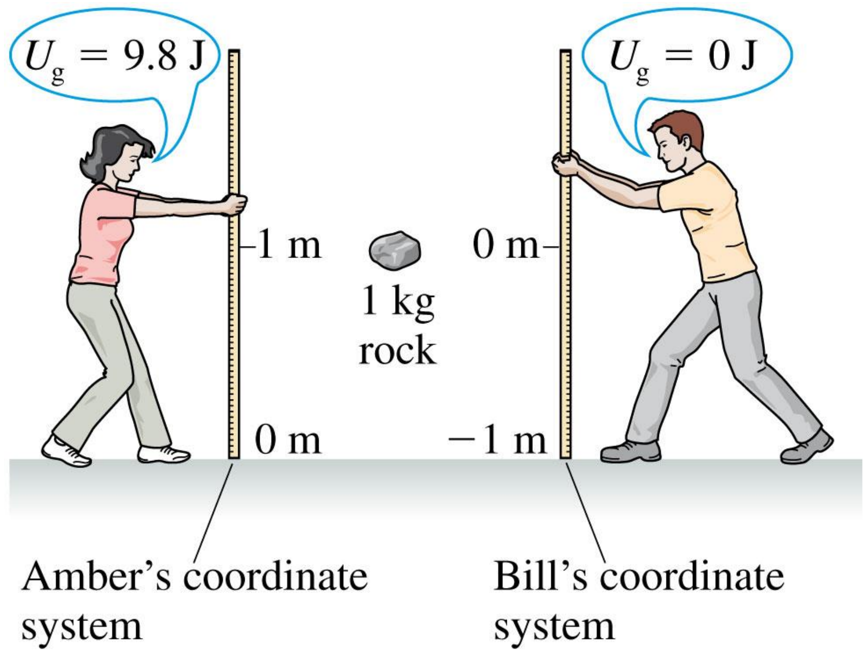
1. The distance from the axis to the PE curve is the particle's potential energy. The distance from the PE curve to the TE line is its kinetic energy. These are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ doesn't change.
2. A point where the TE line crosses the PE curve is a turning point. The particle reverses direction.
3. The particle cannot be at a point where the PE curve is above the TE line.
4. The PE curve is determined by the properties of the system—mass, spring constant, and the like. You cannot change the PE curve. However, you can raise or lower the TE line simply by changing the initial conditions to give the particle more or less total energy.
5. A minimum in the PE curve is a point of stable equilibrium. A maximum in the PE curve is a point of unstable equilibrium.

Exercises 18–20

Launching a Pebble

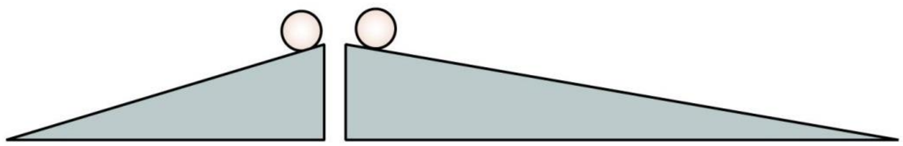
Q.2 Bob uses a slingshot to shoot a 20 g pebble straight up with a speed of 25 m/s. How high does the pebble go?

The Speed of a Falling Rock



Q.3 The 1.0 kg rock shown above is released from rest. Use both Amber's and Bill's perspectives to calculate its speed just before it hits the ground.

Kinetic Energy and Gravitational Potential Energy



Q.4 Starting from rest, a marble first rolls down a steeper hill, then down a less steep hill of the same height. For which is it going faster at the bottom?

a) Faster at the bottom of the steeper hill.

b) Faster at the bottom of the less steep hill.

c) Same speed at the bottom of both hills.

d) Can't say without knowing the mass of the marble.

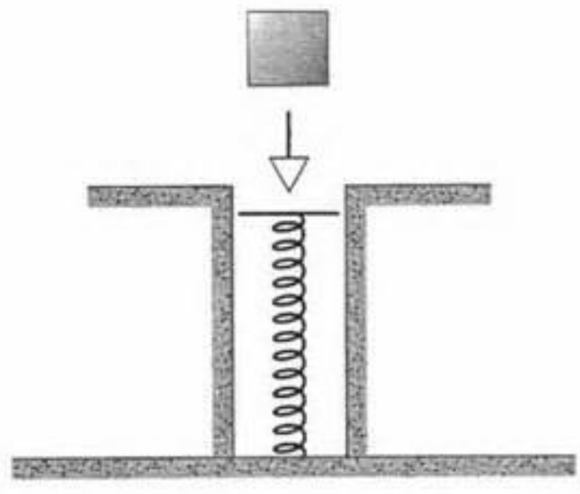
The Speed of a Sled

Q.5 Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0-m-high, very slippery slope. What is her speed at the bottom?

A Spring-Launched Plastic Ball

Q.6 A spring-loaded toy gun launches a 10 g plastic ball. The spring, with spring constant 10 N/m is compressed by 10 cm as the ball is pushed into the barrel. When the trigger is pulled, the spring is released and shoots the ball back out. What is the ball's speed as it leaves the barrel? Assume friction is negligible.

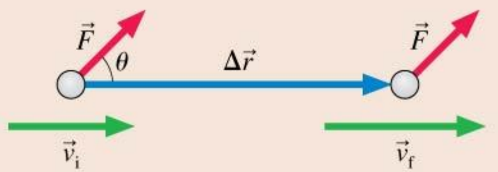
Balancing a Mass on a Spring



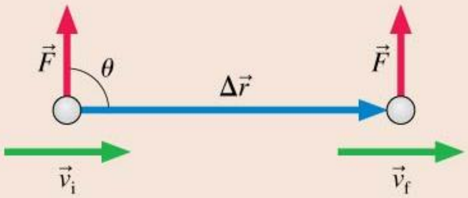
Q.7 A spring of length L_0 and spring constant k is standing on one end. A block of mass m is placed on the spring, compressing it. What is the length of the compressed spring?

TACTICS BOX 11.1

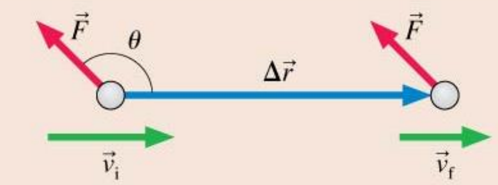
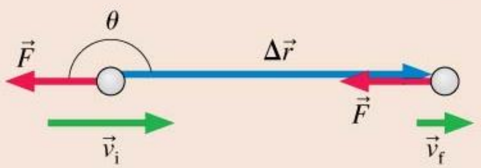
Calculating the work done by a constant force

Force and displacement	θ	Work W	Sign	Energy transfer
	$0°$	$F(\Delta r)$	+	Energy is transferred into the system. The particle speeds up. K increases.
	$<90°$	$F(\Delta r)\cos\theta$	+	

TACTICS BOX 11.1 Calculating the work done by a constant force

Force and displacement	θ	Work W	Sign	Energy transfer
	90°	0	0	No energy is transferred. Speed and K are constant.

TACTICS BOX 11.1 Calculating the work done by a constant force

Force and displacement	θ	Work W	Sign	Energy transfer
	$>90°$	$F(\Delta r)\cos\theta$	–	Energy is transferred out of the system. The particle slows down. K decreases.
	$180°$	$-F(\Delta r)$	–	

Exercises 3–10

Law of conservation of energy The total energy $E_{sys} = E_{mech} + E_{th}$ of an isolated system is a constant. The kinetic, potential, and thermal energy within the system can be transformed into each other, but their sum cannot change. Further, the mechanical energy $E_{mech} = K + U$ is conserved if the system is both isolated and nondissipative.

PROBLEM-SOLVING STRATEGY 11.1 **Solving energy problems**

MODEL Identify which objects are part of the system and which are in the environment. When possible, choose a system without friction or other dissipative forces. Some problems may need to be subdivided into two or more parts.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart. A free-body diagram is helpful if you're going to calculate work.

PROBLEM-SOLVING STRATEGY 11.1 **Solving energy problems**

SOLVE If the system is both isolated and nondissipative, then the mechanical energy is conserved:

$$K_f + U_f = K_i + U_i$$

If there are external or dissipative forces, calculate W_{ext} and ΔE_{th}. Then use the more general energy equation

$$K_f + U_f + \Delta E_{th} = K_i + U_i + W_{ext}$$

Kinematics and/or other conservation laws may be needed for some problems.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Pulling a Suitcase

Q.8 A rope inclined upward at a 45° angle pulls a suitcase through the airport. The tension in the rope is 20 N. How much work does the tension do if the suitcase is pulled 100 m?

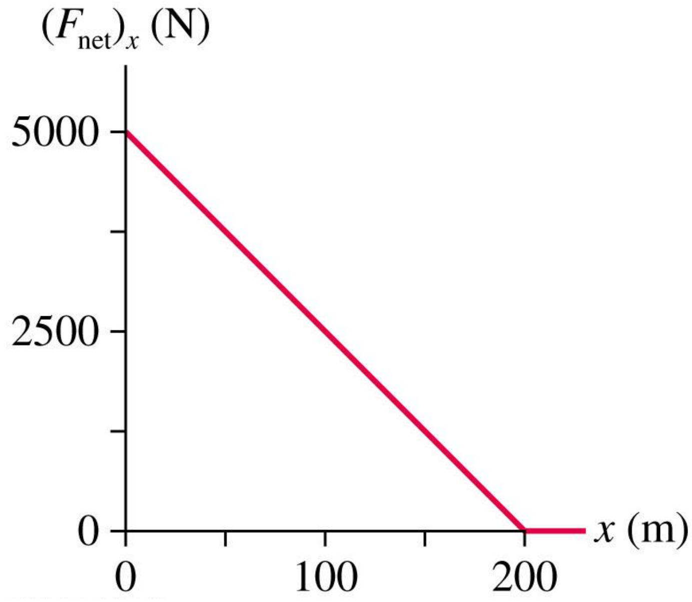
Work During a Rocket Launch

Q.9 A 150,000 kg rocket is launched straight up. The rocket motor generates a thrust of 4.0×10^6 N. What is the rocket's speed at a height of 500 m? Ignore air resistance and any slight mass loss.

Calculating Work Using the Dot Product

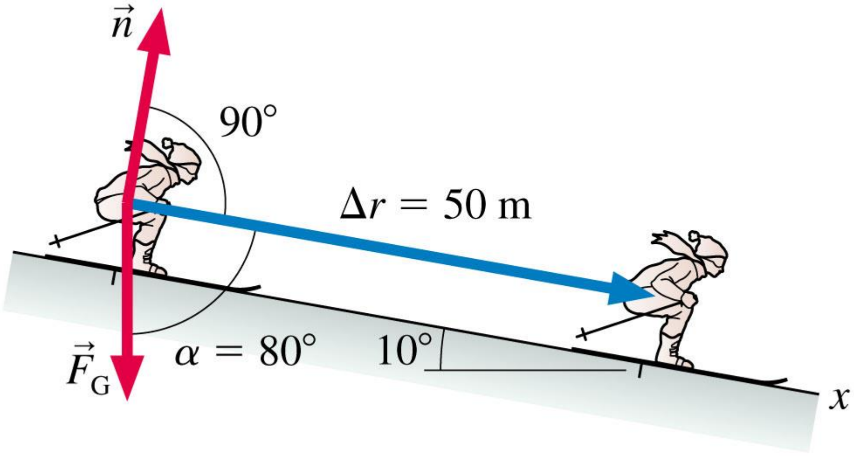
Q.10 A 70 kg skier is gliding at 2.0 m/s when he starts down a very slippery 50-m-long, 10° slope. What is his speed at the bottom?

Using Work to Find the Speed of a Car



Q.11 A 1500 kg car accelerates from rest. The figure shows the net force on the car (propulsion force minus any drag forces) as it travels from x = 0 m to x = 200 m. What is the car's speed after travelling 200 m?

Using Work and Potential Energy



Before:
$x_0 = 0$ m
$v_0 = 2.0$ m/s
$m = 70$ kg

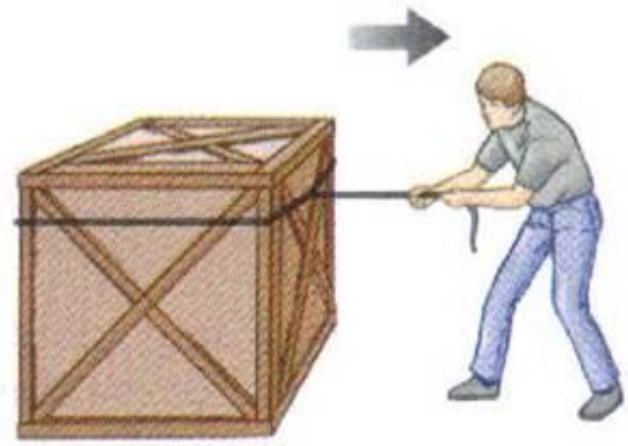
After:
$x_1 = 50$ m
v_1

Find: v_1

Q.12

The skier from question 5 repeats his run after the wind comes up. Recall that the 70 kg skier was gliding at 2.0 m/s when he started down a 50-m-long, 10°, frictionless slope. What is his speed at the bottom if the wind exerts a steady 50 N retarding force opposite his motion?

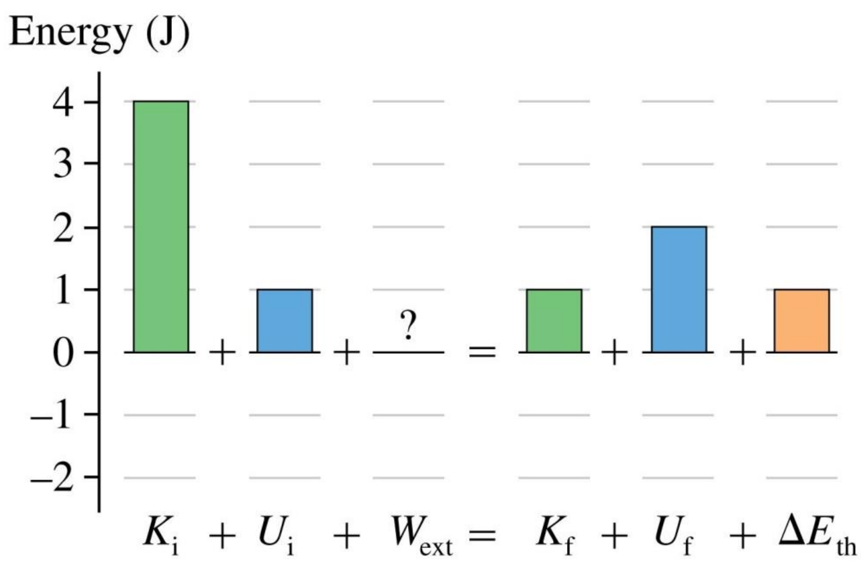
Calculating the Increase in Thermal Energy



Q.13 A rope pulls a 10 kg wooden crate 3.0 m across a wood floor. What is the change in thermal energy? The coefficient of kinetic friction is 0.20.

Energy Bar Charts

Q.14 How much work is done by the environment in the process represented by the energy bar chart?



a) –2 J

b) –1 J

c) 0 J

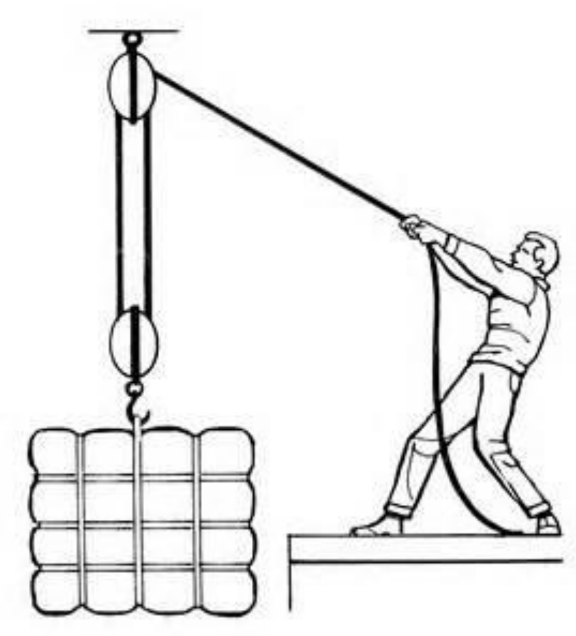
d) 1 J

e) 2 J

Energy Bar Charts

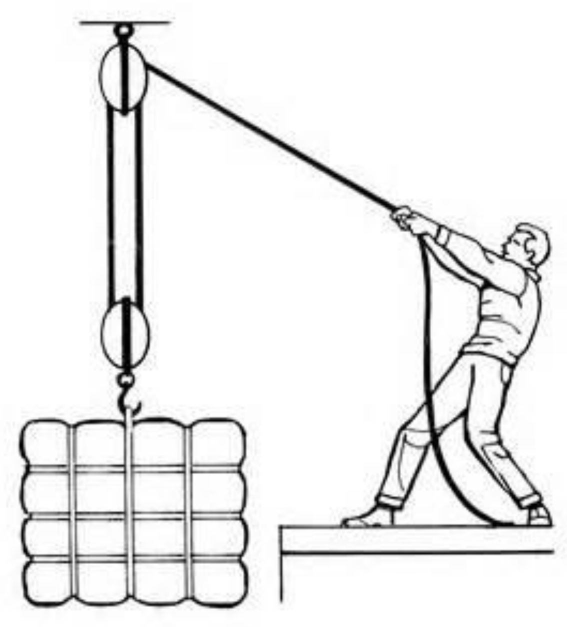
Q.15 A speeding car skids to a halt. Show the energy transfers and transformations on an energy bar chart.

Energy Bar Charts



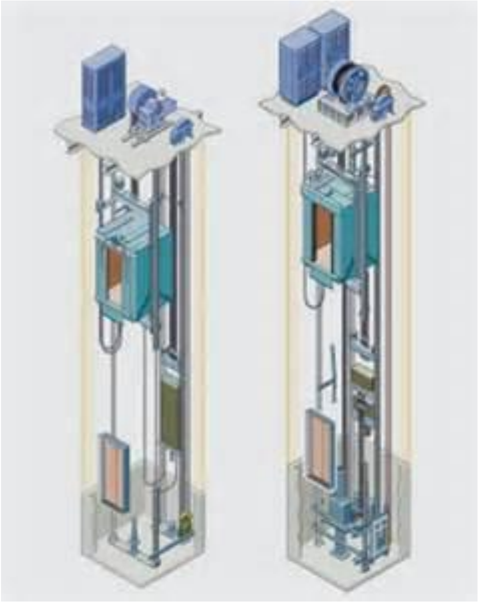
Q.16 A rope lifts a box at constant speed. Show the energy transfers and transformations on an energy bar chart.

Energy Bar Charts



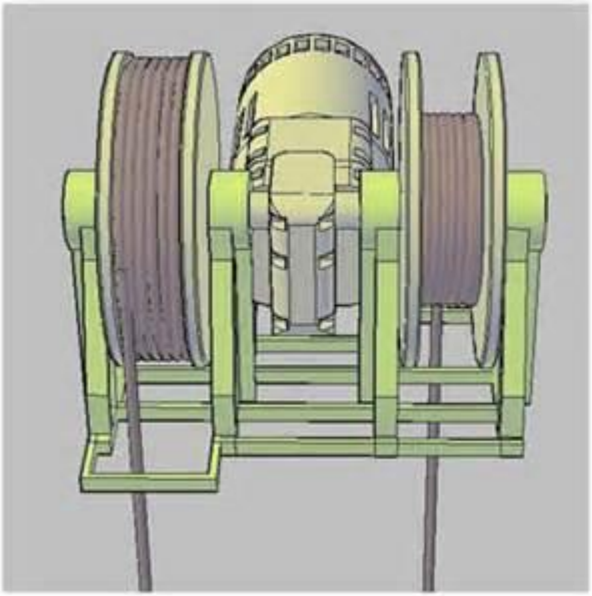
Q.17 The box that was lifted in question 11 now falls at a steady speed as the rope spins a generator and causes a lightbulb to glow. Air resistance is negligible. Show the energy transfers and transformations on an energy bar chart.

Choosing a Motor



Q.18 What power motor is needed to lift a 2000 kg elevator at a steady 3.0 m/s?

Power Output of a Motor



Q.19 A factory uses a motor and a cable to drag a 300 kg machine to the proper place on the factory floor. What power must the motor supply to drag the machine at a speed of 0.50 m/s? The coefficient of friction between the machine and the floor is 0.60.

TACTICS BOX 9.1 Drawing a before-and-after pictorial representation

1. **Sketch the situation.** Use two drawings, labeled "Before" and "After," to show the objects *before* they interact and again *after* they interact.
2. **Establish a coordinate system.** Select your axes to match the motion.
3. **Define symbols.** Define symbols for the masses and for the velocities before and after the interaction. Position and time are not needed.

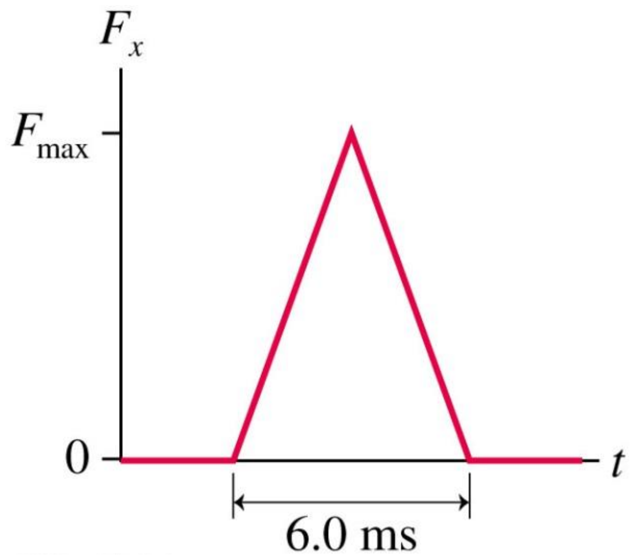
Exercises 17–19

TACTICS BOX 9.1 Drawing a before-and-after pictorial representation

4. **List known information.** Give the values of quantities that are known from the problem statement or that can be found quickly with simple geometry or unit conversions. Before-and-after pictures are simpler than the pictures for dynamics problems, so listing known information on the sketch is adequate.
5. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined in step 3.
6. If appropriate, **draw a momentum bar chart** to clarify the situation and establish appropriate signs.

Exercises 17–19

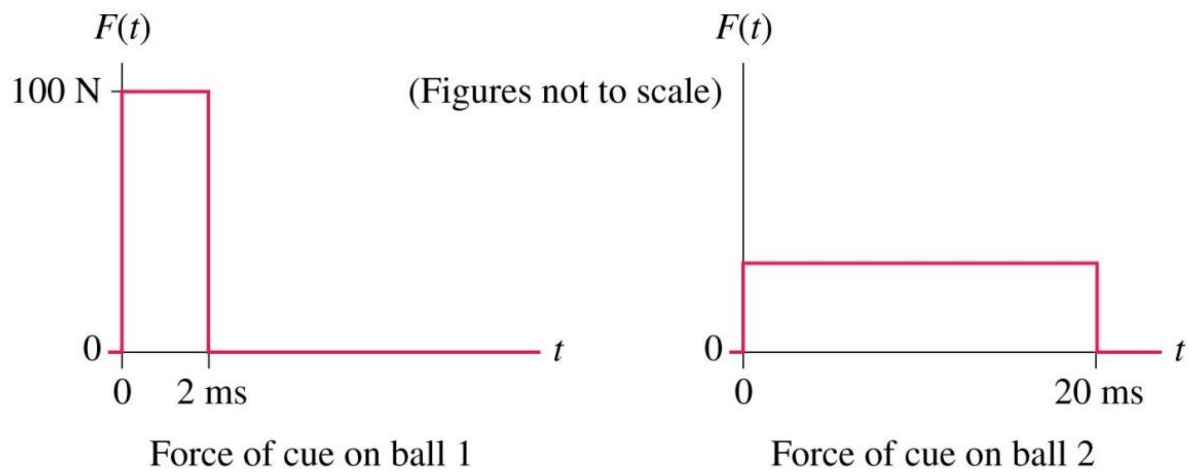
Hitting a Baseball



Q.20 A 150 g baseball is thrown with a speed of 20 m/s. It is hit straight back toward the pitcher at a speed of 40 m/s. The interaction force between the ball and the bat is shown in the figure above. What *maximum force* F_{max} does the bat exert on the ball? What is the *average force* of the bat on the ball?

The Impulse-Momentum Theorem

Q.21 Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?



a) Ball 1.

b) Ball 2.

c) Both balls have the same final speed.

PROBLEM-SOLVING STRATEGY 9.1

Conservation of momentum

MODEL Clearly define *the system.*

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or, as you'll learn in Chapters 10 and 11, conservation of energy.

PROBLEM-SOLVING STRATEGY 9.1

Conservation of momentum

MP

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_f = \vec{P}_i$. In component form, this is

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 16

Law of Conservation of Momentum

Q.22 A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

a) The mosquito.

b) The truck.

c) They have the same change of momentum.

d) Can’t say without knowing their initial velocities.

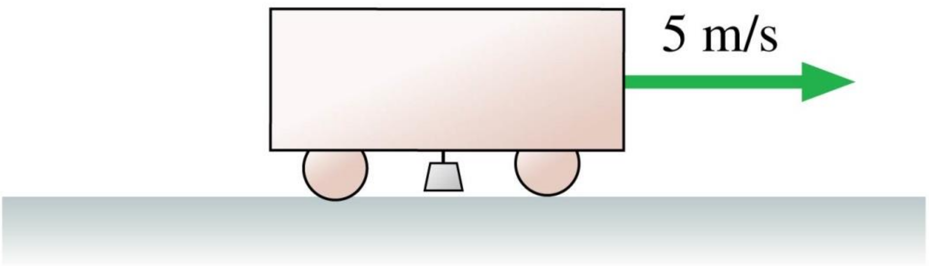
Recoil

Q.23

A 10 g bullet is fired from a 3.0 kg rifle with a speed of 500 m/s. What is the recoil speed of the rifle?

Momentum in Two Dimensions

Q.24 A cart is rolling at 5 m/s. A heavy lead weight is suspended by a thread beneath the cart. Suddenly the thread breaks and the weight falls. Immediately afterward, the speed of the cart is



a) Less than 5 m/s.

b) Still 5 m/s.

c) More than 5 m/s.