CELEN037 Seminar 1

Topics

- Derivatives using First Principles
- The Sum and Difference Rules
- The Product Rule
- Extension of the Product Rule
- The Quotient Rule

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Definition of derivative

The derivative of a function $y = f(x)$ is given by:

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{(x+h+1)^2 - (x+1)^2}{h} \end{aligned}$$

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&= \lim_{h \to 0} \frac{x^2 + h^2 + 1 + 2xh + 2x + 2h - (x^2 + 2x + 1)}{h} \\
&= \lim_{h \to 0} \frac{h^2 + 2xh + 2h}{h} \\
&= \lim_{h \to 0} \frac{h(h + 2x + 2)}{h}
\end{aligned}$$

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&= \lim_{h\to 0} \frac{h^2 + 2xh + 2h}{h} \\
&= \lim_{h\to 0} \frac{h(h + 2x + 2)}{h} \\
&= \lim_{h\to 0} (h + 2x + 2) = 2x + 2
\end{aligned}$$

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Practice Problems on Worksheet:

1: Q1(iii)
2: Q1(iv)
3: Q1(v)
4: Q1(vi)

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Practice Problems on Worksheet:

1: Q1(iii)
2: Q1(iv)
3: Q1(v)
4: Q1(vi)

Answers:

1: $-\dfrac{2}{x^3}$

2: $\dfrac{1}{2\sqrt{x}}$

3: $\dfrac{1}{2\sqrt{x+1}}$

4: $-\dfrac{1}{2\sqrt{x^3}}$

Practice Problems on Worksheet (Cont'd):

1: Q1(ix)
2: Q1(x)
3: Q1(xi)
4: Q1(xii)

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Practice Problems on Worksheet (Cont'd):

1: Q1(ix)
2: Q1(x)
3: Q1(xi)
4: Q1(xii)

Answers:

1: $-\sin x$
2: $\sec^2 x$
3: $-\sin(x+1)$
4: $2\cos 2x$

The Sum and Difference Rules

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

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Solution:

$$y = (x+5) \cdot (3x-1) = 3x^2 + 14x - 5$$

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$$\text{Hence} \quad \frac{dy}{dx} = \frac{d}{dx}(3x^2 + 14x - 5)$$

$$= 3\frac{d}{dx}(x^2) + 14\frac{d}{dx}(x) - 5\frac{d}{dx}(1)$$

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$$= 3\frac{d}{dx}(x^2) + 14\frac{d}{dx}(x) - 5\frac{d}{dx}(1)$$

$$= 6x + 14$$

Practice Problems on Worksheet:

1: Q2(v)
2: Q2(vi)
3: Q2(vii)
4: Q2(viii)

Practice Problems on Worksheet:

1: Q2(v)
2: Q2(vi)
3: Q2(vii)
4: Q2(viii)

Answers:

1: $4x^3 + \dfrac{1}{x^2}$

2: $2 - \dfrac{1}{x^2} + \dfrac{6}{x^3}$

3: $2x + \dfrac{8}{3x^{\frac{5}{3}}}$

4: $1 - \dfrac{6}{x^2}$

Practice Problems on Worksheet (Cont'd):

1: Q2(xix)
2: Q2(xx)
3: Q2(xxi)
4: Q2(xxii)

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Practice Problems on Worksheet (Cont'd):

1: Q2(xix)
2: Q2(xx)
3: Q2(xxi)
4: Q2(xxii)

Answers:

1: $-\frac{5}{3}x^{-\frac{4}{3}} + 3\sin x$

2: $\frac{3}{4}x^{-\frac{1}{4}} + 2\sec^2 x$

3: $\frac{1}{x-5} - \frac{1}{x+1}$

4: $\frac{1}{2\sqrt{x}} + 2^x \ln 2 + \operatorname{cosec}^2 x + \frac{1}{x^2}$

The Product Rule

$$\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

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Example: Given $y = \ln x \cdot \sec x$, find $\frac{dy}{dx}$

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Example: Given $y = \ln x \cdot \sec x$, find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \ln x \cdot \frac{d}{dx}(\sec x) + \sec x \cdot \frac{d}{dx}(\ln x)$$

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$$\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

Example: Given $y = \ln x \cdot \sec x$, find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \ln x \cdot \frac{d}{dx}(\sec x) + \sec x \cdot \frac{d}{dx}(\ln x)$$

$$= \ln x \cdot \sec x \cdot \tan x + \sec x \cdot \frac{1}{x}$$

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Example: Given $y = \ln x \cdot \sec x$, find $\frac{dy}{dx}$

Solution:

$$\begin{aligned} \frac{dy}{dx} &= \ln x \cdot \frac{d}{dx}(\sec x) + \sec x \cdot \frac{d}{dx}(\ln x) \\ &= \ln x \cdot \sec x \cdot \tan x + \sec x \cdot \frac{1}{x} \\ &= \sec x \left(\ln x \cdot \tan x + \frac{1}{x} \right) \end{aligned}$$

Practice Problems on Worksheet:

1: Q3(v)
2: Q3(vi)
3: Q3(vii)
4: Q3(viii)

Practice Problems on Worksheet:

1: Q3(v)
2: Q3(vi)
3: Q3(vii)
4: Q3(viii)

Answers:

1: $\sec x \left(\dfrac{\tan x}{x} - \dfrac{1}{x^2} \right)$

2: $\dfrac{\cos x}{x} - \dfrac{\sin x}{x^2}$

3: $\dfrac{1}{x^2} - \dfrac{\ln x}{x^2}$

4: $2 \cos 2x$

Extension of the Product Rule

$$\frac{d}{dx}(u \cdot v \cdot w) = u \cdot v \cdot \frac{dw}{dx} + v \cdot w \cdot \frac{du}{dx} + w \cdot u \cdot \frac{dv}{dx}$$

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Practice Problems on Worksheet:

1: Q4(i)

2: Q4(iv)

$$\frac{d}{dx}(u \cdot v \cdot w) = u \cdot v \cdot \frac{dw}{dx} + v \cdot w \cdot \frac{du}{dx} + w \cdot u \cdot \frac{dv}{dx}$$

Practice Problems on Worksheet:

1: Q4(i)

2: Q4(iv)

Answers:

1: $e^x \left(\sin x \cdot \ln x + \cos x \cdot \ln x + \frac{\sin x}{x} \right)$

2: $\sin x \left(x \sec^2 x + \tan x + x \right)$

The Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \dfrac{du}{dx} - u \cdot \dfrac{dv}{dx}}{v^2}$$

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Example: Given $y = \dfrac{\sec x}{e^x}$, find $\frac{dy}{dx}$

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Solution:

$$\frac{dy}{dx} = \frac{e^x \cdot \frac{d}{dx}(\sec x) - \sec x \cdot \frac{d}{dx}(e^x)}{(e^x)^2}$$

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$$= \frac{e^x \cdot \sec x \cdot \tan x - \sec x \cdot e^x}{\left(e^x\right)^2}$$

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Solution:

$$\frac{dy}{dx} = \frac{e^x \cdot \frac{d}{dx}(\sec x) - \sec x \cdot \frac{d}{dx}(e^x)}{(e^x)^2}$$

$$= \frac{e^x \cdot \sec x \cdot \tan x - \sec x \cdot e^x}{(e^x)^2}$$

$$= \frac{\sec x \cdot \tan x - \sec x}{e^x}$$

Practice Problems on Worksheet:

1: Q5(ix)
2: Q5(x)
3: Q5(xi)
4: Q5(xii)

Practice Problems on Worksheet:

1: Q5(ix)
2: Q5(x)
3: Q5(xi)
4: Q5(xii)

Answers:

1: $-e^{-x}$
2: $e^{-x}(1-x)$
3: $\dfrac{2}{(1-x)^2}$
4: $-\dfrac{4x}{(1+x^2)^2}$