Exercise 4.1

1. $(b+c)^* a (b+c)^*$

2. $(a+b+c)^* b (a+b+c)^* b (a+b+c)^*$

3. $(a+b)^* + (a+b)^* c (a+b)^* + (a+b)^* c (a+b)^* c (a+b)^*$

4. $(a+b)^* (a+c)^*$

5. $a^* b a^* c a^* + a^* c a^* b a^*$

6. $c^*(a+b) c^* ((a+b) c^* (a+b))^* c^*$

7. $(a+b+c)^* abba (a+b+c)^*$

· Exercise 4.2

<u>lemma 1</u>: $L(aa) = L(a)L(a)$ semantics of RE concat.

$= \{uv \mid u \in L(a) \wedge v \in L(a)\}$ def. of language concat.

$= \{uv \mid u \in \{a\} \wedge v \in \{a\}\}$ semantics of RE

$= \{aa\}$

<u>lemma 2</u>: $L(\varepsilon b^* \phi) = L((\varepsilon b^*)\phi)$ $EFG = E(FG)$

$= L(\phi)$ $E\phi = \phi$

$= \phi$ semantics of RE ϕ.

<u>lemma 3</u>: $L(b+c) = L(b) \cup L(c)$ semantics of RE (+)

$= \{x \mid x \in L(b) \vee x \in L(c)\}$ def. of set union

$= \{x \mid x \in \{b\} \vee x \in \{c\}\}$ semantics of RE

$= \{b, c\}$ def. of set union.

$L((aa+\varepsilon b^*\phi)(b+c)) = L(aa+\varepsilon b^*\phi)L(b+c)$ semantics of RE concat.

def. of language concat.

$= \{uv \mid u \in L(aa+\varepsilon b^*\phi) \wedge v \in L(b+c)\}$

semantics of RE (+)

$= \{uv \mid u \in L(aa) \cup L(\varepsilon b^*\phi) \wedge v \in L(b+c)\}$

lemma 1, lemma 2, lemma 3

$= \{uv \mid u \in \{aa\} \cup \phi \wedge v \in \{b, c\}\}$

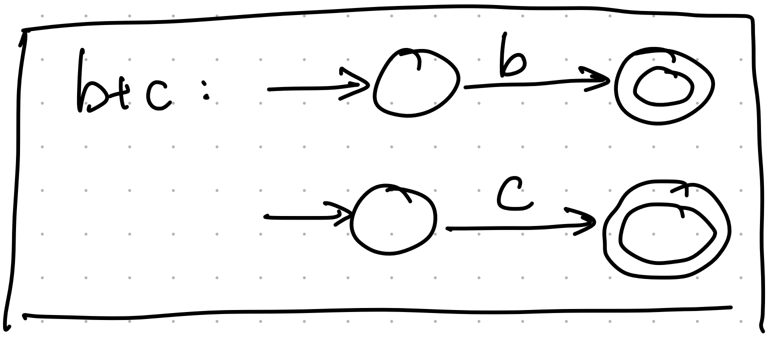
def. of set union.

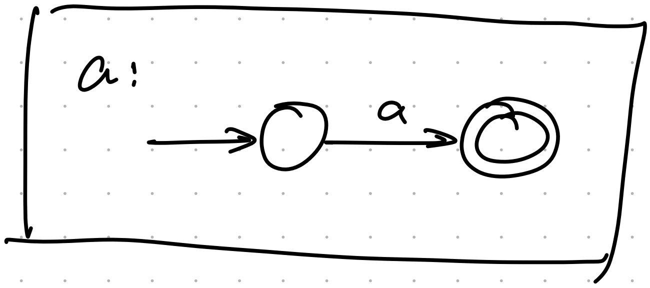
$= \{uv \mid u \in \{aa\} \wedge v \in \{b, c\}\}$

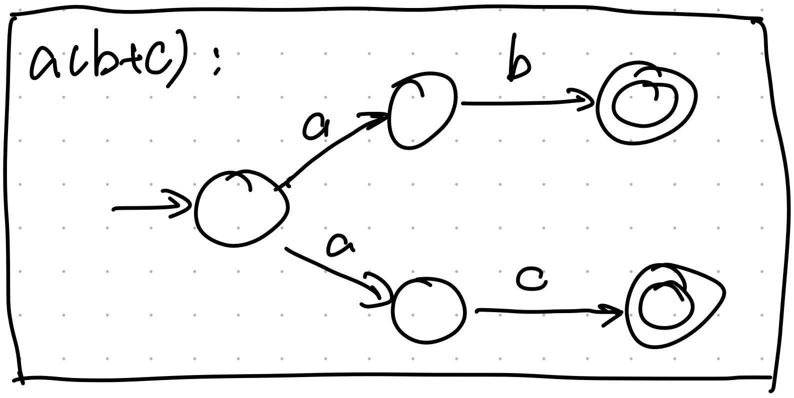
$= \{aab, aac\}$

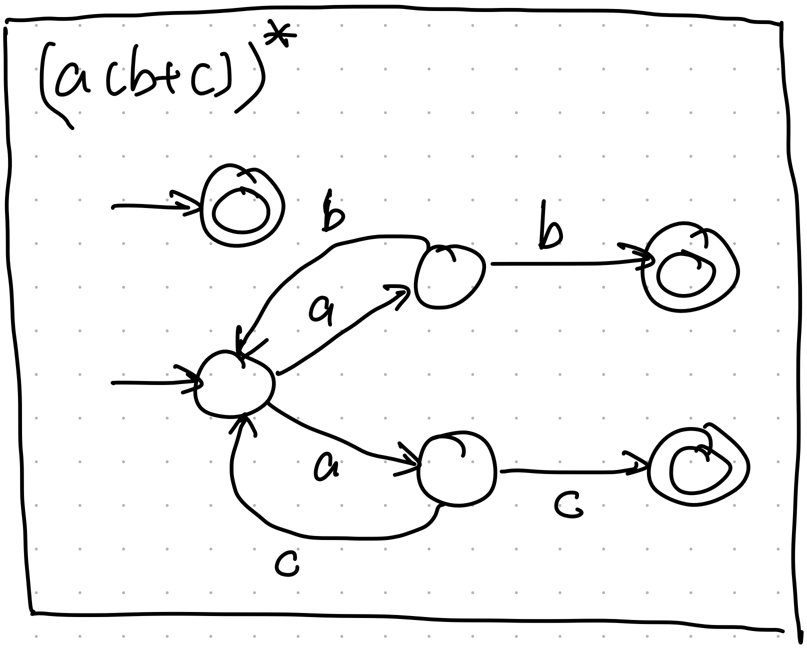
$\therefore (aa+\varepsilon b^*\phi)(b+c) = aab+aac$

• Exercise 4.3



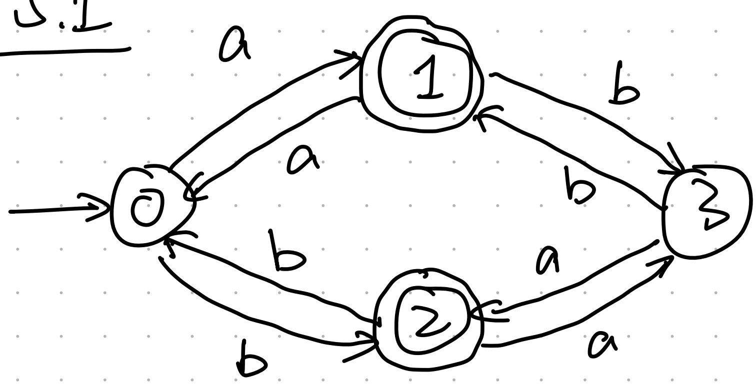






Exercise 5.1

DFA:



0	1	2	
~~01~~			1
~~02~~	12		2
03	~~13~~	~~23~~	3

$(0,1)$: $0 \notin F \wedge 1 \in F$ distinguishable.

$(0,2)$: $0 \notin F \wedge 2 \in F$ distinguishable.

$(1,3)$: $1 \in F \wedge 3 \notin F$ distinguishable.

$(2,3)$: $2 \in F \wedge 3 \notin F$ distinguishable.

$(0,3)$: $(\delta(0,a), \delta(3,a)) = (1,2)$ no info.

$(\delta(0,b), \delta(3,b)) = (2,1)$ no info.

$(1,2)$: $(\delta(1,a), \delta(2,a)) = (0,3)$ no info.

$(\delta(1,b), \delta(2,b)) = (3,0)$ no info

We've checked all pairs and there still remains $(0,3)$, $(1,2)$

Therefore: $1 \equiv 2$, $0 \equiv 3$.

DFA.