Foundation Algebra (CELEN036)

Problem Sheet 4 **Topics: Trigonometry II**

Topic 1: Addition and Factor formulae

1. Prove the following results:

   (i) $\cos(270° - \theta) = -\sin\theta$.

   (ii) $\cot x + \cot y = \csc x \csc y \sin(x + y)$.

   (iii) $\sin(n + 1)A\cos(n + 2)A - \cos(n + 1)A\sin(n + 2)A = -\sin A$.

   (iv) $\tan\left(\frac{\pi}{4} + \alpha\right) = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}$.

   (v) $\tan 63° = \frac{\cos 18° + \sin 18°}{\cos 18° - \sin 18°}$.

   (vi) $\frac{\cos 11° - \sin 11°}{\cos 11° + \sin 11°} = \tan 34°$.

   (vii) $\cot 5° = \frac{\sqrt{3}\cos 25° + \sin 25°}{\cos 25° - \sqrt{3}\sin 25°}$.

2. Given $3\sin(x - y) - \sin(x + y) = 0$. Show that $\tan x = 2\tan y$.

3. Prove that $A+B = \frac{\pi}{4} \Rightarrow (1+\tan A)(1+\tan B) = 2$. Hence deduce the value of $\tan\frac{45°}{2}$.

4. Prove the following results:

   (i) $\frac{\sin 6\theta - \sin 4\theta}{\sin\theta} = 2\cos 5\theta$.

   (ii) $\frac{\sin 80° + \sin 20°}{\cos 20° - \cos 80°} = \sqrt{3}$.

   (iii) $\frac{\sin 75° - \cos 75°}{\cos 75° + \sin 75°} = \frac{1}{\sqrt{3}}$.

(iv) $\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right) = 0.$

(v) $\dfrac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta.$

(vi) $\cos 80^\circ + \sin 50^\circ = \cos 20^\circ.$

(vii) $\sin 19^\circ + \cos 11^\circ = \sqrt{3}(\cos 19^\circ - \sin 11^\circ).$

Topic 2: Multi-angle formulae

5. Prove the following results:

(i) $\cos^4 A - \sin^4 A = \cos 2A.$

(ii) $\dfrac{\tan\theta}{1+\tan^2\theta} = \dfrac{1}{2}\sin 2\theta.$

(iii) $\dfrac{1-\cos 2\theta + \sin 2\theta}{1+\cos 2\theta + \sin 2\theta} = \tan\theta.$

(iv) $\sin 3A - \cos 3A = (\sin A + \cos A)(4\sin A\cos A - 1).$

(v) $1 + \dfrac{4\tan^2\theta}{(1-\tan^2\theta)^2} = \dfrac{1}{1-4\sin^2\theta\cos^2\theta}.$

(vi) $16\sin^2\theta\cos^3\theta = 2\cos\theta - \cos 3\theta - \cos 5\theta.$

6. Simplify: $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ (Take $\sqrt{X^2} = X$)

7. Prove that $\cos\theta = \sqrt{\dfrac{1}{2} + \sqrt{\dfrac{1}{8} + \dfrac{1}{8}\cos 4\theta}}$ (Take $\sqrt{X^2} = X$)

Topic 3: Inverse Trigonometric functions

8. Without using a calculator, find the values of:

(i) $\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$

(ii) $\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$

(iii) $\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right]$

(iv) $\sin\left[2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

Topic 4: Expressing $a\cos x + b\sin x$ in the form $r\cos(x-\theta)$

9. Given that $5\sin\theta + 12\cos\theta \equiv R\cos(\theta-\alpha)$, find $R\,(R>0)$ and $\alpha \in \left[0, \dfrac{\pi}{2}\right]$.

10. Given that $2\sin\theta - \sqrt{5}\cos\theta \equiv -3\cos(\theta+\alpha)$, where $0 < \alpha < 90°$, find the value of α.

11. Show that $\sqrt{3}\sin 2\theta - \cos 2\theta \equiv 2\sin\left(2\theta - \dfrac{\pi}{6}\right)$.

12. Express $5\sin x + 12\cos x$ in the form $R\sin(x+\theta)$, where R and θ are to be determined.

13. Express $2\cos\theta + 5\sin\theta$ in the form $R\cos(\theta-\alpha)$, where $R>0$, and $0<\alpha<90°$. Hence solve the equation $2\cos\theta + 5\sin\theta = 3 \quad (0<\theta<360°)$.

14. Show that $\cos\theta - \sqrt{3}\sin\theta$ can be written in the form $R\cos(\theta+\alpha)$ where $R>0$ and $0<\alpha<\dfrac{\pi}{2}$. Hence sketch the graph of $f(\theta) = \cos\theta - \sqrt{3}\sin\theta \quad (0<\theta<2\pi)$.

Answers

3. $\sqrt{2} - 1$

8. (i) $\dfrac{\sqrt{3}}{2}$ (ii) $-\sqrt{3}$ (iii) $\dfrac{\pi}{3}$ (iv) 1

9. $R = 13,\ \alpha = 0.3948$ radians

10. $41.81°$

12. $R = 13,\ \theta = 1.18$ radians or $67.38°$

13. $R = 2,\ \alpha = 86.2°$, Roots $\theta = 124.35°$ or $12.05°$

14. $R = 2,\ \alpha = \dfrac{\pi}{3}$