University of Nottingham
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Foundation Calculus and Mathematical Techniques (CELEN037)

Answers to Worksheet #10

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1. **Solving ODEs of Variable-Separable Form**

(i) $\dfrac{x^2}{2} = -\dfrac{y^2}{2} + C$

(ii) $\ln|y| = \ln|x| + C$

(iii) $\tan^{-1} y = \tan^{-1} x + C$

(iv) $\dfrac{y^3}{3} + y = \dfrac{x^3}{3} + x + C$

(v) $-\dfrac{1}{y} = \ln|x+1| + C$

(vi) $\ln|y+1| = -\dfrac{1}{x} + C$

(vii) $\tan^{-1} y = \dfrac{x^3}{3} + C$

(viii) $\ln|y| = \ln|x| + x + C$

(ix) $\ln|y| + \dfrac{y^2}{2} = e^x + C$

(x) $\dfrac{1}{2}\ln(1+y^2) = -\ln|\cos x| + C$

(xi) $\ln|\sin y| = -\ln|\cos x| + C$

(xii) $y = \ln|\ln(\sin x)| + C$

(xiii) $\ln|y| = \tan x - x + C$

(xiv) $\ln|\tan y| = \ln|x| + C$

(xv) $y = \ln\left(e^x + e^{-x}\right) + C$

(xvi) $\dfrac{1}{3}(1+y^2)^{\frac{3}{2}} = e^x(x-1) + C$

(xvii) $e^y = e^x + \dfrac{x^3}{3} + C$

(xviii) $\ln|y| = \ln(1+\sin x) + C$

(xix) $y = x^2 + \ln|x| + C$

(xx) $y - \ln|y+1| = \ln(e^x + 1) + C$

(xxi) $\dfrac{y^3}{3} + \dfrac{y^2}{2} = \dfrac{x^2}{2} + \ln|x| + C$

(xxii) $\ln|x| = \ln|\ln y| + C$

(xxiii) $\tan x = \cos y + C$

(xxiv) $\ln|y-1| = \ln|x+1| + C$

2. **Solving IVPs of Variable-Separable Form**

(i) $y = \sqrt[3]{x^3+8}$

(ii) $y = \dfrac{1}{2x^2+1}$

(iii) $\ln y + \dfrac{y^2}{2} + \cos x = \dfrac{3}{2}$

(iv) $y = \sec x$

(v) $y = \sec x$

(vi) $y = \sqrt{2 - \sqrt{x^2+1}}$

(vii) $\sin y = \sqrt{\dfrac{2}{x^2+1}}$

(viii) $y = (x+1)\ln(x+1) - x + 3$

4. **Applications of Differential Equations**

(ii)(b) 87.06%

(iii) $50\ln 2$ (≈ 34.66)