Seminar 9 Answers: Lectures 19-21

Capacitors:

1. The charge on a capacitor increases by 26 μC when the voltage across it increases from 28 V to 78 V. What is the capacitance of the capacitor?

Answer:

Let Q_1 and V_1 be the initial charge and voltage on the capacitor, and let Q_2 and V_2 be the final charge and voltage on the capacitor. We can use the equations below to relate the charges and voltages to the capacitance.

$$Q_1 = CV_1 \qquad Q_2 = CV_2 \qquad Q_2 - Q_1 = CV_2 - CV_1 = C\left(V_2 - V_1\right) \quad \rightarrow$$

$$C = \frac{Q_2 - Q_1}{V_2 - V_1} = \frac{26\times10^{-6}\,\mathrm{C}}{50\,\mathrm{V}} = 5.2\times10^{-7}\,\mathrm{F} = \boxed{0.52\,\mu\mathrm{F}}$$

Determination of Capacitance:

2. What is the capacitance per unit length (F/m) of a coaxial cable whose inner conductor has a 1.0-mm diameter and the outer cylindrical sheath has a 5.0-mm diameter? Assume the space between is filled with air.

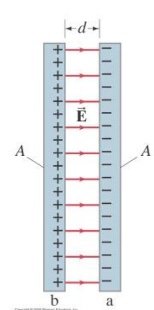
Answer:

The capacitance per unit length of a coaxial cable is derived in Example 24-2 in your textbook, page 731.

$$\frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln\left(R_{\mathrm{outside}}/R_{\mathrm{inside}}\right)} = \frac{2\pi\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}{\ln\left(5.0\,\mathrm{mm}/1.0\,\mathrm{mm}\right)} = \boxed{3.5\times10^{-11}\,\mathrm{F/m}}$$

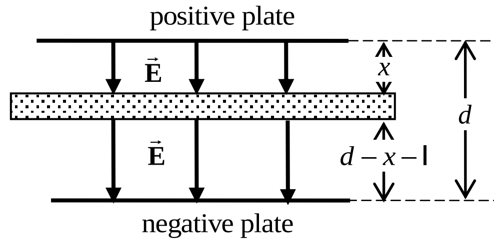
3. A large metal sheet of thickness l is placed between, and parallel to, the plates of the parallel-plate capacitor as shown in the figure below. It does not touch the plates, and extends beyond their edges. (a) What is now the net capacitance in terms of A, d, and l? (b) If $l = 0.40d$, by what factor does the capacitance change when the sheet is inserted?

Figure 24.4



Answer:

(a) The uncharged plate will polarize so that negative charge will be drawn towards the positive capacitor plate, and positive charge will be drawn towards the negative capacitor plate. The same charge will be on each face of the plate as on the original capacitor plates. The same electric field will be in the gaps as before the plate was inserted. Use that electric field to determine the potential difference between the two original plates, and the new capacitance. Let *x* be the distance from one original plate to the nearest face of the sheet, and so *d* = 1- *x* is the distance from the other original plate to the other face of the sheet.



$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{A\varepsilon_0}\ ;\ V_1=Ex=\frac{Qx}{A\varepsilon_0}\ ;\ V_2=E\left(d-l-x\right)=\frac{Q\left(d-l-x\right)}{A\varepsilon_0}$$

$$\Delta V=V_1+V_2=\frac{Qx}{A\varepsilon_0}+\frac{Q\left(d-l-x\right)}{A\varepsilon_0}=\frac{Q\left(d-l\right)}{A\varepsilon_0}=\frac{Q}{C}\ \rightarrow\ C=\boxed{\varepsilon_0\frac{A}{\left(d-l\right)}}$$

(*b*) $$C_{\text{initial}}=\varepsilon_0\frac{A}{d}\ ;\ C_{\text{final}}=\varepsilon_0\frac{A}{\left(d-l\right)}\ ;\ \frac{C_{\text{final}}}{C_{\text{initial}}}=\frac{\varepsilon_0\frac{A}{\left(d-l\right)}}{\varepsilon_0\frac{A}{d}}=\frac{d}{d-l}=\frac{d}{d-0.40d}=\frac{1}{0.60}=\boxed{1.7}$$

Capacitors in Series and Parallel:

4. (a) Six 3.8 μF capacitors are connected in parallel. What is the equivalent capacitance? (b) What is their equivalent capacitance if connected in series?

Answer:

(*a*) Capacitors in parallel add according to the equation below.

$$C_{\text{eq}}=C_1+C_2+C_3+C_4+C_5+C_6=6\left(3.8\times10^{-6}\,\text{F}\right)=\boxed{2.28\times10^{-5}\,\text{F}}=22.8\,\mu\text{F}$$

(*b*) Capacitors in series add according to the equation below.

$$C_{\text{eq}}=\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}+\frac{1}{C_5}+\frac{1}{C_6}\right)^{-1}=\left(\frac{6}{3.8\times10^{-6}\,\text{F}}\right)^{-1}=\frac{3.8\times10^{-6}\,\text{F}}{6}=\boxed{6.3\times10^{-7}\,\text{F}}$$
$$=0.63\,\mu\text{F}$$

5. Three conducting plates, each of area A, are connected as shown in the figure below. (a) Are the two capacitors thus formed connected in series or in parallel? (b) Determine C as a function of d_1, d_2, and A. Assume $d_1 + d_2$ is much less than the dimensions of the plates. (c) The middle plate can be moved (changing the values of d_1 and d_2), so as to vary the capacitance. What are the minimum and maximum values of the net capacitance?

Figure 24.22

Answer:

(a) The two capacitors are in $\boxed{\text{parallel}}$. Both capacitors have their high voltage plates at the same potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).

(b) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0 A}{d_2} = \boxed{\varepsilon_0 A\left(\frac{1}{d_1} + \frac{1}{d_2}\right)} = \varepsilon_0 A\left(\frac{d_1 + d_2}{d_1 d_2}\right)$$

(c) Let $\ell = d_1 + d_2 = \text{constant}$. Then $C = \dfrac{\varepsilon_0 A\ell}{d_1 d_2} = \dfrac{\varepsilon_0 A\ell}{d_1(\ell - d_1)}$. We see that $C \to \infty$ as $d_1 \to 0$ or $d_1 \to \ell$ (which is $d_2 \to 0$). Of course, a real capacitor would break down as the plates got too close to each other. To find the minimum capacitance, set $\dfrac{dC}{d(d_1)} = 0$ and solve for d_1.

$$\frac{dC}{d(d_1)} = \frac{d}{d(d_1)}\left[\frac{\varepsilon_0 A\ell}{d_1\ell - d_1^2}\right] = \varepsilon_0 A\ell\,\frac{(\ell - 2d_1)}{(d_1\ell - d_1^2)^2} = 0 \;\rightarrow\; d_1 = \tfrac{1}{2}\ell = d_2$$

$$C_{\min} = \varepsilon_0 A\left(\frac{d_1 + d_2}{d_1 d_2}\right)_{d_1 = \frac{1}{2}\ell} = \varepsilon_0 A\left(\frac{\ell}{\left(\frac{1}{2}\ell\right)\left(\frac{1}{2}\ell\right)}\right) = \varepsilon_0 A\left(\frac{4}{\ell}\right) = \varepsilon_0 A\left(\frac{4}{d_1 + d_2}\right)$$

$$\boxed{C_{\min} = \frac{4\varepsilon_0 A}{d_1 + d_2}\;;\; C_{\max} = \infty}$$

Electric Energy Storage:

6. How much energy must a 28 V battery expend to charge a 0.45 μF and a 0.20 μF capacitor fully when they are placed (a) in parallel, (b) in series? (c) How much charge flowed from the battery in each case?

Answer:

(*a*) Using the equations below:

$$U_{\text{parallel}} = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(C_1 + C_2)V^2 = \tfrac{1}{2}\left(0.65\times10^{-6}\,\text{F}\right)\left(28\,\text{V}\right)^2 = 2.548\times10^{-4}\,\text{J} \approx \boxed{2.5\times10^{-4}\,\text{J}}$$

(*b*) Using the equations below:

$$U_{\text{series}} = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)V^2 = \tfrac{1}{2}\left(\frac{\left(0.45\times10^{-6}\,\text{F}\right)\left(0.20\times10^{-6}\,\text{F}\right)}{0.65\times10^{-6}\,\text{F}}\right)\left(28\,\text{V}\right)^2$$

$$= 5.428\times10^{-5}\,\text{J} \approx \boxed{5.4\times10^{-5}\,\text{J}}$$

(*c*) The charge can be found from the equation below:

$$U = \tfrac{1}{2}QV \quad \rightarrow \quad Q = \frac{2U}{V} \quad \rightarrow \quad Q_{\text{parallel}} = \frac{2\left(2.548\times10^{-4}\,\text{J}\right)}{28\,\text{V}} = \boxed{1.8\times10^{-5}\,\text{C}}$$

$$Q_{\text{series}} = \frac{2\left(5.428\times10^{-5}\,\text{J}\right)}{28\,\text{V}} = \boxed{3.9\times10^{-6}\,\text{C}}$$

Electric Current, Resistance, Ohm's Law:

7. A service station charges a battery using a current of 6.7 A for 5.0 h. How much charge passes through the battery?

Answer:

Use the definition of current, and the equation below:

$$I = \frac{\Delta Q}{\Delta t} \quad \rightarrow \quad \Delta Q = I\Delta t = (6.7\,\text{A})(5.0\,\text{h})(3600\,\text{s/h}) = \boxed{1.2\times 10^{5}\,\text{C}}$$

8. A bird stands on a dc electric transmission line carrying 3100 A as shown in the figure below. The line has 2.5×10^{-5} Ω resistance per meter, and the bird's feet are 4.0 cm apart. What is the potential difference between the bird's fleet?

Figure 25.34

Answer:

Find the potential difference from the resistance and the current.

$$R = (2.5\times 10^{-5}\,\Omega/\text{m})(4.0\times 10^{-2}\,\text{m}) = 1.0\times 10^{-6}\,\Omega$$

$$V = IR = (3100\,\text{A})(1.0\times 10^{-6}\,\Omega) = \boxed{3.1\times 10^{-3}\,\text{V}}$$

Resistivity:

9. How much would you have to raise the temperature of a copper wire (originally at 20°C) to increase its resistance by 15%?

Answer:

Use the equation below multiplied by l/A so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] = 1.15R_0 \quad\rightarrow\quad 1+\alpha\left(T-T_0\right) = 1.15 \quad\rightarrow$$

$$T - T_0 = \frac{0.15}{\alpha} = \frac{0.15}{0.0068\left(^\circ\mathrm{C}\right)^{-1}} = \boxed{22\,^\circ\mathrm{C}}$$

So raise the temperature by 22°C to a final temperature of 42°C.

10. Compute the voltage drop along a 26 m length of household no. 14 copper wire (used in 15 A circuits). The wire has diameter 1.628 mm and carries a 12 A current.

Answer:

Calculate the voltage drop by combining Ohm's Law with the expression for resistivity:

$$V = IR = I\frac{\rho l}{A} = I\frac{4\rho l}{\pi d^2} = \left(12\,\mathrm{A}\right)\frac{4\left(1.68\times10^{-8}\,\Omega\cdot\mathrm{m}\right)\left(26\,\mathrm{m}\right)}{\pi\left(1.628\times10^{-3}\,\mathrm{m}\right)^2} = \boxed{2.5\,\mathrm{V}}$$

Electric Power:

11. (a) Determine the resistance of, and current through, a 75 W lightbulb connected to its proper source voltage of 110V. (b) Repeat for a 440 W bulb.

Answer:

Find the resistance to find the current.

(*a*) $$P = \frac{V^2}{R} \quad\rightarrow\quad R = \frac{V^2}{P} = \frac{\left(110\,\mathrm{V}\right)^2}{75\,\mathrm{W}} = 161.3\,\Omega \approx \boxed{160\,\Omega}$$

$$P = IV \quad\rightarrow\quad I = \frac{P}{V} = \frac{75\,\mathrm{W}}{110\,\mathrm{V}} = 0.6818\,\mathrm{A} \approx \boxed{0.68\,\mathrm{A}}$$

(*b*) $$P = \frac{V^2}{R} \quad\rightarrow\quad R = \frac{V^2}{P} = \frac{\left(110\,\mathrm{V}\right)^2}{440\,\mathrm{W}} = 27.5\,\Omega \approx \boxed{28\,\Omega}$$

$$P = IV \quad\rightarrow\quad I = \frac{P}{V} = \frac{440\,\mathrm{W}}{110\,\mathrm{V}} = \boxed{4.0\,\mathrm{A}}$$

Alternating Current:

12. A heater coil connected to a 240 V_{rms} ac line has a resistance of 44 Ω. (a) What is the average power used? (b) What are the maximum and minimum values of the instantaneous power?

Answer:

(a) The average power used can be found from the resistance and the rms voltage by using the equations below:

$$\bar{P} = \frac{V_{rms}^2}{R} = \frac{(240\,\text{V})^2}{44\,\Omega} = 1309\,\text{W} \approx \boxed{1300\,\text{W}}$$

(b) The maximum power is twice the average power, and the minimum power is 0.

$$P_{max} = 2\bar{P} = 2(1309\,\text{W}) \approx \boxed{2600\,\text{W}} \qquad P_{min} = \boxed{0\,\text{W}}$$