

Exercise 6.1

1. Assume L_1 is regular, $\exists n_0 \in \mathbb{N}$

Consider $w = a^{n_0} b^m c^{n_0+m} \in L$

Since $|w| = 2n_0 + 2m \geq n_0$, by pumping lemma, $w = xyz$.

$\therefore |xy| \leq n_0$, and $|y| > 0$

$\therefore y$ must contain at least one a , let $y = a^{n_1}$, $n_1 > 0$

By pumping lemma, $\forall k \in \mathbb{N}$, $xy^kz \in L$

take $k=0$, $xz = a^{n_0-n_1} b^m c^{n_0+m} \notin L$, $n_1 > 0$

\therefore We have a contradiction. L_1 is not regular.

2. Assume L_2 is regular, $\exists n \in \mathbb{N}$.

Consider $w = c^n b^{2n} a^{4n} \in L$.

Since $|w| = 7n \geq n$, by pumping lemma, $w = xyz$.

$\therefore |xy| \leq n$ and $|y| > 0$.

$\therefore y$ contains at least 1 c . Let $y = c^{n_0}$, $n_0 > 0$

$\therefore xy^kz \in L$

Let $k=0$, $xz = c^{n-n_0} b^{2n} a^{4n} \notin L$, $n_0 > 0$

\therefore We have a contradiction. L_2 is not regular.

Exercise 7.1

$$1. a) : S \xRightarrow[G]{ } X \xRightarrow[G]{ } \varepsilon \quad \text{or} \quad S \xRightarrow[G]{ } Y \xRightarrow[G]{ } \varepsilon$$

$$b) \quad S \xRightarrow[G]{ } X \xRightarrow[G]{ } aXb \xRightarrow[G]{ } aaXbb \xRightarrow[G]{ } aabb$$

$$c) \quad S \xRightarrow[G]{ } Y \xRightarrow[G]{ } cYd \xRightarrow[G]{ } ccYdd \xRightarrow[G]{ } cccYddd \xRightarrow[G]{ } cccddd$$

2. No.

S could either derive into X or Y.

X could only derive into X with more a and b.

Y could only derive into Y with more c and d.

\therefore S could not derive with both a and d.

$$3. \quad L(G) = \{a^n b^n \mid n \in \mathbb{N}\} \cup \{c^n d^n \mid n \in \mathbb{N}\}$$

Exercise 7.2

$$G = (\{S, A, B, C, D\}, \{a, b, c, d\}, P, S)$$

$$P = \{S \rightarrow AC \mid D$$

$$A \rightarrow abAba \mid abBba$$

$$B \rightarrow bc \bar{c}b \mid bcBcb$$

$$C \rightarrow \varepsilon \mid dC$$

$$D \rightarrow dd \mid dD\}$$

S: the start nonterminal that produces AC or D

A: produces at least one pair of ab and ba on each side, or produces B

B: produces at least one pair of bc and cb on each side

C: produces any number of d, and concatenates with A

D: produces at least 2 'd' s