Foundation Calculus and Mathematical Techniques (CELEN037)

Problem Sheet 5   Topics: Techniques of integration

Topic 1: Simple Integration

1. Evaluate the following integrals:

(i) $\displaystyle\int \left(\frac{3}{x} + \sqrt[3]{x} - \frac{4}{x} \right) \, dx$

(ii) $\displaystyle\int \left(\frac{x^2 + 3x - 2}{\sqrt{x}} \right) \, dx$

(iii) $\displaystyle\int \frac{x^3 - 1}{x - 1} \, dx$

(iv) $\displaystyle\int \left(2e^x + \frac{6}{x} + \ln 2 \right) \, dx$

(v) $\displaystyle\int \frac{1}{\tan 2x \cdot \sin 2x} \, dx$

(vi) $\displaystyle\int \sin^2 x \, dx$

(vii) $\displaystyle\int \sin \left(\frac{6}{x} \right) \cdot \cos^2 \left(\frac{x}{6} \right) \, dx$

(viii) $\displaystyle\int (x^3 - 2x^2) \left(\frac{1}{x} - 5 \right) \, dx$

Topic 2: The method of substitution for integration

2. Evaluate the following integrals by using appropriate substitutions:

(i) $\displaystyle\int (3x + 2)^4 \, dx$

(ii) $\displaystyle\int \sin(7x - 3) \, dx$

(iii) $\displaystyle\int x^3 \cdot \cos(x^4 - 1) \, dx$

(iv) $\displaystyle\int \frac{1}{7x + 5} \, dx$

(v) $\displaystyle\int \frac{e^{3\sqrt{x}}}{\sqrt{x}} \, dx$

(vi) $\displaystyle\int \frac{4x}{\sqrt{2x^2 + 1}} \, dx$

(vii) $\displaystyle\int e^{\sec x} \cdot \sec x \cdot \tan x \, dx$

(viii) $\displaystyle\int 2x \cdot e^{x^2 - 5} \, dx$

(ix) $\displaystyle\int \sec^2 x \cdot \sqrt{\tan x} \, dx$

(x) $\displaystyle\int \frac{\cos x}{1 + \sin x} \, dx$

(xi) $\displaystyle\int (x + 3)^2 (x - 5)^5 \, dx$

(xii) $\displaystyle\int \frac{1}{\sqrt{x} \cdot (1 + \sqrt{x})^2} \, dx$

(xiii) $\displaystyle\int \frac{5^x}{(5^x + 1)^3} \, dx$

(xiv) $\displaystyle\int e^{\cos x} \cdot \sin x \, dx$

(xv) $\displaystyle\int \frac{(\sqrt{x} + 3)^3 - 27}{x} \, dx$

(xvi) $\displaystyle\int x \cdot \sin(2x^2) \, dx$

Topic 3: More standard integrals

3. Evaluate the following integrals:

(i) $\displaystyle\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 9}}\ dx$

(ii) $\displaystyle\int \frac{\cot x}{\sqrt{\sin^2 x - 16}}\ dx$

(iii) $\displaystyle\int \frac{e^{2x}}{1 + e^{4x}}\ dx$

(iv) $\displaystyle\int \frac{3x^3}{1 + x^8}\ dx$

(v) $\displaystyle\int \frac{\sec x \cdot \tan x}{\sqrt{3 - \sec^2 x}}\ dx$

(vi) $\displaystyle\int \frac{1}{x \cdot (\ln x + 3)(\ln x - 3)}\ dx$

(vii) $\displaystyle\int \frac{3^x}{1 - 9^x}\ dx$

(viii) $\displaystyle\int \frac{1}{\sqrt{e^{2x} - 3}}\ dx$

Topic 4: Integrals of the form $\displaystyle\int f(ax + b)\ dx$

4. Evaluate the following integrals:

(i) $\displaystyle\int \sec(5x - 1) \cdot \tan(5x - 1)\ dx$

(ii) $\displaystyle\int \sin 2x \cdot \cos x\ dx$

(iii) $\displaystyle\int \frac{1}{9x^2 + 24x + 15}\ dx$

(iv) $\displaystyle\int \cos 14x \cdot \cos 3x\ dx$

(v) $\displaystyle\int e^{1-2x}\ dx$

(vi) $\displaystyle\int \sec^2(-5x - 2)\ dx$

Answers

1. (i) $3\ln|x| + \dfrac{3}{4}x^{\frac{4}{3}} - 8\sqrt{x} + C$

 (ii) $\dfrac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} - 4\sqrt{x} + C$

 (iii) $\dfrac{1}{3}x^3 + \dfrac{1}{2}x^2 + x + C$

 (iv) $2e^x + 6\ln|x| + x\cdot\ln 2 + C$

 (v) $-\dfrac{1}{4}(\cot x + \tan x) + C$

 (vi) $\dfrac{1}{2}x - \dfrac{1}{4}\sin 2x + C$

 (vii) $-\dfrac{1}{2}\cos\left(\dfrac{x}{2}\right) - \dfrac{3}{2}\cos\left(\dfrac{x}{6}\right) + C$

 (viii) $-\dfrac{5}{4}x^4 + \dfrac{11}{3}x^3 - x^2 + C$

2. (i) $\dfrac{(3x+2)^5}{15} + C$

 (ii) $-\dfrac{1}{7}\cos(7x-3) + C$

 (iii) $\dfrac{1}{4}\sin(x^4 - 1) + C$

 (iv) $\dfrac{1}{7}\ln|7x+5| + C$

 (v) $\dfrac{2}{3}e^{3\sqrt{x}} + C$

 (vi) $2\sqrt{2x^2+1} + C$

 (vii) $e^{\sec x} + C$

 (viii) $e^{x^2-5} + C$

 (ix) $\dfrac{2}{3}(\tan x)^{\frac{3}{2}} + C$

 (x) $\ln|1 + \sin x| + C$

 (xi) $\dfrac{1}{8}(x-5)^8 + \dfrac{16}{7}(x-5)^7 + \dfrac{32}{3}(x-5)^6 + C$　　(xii) $-\dfrac{2}{1+\sqrt{x}} + C$

 (xiii) $-\dfrac{\ln 5}{2(5^x+1)^2} + C$

 (xiv) $-e^{\cos x} + C$

 (xv) $\dfrac{2}{3}(\sqrt{x}+3)^3 + 3(\sqrt{x}+3)^2 + 18(\sqrt{x}+3) + C$

 (xvi) $-\dfrac{1}{4}\cos(2x^2) + C$

3. (i) $\ln\left|\tan x + \sqrt{\tan^2 x - 9}\right| + C$

 (ii) $\dfrac{1}{4}\sec^{-1}\left(\dfrac{\sin x}{4}\right) + C$

 (iii) $\dfrac{1}{2}\tan^{-1}(e^{2x}) + C$

 (iv) $\dfrac{3}{4}\tan^{-1}(x^4) + C$

 (v) $\sin^{-1}\left(\dfrac{\sec x}{\sqrt{3}}\right) + C$

 (vi) $\dfrac{1}{6}\ln\left|\dfrac{\ln x - 3}{\ln x + 3}\right| + C$

 (vii) $\ln 3 \cdot \ln\left|\dfrac{3^x + 1}{3^x - 1}\right| + C$

 (viii) $\dfrac{1}{\sqrt{3}}\sec^{-1}\left|\dfrac{e^x}{\sqrt{3}}\right| + C$

4.   (i) $\dfrac{1}{5}\sec(5x-1)+C$ (ii) $-\dfrac{1}{6}\cos 3x - \dfrac{1}{2}\cos x + C$

    (iii) $\dfrac{1}{6}\ln\left|\dfrac{3x+3}{3x+5}\right| + C$ (iv) $\dfrac{1}{34}\sin 17x + \dfrac{1}{22}\sin 11x + C$

    (v) $-\dfrac{1}{2}e^{1-2x} + C$ (vi) $-\dfrac{1}{5}\tan(-5x-2) + C$