COMP2054 Tutorial Session 4: Recurrence Relations

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Session outcomes

- Solve recurrence relations to provide exact solutions.
- Use induction to prove recurrence relation definitions.

Exact Solutions

Resolving exact solutions from recurrence relations

Q1. $T(n) = T(n-1) + 1$ and $T(1) = 1$

Q2. $T(n) = 2 \cdot T(n-1)$ and $T(1) = 1$

Q3. $T(n) = 2 \cdot T(n/2)$ and $T(1) = 1$

Resolve the exact solutions for the following:

- Q4. $T(n) = 3 \cdot T(n-1)$ and $T(1) = 1$
- Q5. $T(n) = 3 \cdot T(n/3)$ and $T(1) = 1$
- Q6. $T(n) = 2 \cdot T(n/4)$ and $T(1) = 1$

Recurrence Proofs

Proving exact solutions are the same as their recursive definitions

Q1. Proof

Given: $T(n) = T(n-1) + 1$ and $T(1) = 1$

Prove: $T(n) = n$

Q2. Proof

Given: $T(n) = 2 \cdot T(n-1)$ and $T(1) = 1$

Prove: $T(n) = 2^{n-1}$

Recurrence proofs

- Q3. Given $T(n) = 2 \cdot T(n/2)$ and $T(1) = 1$

  Prove that $T\left(2^k\right) = 2^k$
- Q4. Given $T(n) = 3 \cdot T(n-1)$ and $T(1) = 1$

  Prove that $T(n) = 3^{n-1}$
- Q5. $T(n) = 3 \cdot T(n/3)$ and $T(1) = 1$

  Prove that $T\left(3^k\right) = 3^k$
- Q6. $T(n) = 2 \cdot T(n/4)$ and $T(1) = 1$

  Prove that $T\left(4^k\right) = 2^k$

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Additional Practice Questions

For each of the following:

1. Find the exact solution
2. Prove by induction

Assume you are given $T(1) = 1$

- Q7. $T(n) = 4 \cdot T(n/4)$
- Q8. $T(n) = 4 \cdot T(n/2)$
- Q9. $T(n) = T(n-1) + n$
- Q10. $T(n) = 2 \cdot T(n/2) + 1$
- Q11. $T(n) = n \cdot T(n-1)$

Thank you