



## Foundation Calculus and Mathematical Techniques (CELEN037)

### Problem Sheet 9

### Topics: Definite Integrals

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#### Topic 1: Area Calculation Using Definite Integrals

1. Find the area of the region bounded by the given curves.

- (i)  $y = x^2$ , lines  $x = 3$ ,  $x = 4$  and the  $X$ -axis.
- (ii)  $y = x^3 + 3$ , lines  $x = 2$ ,  $x = 4$  and the  $X$ -axis.
- (iii)  $x = 4y - y^2$ , lines  $y = 1$ ,  $y = 3$  and the  $Y$ -axis.
- (iv)  $x = 1 + y^2$  and line  $x = 10$ .
- (v)  $x = 3y^2 - 9$ , lines  $y = 0$ ,  $y = 1$  and the  $Y$ -axis.
- (vi)  $x = y^2 + 4y$  and line  $x = 0$ .
- (vii)  $y = \sin x$ , lines  $x = 0$ ,  $x = 2\pi$  and the  $X$ -axis.
- (viii)  $y = \cos x$ , lines  $x = 0$ ,  $x = 2\pi$  and the  $X$ -axis.
- (ix)  $y = x^2 - 4$  and  $y = 8 - 2x^2$ .
- (x)  $x = y^2 - 6$  and  $x = -y + 6$ .

#### Topic 2: Volume Calculation Using Definite Integrals

2. Find the volume of solid of revolution when region bounded by the following curve(s) is revolved about the given axis/line. Use any appropriate method.

- (i) The region bounded by  $y = 2x^2$ , lines  $x = 0$ ,  $x = 5$  and the  $X$ -axis is revolved about the  $X$ -axis.
- (ii) The region bounded by  $y = e^{2x}$ , lines  $x = 1$ ,  $x = 2$  and the  $X$ -axis is revolved about the  $X$ -axis.
- (iii) The region bounded by  $x^2 - y^2 = 16$ , line  $x = 8$  and the  $X$ -axis is revolved about the  $X$ -axis.
- (iv) The region bounded by  $x = y^3$ , lines  $y = 1$ ,  $y = 2$  and the  $Y$ -axis is revolved about the  $Y$ -axis.
- (v) The region bounded by  $x = \cos y$ , lines  $y = 0$ ,  $y = \pi$  and the  $Y$ -axis is revolved about the  $Y$ -axis.
- (vi) The region bounded by  $y = 4x^2$ , line  $y = 16$  and the  $X$ -axis is revolved about line  $y = 16$ .

- (vii) The region bounded by  $y = x^3$ ,  $y = \sqrt[3]{x}$ , lines  $x = 0$  and  $x = 1$  is revolved about the  $X$ -axis.
  - (viii) The region bounded by  $y = e^{-x^2}$ , lines  $x = 0$ ,  $x = 1$  and the  $X$ -axis is revolved about the  $Y$ -axis.
  - (ix) The region bounded by  $y = 3x^2$  and  $y = 3x$  is revolved about the  $X$ -axis.
  - (x) The region bounded by  $x^2 + y^2 = 9$  and the  $Y$ -axis is revolved about the  $Y$ -axis.
3. Consider the region  $R$  bounded by  $y = -x^2 - 3x + 6$  and  $x + y = 3$ . Find the volume of the solid generated by revolving  $R$  about:
- (i) the  $X$ -axis;
  - (ii) line  $y = 2$ .

### Topic 3: Numerical Integration

4. Evaluate  $\int_0^1 x^3 dx$  by using:
- (a) Trapezoidal rule with 10 sub-intervals of equal width;
  - (b) Simpson's rule with 10 sub-intervals of equal width.

Give approximation to 4 d.p.

5. Evaluate  $\int_1^2 x^3 \sqrt{x^5 + 2x^2 - 1} \, dx$  by using:
- (a) Trapezoidal rule with 4 sub-intervals of equal width;
  - (b) Simpson's rule with 4 sub-intervals of equal width.

Give approximation to 6 d.p.

## Answers

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1. (i)  $\frac{37}{3}$

(ii) 66

(iii)  $\frac{22}{3}$

(iv) 36

(v) 8

(vi)  $\frac{32}{3}$

(vii) 4

(viii) 4

(ix) 32

(x)  $\frac{343}{6}$

2. (i)  $2500\pi$

(ii)  $\frac{(e^8 - e^4)\pi}{4}$

(iii)  $\frac{256\pi}{3}$

(iv)  $\frac{127\pi}{7}$

(v)  $\frac{\pi^2}{2}$

(vi)  $\frac{8192\pi}{15}$

(vii)  $\frac{16\pi}{35}$

(viii)  $\left(1 - \frac{1}{e}\right)\pi$

(ix)  $\frac{6\pi}{5}$

(x)  $36\pi$

3. (i)  $\frac{1792\pi}{15}$

(ii)  $\frac{384\pi}{5}$

4. (a) 0.2525

(b) 0.2500

5. (a) 16.561138

(b) 15.926800