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Foundation Algebra (CELEN036)

Problem Sheet 11 **Topics: Series**

Topic 1: Finding the n^{th} term

1. Write down the first three terms of the following series:

   (i) $\displaystyle\sum_{r=1}^{\infty} \frac{x^{2r}}{(2r-1)(2r+1)}$

   (ii) $\displaystyle\sum_{r=0}^{\infty} \frac{(-1)^{r-1}\, 2^{2r}\, x^{2r-1}}{(2r)!}$

   (iii) $\displaystyle\sum_{n=1}^{\infty} 2^{n/2} \cdot \sin\left(\frac{n\pi}{4}\right) \cdot \frac{x^n}{n!}$

2. From the following formula for the series S_n, obtain the formula for the corresponding sequence:

   (i) $S_n = n^3 - 2n$ (ii) $S_n = \dfrac{1-2^n}{3}$ (iii) $S_n = \dfrac{a(r^n-1)}{r-1}, \quad r \neq 1$

3. Find an Arithmetic Progression (A.P.) the sum of whose first n terms is $2n^2 + n$.

Topic 2: Arithmetic Series

4. If the sixth and the tenth term of an A.P. are 23 and 39 respectively, find a_{16} and S_{19}.

5. The eighth term of an A.P. is 5 and the sum of the first 14 terms is 49. Find the first term.

6. Obtain an A.P. whose fourth term is 4 and the sum of the first eight terms is $\dfrac{2}{5}$ times the sum of the first four terms.

7. The sequence obtained by taking successive differences of $4,\ 6,\ 11,\ 19,\ 30, \cdots$ (for example, $6-4=2$, $11-6=5$) is an A.P. Find the sequence and the sum of the first n terms.

8. Find the sum of all the integers between 100 and 600 that are multiples of 11.

9. If the sums of the first n, $2n$, and $3n$ terms of an A.P. are k_1, k_2, and k_3 respectively, prove that $k_3 = 3(k_2 - k_1)$.

Topic 3: Geometric Series

10. If $a = 25$, $r = \dfrac{1}{5}$, and $a_n = \dfrac{1}{625}$, find n and S_4.

11. If $r = \frac{1}{3}$ and $S_4 = 150$, find the first term a.

12. If $a = 16$ and $a_5 = 81$, find $r > 0$ and S_3.

13. If for a geometric sequence, $a_2 = 6$ and $a_5 = 48$, find S_5.

14. The second term of a G.P. is $\frac{1}{4}$ and the sum of the first 4 terms is $\frac{1}{16}$ times the sum of the next tour terms. Find the G.P.

15. Find the sum of the following infinite geometric series:
    (i) $\frac{1}{4} + \frac{1}{20} + \frac{1}{100} + \cdots$ (ii) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \cdots$

Topic 4: Power series

16. Find the sum of the integers from 1 to 1000.

17. Find the sum: $1 + 3 + 5 + 7 + 9 + 11 + \cdots$ (up to n terms).

18. Find the sum: $1 + (1 + 3) + (1 + 3 + 5) + \cdots$ (up to n terms).

19. Find the sum: $1 + (3 + 5) + (7 + 9 + 11) + \cdots$ (up to n terms).

20. Find the sum: $1 \cdot 3 \cdot 7 + 2 \cdot 5 \cdot 11 + 3 \cdot 7 \cdot 15 + \cdots$ (up to n terms).

Answers

1. (i) $\frac{x^2}{3}, \quad \frac{x^4}{15}, \quad \frac{x^6}{35}$ (ii) $-\frac{1}{x}, \quad 2x, \quad -\frac{2x^3}{3}$ (iii) $x, \quad x^2, \quad \frac{x^3}{3}$

2. (i) $a_n = 3n^2 - 3n - 1$ (ii) $-\frac{2^{n-1}}{3}$ (iii) $a\, r^{n-1}$

3. $a_n = 4n - 1$

4. $a_{16} = 63, \quad S_{19} = 741$

5. $a = -16$

6. $a_n = -4n + 20$

7. $a_n = 3n - 1, \quad S_n = \frac{n(3n+1)}{2}$

8. 15840

10. $n = 7, \quad S_4 = \frac{156}{5}$

11. $\dfrac{405}{4}$

12. $r = \dfrac{3}{2}, \quad S_3 = 76$

13. 93

14. $a_n = \dfrac{2^n}{16}$ or $a_n = \dfrac{(-2)^n}{16}$

15. (i) $\dfrac{5}{16}$ (ii) $\dfrac{2}{3}$

16. 500500

17. n^2

18. $\dfrac{n(n+1)(2n+1)}{6}$

19. $\dfrac{n^2(n+1)^2}{4}$

20. $\dfrac{1}{6}\, n\,(n+1)(12n^2+32n+19)$