CELEN037 Seminar 3

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Topics

- Parametric Differentiation
- Maclaurin's Series
- Equations of Tangent and Normal Lines
- Newton-Raphson Method
- Increasing and Decreasing Functions

Example: Given $x = a(t - \sin t)$ and $y = a(1 - \cos t)$ (a is a constant), find $\left.\dfrac{dy}{dx}\right|_{t=\frac{\pi}{2}}$.

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$$\begin{aligned} \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}} &= \frac{\sin \frac{\pi}{2}}{1 - \cos \frac{\pi}{2}} \\ &= \frac{1}{1 - 0} \\ &= 1 \end{aligned}$$

Practice Problems on Worksheet:

1: Q1(ii)
2: Q1(iv)
3: Q1(v)
4: Q1(vi)

Practice Problems on Worksheet:

1: Q1(ii)

2: Q1(iv)

3: Q1(v)

4: Q1(vi)

Answers:

1: -1

2: -1

3: $\dfrac{v_0 \sin\alpha - gt}{v_0 \cos\alpha}$

4: $-\dfrac{2t}{1-t^2}$

Example: Obtain the Maclaurin's Series expansions of the following function:

$$f(x) = \frac{1}{1-x}; \quad -1 < x < 1$$

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$$f''(x) = \frac{-2}{(1-x)^3} \cdot (-1) = \frac{2}{(1-x)^3} \implies f''(0) = 2$$

Example: Obtain the Maclaurin's Series expansions of the following function:

$$f(x) = \frac{1}{1-x}; \quad -1 < x < 1$$

Solution:

$$\begin{aligned}
f(0) &= 1 \\
f'(x) &= \frac{-1}{(1-x)^2} \cdot (-1) = \frac{1}{(1-x)^2} \implies f'(0) = 1 \\
f''(x) &= \frac{-2}{(1-x)^3} \cdot (-1) = \frac{2}{(1-x)^3} \implies f''(0) = 2 \\
f'''(x) &= \frac{-6}{(1-x)^4} \cdot (-1) = \frac{6}{(1-x)^4} \implies f'''(0) = 6
\end{aligned}$$

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f'''(x) &= \frac{-6}{(1-x)^4} \cdot (-1) = \frac{6}{(1-x)^4} \implies f'''(0) = 6 \\
f^{(4)}(x) &= \frac{-24}{(1-x)^5} \cdot (-1) = \frac{24}{(1-x)^5} \implies f^{(4)}(0) = 24
\end{aligned}$$

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$$\Rightarrow \frac{1}{1-x} = 1 + x \cdot (1) + \frac{x^2}{2!} \cdot (2) + \frac{x^3}{3!} \cdot (6) + \frac{x^4}{4!} \cdot (24) + \cdots$$

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&= 1 + x + x^2 + x^3 + x^4 + \cdots
\end{aligned}$$

Practice Problems on Workbook:

1: Q2(i)
2: Q2(iii)
3: Q2(vi)
4: Q2(vii)

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Practice Problems on Workbook:

1: Q2(i)
2: Q2(iii)
3: Q2(vi)
4: Q2(vii)

Answers:

1: $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

2: $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} + \cdots$

3: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$

4: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$

Example: The equation of the curve is given by $y = \frac{1}{x}$. Obtain the euqation of the tangent line and the normal line to the curve at point $(-1, -1)$.

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$$\left.\frac{dy}{dx}\right|_{x=-1} = -\frac{1}{(-1)^2} = -\frac{1}{1} = -1 = m$$

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$$y + x + 2 = 0$$

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$$y + x + 2 = 0$$

Equation of normal line: $y - y_1 = -\dfrac{1}{m}(x - x_1)$

$$y = x$$

Practice Problems on Workbook:

1: Q3(ii)
2: Q3(iii)

Practice Problems on Workbook:

1: Q3(ii)

2: Q3(iii)

Answers:

1: Tangent line $y - 2x + 3 = 0$

Normal line $2y + x + 1 = 0$

2: Tangent line $2y + 3x - 6 = 0$

Normal line $3y - 2x - 9 = 0$

Newton-Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (n = 0, 1, 2, 3, \cdots)$$

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Solution:

$f(x) = x^4 - \sin x - 1; \quad f'(x) = 4x^3 - \cos x;$

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Steps on calculator

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Steps on calculator

(i) Set calculator to RADIAN mode:

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Steps on calculator

(i) Set calculator to RADIAN mode: **Shift** **Mode** **4**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (n = 0, 1, 2, 3, \cdots)$$

Example: Use the Newton-Raphson method to approximate the root of $x^4 - \sin x = 1$, correct to 7 d.p., by starting with $x_0 = 1.5$.

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$f(x) = x^4 - \sin x - 1; \quad f'(x) = 4x^3 - \cos x; \quad x_{n+1} = x_n - \frac{x_n^4 - \sin x_n - 1}{4x_n^3 - \cos x_n}$

Steps on calculator

(i) Set calculator to RADIAN mode: **Shift** **Mode** **4**

(ii) Fix calculator to **7** digits:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (n = 0, 1, 2, 3, \cdots)$$

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Steps on calculator

(i) Set calculator to RADIAN mode: **Shift** **Mode** **4**

(ii) Fix calculator to **7** digits: **Shift** **Mode** **6** **7**

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(iii) Press **1.5** and $\boxed{=}$ on calculator

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(iv) Enter the formula on calculator as:

$$\textbf{Ans} - \frac{(\textbf{Ans}^4 - \sin(\textbf{Ans}) - 1)}{(4 \times \textbf{Ans}^3 - \cos(\textbf{Ans}))}$$

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(v) Press $\boxed{=}$ button successively, and write down all x_n in a table

$$x_{n+1} = x_n - \frac{f(x_n)}{\ldots} \quad (\ldots, 1, 2, 3, \cdots)$$

Calculation results

n	x_n
0	1.5000000
1	1.2717667
2	1.1885303
3	1.1778677
4	1.1777035
5	1.1777035

Example: Use the Newto... ...pproximate the root of $x^4 - \sin x = 1$, correct to ... $x_0 = 1.5$.

Solution:

$f(x) = x^4 - \sin x - 1;$ ... $x_{n+1} = x_n - \frac{x_n^4 - \sin x_n - 1}{4x_n^3 - \cos x_n}$

Steps on calculator

(i) Set calculator to RA... mode: **Shift** **Mo**...

(ii) Fix calculator to **7** d... **Shift** **Mode** **6** **7**

(iii) Press **1.5** and $\boxed{=}$ on calculator $(x_0 = 1.5)$

... the formula on calculator

$$\frac{(\textbf{Ans}^4 - \sin(\textbf{Ans}) - 1)}{(4 \times \textbf{Ans}^3 - \cos(\textbf{Ans}))}$$

(v) Press $\boxed{=}$ button successively, and write down all x_n in a table

$$x_{n+1} = x_n - \frac{f(x_n)}{\ldots} \quad (\ldots 1, 2, 3, \cdots)$$

Example: Use the Newto… …pproximate the root of $x^4 - \sin x = 1$, correct to … $x_0 = 1.5$.

Solution:

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n	x_n
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1	1.2717667
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Thus, the root is $x^* = 1.1777035$

Steps on calculator

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (n = 0, 1, 2, 3, \cdots)$$

Practice Problems on Workbook:

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2: Q4(iii)
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Practice Problems on Workbook:

1: Q4(ii)

2: Q4(iii)

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4: Q4(vi)

Answers:

1: $x = 1.895494$

2: $x = 1.021690$

3: $x = -1.272020$

4: $x = 2.09455148$

Example: Show that $f(x) = e^{-2x} + 1$ is always decreasing.

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Therefore, $x \in (-\infty, -\sqrt{\frac{5}{3}})$ and $(\sqrt{\frac{5}{3}}, +\infty)$ $\quad f$ is increasing;

$x \in (-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}})$ $\quad f$ is decreasing.

Practice Problems on Workbook:

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2: Q5(ii)
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4: Q5(vi)

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Answers:

1: $f' = \sec^2 x > 0$ for all $x \in \mathbb{R}$

2: $f' = -\sin x > 0$ for all $x \in$ Fourth Quadrant

3: $2x + 2 < 0$ for all $x \in (-\infty, -1)$

4: $x \in (-\infty, \frac{-3\sqrt{73}-9}{16})$, f is decreasing; $x \in (\frac{-3\sqrt{73}-9}{16}, 0)$, f is increasing; $x \in (0, \frac{3\sqrt{73}-9}{16})$, f is decreasing; $x \in (\frac{3\sqrt{73}-9}{16}, +\infty)$, f is increasing;

Office Hours

Day	Time	Venue
Tuesday	12:00 to 14:00	Trent 314a
	13:00 to 14:00	Trent 322
Thursday	16:30 to 17:30	TB 417
	17:00 to 18:00	IAMET 315
Friday	14:00 to 15:00	PB 330
	17:00 to 18:00	TB 417

Weekly drop-in Session: Wednesdays 16:00-17:00 at PB115+.