FOUNDATION SCIENCE A

SEMINAR 6: THE ELECTRIC FLUX & ELECTRIC POTENTIALS

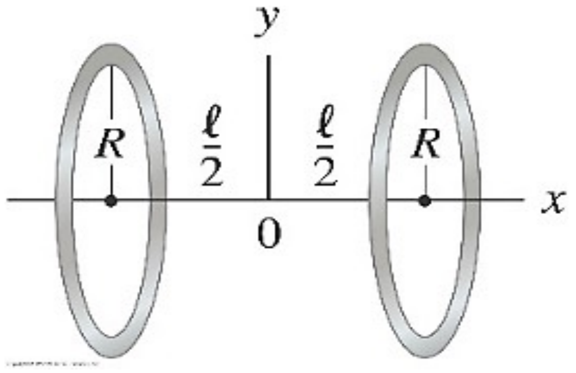
University of Nottingham
UK | CHINA | MALAYSIA

LEARNING OUTCOMES

- To be able to understand the use of **Gauss's Law** in terms of solving a variety of cases for the electric flux, electric field and electric potentials.
- To be able to analyse the equipotential problems for a given charge.

QUESTION 10 (Seminar 5):

Two parallel circular rings of radius R have their centers on the x axis separated by a distance l as shown in the figure below. If each ring carries a uniformly distributed charge Q, find the electric field, $\vec{E}(x)$, at points along the x axis.



Answer:

- Consider Example 21-9 in chapter 21 of your physics textbook.
- Use the result from this example, but shift the center of the ring to be at $\boldsymbol{x} = {}^{l}\!/_{2}$ **for the ring on the right**, and at $\boldsymbol{x} = {}^{l}\!/_{2}$ **for the ring on the left**.
- The fact that the original expression has a factor of x results in the interpretation that the sign of the field expression will give the direction of the field. No special consideration needs to be given to the location of the point at which the field is to be calculated.

QUESTION 10 (Seminar 5):

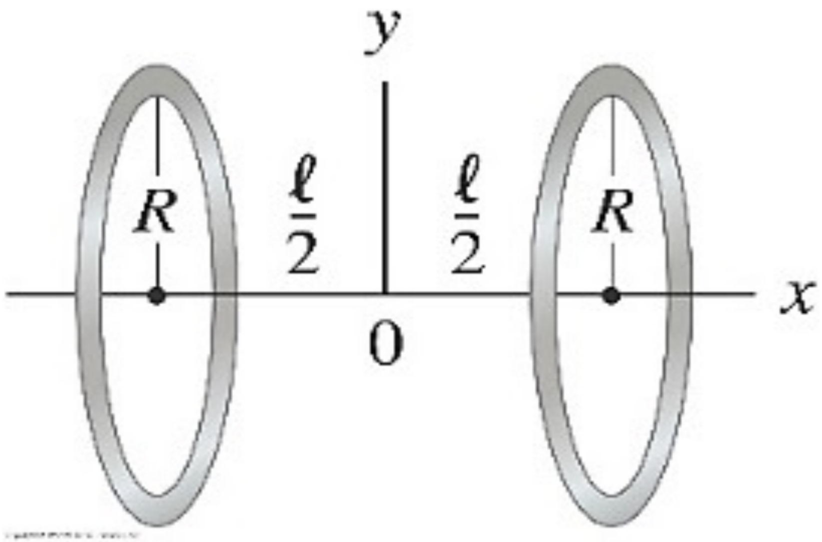
Two parallel circular rings of radius R have their centers on the x axis, separated by a distance, l. If each ring carries a uniformly distributed charge Q, find the electric field, $\vec{E}(x)$, at points along the x axis.

Answer: So, electric field at any point **along x-axis** will be:

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_{\text{right}} + \vec{\boldsymbol{E}}_{\text{left}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q\left(x-\frac{1}{2}l\right)}{\left[\left(x-\frac{1}{2}l\right)^2+R^2\right]^{3/2}}\right)\hat{\mathbf{i}} + \frac{1}{4\pi\varepsilon_0}\left(\frac{Q\left(x+\frac{1}{2}l\right)}{\left[\left(x+\frac{1}{2}l\right)^2+R^2\right]^{3/2}}\right)\hat{\mathbf{i}}$$

$$= \hat{\mathbf{i}}\frac{Q}{4\pi\varepsilon_0}\left\{\frac{\left(x-\frac{1}{2}l\right)}{\left[\left(x-\frac{1}{2}l\right)^2+R^2\right]^{3/2}} + \frac{\left(x+\frac{1}{2}l\right)}{\left[\left(x+\frac{1}{2}l\right)^2+R^2\right]^{3/2}}\right\}$$



Electric Flux:

QUESTION 1:

The Earth possesses an electric field of (average) magnitude 150 $N.C^{-1}$ near its surface. The field points radially inward.
Calculate the net electron flux outward through a spherical surface surrounding, and just beyond, the Earth's surface.

Answer:

Use the equation below for the electric flux of a uniform field. Note that the surface area vector points radially outward, and the electric field vector points radially inward. Thus, the angle between the two is 180°.

$$\phi_E = \vec{E}.\vec{A} \quad = E.A.\cos\theta$$

$$=(150\ N.C^{-1})\times 4\pi \times R_E{}^2 \times \cos 180°$$

$$\mathbf{= -7.7 \times 10^{16}\ N.m^2.C^{-1}}$$

QUESTION 2:

The total electric flux from a cubical box 28.0 cm on a side is 1.84 × 103 $\mathrm{N \cdot m^2/C}$. What charge is enclosed by the box?

Answer:

Use Gauss's law to determine the enclosed charge.

$$\phi_E = \frac{Q_{encl}}{\varepsilon_0}$$

$$Q_{encl} = \phi_E . \varepsilon_0$$

$$= (1840\ \mathrm{N.m^2.C^{-1}})\times(8.85\times10^{-12}\ \mathrm{C^2.N^{-1}.m^{-2}})$$

$$= \mathbf{1.63\times10^{-8}\ C}$$

Applications of Gauss's Law:

QUESTION 3:

A flat square sheet of thin aluminum foil, 25 cm on a side, carries a uniformly distributed 275 nC charge. What, approximately, is the electric field

a) 1.0 cm above the centre of the sheet and
b) 15 m above the centre of the sheet?

Answer:

a) When close to the sheet, approximate it to an infinite sheet, using the following equation. Assume the charge is over both surfaces of the aluminum.

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{\dfrac{275 \times 10^{-9}\ \mathrm{C}}{(0.25\ \mathrm{m})^2}}{2(8.85 \times 10^{-12}\ \mathrm{C^2.N^{-1}.m^2})} = 2.5 \times 10^5\ \mathrm{N.C^{-1}}, \text{ away from the sheet}$$

b) When far from the sheet, approximate it as a point charge.

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = (8.988 \times 10^9\ \mathrm{N.m^2.C^{-2}}).\frac{275 \times 10^{-9}\ \mathrm{C}}{(15\ \mathrm{m})^2} = 11\ \mathrm{N.C^{-1}}, \text{ away from the sheet}$$

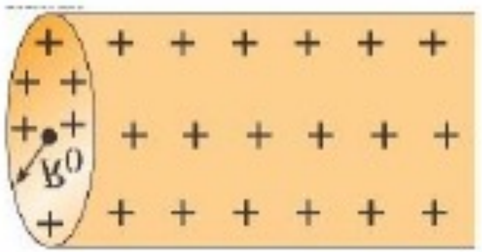
Determination of Capacitance

QUESTION 4:

A very long solid non-conducting cylinder of radius R_0 , and length l (R_0 << l) possesses a uniform volume charge density ρ_E (Cm^{-3}). Determine the electric field at points

a) outside the cylinder ($R > R_0$) and

b) inside the cylinder ($R < R_0$). Do only for points far from the ends and for which $R << l$.



Answer:

Use the following expression

$$\oint \boldsymbol{E}\cdot \boldsymbol{dA} = E(2\pi R l) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho_E V_{encl}}{\varepsilon_0} \rightarrow E = \frac{\rho_E V_{encl}}{2\pi\varepsilon_0 R l}$$

a) For $R > R_0$ the enclosed volume of the shell is

$$V_{encl} = \pi . R_0^2 . l \quad = \frac{\rho_E V_{encl}}{2\pi\varepsilon_0 R l} \quad = \frac{\rho_E . R_0^2}{2\varepsilon_0 R}$$, radially downwards

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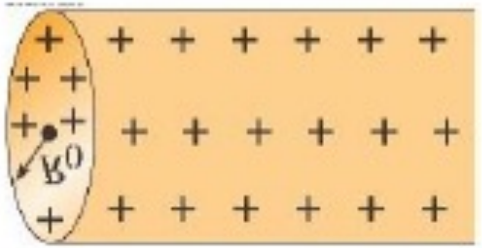
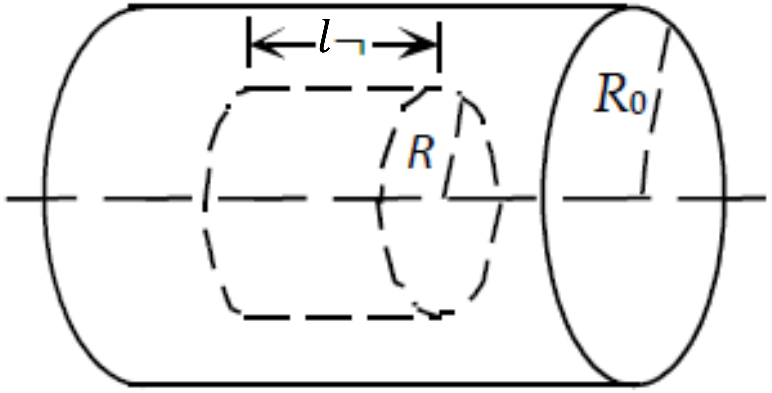


a) outside the cylinder ($R > R_0$) and
b) inside the cylinder ($R < R_0$). Do only for points far from the ends and for which $R << l$.

Answer:

b) For $R < R_0$ the enclosed volume of the shell is $V_{encl} = \pi . R^2 . l$

$$E = \frac{\rho_E . V_{encl}}{2\pi\varepsilon_0 R l} = \frac{\rho_E . R}{2\varepsilon_0} \text{, radially outwards}$$



Electric Potential

QUESTION 5:

How much work does the electric field do in moving a proton from a point with a potential of +185 V to a point where it is -55 V?

Answer:

The work done by the electric field can be found by using the following equation.

$$V_{ba} = -\frac{W_{ba}}{q} \qquad W_{ba} = -q.V_{ba}$$

$$= -(1.60 \times 10^{-19}\ \mathrm{C}).[-55\ \mathrm{V} - 185\ \mathrm{V}]$$

$$\mathbf{= 3.84 \times 10^{-17}\ J}$$

Electric Potential

QUESTION 6:

The work done by an external force to move a -9.10 μC charge from point *a,* to point *b,* is 7.00 $\times 10^{-4}$ J. If the charge was started from rest and had 7.00 $\times 10^{-4}$ J of kinetic energy when it reached point *b*, what must be the potential difference between *a* and *b*?

Answer

By the work energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by the following equation

$$W_{ext} + W_{elec} = KE_{final} - KE_{initial}$$

$$W_{ext} - q(V_b - V_a) = KE_{final}$$

$$(V_b - V_a) = \frac{W_{ext} - KE_{final}}{q} = \frac{7.00 \times 10^{-4}\ \mathrm{J} - 2.10 \times 10^{-4}\ \mathrm{J}}{-9.10 \times 10^{-6}\ \mathrm{C}} = \mathbf{-53.8\ V}$$

Since the potential difference is negative, note that $V_a > V_b$

Potential Related to Electric Field

QUESTION 7:

The electric field between two parallel plates connected to a 45 V battery is 1300 $V.m^{-1}$. How far apart are the plates?

Answer:

The distance between the plates is found, using the magnitude of the electric field.

$$|E| = \frac{V_{ba}}{d}$$

$$\mathrm{d} = \frac{V_{ba}}{|E|}$$

$$= \frac{45\ \mathrm{V}}{1300\ \mathrm{V.m^{-1}}}$$

$$\mathbf{= 3.5 \times 10^{-2}\ m}$$

Potential Related to Electric Field

QUESTION 8:
A manufacturer claims that a carpet will not generate more than 5.0 kV of static electricity. What magnitude of charge would have to be transferred between a carpet and a shoe for there to be a 5.0 kV potential difference between the shoe and the carpet? Approximate the shoe and the carpet as large sheets of charge separated by a distance d = 1.0 mm.

Answer:
Assume the electric field is uniform, then use the equation below to estimate the magnitude of the electric field. Use the expression for the electric field due to a pair of oppositely charged planes. Approximate the area of a shoe as 30 cm x 8 cm.

$$E = \frac{V}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 . A}$$

$$Q = \frac{\varepsilon_0 . A . V}{d} = \frac{(8.85 \times 10^{-12} \mathrm{C}^2 . \mathrm{N}^{-1} . \mathrm{m}^2)(0.024\ \mathrm{m}^2)(5.0 \times 10^{-3}\ \mathrm{V})}{1.0 \times 10^{-3}\ \mathrm{m}}$$

$$= 1.1 \times 10^{-6}\ \mathrm{C}$$

Potential Related to Electric Field

Question 9:

A point charge *Q* creates an electric potential of +185 V at a distance of 15 cm. What is *Q* (let V = 0 at r = ∞)?

Answer:

Use the equation below to find the charge.

$$V = \frac{1}{4\pi\varepsilon_0} . \frac{Q}{r^2}$$

$$Q = 4.\pi.\varepsilon_0.r.V$$

$$= \frac{1}{8.988 \times 10^9\ \mathrm{N.m^2.C^{-2}}} \times 0.15\ \mathrm{m} \times 185\ \mathrm{V}$$

$$= \mathbf{3.1 \times 10^{-9}\ C}$$

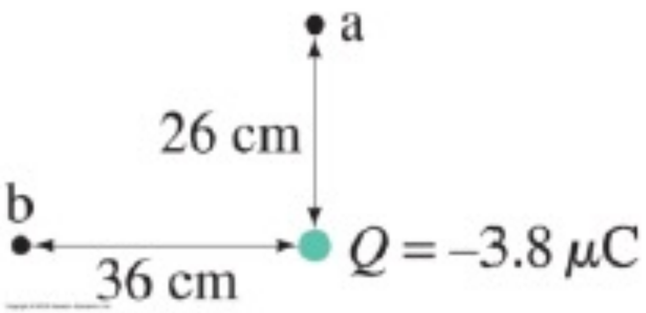
Potential Due to Point Charges

Question 10:

Point a is 26 cm north of a -3.8 μC point charge, and point b is 36 cm west of the charge. Determine

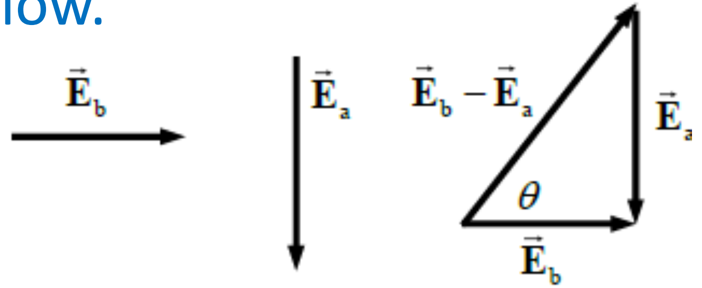
a) V_b - V_a and

b) $\vec{E}_b$- $\vec{E}_a$ (magnitude and direction).



Answer:

a) The potential due to a point charge is given by the equation below.



$$V_{ba} = V_b - V_a = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r_b} - \frac{1}{4\pi\varepsilon_0}.\frac{q}{r_a} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$= 8.988 \times 10^9\ \text{N.m}^2.\text{C}^{-2} \times (-3.8 \times 10^{-6}\ \text{C}) \times \left(\frac{1}{0.36\ \text{m}} - \frac{1}{0.26\ \text{m}}\right)$$

$$= \mathbf{3.6 \times 10^4\ V}$$

b) The direction of the electric field due to a negative charge is towards the charge, so the field at point a will point downward, and the field at point b will point to the right.

Potential Due to Point Charges:

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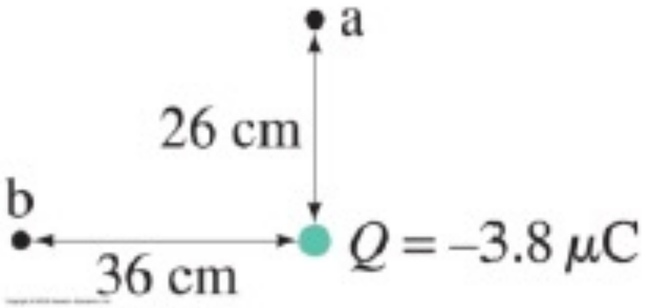
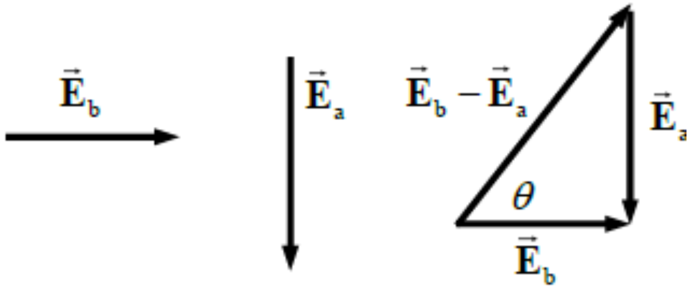


Answer:

$$\vec{E}_b = \frac{1}{4\pi\varepsilon_0}.\frac{|q|}{r_b^2}.\hat{\mathbf{i}} = \frac{8.988\times10^{9}\ \mathrm{N.m^2.C^{-2}}\ (3.8\times10^{-6}\ \mathrm{C})}{(0.36\ \mathrm{m})^2}.\hat{\mathbf{i}} = 2.636\times10^{5}\ \mathrm{V.m^{-1}}.\ \hat{\mathbf{i}}$$

$$\vec{E}_a = -\frac{1}{4\pi\varepsilon_0}.\frac{|q|}{r_a^2}.\hat{\mathbf{j}} = -\frac{8.988\times10^{9}\ \mathrm{N.m^2.C^{-2}}\ (3.8\times10^{-6}\ \mathrm{C})}{(0.26\ \mathrm{m})^2}.\hat{\mathbf{j}} = -5.054\times10^{5}\ \mathrm{V.m^{-1}}.\ \hat{\mathbf{j}}$$



$$\vec{E}_b - \vec{E}_a = 2.636\times10^{5}\ \mathrm{V.m^{-1}}.\ \hat{\mathbf{i}} + 5.054\times10^{5}\ \mathrm{V.m^{-1}}.\ \hat{\mathbf{j}}$$

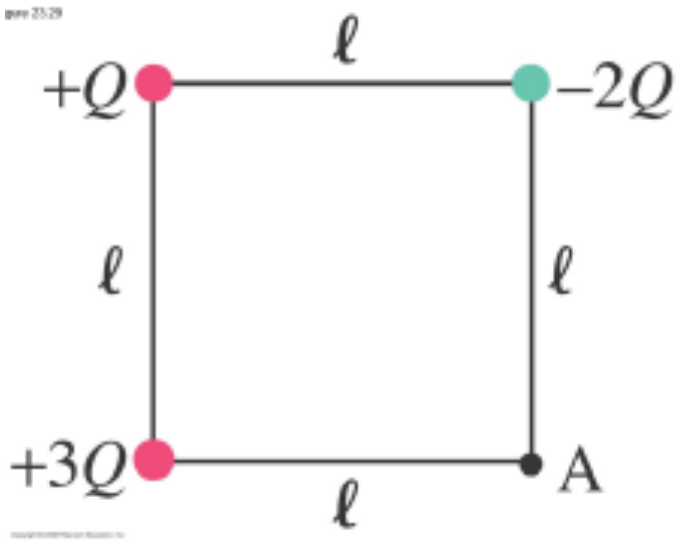
$$\left|\vec{E}_b - \vec{E}_a\right| = \sqrt{(2.636\times10^{5}\ \mathrm{V.m^{-1}})^2 + (5.054\times10^{5}\ \mathrm{V.m^{-1}})^2} = 5.7\times10^{5}\ \mathrm{V.m^{-1}}$$

$$\theta = \tan^{-1}\frac{-E_a}{E_b} = \tan^{-1}\frac{5.054\times10^{5}}{2.636\times10^{5}} = 62°$$

Potential Due to Charge Distribution:

QUESTION 11:

Three point charges are arranged at the corners of a square of side, l. What is the potential at the fourth corner (point A), taking $V = 0$ at a great distance?



Answer:

The potential at the corner is the sum of the potentials due to each of the charges, using the equation below.

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{3Q}{l} + \frac{1}{4\pi\varepsilon_0}.\frac{Q}{\sqrt{2}l} + \frac{1}{4\pi\varepsilon_0}.\frac{-2Q}{l}$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{Q}{l}.\left(1 + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{\sqrt{2}Q}{2l}.(\sqrt{2} + 1)$$

Equipotentials:

QUESTION 12:

A metal sphere of radius, r_0 = 0.44 m, carries a charge Q = 0.50 μC. Equipotential surfaces are to be drawn for 100 V intervals outside the sphere.

Determine the radius *r*, of

a) the first,

b) the tenth, and

c) the 100th equipotential from the surface.

Answer:

The potential at the surface of the sphere is $V_0 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_0}$

The potential outside the sphere is $V = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} = V_0.\frac{r_0}{r}$ and decreases as you move away from the surface.

Equipotentials:

QUESTION 12:

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Determine the radius r, of

a) the first,
b) the tenth, and
c) the 100th equipotential from the surface.

Answer:

The difference in potential between a given location and the surface is to be a multiple of 100 V.

$$V_0 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_0} = 8.988 \times 10^9 \text{ N.m}^2.\text{C}^{-2} \times \left(\frac{0.50 \times 10^{-6}\text{C}}{0.44 \text{ m}}\right) = 10.216 \text{ kV}$$

$$V_0 - V = V_0 - V_0.\frac{r_0}{r} = (100\ V)n \qquad r = \frac{V_0}{[V_0 - (100\text{ V})n]}.\, r_0$$

Equipotentials:

QUESTION 12:

Determine the radius *r,* of

a) the first,

b) the tenth, and

c) the 100th equipotential from the surface.

Note that to within the appropriate number of significant figures, this location is at the surface of the sphere. That can be interpreted that we don't know the voltage well enough to be working with a 100-V difference.

$$r_1 = \frac{V_0}{[V_0-(100\text{ V})n]}.\, r_0 = \frac{10{,}216\text{ V}}{[10{,}116\text{ V}]} \times 0.44\text{ m} \quad \mathbf{= 0.444\ m}$$

$$r_{10} = \frac{V_0}{[V_0-(100\text{ V})n]}.\, r_0 = \frac{10{,}216\text{ V}}{[9{,}216\text{ V}]} \times 0.44\text{ m} \quad \mathbf{= 0.49\ m}$$

$$r_{100} = \frac{V_0}{[V_0-(100\text{ V})n]}.\, r_0 = \frac{10{,}216\text{ V}}{[216\text{ V}]} \times 0.44\text{ m} \quad \mathbf{= 21\ m}$$

Q&A?
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