The University of Nottingham

School of Computer Science

A Level 2 Module, Spring Semester 2018-2019

Exam of Introduction to Image Processing (COMP2047)

Time Allowed: 1.5 Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer ALL questions

No calculators are permitted in this examination

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject-specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

INFORMATION FOR INVIGILATORS:
Collect both the exam papers and the answer booklets at the end of the exam.

Q 1: Define the first order derivative of a one-dimensional signal $f(x)$ as

$$\frac{\partial f}{\partial x} = f(x+1) - f(x),$$

(i) derive the second-order derivative of $f(x)$; (4)

(ii) verify that the second order derivative of $f(x)$ can be interpreted as filtering $f(x)$ with filter $h = [1, -2, \underline{1}]$, whose zero index is denoted by the position of the underlined element; (4)

(iii) calculate the Discrete-time Fourier Transform (DTFT) of h, state whether filter h is a low-pass filter or not, and justify your statement. (5)

Q 2: The pixel values of a small 5 x 5 image patch are shown in the following table. Calculate the output of the center (underlined) pixel for the following spatial filtering operations.

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & \underline{4} & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{array}$$

(i) 3 x 3 Laplacian with the following filtering mask; (4)

$$\begin{array}{ccc} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{array}$$

(ii) 3 x 3 average filtering; (4)

(iii) is the following filtering mask a reasonable Laplacian filtering mask? Explain your answer as well. (5)

$$\begin{array}{ccc} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{array}$$

Q 3: This problem is about the thresholding operator which is widely used in image processing applications, for example in image de-noising and image segmentation. The following equation defines the hard thresholding operator $H_s(y)$ (10)

$$H_s(y) = \begin{cases} y & \text{if } |y| > s; \\ 0 & \text{if } |y| \leq s, \end{cases}$$

where $s > 0$ is a predefined threshold value. Though intuitively straightforward, the hard thresholding operator can also be justified as the optimizer to the following problem

$$\underset{x}{\text{minimize}}\ (x-y)^2 + s^2|x|_0,$$

where

$$|x|_0 = \begin{cases} 1 & \text{if } x \neq 0; \\ 0 & \text{otherwise.} \end{cases}$$

Please prove this statement.

Q 4: This problem is about histogram equalization. The following fact is the key ingredient for deriving the histogram equalization algorithm. Let r be a positive random variable with probability density function given as $p_r(r)$. Define the cumulative density function $T(r)$ of $p_r(r)$ as

$$T(r) = \int_0^r p_r(w)\mathrm{d}w,$$

then it is true that $s = T(r)$ is uniformly distributed within interval $[0, 1]$. Please prove this statement. (12)

Q 5: This question is about Huffman coding which can be exploited for image compression. Suppose that we have five grey levels denoted as $\{a, b, c, d, e\}$, and their frequencies appearing in the image are given as $\{0.87, 0.04, 0.04, 0.03, 0.02\}$. Apply Huffman coding scheme to encode different grey levels, and calculate the average length of the obtained Huffman code. (12)