University of Nottingham
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Foundation Calculus and Mathematical Techniques (CELEN037)

Answers to Worksheet #2

1.

(i) $-e^x \sin(e^x)$

(ii) $-\cos(\cos x)\sin x$

(iii) $-\sin(\sin x)\cos x$

(iv) $\dfrac{\sec^2(\ln x)}{x}$

(v) $\tan x$

(vi) $-\dfrac{\cos\Big(\cos(\ln x)\Big)\sin(\ln x)}{x}$

(vii) $-\cos\Big(\ln(\cos x)\Big)\tan x$

(viii) $-\dfrac{\sec^2\Big(\cos(\sqrt{x})\Big)\sin(\sqrt{x})}{2\sqrt{x}}$

(ix) $e^x \cot(e^x)$

2.

(i) $-e^x \tan(e^x)$

(ii) $-\dfrac{\cos\Big(\cos(\ln x)\Big)\sin(\ln x)}{x}$

(iii) $-\cos\Big(\ln(\cos x)\Big)\tan x$

(iv) $-\dfrac{\sec^2\Big(\cos(\sqrt{x})\Big)\sin(\sqrt{x})}{2\sqrt{x}}$

(v) $e^x \cot(e^x)$

(vi) $-\dfrac{\cos(e^{\cos x})\, e^{\cos x}\sin x}{2\sqrt{\sin(e^{\cos x})}}$

3.

(i) $(\tan x)^{\sin x}\Big(\sec x + \ln(\tan x)\cos x\Big)$

(ii) $(\cos x)^{\sin x}\Big(\ln(\cos x)\cos x - \sin x\tan x\Big)$

(iii) $(\sin x)^{\cos x}\Big(\cos x\cot x - \ln(\sin x)\sin x\Big)$

(iv) $(\cos x)^{x}\Big(\ln(\cos x) - x\tan x\Big)$

(v) $x^{\cos x}\left(\dfrac{\cos x}{x} - \sin x\ln x\right)$

(vi) $(\ln x)^{\tan x}\left(\ln(\ln x)\sec^2 x + \dfrac{\tan x}{x\ln x}\right)$

(vii) $x^x(\ln x + 1)$

(viii) $\dfrac{\sqrt[x]{x}(1-\ln x)}{x^2}$

(ix) $\cos(x^x)\, x^x(\ln x + 1)$

(x) $\dfrac{\sqrt[3]{x}\tan^4 x}{\cos(e^x)}\left(\dfrac{1}{3x} + \dfrac{4\sec^2 x}{\tan x} + e^x\tan(e^x)\right)$

4.

(i) $\dfrac{x^2 - y}{x - y^2}$

(ii) $-\dfrac{\sin(x+y)+2x}{\sin(x+y)+2y}$

(iii) $-\dfrac{2y\sin(xy)\sqrt{x+y}+1}{2x\sin(xy)\sqrt{x+y}+1}$

(iv) $\dfrac{2x - y\cos(xy)}{2y + x\cos(xy)}$

(v) $-\dfrac{y^2}{x^2}$

(vi) $\dfrac{x - y\left(2+\sec^2(xy)\right)\sqrt{x^2-y^2}}{y + x\left(2+\sec^2(xy)\right)\sqrt{x^2-y^2}}$

(vii) $\dfrac{2y^2-2xy-x}{x^2-4xy+y}, \quad \left.\dfrac{dy}{dx}\right|_{(1,1)} = \dfrac{1}{2}$

(viii) $\dfrac{dy}{dx} = 1, \quad \left.\dfrac{dy}{dx}\right|_{(0,0)} = 1$

(ix) $\dfrac{3x^2-2xy-y^2}{x^2+2xy-3y^2}, \quad \left.\dfrac{dy}{dx}\right|_{(1,-1)} = -1$

(x) $\dfrac{-2xy^3+5y^2+3y-2}{3x^2y^2-10xy-3x}, \quad \left.\dfrac{dy}{dx}\right|_{(2,1)} = -\dfrac{1}{7}$

5.

(i) $-\dfrac{1}{|x|\sqrt{x^2-1}}$

(ii) $\dfrac{1}{1+x^2}$

(iii) $\dfrac{1}{2\left(1+x^2\right)}$

(iv) $\dfrac{1}{2}$

(v) $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$

(vi) $\dfrac{2(1-x^2)}{x^4+6x^2+1}$

(vii) $\dfrac{2}{1+x^2}$

(viii) (a) -2 (b) 0

Solution to Problem 3(x).

$$\ln y = \ln \frac{\sqrt[3]{x}\cdot\tan^4 x}{\cos\left(e^x\right)} = \ln\sqrt[3]{x} + \ln\left(\tan^4 x\right) - \ln\cos\left(e^x\right) = \frac{1}{3}\ln x + 4\ln(\tan x) - \ln\cos\left(e^x\right)$$

$$\Rightarrow \quad \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3}\cdot\frac{1}{x} + 4\cdot\frac{1}{\tan x}\cdot\sec^2 x - \frac{1}{\cos\left(e^x\right)}\cdot\left(-\sin\left(e^x\right)\right)\cdot e^x = \frac{1}{3x} + \frac{4\sec^2 x}{\tan x} + e^x\tan\left(e^x\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left(\frac{1}{3x} + \frac{4\sec^2 x}{\tan x} + e^x\tan\left(e^x\right)\right) = \frac{\sqrt[3]{x}\cdot\tan^4 x}{\cos\left(e^x\right)}\left(\frac{1}{3x} + \frac{4\sec^2 x}{\tan x} + e^x\tan\left(e^x\right)\right)$$

Solution to Problem 5(viii).

$$\frac{d}{dx}\left[\sin^{-1}(\cos x) + \cos^{-1}(\sin x)\right] = \frac{d}{dx}\sin^{-1}(\cos x) + \frac{d}{dx}\cos^{-1}(\sin x)$$

$$= \frac{-\sin x}{\sqrt{1-\cos^2 x}} - \frac{\cos x}{\sqrt{1-\sin^2 x}} = \frac{-\sin x}{|\sin x|} - \frac{\cos x}{|\cos x|} = \begin{cases} -1-1=-2, \text{ if } & 0 < x < \dfrac{\pi}{2} \\ -1+1=0, \text{ if } & \dfrac{\pi}{2} < x < \pi \end{cases}$$