The University of Nottingham Ningbo China

SCHOOL OF COMPUTER SCIENCE

A LEVEL 1 MODULE, AUTUMN SEMESTER 2019–2020

MATHEMATICS FOR COMPUTER SCIENTISTS

Time allowed ONE hour

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer all THREE questions. Each question is worth 25 marks, hence the total mark is 75.

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject-specific translation directories are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

Question 1 This question is about logic and inference. (25 marks)

(a) Show that $(p \wedge q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent, using truth tables or logical equivalences laws. (2 marks)

(b) Prove that $(p \rightarrow q) \vee (q \rightarrow r)$ is a tautology, using truth tables or logical equivalences laws. (2 marks)

(c) Use the disjunctive syllogism to show that if p and $p \wedge q \rightarrow r$ are true, then $q \rightarrow r$ can be inferred. (3 marks)

(d) Consider this expression,

$$\forall x(P(x, z) \rightarrow ((\exists y Q(x, y)) \wedge R(z, x) \wedge S(x, y))) :$$

(i) What is the scope of the existential quantifier? (1 mark)

(ii) Which variables are bound? (2 marks)

(iii) Which variables are free? (2 marks)

(e) In the domain of polygons, let the predicate $P(x, y)$ be the statement "x has more sides than y", $Q(x)$ be "x is a square" and $R(x)$ be "x is a hexagon".

(i) Translate these two sentences into predicate logic: (4 marks)

[a] No square is a hexagon.

[b] All hexagons have more sides than any square.

(ii) Translate this statement into English: (2 marks)

[c] $\exists x(Q(x) \wedge \forall y(P(y, x) \rightarrow \neg Q(y)))$

(iii) Can statement [a] be inferred from statements [b] and [c]? If yes, then give a prove. If no, give a counter-example. (7 marks)

Question 2 This question is about proofs. (25 marks)

(a) Let a and b be any two real numbers. Prove that $(a+b)^2 \geq a(a+2b)$. (4 marks)

(b) Prove that for an integer n, these four statements are equivalent: (1) n is odd, (2) n^2 is odd, (3) n^2+1 is even, and (4) $3n^2+4$ is odd. (13 marks)

(c) Use mathematical induction to prove that

$$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

is true for all integers $n \geq 1$ and real numbers $r \neq 1$. (8 marks)

Question 3 This question is about combinations and probability.
(25 marks)

Please show both the final results and how they are calculated in your answer.

(a) Give Laplace's definition of the probability of an event with finitely many, equally likely, possible outcomes. (3 marks)

(b) Six fair coins are thrown together. Each has an equal chance of coming up Heads or Tails.

(i) What is the size of the sample space for the outcomes? (2 marks)

(ii) What is the probability of getting 6 Tails? (2 marks)

(iii) What is the probability of getting exactly 4 Heads? (2 marks)

(iv) What is the probability of getting less than 4 Heads? (2 marks)

(v) Suppose three of the coins are 20p coins and the other three are 10p coins. What is the probability that the total value of coins landing Heads will be 60p or more? (4 marks)

(c) Suppose at a party, the host of the party Helen has 10 small cakes, each of which she is equally likely to give to Tom. Tom is given one *and only one* cake to eat. It is either a chocolate cake, a fruit cake or a cheesecake. Further suppose the probability he gets a fruit cake is 0.2 and the probability he gets either a fruit cake or a cheesecake is 0.6.
When answering the following questions, state any probability theorems you use.

(i) What is the probability Tom does not get a chocolate cake?
(1 mark)

(ii) What is the probability Tom gets a chocolate cake or a fruit cake?
(4 marks)

(iii) What is the probability Tom gets a cheesecake? (2 marks)

(iv) After giving Tom his cake, what is the probability Helen still has two or more fruit cakes left?
(3 marks)