In [ ]:
```
import math
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [ ]:
```
def plot_oh(f, g, c, n_0, min_x, max_x):
    xs = np.arange(min_x,max_x,1, dtype='int64')
    fn_ys = f(xs)
    gn_ys = c*g(xs)
    plt.axvline(x = n_0, color = 'k', linestyle='--')
    plt.scatter(x=xs, y=fn_ys, marker='x')
    plt.scatter(x=xs, y=gn_ys, marker='.')
    plt.xlabel('n')
    plt.ylabel('f(n)')
    plt.legend(['n_0 = '+str(n_0),'f(n)', 'cg(n)'])
```

Big-Oh Definition

Given positive functions $f(n)$ and $g(n)$, we can say that $f(n)$ is $O(g(n))$ if and only if there exists positive constants c and n_0 such that $f(n) \leq c \cdot g(n), \forall n \geq n_0$.

Q1. Prove that 5 is $O(1)$

Q2. Prove that $2n + 1$ is $O(3n)$

Basic Questions

Q3. Prove that 4 is $O(2)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Q4. Prove that $2n + 1$ is $O(n)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Medium Difficulty Questions

To recall the definition of Big-Oh: Given positive functions $f(n)$ and $g(n)$, we can say that $f(n)$ is $O(g(n))$ if and only if there exists positive constants c and n_0 such that $f(n) \leq c \cdot g(n), \forall n \geq n_0$.

Q5. Prove that n^2 is $O(2n^2)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Q6. Prove that $n^2 - 3$ is $O(n^2)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Q7. Prove that $n^2 - 5n$ is $O(n^2)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Q8. Prove that $n^2 + 1$ is $O(n^2)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Conceptually Challenging Questions

To recall the definition of Big-Oh: Given positive functions $f(n)$ and $g(n)$, we can say that $f(n)$ is $O(g(n))$ if and only if there exists positive constants c and n_0 such that $f(n) \leq c \cdot g(n), \forall n \geq n_0$.

Q9. From the definitions, prove or disprove that 1 is $O(n)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Q10. From the definitions, prove or disprove that n is $O(1)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Q11. From the definitions, prove or disprove that n^2 is $O(n)$

In [ ]:
```
# c = ???
# n_0 = ???
# f = lambda n: ???
# g = lambda n: ???

plot_oh(f, g, c, n_0, 0, 50)
#plt.rcParams["figure.figsize"] = (4,4)
```

Q12. Given that $f(n) = \text{ IF } even(n) \text{ THEN } n+3 \text{ ELSE } n^2+5$ state the Big-Oh Behaviour and prove it from the definition

Algebraically Challenging

Work out the Big-Oh of the following functions and prove them using the definitions.

Q13. $3n^3 + 10000n$

Q14. $n \; log(n) + 2n$

Q15. $2^n + n$

Summary: Venn Diagram

Draw a Venn diagram of the sets $O(1)$, $O(n)$ and $O(n^2)$, and place the following functions on the diagram:

- $f1(n) = 1$
- $f2(n) = 42$
- $f3(n) = n$
- $f4(n) = 3n + 5$
- $f5(n) = n^2$
- $f6(n) = n^2 + log(n)$