University of Nottingham
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Foundation Calculus & Mathematical Techniques CELEN037

Weekly Worksheet-1 **Topics:** Derivatives using First Principles, Rules of Differentiation

Type 1: Differentiation using First Principles

1. Use first principles (definition of derivative) to find $\frac{dy}{dx}$ where,

(i) $y = x^2 + 1$ (ii) $y = (x+1)^2$ (iii) $y = \frac{1}{x^2}$

(iv) $y = \sqrt{x}$ (v) $y = \sqrt{x+1}$ (vi) $y = \frac{1}{\sqrt{x}}$

(vii) $y = a^x$ $(viii)$ $y = e^{1-x}$ (ix) $y = \cos x$

(x) $y = \tan x$ (xi) $y = \cos(x+1)$ (xii) $y = \sin 2x$

$(xiii)$ $y = \sin^2 x$ Use: $\sin^2 A - \sin^2 B = \sin(A+B) \cdot \sin(A-B)$

(xiv) $y = \sin x^2$ Use: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(xv) $y = x^n$ Use: Binomial Theorem.

Type 2: Rules of Differentiation (Sum and Difference rules)

2. Find $\frac{dy}{dx}$ for the following functions:

(i) $y = x^5 + x + 99$ (ii) $y = 5\,x - x \cdot (4x+3)$

(iii) $(x+5) \cdot (3x-1)$ (iv) $(x-2)^3$

(v) $y = x^4 - \frac{1}{x}$ (vi) $y = \frac{2x^3 + x - 3}{x^2}$

(vii) $y = x^2 - \frac{4}{\sqrt[3]{x^2}} - 1$ $(viii)$ $y = \frac{(x-2)\cdot(x-3)}{x}$

(ix) $y = \dfrac{x^2 - x - 2}{x^2 + x}$ (x) $y = \dfrac{\left(x^2 - 4\right) \cdot (x + 3)}{x^2 - 2x}$

(xi) $y = \dfrac{1}{\sqrt{x}} - e^x + x$ (xii) $y = \ln x - x^3 + \dfrac{1}{x}$

$(xiii)$ $y = \sec x - \text{cosec} x$ (xiv) $y = \dfrac{\cos x + 1}{\sin x}$

(xv) $y = \dfrac{1}{x^3} + 2\,\tan x$ (xvi) $\sec x \cdot (\cos x + 1)^2$

$(xvii)$ $y = \dfrac{\cos^2 x + \sin x \cdot (\sin x + 1)}{\cos x}$ $(xviii)$ $y = x^{-2} + 4\,\sin x + x^{2/3} - \cot x$

(xix) $y = 5x^{-\frac{1}{3}} - 3\,\cos x$ (xx) $y = \sqrt[4]{x^3} + 2\,\tan x$

(xxi) $y = \ln\left(\dfrac{x - 5}{x + 1}\right)$ $(xxii)$ $y = \sqrt{x} + 2^x - \cot x - \dfrac{1}{x}$

Type 3A: Rules of Differentiation (The Product rule)

3. Find $\dfrac{dy}{dx}$ for the following functions:

(i) $y = x^2 \cdot \cos x$ (ii) $y = x^3 \cdot \ln x$

(iii) $y = \cos x \cdot \ln x$ (iv) $y = \ln x \cdot \sec x$

(v) $y = \dfrac{1}{x} \cdot \sec x$ **Hint**: $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$ (vi) $y = \dfrac{1}{x} \cdot \sin x$ **Hint**: $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$

(vii) $y = \dfrac{1}{x} \cdot \ln x$ **Hint**: $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$ $(viii)$ $y = \sin 2x$ **Hint**: Use formula for $\sin 2\theta$

(ix) $y = \sec x \cdot (1 + \tan x)$ (x) $y = e^x \cdot \sec x$

Type 3B: Rules of Differentiation (Extension of the Product Rule)

4. Find $\dfrac{dy}{dx}$ for the following functions:

(i) $y = e^x \cdot \sin x \cdot \ln x$ (ii) $y = \tan x \cdot x^2 \cdot e^x$

(iii) $y = x^3 \cdot e^x \cdot \ln x$ (iv) $y = x \cdot \sin x \cdot \tan x$

Type 4: Rules of Differentiation (The Quotient Rule)

5. Find $\dfrac{dy}{dx}$ for the following functions:

(i) $y = \tan x = \dfrac{\sin x}{\cos x}$ (ii) $y = \dfrac{x}{\sin x}$

(iii) $y = \dfrac{\sec x}{e^x}$ (iv) $y = \dfrac{1 + \tan x}{\sec x}$

(v) $y = \dfrac{x}{\ln x}$ (vi) $y = \dfrac{x^2}{\tan x}$

(vii) $y = \dfrac{\sin x}{e^x}$ $(viii)$ $y = \dfrac{e^x}{x^3}$

(ix) $y = e^{-x} = \dfrac{1}{e^x}$ (x) $y = x \cdot e^{-x}$

(xi) $y = \dfrac{1 + x}{1 - x}$ (xii) $y = \dfrac{1 - x^2}{1 + x^2}$