COMP3055
Machine Learning

Topic 11 – Perceptron, ADLINE, and Delta Rule

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Recall: Machine Learning Concept

- Artificial Intelligence: AI is the science of making machines do things that require intelligence if done by men (Minsky 1986).

- Machine Learning is an area of AI concerned with development of techniques which allow machines to learn.

- Why Machine Learning?
  - To build machines exhibiting intelligent behaviour (i.e., able to reason, predict, and adapt) while helping humans work, study, and entertain themselves

Machine Learning Algorithms

- The success of machine learning system depends on the algorithms.
- The algorithms control the search to find and build the knowledge structures.
- The learning algorithms should extract useful information from training examples.

Machine Learning Algorithms

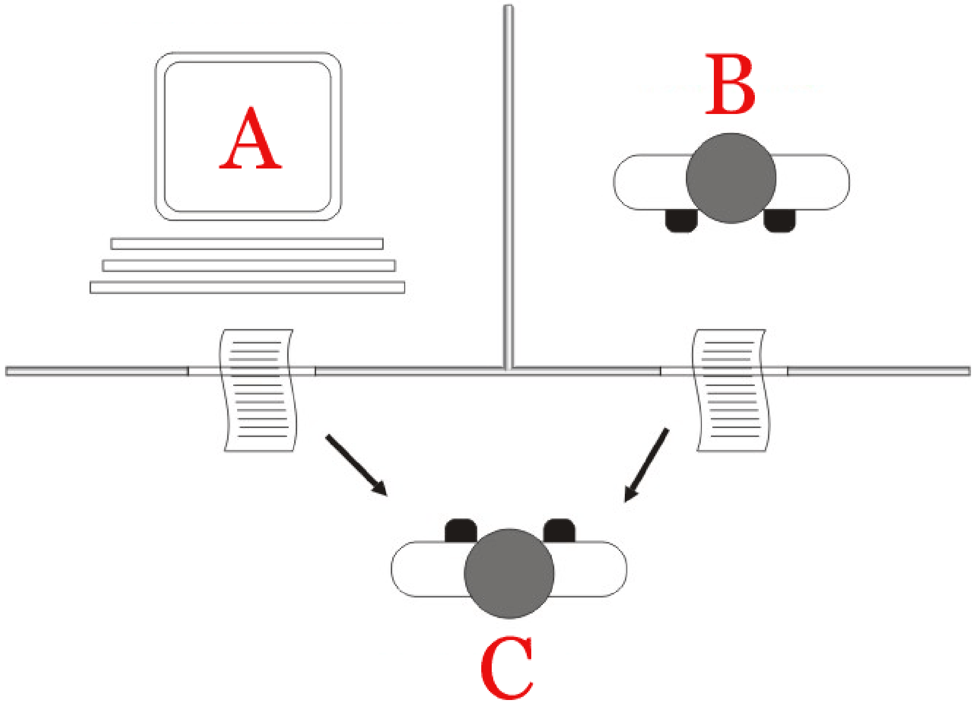
- **Supervised learning** (generates a function that maps inputs to desired outputs)
  - Prediction
  - Classification (discrete labels), Regression (real values)

- **Unsupervised learning** (models a set of inputs, labelled examples are not available)
  - Probability distribution estimation
  - Finding association (in features)
  - Dimension reduction
  - Clustering

- **Reinforcement learning** (learning by education / experience)
  - Decision making (robot, chess machine)

Artificial General Intelligence

- **Artificial General Intelligence** (AGI) is the hypothetical intelligence of a machine that has the capacity to understand or learn any intellectual task that a human being can. AGI can also be referred to as **strong AI**
- Example: Turing Test



Purposeful Problem Solver

- In contrast to strong AI, **weak AI** is not intended to perform human cognitive abilities, rather, weak AI is limited to the use of software to study or accomplish specific problem solving or reasoning tasks.
- Examples: face recognition, speech recognition, games, etc.
- Application domains: security, medicine, education, finances, genetics, etc.

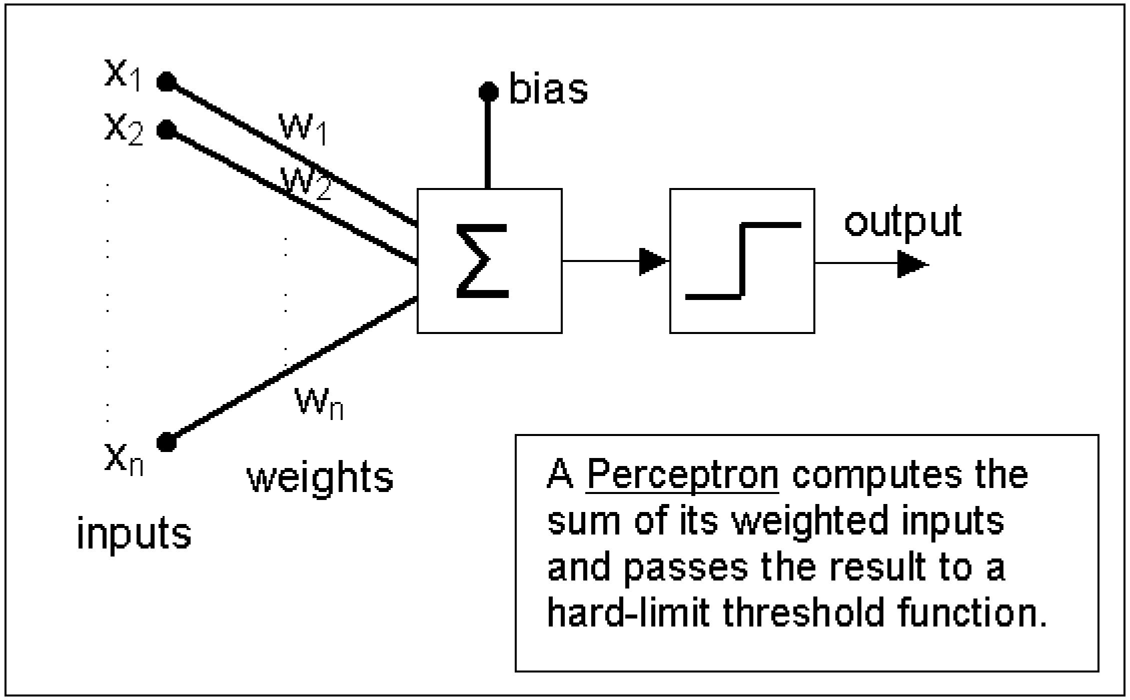
Feature Engineering

- Feature engineering is the process of using domain knowledge to extract features from raw data via data mining techniques. These features can be used to improve the performance of machine learning algorithms.

- Process:
  - Brainstorming or testing features;
  - Deciding what features to create;
  - Creating features;
  - Checking how the features work with your model;
  - Improving your features if needed;
  - Go back to brainstorming/creating more features until the work is done.

Perceptron - Basic

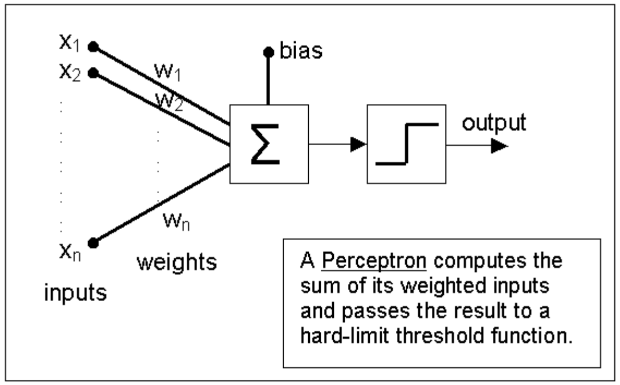
Perceptron is a type of Artificial Neural Network (ANN)



A Perceptron computes the sum of its weighted inputs and passes the result to a hard-limit threshold function.

Perceptron - Operation

Perceptron takes a vector of real-valued inputs, calculates a *linear combination* of these inputs. Then it outputs 1 if the result is greater than some threshold and -1 otherwise.



A Perceptron computes the sum of its weighted inputs and passes the result to a hard-limit threshold function.

$$R = w_0 + w_1x_1 + w_2x_2 + \cdots + w_nx_n = w_0 + \sum_{i=1}^{n} w_ix_i$$

$$output = o = sign(R) = \begin{cases} +1, & if\ R > 0 \\ -1, & otherwise \end{cases}$$

Perceptron – Decision Surface

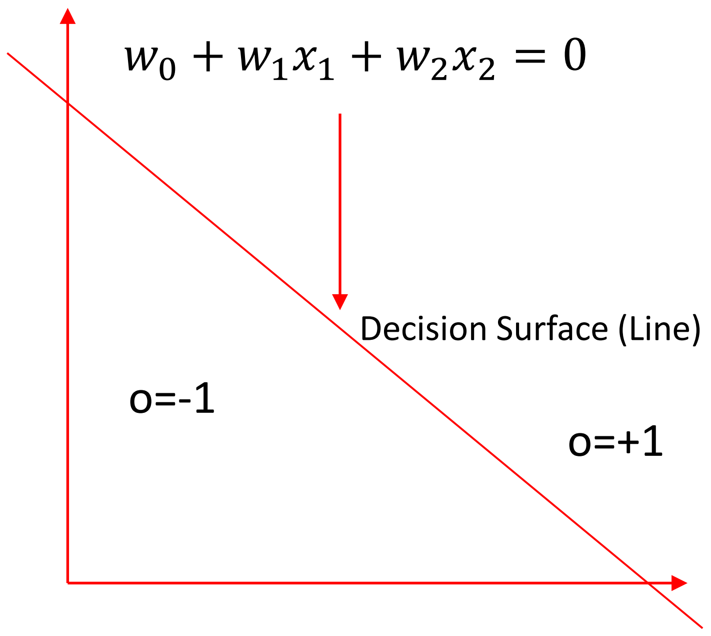
- Perceptron can be regarded as representing a hyperplane decision surface in the n-dimensional **feature space** of instances.

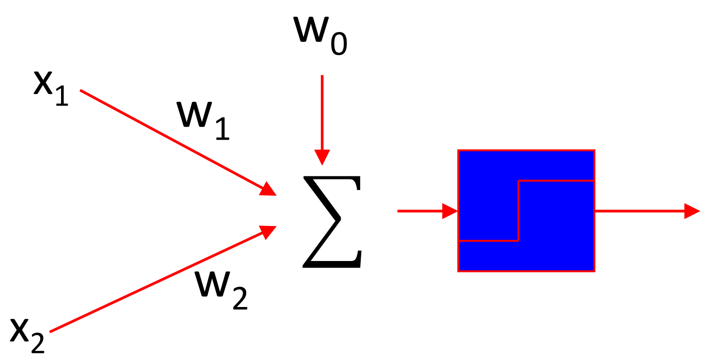
- The perceptron outputs a 1 for instances lying on one side of the hyperplane and a -1 for instances lying on the other side.

- This hyperplane is called the **Decision Surface**.

Perceptron – Decision Surface

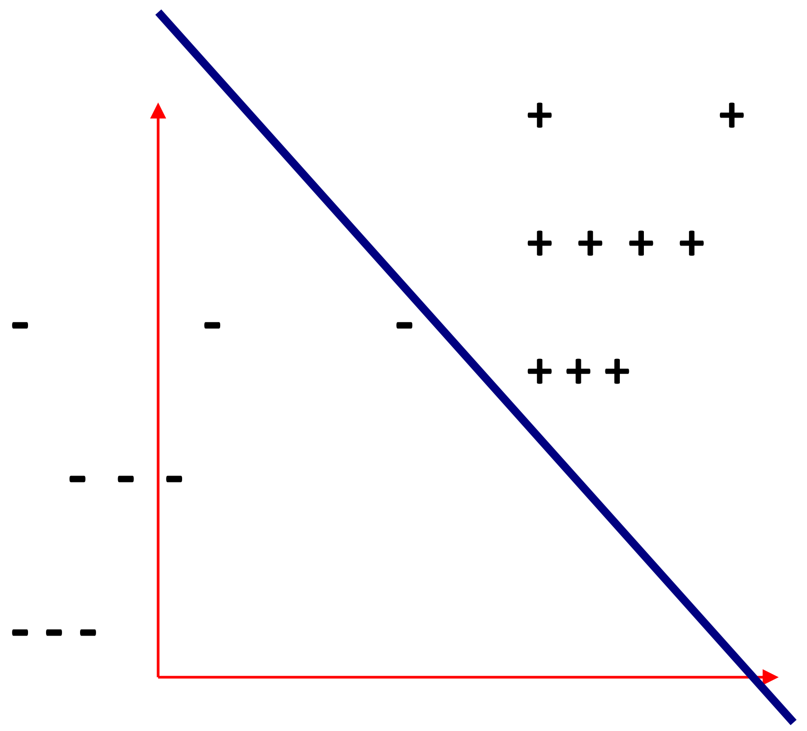
In 2-dimensional space:





Perceptron – Representation Power

- The Decision Surface is linear.
- Perceptron can only solve **Linearly Separable Problems**.

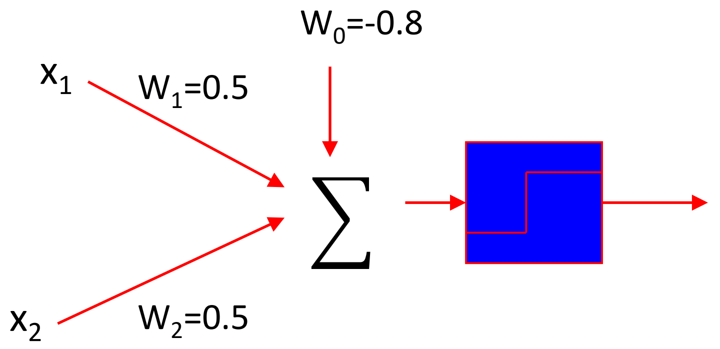


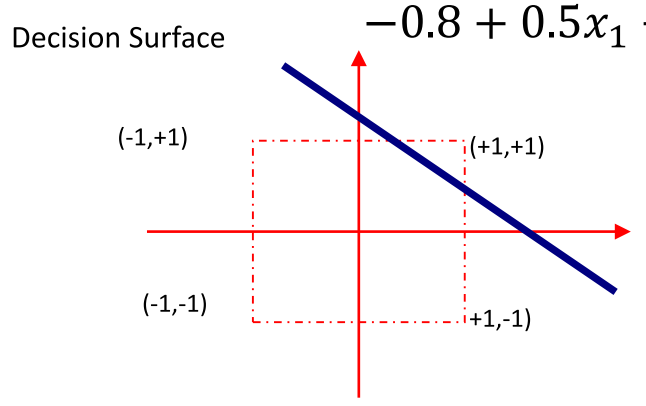
Perceptron – Representation Power

Can represent many Boolean functions, assuming Boolean values of 1 (true) and -1 (false).

AND

x1	x2	D
-1	-1	-1
-1	+1	-1
+1	-1	-1
+1	+1	+1





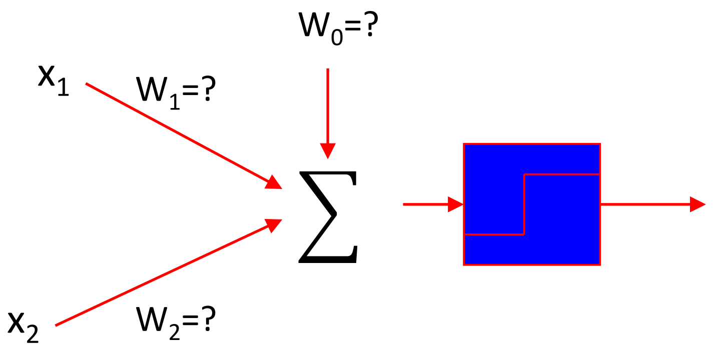
$$-0.8 + 0.5x_1 + 0.5x_2 = 0$$

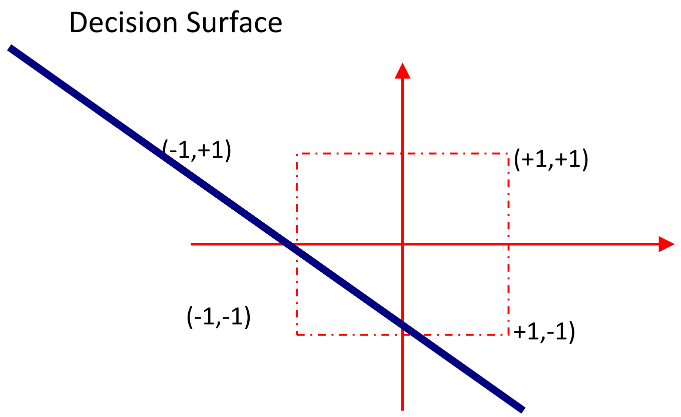
Perceptron – Representation Power

Can represent many Boolean functions, assuming Boolean values of 1 (true) and -1 (false).

OR

x1	x2	D
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	+1





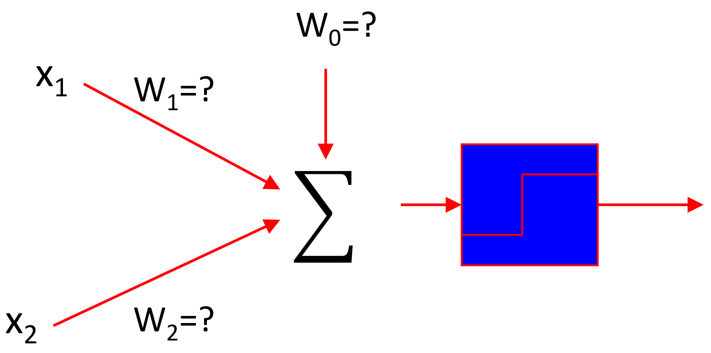
Perceptron – Representation Power

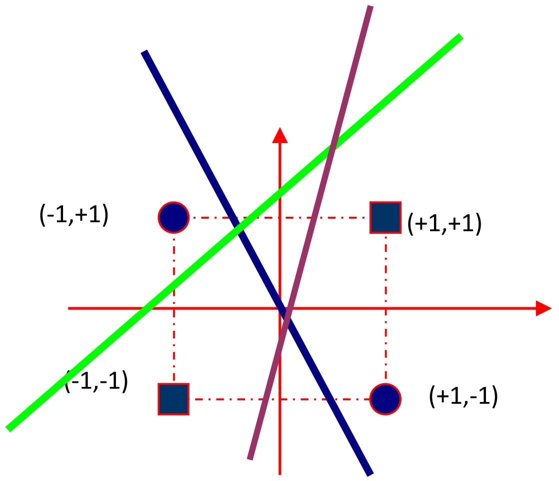
Some problems are linearly non-separable.

XOR

x1	x2	D
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	-1

Decision Surface:
It doesn't matter where you place the line (decision surface). It is impossible to separate the space such that on one side we have o = 1 and on the other we have o = -1.





Perceptron Cannot Solve such Problem!

Perceptron – Training Algorithm

Training sample pairs (X, d), where X is the input vector, d is the input vector's classification (+1 or -1) is iteratively presented to the network for training, *one at a time*, until the process converges.

Perceptron – Training Algorithm

The Procedure is as follows:

1. Set the weights to small random values, e.g., in the range (-1, 1)

2. Present X, and calculate

$$R = \sum_{i=0}^{n} w_i x_i \quad , \quad o = sign(R) = \begin{cases} +1, & if\ R > 0 \\ -1, & otherwise \end{cases}$$

let x_0=1

1. Update the weights

$$w_i \leftarrow w_i + \eta(d - o)x_i, i = 0,1, \dots, n$$

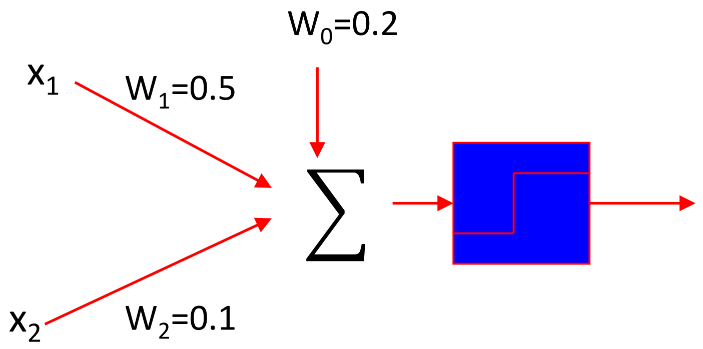
$$0 < \eta < 1, is\ the\ training\ rate$$

2. Repeat by going to step 2

Perceptron – Training Algorithm

Example

x1	x2	D
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	+1



$$w_i \leftarrow w_i + \eta(d - o)x_i, i = 1,2, \dots, n$$

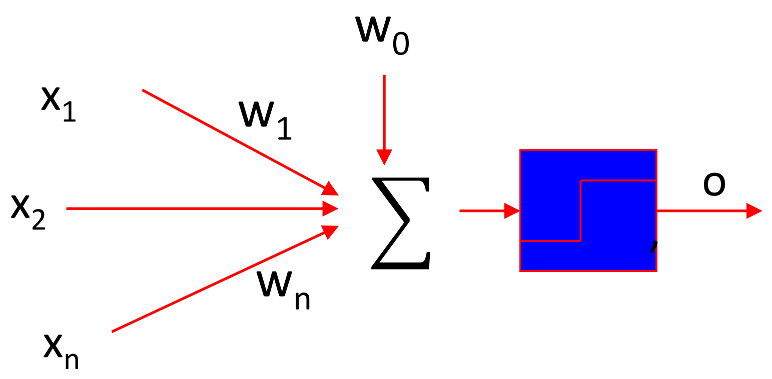
Perceptron – Training Algorithm

Convergence Theorem

The perceptron training rule will converge (finding a weight vector correctly classifies all training samples) within a finite number of iterations, **provided the training examples are linearly separable** and provided a sufficiently small η is used.

Adaptive Linear Element (ADLINE)

Perceptron

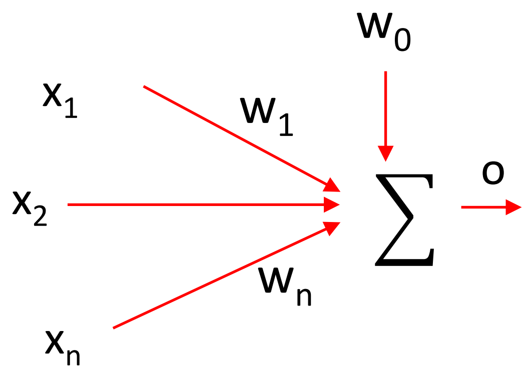


$$R = w_0 + \sum_{i=1}^{n} w_i x_i$$

$$o = sign(R) = \begin{cases} +1, & if\ R > 0 \\ -1, & otherwise \end{cases}$$

VS

ADLINE



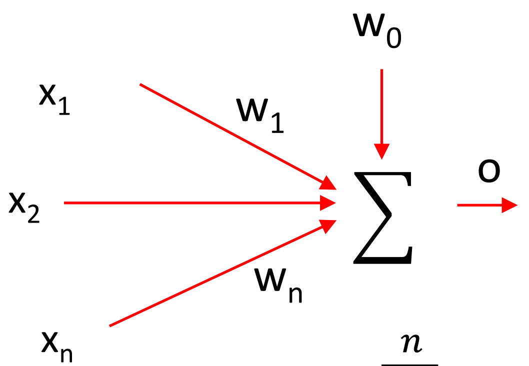
$$o = w_0 + \sum_{i=1}^{n} w_i x_i$$

ADLINE VS Perceptron

- When the problem is not linearly separable, perceptron will fail to converge.
- ADLINE can overcome this difficulty by finding a best fit approximation to the target.

The ADLINE Error Function

- We have training pairs *(X(k), d(k), k =1, 2, …, K)*, where *K* is the number of training samples, the **training error** specifies the difference between the output of the ALDLINE and the desired target.



$$o = w_0 + \sum_{i=1}^{n} w_i x_i$$

- The error is defined as

$$E(W) \equiv \frac{1}{2} \sum_{k=1}^{K} (d(k) - o(k))^2$$

$o(k) = W^T X(k)$ is the output of presenting the training input *X(k)*

The ADLINE Error Function

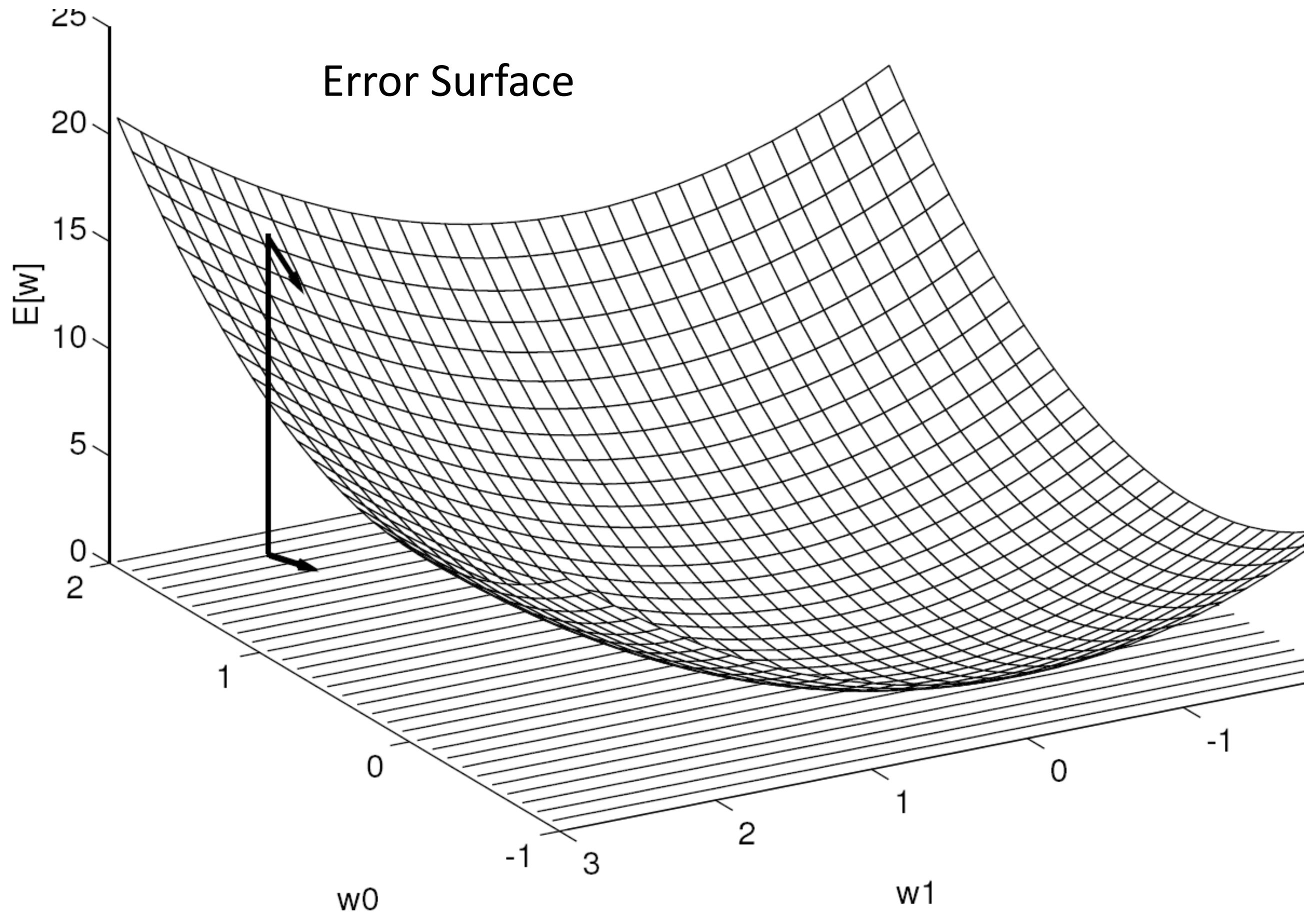
- The error is defined as

$$E(W) \equiv \frac{1}{2}\sum_{k=1}^{K}(d(k) - o(k))^2$$

- The smaller $E(W)$ is, the closer is the approximation.

- We need to find W, based on the given training set, that minimizes the error $E(W)$.

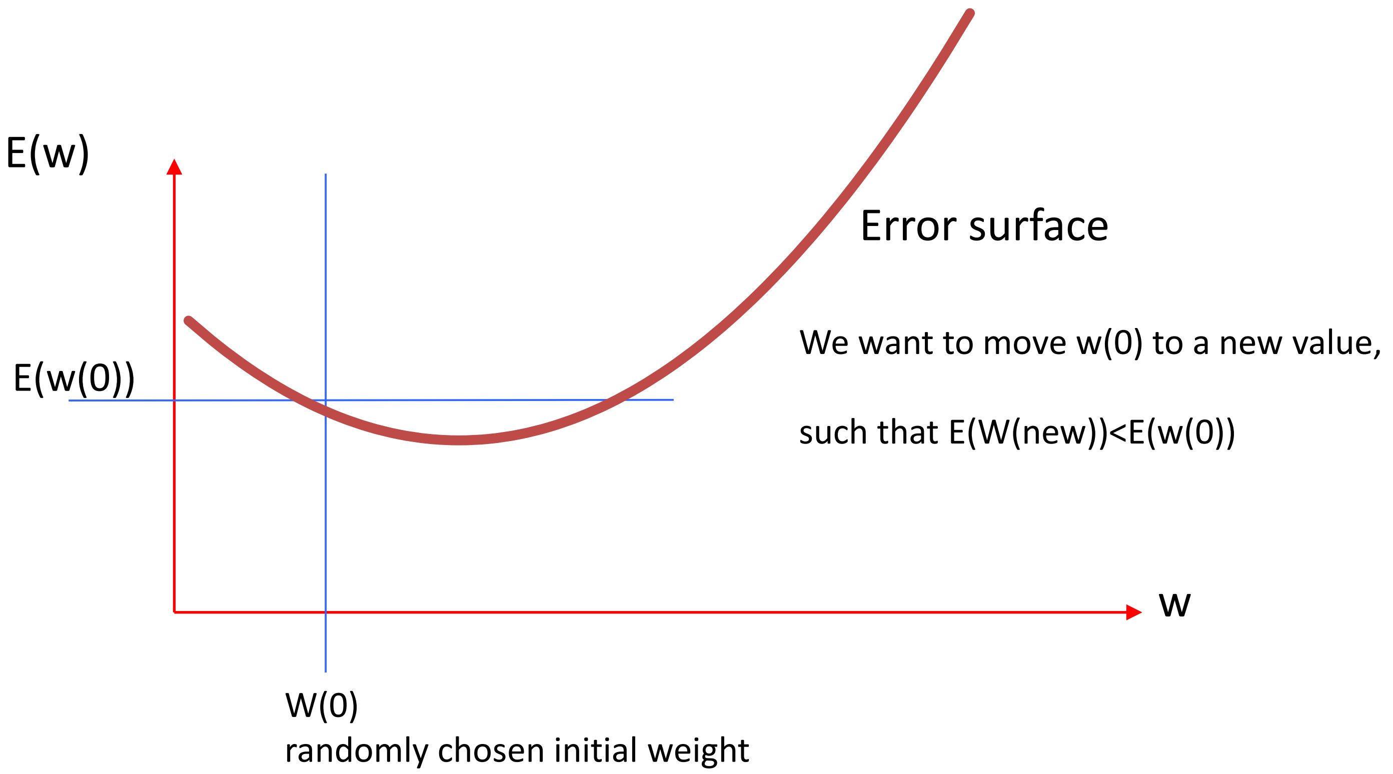
The ADLINE Error Function



The Gradient Descent Rule

An intuition

Before we formally derive the gradient decent rule, here is an intuition of what we should be doing.


E(w)
Error surface
We want to move w(0) to a new value,
such that E(W(new))<E(w(0))
E(w(0))
w
W(0)
randomly chosen initial weight

The Gradient Descent Rule

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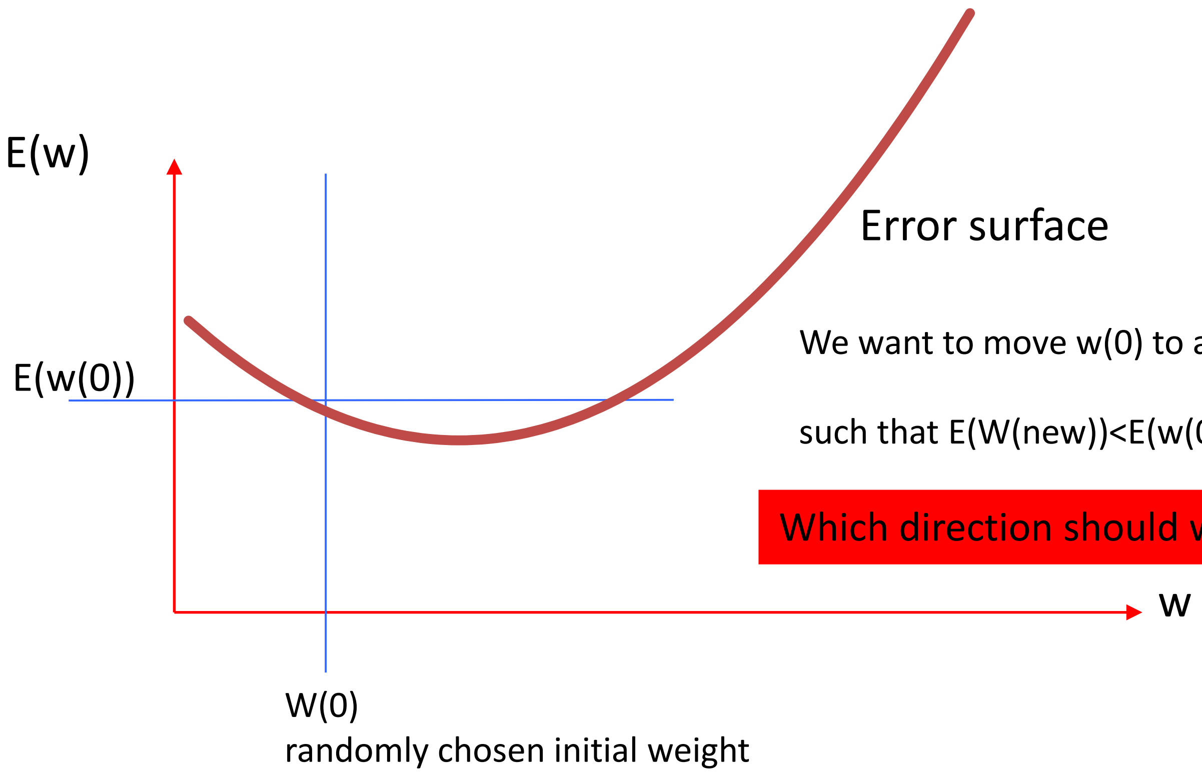

We want to move w(0) to a new value,

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Which direction should we move w(0)

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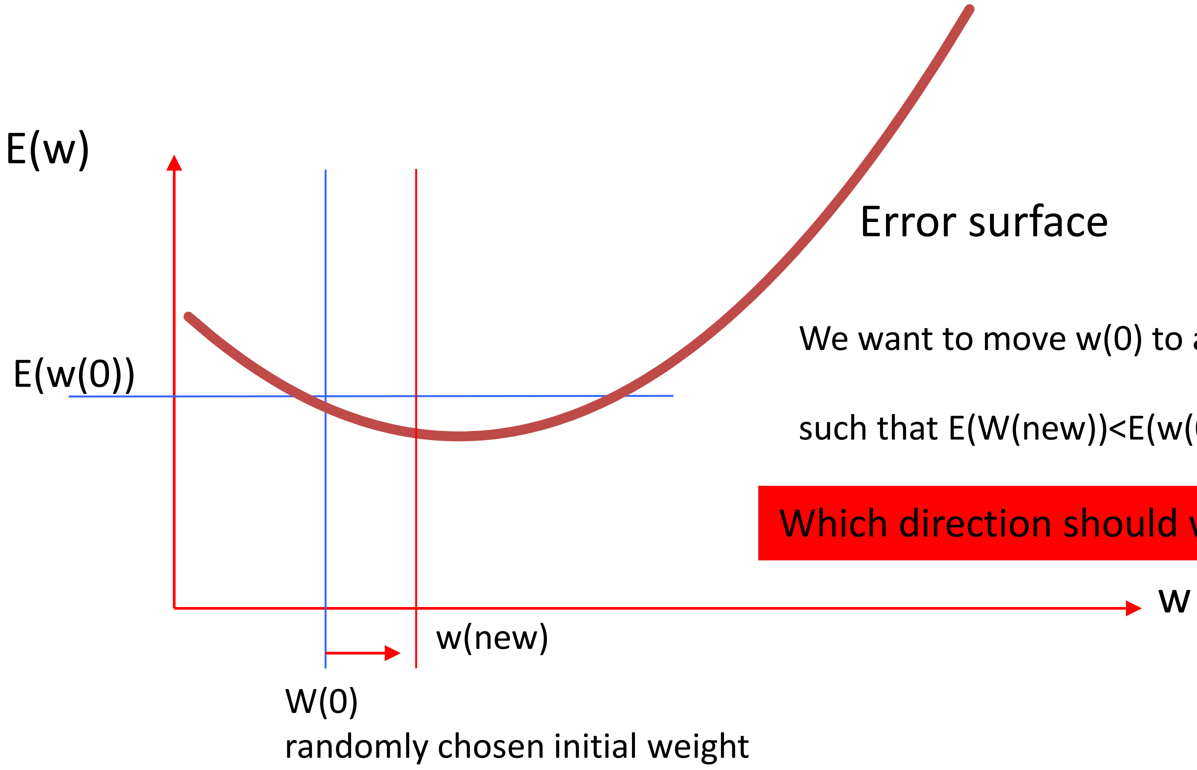

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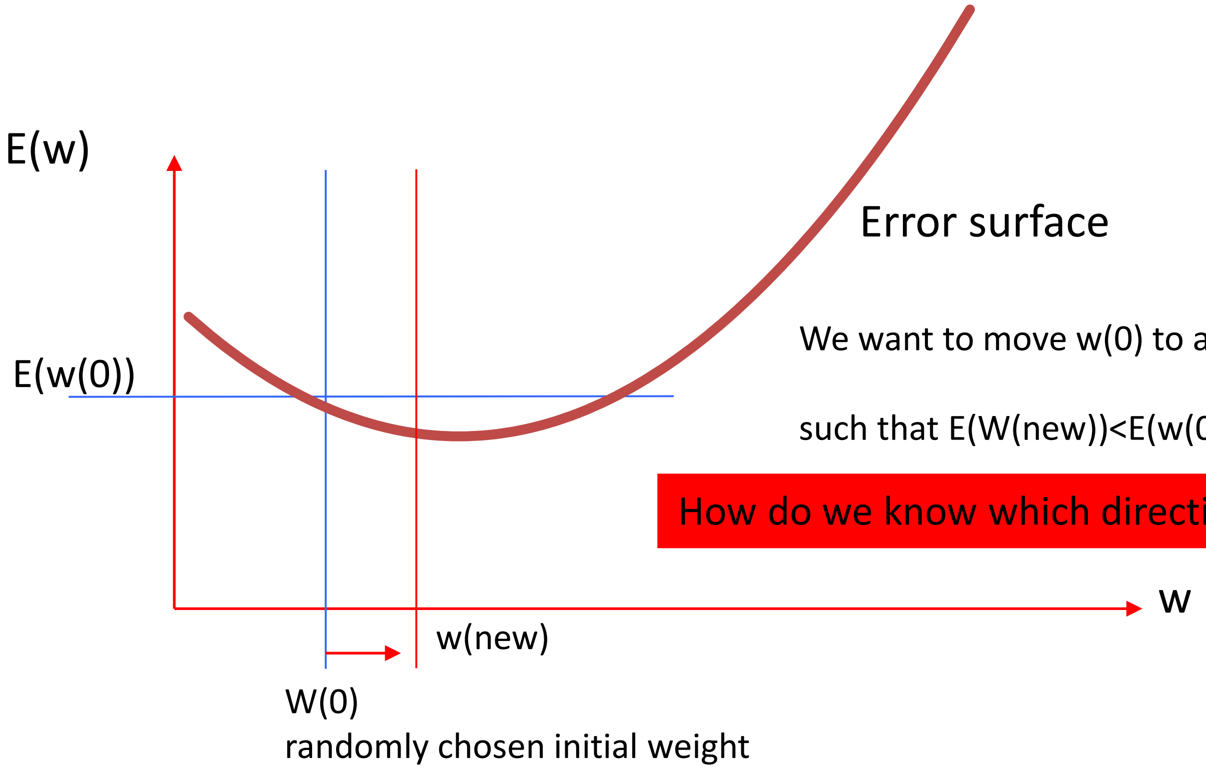

We want to move w(0) to a new value,

such that E(W(new))<E(w(0))

How do we know which direction to move?

The Gradient Descent Rule

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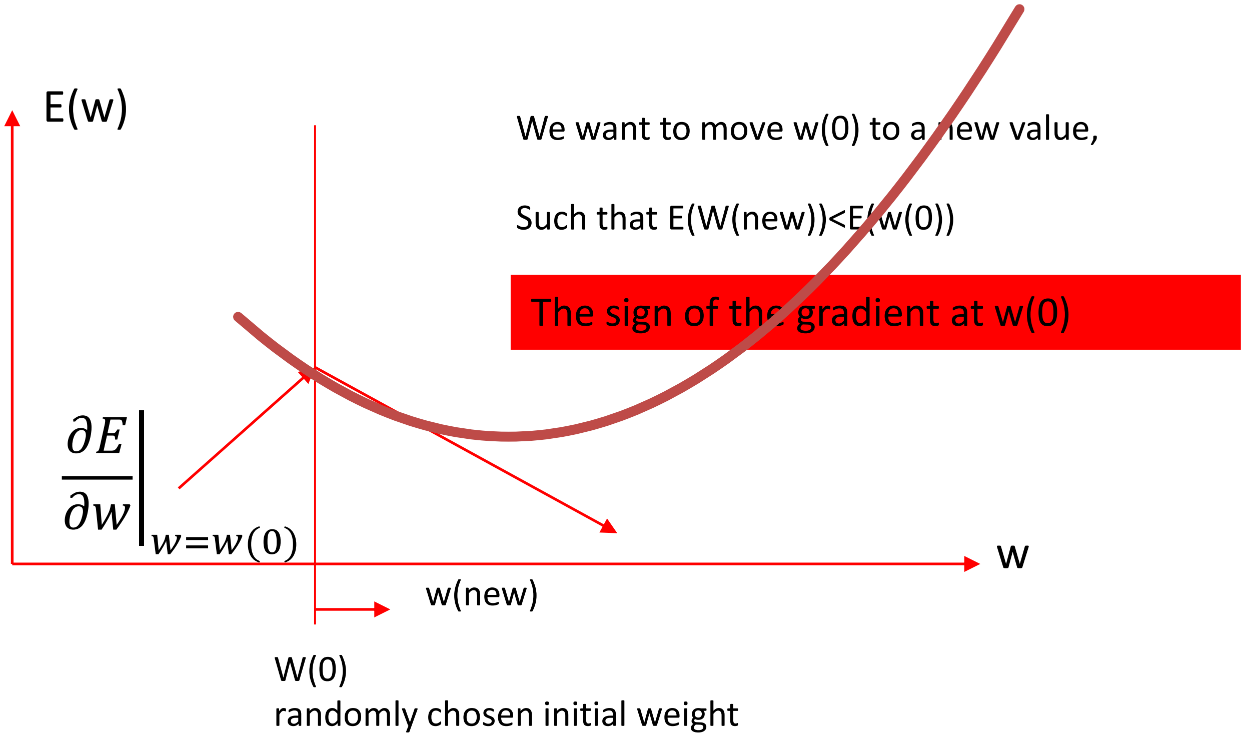
E(w)
We want to move w(0) to a new value,
Such that E(W(new))<E(w(0))
The sign of the gradient at w(0)
$\left.\frac{\partial E}{\partial w}\right|_{w=w(0)}$
w
w(new)
W(0)
randomly chosen initial weight

The Gradient Descent Rule

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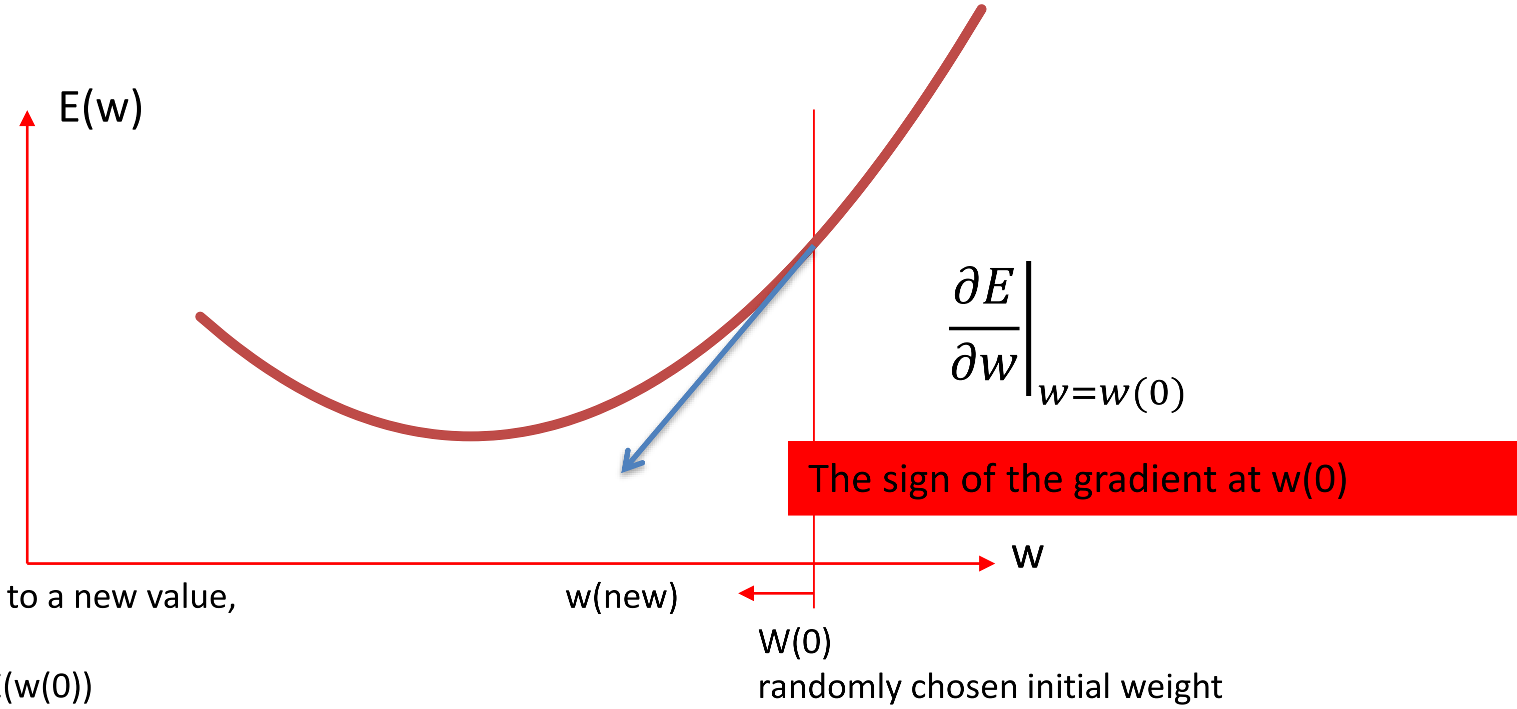

E(w)
$\left.\frac{\partial E}{\partial w}\right|_{w=w(0)}$
The sign of the gradient at w(0)
w
w(new)
W(0)
randomly chosen initial weight

We want to move w(0) to a new value,

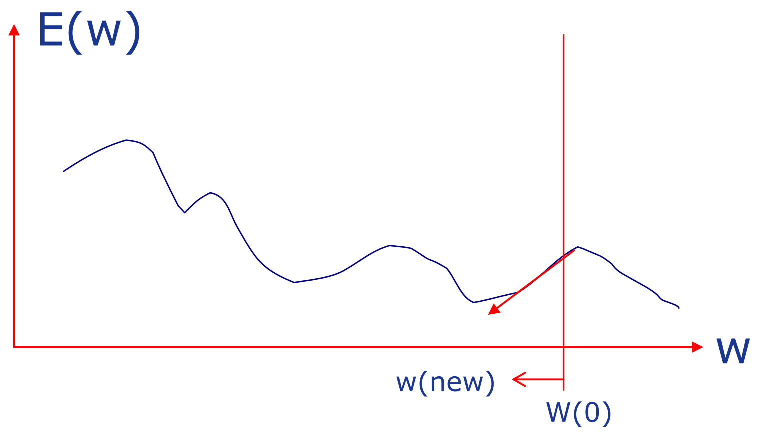
Such that E(W(new))<E(w(0))

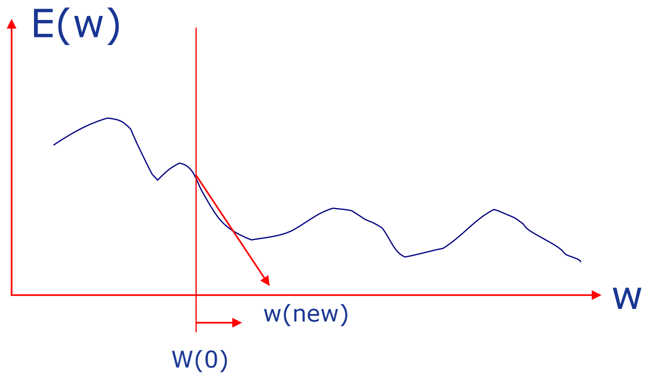
The Gradient Descent Rule

An intuition

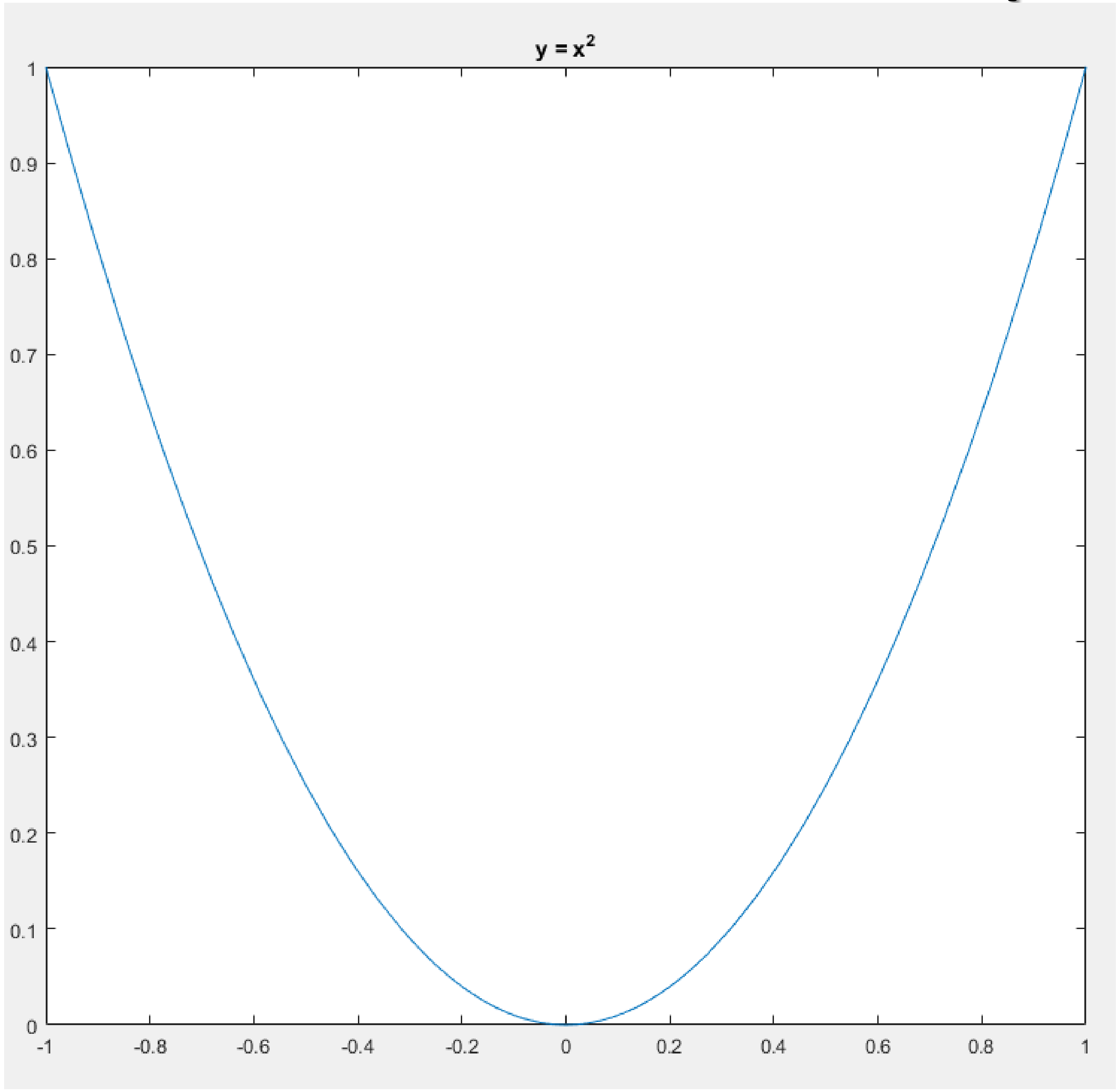
The intuition leads to the following rule:

$$w(new) \leftarrow w(old) - \Delta sign\left(\left.\frac{\partial E}{\partial w}\right|_{w=w(old)}\right)$$

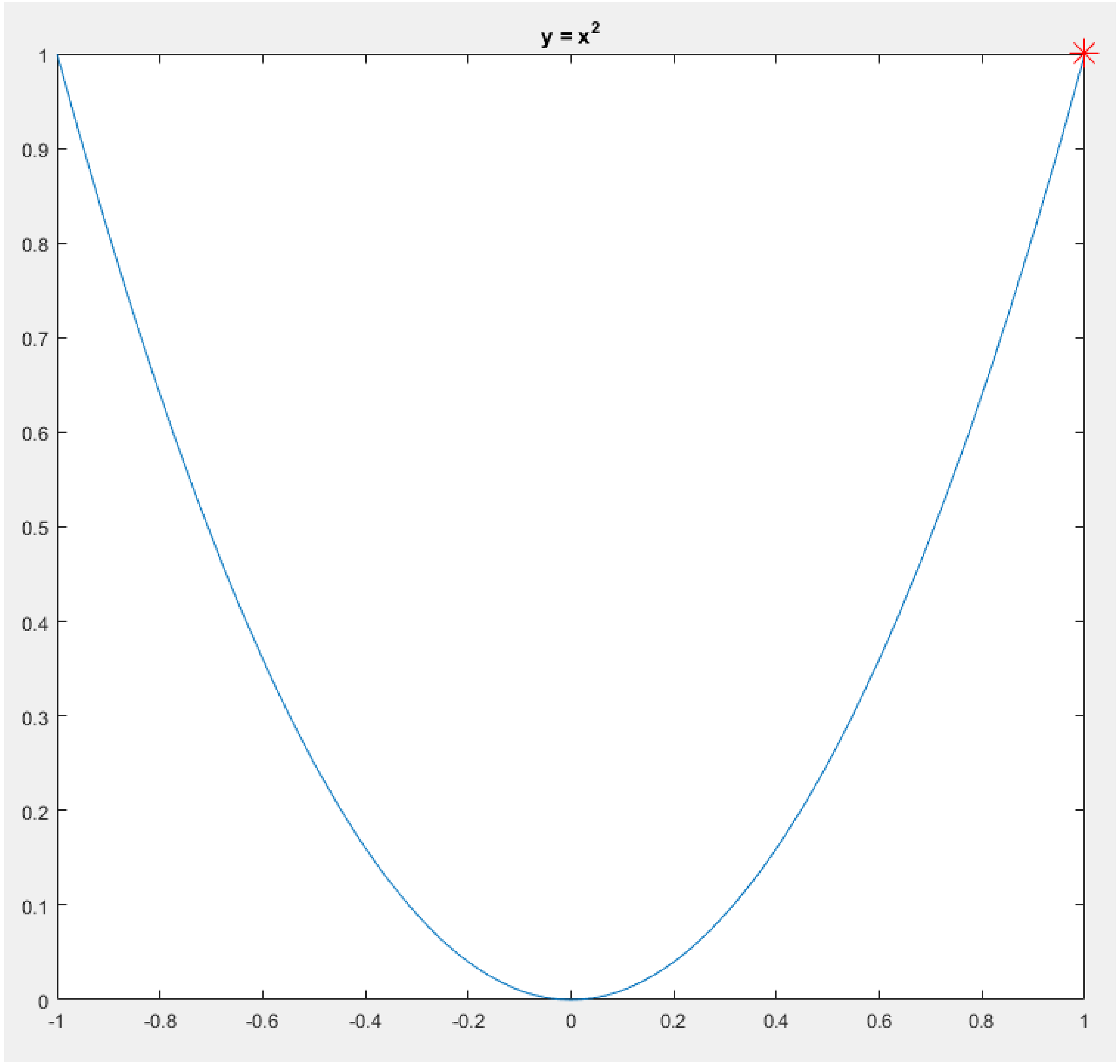




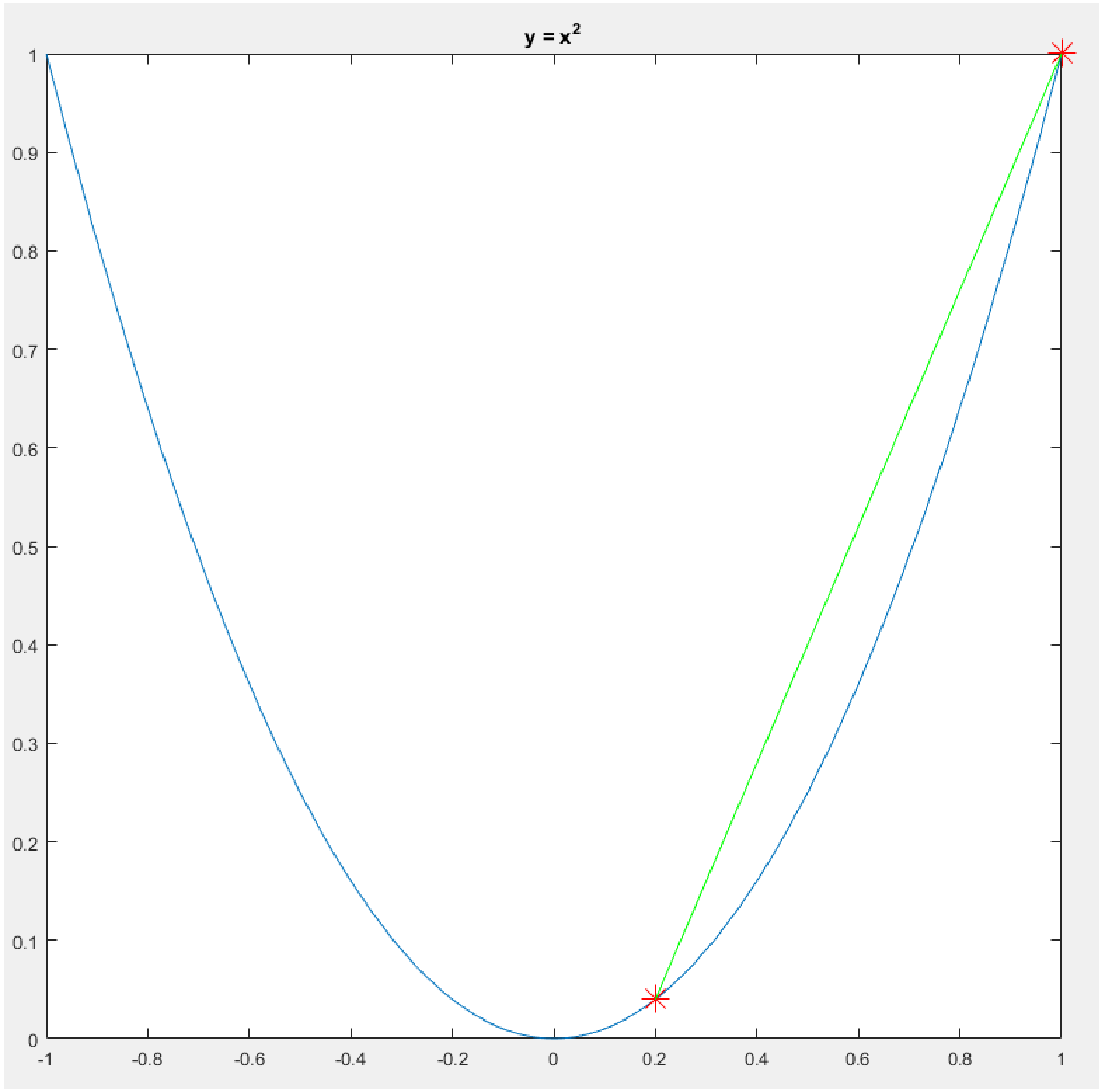
Gradient Descent Example



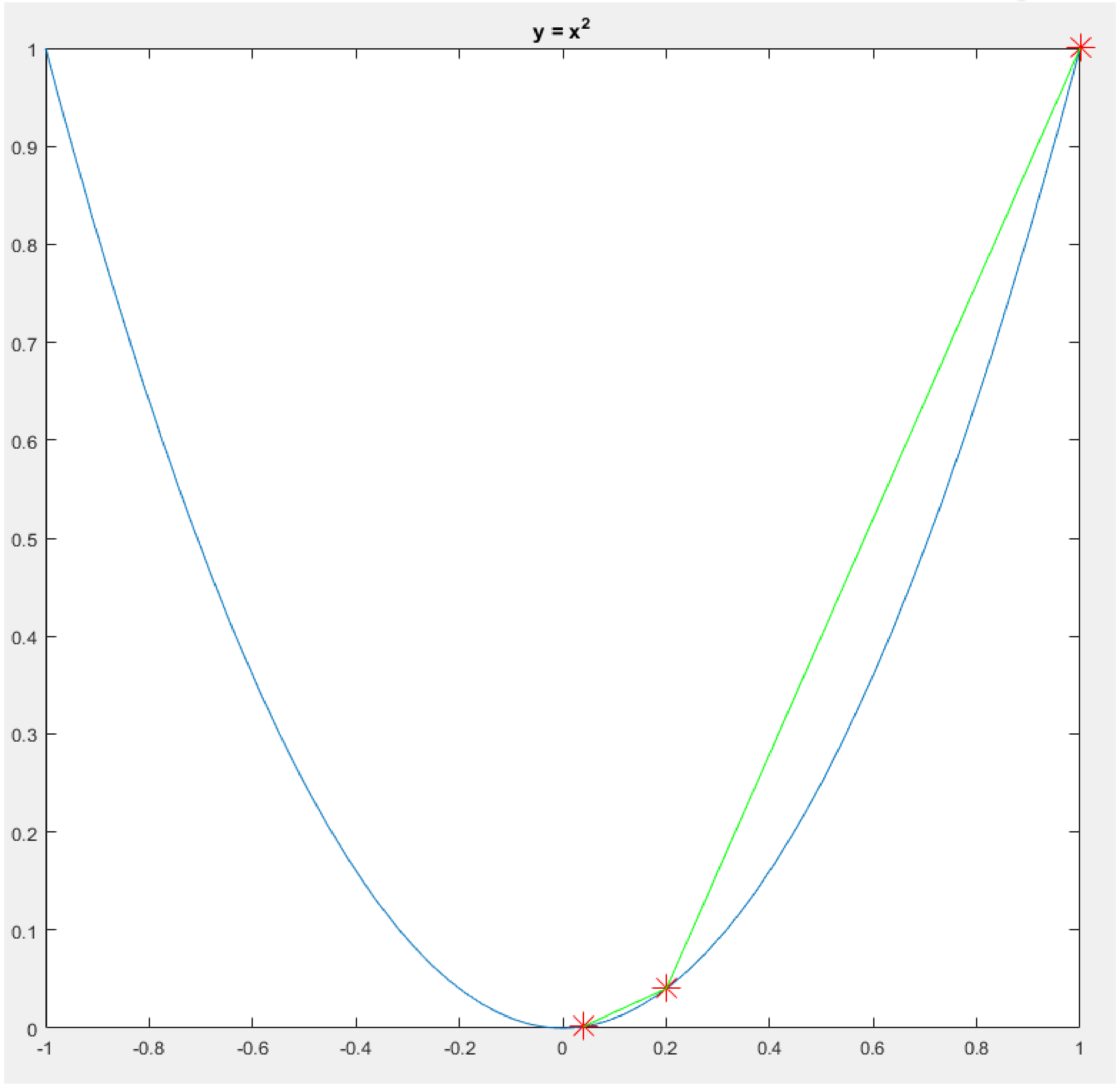
Gradient Descent Example



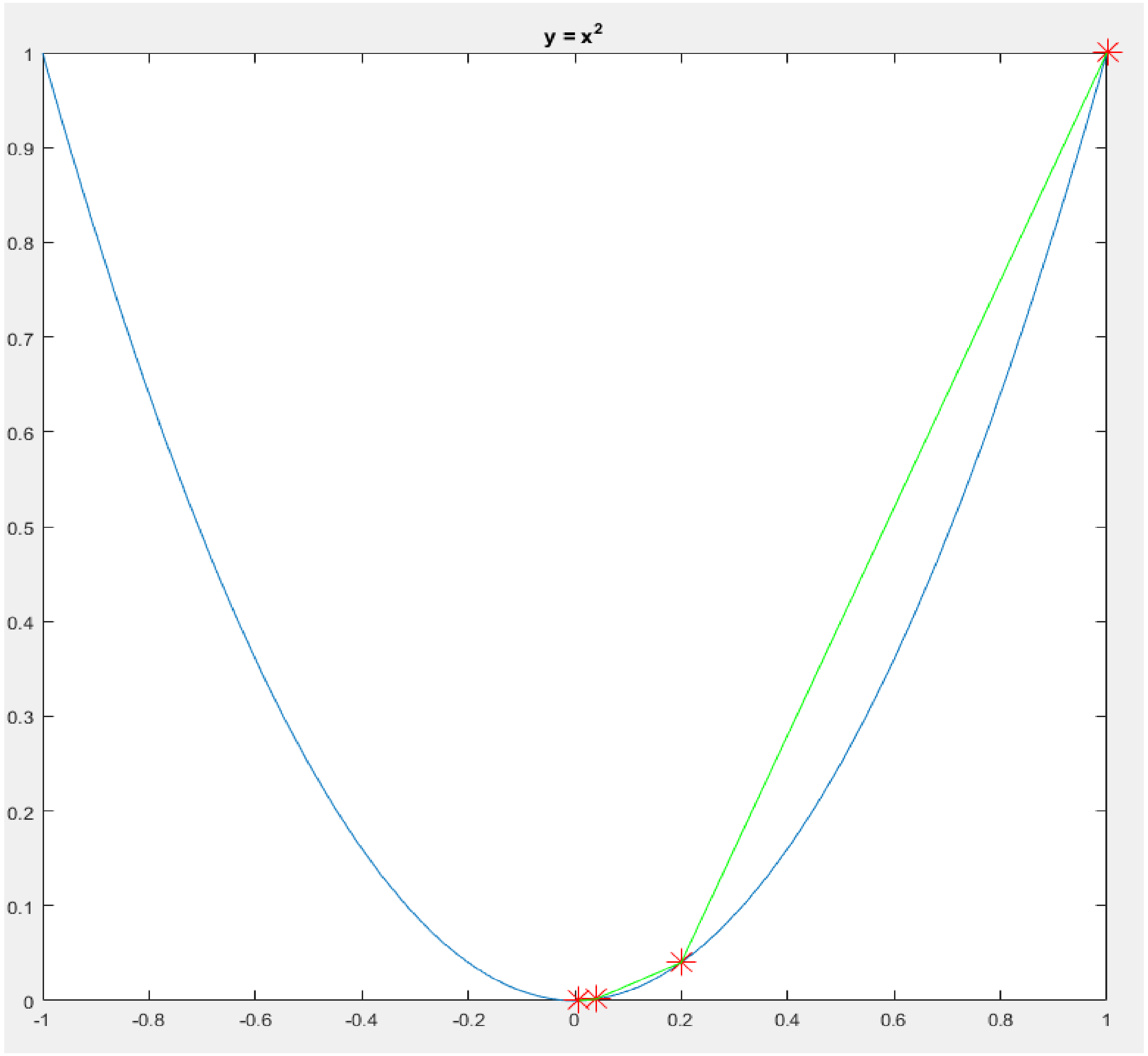
Gradient Descent Example



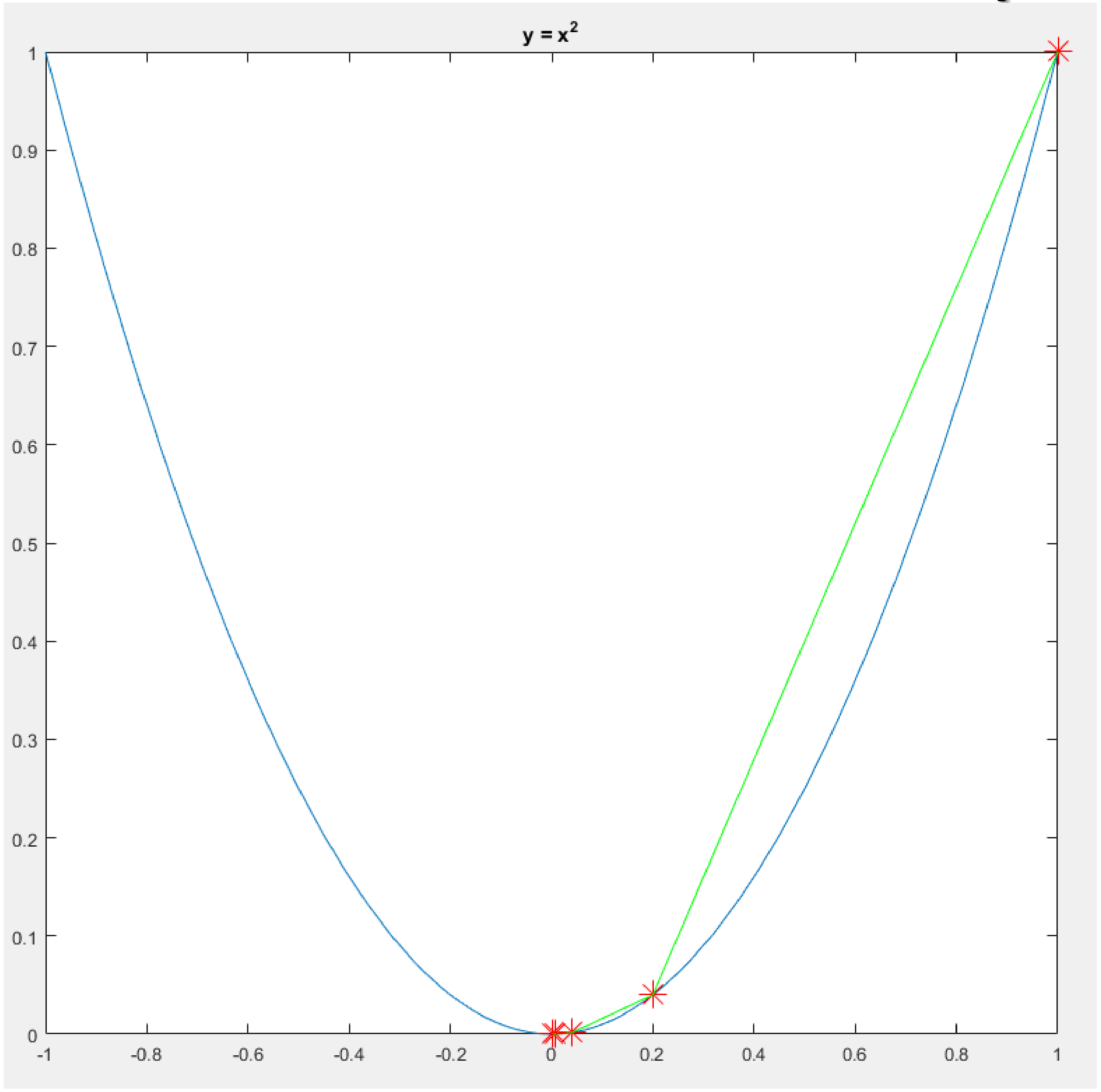
Gradient Descent Example



Gradient Descent Example



Gradient Descent Example



The Gradient Descent Rule

Formal Derivation of Gradient Descent

$$\nabla E(W) = \left[\frac{\partial E}{\partial w_1}, \frac{\partial E}{\partial w_2}, \dots, \frac{\partial E}{\partial w_n}\right]$$

- The gradient of E is a vector, whose components are the partial derivatives of E with respect to each of the w_i's
- The gradient specifies the direction that produces the steepest increase in E. (i.e., speed of change (slope) of the function in one direction.)
- Negative of the vector gives the direction of steepest decrease in E.

The Gradient Descent Rule

The gradient training rule is

$$W \leftarrow W - \eta \nabla E(W)$$

$$w_i \leftarrow w_i - \eta \frac{\partial E}{\partial w_i}$$

η is the training rate

The Gradient Descent Rule

Gradient of ADLINE Error Functions

$$E(W) \equiv \frac{1}{2}\sum_{k=1}^{K}(d(k)-o(k))^2$$

$$\frac{\partial E}{\partial w_i} = \frac{\partial E}{\partial w_i}\left(\frac{1}{2}\sum_{k=1}^{K}(d(k)-o(k))^2\right)$$

$$= \frac{1}{2}\sum_{k=1}^{K}\left(\frac{\partial E}{\partial w_i}(d(k)-o(k))^2\right)$$

$$= \frac{1}{2}\sum_{k=1}^{K}\left(2(d(k)-o(k))\frac{\partial E}{\partial w_i}(d(k)-o(k))\right)$$

$$= \sum_{k=1}^{K}\left((d(k)-o(k))\frac{\partial E}{\partial w_i}(d(k)-w_0-\sum_{i=1}^{n}w_i x_i(k))\right)$$

$$= \sum_{k=1}^{K}((d(k)-o(k))(-x_i(k)\,)$$

$$= -\sum_{k=1}^{K}(d(k)-o(k))x_i(k)$$

The Gradient Descent Rule

ADLINE weight updating using **gradient descent rule**

$$w_i \leftarrow w_i + \eta \sum_{k=1}^{K} (d(k) - o(k)) x_i(k)$$

The Gradient Descent Rule

Gradient descent training procedure

- Initialise w_i to small vales, e.g., in the range of *(-1, 1)*, choose a learning rate, e.g., $\eta = 0.2$

- Until the termination condition is met, Do
  - For all training sample pair *(X(k), d(k))*, input the instance *X(k)* and compute

$$\delta_i = -\sum_{k=1}^{K} (d(k) - o(k)) x_i(k)$$

  - For each weight w_i, Do

$$w_i \leftarrow w_i - \eta \delta_i$$

Batch Mode:

gradients accumulated over **ALL** samples first

Then update the weights

Stochastic (Incremental) Gradient Descent

Also called online mode, Least Mean Square (LMS), Widrow-Hoff, and Delta Rule

- Initialise w_i to small vales, e.g., in the range of *(-1, 1)*, choose a learning rate, e.g., $\eta = 0.01$ (should be smaller than batch mode)
- Until the termination condition is met, Do
  - For EACH training sample pair *(X(k), d(k))*, compute

$$\delta_i = -(d(k) - o(k))x_i(k)$$

  - For each weight w_i, Do

$$w_i \leftarrow w_i - \eta\delta_i$$

Online Mode:

Calculate gradient for **EACH (or a SUBSET of)**samples

Then update the weights

Training Iterations, Epochs

- Training is an iterative process; training samples will have to be used repeatedly for training.

- Assuming we have K training samples $[(X(k), d(k)), k=1, 2, \ldots, K]$; then an epoch is the presentation of all K sample for training once.

  – First epoch: Present training samples: $(X(1), d(1)), (X(2), d(2)), \ldots (X(K), d(K))$
  – Second epoch: Present training samples: $(X(K), d(K)), (X(K-1), d(K-1)), \ldots (X(1), d(1))$

  – Note the order of the training sample presentation between epochs can (and should normally) be different.

- Normally, training will take many epochs to complete.

Termination of Training

To terminate training, there are normally two ways

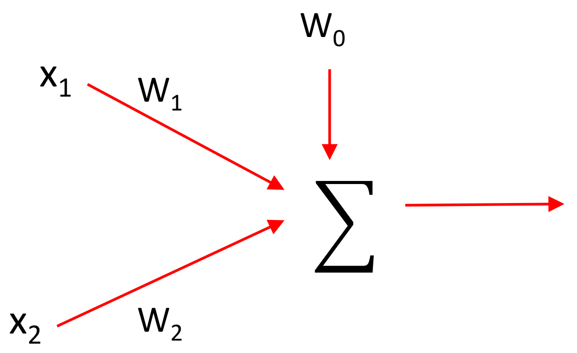
- When a pre-set number of training epochs is reached.
- When the error is smaller than a pre-set value.

$$E(W) \equiv \frac{1}{2}\sum_{k=1}^{K}(d(k) - o(k))^2$$

Gradient Descent Training

Example

x1	x2	D
-1	-1	-1
-1	+1	+1
+1	-1	+1
+1	+1	+1



Initialization

$W_0(0)=0.1$; $W_1(0)=0.2$; $W_2(0)=0.3$;

$\eta=0.5$

Further Reading

Chapter 4, T. M. Mitchell, Machine Learning, McGraw-Hill International Edition, 1997