FOUNDATION SCIENCE A

SEMINAR 4: SIMPLE MACHINES, FLUIDS AND LIGHT

University of Nottingham
UK | CHINA | MALAYSIA

LEARNING OUTCOMES

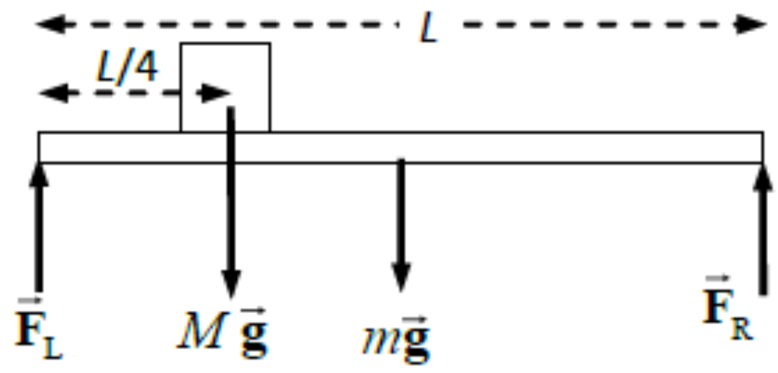
To better understand and analyse the following problems:

- The use of simple machines
- Fluid Flow
- Light

QUESTION 1:

A 110 kg horizontal beam is supported at each end. A 320 kg piano rests a quarter of the way from one end. What is the vertical force on each of the supports?

Answer:

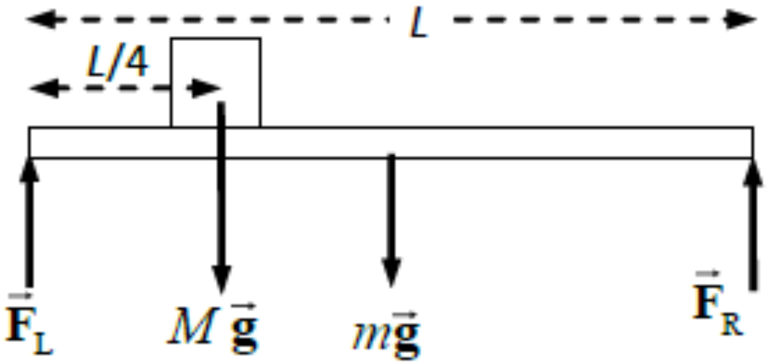


- Let m be the mass of the beam, and M be the mass of the piano.
- Calculate torques about the left end of the beam, with counterclockwise torques positive.
- The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.

QUESTION 1:

A 110 kg horizontal beam is supported at each end. A 320 kg piano rests a quarter of the way from one end. What is the vertical force on each of the supports?

Answer:



$$\sum \tau = F_R L - mg\left(\tfrac{1}{2}L\right) - Mg\left(\tfrac{1}{4}L\right) = 0$$

$$F_R = \left(\tfrac{1}{2}m + \tfrac{1}{4}M\right)g = \left[\tfrac{1}{2}(110\,\mathrm{kg}) + \tfrac{1}{4}(320\,\mathrm{kg})\right]\left(9.80\,\mathrm{m/s^2}\right) = 1.32\times10^3\,\mathrm{N}$$

$$\sum F_y = F_L + F_R - mg - Mg = 0$$

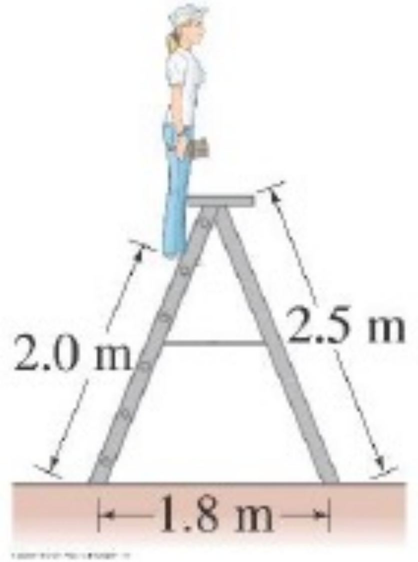
$$F_L = (m+M)g - F_R = (430\,\mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right) - 1.32\times10^3\,\mathrm{N} = 2.89\times10^3\,\mathrm{N}$$

The forces on the supports are equal in magnitude and opposite in direction to the above two results.

$$\boxed{F_R = 1300\,\mathrm{N\ down}} \qquad \boxed{F_L = 2900\,\mathrm{N\ down}}$$

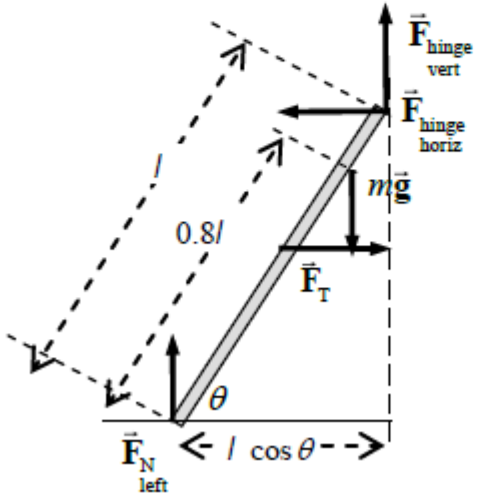
QUESTION 2:

A 56.0 kg person stands 2.0 m from the bottom of the stepladder shown in the figure below. Determine (*a*) the tension in the horizontal tie rod, which is halfway up the ladder, (*b*) the normal force the ground exerts on each side of the ladder, and (c) the force (magnitude and direction) that the left side of the ladder exerts on the right side at the hinge on the top. Ignore the mass of the ladder and assume the ground is frictionless. [*Hint:* Consider free-body diagrams for each section of the ladder.]



Answer:

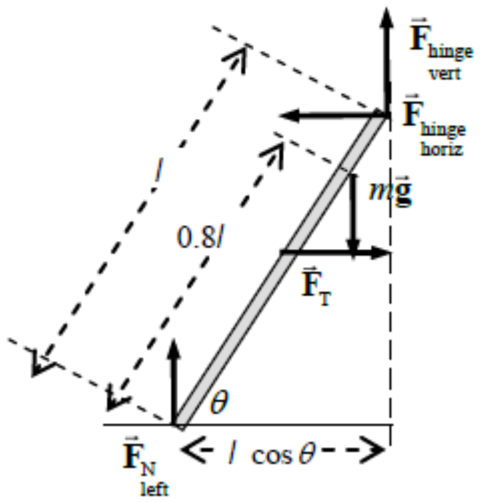
(a) Consider the free-body diagram for each side of the ladder. Because the two sides are not identical, we must have both horizontal and vertical components to the hinge force of one side of the ladder on the other.



Equilibrium

Answer:

(a) First determine the angle from $\cos\theta = \frac{\frac{1}{2}d}{l} = \frac{d}{2l}$.



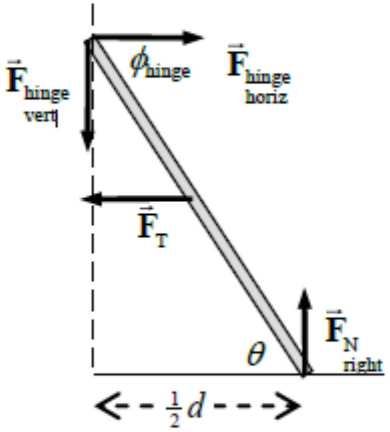
$$\theta = \cos^{-1}\frac{\frac{1}{2}d}{l} = \cos^{-1}\frac{0.9\,\text{m}}{2.5\text{m}} = 68.9°$$

Write equilibrium equations for the following conditions:

Vertical forces on total ladder:

$$\sum F_{\text{vert}} = F_{\substack{\text{N}\\\text{left}}} - mg + F_{\substack{\text{hinge}\\\text{vert}}} - F_{\substack{\text{hinge}\\\text{vert}}} + F_{\substack{\text{N}\\\text{right}}} = 0 \rightarrow$$

$$F_{\substack{\text{N}\\\text{left}}} + F_{\substack{\text{N}\\\text{right}}} = mg$$



Torques on left side, about top, clockwise positive.

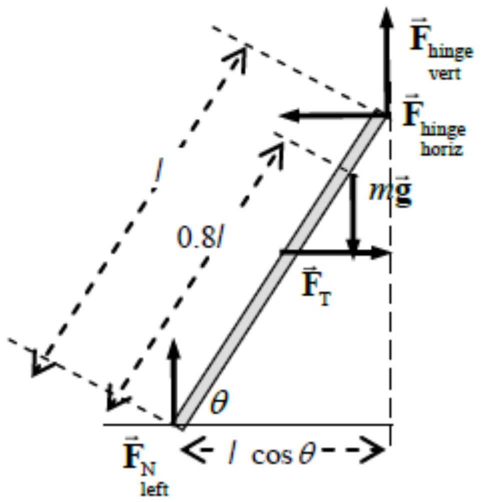
$$\sum \tau = F_{\substack{\text{N}\\\text{left}}}(l\cos\theta) - mg(0.2l)\cos\theta - F_T\left(\tfrac{1}{2}l\right)\sin\theta = 0$$

Equilibrium

Answer:

(a) Torques on right side, about top, clockwise positive.

$$\sum \tau = -F_{\underset{\text{right}}{\text{N}}}(l\cos\theta) + F_{\text{T}}\left(\tfrac{1}{2}l\right)\sin\theta = 0$$



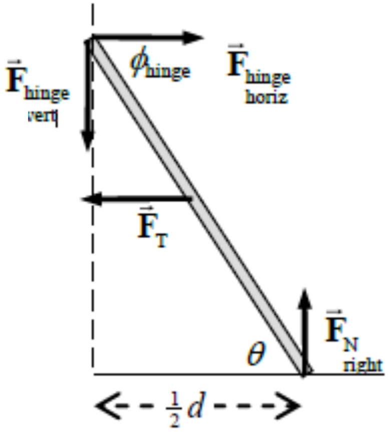
Subtract the second torque equation from the first.

$$\left(F_{\underset{\text{left}}{\text{N}}} + F_{\underset{\text{right}}{\text{N}}}\right)(l\cos\theta) - mg(0.2l)\cos\theta - 2F_{\text{T}}\left(\tfrac{1}{2}l\right)\sin\theta = 0$$

Substitute in from the vertical forces equation, and solve for the tension.

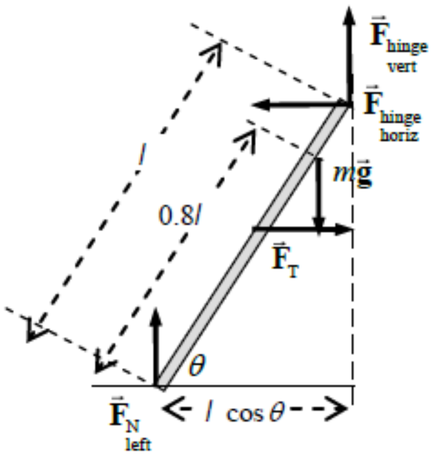
$$mg(l\cos\theta) - mg(0.2l)\cos\theta - 2F_{\text{T}}\left(\tfrac{1}{2}l\right)\sin\theta = 0 \quad \rightarrow$$

$$F_{\text{T}} = \frac{mg}{\sin\theta}(0.8\cos\theta) = \frac{0.8mg}{\tan\theta} = \frac{0.8(56.0\,\text{kg})(9.80\,\text{m/s}^2)}{\tan 68.9^\circ} = 169.4\,\text{N} \approx \boxed{170\,\text{N}}$$



Answer:

(b) To find the normal force on the right side, use the torque equation for the right side.



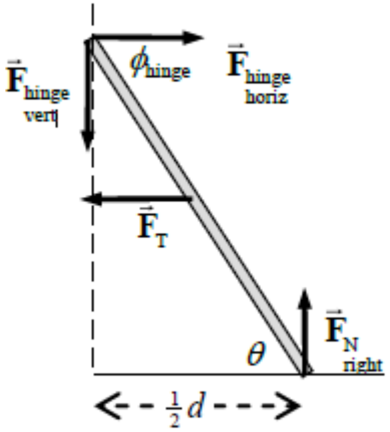
$$-F_{\substack{N\\\text{right}}}(l\cos\theta)+F_T\left(\tfrac{1}{2}l\right)\sin\theta=0\quad\rightarrow$$

$$F_{\substack{N\\\text{right}}}=\tfrac{1}{2}F_T\tan\theta=\tfrac{1}{2}(169.4\,\text{N})\tan 68.9^\circ=219.5\,\text{N}\approx\boxed{220\,\text{N}}$$

To find the normal force on the left side, use the vertical force equation for the entire ladder.

$$F_{\substack{N\\\text{left}}}+F_{\substack{N\\\text{right}}}=mg\quad\rightarrow$$

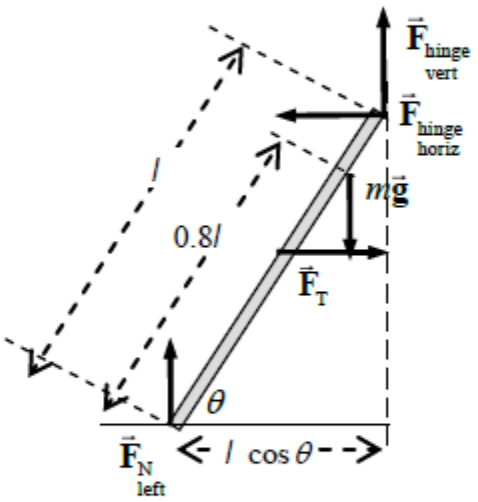
$$F_{\substack{N\\\text{left}}}=mg-F_{\substack{N\\\text{right}}}=(56.0\,\text{kg})(9.80\,\text{m/s}^2)-219.5\,\text{N}=329.3\,\text{N}\approx\boxed{330\,\text{N}}$$



Equilibrium

Answer:

(c) We find the hinge force components from the free-body diagram for the right side.

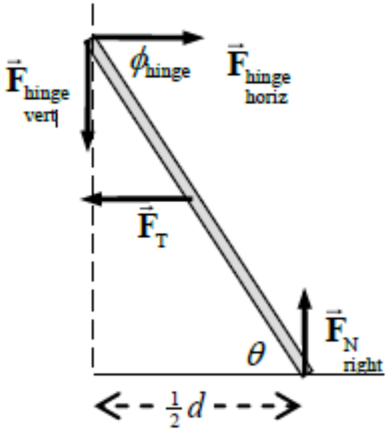


$$\sum F_{\text{vert}} = F_{\substack{N\\\text{right}}} - F_{\substack{\text{hinge}\\\text{vert}}} = 0 \;\rightarrow\; F_{\substack{\text{hinge}\\\text{vert}}} = F_{\substack{N\\\text{right}}} = 219.5\,\text{N}$$

$$\sum F_{\text{horiz}} = F_{\substack{\text{hinge}\\\text{horiz}}} - F_T = 0 \;\rightarrow\; F_{\substack{\text{hinge}\\\text{horiz}}} = F_T = 169.4\,\text{N}$$

$$F_{\text{hinge}} = \sqrt{F^2_{\substack{\text{hinge}\\\text{horiz}}} + F^2_{\substack{\text{hinge}\\\text{vert}}}} = \sqrt{(169.4\,\text{N})^2 + (219.5\,\text{N})^2} = 277.3\,\text{N} \approx \boxed{280\,\text{N}}$$

$$\phi_{\text{hinge}} = \tan^{-1}\frac{F_{\substack{\text{hinge}\\\text{vert}}}}{F_{\substack{\text{hinge}\\\text{horiz}}}} = \tan^{-1}\frac{219.5\,\text{N}}{169.4\,\text{N}} = \boxed{52^\circ}$$



Pressure; Pascal's Principle:

QUESTION 3:

Estimate the pressure needed to raise a column of water to the same height as a 35 m tall oak tree.

Answer:

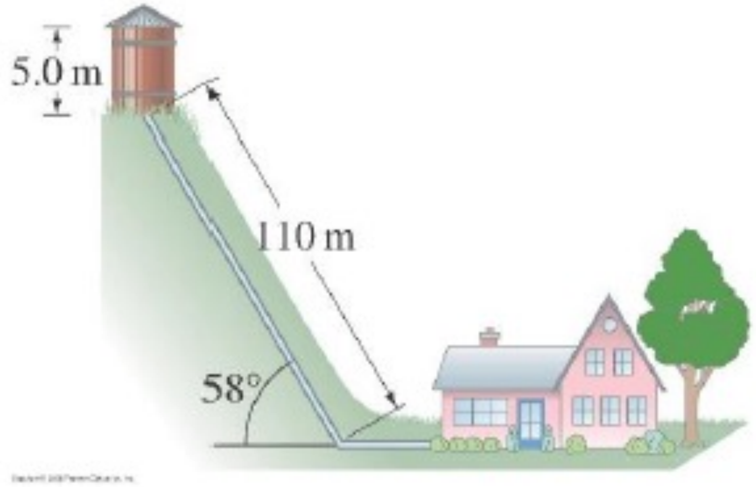
The pressure is given by the equation below.

$$P = \rho g h = (1000)\left(9.80\,\mathrm{m/s^2}\right)(35\,\mathrm{m}) = \boxed{3.4\times10^5\ \mathrm{N/m^2}} \approx 3.4\,\mathrm{atm}$$

Pressure; Pascal's Principle:

QUESTION 4:

A house at the bottom of a hill is fed by a full tank of water 5.0 m deep and connected to the house by a pipe that is 110 m long at an angle of 58° from the horizontal (see the figure below). (*a*) Determine the water gauge pressure at the house (*b*) How high could the water shoot if it came vertically out c a broken pipe in front of the house?



Answer:

(*a*) The gauge pressure is given by the equation below. The height is the height from the bottom of the hill to the top of the water tank.

$$P_G = \rho g h = \left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\ \mathrm{m/s^2}\right)\left[5.0\ \mathrm{m} + (110\ \mathrm{m})\sin 58^\circ\right] = \boxed{9.6\times10^5\ \mathrm{N/m^2}}$$

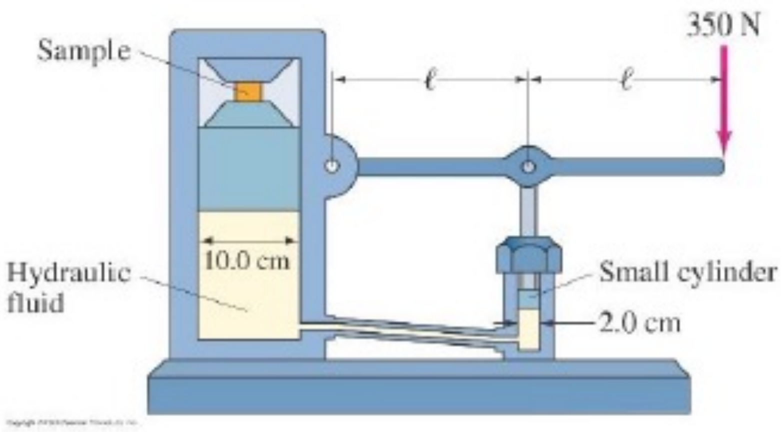
(*b*) The water would be able to shoot up to the top of the tank (ignoring any friction).

$$h = 5.0\,\mathrm{m} + (110\,\mathrm{m})\sin 58^\circ = \boxed{98\,\mathrm{m}}$$

Pressure; Pascal's Principle:

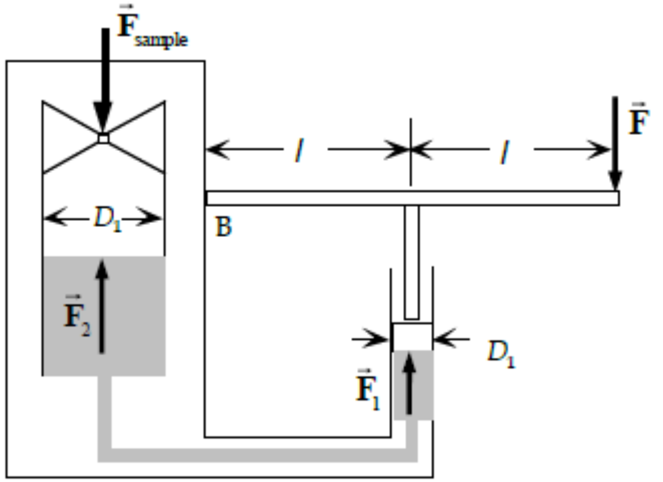
QUESTION 5:

A hydraulic press for compacting powdered samples has a large cylinder which is 10.0 cm in diameter, and a small cylinder with a diameter of 2.0 cm (see the figure below). A lever is attached to the small cylinder as shown. The sample, which is placed on the large cylinder, has an area of 4.0 cm^2. What is the pressure on the sample if 350 N is applied to the lever?



Answer:

- Consider the lever (handle) of the press.
- The net torque on that handle is 0.
- Use that to find the force exerted by the hydraulic fluid upwards on the small cylinder (and the lever).
- Then Pascal's principle can be used to find the upwards force on the large cylinder, which is the same as the force on the sample.



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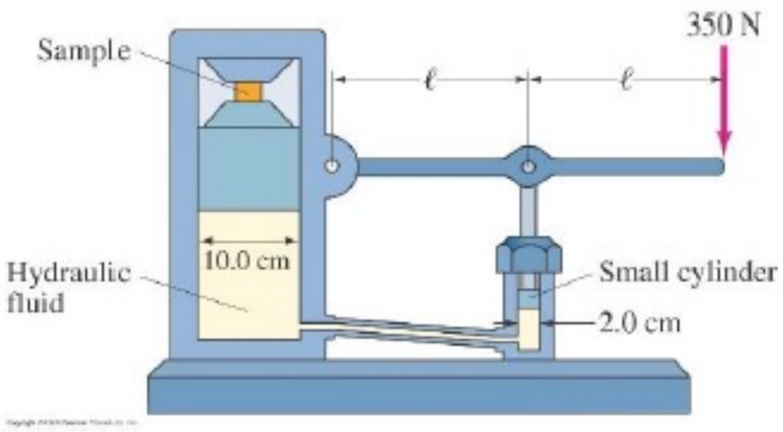


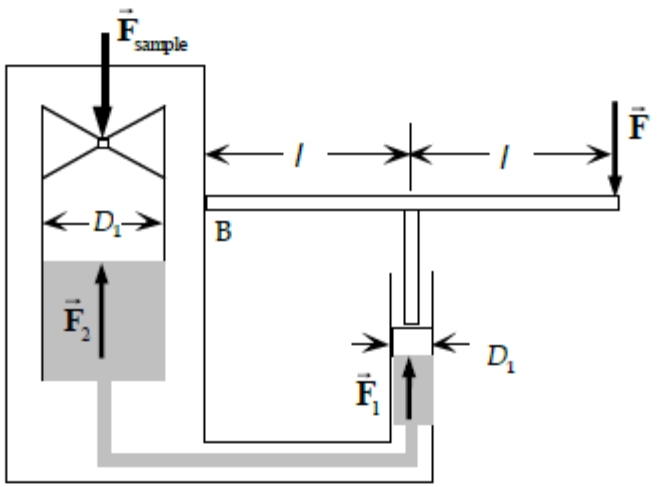
Answer:

$$\sum \tau = F(2l) - F_1 l = 0 \rightarrow F_1 = 2F$$

$$P_1 = P_2 \rightarrow \frac{F_1}{\pi\left(\frac{1}{2}d_1\right)^2} = \frac{F_2}{\cancel{\pi}\left(\frac{1}{2}d_2\right)^2} \rightarrow$$

$$F_2 = F_1\left(d_2/d_1\right)^2 = 2F\left(d_2/d_1\right)^2 = F_{\text{sample}} \rightarrow$$

$$P_{\text{sample}} = \frac{F_{\text{sample}}}{A_{\text{sample}}} = \frac{2F\left(d_2/d_1\right)^2}{A_{\text{sample}}} = \frac{2(350\,\mathrm{N})(5)^2}{4.0\times10^{-4}\,\mathrm{m}^2} = \boxed{4.4\times10^7\,\mathrm{N/m}^2} \approx 430\,\mathrm{atm}$$



Buoyancy and Archimedes' Principle:

QUESTION 6:

A crane lifts the 16,000 kg steel hull of a sunken ship out of the water. Determine (*a*) the tension in the crane's cable when the hull is fully submerged in the water, and (b) the tension when the hull is completely out of the water.

Answer:

(*a*) When the hull is submerged, both the buoyant force and the tension force act upward on the hull, and so their sum is equal to the weight of the hull, if the hull is not accelerated as it is lifted. The buoyant force is the weight of the water displaced.

$$T + F_{\text{buoyant}} = mg \quad \rightarrow$$

$$T = mg - F_{\text{buoyant}} = m_{\text{hull}}g - \rho_{\text{water}}V_{\text{sub}}g = m_{\text{hull}}g - \rho_{\text{water}}\frac{m_{\text{hull}}}{\rho_{\text{hull}}}g = m_{\text{hull}}g\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{hull}}}\right)$$

$$= \left(1.6\times10^{4}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(1 - \frac{1.00\times10^{3}\,\text{kg/m}^3}{7.8\times10^{3}\,\text{kg/m}^3}\right) = 1.367\times10^{5}\,\text{N} \approx \boxed{1.4\times10^{5}\,\text{N}}$$

QUESTION 6:

A crane lifts the 16,000 kg steel hull of a sunken ship out of the water. Determine (*a*) the tension in the crane's cable when the hull is fully submerged in the water, and (b) the tension when the hull is completely out of the water.

Answer:

(*b*) When the hull is completely out of the water, the tension in the crane’s cable must be equal to the weight of the hull.

$$T = mg = \left(1.6\times10^{4}\,\text{kg}\right)\left(9.80\,\text{m/s}^{2}\right) = 1.568\times10^{5}\,\text{N} \approx \boxed{1.6\times10^{5}\,\text{N}}$$

Buoyancy and Archimedes' Principle:

QUESTION 7:

A cube of side length 10.0 cm and made of unknown material floats at the surface between water and oil. The oil has a density of 810 kg/m3. If the cube floats so that it is 72% in the water and 28% in the oil, what is the mass of the cube and what is the buoyant force on the cube?

Answer:

The weight of the object must be balanced by the two buoyant forces, one from the water and one from the oil. The buoyant force is the density of the liquid, times the volume in the liquid, times the acceleration due to gravity. We represent the edge length of the cube by

$$mg = F_{\underset{\text{oil}}{\text{B}}} + F_{\underset{\text{water}}{\text{B}}} = \rho_{\text{oil}} V_{\text{oil}} g + \rho_{\text{water}} V_{\text{water}} g = \rho_{\text{oil}} l^2 (0.28 l) g + \rho_{\text{water}} l^2 (0.72 l) g \rightarrow$$

$$m = l^3 (0.28 \rho_{\text{oil}} + 0.72 \rho_{\text{water}}) = (0.100\,\text{m})^3 \left[0.28 (810\,\text{kg/m}^2) + 0.72 (1000\,\text{kg/m}^2) \right]$$

$$= 0.9468\,\text{kg} \approx \boxed{0.95\,\text{kg}}$$

The buoyant force is the weight of the object, $mg = (0.9468\,\text{kg})(9.80\,\text{m/s}^2) = \boxed{9.3\,\text{N}}$

Fluid Flow, Bernoulli's Equation:

QUESTION 8:

A fish tank has dimensions 36 cm wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 4.0 h, what should the flow speed be in the 3.0 cm diameter input tube for the filter?

Answer:

The flow speed is the speed of the water in the input tube. The entire volume of the water in the tank is to be processed in 4.0 h. The volume of water passing through the input tube per unit time is the volume rate of flow, as expressed below:

$$\frac{V}{\Delta t} = Av \quad \rightarrow \quad v = \frac{V}{A\Delta t} = \frac{lwh}{\pi r^2 \Delta t} = \frac{(0.36\,\text{m})(1.0\,\text{m})(0.60\,\text{m})}{\pi (0.015\,\text{m})^2 (4.0\,\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)} = 0.02122\,\text{m/s} \approx \boxed{2.1\,\text{cm/s}}$$

Fluid Flow, Bernoulli's Equation:

QUESTION 9:

What is the lift (in Newtons) due to Bernoulli's principle on a wing of area 88 m^2 if the air passes over the top and bottom surfaces at speeds of 280 $m \cdot s^{-1}$ and 150 $m \cdot s^{-1}$, respectively?

Answer:

The lift force would be the difference in pressure between the two wing surfaces, times the area of the wing surface. The difference in pressure can be found from Bernoulli's equation. We consider the two surfaces of the wing to be at the same height above the ground. Call the bottom surface of the wing point 1, and the top surface point 2.

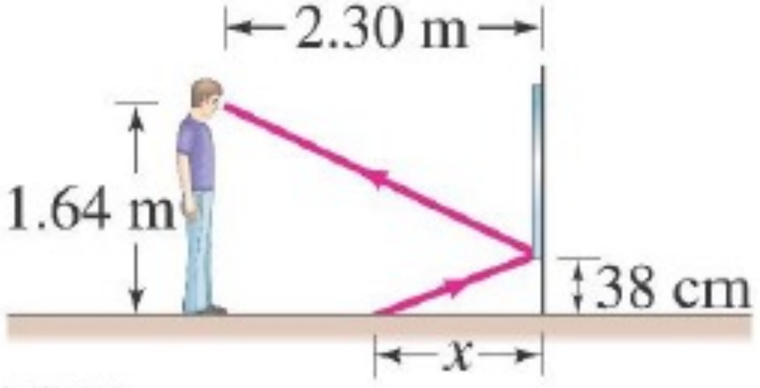
$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow \quad P_1 - P_2 = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$F_{\text{lift}} = \left(P_1 - P_2\right)\left(\text{Area of wing}\right) = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$$

$$= \tfrac{1}{2}\left(1.29\,\text{kg/m}^3\right)\left[\left(280\,\text{m/s}\right)^2 - \left(150\,\text{m/s}\right)^2\right]\left(88\,\text{m}^2\right) = \boxed{3.2\times10^6\,\text{N}}$$

QUESTION 10:

A person whose eyes are 1.64 m above the floor stands 2.30 m in front of a vertical plane mirror whose bottom edge is 38 cm above the floor, as shown in the figure below. What is the horizontal distance *x* to the base of the wall supporting the mirror of the nearest point on the floor that can be seen reflected in the mirror?

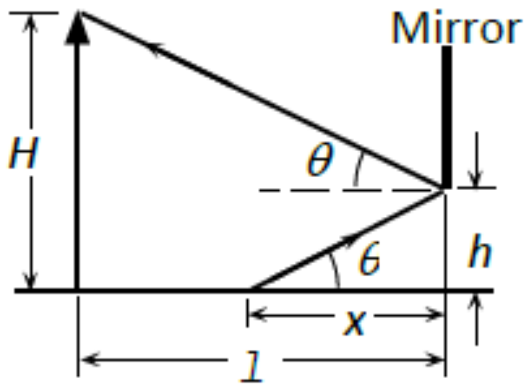


Answer:

The angle of incidence is the angle of reflection. See diagram for the appropriate lengths.

$$\tan\theta = \frac{(H-h)}{l} = \frac{h}{x} \rightarrow$$

$$\frac{(1.64\,\text{m} - 0.38\,\text{m})}{(2.30\,\text{m})} = \frac{(0.38\,\text{m})}{x} \rightarrow x = \boxed{0.69\,\text{m}}$$



Index of Refraction:

QUESTION 11:

The speed of light in ice is $2.29\times10^{8}\ ms^{-1}$.What is the index of refraction of ice?

Answer:

We find the index of refraction from the equation below

$$n=\frac{c}{v}=\frac{3.00\times10^{8}\ \text{m/s}}{2.29\times10^{8}\ \text{m/s}}=\boxed{1.31}$$

Refraction

TABLE 23.1 Indices of refraction

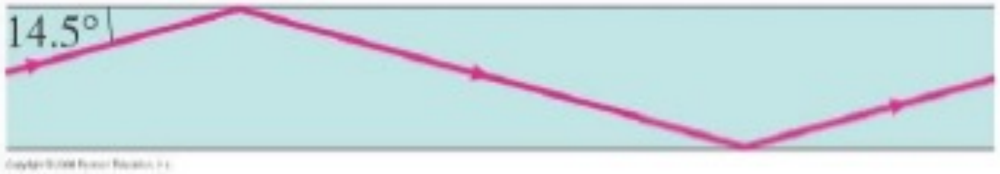
Medium	n
Vacuum	1.00 exactly
Air (actual)	1.0003
Air (accepted)	1.00
Water	1.33
Ethyl alcohol	1.36
Oil	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.41
Silicon (infrared)	3.50

$$n = \frac{c}{v_{\mathrm{medium}}}$$

Total Internal Reflection:

QUESTION 12:

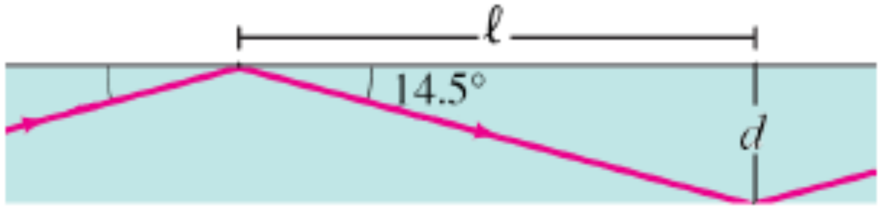
A ray of light, after entering a light fiber, reflects at an angle of 14.5° with the long axis of the fiber, as shown in the figure below. Calculate the distance along the axis of the fiber that the light ray travels between successive reflections off the sides of the fiber. Assume that the fiber has an index of refraction of 1.55 and is 1.40×10^{-4} m in diameter.



Answer:

The ray reflects at the same angle, so each segment makes a 14.5° angle with the side. We find the distance l between reflections from the definition of the tangent function.

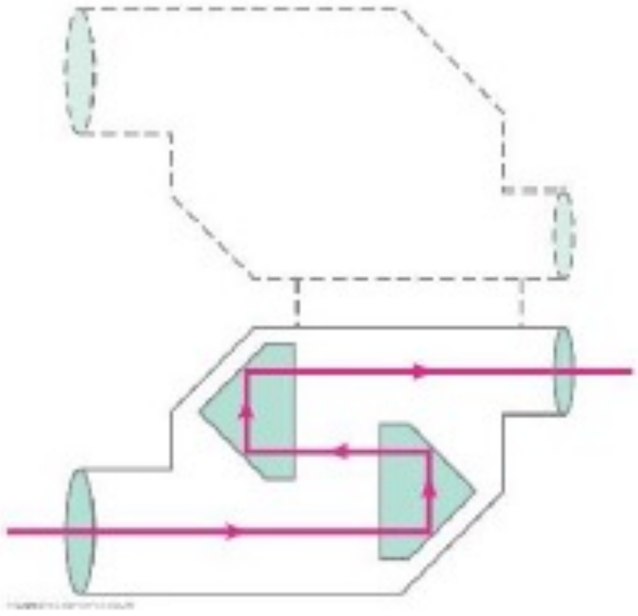
$$\tan\theta = \frac{d}{l} \rightarrow l = \frac{d}{\tan\theta} = \frac{1.40\times10^{-4}\ \text{m}}{\tan\ 14.5°} = \boxed{5.41\times10^{-4}\ \text{m}}$$



Total Internal Reflection:

QUESTION 13:

(a) What is the minimum index of refraction for a glass or plastic prism to be used in binoculars (as shown in the figure below) so that total internal reflection occurs at 45'? (b) Will binoculars work if their prisms (assume *n* = 1.58) are immersed in water? (c) What minimum *n* is needed if the prisms are immersed in water?



Answer:

(a) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have the following. (Snell's Law of Refraction)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow| \quad n_1 \sin 45° = (1.00)\sin\theta_2$$

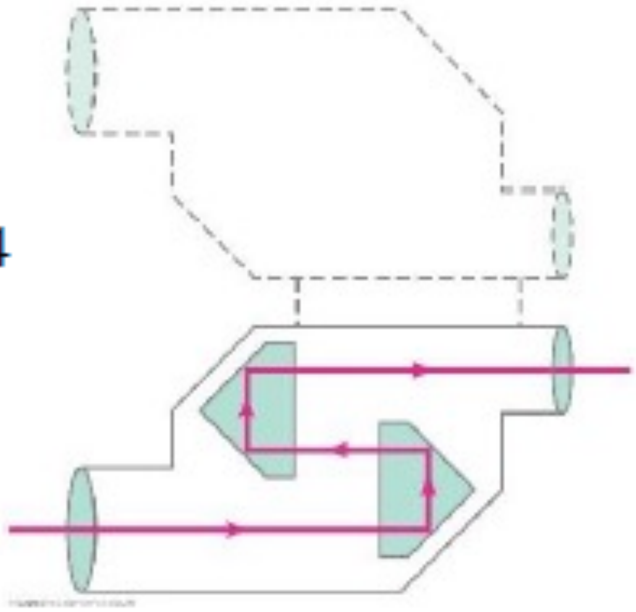
For total internal reflection to occur, $\sin\theta_2 \geq 1$, and so $n_1 \geq \dfrac{1}{\sin 45°} = \boxed{1.41}$.

Total Internal Reflection:

Answer:

(*b*) When there is water outside the surface, we have the following.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad (1.58)\sin 45° = (1.33)\sin\theta_2 \quad \rightarrow \quad \sin\theta_2 = 0.84$$



Because $\sin\theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting.}}$

(*c*) For total reflection when there is water outside the surface, we have the following.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad n_1 \sin 45° = (1.33)\sin\theta_2$$

$$n_1 \sin 45° = (1.33)\sin\theta_2.$$

For total internal reflection to occur, $\sin\theta_2 \geq 1$.

$$n_1 \geq \frac{1.33}{\sin 45°} = \boxed{1.88}$$

Q&A?
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