The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Science A Physics

Lectures 4-6:

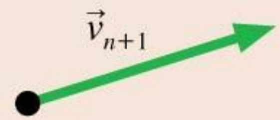
Additional Problems: Free-body Diagrams, Energy, and Momentum

The Acceleration Vector

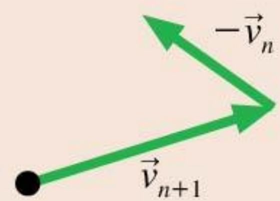
TACTICS BOX 4.1 **Finding the acceleration vector**

To find the acceleration between velocity $\vec{v}_n$ and velocity $\vec{v}_{n+1}$:

❶ Draw the velocity vector $\vec{v}_{n+1}$.

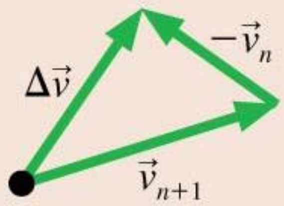


❷ Draw $-\vec{v}_n$ at the tip of $\vec{v}_{n+1}$.

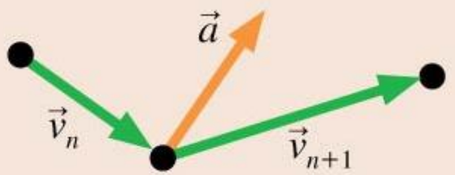


The Acceleration Vector

❸ Draw $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$
$$= \vec{v}_{n+1} + (-\vec{v}_n)$$
This is the direction of $\vec{a}$.



❹ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration between $\vec{v}_n$ and $\vec{v}_{n+1}$.



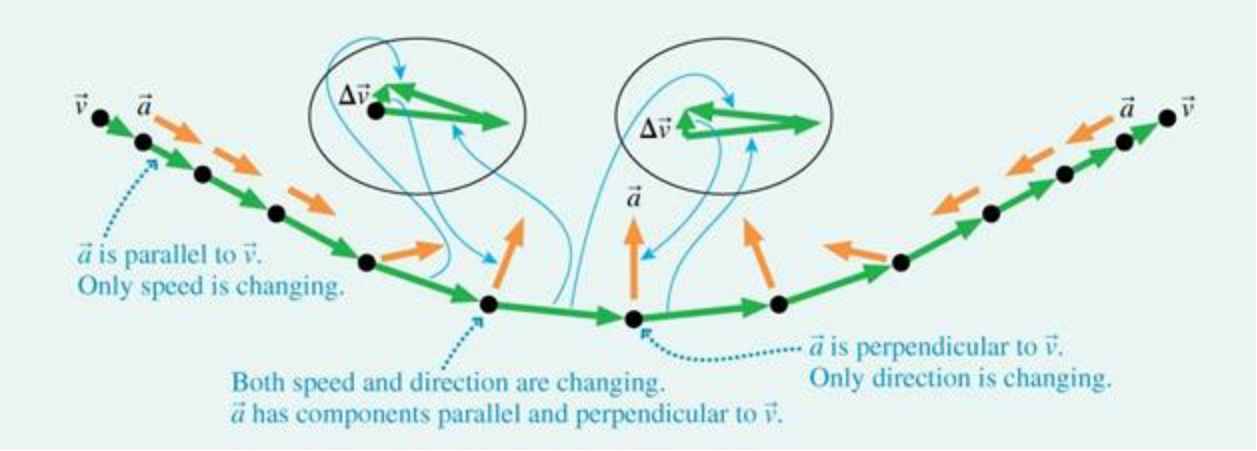
Exercises 1–4

Through the Valley

Q.1 A ball rolls down a long hill, through the valley, and back up the other side. Draw a complete motion diagram of the ball, showing velocity and acceleration vectors.

MODEL: Model the ball as a particle

Through the Valley



VISUALISE:

Where the particle moves along a straight line, it speeds up if $\vec{a}$ and $\vec{v}$ point in the same direction and slows down if $\vec{a}$ and $\vec{v}$ point in the opposite direction.

For linear motion, acceleration is a change of speed. When the direction of $\vec{v}$ changes, as it does when the ball goes through the valley, we need to use vector subtraction to find the direction of $\Delta\vec{v}$ and thus of $\vec{a}$. The procedure is shown at two points in the motion diagram.

PROBLEM-SOLVING STRATEGY 10.1

Conservation of mechanical energy

MODEL Choose a system that is isolated and has no friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + U_f = K_i + U_i$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 8

TACTICS BOX 10.1 **Interpreting an energy diagram**

1. The distance from the axis to the PE curve is the particle's potential energy. The distance from the PE curve to the TE line is its kinetic energy. These are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ doesn't change.
2. A point where the TE line crosses the PE curve is a turning point. The particle reverses direction.
3. The particle cannot be at a point where the PE curve is above the TE line.
4. The PE curve is determined by the properties of the system—mass, spring constant, and the like. You cannot change the PE curve. However, you can raise or lower the TE line simply by changing the initial conditions to give the particle more or less total energy.
5. A minimum in the PE curve is a point of stable equilibrium. A maximum in the PE curve is a point of unstable equilibrium.

Exercises 18–20

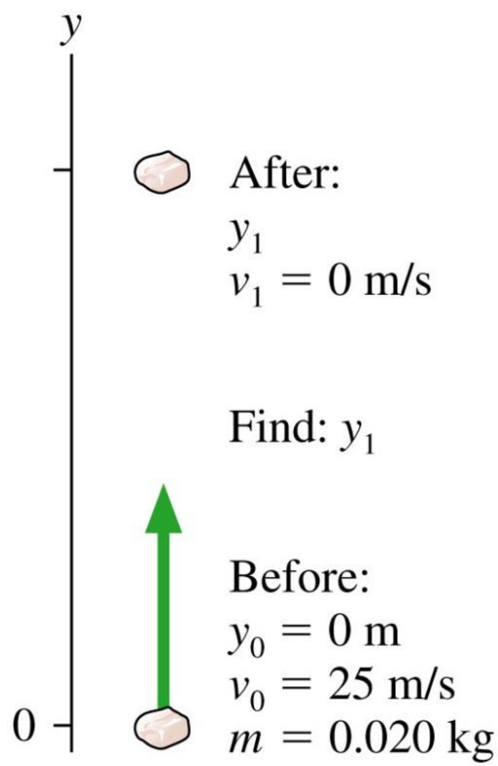
Launching a Pebble

Q.2 Bob uses a slingshot to shoot a 20 g pebble straight up with a speed of 25 m/s. How high does the pebble go?

MODEL:

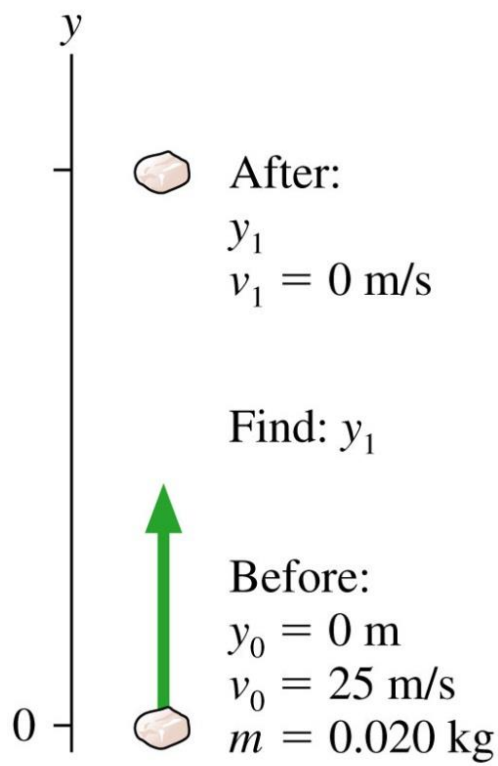
This is free-fall motion, so the sum of the kinetic and gravitational potential energy does not change as the pebble rises.

Launching a Pebble



VISUALISE: The figure above shows a before-and-after pictorial representation. The pictorial representation for energy problems is essentially the same as the pictorial representation you learned for momentum problems. We'll use numerical subscripts 0 and 1 for the initial and final points.

Launching a Pebble

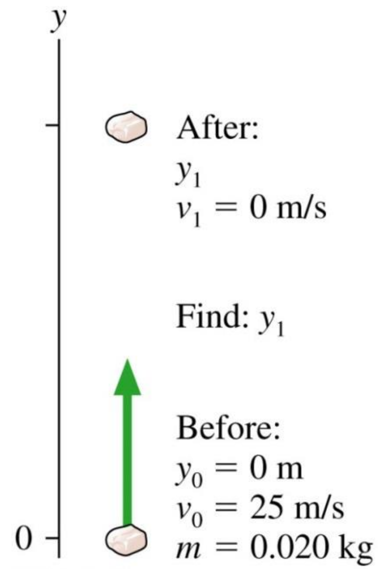


SOLVE:

Equation $K_1 + U_{g1} = K_0 + U_{g0}$ tells us that the sum $K + U_g$ is not changed by the motion. Using the definitions of K and U_g, we have

$$\tfrac{1}{2} mv_1^2 + mgy_1 = \tfrac{1}{2} mv_0^2 + mgy_0$$

Launching a Pebble



$$\tfrac{1}{2} mv_1^2 + mgy_1 = \tfrac{1}{2} mv_0^2 + mgy_0$$

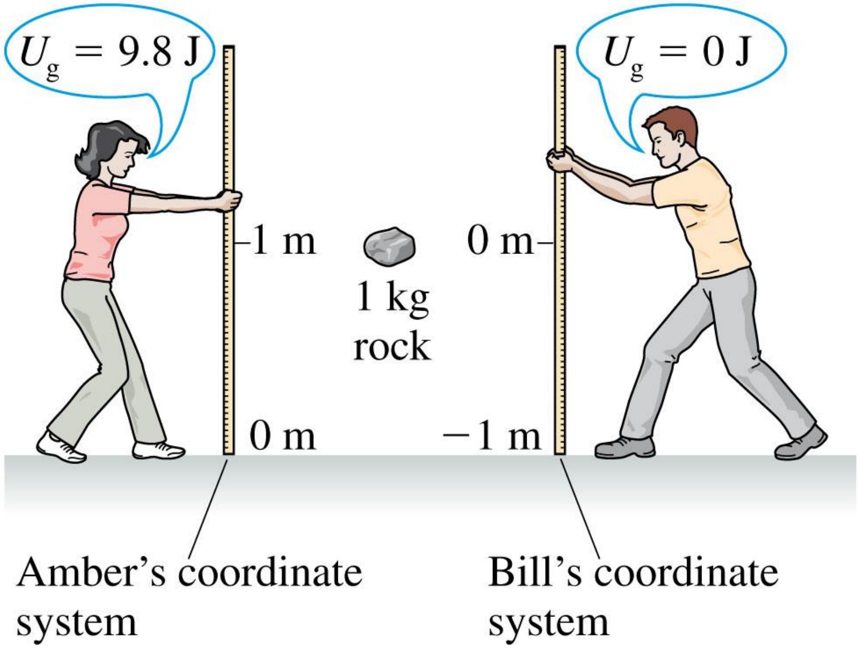
Here $y_0 = 0$ m and $v_1 = 0$ m/s, so the equation simplifies to

$mgy_1 = \tfrac{1}{2} mv_0^2$.

This is easily solved for the height y_1:

$$y_1 = \frac{v_0{}^2}{2g} = \frac{(25\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})} = 32\ \mathrm{m}$$

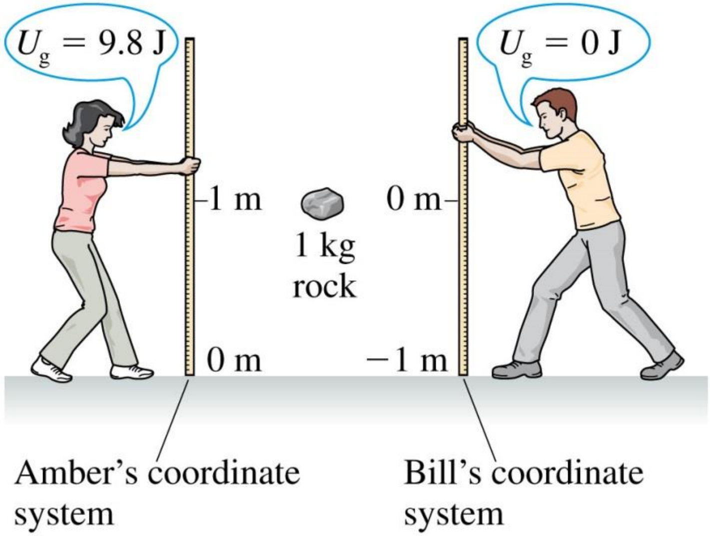
The Speed of a Falling Rock



Q.3 The 1.0 kg rock shown above is released from rest. Use both Amber's and Bill's perspectives to calculate its speed just before it hits the ground.

MODEL: This is free-fall motion, so the sum of the kinetic and gravitational potential energy does not change as the rock falls.

The Speed of a Falling Rock



SOLVE:

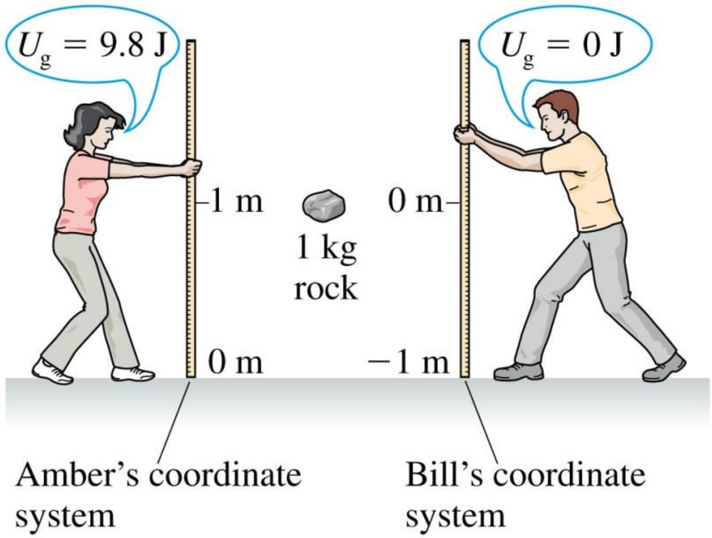
The energy equation is $K_f + U_{gf} = K_i + U_{gi}$. Bill and Amber both agree that $K_i = 0$ because the rock was released from rest, so we have

$$K_f = \frac{1}{2} m v_f^2 = -(U_{gf} - U_{gi}) = - \Delta U$$

According to Amber, $U_{gi} = mgy_i = 9.8$ J and $U_{gf} = mgy_f = 0$ J. Thus

$$\Delta U_{\text{Amber}} = U_{gf} - U_{gi} = - 9.8 \text{ J}$$

The Speed of a Falling Rock



SOLVE:

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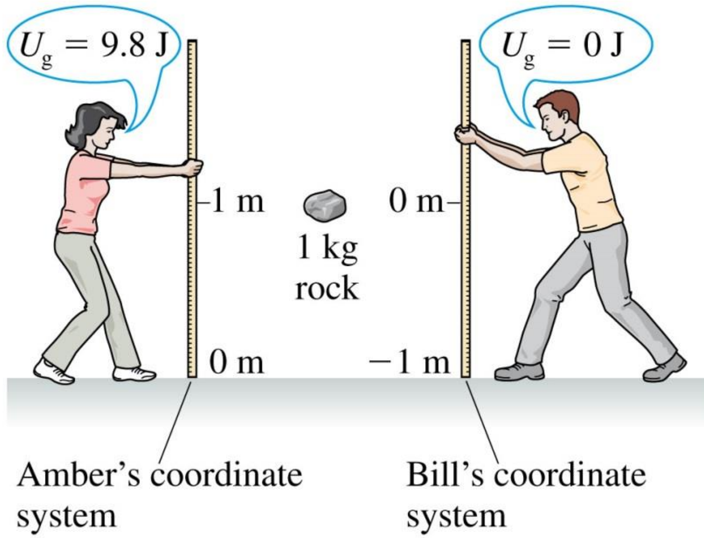
$$K_f = \frac{1}{2} m v_f^2 = -(U_{gf} - U_{gi}) = -\Delta U$$

According to Bill, $U_{gi} = mgy_i = 0$ J and $U_{gf} = mgy_f = -9.8$ J. Thus

$$\Delta U_{Bill} = U_{gf} - U_{gi} = -9.8 \text{ J}$$

The Speed of a Falling Rock

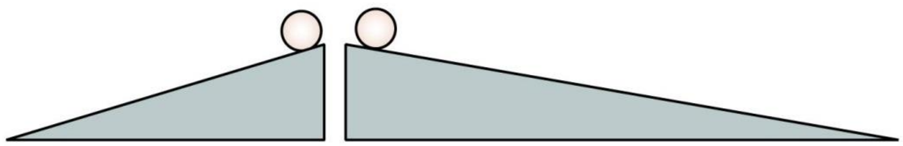


SOLVE:

Bill has different values for U_{gi} and U_{gf} but the **same** value for ΔU. Thus, they both agree that the rock hits the ground with speed

$$v_f = \sqrt{\frac{-2\Delta U}{m}} = \sqrt{\frac{-2(-9.8\ \text{J})}{1.0\ \text{kg}}} = 4.4\ \text{m/s}$$

Kinetic Energy and Gravitational Potential Energy



Q.4 Starting from rest, a marble first rolls down a steeper hill, then down a less steep hill of the same height. For which is it going faster at the bottom?

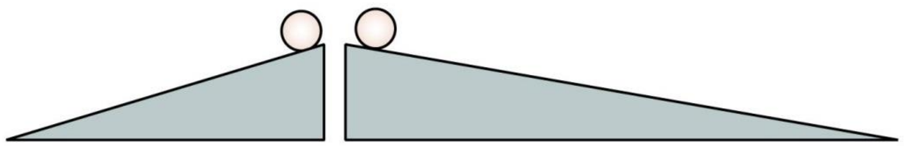
a) Faster at the bottom of the steeper hill.

b) Faster at the bottom of the less steep hill.

c) Same speed at the bottom of both hills.

d) Can't say without knowing the mass of the marble.

Kinetic Energy and Gravitational Potential Energy



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The Speed of a Sled

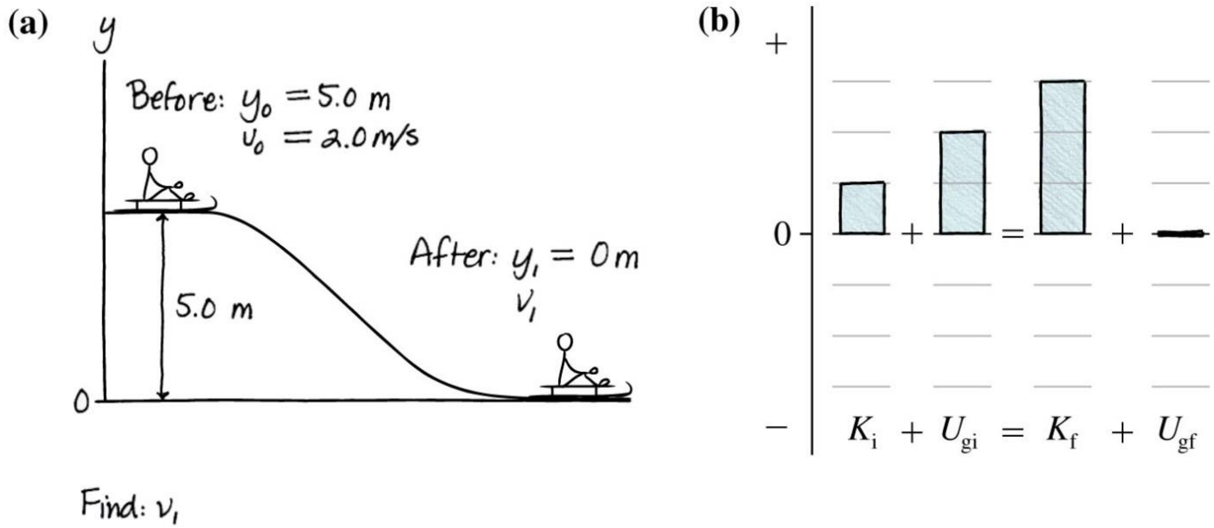
Q.5 Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0-m-high, very slippery slope. What is her speed at the bottom?

The Speed of a Sled

Q.5 Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0-m-high, very slippery slope. What is her speed at the bottom?

MODEL: Model Christine and the sled as a particle. Assume the slope is frictionless. In that case, the sum of her kinetic and gravitational energy does not change as she slides down.

The Speed of a Sled

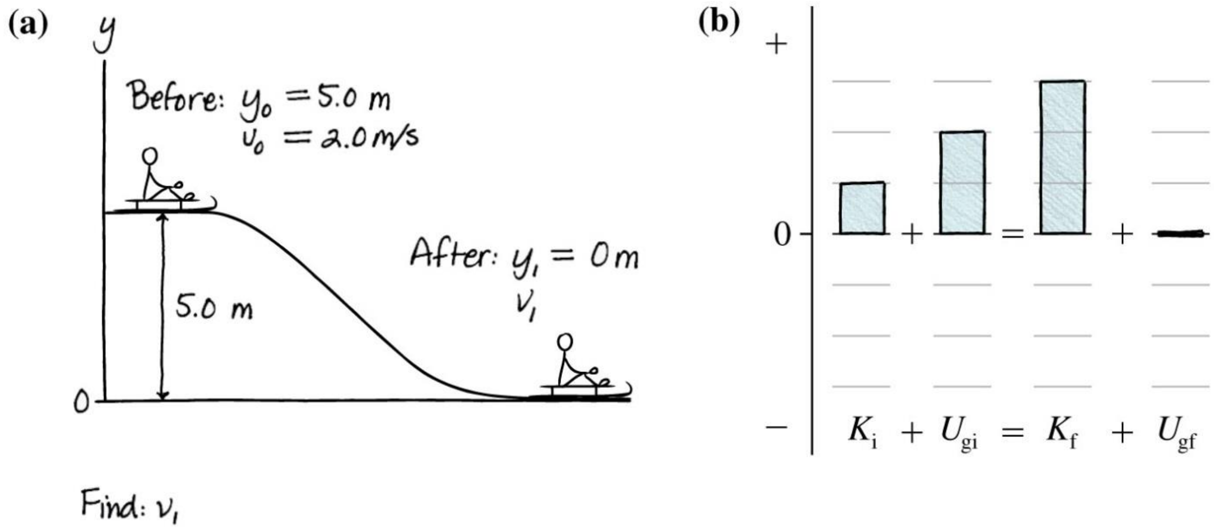


VISUALISE:

Figure (a) shows a before-and-after pictorial representation. We are not told the angle of the slope, or even if it is a straight slope, but the change in potential energy depends only on the height Christine descends and not on the shape of the hill.

Figure (b) is an energy bar chart in which we see an initial kinetic and potential energy as she goes down the slope.

The Speed of a Sled



SOLVE:

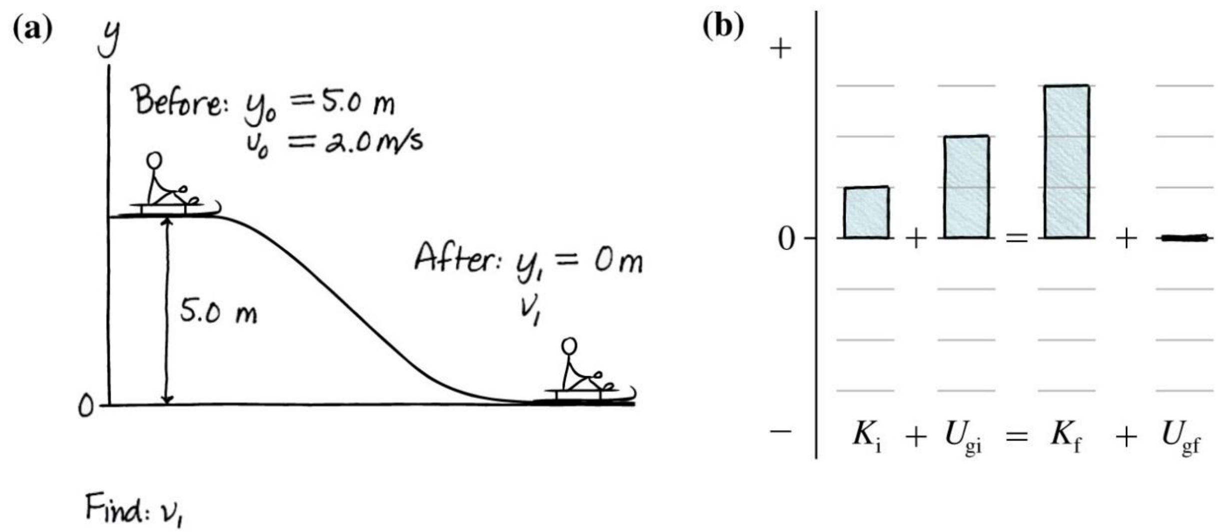
The quantity $K + U_g$ is the same at the bottom of the hill as it was at the top. Thus

$$\tfrac{1}{2} mv_1^2 + mgy_1 = \tfrac{1}{2} mv_0^2 + mgy_0$$

This is easily solved for Christine's speed at the bottom:

$$v_f = \sqrt{v_0{}^2 + 2g(y_0 - y_1)} = \sqrt{v_0{}^2 + 2gh)} = 10 \text{ m/s}$$

The Speed of a Sled



ASSESS:

We did not need the mass of Christine or the sled.

$$v_f = \sqrt{{v_0}^2 + 2g(y_0 - y_1)} = \sqrt{{v_0}^2 + 2gh)} = 10\ \mathrm{m/s}$$

A Spring-Launched Plastic Ball

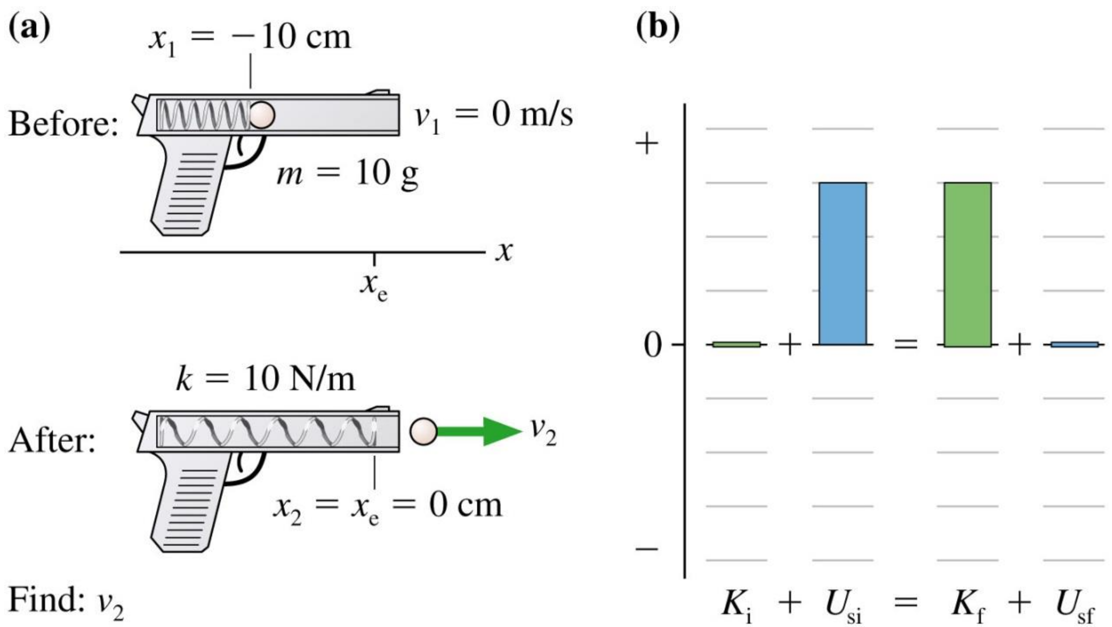
Q.6 A spring-loaded toy gun launches a 10 g plastic ball. The spring, with spring constant 10 N/m is compressed by 10 cm as the ball is pushed into the barrel. When the trigger is pulled, the spring is released and shoots the ball back out. What is the ball's speed as it leaves the barrel? Assume friction is negligible.

A Spring-Launched Plastic Ball

MODEL:

Assume an ideal spring that obeys Hooke's law. Also assume that the gun is held firmly enough to present recoil. There's no friction; hence the mechanical energy $K + U_s$ is conserved.

A Spring-Launched Plastic Ball

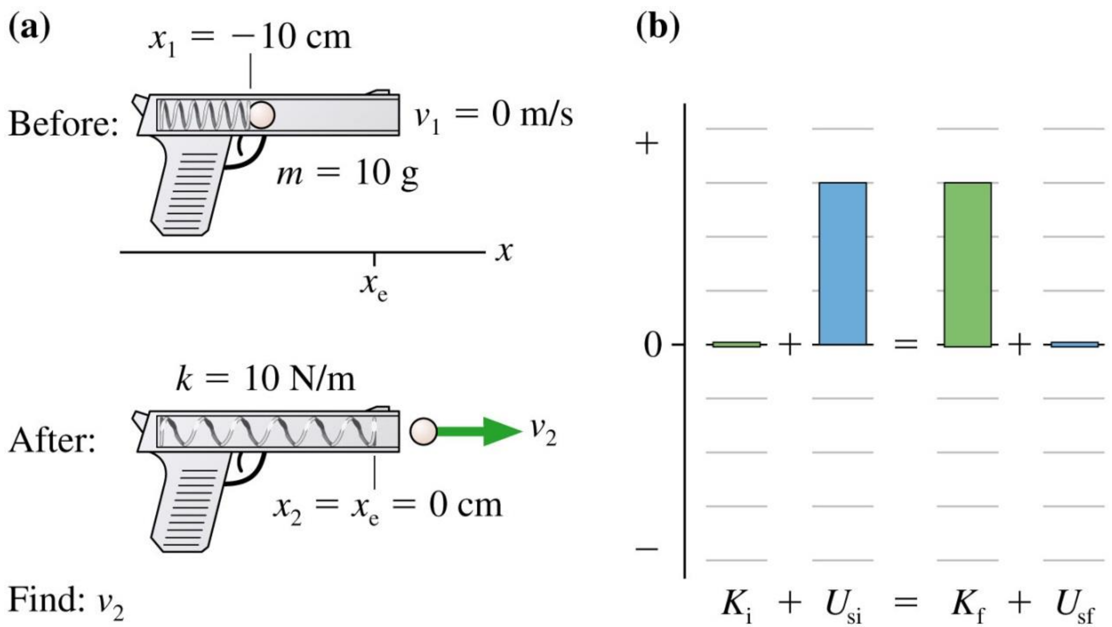


VISUALISE:

Figure (a) shows a before-and-after pictorial representation. We have chosen to put the origin of the coordinate system at the equilibrium position of the free end of the spring.

The bar chart of figure (b) shows the potential energy stored in the compressed spring being entirely transformed into the kinetic energy of the ball.

A Spring-Launched Plastic Ball

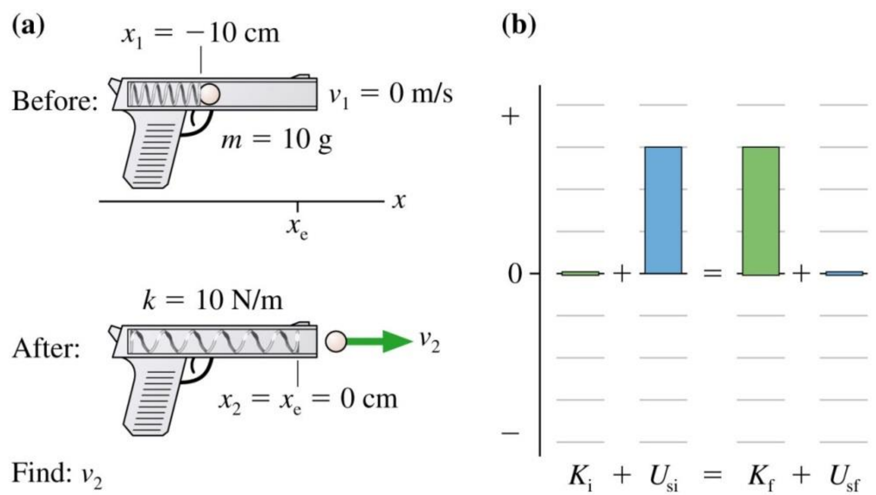


SOLVE:

The energy conservation equation is $K_2 + U_{s2} = K_1 + U_{s1}$. We can use the elastic potential energy of the spring to write this as

$$\tfrac{1}{2} mv_2^2 + \tfrac{1}{2} k(x_2 - x_e)^2 = \tfrac{1}{2} mv_1^2 + \tfrac{1}{2} k(x_1 - x_e)^2$$

A Spring-Launched Plastic Ball



SOLVE:

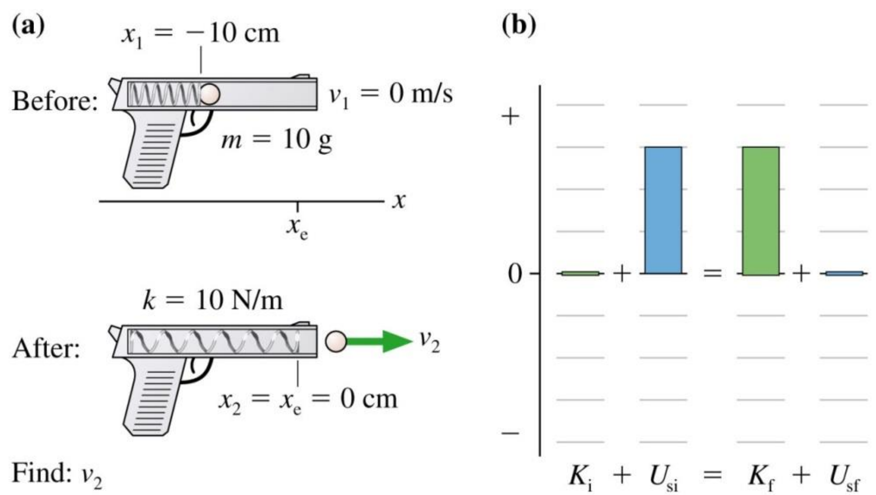
The energy conservation equation is $K_2 + U_{s2} = K_1 + U_{s1}$. We can use the elastic potential energy of the spring to write this as

$$\tfrac{1}{2} mv_2^2 + \tfrac{1}{2} k(x_2 - x_e)^2 = \tfrac{1}{2} mv_1^2 + \tfrac{1}{2} k(x_1 - x_e)^2$$

Notice that we used *x*, rather than the generic *s*, and that we explicitly wrote the meaning of Δx_1 and Δx_2.
Using $x_2 = x_e = 0$ m and $v_1 = 0$ m/s, this simplifies to

$$\tfrac{1}{2} mv_2^2 = \tfrac{1}{2} k\, x^2$$

A Spring-Launched Plastic Ball



SOLVE:

Notice that we used *x*, rather than the generic *s*, and that we explicitly wrote the meaning of Δx_1 and Δx_2.

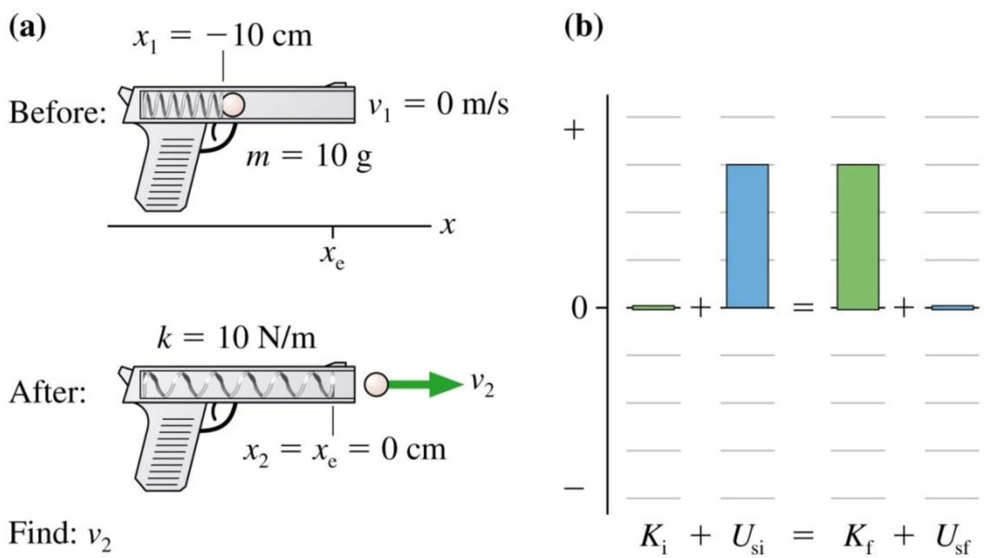
Using $x_2 = x_e = 0$ m and $v_1 = 0$ m/s, this simplifies to

$$\tfrac{1}{2} m v_2^2 = \tfrac{1}{2} k x^2$$

It is now straightforward to solve for the ball's speed;

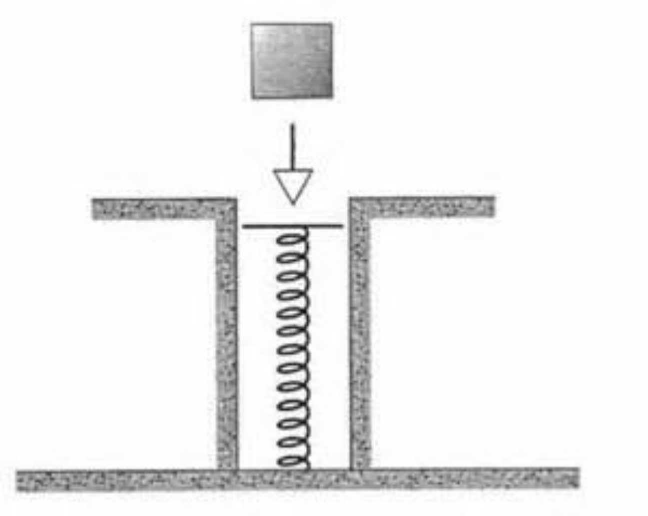
$$v_2 = \sqrt{\frac{k{v_1}^2}{m}} = \sqrt{\frac{(10\ \mathrm{N/m})(-0.10\ \mathrm{m})^2}{0.010\ \mathrm{kg}}} = 3.2\ \mathrm{m/s}$$

A Spring-Launched Plastic Ball



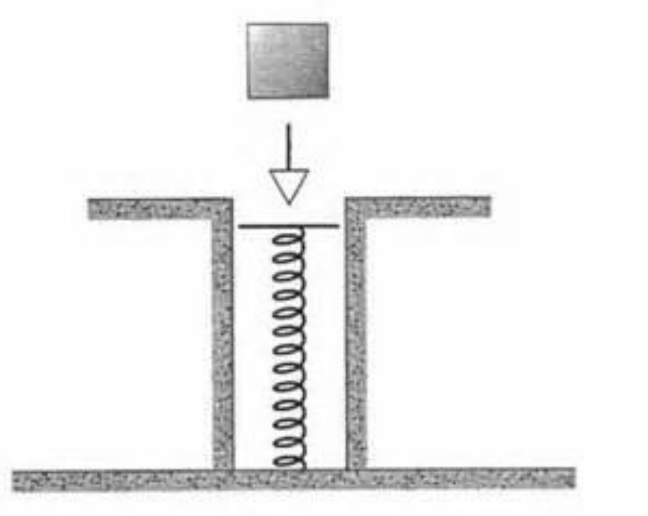
ASSESS: This a problem that we could not have solved with Newton's laws. The acceleration is not constant, and we have not learned how to handle the kinetics of non-constant acceleration. But with conservation of energy – it's easy! The result, 3.2 m/s, seems reasonable for a toy gun.

Balancing a Mass on a Spring



Q.7 A spring of length L_0 and spring constant k is standing on one end. A block of mass m is placed on the spring, compressing it. What is the length of the compressed spring?

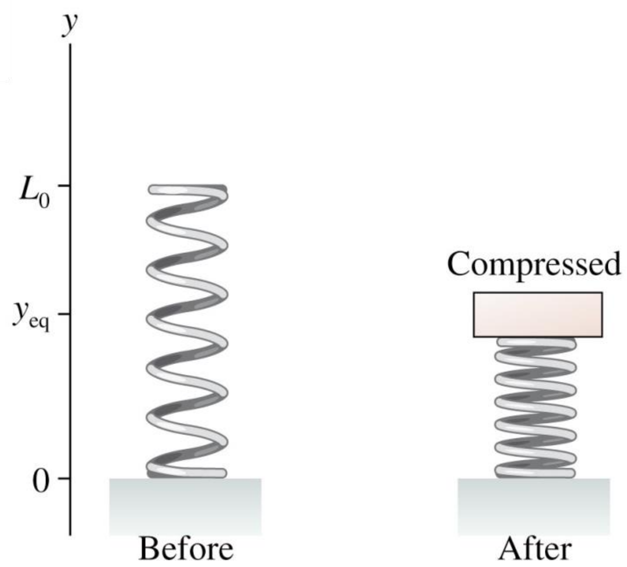
Balancing a Mass on a Spring



MODEL:

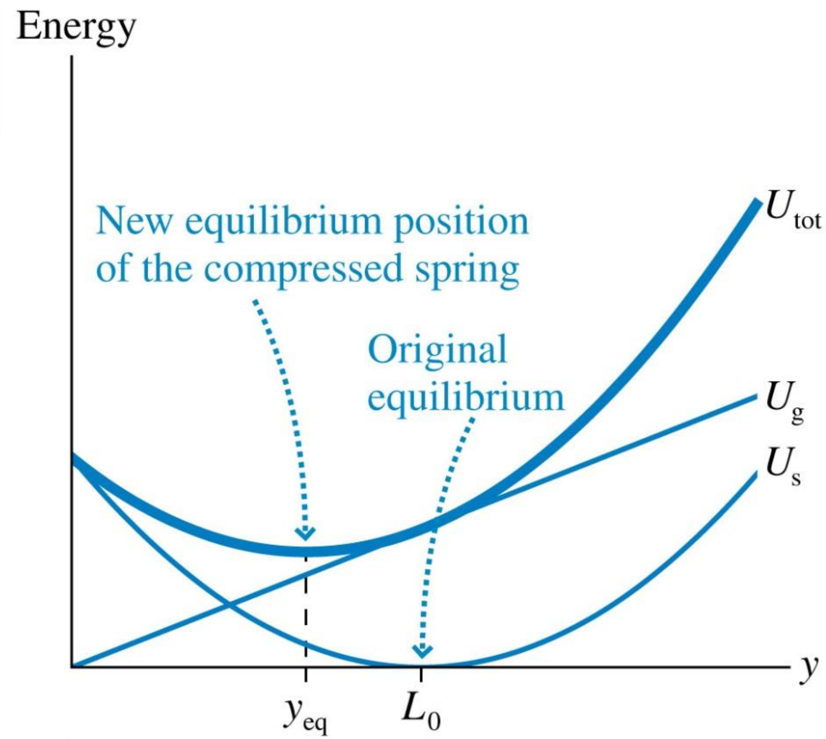
Assume an ideal spring obeying Hooke's law. The block + spring system has both gravitational potential energy U_g ***and*** elastic potential energy U_s. The block sitting on top of the spring is at a point of stable equilibrium (small disturbance cause the block to oscillate slightly around the equilibrium position), so we can solve this problem by looking at the **energy diagram**.

Balancing a Mass on a Spring



VISUALISE: Above is a pictorial representation. We've used a coordinate system with the origin at ground level, so the equilibrium position of the uncompressed spring is $y_e = L_0$.

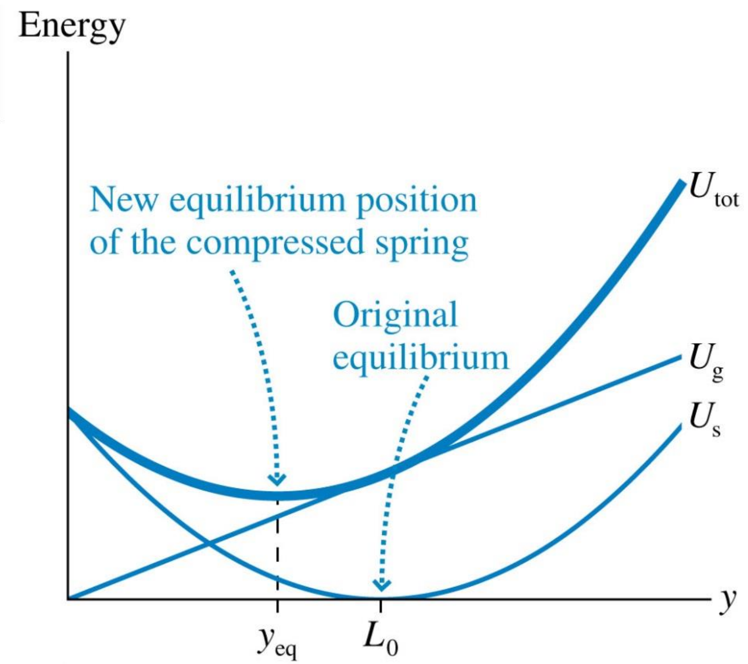
Balancing a Mass on a Spring



SOLVE: The figure shows the two potential energies separately, and also shows the total potential energy:

$$U_{tot} = U_g + U_s = mgy + \tfrac{1}{2}k(y - L_0)^2$$

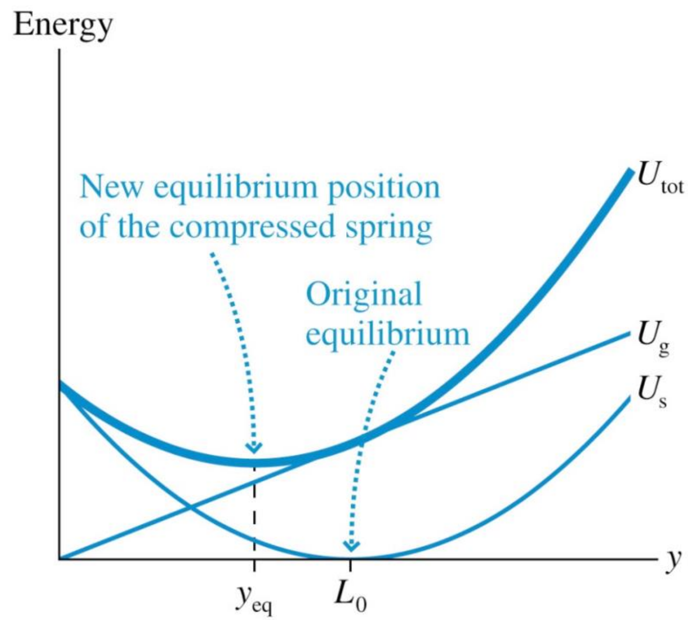
Balancing a Mass on a Spring



The equilibrium position (the minimum of U_{tot}) has shifted from L_0 to a smaller value of y, closer to the ground. We can find the equilibrium by locating the position of the minimum in the PE curve. You may know from calculus that the minimum of a function is at the point where the derivative (or slope) is zero. The derivative of U_{tot} is:

$$\frac{dU_{\mathrm{tot}}}{dy} = mg + k(y - L_0)$$

Balancing a Mass on a Spring



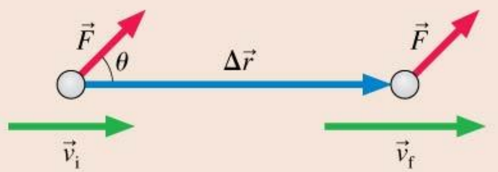
The derivative is zero at the point y_{eq}, so we can easily find:

$$mg + k(y_{\text{eq}} - L_0) = 0$$

$$y_{\text{eq}} = L_0 - \frac{mg}{k}$$

The block compresses the spring by the length mg/k from its original length L_0, giving it a new equilibrium length.

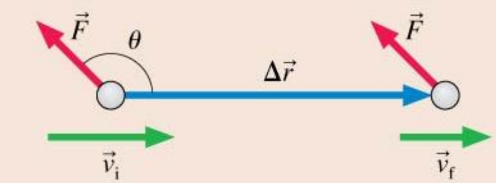
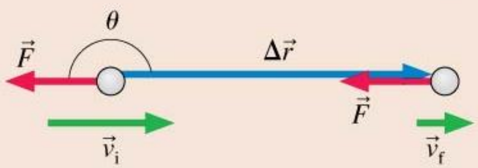
TACTICS BOX 11.1 Calculating the work done by a constant force

Force and displacement	θ	Work W	Sign	Energy transfer
	$0°$	$F(\Delta r)$	+	Energy is transferred into the system. The particle speeds up. K increases.
	$<90°$	$F(\Delta r)\cos\theta$	+	

TACTICS BOX 11.1 **Calculating the work done by a constant force**

Force and displacement	θ	Work W	Sign	Energy transfer
	90°	0	0	No energy is transferred. Speed and K are constant.

TACTICS BOX 11.1 **Calculating the work done by a constant force**

Force and displacement	θ	Work W	Sign	Energy transfer
	$>90°$	$F(\Delta r)\cos\theta$	$-$	Energy is transferred out of the system. The particle slows down. K decreases.
	$180°$	$-F(\Delta r)$	$-$	

Exercises 3–10

Law of conservation of energy The total energy $E_{sys} = E_{mech} + E_{th}$ of an isolated system is a constant. The kinetic, potential, and thermal energy within the system can be transformed into each other, but their sum cannot change. Further, the mechanical energy $E_{mech} = K + U$ is conserved if the system is both isolated and nondissipative.

PROBLEM-SOLVING STRATEGY 11.1

Solving energy problems

MODEL Identify which objects are part of the system and which are in the environment. When possible, choose a system without friction or other dissipative forces. Some problems may need to be subdivided into two or more parts.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart. A free-body diagram is helpful if you're going to calculate work.

PROBLEM-SOLVING STRATEGY 11.1 Solving energy problems

SOLVE If the system is both isolated and nondissipative, then the mechanical energy is conserved:

$$K_f + U_f = K_i + U_i$$

If there are external or dissipative forces, calculate W_{ext} and ΔE_{th}. Then use the more general energy equation

$$K_f + U_f + \Delta E_{th} = K_i + U_i + W_{ext}$$

Kinematics and/or other conservation laws may be needed for some problems.

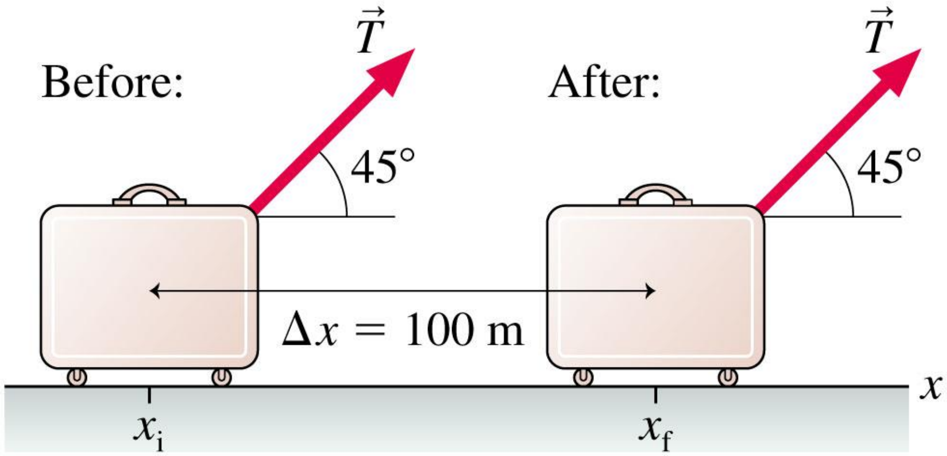
ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Pulling a Suitcase

Q.8 A rope inclined upward at a 45° angle pulls a suitcase through the airport. The tension in the rope is 20 N. How much work does the tension do if the suitcase is pulled 100 m?

MODEL:
Model the suitcase as a particle.

Pulling a Suitcase



SOLVE:

The motion is along the x-axis, so in this case $\Delta r = \Delta x$. We can find that the tension does work:

$$W = T(\Delta x)\cos\theta = (20\ \text{N})(100\ \text{m})\cos 45^\circ \quad = 1400\ \text{J}$$

ASSESS:

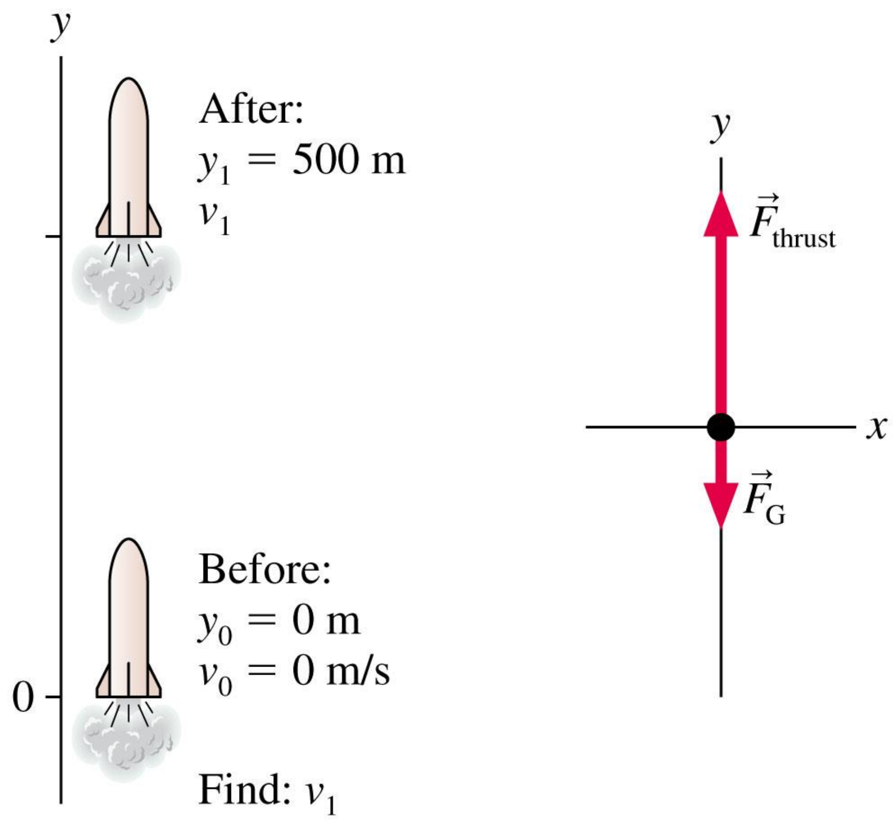
Because a person pulls the rope, we would say informally that the person does 1400 J of work on the suitcase.

Work During a Rocket Launch

Q.9 A 150,000 kg rocket is launched straight up. The rocket motor generates a thrust of 4.0×10^6 N. What is the rocket's speed at a height of 500 m? Ignore air resistance and any slight mass loss.

MODEL: Model the rocket as a particle. The thrust and gravity are constant forces that do work on the rocket.

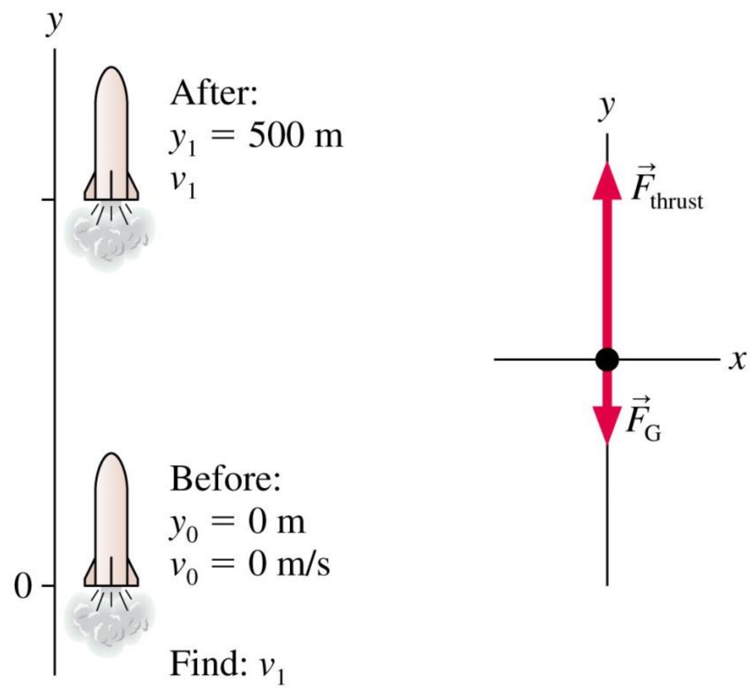
Work During a Rocket Launch



VISUALISE:

The figure above shows a pictorial representation and a free-body diagram.

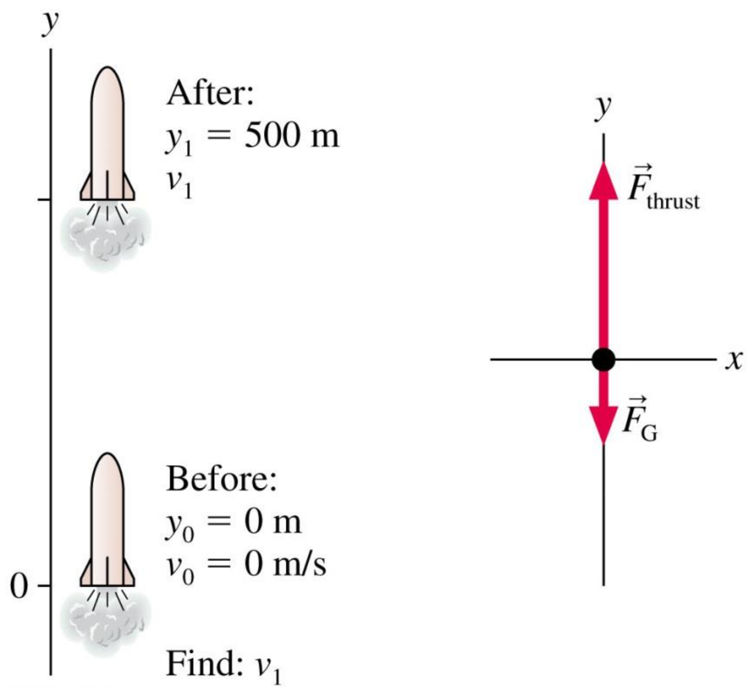
Work During a Rocket Launch



SOLVE: We can solve this problem with the work-kinetic energy theorem $\Delta K = W_{net}$. Both forces do work on the rocket. The thrust is in the direction of motion, with $\theta = 0°$, and thus

$$W_{thrust} = F_{thrust}\,(\Delta r) = (4.0\times10^6 \text{ N})(500 \text{ m}) = 2.00\times10^9 \text{ J}$$

Work During a Rocket Launch

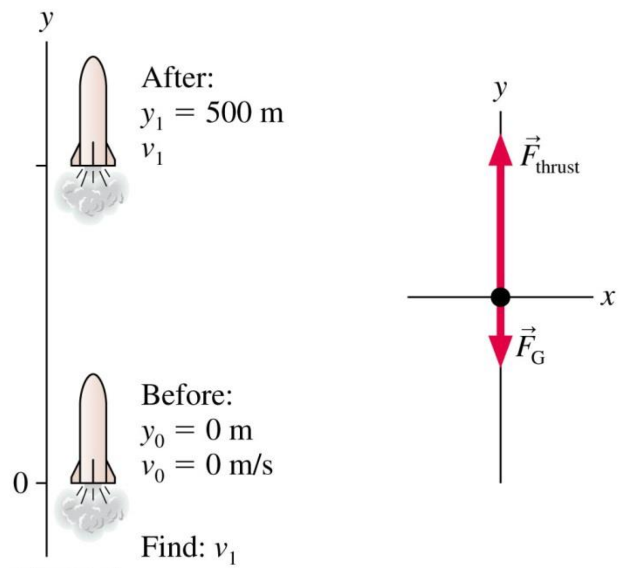


SOLVE: The gravitational force points downward, opposite the displacement $\Delta\vec{r}$, so $\theta = 180°$. Thus, the work done by gravity is

$$W_{grav} = -F_G\,(\Delta r) = -mg\,(\Delta r)$$

$$= -(1.5\times10^5 \text{ kg})(9.8 \text{ m/s}^2)(500 \text{ m}) = -0.74\times10^9 \text{ J}$$

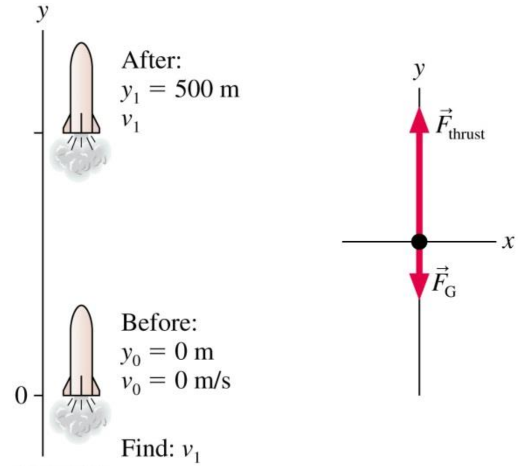
Work During a Rocket Launch



The work done by the thrust is positive. By itself, the thrust would cause the rocket to speed up.

The work done by gravity is negative, not because $\vec{F}_G$ points down ,but because $\vec{F}_G$ is opposite the displacement. By itself, gravity would cause the rocket to slow down.

Work During a Rocket Launch



- The work-kinetic energy theorem, using v_0 = 0 m/s, is

$$\Delta K = \tfrac{1}{2} m v_1^2 - 0 = W_{\text{net}} = W_{\text{thrust}} + W_{\text{grav}} = 1.26 \times 10^9 \text{ J}$$

- This is easily solved for the speed:

$$v_1 = \sqrt{\frac{2W_{\text{net}}}{m}} = 130 \text{ m/s}$$

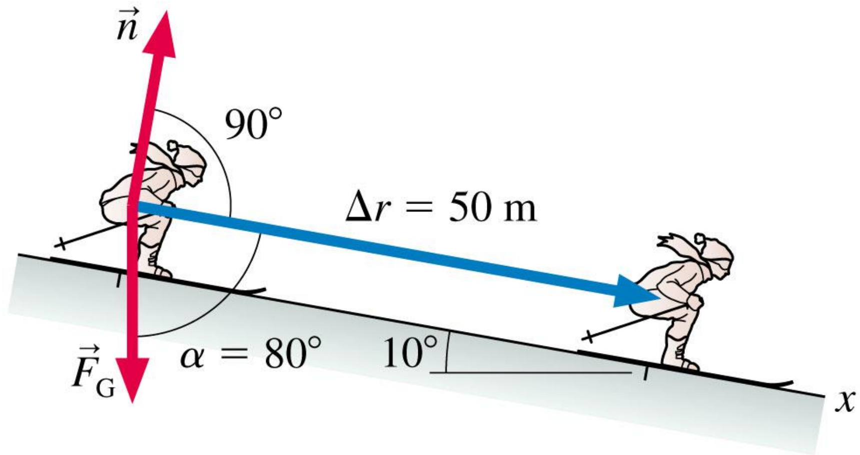
ASSESS: The net work is positive, meaning that energy is transferred to the rocket. In response, the rocket speeds up.

Calculating Work Using the Dot Product

Q.10 A 70 kg skier is gliding at 2.0 m/s when he starts down a very slippery 50-m-long, 10° slope. What is his speed at the bottom?

MODEL: Model the skier as a particle and interpret "very slippery" to mean frictionless. Use the work-kinetic energy theorem to find his final speed.

Calculating Work Using the Dot Product



Before:

$x_0 = 0$ m
$v_0 = 2.0$ m/s
$m = 70$ kg

After:

$x_1 = 50$ m
v_1

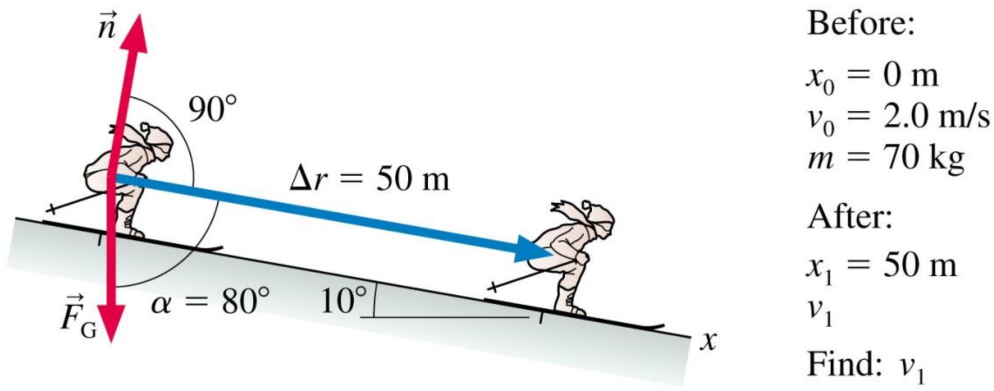
Find: v_1

SOLVE: The only forces on the skier are $\vec{F}_G$ and $\vec{n}$. The normal force is perpendicular to the motion and thus does no work. The work done by gravity is easily calculated as a product:

$$W = \vec{F}_G \cdot \Delta\vec{r} = \mathrm{mg}(\Delta r)\cos\alpha$$

$$= (70\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(50\ \mathrm{m})\cos 80° = 5960\ \mathrm{J}$$

Calculating Work Using the Dot Product



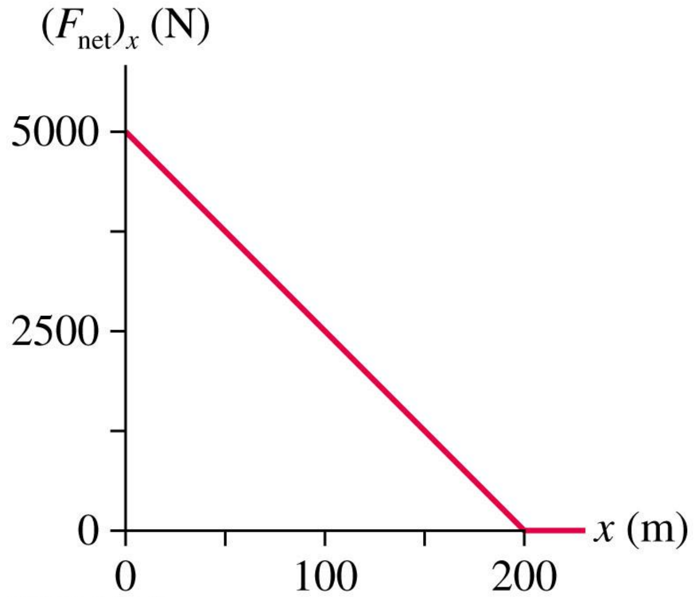
$$= (70\text{ kg})(9.8\text{ m/s}^2)(50\text{ m})\cos 80° = 5960\text{ J}$$

Notice that the angle between the vectors is 80°, not 10°. Then, from the work-kinetic energy theorem, we find

$$\Delta K = \frac{1}{2}m{v_1}^2 - \frac{1}{2}m{v_0}^2 = W$$

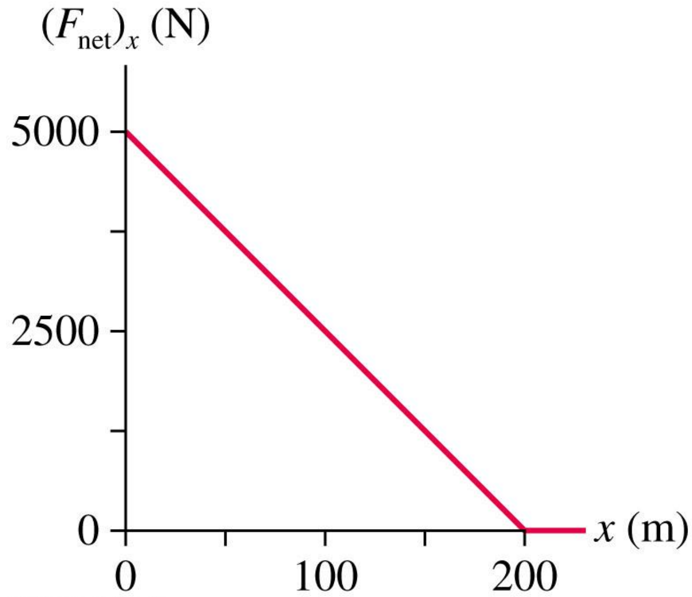
$$v_1 = \sqrt{{v_0}^2 + \frac{2W}{m}} = \sqrt{(2.0\text{ m/s})^2 + \frac{2(56960\text{ J})}{70\text{ kg}}} = 13\text{ m/s}$$

Using Work to Find the Speed of a Car



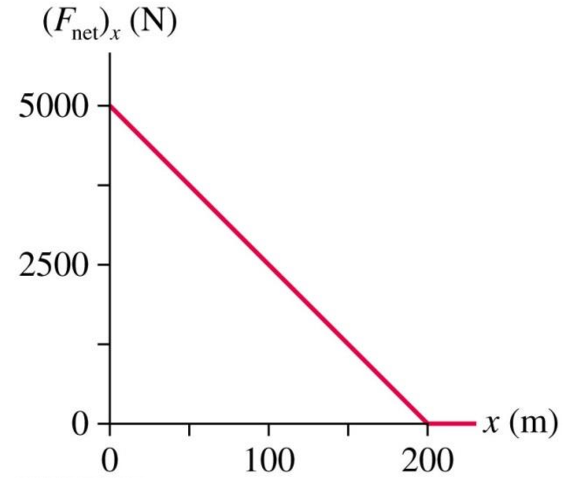
Q.11 A 1500 kg car accelerates from rest. The figure shows the net force on the car (propulsion force minus any drag forces) as it travels from x = 0 m to x = 200 m. What is the car's speed after travelling 200 m?

Using Work to Find the Speed of a Car



SOLVE: The acceleration $a_x = (F_{net})/m$ is high as the car starts, but decreases as the car picks up speed because of increasing drag. The figure is a realistic portrayal of the net force on a car. But a variable force means that we cannot use the familiar constant-acceleration kinematics. Instead, we can use the work-kinetic energy theorem.

Using Work to Find the Speed of a Car



SOLVE:

Because v_i = 10 m/s, we have

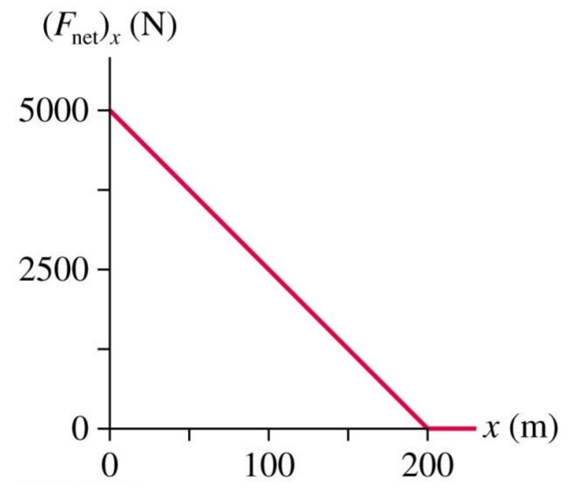
$$\Delta K = \frac{1}{2} m v_f^2 - 0 = W_{net}$$

Starting from x_i = 0 m, the work is

$$W_{net} = \int_{0\text{ m}}^{x_f} (F_{net})_x dx$$

$$= \text{area under the} (F_{net})_x \text{−versus} - x \text{ graph from 0 m to } x_f$$

Using Work to Find the Speed of a Car



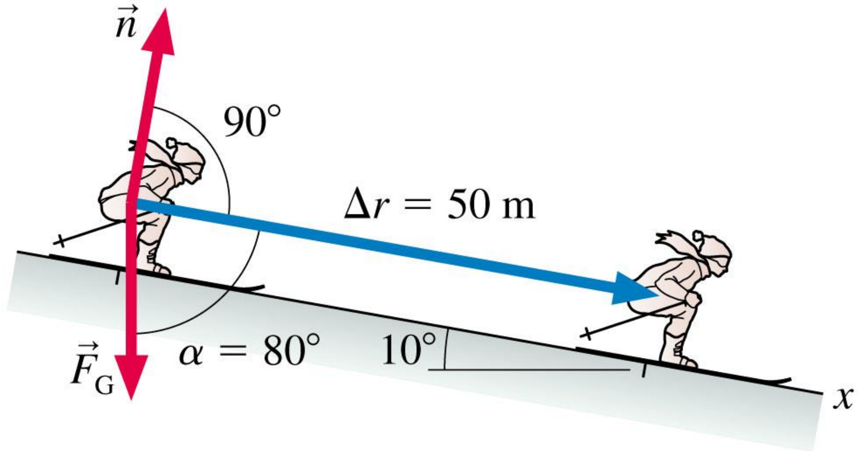
The area under the curve is that of a triangle of width 200 m. Thus

$$W_{net} = area = \frac{1}{2}(5000\ \mathrm{N})(200\ \mathrm{m}) = 500{,}000\ \mathrm{J}$$

The work-kinetic energy theorem then gives

$$v_f = \sqrt{\frac{2W_{\mathrm{net}}}{m}} = \sqrt{\frac{2(500{,}000\ \mathrm{J})}{1500\ \mathrm{kg}}} = 26\ \mathrm{m/s}$$

Using Work and Potential Energy



Before:

$x_0 = 0$ m
$v_0 = 2.0$ m/s
$m = 70$ kg

After:

$x_1 = 50$ m
v_1

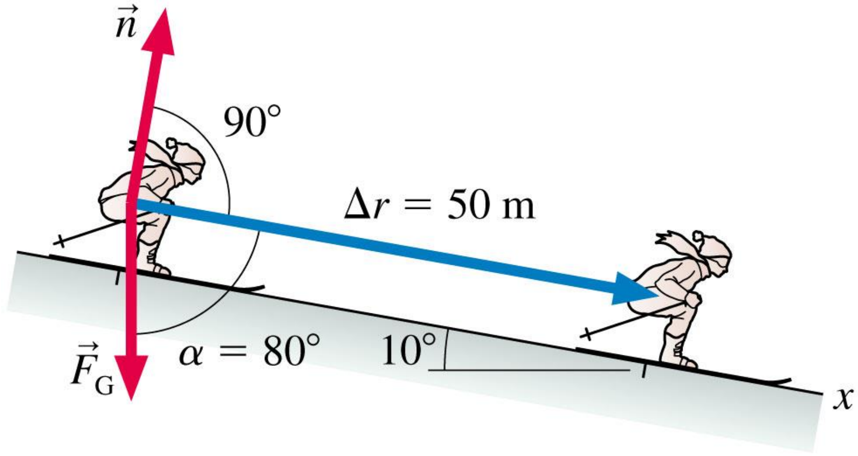
Find: v_1

Q.12

The skier from question 5 repeats his run after the wind comes up. Recall that the 70 kg skier was gliding at 2.0 m/s when he started down a 50-m-long, 10°, frictionless slope. What is his speed at the bottom if the wind exerts a steady 50 N retarding force opposite his motion?

MODEL: This time let the system be the skier and the earth.

Using Work and Potential Energy



Before:
$x_0 = 0$ m
$v_0 = 2.0$ m/s
$m = 70$ kg

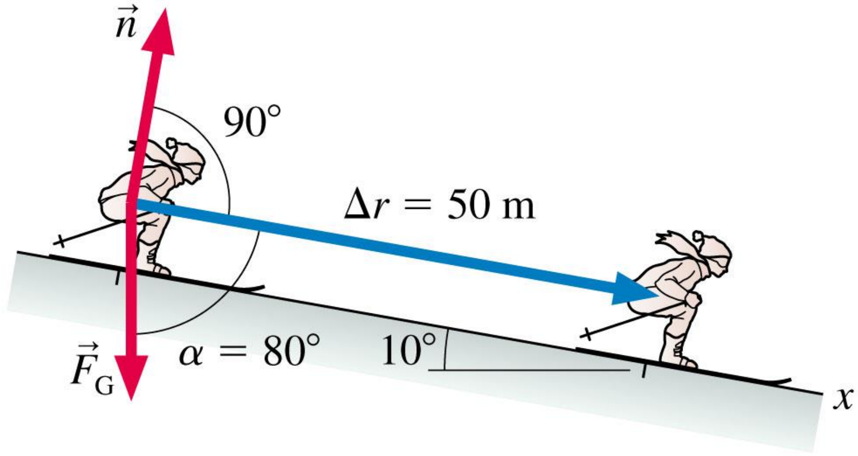
After:
$x_1 = 50$ m
v_1

Find: v_1

SOLVE: In solving this problem with the work-kinetic energy theorem, we had to calculate the work done by gravity. Gravity is a conservative force that we can associate with the gravitational potential energy U_g. The retarding force of the wind is non-conservative. Thus

$$\Delta K + \Delta U_g = W_{nc} = W_{wind}$$

Using Work and Potential Energy



Before:
$x_0 = 0$ m
$v_0 = 2.0$ m/s
$m = 70$ kg

After:
$x_1 = 50$ m
v_1

Find: v_1

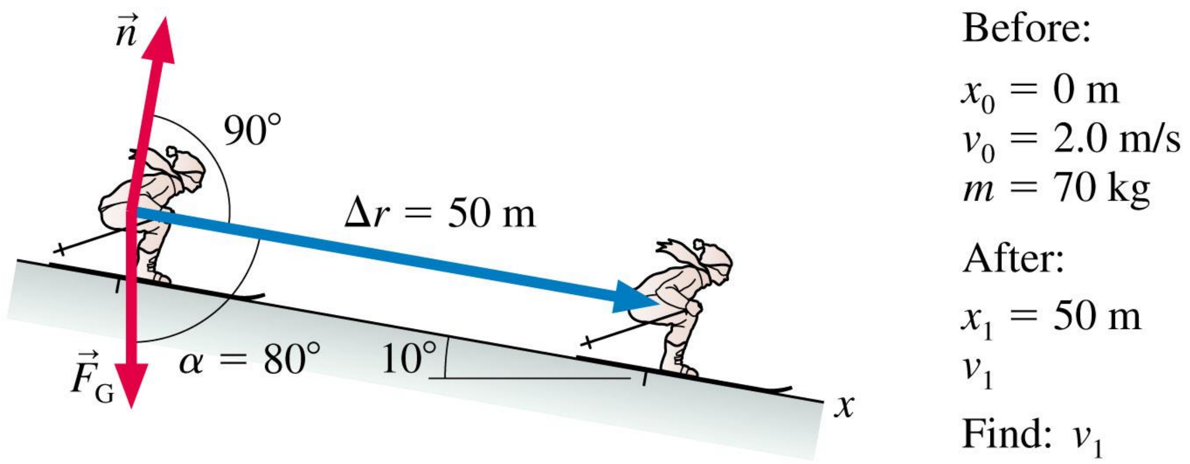
The gravitational potential energy is $U_g = mgy$. Because the wind force is opposite the skier's motion, with $\theta = 180°$, it does work $W_{wind} = \vec{F}_{wind} \cdot \Delta\vec{r} = -F_{wind} \cdot \Delta r$.
Thus, the energy equation becomes

$$\frac{1}{2}mv_1{}^2 - \frac{1}{2}mv_0{}^2 + mgy_1 - mgy_0 = -F_{wind} \cdot \Delta r$$

Using the values given, we find

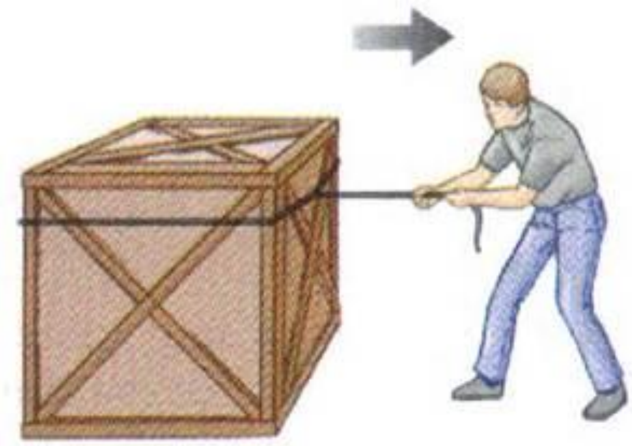
$$v_1 = \sqrt{v_0{}^2 + 2gy_0 - 2F_{\text{wind}} \cdot \Delta r\,/m} = 10\ \text{m/s}$$

Using Work and Potential Energy



ASSESS: What appeared to be a difficult problem, with both gravity and a retarding force, turned out to be straightforward when analyzed with energy and work. The skier's final speed is about 25% slower when the wind is blowing.

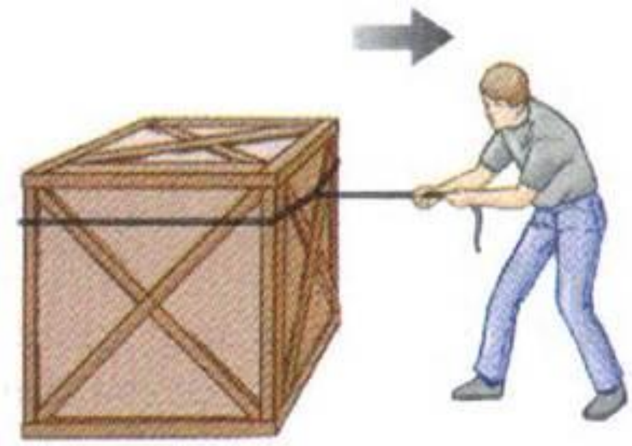
Calculating the Increase in Thermal Energy



Q.13 A rope pulls a 10 kg wooden crate 3.0 m across a wood floor. What is the change in thermal energy? The coefficient of kinetic friction is 0.20.

MODEL: Let the system be crate + floor. Assume the floor is horizontal.

Calculating the Increase in Thermal Energy



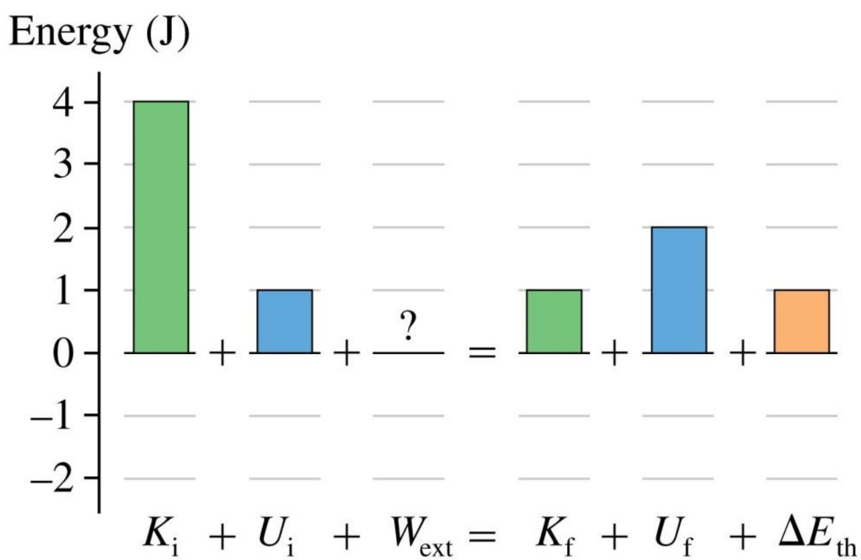
SOLVE: The friction force on an object moving on a horizontal surface is $F_k = \mu_k n = \mu_k mg$. Thus, the change in thermal energy, is

$$\Delta E_{th} = f_k \Delta s = \mu_k mg\Delta s = (0.20)(10\ \text{kg})(9.80\ \text{m/s}^2)(3.0\ \text{m}) = 59\ \text{J}$$

ASSESS: The thermal energy of the crate and floor increases by 59 J. We cannot determine ΔE_{th} for the crate (or floor) alone.

Energy Bar Charts

Q.14 How much work is done by the environment in the process represented by the energy bar chart?



a) −2 J

b) −1 J

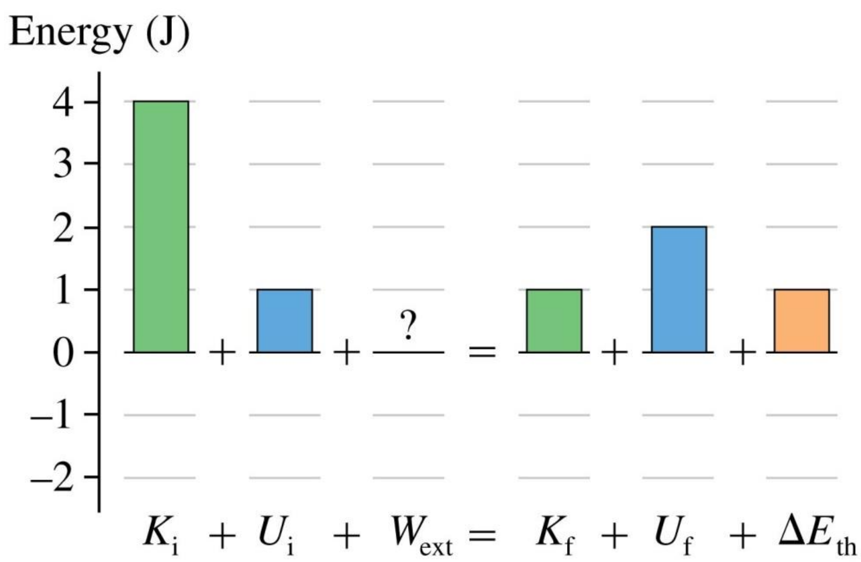
c) 0 J

d) 1 J

e) 2 J

Energy Bar Charts

Q.14 How much work is done by the environment in the process represented by the energy bar chart?



a) –2 J

b) –1 J

c) 0 J

d) 1 J

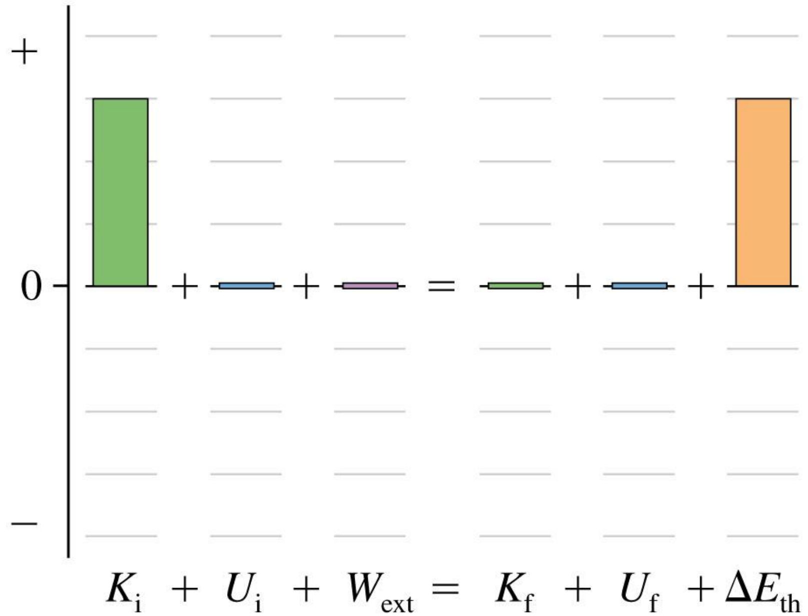
e) 2 J

The system started with 5 J but ends with 4 J.
1 J must have been transferred from the system to the environment as work.

Energy Bar Charts

Q.15 A speeding car skids to a halt. Show the energy transfers and transformations on an energy bar chart.

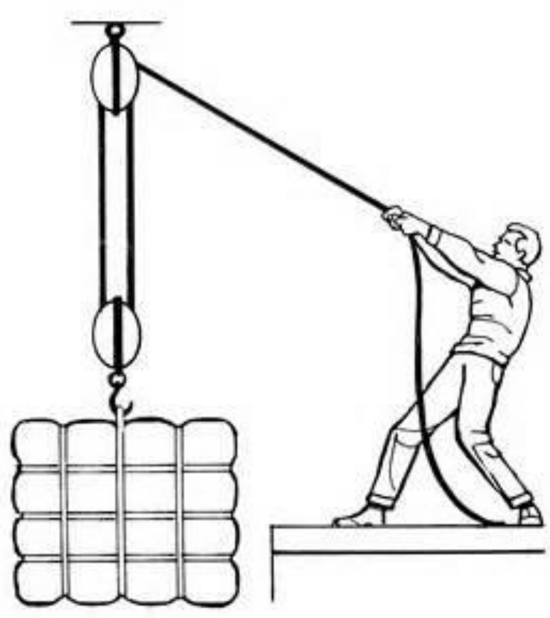
Energy Bar Charts



SOLVE:

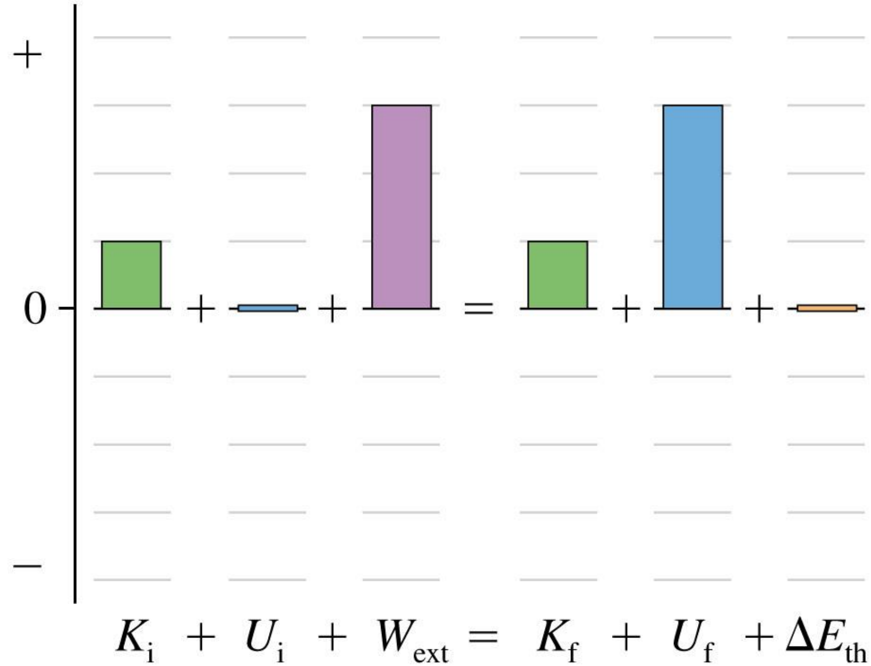
The car has an initial kinetic energy K_i. That energy is transformed into the thermal energy of the car and the road. The potential energy doesn't change, and no work is done by external forces, so the process is an energy transformation $K_i \rightarrow E_{th}$. This is shown in the above figure. E_{sys} is conserved, but E_{mech} is not.

Energy Bar Charts



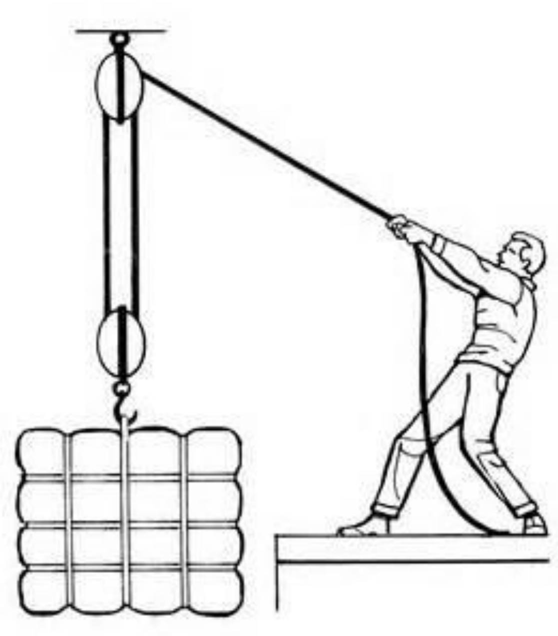
Q.16 A rope lifts a box at constant speed. Show the energy transfers and transformations on an energy bar chart.

Energy Bar Charts



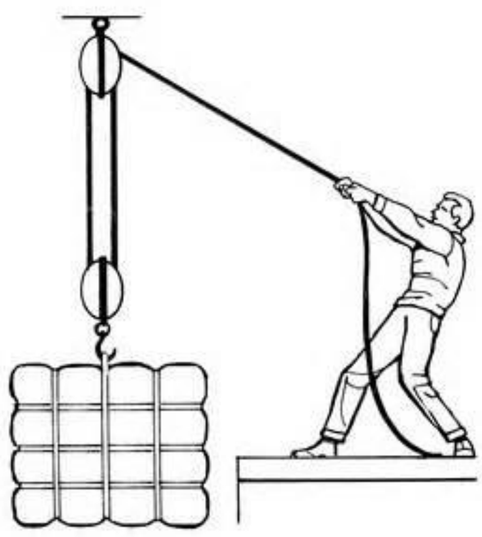
SOLVE: The tension in the rope is an external force that does work on the box, increasing the potential energy of the box. The kinetic energy is unchanged because the speed is constant. The process is an energy transfer $W_{ext} \rightarrow U_f$, as the figure shows. This is not an isolated system, so E_{sys} is not conserved.

Energy Bar Charts



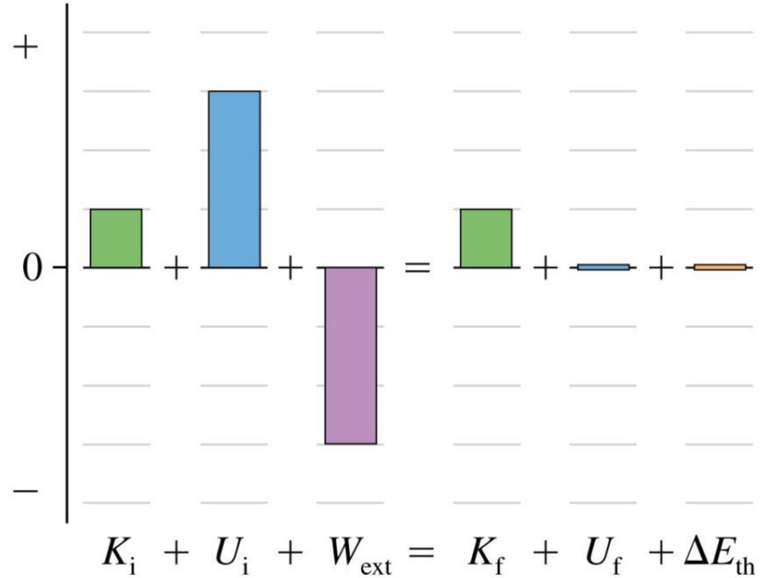
Q.17 The box that was lifted in question 16 now falls at a steady speed as the rope spins a generator and causes a lightbulb to glow. Air resistance is negligible. Show the energy transfers and transformations on an energy bar chart.

Energy Bar Charts



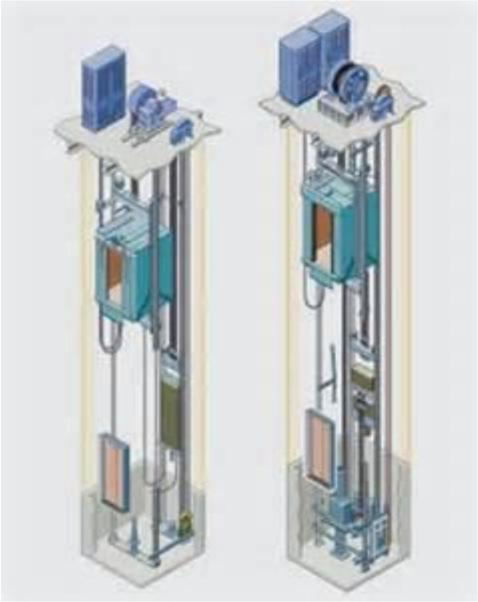
SOLVE: The initial potential energy decreases, but K does not change and $E_{th} = 0$. The tension in the rope is an external force that does work, but W_{ext} is negative in this case because $\vec{T}$ points up while the displacement $\Delta\vec{r}$ is down. Negative work means that energy is transferred from the system to the environment or, in more informal terms, that the system does work on the environment.

Energy Bar Charts



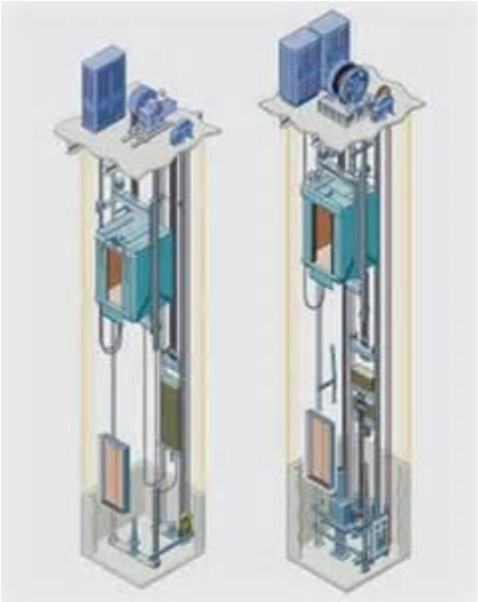
SOLVE: The falling box does work on the generator to spin it and light the bulb. Energy is transferred out of the system and eventually ends up in the lightbulb as electrical energy. The process is $U_i \rightarrow W_{ext}$. This is shown in the above figure.

Choosing a Motor



Q.18 What power motor is needed to lift a 2000 kg elevator at a steady 3.0 m/s?

Choosing a Motor

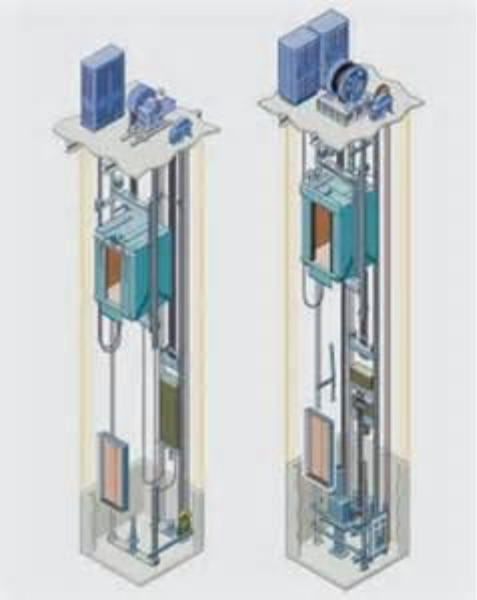


SOLVE:

The tension in the cable does work on the elevator to lift it. Because the cable is pulled by the motor, we say that the motor does the work of lifting the elevator. The net force is zero because the elevator moves at constant velocity, so the tension is simply $T = mg = 19{,}600$ N. The energy gained by the elevator is

$$\Delta E_{\mathrm{sys}} = W_{\mathrm{ext}} = T\Delta y$$

Choosing a Motor



SOLVE:

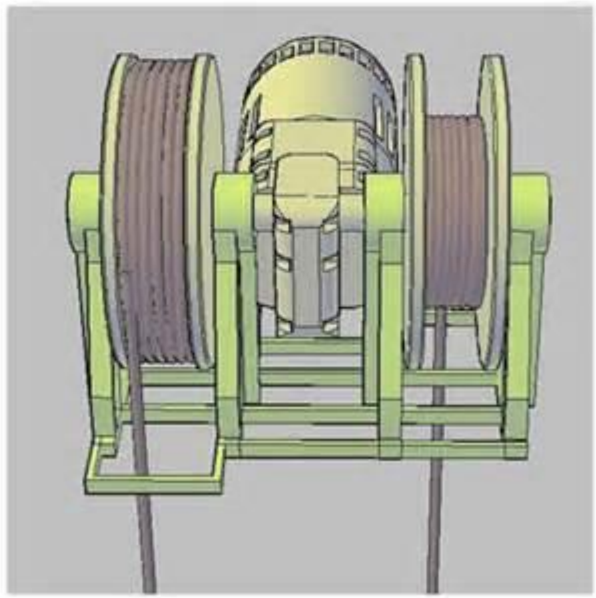
The power required to give the system this much energy in a time interval Δt is

$$P = \frac{\Delta E_{\text{sys}}}{\Delta \text{t}} = \frac{T\Delta y}{\Delta \text{t}}$$

But $\Delta y = v\Delta \text{t}$, so

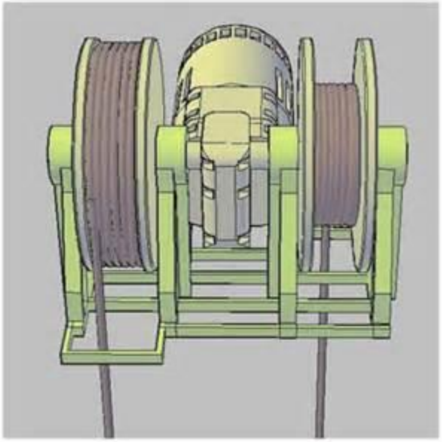
$P = Tv = (19{,}600\ \text{N})(3.0\ \text{m/s}) = 58{,}800\ \text{W} = 79\ \text{hp}$

Power Output of a Motor



Q.19 A factory uses a motor and a cable to drag a 300 kg machine to the proper place on the factory floor. What power must the motor supply to drag the machine at a speed of 0.50 m/s? The coefficient of friction between the machine and the floor is 0.60.

Power Output of a Motor



SOLVE: The force applied by the motor, through the cable, is the tension force $\vec{T}$. This force does work on the machine with power $P = Tv$. The machine is in equilibrium because the motion is at constant velocity; hence the tension in the rope balances the friction and is $T = f_k = \mu_k mg$

The motor's power output is

$$P = Tv = \mu_k mgv = 882\ \text{W}$$

TACTICS BOX 9.1 **Drawing a before-and-after pictorial representation**

1. **Sketch the situation.** Use two drawings, labeled "Before" and "After," to show the objects *before* they interact and again *after* they interact.
2. **Establish a coordinate system.** Select your axes to match the motion.
3. **Define symbols.** Define symbols for the masses and for the velocities before and after the interaction. Position and time are not needed.

Exercises 17–19

TACTICS BOX 9.1 **Drawing a before-and-after pictorial representation**

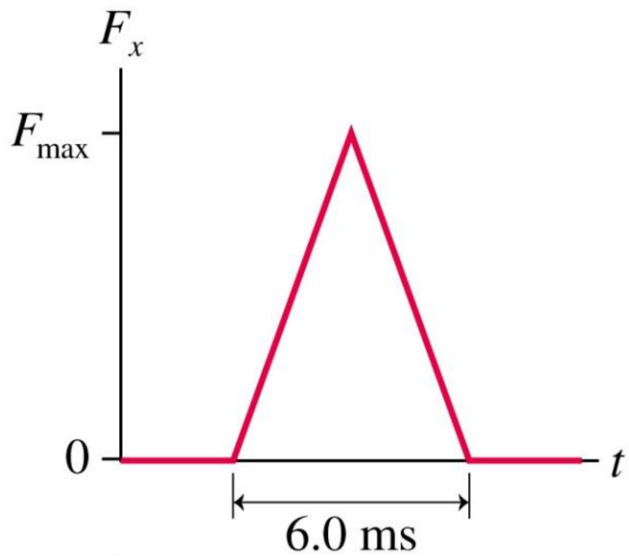
❹ **List known information.** Give the values of quantities that are known from the problem statement or that can be found quickly with simple geometry or unit conversions. Before-and-after pictures are simpler than the pictures for dynamics problems, so listing known information on the sketch is adequate.

❺ **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined in step 3.

❻ If appropriate, **draw a momentum bar chart** to clarify the situation and establish appropriate signs.

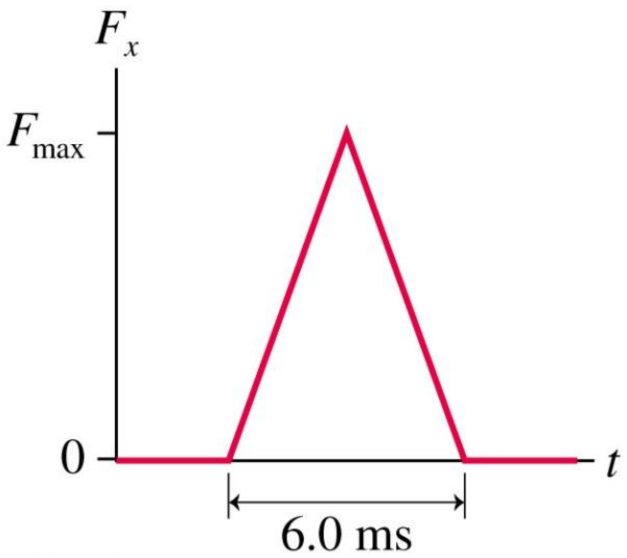
Exercises 17–19

Hitting a Baseball



Q.20 A 150 g baseball is thrown with a speed of 20 m/s. It is hit straight back toward the pitcher at a speed of 40 m/s. The interaction force between the ball and the bat is shown in the figure above. What *maximum force* F_{max} does the bat exert on the ball? What is the *average force* of the bat on the ball?

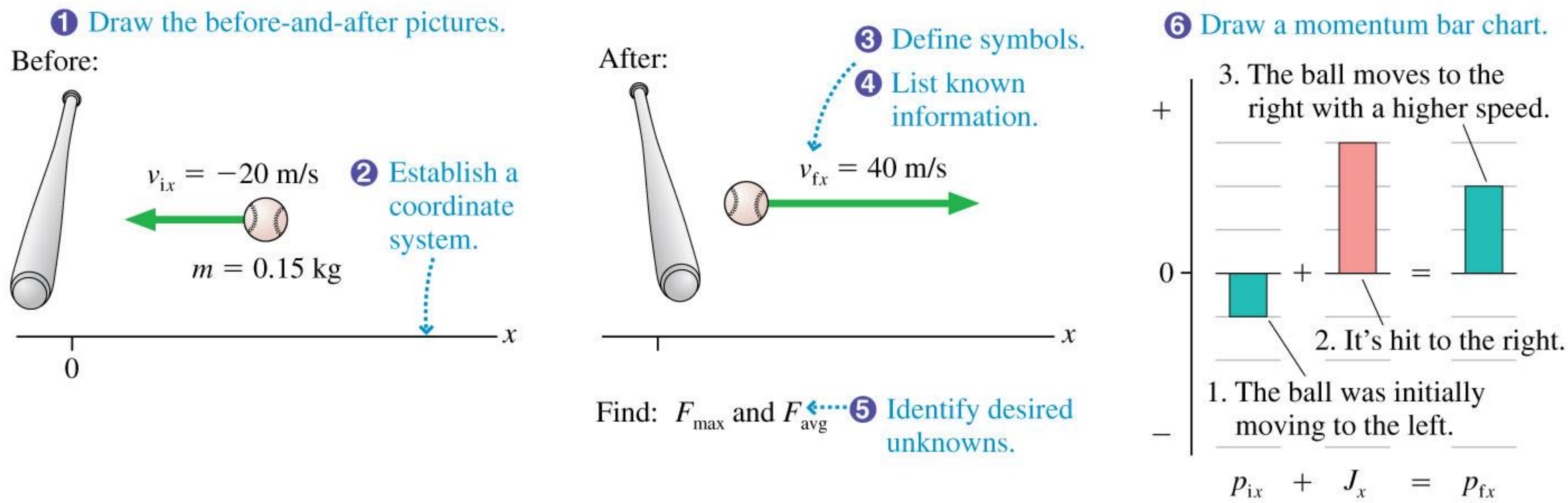
Hitting a Baseball



MODEL:

Model the baseball as a particle and the interaction as a collision.

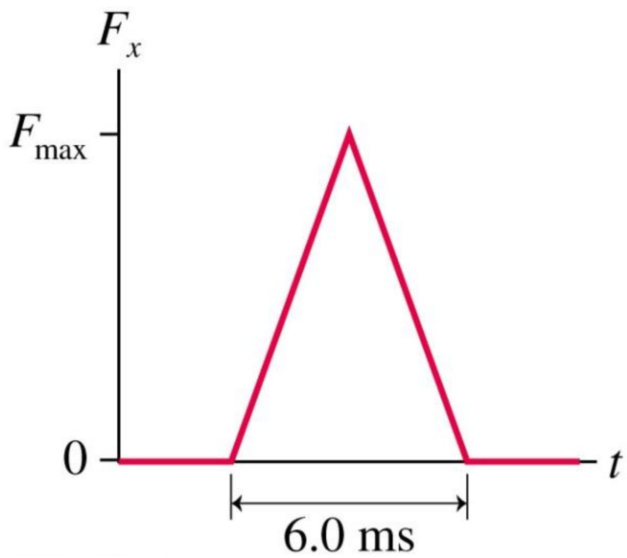
Hitting a Baseball



VISUALIZE:

The figure above is a before-and-after pictorial representation. Because F_x is positive (a force to the right), we know the ball was initially moving toward the left, and is hit back toward the right. Thus, we converted the statements about speeds into information about velocities, with v_{ix} negative.

Hitting a Baseball



SOLVE:

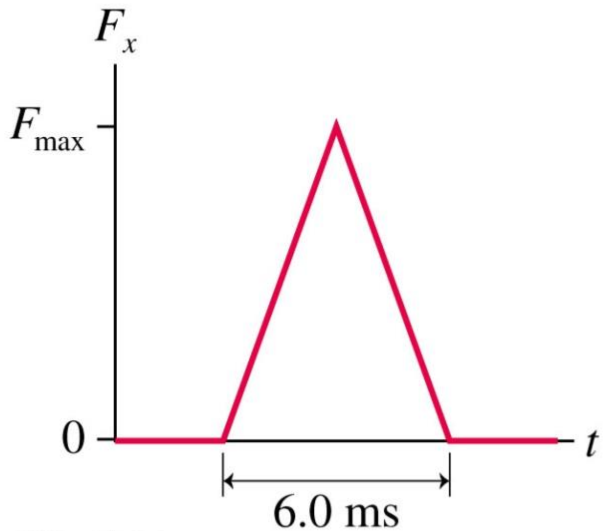
Until now we've consistently started the mathematical representation with Newton's second law. Now we want to use the impulse-momentum theorem:

$$\Delta p_x = J_x = \text{area under the froce curve}$$

We know the velocities before and after the collision, so we can calculate the ball's momenta:

$$p_{ix} = mv_{ix} = (0.15\ \text{kg})(-20\ \text{m/s}) = -3.0\ \text{kg m/s}$$
$$p_{fx} = mv_{fx} = (0.15\ \text{kg})(40\ \text{m/s}) = 6.0\ \text{kg m/s}$$

Hitting a Baseball



Thus, the change in momentum is:

$$\Delta p_x = p_{fx} - p_{ix} = 9.0 \text{ kg m/s}$$

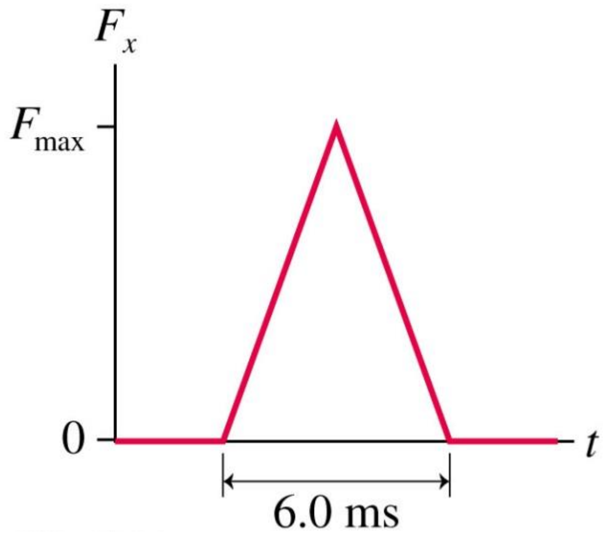
The force curve is a triangle with height F_{max} and width 6.0 ms. The area under the curve is

$$J_x = area = \frac{1}{2} \times F_{max} \times (0.0060 \text{ s}) = F_{max} \times (0.0030 \text{ s})$$

According to the impulse-momentum theorem,

$$9.0 \text{ kg m/s} = F_{max} \times (0.0030 \text{ s})$$

Hitting a Baseball



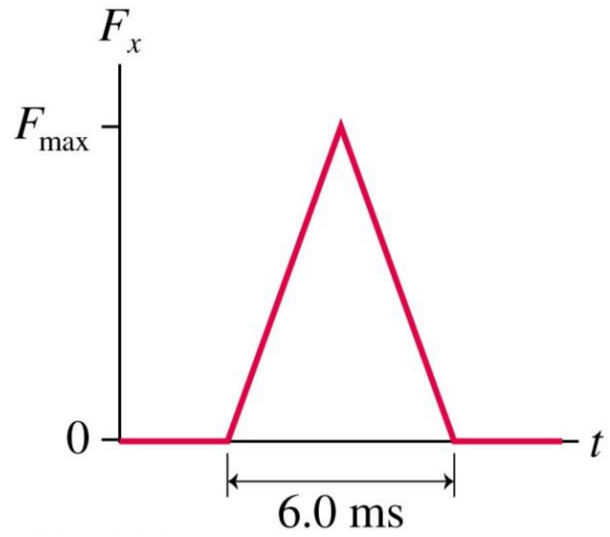
Thus, the maximum force is:

$$F_{max} = \frac{9.0 \text{ kg m/s}}{0.0030 \text{ s}} = 3000 \text{ N}$$

The average force, which depends on the collision duration $\Delta t = 0.0060$ s, has the smaller value:

$$F_{avg} = \frac{J_x}{\Delta t} = \frac{\Delta p_x}{\Delta t} = \frac{9.0 \text{ kg m/s}}{0.0060 \text{ s}} = 1500 \text{ N}$$

Hitting a Baseball

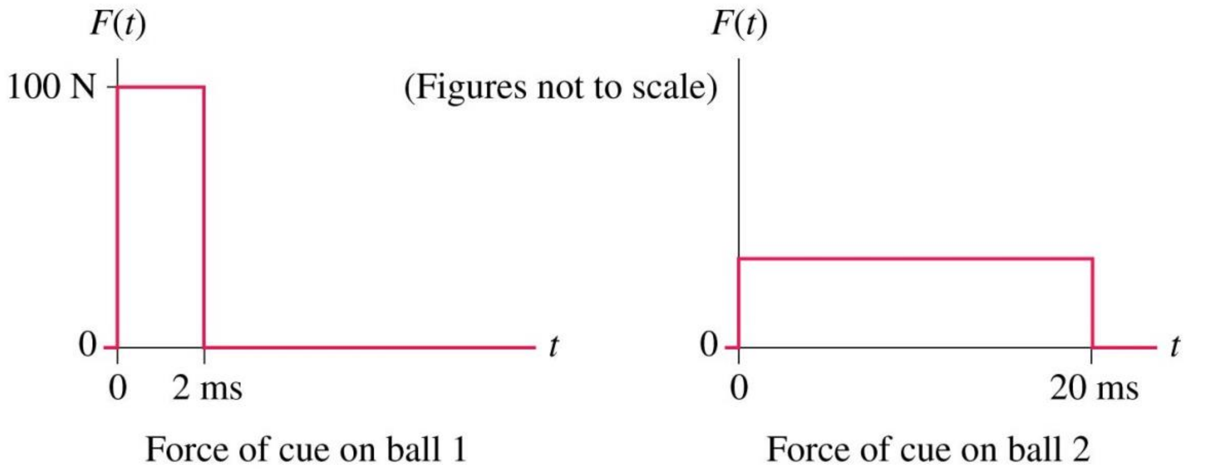


ASSESS:

F_{max} is a large force, but quite typical of the impulsive forces during collisions. The main thing to focus on is our new perspective: an impulse changes the momentum of an object.

The Impulse-Momentum Theorem

Q.21 Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?



a) Ball 1.

b) Ball 2.

c) Both balls have the same final speed.

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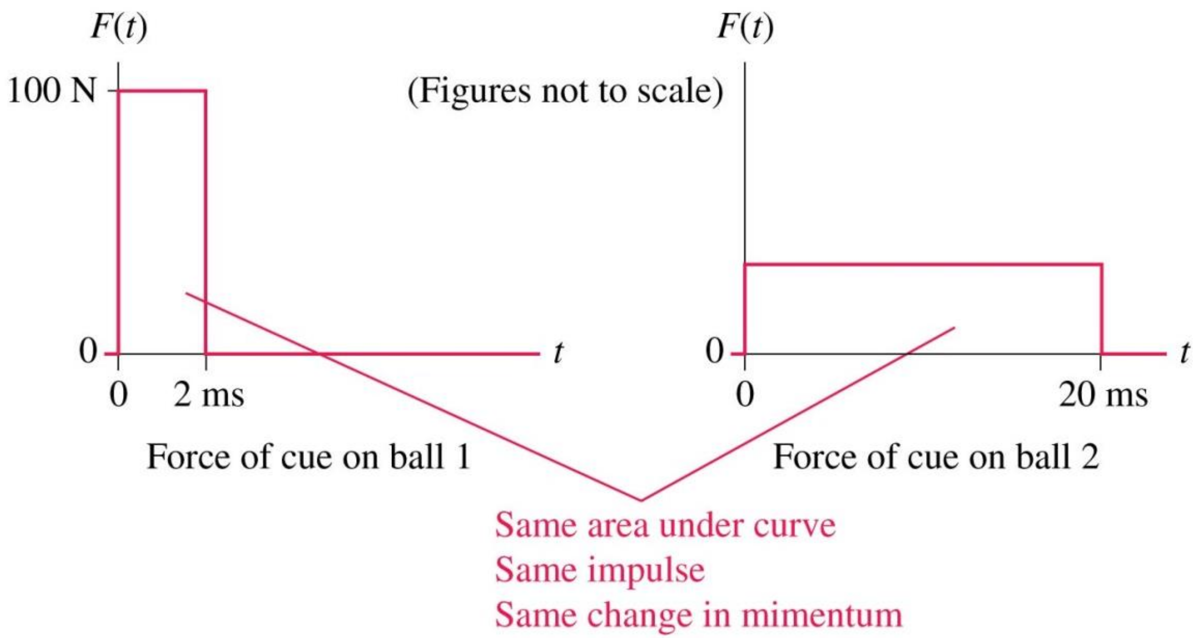

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b) Ball 2.

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PROBLEM-SOLVING STRATEGY 9.1 Conservation of momentum

MODEL Clearly define *the system.*

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or, as you'll learn in Chapters 10 and 11, conservation of energy.

PROBLEM-SOLVING STRATEGY 9.1

Conservation of momentum

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_f = \vec{P}_i$. In component form, this is

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 16

Law of Conservation of Momentum

Q.22 A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

a) The mosquito.

b) The truck.

c) They have the same change of momentum.

d) Can’t say without knowing their initial velocities.

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Momentum is conserved, so $\Delta p_{\text{mosquito}} + \Delta p_{\text{truck}} = 0$.

Equal magnitude (but opposite sign) changes in momentum.

Recoil

Q.23

A 10 g bullet is fired from a 3.0 kg rifle with a speed of 500 m/s. What is the recoil speed of the rifle?

Recoil

MODEL:
The rifle causes a small mass of gunpowder to explode, and the expanding gas then exerts forces on both the bullet and the rifle.

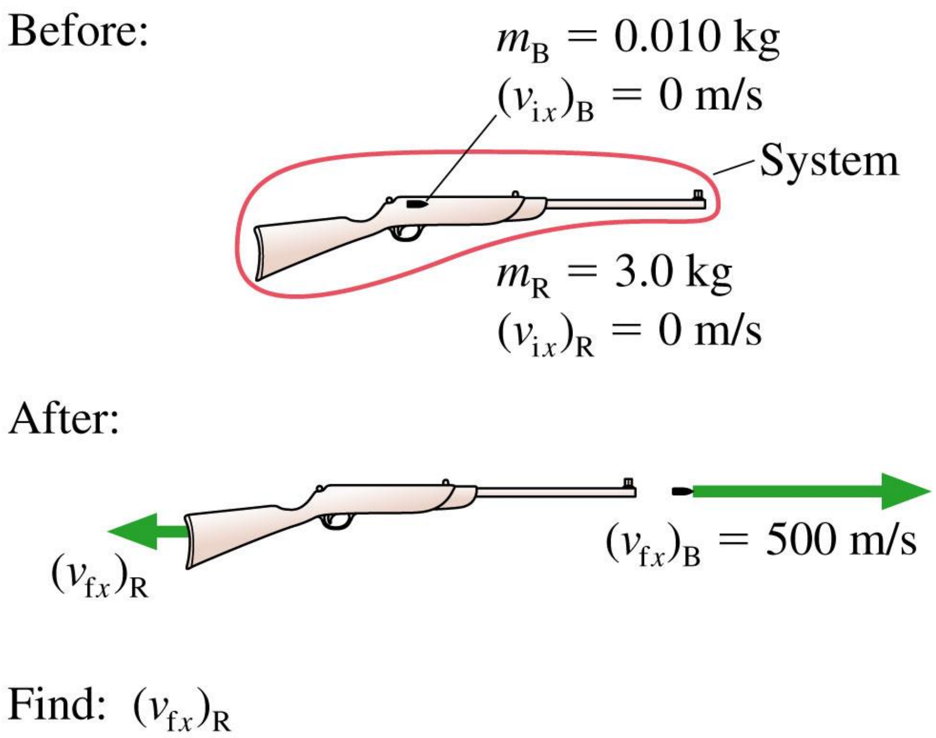
Let's define the **system** to be bullet + gas + rifle. The forces due to the expanding gas during the explosion are internal forces, within the system. Any friction forces between the bullet and the rifle as the bullet travels down the barrel are also internal forces.

Recoil

MODEL:

Gravity, the only external forces, is balanced by the normal forces of the barrel on the bullet and the person holding the rifle, so $\vec{F}_{net} = \vec{0}$. This is an isolated system and the law of conservation of momentum applies.

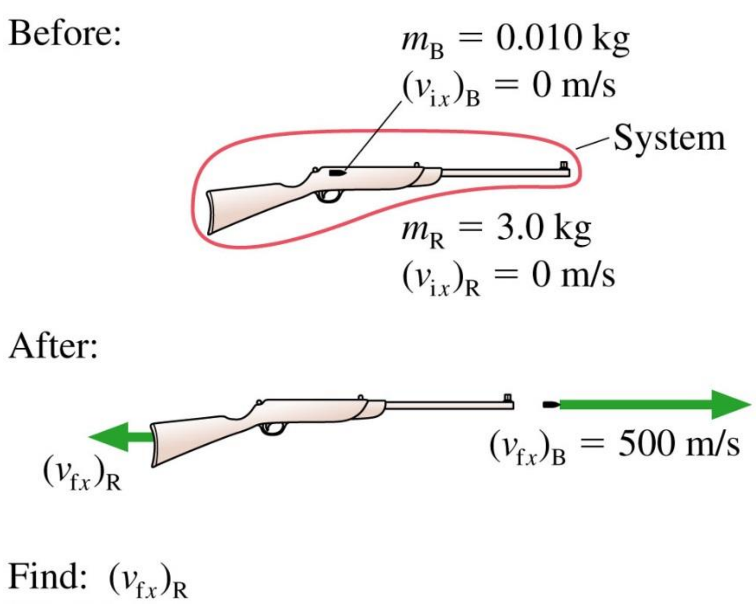
Recoil



VISUALISE:

The figure above shows a pictorial representation before and after the bullet is fired.

Recoil

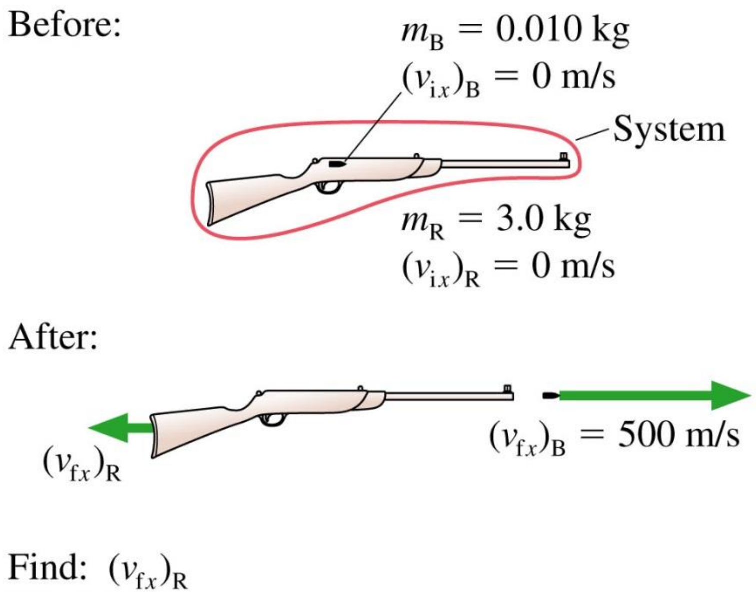


SOLVE:

The x-component of the total momentum is $P_x = (p_x)_B + (p_x)_R + (p_x)_{gas}$. Everything is at rest before the trigger is pulled, so the initial momentum is zero. The law of conservation of momentum is thus

$$P_{fx} = = m_B(v_{fx})_B + m_R(v_{fx})_R = P_{ix} = 0$$

Recoil

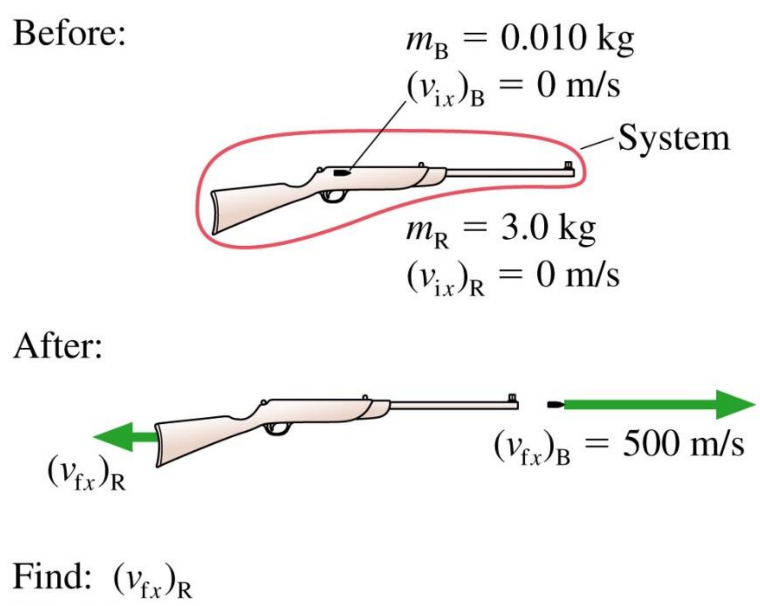


SOLVE:

The rifle's velocity is thus

$$(v_{fx})_R = -\frac{m_B}{m_R}(v_{fx})_R = -\frac{0.010 \text{ kg}}{3.0 \text{ kg}} \times 500 \text{ m/} = -1.7 \text{ m/s}$$

Recoil



SOLVE:

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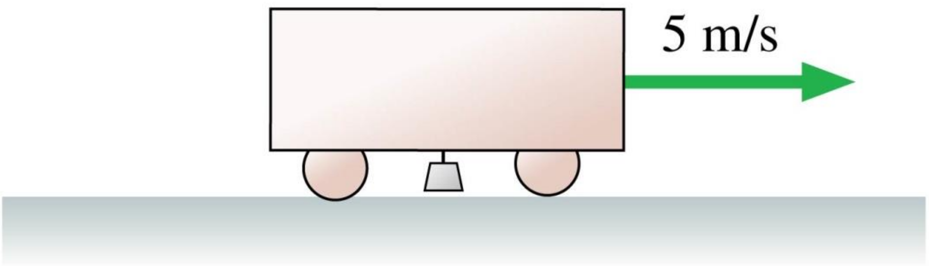
- The minus sign indicates that the rifle’s recoil is to the left. The recoil speed is 1.7 m/s.

Momentum in Two Dimensions

Q.24 A cart is rolling at 5 m/s. A heavy lead weight is suspended by a thread beneath the cart. Suddenly the thread breaks and the weight falls. Immediately afterward, the speed of the cart is



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b) Still 5 m/s.

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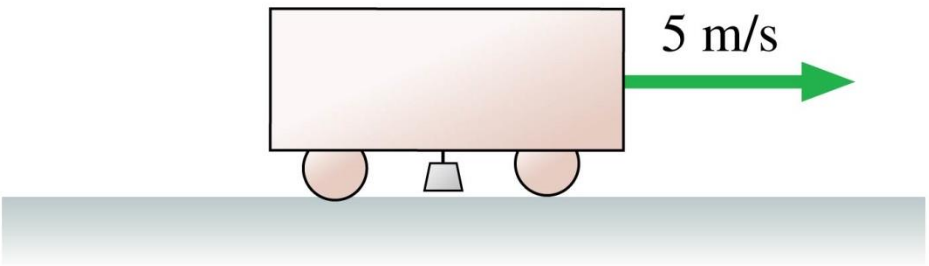

a) Less than 5 m/s.

b) Still 5 m/s.

c) More than 5 m/s.

No external forces to exert an impulse.
The falling weight still has forward momentum.