University of Nottingham
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Foundation Algebra for Physical Sciences & Engineering (CELEN036)

Homework 1

Topic 1: Composition of functions

1. For the following functions, find $(f \circ g)(x)$ and $(g \circ f)(x)$. Also determine their domains.

   (i) $f(x) = \sqrt{x-4}, \quad g(x) = 3x + 2$

   (ii) $f(x) = \dfrac{3x}{x-1}, \quad g(x) = \dfrac{2}{x}$

   (iii) $f(x) = \sqrt{2x+7}, \quad g(x) = \dfrac{2x}{x-1}$

   (iv) $f(x) = x^2 - 3x, \quad g(x) = x + 2$

2. For $f(x) = x^2 - 8, \quad g(x) = x + 2$, find $(f \circ g)(5)$.

3. For $h(x) = \sqrt{2x+3} - 5$ and $f(x) = \sqrt{x} - 5$, find the function g such that $h(x) = (f \circ g)(x)$.

4. For $f(x) = 2x - 1, g(x) = x^2 - 1$, and $h(x) = x + 4$, find $(f \circ g \circ h)(x)$ and $(g \circ f \circ h)(x)$.

5. Given the values of functions f and g in the following tables, find $(f \circ g)(4)$ and $(g \circ f)(4)$.

x	2	3	4
$f(x)$	-1	2	5

x	3	4	5
$g(x)$	3	3	0.5

Topic 2: Inverse functions

6. Find the inverse of the following functions:

   (i) $f(x) = \sqrt[3]{x+5}$

   (ii) $f(x) = \dfrac{2x}{x+1}, \quad x \neq -1$

   (iii) $f(x) = \dfrac{(x-1)^3}{8}$

   (iv) $f(x) = x^2 + 3, \quad x \geq 0$

7. Find: $(g^{-1} \circ f^{-1})(2), \quad (g \circ f^{-1})(-1), \quad$ and $(f^{-1} \circ g^{-1})(3)$ based on the following tables:

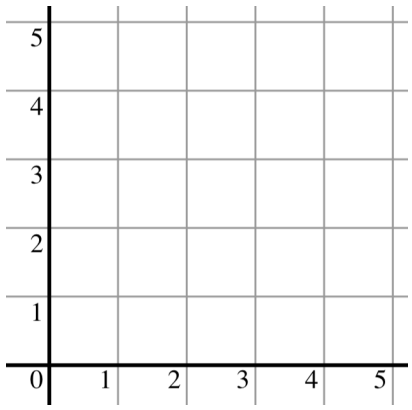
x	2	3	4	5	6
$f(x)$	-1	0	1	2	3

x	-1	2	3
$g(x)$	1	3	5

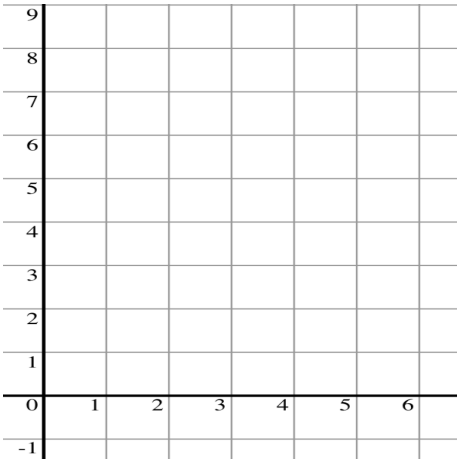
Topic 3: Sketching graphs of functions

8. Sketch the graphs of the following functions:

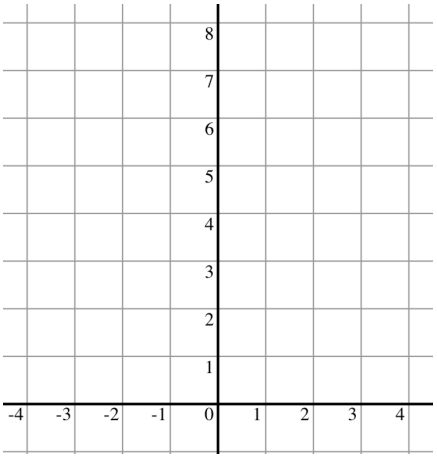
(i) $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2, & 1 < x \leq 2 \\ -x+5, & 2 < x \leq 5 \end{cases}$



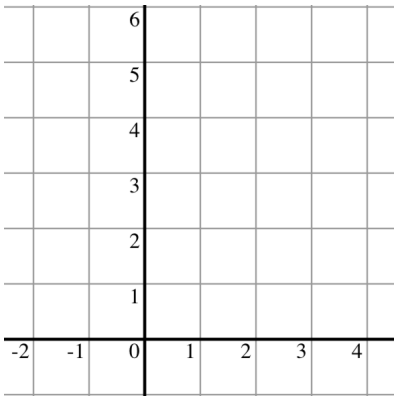
(ii) $f(x) = (x-3)^2 - 1$



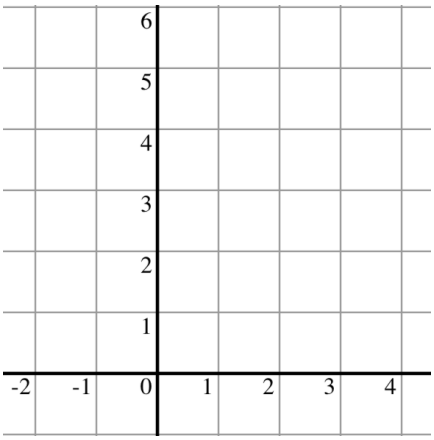
(iii) $f(x) = |x^2 - 4|$



(iv) $f(x) = |x-2| + 1$



9. Sketch the graph of $f(x) = (x-1)^2 - 1,\ x \geq 1$. Use this information to draw the graph of $f^{-1}(x)$ without finding the inverse function f^{-1}.



Topic 4: Modulus inequalities

10. Solve the following inequalities for $x \in \mathbb{R}$:

(i) $\left|\frac{4x+5}{3} - \frac{1}{2}\right| \leq \frac{7}{6}$

(ii) $\frac{|x|}{2} - \frac{5}{6} \geq \frac{1}{3}$

(iii) $|3x - 11| + 6 \leq 9$

(iv) $|4 - 3x| + 12 > 7$

(iii) $|x - 2| \leq \frac{x}{2}$

(iv) $|3 + x| \geq 6x$

(iii) $-\frac{2}{3}|x + 1| - 5 < -7$

(iv) $\frac{1}{2}|x - 3| + \frac{5}{4} \leq \frac{7}{4}$