FOUNDATION SCIENCE A

SEMINAR 10: INDUCTANCE, EM OSCILLATIONS, AC CIRCUIT

University of Nottingham
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LEARNING OUTCOMES

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- To understand the inductance of different types of conductors and inductors.
- To be able to measure parameters of various circuits, *i.e.* LR, LC, LCR. Parameters include resistance, inductance, capacitance, angular velocity, time, voltage, and energy.
- To solve problems of an AC circuit.

Seminar 10

Spend the first 15-20 minutes to solve questions: #3, #5, #6, #8, #10, #11 and #12 with your group fellows.

Inductance in Series and Parallel:

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

$$\frac{e}{L_{eq}} = \frac{e}{L_1} + \frac{e}{L_2}$$

Reactance of A Capacitor:

$$X_C = \frac{1}{2\pi f C}$$

LC Circuit:

$$I = Q_0\omega$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$U = \frac{1}{2}LI^2$$

LCR Circuit:

$$Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + (2\pi f L - 1/(2\pi f C))^2}$$

$$L = \frac{1}{2\pi f}\sqrt{\left(\frac{V_{rms}}{I_{rms}}\right)^2 - R^2}$$

Current in LR Circuit:

$$I = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

Mutual Inductance:

(I) Determine the mutual inductance per unit length between two long solenoids, one inside the other, whose radii are r_1 and r_2 $(r_2 < r_1)$ and whose turns per unit length are n_1 and n_2. The value for the permeability of free space (μ_0) is $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$.

Answer:

If we assume the outer solenoid is carrying current I_1 then the magnetic field inside the outer solenoid is $B = \mu_0 n_1 I_1$. The flux in each turn of the inner solenoid is $\Phi_{21} = B\pi r_2^2 = \mu_0 n_1 I_1 \pi r_2^2$. The mutual inductance is given by the equation below.

$$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{n_2 l \mu_0 n_1 I_1 \pi r_2^2}{I_1} \rightarrow \frac{M}{l} = \mu_0 n_1 n_2 \pi r_2^2$$

Self-Inductance:

(2) There is a solenoid with an inductance $0.285\ \mathrm{mH}$, a length of $36\ cm$, and a cross-sectional area $6\times10^{-4}\ \mathrm{m}^2$. (a) Find the number of turns of the solenoid. (b) Suppose at a specific time the emf is $-12.5\ \mathrm{mV}$, find the rate of change of the current at that time.

Answer:

Use the definition of the inductance of a solenoid.

$$L = \frac{\mu_0 N^2 A}{l} \quad \text{Therefore} \quad N^2 = \frac{Ll}{\mu_0 A} = \frac{(0.285\times10^{-3})(0.36)}{(1.26\times10^{-6})(6\times10^{-4})}$$

$$N = 368.4 \approx 369\ turns$$

The induced EMF is given by

$$\varepsilon = -L\frac{di}{dt} \quad \text{Therefore} \quad -12.5\times10^{-3} = -(0.285\times10^{-3})\frac{di}{dt} \qquad \frac{di}{dt} = 43.9\ \mathrm{A}\cdot\mathrm{s}^{-1}$$

Self-Inductance:

(3) Ignoring any mutual inductance, what is the equivalent inductance of two inductors connected (*a*) in series, (*b*) in parallel?

Answer:

(a) When connected in series the voltage drops across each inductor will add, while the currents in each inductor are the same.

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} = -(L_1 + L_2)\frac{dI}{dt} \boxed{= -\boldsymbol{L_{eq}} = \boldsymbol{L_1} + \boldsymbol{L_2}}$$

(b) When connected in parallel the currents in each inductor add to the equivalent current, while the voltage drop across each inductor is the same as the equivalent voltage drop.

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \qquad \rightarrow \frac{e}{L_{eq}} = \frac{e}{L_1} + \frac{e}{L_2} \qquad \boxed{\rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

Magnetic Energy Storage:

(4) Typical large values for electric and magnetic fields attained in laboratories are about 1.0×10^4 V/m and 2.0 T. (a) Determine the energy density for each field and compare. (*b*) What magnitude electric field would be needed to produce the same energy density as the 2.0 T magnetic field?

Answer:

(a) Calculate the energy density:

$$\boxed{\boldsymbol{u_E = \frac{1}{2}\varepsilon_0 E^2}} = \frac{1}{2}(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(1.0\times10^4\ \mathrm{N/C})^2 = \boxed{4.4\times10^{-4}\ \mathrm{J/m^3}}$$

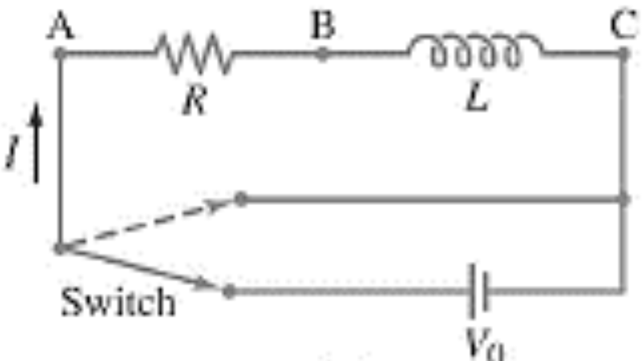
$$\boxed{\boldsymbol{u_B = \frac{B^2}{2\mu_0}}} = \frac{(2.0\ T)^2}{2(4\pi\times10^{-7} T\cdot m/A)} = 1.592\times10^6\ \mathrm{J/m^3} \quad \boxed{\approx 1.6\times10^6\ \mathrm{J/m^3}}$$

(b) Calculate the energy stored in the electric and magnetic energy fields.

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B \rightarrow E = \sqrt{\frac{2u_B}{\varepsilon_0}} = \sqrt{\frac{2(1.592\times10^6\ J/m^3)}{(8.85\times10^{-12}\ C^2/N\cdot m^2)}} \quad \boxed{= 6.0\times10^8\ \mathrm{N/C}}$$

LR Circuits:

(5) After how many time constants does the current in the figure below reach within (*a*) 5.0%, (*b*) 1.0%, and (c) 0.10% of its maximum value?

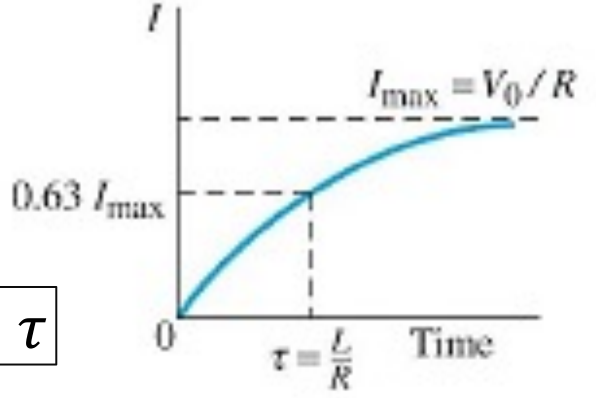


(a)

Answer:

For an *LR* circuit, we have $I = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$. Solving for t:

$$I = I_0\left(1 - e^{-\frac{t}{\tau}}\right) \rightarrow e^{-\frac{t}{\tau}} = 1 - \frac{I}{I_{max}} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{max}}\right)$$



(b)

(a) $I = 0.95\, I_{max} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{max}}\right)$ = - τ $\ln(1 - 0.95)$ $\boxed{= 3.0\,\tau}$

(b) $I = 0.99\, I_{max} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{max}}\right)$ = - τ $\ln(1 - 0.99)$ $\boxed{= 4.6\,\tau}$

(c) $I = 0.999\, I_{max} \rightarrow t = -\tau \ln\left(1 - \frac{I}{I_{max}}\right)$ = - τ $\ln(1 - 0.999)$ $\boxed{= 6.9\,\tau}$

LC Circuits and Oscillations:

(6) A 425-pF capacitor is charged to 135 V and then quickly connected to a 175 mH inductor. Determine (a) the frequency of oscillation, (b) the peak value of the current, and (c) the maximum energy stored in the magnetic field of the inductor.

Answer:

(a) We calculate the resonant frequency using the equation below.

$$\boxed{\boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}}} = \frac{1}{2\pi}\sqrt{\frac{1}{(0.175\ H)(425\times10^{-12}\ F)}} = 18{,}450\ Hz \quad \boxed{\approx 18.5\ kHz}$$

(*b*) As shown in the equation below, we set the **peak current equal to the maximum charge multiplied by the angular frequency**.

$$\boldsymbol{I = Q_0\omega = CV(2\pi f)} = (425\times10^{-12}\ \mathrm{F})(135\ \mathrm{V})(2\pi)(18{,}450\ \mathrm{Hz})$$

$$= 6.653\times10^{-3}\ \mathrm{A} \quad \boxed{\approx 6.65\ \mathrm{mA}}$$

(c) We use the equation below to calculate the maximum energy stored in the inductor.

$$\boldsymbol{U = \frac{1}{2}LI^2} = \frac{1}{2}(0.175\ \mathrm{H})(6.653\times10^{-3}\ \mathrm{A})^2 \quad \boxed{\approx 3.87\ \mu\mathrm{J}}$$

LC Oscillations with Resistance:

(7) How much resistance must be added to a pure *LC* circuit (L = 350 mH, C = 1800 pF) to change the oscillator's frequency by 0.25%? Will it be increased or decreased?

Answer:

As shown by $\boldsymbol{\omega'} = \sqrt{\frac{\mathbf{1}}{\boldsymbol{LC}} - \frac{\boldsymbol{R^2}}{\mathbf{4}\boldsymbol{L^2}}}$ adding resistance will decrease the oscillation frequency.

We use $\omega = 2\pi f = \sqrt{\frac{1}{LC}}$ for the pure LC circuit frequency:

$$\omega' = (1 - 0.0025)\omega \rightarrow \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 0.9975\sqrt{\frac{1}{LC}}$$

$$\rightarrow R = \sqrt{\frac{4L}{C}(1 - 0.9975^2)} = \sqrt{\frac{4(0.350\ H)}{(1.900 \times 10^{-9}\ \mathrm{F})}(1 - 0.9975^2)} \quad \boxed{= 2.0\ \mathrm{k\Omega}}$$

AC Circuits, Reactance:

(8) What is the reactance of a 9.2 μF capacitor at a frequency of (*a*) 60.0 Hz, (*b*) 1.00 MHz?

Answer:

The reactance of a capacitor is given by: $X_C = \dfrac{1}{2\pi f C}$

(a) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60\text{ Hz})(9.2\times10^{-6}\text{ F})}$ $\boxed{= 290\ \Omega}$

(b) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(1.00\times10^{6}\text{ Hz})(9.2\times10^{-6}\text{ F})}$ $\boxed{= 1.7\times10^{-2}\ \Omega}$

AC Circuits, Reactance:

(9) What is the inductance *L* of the primary of a transformer whose input is 110 V at 60 Hz and the current drawn is 3.1 A? Assume no current in the secondary.

Answer:

We use $V_{rms} = I_{rms}\omega L$ to solve for the impedance:

$$V_{rms} = I_{rms}\omega L \qquad \rightarrow L = \frac{V_{rms}}{I_{rms}\omega} \quad = \frac{110\ \text{V}}{(3.1\ \text{A})2\pi(60\ \text{Hz})} \boxed{= 94\ \text{mH}}$$

LRC Series AC Circuit:

(10) For a 120 V, 60 Hz voltage, a current of 70 mA passing through the body for 1.0 s could be lethal. What must be the impedance of the body for this to occur?

Answer:

We find the impedance from the equation below:

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{120\ V}{70\times10^{-3}\mathrm{A}} \boxed{= 1700\ \Omega}$$

LRC Series AC Circuit:

(11) A 75 W lightbulb is designed to operate with an applied ac voltage of 120 V rms. The bulb is placed in series with an inductor *L*, and this series combination is then connected to a 60 Hz 240 V rms voltage source. For the bulb to operate properly, determine the required value for *L*. Assume the bulb has resistance *R* and negligible inductance.

Answer:

The light bulb acts like a resistor in series with the inductor. Using the desired rms voltage across the resistor and the power dissipated by the light bulb we calculate the rms current in the circuit and the resistance. Then using this current and the rms voltage of the circuit we calculate the impedance of the circuit and the required inductance.

$$I_{rms} = \frac{P}{V_{R,rms}} = \frac{75\ \mathrm{W}}{120\ \mathrm{V}}\ 0.625\ \mathrm{A}$$

$$R = \frac{V_{R,rms}}{I_{rms}} = \frac{120\ V}{0.625\ A} = 192\ \Omega$$

$$Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + (2\pi f L)^2}$$

$$L = \frac{1}{2\pi f}\sqrt{\left(\frac{V_{rms}}{I_{rms}}\right)^2 - R^2} = \frac{1}{2\pi(60\ \mathrm{Hz})}\sqrt{\left(\frac{240\ \mathrm{V}}{0.625\ \mathrm{A}}\right)^2 - (192\ \Omega)^2}\ \boxed{= 0.88\ \mathrm{H}}$$

Resonance in AC Circuit:

(12) An *LRC* circuit has *L* = 4.15 mH and *R* = 3.80 kΩ. (a) What value must *C* have to produce resonance at 33.0 kHz? (*b*) What will be the maximum current at resonance if the peak external voltage is 136 V?

Answer:

(a) We find the capacitance from the resonant frequency:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \rightarrow C = \frac{1}{4\pi^2 L f_0^2} = \frac{1}{4\pi^2 (4.15 \times 10^{-3}\ \mathrm{H})(33.0 \times 10^3\ \mathrm{Hz})}$$

$$\boxed{= 5.60 \times 10^{-9}\ \mathrm{F}}$$

(b) At resonance, the impedance is the resistance, so the current is given by Ohm's law:

$$I_{peak} = \frac{V_{peak}}{R} = \frac{136\ V}{3800\ \Omega} \quad \boxed{= 35.8\ \mathrm{mA}}$$

Q&A?
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