Support Session

Wednesdays from 13:00 to 14:00.

Venue: TB 318+

The support session is another opportunity (in addition to the office hours) for you to get your questions answered by tutors.

Mid-Semester Examination

Wednesday 3rd November from 13:30 to 14:30.

Confirm your venue from CPSO

Topics covered would be those topics treated in Seminars 1 to 4

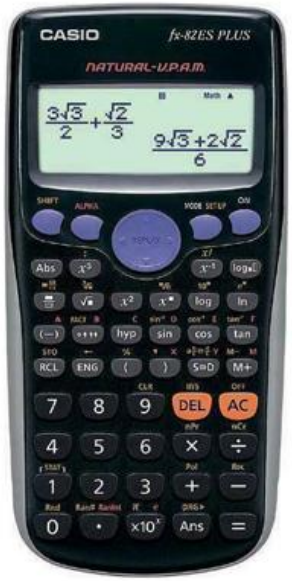
Revision Materials would be made available on Moodle, and you would be informed when they are live

Please make sure you bring a permissible calculator to the examination

Permissible Calculator

CASIO fx-82E family

fx – 82ES PLUS



fx – 82ES PLUS 2nd edition

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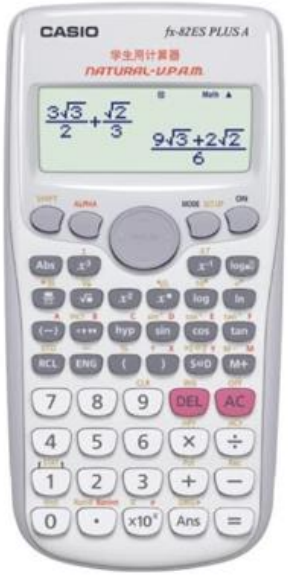


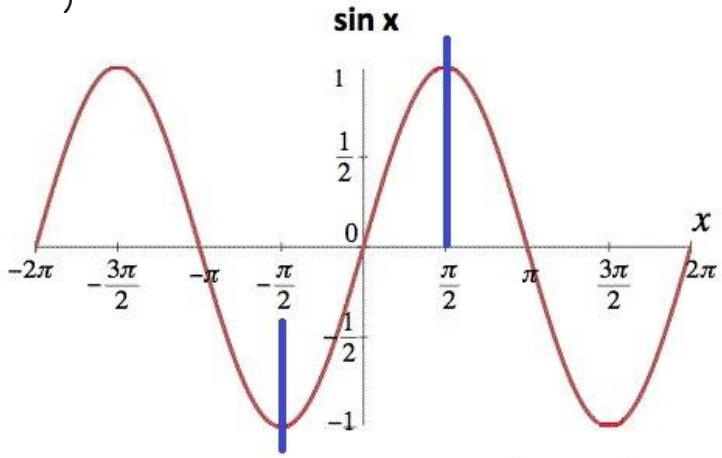
Lecture 4

Topics covered in this lecture session

1. Inverse Trigonometric functions.
2. Expressing $a\cos x + b\sin x$ in the form $r\cos(\theta - x)$.

Inverse Trigonometric Functions

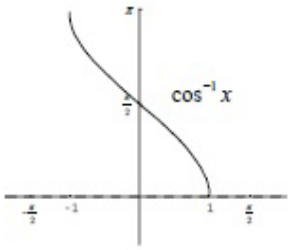
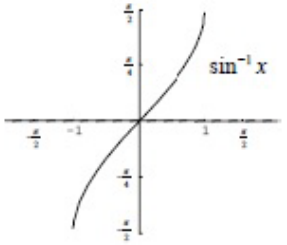
The graph of the sine function over $\mathbb{R}$, indicates that it is not one-one however, if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then sine function is one-one and its inverse exists.



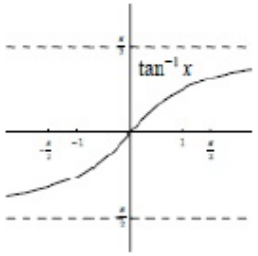
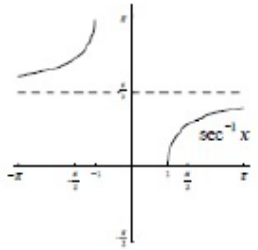
It is denoted by $\sin^{-1}$ **or** $\arcsin$ and is defined by

$$y = \sin x \quad \Leftrightarrow \quad x = \sin^{-1} y \quad ; \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

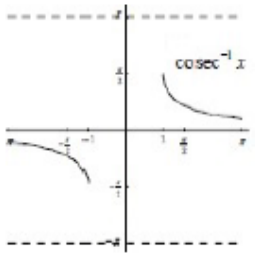
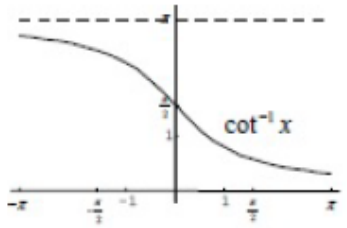
Inverse Trigonometric Functions

Inverse Function	Domain of Inverse function ≡ Range of Trigonometric function	Range of Inverse function *i.e.* Restricted Domain for Trigonometric function	Graph of Inverse Trigonometric function
$\cos^{-1} x$ or arccos	$[-1, 1]$	$[0, \pi]$	
$\sin^{-1} x$ or arcsin	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	

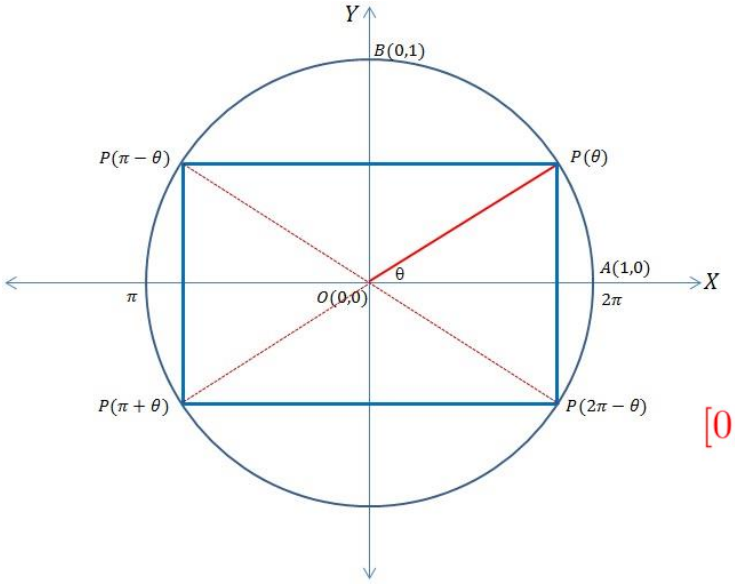
Inverse Trigonometric Functions

Inverse Function	Domain of Inverse function ≡ Range of Trigonometric function	Range of Inverse function *i.e.* Restricted Domain for Trigonometric function	Graph of Inverse Trigonometric function
$\tan^{-1} x$ or arctan	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	
$\sec^{-1} x$ or arcsec	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	

Inverse Trigonometric Functions

Inverse Function	Domain of Inverse function ≡ Range of Trigonometric function	Range of Inverse function *i.e.* Restricted Domain for Trigonometric function	Graph of Inverse Trigonometric function
$\text{cosec}^{-1}x$ or arccosec	$\mathbb{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	
$\cot^{-1}x$ or arccot	$\mathbb{R}$	$(0, \pi)$	

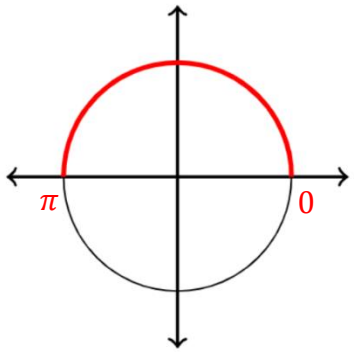
Inverse Trigonometric Functions



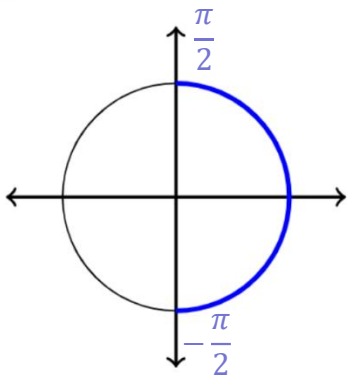
Reference Angle $\theta \in (0, 2\pi)$

Restricted domain for Inverse Trigonometric functions

$[0, \pi]$, used for $\cos^{-1}$



$[-\frac{\pi}{2}, \frac{\pi}{2}]$, used for $\sin^{-1}$



$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, used for $\tan^{-1}$

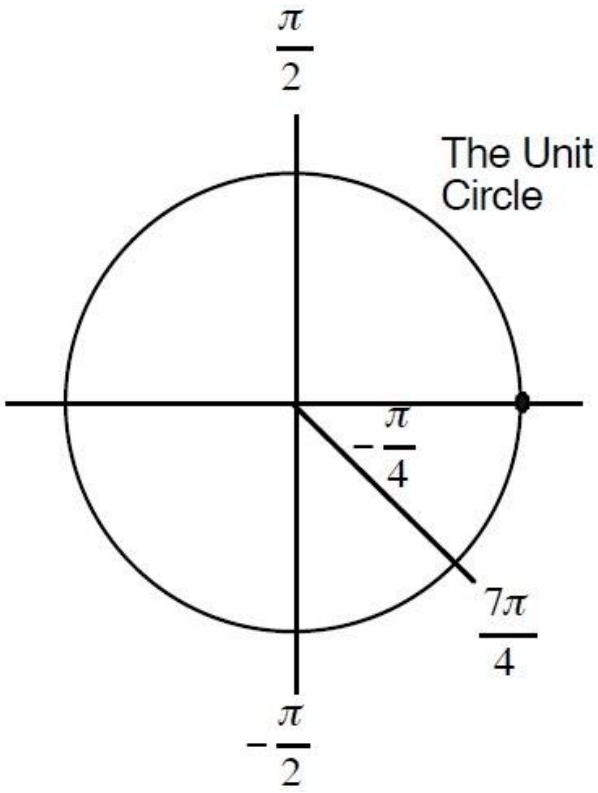
Inverse Trigonometric Functions

Find the values of:

(i) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(ii) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

(iii) $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$



Inverse Trigonometric Functions

Question 4-N36-Q1

Solve $\cos^{-1}\left(\cos\left(\frac{14\pi}{3}\right)\right) - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

A $\frac{\pi}{2}$

B π

C 2π

Expressing $a\cos x + b\sin x$ in the form $r\cos(\theta - x)$

Sometimes it is important to express

$f(x) = a\cos x + b\sin x$ in the form $r\cos(\theta - x)$,

so as to

- determine the range of f;
- find the period of f;
- sketch the graph of the function f.

Expressing $a\cos x + b\sin x$ in the form $r\cos(\theta - x)$

The Method: Let $a = r\cos\theta$

$b = r\sin\theta$

where θ and r are to be determined.

- Squaring and adding $\Rightarrow$ $a^2 + b^2 = r^2$ $\Rightarrow$ $r = \sqrt{a^2 + b^2}$

- Dividing the second equation by the first equation, gives

$$\frac{r\sin\theta}{r\cos\theta} = \frac{b}{a} \quad \Rightarrow \quad \tan\theta = \frac{b}{a}$$ (from which θ can be found)

Expressing $a\cos x + b\sin x$ in the form $r\cos(\theta - x)$

Thus, $f(x) = a\cos x + b\sin x$

$$= r\cos\theta\cos x + r\sin\theta\sin x$$

$$= r\left[\cos\theta\cos x + \sin\theta\sin x\right]$$

$$= r\cos(\theta - x) \quad \textbf{or} \quad r\cos(x - \theta)$$

because, $\cos(-\theta) = \cos\theta$

Expressing $a\cos x + b\sin x$ in the form

$$r\ \cos(\theta - x)$$

1. Prove that $\cos 2x - \sqrt{3}\sin 2x = 2\cos\left(2x + \frac{\pi}{3}\right)$.

2. Express $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \pi/2$.

   Hence (i) sketch the graph of $y = f(x) = \sin x - \sqrt{3}\cos x$;

   (ii) find the range of f;

   (iii) find the period of f.

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Inverse Trigonometric Functions

Question 4-N36-Q2

Express $4\sin x - 3\cos x$ in the form $r\sin(x-\alpha)$

A $5\sin(x+45.00^o)$

B $5\sin(x-30.00^o)$

C $5\sin(x+36.87^o)$

Suggested Reading

Foundation Algebra by P. Gajjar.

(Chapter 6)