Lecture 1

Lecture Content

- An Introduction to Derivative
- Formal definition of the Derivative
- Derivatives of basic algebraic expressions
- Derivatives of standard functions:
  - Trigonometric
  - Exponential
  - Logarithmic
- Rules of Differentiation: Sum and Difference, Product
- Extension of the Product Rule
- Quotient Rule for Differentiation

Rate of change

Many real-world phenomena involve changing quantities:

- the speed of a rocket
- the inflation of currency
- the number of bacteria in a culture
- the voltage of an electrical signal

and so forth.

Rate of change

We introduce the concept of a derivative, (a mathematical tool for studying the rate at which one quantity changes relative to another).

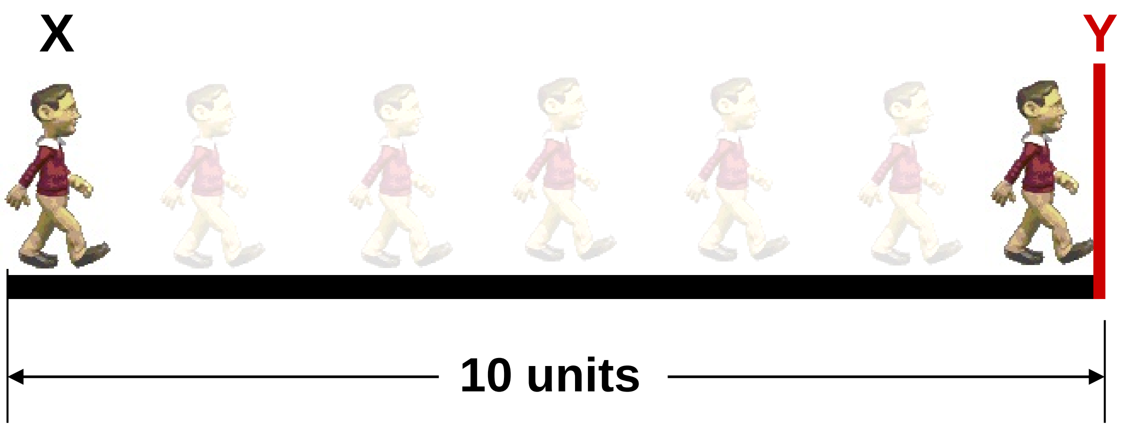
We would do this by noting that the study of rates of change is closely related to the geometric concept of a tangent line to the curve.

Limit – An Introduction

Historical Note: Two geometric problems were largely responsible for the development of Calculus, these are

1. Finding tangent lines to curves
2. Finding areas of plane regions

   These two problems are closely related to a fundamental concept of calculus known as a Limit



Question:

What is the distance travelled by X?

Limit – An Introduction

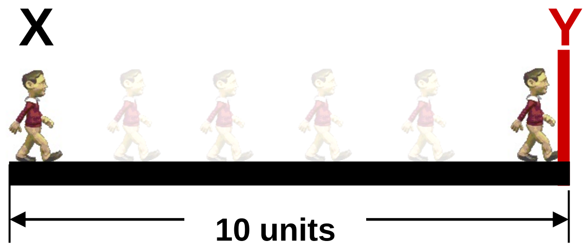
Distance travelled $\approx$ 10 units.

In mathematical terms, we write,

Limit (Distance travelled) $=$ 10 units.

$$\lim_{X \to Y} \text{(Distance travelled)} = 10 \text{ units}$$

where X is the person X and Y is the red wall.



Example

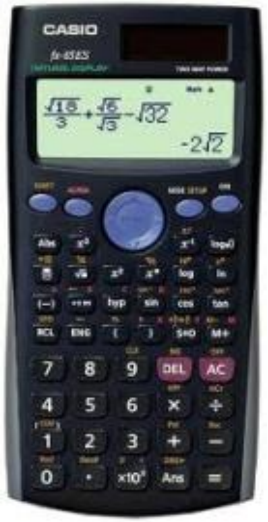
Evaluate the limit, numerically:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

$\longrightarrow$ **2** $\longleftarrow$

x	1.9	1.99	1.999	2.001	2.01	2.1
Limit	3.9	3.99	3.999	4.001	4.01	4.1

$\longrightarrow$ **4** $\longleftarrow$



Thus, $\displaystyle\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$

Example

In general we write the limit as:

$$\lim_{x \to a} f(x) = A$$

Which reads "the limit of $f(x)$ as x tends to a is A."

From this we can identify an important property of a Limit: $\lim_{x \to a} f(x)$ exists if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = A$ (where A is a finite number).

$$\lim_{x \to 2^-} \frac{x^2 - 4}{x - 2} = 4 \qquad \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2} = 4$$

$\longrightarrow$ **2** $\longleftarrow$

Left-hand Limit   Right-hand Limit

x	1.9	1.99	1.999	2.001	2.01	2.1
$Limit$	3.9	3.99	3.999	4.001	4.01	4.1

$\longrightarrow$ **4** $\longleftarrow$

Some Results on Limits

If L, M, c, and k are real numbers and

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M, \quad \text{then}$$

1. *Sum Rule:* $\lim_{x \to c}(f(x) + g(x)) = L + M$
2. *Difference Rule:* $\lim_{x \to c}(f(x) - g(x)) = L - M$
3. *Constant Multiple Rule:* $\lim_{x \to c}(k \cdot f(x)) = k \cdot L$
4. *Product Rule:* $\lim_{x \to c}(f(x) \cdot g(x)) = L \cdot M$
5. *Quotient Rule:* $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \quad M \neq 0$
6. *Power Rule:* $\lim_{x \to c} [\, f(x)\,]^n = L^n$, n a positive integer
7. *Root Rule:* $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer

(If n is even, we assume that $f(x) \geq 0$ for x in an interval containing c.)

Definition of Slope and Tangent Line

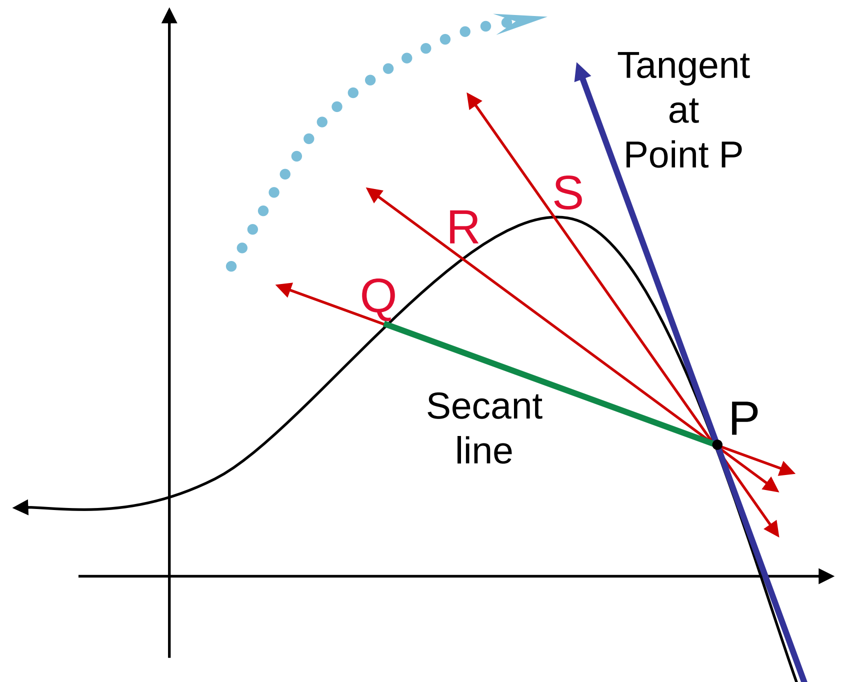
Slope or **Gradient** of a line describes its steepness.

Slope (given two points on line)

$$= \frac{\text{Difference of Y-coordinates}}{\text{Difference of X-coordinates}}$$

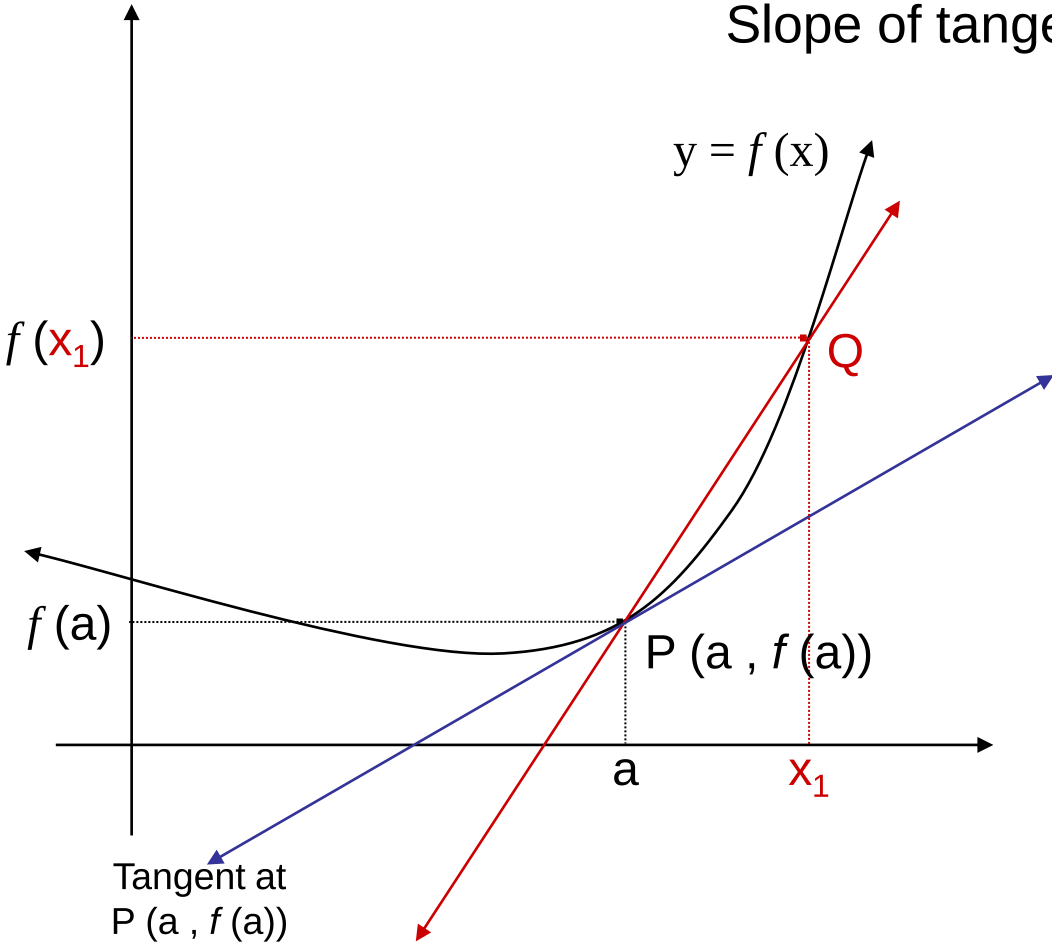
(Difference to be taken in the same order)

Tangent line (Tangent) to a curve at a given point is the straight line that '***just touches***' the curve at that point.



Secant line joins <u>two</u> points on a curve

Derivative – An application of Limit



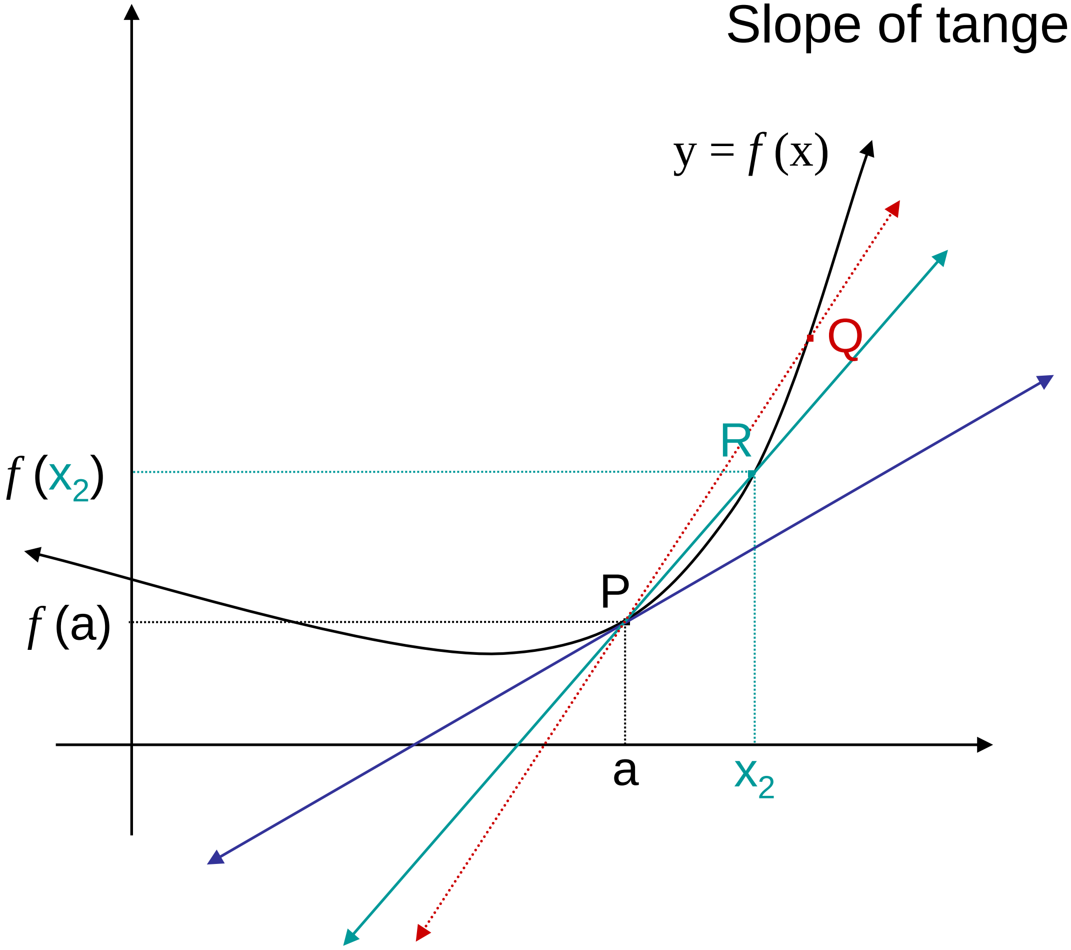
Slope of tangent ≈ Slope of secant line

$$= \frac{\text{Difference of Y-coordinates}}{\text{Difference of X-coordinates}}$$

$$= \frac{f(x_1) - f(a)}{x_1 - a}$$

First approximation

Derivative – An application of Limit



Slope of tangent $\approx$ Slope of secant line

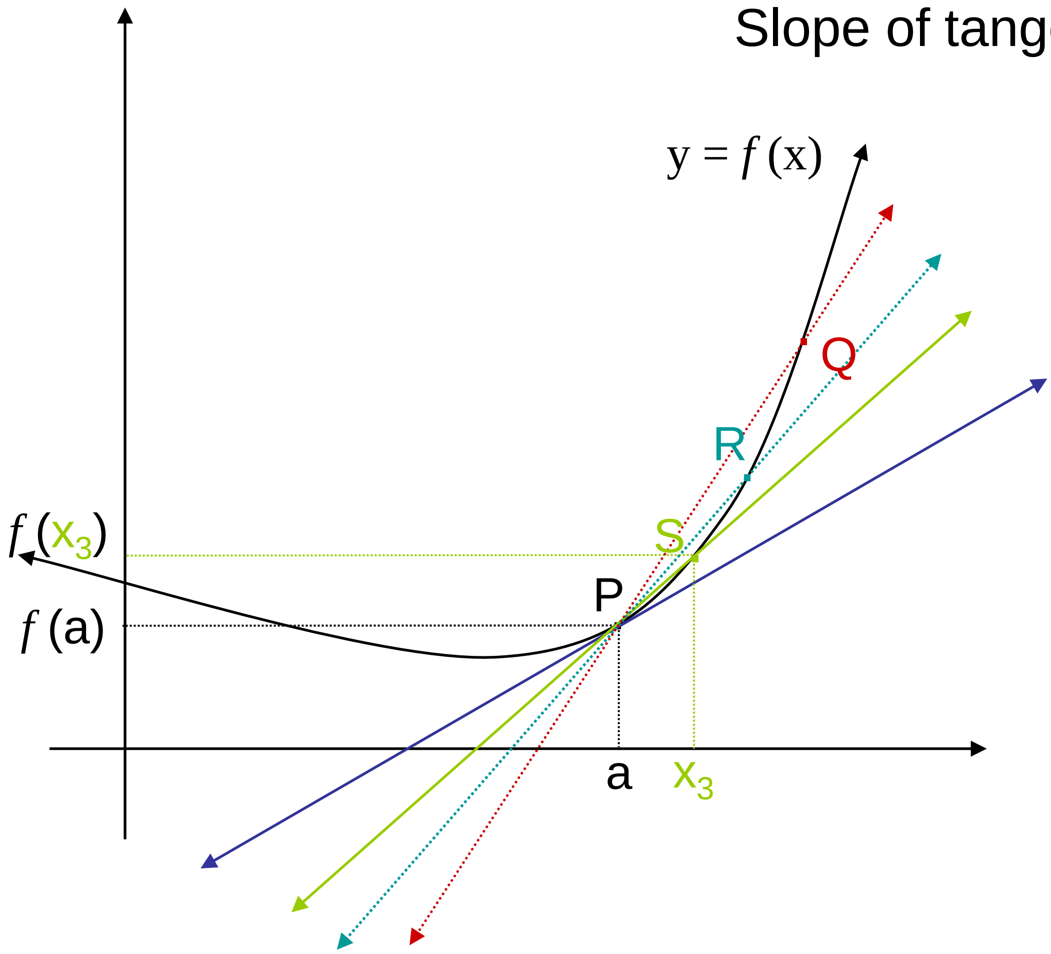
$$= \frac{\text{Difference of Y-coordinates}}{\text{Difference of X-coordinates}}$$

$$= \frac{f(x_2) - f(a)}{x_2 - a}$$

Second approximation

better than the first one

Derivative – An application of Limit



Slope of tangent ≈ Slope of secant line

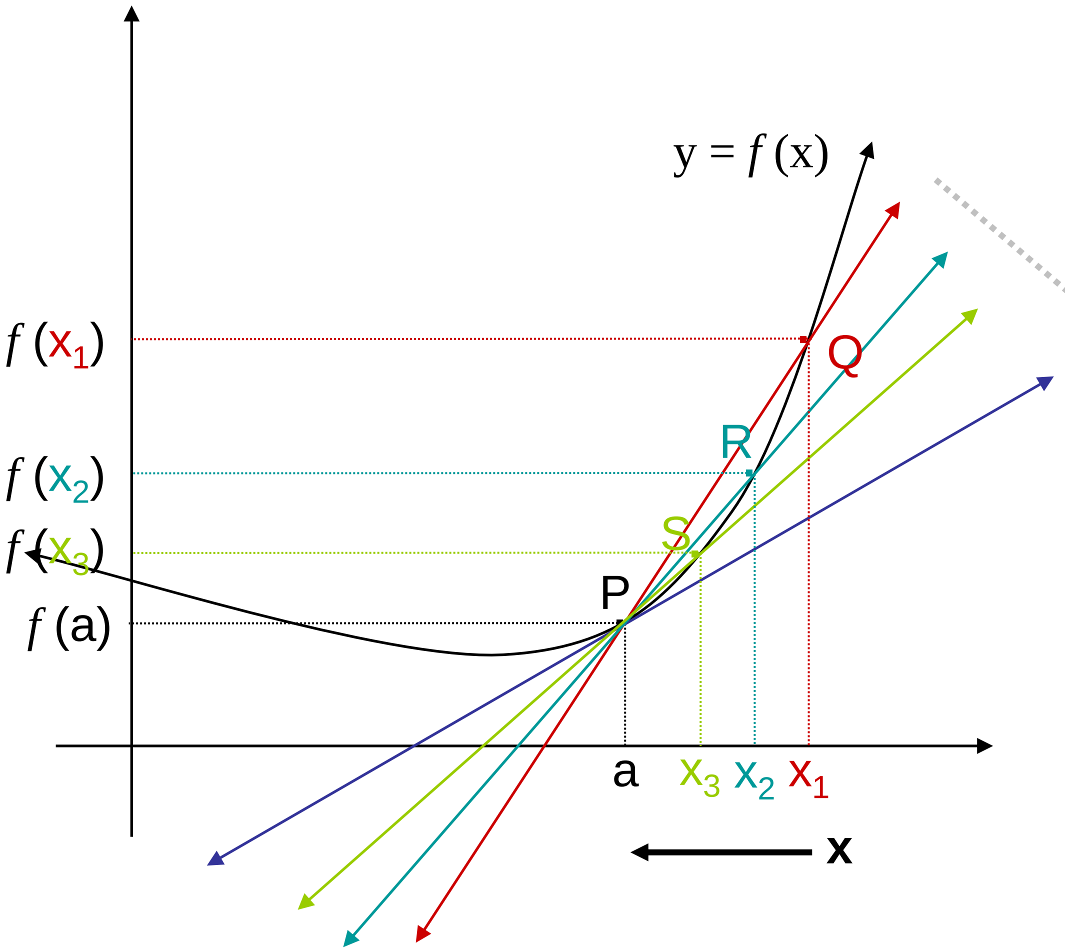
$$= \frac{\text{Difference of Y-coordinates}}{\text{Difference of X-coordinates}}$$

$$= \frac{f(x_3) - f(a)}{x_3 - a}$$

Third approximation

better than the second one

Derivative – An application of Limit



Observation

Secant lines → Tangent line
as
x → a

$$\frac{f(x_1) - f(a)}{x_1 - a}$$

$$\frac{f(x_2) - f(a)}{x_2 - a}$$

$$\frac{f(x_3) - f(a)}{x_3 - a}$$

Better approximation for Slope

Derivative – An application of Limit

In Summary:

$$\text{As } x \to a, \text{ Secant line} \to \text{Tangent line}$$

$$\therefore \text{ Slope of tangent line} \approx \text{Slope of secant line}$$

$$\therefore \text{ Slope of tangent line} = \lim_{x \to a} (\text{Slope of secant line})$$

$$\text{i.e. Slope of tangent line} = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

Definition of the Derivative

The slope (gradient) of the tangent at point $P(a)$ on the curve $y = f(x)$ is defined as the derivative of y with respect to x (at point a).

It is denoted by $f'(x)$ or $\dfrac{dy}{dx}$

At point $x = a$, it is denoted by $f'(a)$

Thus,

$$\boxed{\left.\frac{dy}{dx}\right)_{(\text{at point } x=a)} = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

Other useful (equivalent) forms of the Derivative:

1. Let $x = a + h$, so that, as $x \to a$, $h \to 0$.

$$\boxed{\therefore \left.\frac{dy}{dx}\right)_{(\text{at point } x=a)} = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

2. Generalizing,

$$\boxed{\therefore \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

OR

$$\boxed{\lim_{\delta x \to 0} \frac{(y + \delta y) - y}{\delta x}}$$

Other useful (equivalent) forms of the Derivative:

3. Let $t = x + h$, so that, as $h \to 0$, $t \to x$.

$$\boxed{\therefore \; \frac{dy}{dx} = f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}}$$

Derivatives of standard functions

(1) If $y = f(x) = x^2$, find $\dfrac{dy}{dx}$, using the definition of derivative (**first principles**).

(2) If $y = f(x) = 3x + 5$, find $\dfrac{dy}{dx}$, using the definition of derivative

Derivatives of standard functions

(1) If $y = f(x) = x^2$, find $\dfrac{dy}{dx}$, using the definition of derivative (**first principles**).

Solution:

$f(x) = x^2$

Method 1

$$f(x) = x^2 \implies f(x+h) = (x+h)^2$$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x + 0$$

$$\therefore \boldsymbol{f'(x) = 2x}$$

Derivatives of standard functions

(1) If $y = f(x) = x^2$, find $\dfrac{dy}{dx}$, using the definition of derivative (**first principles**).

Method 2

$$f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$$

$$= \lim_{t \to x} \frac{t^2 - x^2}{t - x}$$

$$= \lim_{t \to x} \frac{(t + x)\cancel{(t - x)}}{\cancel{t - x}}$$

$$f'(x) = \lim_{t \to x} (t + x)$$

$$= \boldsymbol{x + x = 2x}$$

Derivatives of standard functions

(2) If $y = f(x) = 3x + 5$, find $\dfrac{dy}{dx}$, using the definition of derivative

Solution:

$$f(x) = 3x + 5$$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h) + 5 - (3x+5)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x} + 3h + \cancel{5} - \cancel{3x} - \cancel{5}}{h}$$

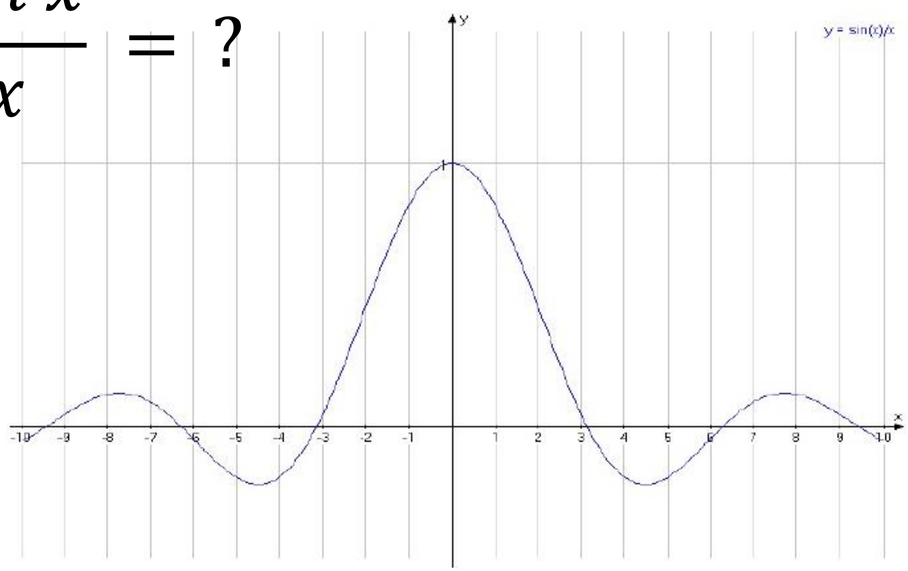
$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}} = 3$$

$$\therefore \boldsymbol{f'(x) = 3}$$

Limit of Trigonometric functions

$$\lim_{x \to 0} \sin x = 0 \qquad \lim_{x \to 0} \cos x = 1$$

$$\lim_{x \to 0} \frac{\sin x}{x} = ?$$



$\lim_{x \to 0^-} \frac{\sin x}{x}$

x	$f(x)$
-0.1	0.99833
-0.05	0.99958
-0.01	0.99998
-0.005	0.99999
0	?
0.005	0.99999
0.01	0.99998
0.05	0.99958
0.1	0.99833

$\lim_{x \to 0^+} \frac{\sin x}{x}$

Thus, $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Derivative of standard functions (Trigonometric)

Show that: $$\frac{d}{dx}(\sin x) = \cos x$$

Proof:

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$$

$$\boxed{\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$$

Derivative of standard functions (Trigonometric)

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h\to 0} \cos\left(\frac{2x+h}{2}\right) \lim_{h\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

Thus,

$$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$

Derivative of Trigonometric functions

$$\boxed{\frac{d}{dx}(\cos x) = -\sin x}$$

$$\boxed{\frac{d}{dx}(\tan x) = \sec^2 x \quad ; \quad x \neq (2k+1)\frac{\pi}{2} \; ; \; k \in \mathbb{Z}}$$

$$\boxed{\frac{d}{dx}(\sec x) = \sec x \tan x \quad ; \quad x \neq (2k+1)\frac{\pi}{2} \; ; \; k \in \mathbb{Z}}$$

$$\boxed{\frac{d}{dx}(\mathrm{cosec} x) = -\mathrm{cosec} x \cot x \quad ; \quad x \neq k\pi \; ; \; k \in \mathbb{Z}}$$

$$\boxed{\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x \quad ; \quad x \neq k\pi \; ; \; k \in \mathbb{Z}}$$

Derivative of standard functions (Exponential)

$$\frac{d}{dx}\left(e^x\right) = e^x$$

Proof:

$$\frac{d}{dx}\left(e^x\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

$$= e^x \cdot \lim_{h\to 0} \left(\frac{e^h - 1}{h}\right)$$

Derivative of standard functions (Exponential)

Result: $$\lim_{h \to 0} \frac{e^h - 1}{h} = \log_e e = 1$$

Therefore, $$\frac{d}{dx}\left(e^x\right) = e^x \cdot (1)$$

Thus,

$$\boxed{\frac{d}{dx}\left(e^x\right) = e^x}$$

Derivative of standard functions (Logarithmic)

Show that:

$$\frac{d}{dx}(\log_e x) = \frac{d}{dx}(\ln x) = \frac{1}{x} \quad ; \quad x \in \mathbb{R}^+$$

Proof:

$$\begin{aligned}
\frac{d}{dx}(\log_e x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\log_e(x+h)-\log_e x}{h} \\
&= \lim_{h\to 0} \frac{\log_e\left(\frac{x+h}{x}\right)}{h} \qquad \because \log a - \log b = \log\left(\frac{a}{b}\right) \\
&= \lim_{h\to 0} \frac{\log_e\left(1+\frac{h}{x}\right)}{h} \\
&= \lim_{h\to 0} \frac{1}{h} \cdot \left[\log_e\left(1+\frac{h}{x}\right)\right] = \lim_{h\to 0} \log_e\left(1+\frac{h}{x}\right)^{1/h} \qquad \because b\log a = \log a^b
\end{aligned}$$

Derivative of standard functions (Logarithmic)

Show that:

$$\frac{d}{dx}(\log_e x) = \frac{d}{dx}(\ln x) = \frac{1}{x} \quad ; \quad x \in \mathbb{R}^+$$

Proof:

$$\frac{d}{dx}(\log_e x) = \lim_{h \to 0} \log_e \left(1 + \frac{h}{x}\right)^{1/h}$$

$$= \log_e \left[\lim_{h \to 0} \left(1 + \frac{h}{x}\right)^{1/h}\right]$$

Use: Limit of a composite function:

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

Let $\frac{h}{x} = m \Longrightarrow h \to 0$ then $m \to 0$.

$$\therefore \log_e \left[\lim_{h \to 0} \left(1 + \frac{h}{x}\right)^{1/h}\right] = \ln\left[\left(\lim_{m \to 0}(1+m)^{\frac{1}{mx}}\right)\right]$$

$$= \ln\left[\boxed{\left(\lim_{m \to 0}(1+m)^{\frac{1}{m}}\right)}^{\frac{1}{x}}\right]$$

$$= \ln\left[e^{\frac{1}{x}}\right] = \frac{1}{x}\ln e$$

$$\therefore \frac{d}{dx}(\log_e x) = \frac{d}{dx}(\ln x) = \frac{1}{x} \quad ; \quad x \in \mathbb{R}^+$$

Result:

$$\boxed{\lim_{x \to 0} (1+x)^{1/x} = e} \quad \textbf{OR} \quad \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

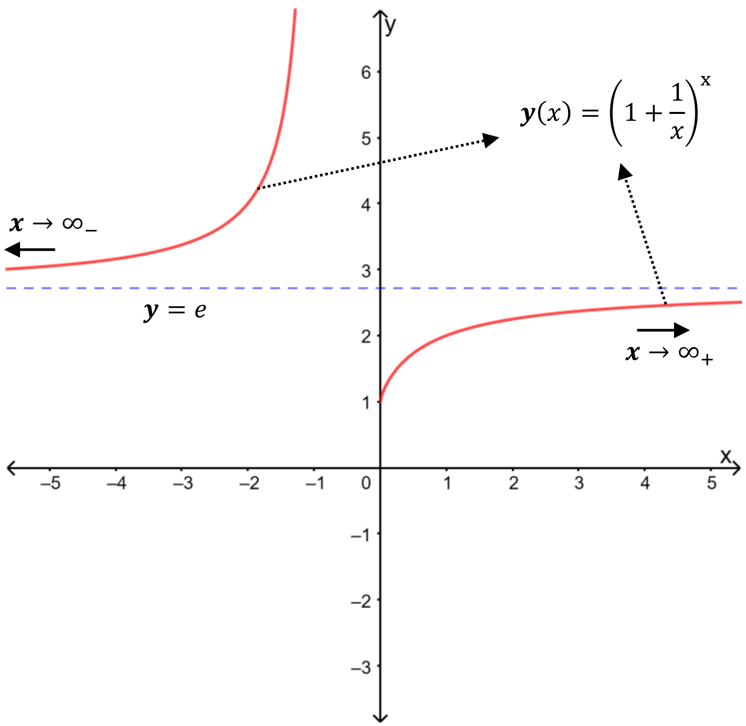


Table of Derivatives of Standard Functions

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\ \sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\ \cot x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x\ \log_e a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Exercise

Given $y(x) = x^n$, using the definition of the derivative (from first principles) show that:

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

Hint: use the binomial expansion for $(x+h)^n$

Rules of Differentiation

Differentiation is the process of finding the derivatives.

The SUM rule

If $u = f(x)$ and $v = g(x)$ are differentiable functions of x, then,

$$\boxed{\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}}$$

The DIFFERENCE rule

$$\boxed{\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}}$$

Example

Given $y = 1 + \dfrac{x^2 - 7x + 4}{x^3}$, find $\dfrac{dy}{dx}$

$$\boxed{\frac{d}{dx}(k) = 0}$$ k is a constant

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}\left(1 + \frac{x^2 - 7x + 4}{x^3}\right) \Rightarrow \boxed{\frac{d}{dx}(1)} + \frac{d}{dx}(x^{-1}) - \boxed{7\frac{d}{dx}(x^{-2})} + \boxed{4\frac{d}{dx}(x^{-3})}$$

$$= 0 + (-1)(x^{-1-1}) - 7(-2)(x^{-2-1}) + 4\,(-3)(x^{-3-1})$$

$$\therefore \frac{dy}{dx} = -\frac{1}{x^2} + \frac{14}{x^3} - \frac{12}{x^4}$$

$$\boxed{\frac{d}{dx}\big(kf(x)\big) = k\frac{d}{dx}\big(f(x)\big)}$$

k is a constant

Note: $\dfrac{d}{dx}(x^n) = nx^{n-1}$

Example

Given $y = x^3 + \sin x - e^x + \ln x$, find $\dfrac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}x^3 + \frac{d}{dx}(\sin x) - \frac{d}{dx}e^x + \frac{d}{dx}(\ln x)$$

$$\frac{dy}{dx} = 3x^2 + \cos x - e^x + \frac{1}{x}$$

Example

Given $y = (x^2 - 1)(x^2 + 1)$ find $\dfrac{dy}{dx}$

Solution:

$$y = (x^2 - 1)(x^2 + 1) = x^4 - 1$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^4 - 1) = \frac{d}{dx}(x^4) + \frac{d}{dx}(1) \quad \Rightarrow \quad 4x^3 + 0 = 4x^3$$

Rules of Differentiation

The PRODUCT rule

If $u = f(x)$ and $v = g(x)$ are differentiable functions of x, then,

$$\boxed{\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}}$$

Examples

Find $\frac{dy}{dx}$ $\quad y = \ln x \cdot \cos x$

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}(\ln x \cdot \cos x) \Rightarrow \frac{d}{dx}(uv) = u\frac{dv}{dx} + \mathrm{v}\frac{du}{dx}$$ **Product Rule**

Let $u = \ln x$; $\frac{du}{dx} = \frac{1}{x}$

and $v = \cos x$; $\frac{dv}{dx} = -\sin x$

$$\therefore \frac{dy}{dx} = \ln x \cdot (-\sin x) + \cos x \cdot \left(\frac{1}{x}\right)$$

$$= -\ln x \cdot \sin x + \frac{\cos x}{x} \qquad x \in \mathbb{R}^{+}$$

Examples

Find $\dfrac{dy}{dx}$ $\quad y = \sin 2x$

Solution:

$$y = \sin 2x = 2\sin x \cos x \quad \Rightarrow \quad \frac{d}{dx}(uv) = u\frac{dv}{dx} + \mathrm{v}\frac{du}{dx}$$ **Product Rule**

$$\frac{dy}{dx} = 2\frac{d}{dx}(\sin x \cdot \cos x)$$

Let $u = \sin x$; $\dfrac{du}{dx} = \cos x$

and $v = \cos x$; $\dfrac{dv}{dx} = -\sin x$

$$\therefore \frac{dy}{dx} = \sin x \cdot (-\sin x) + \cos x \cdot (\cos x)$$

$$= -\sin^2 x + \cos^2 x$$

$$= \cos^2 x - \sin^2 x = \mathbf{2\cos 2x}$$

A faster method would be shown later!

Example

Given $y = (x^2 - 1)(x^2 + 1)$, find $\dfrac{dy}{dx}$ using the product rule.

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}[(x^2 - 1)(x^2 + 1)] \Rightarrow \frac{d}{dx}(uv) = u\frac{dv}{dx} + \mathrm{v}\frac{du}{dx}$$ **Product Rule**

Let $u = (x^2 - 1)$; $\dfrac{du}{dx} = 2x$

and $v = (x^2 + 1)$; $\dfrac{dv}{dx} = 2x$

$$\therefore \frac{dy}{dx} = (x^2 - 1)(2x) + (x^2 + 1)(2x)$$

$$= (2x^2)2x = 4x^3$$

Note: this was solved earlier by first simplifying and then applying the sum/difference rule

Extension of the Product Rule

If $u = f(x)$, $v = g(x)$, and $w = h(x)$ are differentiable functions of x, then

$$\frac{d}{dx}(u \cdot v \cdot w) = \boxed{u\,v \cdot \frac{dw}{dx}} + \boxed{v\,w \cdot \frac{du}{dx}} + \boxed{u\,w \cdot \frac{dv}{dx}}$$

Keep 2 functions fixed, take the derivative of the 3rd

Example

Find $\dfrac{d}{dx}\left(x\, e^{x}\ \cot x\right)$.

Solution:

$$\frac{d}{dx}\left(x\, e^{x}\ \cot x\right) =$$

$$\boxed{x\, e^{x} \cdot \frac{d}{dx}(\cot x)} + \boxed{e^{x}\ \cot x \cdot \frac{d}{dx}(x)} + \boxed{x\ \cot x \cdot \frac{d}{dx}(e^{x})}$$

Keep 2 functions fixed, take the derivative of the 3rd

$$= x\, e^{x}\left(-\mathrm{cosec}^2 x\right) + e^{x}\ \cot x\,(1) + x\ \cot x\, e^{x}$$

$$= e^{x}\ \left(-\,x\,\mathrm{cosec}^2 x + \cot x + x\ \cot x\right).$$

Quotient Rule for Differentiation

If $u = f(x)$ and $v = g(x)$ are differentiable functions of x, then

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,\dfrac{du}{dx} - u\,\dfrac{dv}{dx}}{v^2}$$

Example

Find $\dfrac{d}{dx}\left(\dfrac{x^2-1}{x^2+1}\right)$.

Consider $u = x^2 - 1$ and $v = x^2 + 1$.

Using Quotient Rule

Example: Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,\dfrac{du}{dx} - u\,\dfrac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right) = \frac{(x^2+1)\cdot\dfrac{d}{dx}(x^2-1) - (x^2-1)\cdot\dfrac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\cdot 2x - (x^2-1)\cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x\,(x^2+1-x^2+1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

Practice Problems

(1) Evaluate: $\lim_{x \to 2} \dfrac{x-2}{\sqrt{x^2-4}}$.

(2) Given $y = \sqrt{x-1}$, find $\dfrac{dy}{dx}$,

from first principles.

(3) Given $y = 3x^3 - 4x\cos x$, find $\dfrac{dy}{dx}$.

(4) Given $y = \dfrac{\sin x}{e^x}$, find $\dfrac{dy}{dx}$.