Worksheet 4 Answers

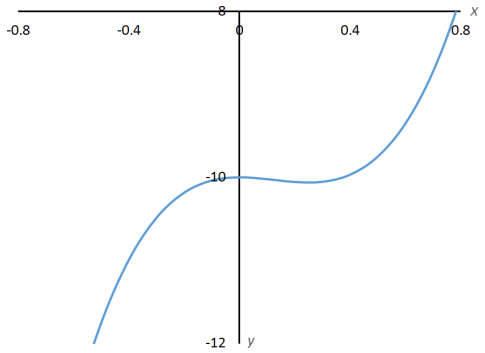
1.

$i)$ $x = 0\,, y = -10$

$f''(0) = -6 \quad \therefore$ maxima

$x = \frac{1}{4}\,, y = -10\frac{1}{16}$

$f''\left(\frac{1}{4}\right) = 6 \quad \therefore$ minima

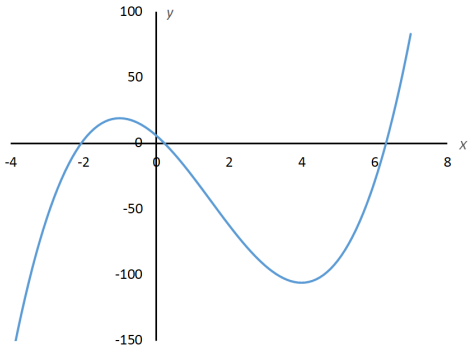


$ii)$ $x = -1\,, y = 19$

$f''(-1) = -30 \quad \therefore$ maxima

$x = 4\,, y = -106$

$f''(4) = 30 \quad \therefore$ minima



$iii)$ $x = 0\,, y = 25$

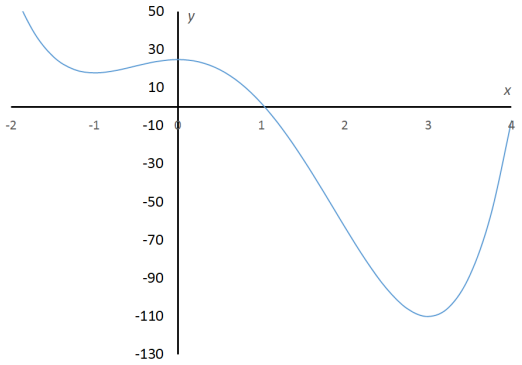
$f''(0) = -36 \quad \therefore$ maxima

$x = -1\,, y = 18$

$f''(-1) = 48 \quad \therefore$ minima

$x = 3\,, y = -110$

$f''(3) = 144 \quad \therefore$ minima



$iv)$ $x = 0\,, y = 1$

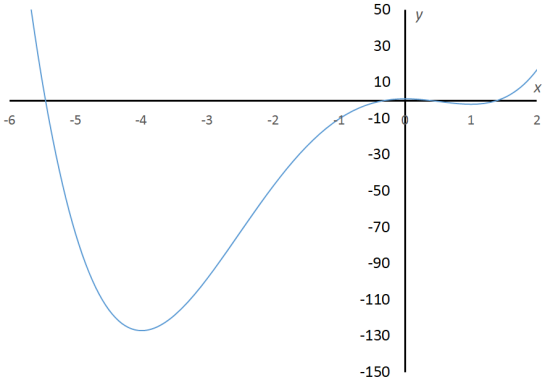
$f''(0) = -16 \quad \therefore$ maxima

$x = 1\,, y = -2$

$f''(1) = 20 \quad \therefore$ minima

$x = -4\,, y = -127$

$f''(-4) = 80 \quad \therefore$ minima



v) $x = 0\,, y = 0$

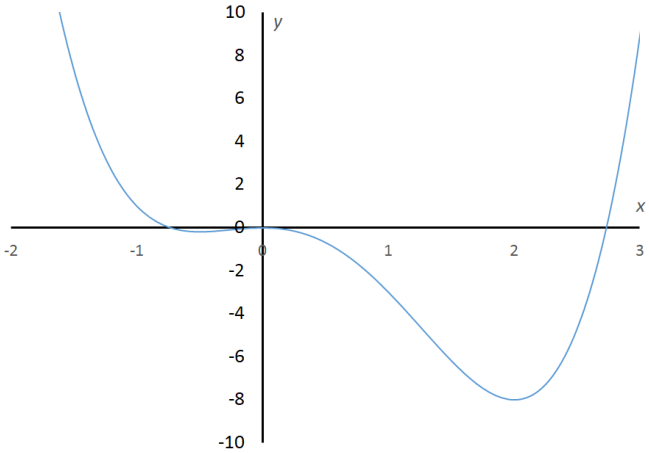
$f''(0) = -4 \quad \therefore$ maxima

$x = 2\,, y = -8$

$f''(2) = 20 \quad \therefore$ minima

$x = -\frac{1}{2}\,, y = -\frac{3}{16}$

$f''\left(-\frac{1}{2}\right) = 5 \quad \therefore$ minima

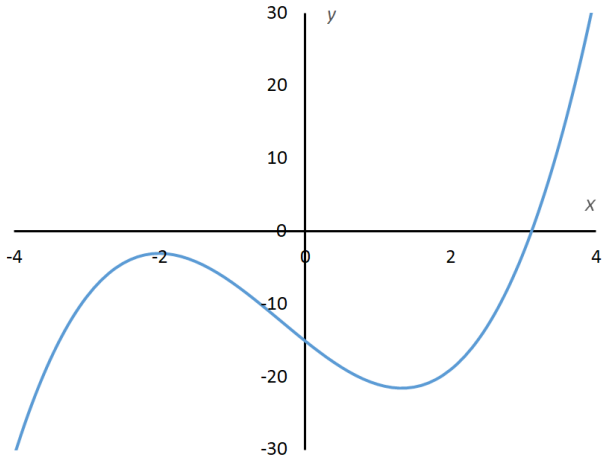


vi) $x = -2\,, y = -3$

$f''(-2) = -10 \quad \therefore$ maxima

$x = \frac{4}{3}\,, y = -21\frac{14}{27}$

$f''\left(\frac{4}{3}\right) = 10 \quad \therefore$ minima



2.

i)

$x = 5\sqrt{2} \quad \& \quad y = 5\sqrt{2}$

$\left.\frac{d^2A}{dx^2}\right|_{x=5\sqrt{2}} = -4 \quad \therefore$ maxima

ii)

$x = 2 \quad \& \quad y = 4$

$\left.\frac{d^2D}{dx^2}\right|_{x=2} = \frac{25}{68}\sqrt{17} \quad \therefore$ minima

iii)

Length $= 8\sqrt{2}$

Width $= 4\sqrt{2}$

$Area = 64$

$\left.\frac{d^2A}{dx^2}\right|_{x=2\sqrt{2}} = -8 \quad \therefore$ maxima

3.

i) $\frac{dV}{dt} = 25\pi \ \text{cm}^3/\text{s}$

ii) $\frac{dr}{dt} = \frac{1}{40\pi} \ \text{cm/s}$

iii) $\frac{dh}{dt} = 350 \ \text{km/h}$

iv) $\frac{dc}{dt} = -\frac{1}{4} \ \text{cm/s}$

4.

$i)\ x^4 - \frac{5x^2}{2} + 6x + c$

$ii)\ \frac{x^8}{8} + \frac{1}{4x^4} + \frac{2x\sqrt{x}}{3} + c$

$iii)\ 2x^4 + 4\,x^3 + 3\,x^2 + x + c$

$iv)\ \frac{12x^{5/2}}{5} + \frac{1}{x} - 5\ln|x| + c$

$v)\ e^x - 4\sqrt{x} + c$

$vi)\ \frac{3}{5}x^{5/3} + 3e^x - \frac{1}{x} + c$

$vii)\ \sin(x) + 2\cos(x) + c$

$viii)\ \frac{x^5}{5} - \tan(x) + c$

$ix)\ \tan(x) + x + c$

$x)\ \sin(x) + \cot(x) + x + c$

$xi)\ e^x - \sec(x) + c$

$xii)\ -\sec(x) + c$

$xiii)\ \tan(x) - x + c$

$xiv)\ \tan^{-1}(x) + \frac{x^2}{2} + c$

$xv)\ \frac{a^x}{\ln(a)} - \frac{x^{a+1}}{a+1} + c$