University of Nottingham
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Foundation Calculus and Mathematical Techniques (CELEN037)

Answers to Worksheet #1

1.

(i) $2x$ (ii) $2x+2$ (iii) $-\dfrac{2}{x^3}$

(iv) $\dfrac{1}{2\sqrt{x}}$ (v) $\dfrac{1}{2\sqrt{x+1}}$ (vi) $-\dfrac{1}{2\sqrt{x^3}}$

(vii) $a^x \ln a$ (viii) $-e^{1-x}$ (ix) $-\sin x$

(x) $\sec^2 x$ (xi) $-\sin(x+1)$ (xii) $2\cos 2x$

(xiii) $\sin 2x$ (xiv) $2x\cos x^2$ (xv) nx^{n-1}

2.

(i) $5x^4+1$ (ii) $-8x+2$

(iii) $6x+14$ (iv) $3x^2-12x+12$

(v) $4x^3+\dfrac{1}{x^2}$ (vi) $2-\dfrac{1}{x^2}+\dfrac{6}{x^3}$

(vii) $2x+\dfrac{8}{3\sqrt[3]{x^5}}$ (viii) $1-\dfrac{6}{x^2}$

(ix) $\dfrac{2}{x^2}$ (x) $1-\dfrac{6}{x^2}$

(xi) $-\dfrac{1}{2\sqrt{x^3}}-e^x+1$ (xii) $\dfrac{1}{x}-3x^2-\dfrac{1}{x^2}$

(xiii) $\tan x\sec x+\cot x\ \mathrm{cosec}\ x$ (xiv) $-\mathrm{cosec}\ x\big(\mathrm{cosec}\ x+\cot x\big)$

(xv) $-\dfrac{3}{x^4}+2\sec^2 x$ (xvi) $\tan x\sec x-\sin x$

(xvii) $\sec x\big(\tan x+\sec x\big)$ (xviii) $-\dfrac{2}{x^3}+4\cos x+\dfrac{2}{3\sqrt[3]{x}}+\mathrm{cosec}^2 x$

(xix) $-\dfrac{5}{3}x^{-\frac{4}{3}}+3\sin x$ (xx) $\dfrac{3}{4}x^{-\frac{1}{4}}+2\sec^2 x$

(xxi) $\dfrac{1}{x-5}-\dfrac{1}{x+1}$ (xxii) $\dfrac{1}{2\sqrt{x}}+2^x\ln 2+\mathrm{cosec}^2 x+\dfrac{1}{x^2}$

3.

(i) $2x\cos x - x^2\sin x$

(ii) $x^2\big(3\ln x + 1\big)$

(iii) $\dfrac{\cos x}{x} - \sin x \ln x$

(iv) $\sec x\left(\dfrac{1}{x} + \tan x \ln x\right)$

(v) $\sec x\left(\dfrac{\tan x}{x} - \dfrac{1}{x^2}\right)$

(vi) $\dfrac{\cos x}{x} - \dfrac{\sin x}{x^2}$

(vii) $\dfrac{1 - \ln x}{x^2}$

(viii) $2\cos 2x$

(ix) $\sec x\left(\tan x + \tan^2 x + \sec^2 x\right)$

(x) $e^x \sec x\big(1 + \tan x\big)$

4.

(i) $e^x\left(\sin x \ln x + \cos x \ln x + \dfrac{\sin x}{x}\right)$

(ii) $xe^x\big(x\sec^2 x + 2\tan x + x\tan x\big)$

(iii) $x^2e^x\big(3\ln x + x\ln x + 1\big)$

(iv) $\sin x\left(\tan x + x + x\sec^2 x\right)$

5.

(i) $\sec^2 x$

(ii) $\dfrac{\sin x - x\cos x}{\sin^2 x}$

(iii) $\dfrac{\sec x(\tan x - 1)}{e^x}$

(iv) $\cos x - \sin x$

(v) $\dfrac{\ln x - 1}{(\ln x)^2}$

(vi) $\dfrac{2x\tan x - x^2\sec^2 x}{\tan^2 x}$

(vii) $\dfrac{\cos x - \sin x}{e^x}$

(viii) $\dfrac{xe^x - 3e^x}{x^4}$

(ix) $-e^{-x}$

(x) $\dfrac{1 - x}{e^x}$

(xi) $\dfrac{2}{(1-x)^2}$

(xii) $\dfrac{-4x}{(1+x^2)^2}$

Solution to Problem 1(xiv).

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sin(x+h)^2 - \sin x^2}{h}$$

$$= \lim_{h\to 0}\frac{2\cos\left(\frac{(x+h)^2+x^2}{2}\right)\sin\left(\frac{(x+h)^2-x^2}{2}\right)}{h} = 2\lim_{h\to 0}\cos\left(\frac{(x+h)^2+x^2}{2}\right)\cdot\lim_{h\to 0}\frac{\sin\left(h\left(x+\frac{h}{2}\right)\right)}{h}$$

$$= 2\cos x^2\cdot\lim_{h\to 0}\left(\frac{\sin\left(h\left(x+\frac{h}{2}\right)\right)}{h}\cdot\frac{x+\frac{h}{2}}{x+\frac{h}{2}}\right) = 2\cos x^2\cdot\lim_{h\to 0}\frac{\sin\left(h\left(x+\frac{h}{2}\right)\right)}{h\left(x+\frac{h}{2}\right)}\cdot\lim_{h\to 0}\left(x+\frac{h}{2}\right)$$

$$= 2\cos x^2\cdot 1\cdot x = 2x\cos x^2$$