FOUNDATION SCIENCE A

SEMINAR 5: THE ELECTRIC FORCE & COULOMB'S LAW

University of Nottingham
UK | CHINA | MALAYSIA

LEARNING OUTCOMES

To better understand and analyse the following problems:

- Electric Charge
- Electric Force Concepts Using Coulomb's Law
- Static Electricity

Coulomb's Law

QUESTION 1:

How many electrons make up a charge of $-38.0\ \mu C$?

Answer:

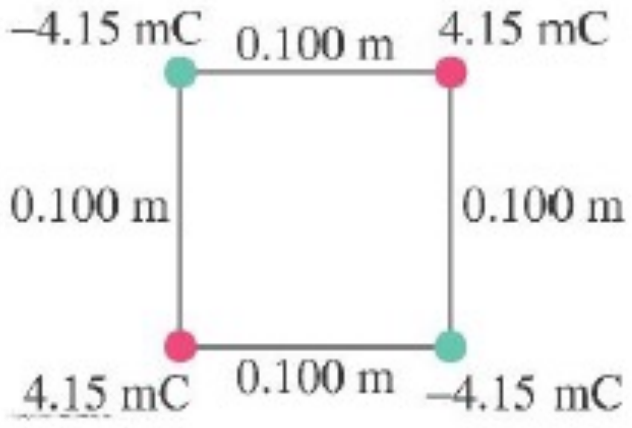
Use the charge per electron to find the number of electrons.

$$\left(-38.0\times10^{-6}\,\mathrm{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\,\mathrm{C}}\right)=\boxed{2.37\times10^{14}\ \text{electrons}}$$

Coulomb’s Law

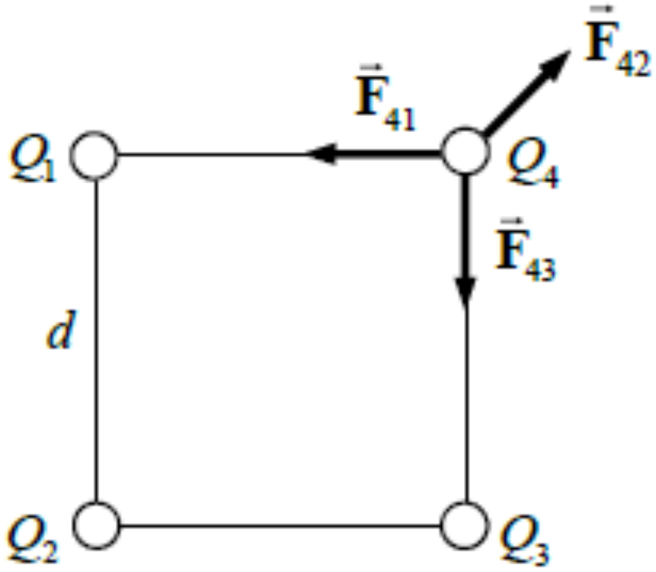
QUESTION 2:

Two negative and two positive point charges (magnitude $Q = 415$ mC) are placed on opposite corners of a square as shown in the figure below. Determine the magnitude and direction of the force on each charge.



Answer:

Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.



The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable, d, represent the 0.100 m length of a side of the square, and let the variable, Q, represent the 4.15 mC charge at each corner.

Coulomb's Law

Answer:

$$F_{41} = k\frac{Q^2}{d^2} \rightarrow F_{41x} = -k\frac{Q^2}{d^2}, F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \rightarrow F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}, F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

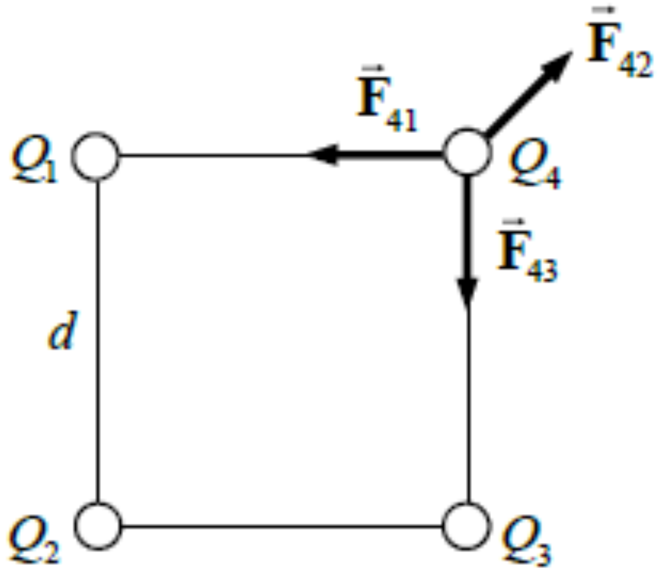
$$F_{43} = k\frac{Q^2}{d^2} \rightarrow F_{43x} = 0, F_{43y} = -k\frac{Q^2}{d^2}$$

Add the x and y components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = -k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(-1 + \frac{\sqrt{2}}{4}\right) = -0.64645k\frac{Q^2}{d^2} = F_{4y}$$

Coulomb's Law

Answer:



$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}(0.64645)\sqrt{2} = k\frac{Q^2}{d^2}(0.9142)$$

$$= \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(4.15\times10^{-3}\,\mathrm{C}\right)^2}{\left(0.100\,\mathrm{m}\right)^2}(0.9142) = \boxed{1.42\times10^7\,\mathrm{N}}$$

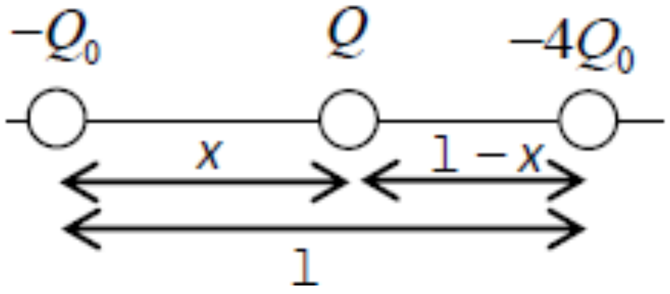
$\theta = \tan^{-1}\dfrac{F_{4y}}{F_{4x}} = \boxed{225^\circ}$ from the x-direction, or exactly towards the center of the square.

For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square. Thus for each charge, the net force will be the magnitude of $\boxed{1.42\times10^7\,\mathrm{N}}$ and will lie along the line from the charge inwards towards the center of the square.

Coulomb's Law

QUESTION 3:

Two charges, $-Q_0$ and $-4Q_0$, are a distance l apart. These two charges are free to move, but do not because there is a third charge nearby. What must be the magnitude of the third charge and its placement in order for the first two to be in equilibrium?



Answer:

The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges.

Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges.

See the diagram for the definition of variables.

For each negative charge, equate the magnitudes of the two forces on the charge.

Also note that $0 < x < 1$.

Coulomb's Law

QUESTION 3:

Two charges, -Q_0 and -4Q_0, are a distance l apart. These two charges are free to move, but do not because there is a third charge nearby. What must be the magnitude of the third charge and its placement in order for the first two to be in equilibrium?

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Two charges, -Q_0 and -4Q_0, are a distance l apart. These two charges are free to move, but do not because there is a third charge nearby. What must be the magnitude of the third charge and its placement in order for the first two to be in equilibrium?

Answer:

$$\text{left: } k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{l^2} \qquad \text{right: } k\frac{4Q_0Q}{(l-x)^2} = k\frac{4Q_0^2}{l^2} \rightarrow$$

$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0Q}{(l-x)^2} \rightarrow x = \tfrac{1}{3}l$$

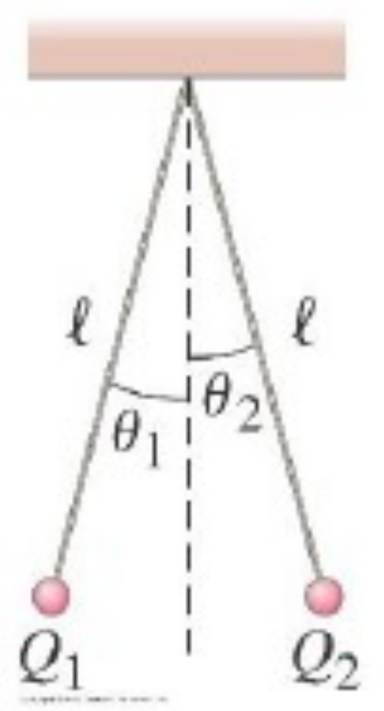
$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{l^2} \rightarrow Q = 4Q_0\frac{x^2}{l^2} = Q_0\frac{4}{(3)^2} = \tfrac{4}{9}Q_0$$

Thus, the charge should be of magnitude $\boxed{\tfrac{4}{9}Q_0}$, and a distance $\boxed{\tfrac{1}{3}l \text{ from } -Q_0 \text{ towards } -4Q_0}$.

Coulomb's Law

QUESTION 4:

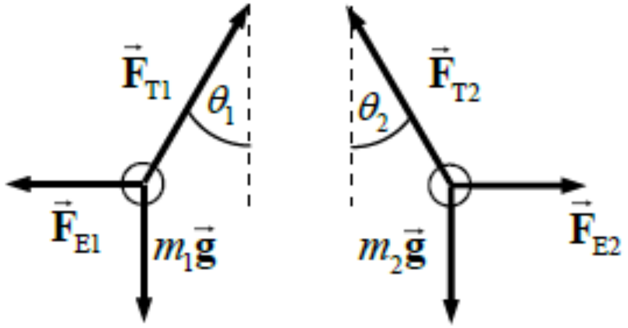
Two small charged spheres hang from cords of equal length l as shown in the figure below and make small angle θ_1 and θ_2 with the vertical. (a) If $Q_1 = Q, Q_2 = 2Q$, and $m_1 = m_2 = m$, determine the ratio θ_1/θ_2. (b) If $Q_1 = Q, Q_2 = 2Q$, and $m_1 = m$ and $m_2 = 2m$, determine the ratio θ_1/θ_2. (c) Estimate the distance between the spheres for each case.



Answer:

If all of the angles to the vertical (in both cases) are assumed to be small, then the spheres only have horizontal displacement, and so the electric force of repulsion is always horizontal.

See the free-body diagram for each sphere, showing the three forces of gravity, tension, and the electrostatic force.

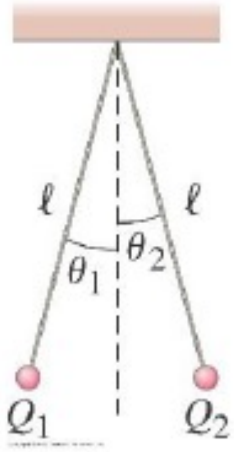


The right is the positive horizontal direction, and up is the positive vertical direction.

Since the spheres are in equilibrium, the net force in each direction is zero.

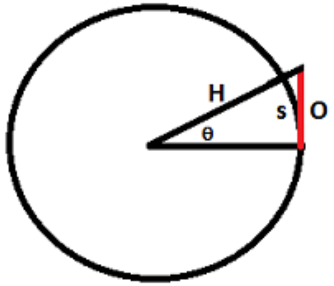
QUESTION 4:

Two small charged spheres hang from cords of equal length l as shown in the figure below and **make small angle θ_1 and θ_2** with the vertical. (a) If $Q_1 = Q$, $Q_2 = 2Q$, and $m_1 = m_2 = m$, determine the ratio θ_1/θ_2. (b) If $Q_1 = Q, Q_2 = 2Q$ and $m_1 = m$ and $m_2 = 2m$, determine the ratio θ_1/θ_2. (c) Estimate the distanc between the spheres for each case.



Answer:

The small angle condition leads to $\tan\theta \approx \sin\theta \approx \theta$ for all small angles.



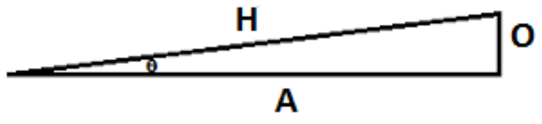
sin θ = O/H

Arc Length:

s = rθ (r ≈ H)

θ = s/H

so,
sin θ ≈ θ

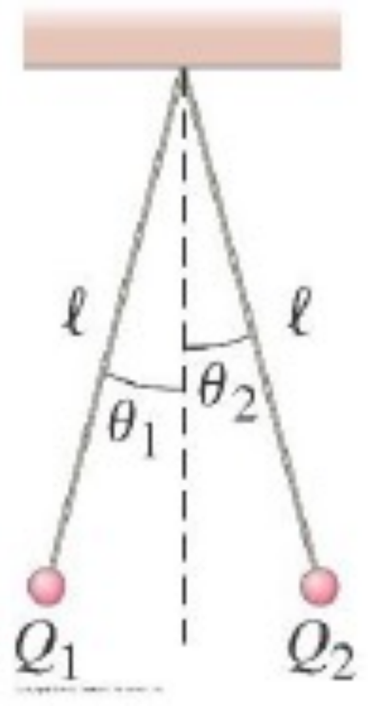


A is similar in length to H,
so sin θ = tan θ = θ

Small angle approximations:

cos θ = 1
sin θ = θ
tanθ = θ

Coulomb's Law



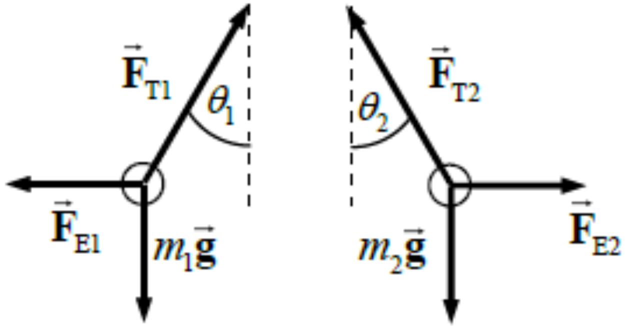
Answer:

(a) $$\sum F_{1x} = F_{T1}\sin\theta_1 - F_{E1} = 0 \;\rightarrow\; F_{E1} = F_{T1}\sin\theta_1$$

$$\sum F_{1y} = F_{T1}\cos\theta_1 - m_1 g \;\rightarrow\; F_{T1} = \frac{m_1 g}{\cos\theta_1} \;\rightarrow\; F_{E1} = \frac{m_1 g}{\cos\theta_1}\sin\theta_1 = m_1 g\tan\theta_1 = m_1 g\theta_1$$

A completely parallel analysis would give F_{E2}=$m_2 g\theta_2$. Since the electric forces are a Newton's Third Law pair, they can be set equal to each other in magnitude.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_2/m_1 = \boxed{1}$$

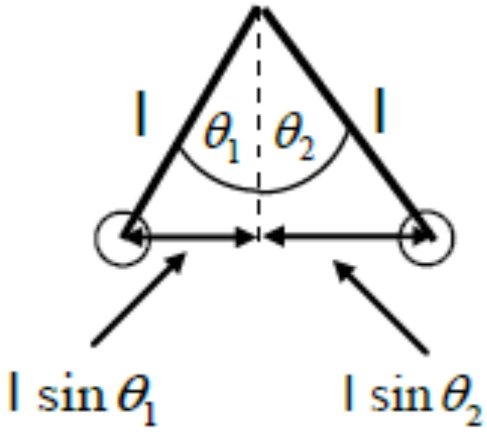


(*b*) The same analysis can be done for this case.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_1/m_1 = \boxed{2}$$

Answer:

(c) The horizontal distance from one sphere to the other is s by the small angle approximation. See the diagram. Use the relationship derived above that $F_E = mg\theta$ to solve for the distance.



Case 1: $d = l(\theta_1 + \theta_2) = 2l\theta_1 \rightarrow \theta_1 = \dfrac{d}{2l}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{d}{2l} \rightarrow \boxed{d = \left(\frac{4lkQ^2}{mg}\right)^{1/3}}$$

Case 2: $d = l(\theta_1 + \theta_2) = \dfrac{3}{2}l\theta_1 \rightarrow \theta_1 = \dfrac{2d}{3l}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{2d}{3l} \rightarrow \boxed{d = \left(\frac{3lkQ^2}{mg}\right)^{1/3}}$$

Electric Field, Field Lines:

QUESTION 5:

A downward electric force of 8.4 N is exerted on a -8.8 μC charge. What are the magnitude and direction of the electric field at the position of this charge?

Answer:

Use the equation below to calculate the electric field.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{8.4\,\mathrm{N\,down}}{-8.8\times10^{-6}\,\mathrm{C}} = \boxed{9.5\times10^{5}\ \mathrm{N/C\ up}}$$

Electric Field, Field Lines:

QUESTION 6:

Determine the magnitude and direction of the electric field at a point midway between a -8.0 μC charge and a +5.8 μC charge 8.0 cm apart. Assume no other charge are nearby.

Answer:

The electric field due to the negative charge will point toward the negative charge, and the electric field due to the positive charge will point away from the positive charge. Thus, both fields point in the same direction, towards the negative charge, and so can be added.

$$E = |E_1| + |E_2| = k\frac{|Q_1|}{r_1^2} + k\frac{|Q_2|}{r_2^2} = k\frac{|Q_1|}{(l/2)^2} + k\frac{|Q_2|}{(l/2)^2} = \frac{4k}{l^2}\left(|Q_1| + |Q_2|\right)$$

$$= \frac{4\left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)}{(0.080\,\mathrm{m})^2}\left(8.0\times10^{-6}\,\mathrm{C} + 5.8\times10^{-6}\,\mathrm{C}\right) = \boxed{7.8\times10^7\ \mathrm{N/C}}$$

The direction is $\boxed{\text{towards the negative charge}}$.

Electric Field, Field Lines:

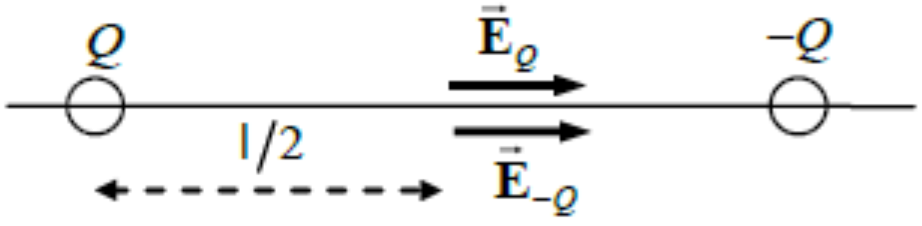
QUESTION 7:

The electric field midway between two equal but opposite point charges is 586 NC^{-1}, and the distance between the charges is 16.0 cm. What is the magnitude of the charge on each?

Answer:

The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus, the magnitudes of the two fields can be added together to find the charges.

$$E_{net} = 2E_Q = 2k\frac{Q}{(l/2)^2} = \frac{8kQ}{l^2} \rightarrow Q = \frac{El^2}{8k} = \frac{(586\,\mathrm{N/C})(0.160\,\mathrm{m})^2}{8(8.988\times10^9\,\mathrm{N\cdot m^2/C^2})} = \boxed{2.09\times10^{-10}\,\mathrm{C}}$$

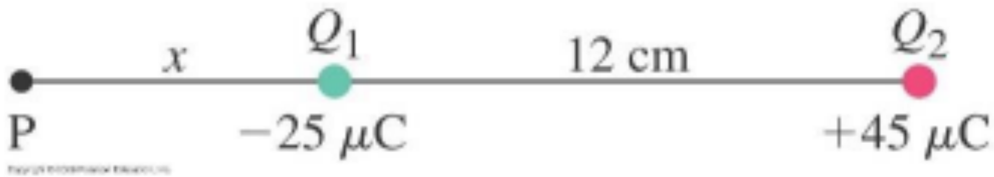


Electric Field, Field Lines:

QUESTION 8:

Two point charges, $Q_1 = -25\ \mu\text{C}$ and $Q_2 = +45\ \mu\text{C}$, are separated by a distance of 12 cm. The electric field at the point P (see the figure below) is zero. How far from Q_1 is P?

Answer:

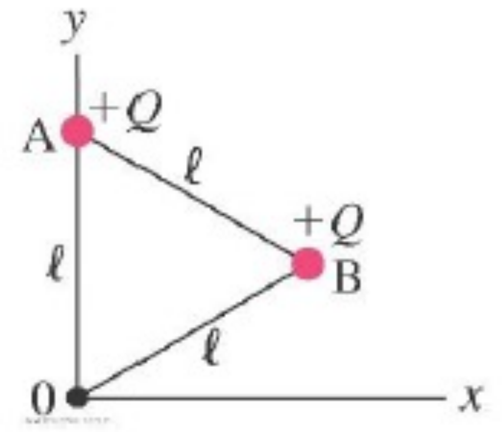


For the net field to be zero at point P, the magnitudes of the fields created by *Q*1 and *Q*2 must be equal. Also, the distance *x* will be taken as positive to the left of *Q*1. That is the only region where the total field due to the two charges can be zero. Let the variable *l* represent the 12 cm distance, and note that $|Q_1| = \frac{1}{2}\, Q_2$.

$$\left|\vec{\mathbf{E}}_1\right| = \left|\vec{\mathbf{E}}_2\right| \quad \rightarrow \quad k\frac{|Q_1|}{x^2} = k\frac{Q_2}{(x+l)^2} \quad \rightarrow \quad x = l\frac{\sqrt{|Q_1|}}{\left(\sqrt{Q_2} - \sqrt{|Q_1|}\right)} = (12\,\text{cm})\frac{\sqrt{25\mu C}}{\left(\sqrt{45\mu C} - \sqrt{25\mu C}\right)} = \boxed{35\,\text{cm}}$$

Electric Field, Field Lines:

QUESTION 9:



(a) Determine the electric field $\vec{\mathbf{E}}$ at the origin, O, in the figure below due to the two charges at A and B. (b) Repeat, but let the charge at B be reversed in sign.

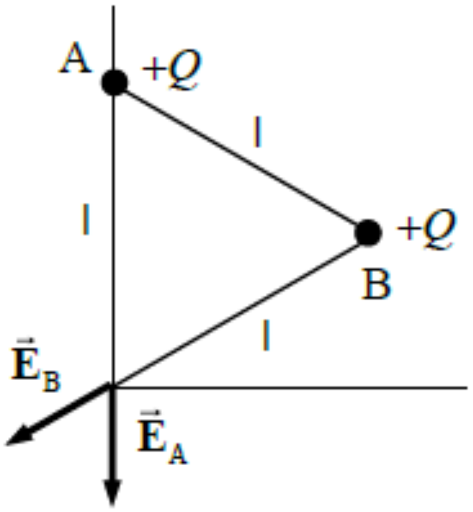
Answer:

(*a*) The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from A to the origin, $30°$ below the negative x axis.

$$E_A = k\frac{Q}{l^2} \rightarrow E_{Ax} = 0\,,\, E_{Ax} = -k\frac{Q}{l^2}$$

$$E_B = k\frac{Q}{l^2} \rightarrow E_{Bx} = -k\frac{Q}{l^2}\cos 30^\circ = -k\frac{\sqrt{3}Q}{2l^2}, \quad E_{By} = -k\frac{Q}{l^2}\sin 30^\circ = -k\frac{Q}{2l^2}$$

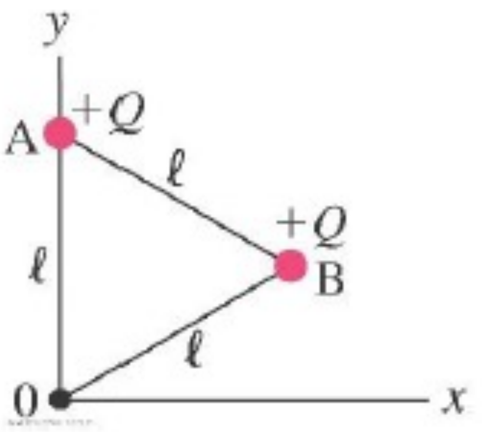
$$E_x = E_{Ax} + E_{Bx} = -k\frac{\sqrt{3}Q}{2l^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{3Q}{2l^2}$$



Electric Field, Field Lines:

QUESTION 9:

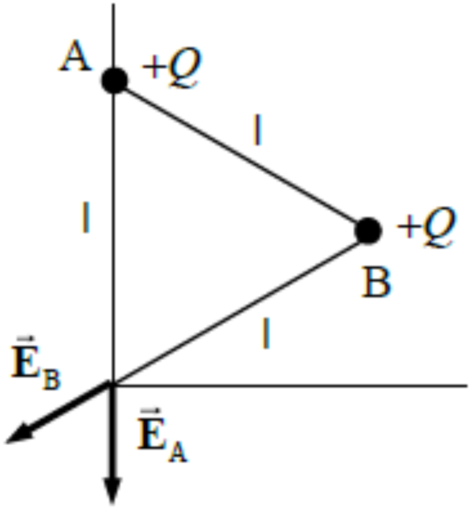
Answer:



$$E_x = E_{Ax} + E_{Bx} = -k\frac{\sqrt{3}Q}{2l^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{3Q}{2l^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4l^4} + \frac{9k^2Q^2}{4l^4}} = \sqrt{\frac{12k^2Q^2}{4l^4}} = \boxed{\frac{\sqrt{3}kQ}{l^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-k\frac{3Q}{2l^2}}{-k\frac{\sqrt{3}Q}{2l^2}} = \tan^{-1}\frac{-3}{-\sqrt{3}} = \tan^{-1}\sqrt{3} = \boxed{240^\circ}$$

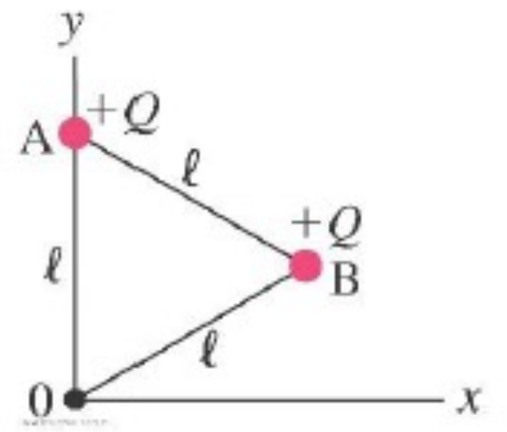


Electric Field, Field Lines:

QUESTION 9:

Answer:

(*b*) Now reverse the direction of $\overrightarrow{E_A}$



$$E_A = k\frac{Q}{l^2} \rightarrow E_{Ax} = 0\,,\; E_{Ax} = -k\frac{Q}{l^2}$$

$$E_B = k\frac{Q}{l^2} \rightarrow E_{Bx} = k\frac{Q}{l^2}\cos 30^\circ = k\frac{\sqrt{3}Q}{2l^2}\,,\; E_{By} = k\frac{Q}{l^2}\sin 30^\circ = k\frac{Q}{2l^2}$$

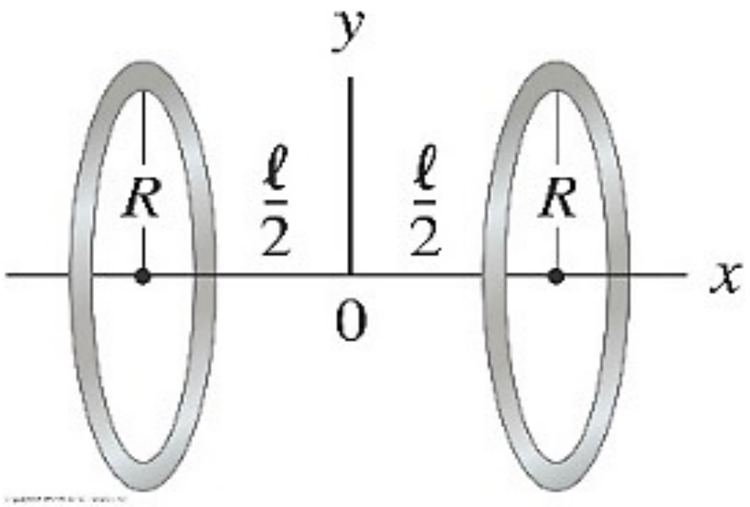
$$E_x = E_{Ax} + E_{Bx} = k\frac{\sqrt{3}Q}{2l^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{Q}{2l^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4l^4} + \frac{k^2Q^2}{4l^4}} = \sqrt{\frac{4k^2Q^2}{4l^4}} = \boxed{\frac{kQ}{l^2}}$$

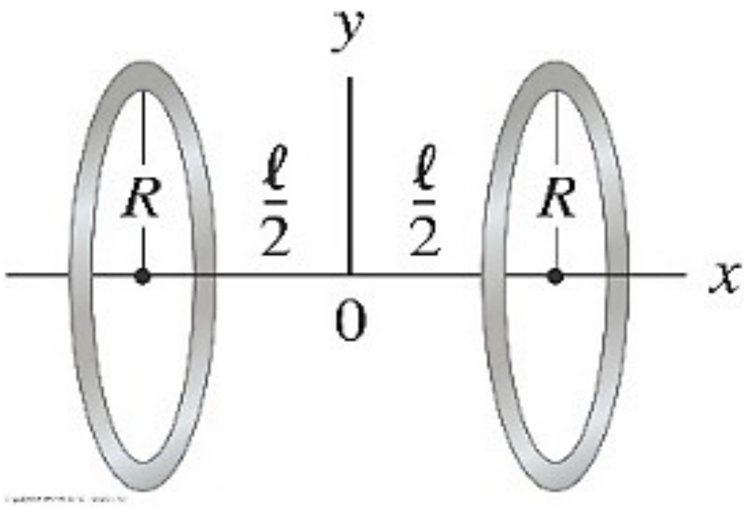
$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{k\frac{Q}{2l^2}}{-k\frac{\sqrt{3}Q}{2l^2}} = \tan^{-1}\frac{1}{-\sqrt{3}} = \boxed{330^\circ}$$

QUESTION 10:

Two parallel circular rings of radius R have their centers on the $\boldsymbol{x}$ axis separated by a distance $\boldsymbol{l}$ as shown in the figure below. If each ring carries a uniformly distributed charge Q, find the electric field, $\vec{\boldsymbol{E}}(\boldsymbol{x})$, at points along the $\boldsymbol{x}$ axis.



QUESTION 10:

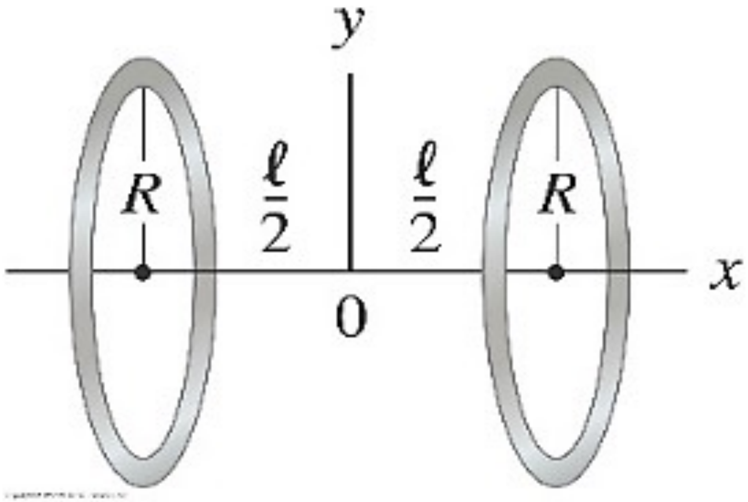


Answer:

- Consider Example 21-9 in chapter 21 of your physics textbook.
- We use the result from this example, but shift the center of the ring to be at $x = {}^{l}/_{2}$ **for the ring on the right**, and at $x = {}^{l}/_{2}$ **for the ring on the left**.
- The fact that the original expression has a factor of x results in the interpretation that the sign of the field expression will give the direction of the field. No special consideration needs to be given to the location of the point at which the field is to be calculated.

Electric Field, Field Lines:

QUESTION 10:



Answer:

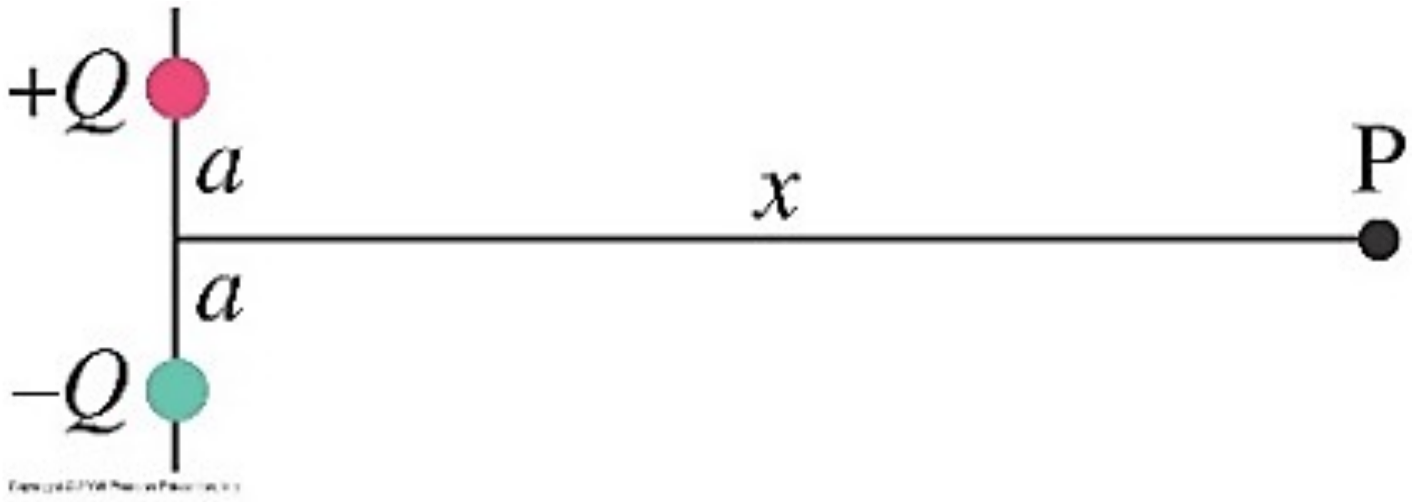
$$\vec{E} = \vec{E}_{\text{right}} + \vec{E}_{\text{left}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q\left(x-\frac{1}{2}l\right)}{\left[\left(x-\frac{1}{2}l\right)^2+R^2\right]^{3/2}}\right)\hat{\mathbf{i}}+\frac{1}{4\pi\varepsilon_0}\left(\frac{Q\left(x+\frac{1}{2}l\right)}{\left[\left(x+\frac{1}{2}l\right)^2+R^2\right]^{3/2}}\right)\hat{\mathbf{i}}$$

$$= \hat{\mathbf{i}}\,\frac{Q}{4\pi\varepsilon_0}\left\{\frac{\left(x-\frac{1}{2}l\right)}{\left[\left(x-\frac{1}{2}l\right)^2+R^2\right]^{3/2}}+\frac{\left(x+\frac{1}{2}l\right)}{\left[\left(x+\frac{1}{2}l\right)^2+R^2\right]^{3/2}}\right\}$$

QUESTION 11:

Determine the direction and magnitude of the electric field at the point P shown in the figure below. The two charges are separated by a distance of $2a$. Point P is on the perpendicular bisector of the line joining the charges, a distance x from the midpoint between them. Express your answer in terms of Q, x, a, and k.



QUESTION 11:

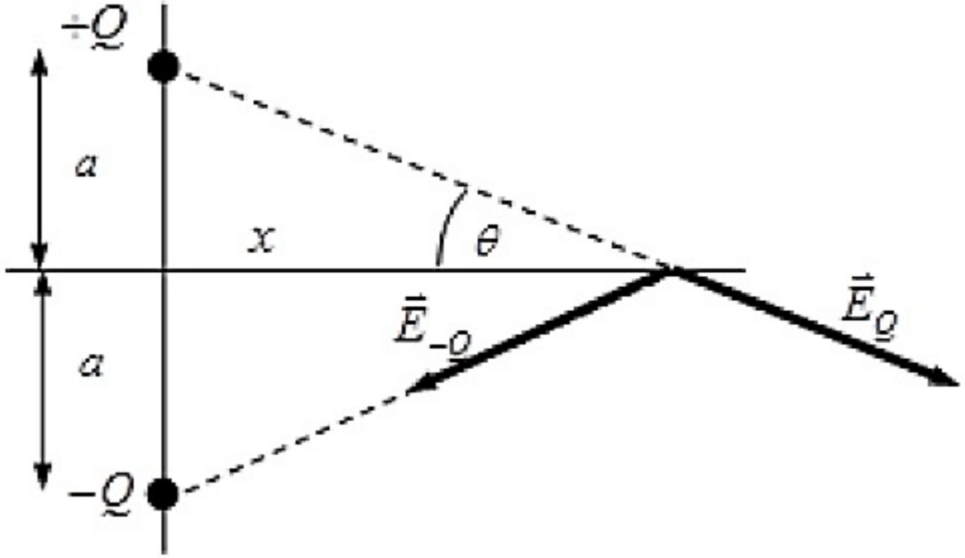
Answer:

From the diagram, we see that the *x* components of the two fields will cancel each other at the point P. Thus, the net electric field will be in the negative *y*-direction, and will be twice the *y*-component of either electric field vector.

$$E_{\text{net}} = 2E\sin\theta = 2\frac{kQ}{x^2+a^2}\sin\theta$$

$$= \left(\frac{2kQ}{x^2+a^2}\right)\frac{a}{(x^2+a^2)^{1/2}}$$

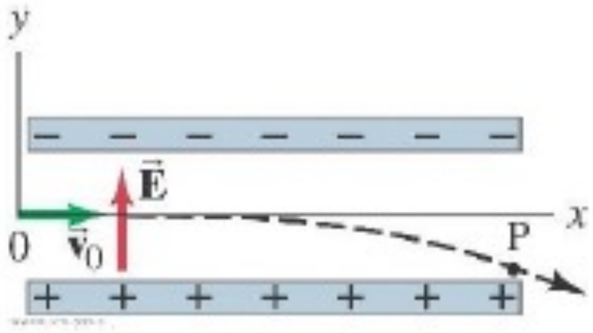
$$= \frac{2kQa}{(x^2+a^2)^{3/2}}$$



in the negative *y* direction.

QUESTION 12:

An electron moving to the right at $7.5 \times 10^5\ \mathrm{ms}^{-1}$ enters a uniform electric field parallel to its direction of motion. If the electron is to be brought to rest in the space of 4.0 cm, (a) what direction is required for the electric field and (b) what is the strength of the field?



Answer:

(a) The electron will experience a force in the opposite direction to the electric field. Since the electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the right .

(b) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$F = qE = ma \quad \rightarrow \quad a = \frac{qE}{m} \qquad v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x \quad \rightarrow$$

$$E = \frac{m\left(v^2 - v_0^2\right)}{2q\Delta x} = \frac{-mv_0^2}{2q\Delta x} = -\frac{\left(9.109\times10^{-31}\,\mathrm{kg}\right)\left(7.5\times10^5\,\mathrm{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\mathrm{C}\right)\left(0.040\,\mathrm{m}\right)} = \boxed{40\,\mathrm{N/C}} \quad (2\text{ sig. fig.})$$

Q&A?
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