University of Nottingham
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Foundation Calculus & Mathematical Techniques CELEN037

Weekly Worksheet-7 **Topics:** Integration by partial fractions, Integration by parts, Definite integrals

Type 1: Integration using Partial Fractions:

Type 1A: Non-repeated linear factors.

1. Evaluate the following integrals by using the method of partial fractions:

(i) $\int \frac{5}{(4x-3)(2x+1)}\,dx$ (ii) $\int \frac{13}{(3x-2)(2x+3)}\,dx$

(iii) $\int \frac{3x}{(x-1)(x+2)}\,dx$ (iv) $\int \frac{x-9}{(x+5)(x-2)}\,dx$

(v) $\int \frac{5x-8}{(x+4)(x-3)}\,dx$ (vi) $\int \frac{3x-1}{x^2+x-12}\,dx$

Type 1B: Non-repeated quadratic factors.

2. Evaluate the following integrals by using the method of partial fractions:

(i) $\int \frac{20}{(x-4)(x^2+4)}\,dx$ (ii) $\int \frac{3}{(x+1)(x^2+2)}\,dx$

(iii) $\int \frac{17}{(x-4)(x^2+1)}\,dx$ (iv) $\int \frac{10}{(x-1)(x^2+9)}\,dx$

Type 1C: Repeated linear factors.

3. Evaluate the following integrals by using the method of partial fractions:

(i) $\int \frac{25}{(x-3)^2(x+2)}\,dx$ (ii) $\int \frac{9}{(x+1)(x-2)^2}\,dx$

Type 2: Integration by parts $\boxed{\int u \cdot \frac{dv}{dx}\ dx = u \cdot v - \int v \cdot \frac{du}{dx}\ dx}$

Note: Choose u to be the function whose category occurs earlier in the LIATE rule and the other function as $\frac{dv}{dx}$.

- **L** $\rightarrow$ Logarithmic
- **I** $\rightarrow$ Inverse (Trigonometric)
- **A** $\rightarrow$ Algebraic
- **T** $\rightarrow$ Trigonometric
- **E** $\rightarrow$ Exponential

4. Evaluate the following integrals by using the method of integration by parts.

(i) $\int x \sin x\ dx$ (ii) $\int x^2 \ln x\ dx$ (iii) $\int x^2 \sin^{-1} x\ dx$

(iv) $\int \cos^{-1} x\ dx$ (v) $\int x \tan^{-1} x\ dx$ (vi) $\int x \sec^2 x\ dx$

Type 3: Integration by parts after appropriate substitution.

5. Evaluate the following by first using appropriate substitution and then integration by parts:

(i) $\int x^3 e^{x^2}\ dx$ (ii) $\int \frac{\ln x}{\sqrt{x}}\ dx$ (iii) $\int \sin(\ln x)\ dx$

Type 4: Evaluating definite integrals.

Fundamental Theorem of Calculus:

If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\boxed{\int_a^b f(x)\ dx = F(b) - F(a)}$$

Practice problems: Evaluate the following definite integrals.

(i) $\int_1^2 \frac{1+x}{x}\ dx$ (ii) $\int_0^{\pi/4} \tan^2 x\ dx$ (iii) $\int_0^{\pi/2} \frac{\sin x}{1+\cos x}\ dx$

(iv) $\int_0^{\pi/2} \frac{\cos x}{\sin x + 2}\ dx$ (v) $\int_0^{\pi/4} \frac{\sec^2 x}{1+\tan x}\ dx$ (vi) $\int_0^1 \frac{3x^2}{1+x^3}\ dx$