University of Nottingham
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Foundation Calculus and Mathematical Techniques (CELEN037)

Problem Sheet 1 **Topics: Derivatives**

Topic 1: Derivatives using First Principles

1. Use First Principles to find the derivative of the following functions

   (i) $y = (x+2)^2$ (ii) $y = e^{3x-2}$

   (iii) $y = \cos 2x$ (iv) $y = \sqrt{x^2+1}$

2. Use First Principles to find the gradient of the curve $y = x^2 - \dfrac{3}{x}$ at point $x = 1$.

Topic 2: The Sum and Difference Rules

3. Use the Sum and Difference Rules to find the derivative of the following functions

   (i) $y = x^5 - 5^x - 5^5$ (ii) $y = \dfrac{3\sin x \cdot \cos x + 6\cot x - 2\sin x}{\cos x}$

   (iii) $y = \dfrac{\sqrt{x} + 1 + \sqrt[5]{x}}{\sqrt[3]{x}}$ (iv) $y = 2x^3 + \sqrt{x} + \left(\dfrac{x^3+2x}{x^2}\right)$

4. Consider the hyperbolic functions $\cosh x = \dfrac{1}{2}\left(e^x + e^{-x}\right)$ and $\sinh x = \dfrac{1}{2}\left(e^x - e^{-x}\right)$. Find $\dfrac{d}{dx}(\cosh x) + \dfrac{d}{dx}(\sinh x)$.

Topic 3: The Product Rule

5. Use the Product Rule to find the derivative of the following functions

   (i) $y = x^e e^{-x}$ (ii) $y = \dfrac{1}{2}\sin 2x$

   (iii) $y = x \cdot \sec x \cdot \ln x$ (iv) $y = \sqrt{x} \cdot e^x \cdot \cos x$

6. Let $y = x^2 + 2x \cdot \cot x$. Find $\dfrac{dy}{dx}$.

Topic 4: The Quotient Rule

7. Use the Quotient Rule to find the derivative of the following functions

   (i) $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ (ii) $y = \dfrac{x^3 - 4x^2 + 2x}{x^2 - x}$

   (iii) $y = \dfrac{x}{1 + \cot x}$ (iv) $y = 1 + x + x^2 + x^3 + \cdots \quad (|x| < 1)$

8. Let $y = \sqrt{\dfrac{1+\cos 2x}{1-\cos 2x}}$, where $\dfrac{\pi}{2} < x < \pi$. Find $\dfrac{dy}{dx}$

Answers

1. (i) $2x + 4$ (ii) $3e^{3x-2}$ (iii) $-2\sin 2x$ (iv) $\dfrac{x}{\sqrt{x^2+1}}$

2. 5

3. (i) $5x^4 + 5^x \ln 5$ (ii) $3\cos x - 6\csc x \cdot \cot x - 2\sec^2 x$
   (iii) $\dfrac{1}{6}x^{-\frac{5}{6}} - \dfrac{1}{3}x^{-\frac{4}{3}} - \dfrac{2}{15}x^{-\frac{17}{15}}$ (iv) $6x^2 + \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^2} + 1$

4. e^x

5. (i) 0 (ii) $\cos 2x$ (iii) $\sec x \cdot (\ln x + x \cdot \tan x \cdot \ln x + 1)$
   (iv) $e^x \left(\dfrac{\cos x}{2\sqrt{x}} + \sqrt{x} \cdot \cos x - \sqrt{x} \cdot \sin x \right)$

6. $2(x + \cot x - x \cdot \csc^2 x)$

7. (i) $-\dfrac{4e^{2x}}{(e^{2x}-1)^2}$ (ii) $\dfrac{x^2 - 2x + 2}{(x-1)^2}$ (iii) $\dfrac{1 + \cot x + x \cdot \csc^2 x}{(1 + \cot x)^2}$
   (iv) $\dfrac{1}{(1-x)^2}$

8. $\csc^2 x$