



Foundation Calculus and Mathematical Techniques (CELEN037)

Problem Sheet 8

Topics: Definite Integrals

Topic 1: Definite Integrals using Substitution

1. Evaluate the following definite integrals using the method of substitution:

$$(i) \int_0^{\frac{\sqrt{\pi}}{2}} x \cdot \sin(x^2) dx$$

$$(ii) \int_{-2}^{-1} \frac{x}{(x^2 + 2)^3} dx$$

$$(iii) \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin \theta}{1 - \cos \theta} d\theta$$

$$(iv) \int_{\pi^2}^{4\pi^2} \frac{\cos \sqrt{x}}{\sqrt{x}} dx$$

$$(v) \int_0^{\frac{\pi}{3}} \sec^3 x \cdot \tan x dx$$

$$(vi) \int_1^2 x^2 \cdot \ln^2 x \cdot (\ln x + 1) dx$$

Topic 2: Integration by Parts for Definite Integrals

2. Evaluate the following integrals using the method of integration by parts:

$$(i) \int_0^{\frac{1}{2}} \sin^{-1} x dx$$

$$(ii) \int_0^1 e^{\sqrt{x}} dx$$

$$(iii) \int_0^{\frac{\pi}{4}} e^x \cdot \sin 2x dx$$

$$(iv) \int_2^e \frac{\ln(\ln x)}{x} dx$$

$$(v) \int_0^1 x \cdot \ln(2 - x) dx$$

$$(vi) \int_1^e \cos(\ln x) dx$$

Topic 3: Use of Properties for Evaluating Definite Integrals

3. Evaluate the following integrals

$$(i) \int_0^3 |4 - x^2| dx$$

$$(ii) \int_2^4 \frac{\sqrt{x}}{\sqrt{6 - x} + \sqrt{x}} dx$$

$$(iii) \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$$

$$(iv) \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \ln \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$$

$$(v) \int_{-1}^1 \frac{x^3 \cdot \sin^{-1} x}{\tan x} dx$$

$$(vi) \int_0^{\pi} x^2 \cdot |\cos x| dx$$

Topic 4: Area Calculation using Definite Integrals

4. Calculate the area of the following regions:

(i) Find the area of the region bounded the curve $y = (x - 1)^3$, the lines $x = 0$, $x = 2$, and the X -axis.

(ii) Find the area of the region bounded by the curve $x = -y^{-2}$, the lines $y = 1$, $y = 4$, and the Y -axis.

(iii) Find the area of the region bounded by the parabola $y = x^2 + 2x + 3$ and the line $y = 6$.

(iv) Find the area of the region bounded by the curve $x = \sec y \cdot \tan y$ and the lines $x = 2$, $y = 0$, and $y = \frac{\pi}{3}$.

Answers

1. (i) $\frac{1}{2} - \frac{\sqrt{2}}{4}$

(ii) $-\frac{1}{48}$

(iii) $\ln 2$

(iv) 0

(v) $\frac{7}{3}$

(vi) $\frac{8}{3} \ln^3 2$

2. (i) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

(ii) 2

(iii) $\frac{e^{\frac{\pi}{4}} + 2}{5}$

(iv) $\ln 2 \cdot [1 - \ln(\ln 2)] - 1$

(v) $2 \ln 2 - \frac{5}{4}$

(vi) $\frac{e}{2} (\sin 1 + \cos 1) - \frac{1}{2}$

3. (i) $\frac{23}{3}$

(ii) 1

(iii) $\frac{\pi}{4}$

(iv) 0

(v) 0

(vi) $\frac{\pi^2}{2} + 2\pi - 4$

4. (i) $\frac{1}{2}$

(ii) $\frac{3}{4}$

(iii) $\frac{32}{3}$

(iv) $\frac{2\pi}{3} - 1$