University of Nottingham
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COMP3055 Machine Learning

Probability Exercises Solutions

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Probability Exercises

Exercise-1: **Nightlights and Myopia**

Assuming these data are representative of a larger population, what is the **approximate probability** that someone from that population who **sleeps with a nightlight** in early childhood **will develop some degree of myopia**?

Slept with:	No Myopia	Myopia	High Myopia	Total
Darkness	155 (90%)	15 (9%)	2 (1%)	172
Nightlight	153 (66%)	72 (31%)	7 (3%)	232
Full Light	34 (45%)	36 (48%)	5 (7%)	75
Total	342 (71%)	123 (26%)	14 (3%)	479

Exercise-2: Tuberculous meningitis

If tuberculous meningitis had a case fatality of 20%,

(a) Find the probability that this disease would be fatal in two randomly selected patients (the two events are independent)

(b) If two patients are selected randomly what is the probability that at least one of them will die?

Exercise-3: We have a population of potential workers. We know that
40% are vocational school graduates (V),
50% are high school grads (H),
10% are college grads (C).
In addition,
10% of the vocational school grads are unemployed (U),
5% of the high school grads are unemployed (U),
2% of the college grads are unemployed (U).

Determine the probability that a randomly selected unemployed person is a college graduate, that is, Pr(C|U)

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Note: 72 + 7 = 79 of the 232 nightlight users developed some degree of myopia. So the probability to be 79/232 = 0.34

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(a) Find the probability that this disease would be fatal in two randomly selected patients (the two events are independent)

(b) If two patients are selected randomly what is the probability that at least one of them will die?

(a) P(first die and second die) = 20% × 20% = 0.04

(b) P(first die or second die)

= P(first die) + P(second die) - P(both die)

= 20% + 20% - 4%

= 36%

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- Convert this information into probability statements.
- Then determine the probability that a randomly selected unemployed person is a college graduate, that is, Pr(C|U).

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Pr(U|V) = 0.10

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Pr(U|V) = 0.10

Pr(U|H) = 0.05

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- $Pr(V) = 0.40$ $Pr(U|V) = 0.10$
- $Pr(H) = 0.50$ $Pr(U|H) = 0.05$
- $Pr(C) = 0.10$ $Pr(U|C) = 0.02$

In order to calculate Pr(C|U), we need to determine the probability that a randomly selected individual is

1. a vocational school grad & unemployed
2. a high school grad & unemployed
3. a college grad & unemployed

Recall that 40% of our population is vocational school grads, & 10% of them are unemployed.

- Then 10% of 40% of our population is vocational school grads & unemployed.
- So $\Pr(V \& U) = \Pr(V \cap U)$
- $= 0.10 \times 0.40 = 0.04.$
- Similarly, $\Pr(H \& U) = \Pr(H \cap U)$
- $= 0.05 \times 0.50 = 0.025.$
- Also, $\Pr(C \& U) = \Pr(C \cap U)$
- $= 0.02 \times 0.10 = 0.002.$

Given

$\Pr(V \& U) = \Pr(V \cap U) = 0.04$,

$\Pr(H \& U) = \Pr(H \cap U) = 0.025$, &

$\Pr(C \& U) = \Pr(C \cap U) = 0.002$,

we can calculate the probability that a randomly selected individual is unemployed, Pr(U).

- $\Pr(U) = \Pr(V \cap U) + \Pr(H \cap U) + \Pr(C \cap U)$
- $= 0.04 + 0.025 + 0.002$
- $= 0.067$

We can finally determine Pr(C|U), using our calculations & the definition of conditional probability.

- $Pr(C|U) = Pr(C \cap U) / Pr(U)$
- $= 0.002 / 0.067$
- $= 0.030.$
- So the probability that a randomly selected unemployed individual is a college graduate is 0.03.

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ED	Given Pr(ED)	Given Pr(U|ED)		
Vocational School Grads	0.40	0.10		
High School Grads	0.50	0.05		
College Grads	0.10	0.02		
All	1.00	------		

We can also do the problem in an easily organized table.

ED	Given Pr(ED)	Given Pr(U|ED)	Pr(ED∩U) = Pr(U|ED) x Pr(ED)	
Vocational School Grads	0.40	0.10	0.04	
High School Grads	0.50	0.05	0.025	
College Grads	0.10	0.02	0.002	
All	1.00	------	Pr(U)=0.067	

We can also do the problem in an easily organized table.

ED	Given Pr(ED)	Given Pr(U|ED)	$Pr(ED \cap U) = Pr(U\|ED) \times Pr(ED)$	$Pr(ED\|U) = \frac{Pr(ED \cap U)}{Pr(U)}$
Vocational School Grads	0.40	0.10	0.04	0.597
High School Grads	0.50	0.05	0.025	0.373
College Grads	0.10	0.02	0.002	0.030
All	1.00	------	Pr(U)=0.067	1.000