



Foundation Calculus and Mathematical Techniques (CELEN037)

Problem Sheet 7

Topics: Indefinite Integrals

Topic 1: Integration using Partial Fractions

Type 1: Non-repeated linear factors

1. Evaluate the following integrals

$$(i) \int \frac{x}{(x+2)(x+3)} dx$$

$$(ii) \int \frac{11x+17}{2x^2+7x-4} dx$$

$$(iii) \int \frac{x^4-4x^2+x+1}{x^2-4} dx$$

$$(iv) \int \frac{2x+3}{x^2+3x-10} dx$$

$$(v) \int \frac{2x^2+1}{(x-1)(x-2)(x-3)} dx$$

$$(vi) \int \frac{x^2-4}{x^3-3x^2-x+3} dx$$

Type 2: Non-repeated quadratic factors

2. Evaluate the following integrals

$$(i) \int \frac{x^2+2x+4}{(2x-1)(x^2-1)} dx$$

$$(ii) \int \frac{1}{x(x^2+5)} dx$$

$$(iii) \int \frac{3}{x^3+1} dx$$

$$(iv) \int \frac{x^4+1}{x^3+9x} dx$$

$$(v) \int \frac{x+2}{(2x+1)(x^2+x+1)} dx$$

Type 3: Repeated linear factors

3. Evaluate the following integrals

$$(i) \int \frac{3x-2}{x^3+x^2} dx$$

$$(ii) \int \frac{2x^2+x+1}{(x-3)(x+1)^2} dx$$

$$(iii) \int \frac{x-4}{(x-2)(x^2-4)} dx$$

$$(iv) \int \frac{2x}{(x-2)^2(x+2)} dx$$

$$(v) \int \frac{x+4}{x^3+6x^2+9x} dx$$

$$(vi) \int \frac{x^4}{x^3-2x^2-7x-4} dx$$

Topic 2: Integration by parts

4. Evaluate the following integrals

$$(i) \int x \cdot e^{-3x} dx$$

$$(ii) \int 3x \cdot \sin x \cdot \cos x dx$$

$$(iii) \int x \cdot \ln(1-x) dx$$

$$(iv) \int x \cdot \cos\left(\frac{x}{2}\right) dx$$

$$(v) \int x \cdot \tan^2 x dx$$

$$(vi) \int \sin^{-1} x dx$$

5. Evaluate the following integrals by appropriate substitution and integration by parts:

$$(i) \int \sqrt{x} \cdot e^{\sqrt{x}} dx$$

$$(ii) \int (\sin^{-1} x)^2 dx$$

$$(iii) \int \cos(\ln x) dx$$

$$(iv) \int \frac{\ln(\ln x)}{x} dx$$

6. Integration by parts for multiple times:

$$(i) \int e^{-x} \cdot \cos x dx$$

$$(ii) \int e^{-2x} \cdot \cos\left(\frac{x}{2}\right) dx$$

$$(iii) \int e^x \cdot \sin^2 x dx$$

$$(iv) \int (x \cdot \ln x)^2 dx$$

$$(v) \int (x^2 - 1) \cdot \sin 2x dx$$

$$(vi) \int x^3 \cdot e^x dx$$

Topic 3: Evaluating definite Integrals

7. Evaluate the following definite integrals:

$$(i) \int_2^3 \frac{1}{7-x^2} dx$$

$$(ii) \int_0^{\ln 5} 5^x dx$$

$$(iii) \int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$$

$$(iv) \int_{\sqrt{2}}^2 \frac{1}{|x| \cdot \sqrt{x^2-1}} dx$$

Answers

1. (i) $-2 \ln |x+2| + 3 \ln |x+3| + C$ (ii) $\frac{5}{2} \ln |2x-1| + 3 \ln |x+4| + C$
- (iii) $\frac{1}{3}x^3 + \frac{3}{4} \ln |x-2| + \frac{1}{4} \ln |x+2| + C$ (iv) $\ln |x-2| + \ln |x+5| + C$
- (v) $\frac{2}{3} \ln |x-1| - 9 \ln |x-2| + \frac{19}{2} |x-3| + C$
- (vi) $\frac{3}{4} \ln |x-1| + \frac{5}{8} \ln |x-3| - \frac{3}{8} \ln |x+1| + C$
2. (i) $-\frac{7}{2} \ln |2x-1| + 2 \ln |x^2-1| + \frac{3}{2} \ln \left| \frac{x-1}{x+1} \right| + C$
- (ii) $\frac{1}{5} \ln |x| - \frac{1}{10} \ln (x^2+5) + C$
- (iii) $\ln |x+1| - \frac{1}{2} \ln (x^2-x+1) + \sqrt{3} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right) + C$
- (iv) $\frac{1}{2}x^2 + \frac{1}{9} \ln |x| - \frac{41}{9} \ln (x^2+9) + C$
- (v) $\ln |2x+1| - \frac{1}{2} \ln (x^2+x+1) + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + C$
3. (i) $5 \ln |x| + \frac{2}{x} - 5 \ln |x+1| + C$
- (ii) $\frac{5}{8} \ln |x+1| + \frac{1}{2(x+1)} + \frac{11}{8} \ln |x-3| + C$
- (iii) $\frac{3}{8} \ln |x-2| + \frac{1}{2(x-2)} - \frac{3}{8} \ln |x+2| + C$
- (iv) $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| - \frac{1}{x-2} + C$ (v) $\frac{4}{9} \ln \left| \frac{x}{x+3} \right| + \frac{1}{3(x+3)} + C$
- (vi) $\frac{1}{2}x^2 + 2x + \frac{256}{25} \ln |x-4| + \frac{19}{25} \ln |x+1| + \frac{1}{5(x+1)} + C$

$$4. \quad (\text{i}) \quad -\frac{1}{3}e^{-3x} \cdot \left(x + \frac{1}{3}\right) + C \qquad (\text{ii}) \quad -\frac{3}{4}x \cdot \cos 2x + \frac{3}{8} \sin 2x + C$$

$$(\text{iii}) \quad \frac{1}{2}x \cdot \left[x \cdot \ln(1-x) - \frac{1}{2}x - 1\right] - \frac{1}{2} \ln|x-1| + C$$

$$(\text{iv}) \quad 2x \cdot \sin\left(\frac{x}{2}\right) + 4 \cos\left(\frac{x}{2}\right) + C \qquad (\text{v}) \quad x \cdot \tan x + \ln|\cos x| - \frac{x^2}{2} + C$$

$$(\text{vi}) \quad x \cdot \sin^{-1} x + \sqrt{1-x^2} + C$$

$$5. \quad (\text{i}) \quad 2e^{\sqrt{x}} \cdot (x - 2\sqrt{x} + 2) + C \qquad (\text{ii}) \quad x \cdot (\sin^{-1} x)^2 + 2\sqrt{1-x^2} \cdot \sin^{-1} x - 2x + C$$

$$(\text{iii}) \quad \frac{1}{2}x \cdot [\sin(\ln x) + \cos(\ln x)] + C \qquad (\text{iv}) \quad \ln x \cdot [\ln(\ln x) - 1] + C$$

$$6. \quad (\text{i}) \quad \frac{1}{2}e^x \cdot (\sin x + \cos x) + C \qquad (\text{ii}) \quad -\frac{2}{17}e^{-2x} \cdot \left[4 \cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right] + C$$

$$(\text{iii}) \quad \frac{1}{10}e^x \cdot (5 - 2 \sin 2x - \cos 2x) + C \qquad (\text{iv}) \quad \frac{1}{27}x^3 \cdot (9 \ln^2 x - 6 \ln x + 2) + C$$

$$(\text{v}) \quad -\left(\frac{x^2}{2} - \frac{3}{4}\right) \cdot \cos 2x + \frac{1}{2}x \cdot \sin 2x + C \qquad (\text{vi}) \quad e^x \cdot (x^3 - 3x^2 + 6x - 6) + C$$

$$7. \quad (\text{i}) \quad \frac{1}{2\sqrt{7}} \ln \left| \frac{-1 - \sqrt{7}}{-1 + \sqrt{7}} \right| \qquad (\text{ii}) \quad \frac{5^{\ln 5} - 1}{\ln 5} \qquad (\text{iii}) \quad \frac{\pi}{12} \qquad (\text{iv}) \quad \frac{\pi}{12}$$