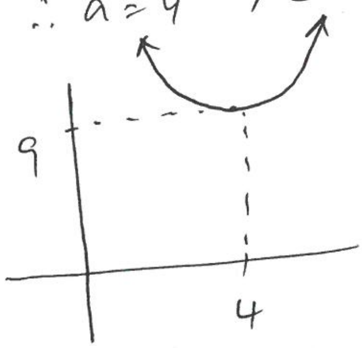
1 a) i) $f(x) = x^2 - 8x + 25$

$= x^2 - 8x + 16 + 9$

$= (x-4)^2 + 9$ ① for completing square

$\therefore a = 4, \ b = 9.$

ii)



① for sketching the curve.

b) $(f \circ g)(2) = 4$

$\Rightarrow f(g(2)) = 4$

$\Rightarrow f(2a+b) = 4$

$\Rightarrow (2a+b) + 3 = 4$

$\Rightarrow 2a + b = 1$ — ①

① Mark

& $g^{-1}(x) = \frac{x-b}{a}$

$\therefore g^{-1}(3) = 1 \Rightarrow \frac{3-b}{a} = 1$

$\Rightarrow a + b = 3$ — ②

① Mark

Solving ① and ②

$\Rightarrow$
$$\begin{array}{r} 2a + \cancel{b} = 1 \\ a + \cancel{b} = 3 \\ \hline \end{array} \text{ sub}$$

$\therefore a = -2$

$\therefore b = 3 - a = 5.$

① for each ~~each~~ correct values of a & b. ~~(both must be right)~~

(c) Let $e^x = m$

$\therefore m^2 - m - 6 = 0$

mark.

$\Rightarrow (m-3)(m+2) = 0$

$\Rightarrow m = 3$ or -2

$\therefore e^x = 3$ or $e^x = -2$ (Not possible)

$\Downarrow$

$\therefore x = \ln 3.$

① for answer. ~~Answer~~

d) $\ln(x+2) + \ln(x-2) = \ln(2x-5)$

~~① Mark~~ $\Rightarrow \ln(x^2-4) = \ln(2x-5)$

$\Rightarrow x^2 - 4 = 2x - 5$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0.$

① Mark. $\Rightarrow x = 1.$

But for $x=1$, $\ln(x-2)$ and $\ln(2x-5)$ are not defined.

① Mark $\therefore$ It is not possible to have solution for $x \in \mathbb{R}$.

2 a) $\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$

① Mark.

$= (2\sin\theta\cos\theta)\cdot\cos\theta + (1-2\sin^2\theta)\cdot\sin\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta.$

b) $A+B+C = 2\pi$

$\Rightarrow A+B = 2\pi - C$

$\Rightarrow \tan(A+B) = \tan(2\pi - C)$ ① Mark.

$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

(2)

$\Rightarrow \tan A + \tan B = -\tan C(1 - \tan A \tan B)$

$\Rightarrow \tan A + \tan B = -\tan C + \tan A \tan B \tan C$

i.e. $\underline{\tan A + \tan B + \tan C = \tan A \tan B \tan C}$. (1) Mark

c) $\cos\theta = 3/5$

i) $\therefore \cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{9}{25}\right) - 1$

$= \frac{-7}{25}$. (1) Mark.

(-0.28)

ii) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\sqrt{\left(1 - \frac{9}{25}\right)} \cdot \frac{3}{5}$

or Δ method

$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5}$ $\quad (\because 0 < \theta < \pi/2)$

$= \frac{24}{25}$. (0.96)

$\therefore \sin 4\theta = 2\sin 2\theta \cos 2\theta$

$= 2 \cdot \left(\frac{24}{25}\right) \cdot \left(\frac{-7}{25}\right)$

$= \frac{-336}{625}$. (-0.5376)

(1) for method.

(1) for answer

d) $f(x) = 2\cos x - \sin x \equiv R\cos(x + \theta)$

$\Rightarrow 2\cos x - \sin x \equiv R\cos x\cos\theta - R\sin x\sin\theta$

$\Rightarrow \left.\begin{array}{l} R\cos\theta = 2 \\ R\sin\theta = +1 \end{array}\right\} \Rightarrow \underline{R = \sqrt{5}}$. (1) Mark

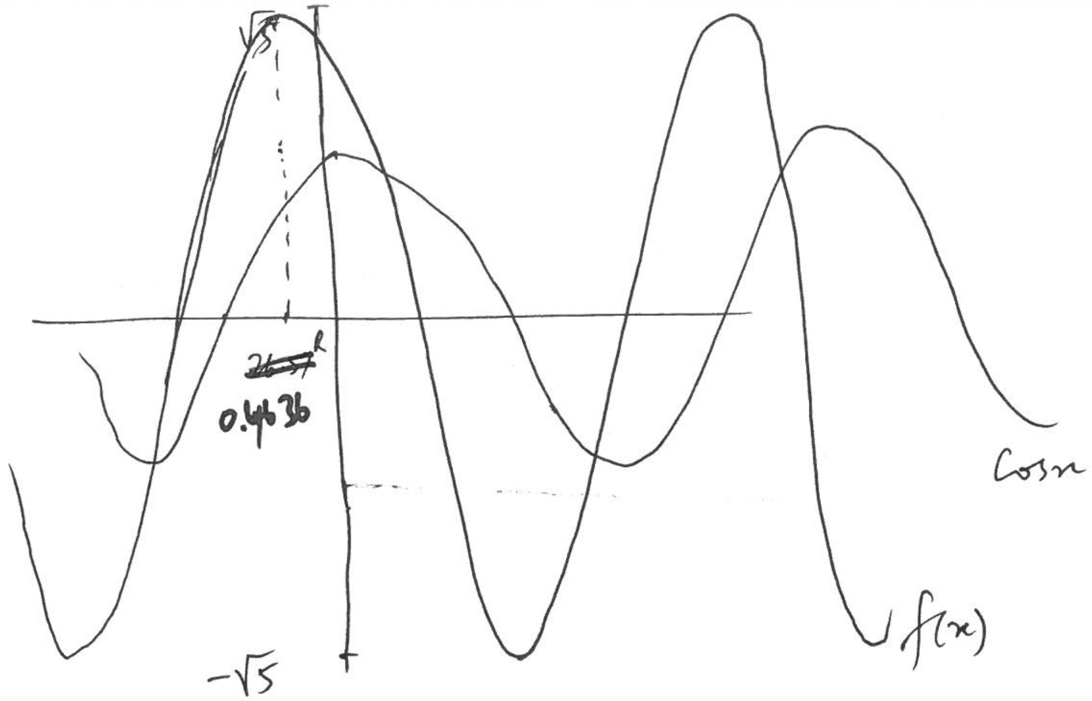
& $\tan\theta = \frac{1}{2} \Rightarrow \underline{\theta = \cancel{26.57}\ 0.4636 \text{ radians}}$. (1) Mark

(3)

$\therefore f(x) = 2\cos x - \sin x = \sqrt{5}\cos(x + \cancel{26.57}\ 0.4636)$.

iii)

① mark.



ii) $R_f = [-\sqrt{5}, \sqrt{5}]$

① mark.

Period of $f = \dfrac{2\pi}{|1|} = 2\pi$.

3 a) $p(x) = 6x^3 - 17x^2 - 30x + 56$

i)

	6	−17	−30	56
−2	↓	−12	+58	−56
	6	−29	28	0

2 Marks for the method & getting zero

As remainder = 0, $(x+2)$ is a factor of $p(x)$.

ii) $\therefore p(x) = (x+2)(6x^2 - 29x + 28)$

$= (x+2)(6x^2 - 21x - 8x + 28)$

$= \underline{(x+2)(3x-4)(2x-7)}$ ① Mark.

④ iii) ① Mark $\therefore p(x) = 0 \Rightarrow x = -2$ or $\frac{4}{3}$ or $\frac{7}{2}$. Answer

3 b) $(x-2)$ is a factor of $p(x)$

$$\Rightarrow p(2)=0 \Rightarrow 2(2)^3+a(2)^2-9(2)+b=0$$

$$\Rightarrow 4a+b+16-18=0$$

① Mark

$$\Rightarrow 4a+b=2 \quad \text{——} \ (1)$$

& $(x+3)$ is a factor of $p(x)$

$$\Rightarrow p(-3)=0 \Rightarrow 2(-3)^3+a(-3)^2-9(-3)+b=0$$

$$\Rightarrow -54+9a+27+b=0$$

① Mark

$$\Rightarrow 9a+b=27 \quad \text{——} \ (2)$$

(2) − (1) gives. $5a=25 \Rightarrow a=5$

$$\therefore b=2-4a=2-20=-18$$

Thus, $a=5$ and $b=-18$. ① Mark.

Any method is ok.
(eg. SD etc.)

c) $$\frac{2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}.$$

(1) Mark

$$\Rightarrow \quad A(x^2+1) + (Bx+C)(x-1) = 2$$

$$\Rightarrow \quad Ax^2 + A + Bx^2 - Bx + Cx - C = 2$$

$\therefore A+B=0 \Rightarrow A=-B$

$A-C=2 \Longrightarrow A-B=2$

$-B+C=0 \Rightarrow B=C$

Thus, $A-B=2$
$A+B=0$

$\therefore 2A=2$

$A=1 \quad \therefore B=-1, \quad C=-1.$

1 Mark.

$$\therefore \frac{2}{(x-1)(x^2+1)} = \frac{1}{(x-1)} - \frac{(x+1)}{x^2+1}.$$

(1) Mark

(If this step is NOT written, but all other working & answers are ok. give full marks).

4 a)

$f(x) = x^3 + 3\cos x - 4 = 0.$

$f(1) = -1.38 < 0$

$f(2) = 2.75 > 0$

Must

Use of IMThm. must be clear.

$\therefore f(1).f(2) < 0$

(1) mark.

$\therefore$ Root lies in $(1, 2)$.

b) $x^3 + 3\cos x - 4 = 0$

$\Rightarrow x^3 = 4 - 3\cos x$

$\Rightarrow x^2 = \dfrac{4 - 3\cos x}{x}$

$\Rightarrow x = \sqrt{\dfrac{4 - 3\cos x}{x}}$

① Mark for the method

$\therefore x_{n+1} = \sqrt{\dfrac{4 - 3\cos x_n}{x_n}}$. ① Mark.

c). With $x_0 = 1.5$, we obtain

① Marks for digit precision.

n	0	1	2	3	4
x_n	1.5	1.589085 ✓	1.597403 ✓	1.598132 ✓	1.598196 ✓

② Marks.

Not required

n	5	6	7
x_n	1.598201	1.598202	1.598202

$\therefore x^* = 1.59820$ (5.d.p.) ① Mark

d).

n	a	b	c	$f(a)$	$f(b)$	$f(c)$	Decision Replace
0	1	2	1.5	<0	>0	<0	a by c
1	1.5	2	1.75	<0	>0	>0	b by c
2	1.5	1.75	1.625				

① Mark.

$\therefore x_1 = 1.75$ and $x_2 = 1.625$ ① for each of x_1 & x_2 values.

⑦

5 a)

① for method

$$(2x+3y)^5 = (2x)^5 + \binom{5}{1}(2x)^4(3y) + \binom{5}{2}(2x)^3(3y)^2 + \binom{5}{3}(2x)^2(3y)^3 + \binom{5}{4}(2x)(3y)^4 + (3y$$

$$= 32x^5 + 5(16x^4)(3y) + 10(8x^3)(9y^2) + 10(4x^2)(27y^3) + 5(2x)(81y^4) + 243y$$

① Mark.

$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5.$$

b) $\dfrac{1}{(1-3x)^4} = \underline{\underline{(1-3x)^{-4}}}$ ① Mark for understanding this step / writing this step.

① Mark for the method.

$$= 1 + (-4)(-3x) + \frac{(-4)(-4-1)}{2!}(-3x)^2 + \frac{(-4)(-4-1)(-4-2)}{3!}(-3x)^3 + \frac{(-4)(-4-1)(-4-2)(-4-3)}{4!}(- + \dots$$

$$= 1 + 12x + 90x^2 + 540x^3 + 2835x^4 + \dots$$

① Mark

$\therefore$ coefficient of x^4 is $\underline{\underline{2835}}$. Answer.

c) ① for the method.

$$\left(x-\frac{5}{x}\right)^8 = x^8 + \binom{8}{1}x^7\left(\frac{-5}{x}\right) + \binom{8}{2}x^6\left(\frac{-5}{x}\right)^2 + \binom{8}{3}x^5\left(\frac{-5}{x}\right)^3 + \binom{8}{4}x^4\left(\frac{-5}{x}\right)^4 + \dots + \text{Last term}$$

$\therefore$ Term independent of x (ie. constant term) $= \binom{8}{4} \cdot 5^4$

$= \underline{43750}$. ① Mark.

8

d) i) $(1.025)^3$

$= (1+0.025)^3$

$\approx 1 + 3(0.025) + \frac{3(3-1)}{2}(0.025)^2$

$= 1 + 0.075 + 0.001875$

$= 1.076875$

$\therefore \underline{(1.025)^3 \approx 1.076875}$ (1) Mark.

ii) $V = \frac{4}{3}\pi r^3$

$\therefore V + \delta V = \frac{4}{3}\pi(r+\delta r)^3$ (1) Mark.

$= \frac{4}{3}\pi(r + 0.025r)^3$

$= \frac{4\pi r^3}{3}(1.025)^3$

$\approx V.(1.076875)$

$= V + 0.076875V$

$\therefore \delta V = 0.076875 V$ (1) for answer

or $\delta V = 7.6875\%$ of V.
7.69 %

6 b) i) $\begin{pmatrix} 3 & 4 \\ 9 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -37 \end{pmatrix}$.

coefficient matrix

Matrix of unknowns.

matrix of constants on the RHS.

① mark.

(ii) $A^{-1} = \dfrac{1}{-21-36}\begin{pmatrix} -7 & -4 \\ -9 & 3 \end{pmatrix}$

$= \dfrac{1}{57}\begin{pmatrix} 7 & 4 \\ 9 & -3 \end{pmatrix}$.

① mark.

(iii) $AX = B$

$\Rightarrow A^{-1}(AX) = A^{-1}B$

$\Rightarrow (A^{-1}A)X = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\Rightarrow X = A^{-1}B$.

① mark.

(iv) $\therefore X = \dfrac{1}{57}\begin{pmatrix} 7 & 4 \\ 9 & -3 \end{pmatrix}\begin{pmatrix} 13 \\ -37 \end{pmatrix}$

$= \dfrac{1}{57}\begin{pmatrix} 91-148 \\ 117+111 \end{pmatrix}$

① for matrix multiplication

$= \dfrac{1}{57}\begin{pmatrix} -57 \\ 228 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \Rightarrow x = -1$ and $y = 4$

① for answer.

10

6 · b)

$$A^2 = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

① for the method.

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\therefore A^2 = I.$$

$$\therefore A^{-1}(A^2) = A^{-1}.I$$

$$\therefore \underline{A = A^{-1}} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}.$$

① Mark.

6. a) ~~$C = (AB)^T - B^{-1}$~~

$$AB = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 5 & 2 \end{pmatrix}$$

① Mark for product matrix

$$\therefore (AB)^T = \begin{pmatrix} 0 & 5 \\ 1 & 2 \end{pmatrix}$$

$$\text{and } B^{-1} = \frac{1}{(-1)}\begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}.$$

① Mark

$$\therefore C = (AB)^T - B^{-1} = \begin{pmatrix} 0 & 5 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 4 \\ 0 & 4 \end{pmatrix}.$$

① Mark Answer.

7 a)

$$(1+i+2i^2)(x+iy) = 3i-5$$

$$\Rightarrow (1+i-2)(x+iy) = 3i-5$$

$$\Rightarrow (-1+i)(x+iy) = 3i-5$$

$$\Rightarrow -x-iy+ix+i^2y = 3i-5$$

1 mark.

$$\Rightarrow -x-iy+ix-y = 3i-5$$

$$\Rightarrow -x-y=-5 \quad \text{or} \quad \boxed{x+y=5}$$

$$ix-iy=3i \quad \text{or} \quad \boxed{x-y=3.}$$

1 mark.

$$\therefore 2x = 8 \Rightarrow x = 4$$

$$\therefore y = 5-x = 1.$$

$$\therefore x=4 \text{ and } y=1.$$

1 mark.

b) $$z^3+8=0$$

1 mark.

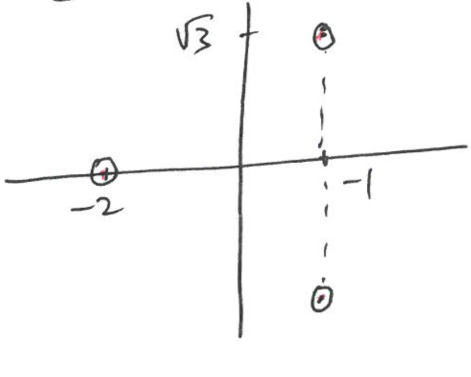
$$\Rightarrow (z+2)(z^2-2z+4)=0$$

$$\Rightarrow z=-2 \quad \text{or} \quad z = \frac{+2 \pm \sqrt{4-4(4)}}{2} = \frac{2 \pm 2\sqrt{3}i}{2}$$

$$\cancel{= \frac{2 \pm 2\sqrt{5}i}{2}} \qquad = 1 \pm i\sqrt{3}.$$

1 mark

$$\therefore z=-2 \quad \text{or} \quad z = -1 \pm \sqrt{3}i$$



1 for the Argand diagram..

c) i) $\left| \frac{\overline{z_1}^2}{\overline{z_3}} \cdot \left(\frac{z_2 \cdot z_3}{z_1} \right) \right| = |z_1|\,|z_2|$. — ① mark for use of properties of modulus

$= \sqrt{(12)^2+(-5)^2} \cdot \sqrt{(3)^2+(4)^2}$

$= 13 \times 5$

① mark $\quad = \underline{\underline{65}}$. Answer.

ii) $z_1 = 12-5i$

$\therefore x=12, \; y=-5 \quad \therefore r = \sqrt{x^2+y^2} = 13.$ ① Mark

$\left.\begin{array}{l} \cos\theta = \frac{x}{r} = \frac{12}{13} > 0 \\ \sin\theta = \frac{y}{r} = \frac{-5}{13} < 0 \end{array}\right\} \therefore \theta \in 4^{th}$ Quadrant

~~$\theta = \pi + \tan^{-1}\left|\frac{y}{x}\right|$~~

$\therefore \theta = -\tan^{-1}\left|\frac{y}{x}\right| = -\tan^{-1}\left(\frac{5}{12}\right)$

$= 0.3948$ radians. ① Mark.

$\therefore z_1 = 12-5i = 13(\cos\theta + i\sin\theta)$

Where $\theta = 0.3948$ radians.

8 a) 4^{th} term $= a + 3d = 10$

12^{th} term $= a + 11d = 66$.

$\therefore 8d = 56 \Rightarrow d = 7$. ~~(1) Mark~~

$\therefore a = 10 - 3d = 10 - 21 = -11$.

$\therefore a = -11$. (1) Mark for both a & d correct values.

i) $\therefore 20^{th}$ term $= a + 19d$

$= -11 + 19(7)$

$= 122$. ~~(1) Mark for S_{20}~~ (1) Mark a_{20}

ii) Sum of first 20 terms $= S_{20}$.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore S_{20} = \frac{20}{2}\left[2(-11) + (20-1)(7)\right]$$

$$= 10\left[-22 + 19(7)\right]$$

$$= 1110.$$ (1) Mark for S_{20}.

b) For G.P; $a = 10$

sum of first 3 terms $= 310$

ie. $a + ar + ar^2 = 310$.

$\Rightarrow 10(1 + r + r^2) = 310$.

$\Rightarrow 1 + r + r^2 = 31$.

$\Rightarrow r^2 + r - 30 = 0$. (1) Mark for Eqn.

$$\Rightarrow \quad r^2 - 5r + 6r - 30 = 0$$

$$\Rightarrow \quad (r-5)(r+6) = 0$$

$$\Rightarrow \quad r = 5, \quad \text{or} \quad r = -6.$$

$\therefore$ GP. are : $10, 50, 250, \ldots$

OR $10, -60, 360, \ldots$

1 Mark for answers.

c) $a = \frac{1}{2}, \quad r = \frac{1}{5} \Rightarrow |r| < 1.$

$$\therefore \quad S = \frac{a}{1-r} = \frac{1/2}{1-1/5} = \frac{1/2}{4/5} = \frac{1}{2}\cdot\frac{5}{4} = \frac{5}{8}.$$

1 Mark. Answer.

d) $$f(n) = \frac{1}{n(n+1)}$$

(i) $$\therefore f(n) - f(n+1) = \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}$$

$$= \frac{(n+2)-(n)}{n(n+1)(n+2)}$$

$$= \frac{2}{n(n+1)(n+2)}.$$ 1 Mark

$$\therefore \frac{1}{n(n+1)(n+2)} = \frac{1}{2}\left[f(n) - f(n+1)\right].$$

(ii) $$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \cdots \text{(upto } n \text{ terms)}$$

$$= \sum_{1}^{n} \frac{1}{n(n+1)(n+2)} = \frac{1}{2}\sum_{1}^{n}\left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right]$$

1 Mark

$$= \frac{1}{2}\left[\frac{1}{1.2} - \cancel{\frac{1}{2.3}} + \cancel{\frac{1}{2.3}} - \cancel{\frac{1}{3.4}} + \ldots + \cancel{\frac{1}{n(n+1)}} - \frac{1}{(n+1)(n+2)}\right].$$

① for Method.

$$= \frac{1}{2}\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right]$$

$$= \frac{n^2+3n\cancel{+2}-\cancel{2}}{4(n+1)(n+2)}$$

$$= \frac{n(n+3)}{4(n+1)(n+2)}$$

~~① Marks~~

(iii) $\therefore \sum_{1}^{\infty} \frac{1}{n(n+1)(n+2)} = \lim_{n\to\infty} \frac{\cancel{n^2}(1+3/n)}{4\cancel{n^2}(1+\frac{1}{n})(1+\frac{2}{n})}$

$$= \frac{1+0}{4(1+0)(1+0)}$$

$$= \frac{1}{4}.$$ Answer.

① Mark.