University of Nottingham
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Foundation Algebra (CELEN036)

Problem Sheet 1 **Topics: Functions**

Topic 1: Composition of functions

1. Given $f(x) = (x+1)(x-2)$ and $g(x) = 2x$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

2. Given $f(x) = x^2 - 1$, $g(x) = 3x + 2$, and $h(x) = \dfrac{1}{x}$. Solve:

   (i) $(f \circ g)(x) = 15$. (ii) $(g \circ g)(x) = h(x)$. (iii) $(g \circ h)(x) = -4$.

3. Given $f(x) = 2x - 1$, and $g(x) = 3x^2 + 2$, $h(x) = ax + b$, where a and b are positive constants. Find a and b such that $(f \circ g \circ h)(x) = 6x^2 + 12x + 9$.

Topic 2: Inverse functions

4. Given $f(x) = 2x^2 + 7$; $x \in \mathbb{R}^+ \cup \{0\}$. Find $f^{-1}(x)$.

5. Given $f(x) = 2x^2 - 3$; $x \geq 0$, $x \in \mathbb{R}$. Find $f^{-1}(x)$.

6. Given $f(x) = \dfrac{x}{x-1}$; $x \in \mathbb{R}$, $x \neq 1$. Find $f^{-1}(x)$.

7. Given $f(x) = \sqrt{2x-1} + 5$; $x \geq \dfrac{1}{2}$. Find $f^{-1}(x)$.

   Also show that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

8. Given $f(x) = 2x - 5$, $g(x) = 1 - x$. Show that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

9. Given $f(x) = 3x + 2$, $g(x) = \dfrac{1}{x}$; $x \neq 0$. Show that $(g \circ f)^{-1}(x) = (f^{-1} \circ g^{-1})(x) = \dfrac{1}{3}\left(\dfrac{1}{x} - 2\right)$.

10. Given $f(x) = \dfrac{10-x}{x+2}$; $x \neq -2$. Find:

    (i) $f^{-1}(2)$.

    (ii) k such that $f(k) = k$.

Topic 3: Sketching graphs of functions

11. A function f is defined by

$$f(x) = \begin{cases} 0 & x < -1 \\ x+1 & -1 \leq x < 0 \\ 1-x & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

    Sketch the graph of $f(x)$.

12. Sketch the graph of $f(x) = x^2 + 2$; $x \in \mathbb{R}$, $x \geq 0$. Use this information to draw the graph of $f^{-1}(x)$ without finding the inverse function f^{-1}.

13. Given $f(x) = (x-2)^2 + 5$; $x \in \mathbb{R}$. Sketch the graph of $f(x)$ for $1 \leq x \leq 5$.

Topic 4: Modulus inequalities

14. Solve the following inequalities for $x \in \mathbb{R}$:

    (i) $|x-1| < 5$.

    (ii) $|x-1| = 3$ and $|x-1| < 3$

    (iii) $|x+3| = 2|x-1|$.

    (iv) $|3x+2| \geq 2-x$.

15. Express the set $\{x \in \mathbb{R} / |2x-1| < 7\}$ as intervals.

16. Determine the values of x for which the following inequalities hold.

    (i) $x^2 > 7x - 12$.

    (ii) $x^2 + 5 < 2x$

    (iii) $b^2 + a^2x^2 > 2abx$; $a \neq 0$.

    (iv) $6x^2 - 5x - 4 < 0$.

    (v) $3x^2 + 5x < 2$.

Answers

1. $2(2x+1)(x-1)$ and $2(x+1)(x-2)$.

2. (i) $\frac{2}{3}$ or -2 (ii) $\frac{1}{9}$ or -1 (iii) $-\frac{1}{2}$

3. $a=1,\, b=1$ or $a=-1,\, b=-1$.

4. $\sqrt{\frac{x-7}{2}}$.

5. $\sqrt{\frac{x+3}{2}}$.

6. $\frac{x}{x-1}$ and $x \neq 1$.

7. $f^{-1}(x) = \frac{1}{2}(x^2 - 10x + 26)$.

10. (i) 2 (ii) $k = -5$ or 2

14. (i) $(-4, 6)$ (ii) No solution (iii) $-\frac{1}{3}\,,\ 5$

    (iv) $x \leq -2$ or $x \geq 0$

15. $-3 < x < 4$

16. (i) $x < 3$ or $x > 4$ (ii) No solution (iii) $x \in \mathbb{R}, x \neq \frac{b}{a}$

    (iv) $-\frac{1}{2} < x < \frac{4}{3}$ (v) $-2 < x < \frac{1}{3}$