COMP2054 Tutorial Session 1: Mathematics for ADE

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Expected Knowledge

$$a < b \Rightarrow 2a < 2b$$
$$a \leq b \Rightarrow 2a \leq 2b$$
$$a < b \Rightarrow -a > b$$
$$a \leq b \Rightarrow -a \geq -b$$

- We expect you to be familiar with the following; if any of the following are unfamiliar, you **must** study the self-study material on Moodle and complete the exercises.

$$a^0 = 1$$
$$a^{b+c} = a^b a^c$$
$$a^{bc} = \left(a^b\right)^c$$
$$a^b / a^c = a^{(b-c)}$$
$$a^{-b} = 1/a^b$$
$$a^{1/b} = \sqrt[b]{a}$$

$$\forall a > 0, \forall b > 0, a < b \Rightarrow 2^a < 2^b$$
$$\forall a > 0, \forall b > 0, a < b \Rightarrow \log(a) < \log(b)$$

Session outcomes

- Understand how to rearrange formulas with logs.
- Understand and apply log rules.
- Understand and calculate the sum of arithmetic and geometric series.

Logs and Exponents

Log rules and rearranging formulas

Recap – Logarithms

$$log_b a = c \Leftrightarrow b^c = a$$

- The result of $\log_b a$ is **how many times do we need to multiply b by itself to get a?**

- Logs are used extensively in data structures and divide-and-conquer sorting algorithms

Example: $\log_2 16$

Recap – Rearranging Logarithms

- $\log_b 1 = 0$
- $\log_a a = 1$
- $b^{\log_b a} = a$
- $\log_b b^a = a$
- $\log_b(a \cdot c) = \log_b a + \log_b c$
- $\log_b(a/c) = \log_b a - \log_b c$
- $\log_b(a^c) = c \cdot \log_b a$
- $\log_b a = \log_b c \cdot \log_c a$
- $\log_b a = 1/\log_a b$

Logarithm Example (1)

$$\log_2 4 \cdot \log_4 64$$

- $\log_b 1 = 0$
- $\log_a a = 1$
- $b^{\log_b a} = a$
- $\log_b b^a = a$
- $\log_b(a \cdot c) = \log_b a + \log_b c$
- $\log_b(a/c) = \log_b a - \log_b c$
- $\log_b(a^c) = c \cdot \log_b a$
- $\log_b a = \log_b c \cdot \log_c a$
- $\log_b a = 1/\log_a b$

Logarithm Example (2)

$$\log_9 3$$

- $\log_b 1 = 0$
- $\log_a a = 1$
- $b^{\log_b a} = a$
- $\log_b b^a = a$
- $\log_b (a \cdot c) = \log_b a + \log_b c$
- $\log_b (a/c) = \log_b a - \log_b c$
- $\log_b (a^c) = c \cdot \log_b a$
- $\log_b a = \log_b c \cdot \log_c a$
- $\log_b a = 1/\log_a b$

Logarithm Exercises

< will be uploaded after the tutorials >

Geometric and Arithmetic Series

Calculating the sum or product of a "sequence of numbers" (series).

Recap – Arithmetic Series

$$S = 1 + 2 + 3 + \cdots + (n - 1) + n$$

To find the sum of S:

$$\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}$$

Example: Find the sum of

$$S = 1 + 2 + 3 + \cdots + 20$$

Recap – Geometric Series

$$S = b^0 + b^1 + b^2 + \cdots + b^{n-1} + b^n$$

Example: Find the sum of

$$S = 3^0 + 3^1 + 3^2 + \cdots + 3^5$$

To find the sum of S:

$$\sum_{i=0}^{n} b^i = \frac{b^{n+1} - 1}{b - 1}$$

Series Exercises

< will be uploaded after the tutorials >