University of Nottingham
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Foundation Algebra (CELEN036)

Problem Sheet 5 **Topics: Polynomial Factorisation**

Topic 1: Remainder and factor theorems

1. Find all values of k for which $(x-1)$ is a factor of the polynomial $p(x) = k^2x^3 - 7kx + 10$.

2. Find constants a and b such that $ax^3 - bx^2 + 45x + 54 = 0$ has a root 3, and yields a remainder of 12 when divided by $(x+1)$.

3. Find the value(s) of k such that $\left(x + \frac{k}{2}\right)$ and $(x + 2k)$ are factors of $x^2 + \left(\frac{15}{2}\right)x + 9$.

Topic 2: Methods of long division and synthetic division

4. Find the quotient $q(x)$ and the remainder $r(x)$ that result when $p(x)$ is divided by $s(x)$.

   (i) $p(x) = x^4 + 3x^3 - 5x + 10$; $s(x) = x^2 - x + 2$

   (ii) $p(x) = 6x^4 + 10x^2 + 5$; $s(x) = 3x^2 - 1$

   (iii) $p(x) = x^5 + x^3 + 1$; $s(x) = x^2 + x$

   (iv) $p(x) = 2x^4 - 3x^3 + 5x^2 + 2x + 7$; $s(x) = x^2 - x + 1$

   (v) $p(x) = 2x^5 + 5x^4 - 4x^3 + 8x^2 + 1$; $s(x) = 2x^2 - x + 1$

   (vi) $p(x) = 5x^6 + 4x^2 + 5$; $s(x) = x^3 + 1$

5. Use the method of synthetic division to find the quotient $q(x)$ and the remainder $r(x)$ that result when $p(x)$ is divided by $s(x)$.

(i) $p(x) = 3x^3 - 4x - 1$; $s(x) = x - 2$

(ii) $p(x) = x^4 - 5x^2 + 4$; $s(x) = x + 5$

(iii) $p(x) = x^5 - 1$; $s(x) = x - 1$

(iv) $p(x) = 2x^3 - x^2 - 2x + 1$; $s(x) = x - 1$

(v) $p(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9$; $s(x) = x + 4$

(vi) $p(x) = x^7 + 1$; $s(x) = x - 1$

6. Given $p_1(x) = x^3 + 4x^2 + x - 6$ and $p_2(x) = x^5 - 1$. Find a polynomial $q(x)$ and a constant r such that:

(i) $p_1(x) = (x - 2)q(x) + r$ (ii) $p_1(x) = (x + 1)q(x) + r$

(iii) $p_2(x) = (x + 1)q(x) + r$ (vi) $p_2(x) = (x - 1)q(x) + r$

7. Use the method of synthetic division to show that $(x - 3)$ is a factor of

$x^3 - 5x^2 + 2x^2y + xy^2 - 8xy - 3y^2 + 6x + 6y$

Topic 3: Polynomial factorisation and solving

8. Factorize the following polynomials completely:

(i) $p(x) = x^3 - 2x^2 - x + 2$ (ii) $p(x) = x^4 + 10x^3 + 36x^2 + 54x + 27$

(iii) $p(x) = 3x^3 + x^2 - 12x - 4$ (iv) $p(x) = x^5 + 4x^4 - 4x^3 - 34x^2 - 45x - 18$

9. Factorize the following polynomials and solve $p(x) = 0$ for $x \in \mathbb{R}$ in each case.

(i) $p(x) = x^3 - x^2 - 10x - 8$ (ii) $p(x) = x^3 - x^2 - 16x - 20$

(iii) $p(x) = x^3 + 4x^2 - 8$ (iv) $p(x) = 2x^3 - 3x^2 - 11x + 6$

Answers

1. $k = 2$ or 5

2. $a = -6,\ b = 3$

3. $k = 3$

4. (i) $q(x) = x^2 + 4x + 2,\ r(x) = -11x + 6$

   (ii) $q(x) = 2x^2 + 4,\ r(x) = 9$

   (iii) $q(x) = x^3 - x^2 + 2x - 2,\ r(x) = 2x + 1$

   (iv) $q(x) = 2x^2 - x + 2,\ r(x) = 5x + 5$

   (v) $q(x) = x^3 + 3x^2 - x + 2,\ r(x) = 3x - 1$

   (vi) $q(x) = 5x^3 - 5,\ r(x) = 4x^2 + 10$

5. (i) $q(x) = 3x^2 + 6x + 8,\ r(x) = 15$

   (ii) $q(x) = x^3 - 5x^2 + 20x - 100,\ r(x) = 504$

   (iii) $q(x) = x^4 + x^3 + x^2 + x + 1,\ r(x) = 0$

   (iv) $q(x) = 2x^2 + x - 1,\ r(x) = 0$

   (v) $q(x) = 2x^3 - 5x^2 + 3x - 39,\ r(x) = 147$

   (vi) $q(x) = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1,\ r(x) = 2$

6. (i) $q(x) = x^2 + 6x + 13,\ r = 20$

   (ii) $q(x) = x^2 + 3x - 2,\ r = -4$

   (iii) $q(x) = x^4 - x^3 + x^2 - x + 1,\ r = -2$

   (iv) $q(x) = x^4 + x^3 + x^2 + x + 1,\ r = 0$

8. (i) $(x - 1)(x + 1)(x - 2)$

   (ii) $(x + 1)(x + 3)^3$

   (iii) $(x - 2)(x + 2)(3x + 1)$

   (iv) $(x + 1)^2(x + 2)(x + 3)(x - 3)$

9. (i) -1 or -2 or 4

(ii) -2 or 5

(iii) -2 or $-1 \pm \sqrt{5}$

(iv) -2 or $\frac{1}{2}$ or 3