Science A Physics

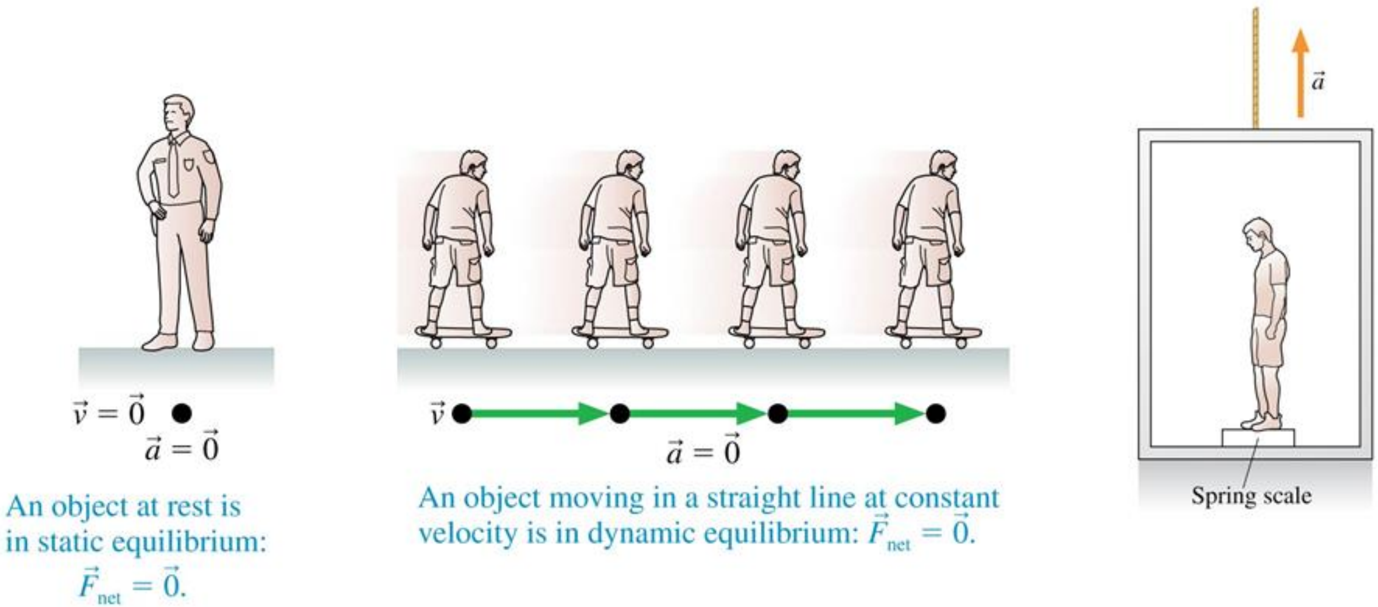
Lecture 5:

Energy & Momentum

Aims of today's lecture

1. Forms of Energy
2. Hooke's Law
3. The Basic Energy Model and Energy Diagrams
4. Conservative and non-conservative systems
5. Power
6. Momentum and Conservation of Momentum
7. Impulse

Analysing the net Force Acting on an Object

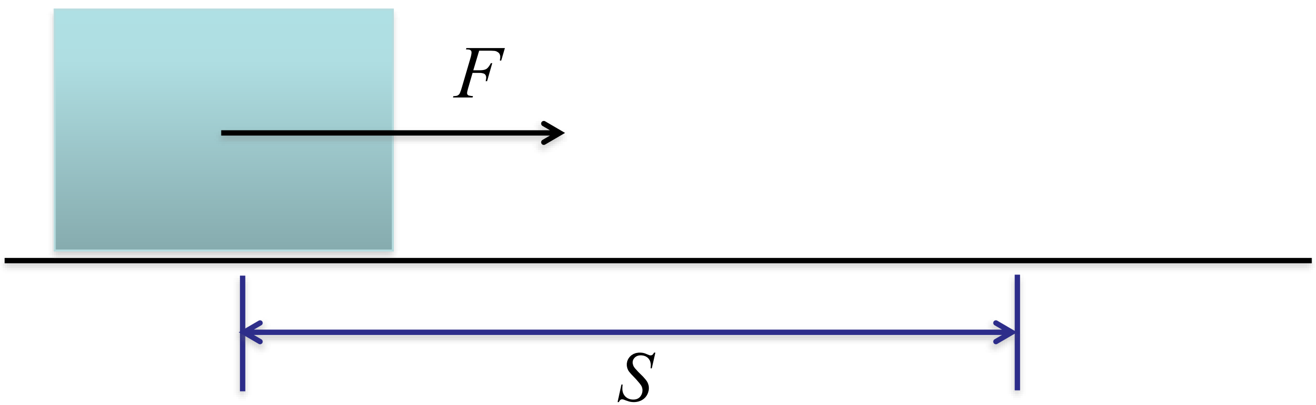


- In this fifth lecture, we are going to look at a relatively new idea used to describe an object which is moving, or which has the potential to move, over a certain distance because of a **net force** acting on the object.
- In general, we call this idea **'energy'**, or **'work'**, depending on the context.
- We will introduce the idea of '**momentum**' to describe the motion of an object. Like energy or force, it is another 'tool' that we can use to solve physics problems.

1. Forms of Energy

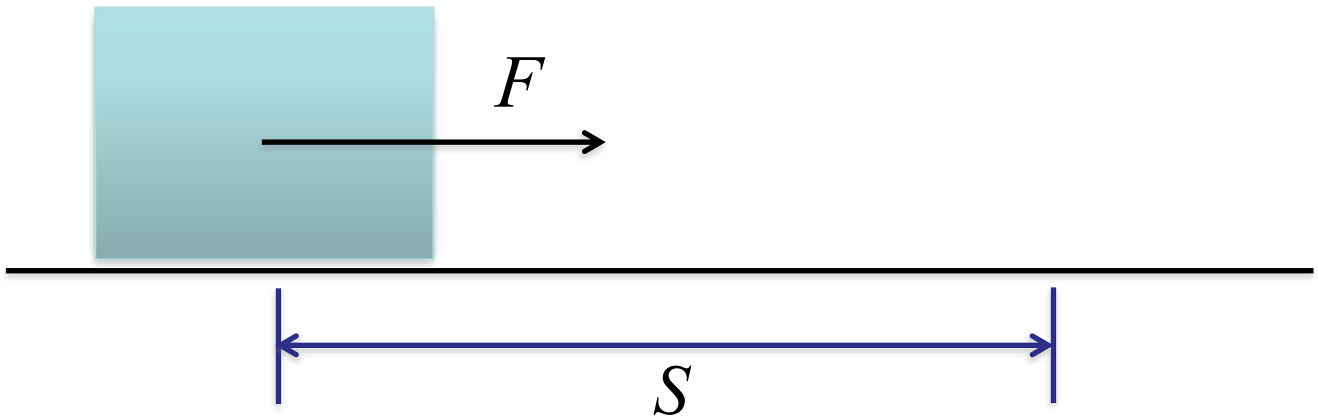
Energy

- Consider the following situation. We have an object at rest on a smooth horizontal surface.
- We then apply a constant net force F, and the object travels a displacement S.



- Clearly, the **net force acting parallel** to the object's displacement causes it to accelerate over this displacement.
- We talk about this in terms of saying that **'work has been done on the object'**, or we have given the object . . . something . . . something we call **energy**.

Energy



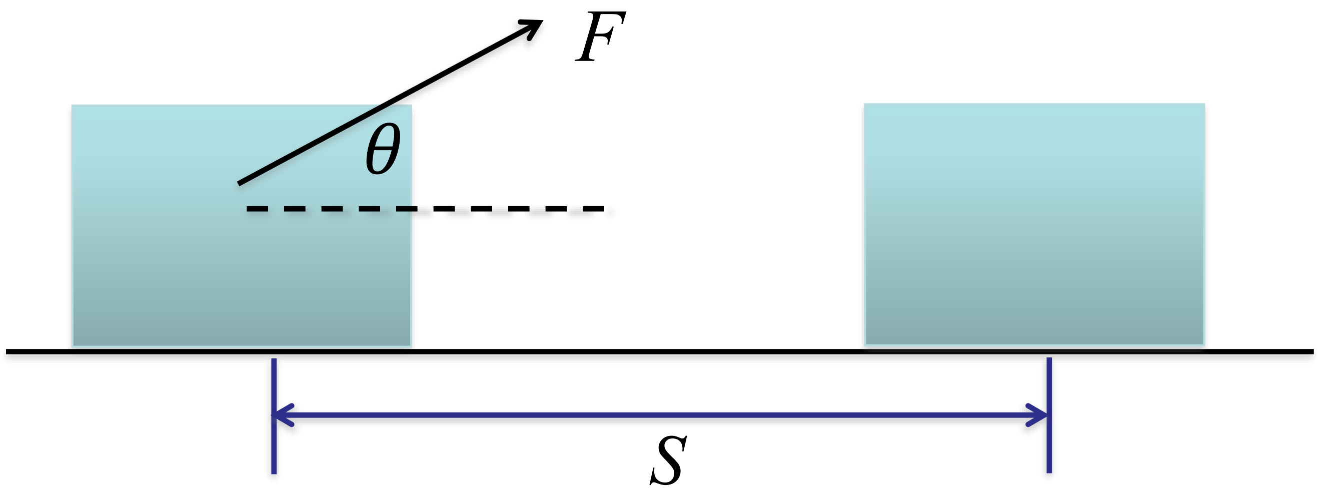
- The work done on the object, or the energy given to the object is defined as

$$W = FS$$

- The unit for work is the **Joule (J)**, 1 Joule being equal to 1 N m; this is also the unit for energy.
- It is worth noting that although work is the product of two vectors, it is still a **scalar quantity**.

Energy

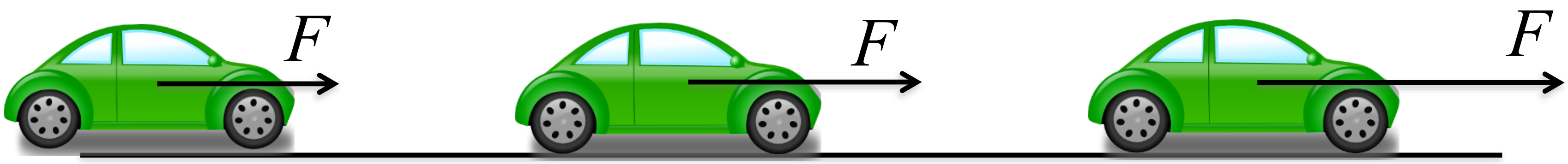
- Consider the same situation, except now the net force F is at an angle θ to the displacement.



- The work done will be given by the horizontal component of F multiplied by S.

$$w = FScos\theta$$

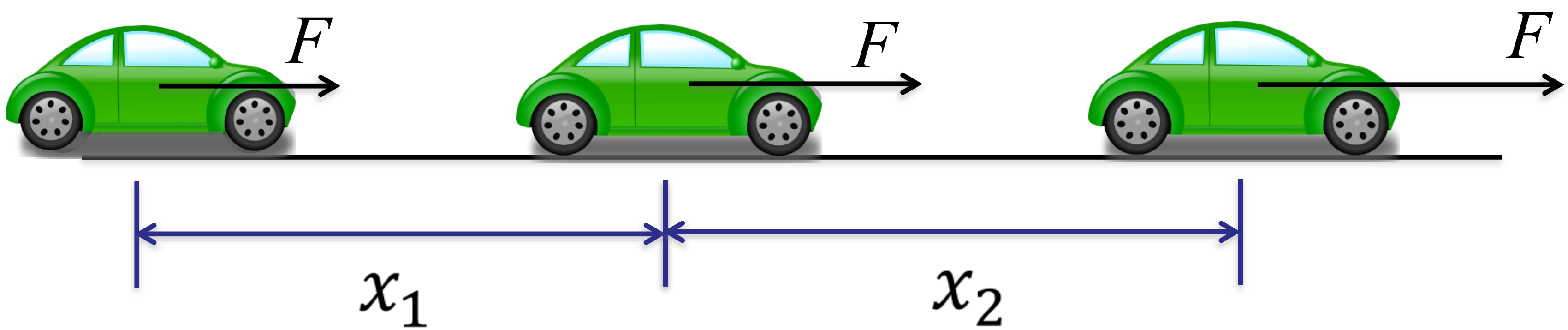
The Work Done by a Varying Force



- Consider a car where the driver is constantly increasing the driving force of the vehicle as it travels along a straight road.
- Clearly the net force acting on the object is not constant, but varies; in this case, it is increasing.

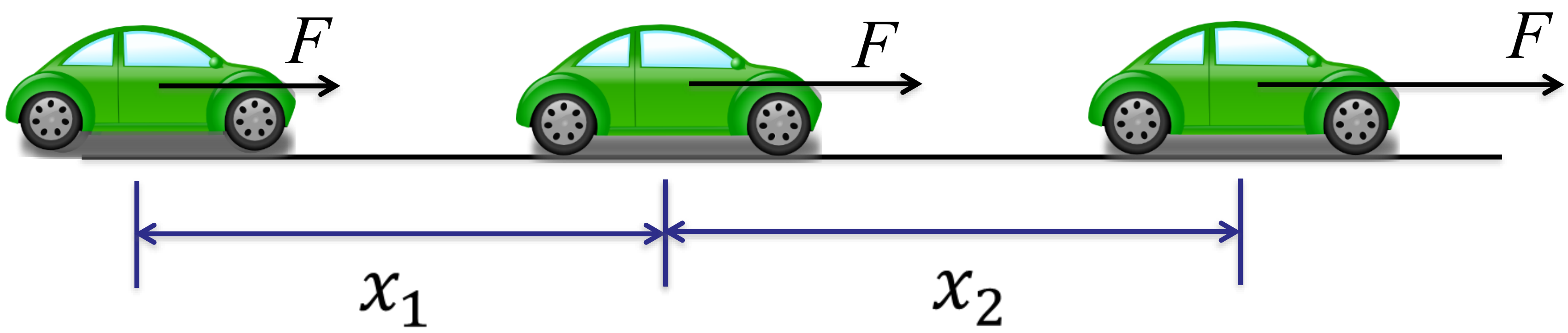
Q. How do we then calculate the work done on the object?; or the energy that is given to the object, in other words?

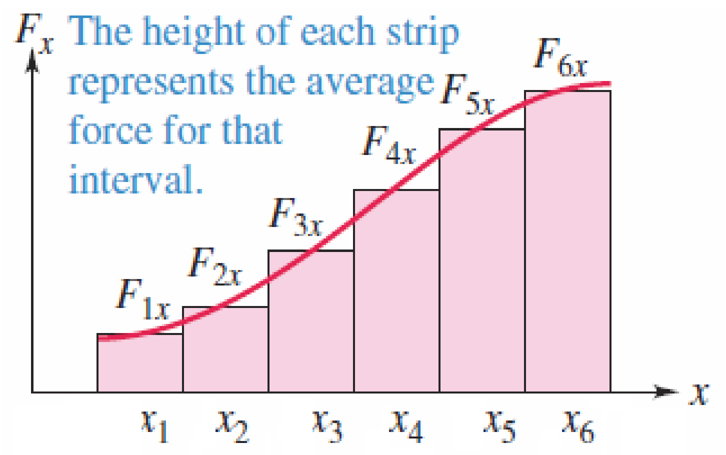
The Work Done by a Varying Force



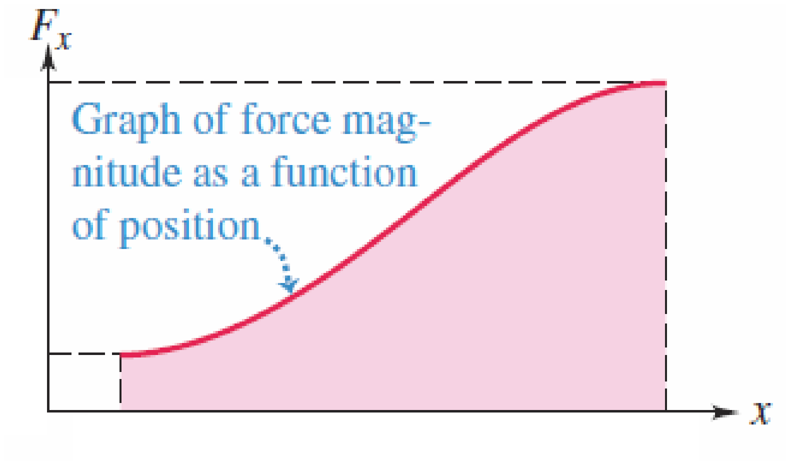
- Well, let's split the road into a number of distinct separate lengths.
- We can now approximate the work done over the distance x_1 by taking the average force over x_1, and the same for x_2, etc.

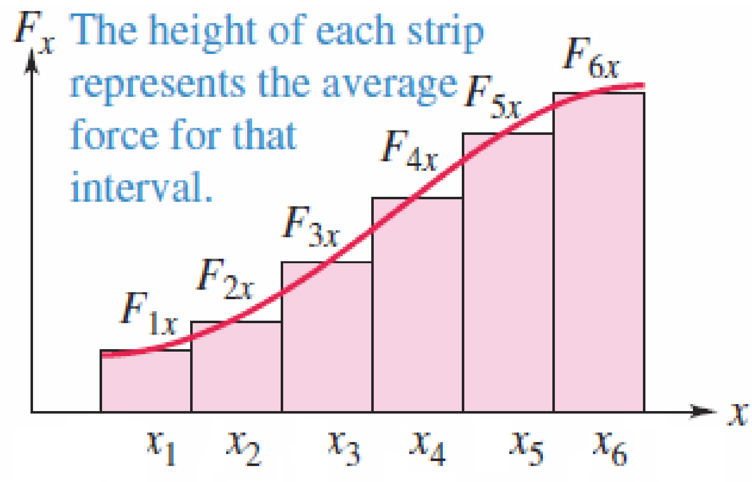
The Work Done by a Varying Force





The Work Done by a Varying Force



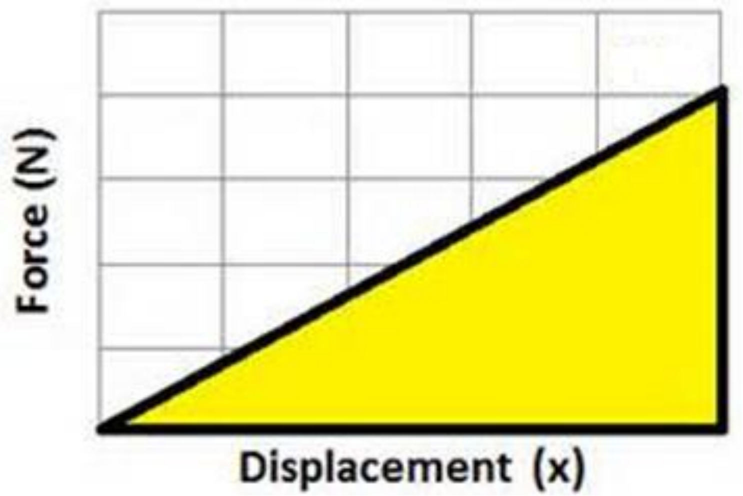


- We can see that the area under the graph actually gives us the work done on the object, or the energy given to the object, in other words. This area is another example of using **integration** in physics.
- Having now defined 'work'/'energy', as well as considered it when the force acting on the object is constant or varies, let's now consider how energy can take different forms, depending on the context.

Kinetic Energy, K

- All moving objects have kinetic energy.
- The more massive an object or the faster it moves, the larger its kinetic energy.
- **Kinetic energy** is the work required to bring an object from rest to some final speed over a certain displacement.

Kinetic Energy, K



- **Kinetic energy** is the work required to bring an object from rest to some final speed over a certain displacement.
- Looking at the above figure, the **integral** of this graph is

$$\frac{1}{2}N.x = \frac{1}{2}m.\frac{x}{s^2}.x = \frac{1}{2}m.v^2 = \text{kinetic energy}$$

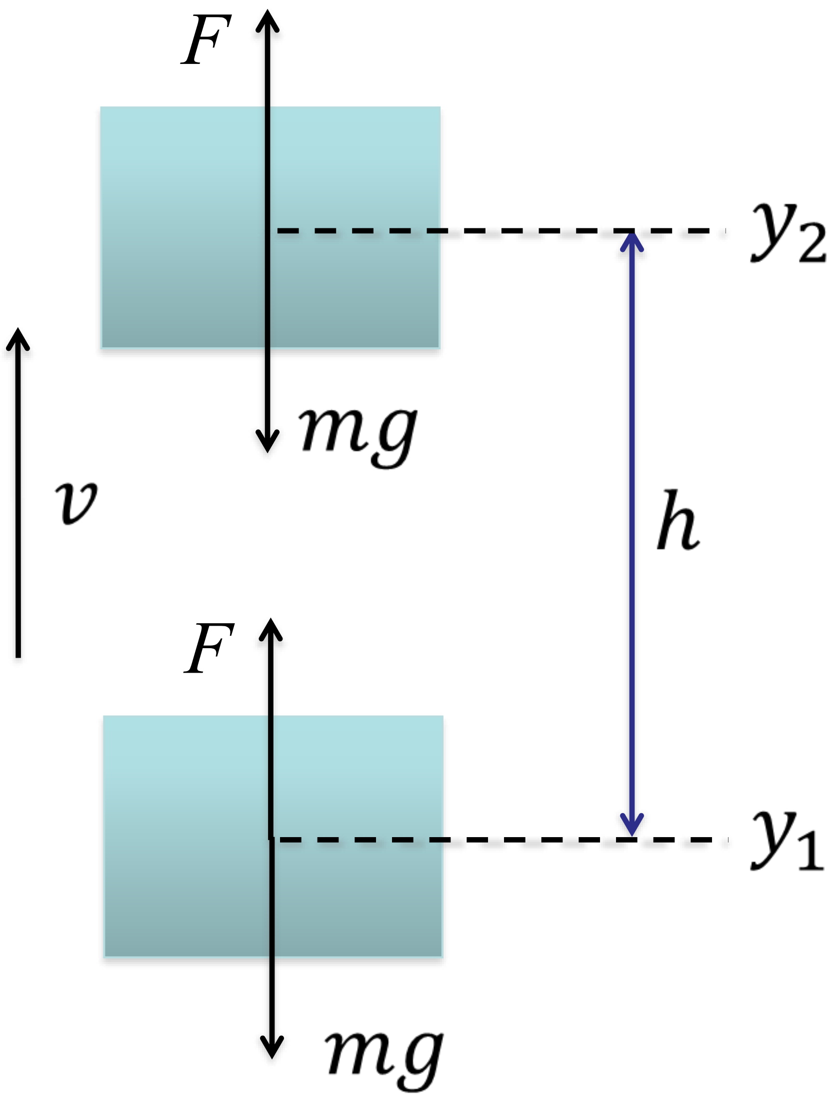
Potential Energy, U

- Potential energy is stored energy associated with an object's position.
- In the above figure, we talk about the roller-coaster having a **gravitational potential energy**, depending on its height above the ground; let's define this type of energy more precisely.

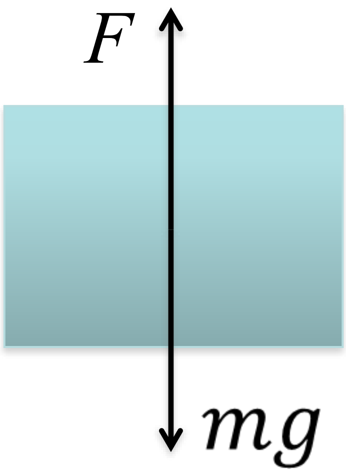
Gravitational Potential Energy, U_{grav}

- Consider an object with mass m which is raised by a constant force F at a constant velocity from a point y_1 to a point y_2.

- Let's label the distance from y_1 to y_2 as h.



Gravitational Potential Energy, U_{grav}



- Since the object is raised at a constant velocity, the forces must be in equilibrium. Therefore,

$$F = mg$$

Gravitational Potential Energy, U_{grav}

- Therefore, the work done in raising an object with a mass of $\boldsymbol{m}$ a distance of $\boldsymbol{h}$ is given by

$$w = mgh$$

- This work has transformed one form of energy (whichever type was used to provide the force $\boldsymbol{F}$) into gravitational potential energy.

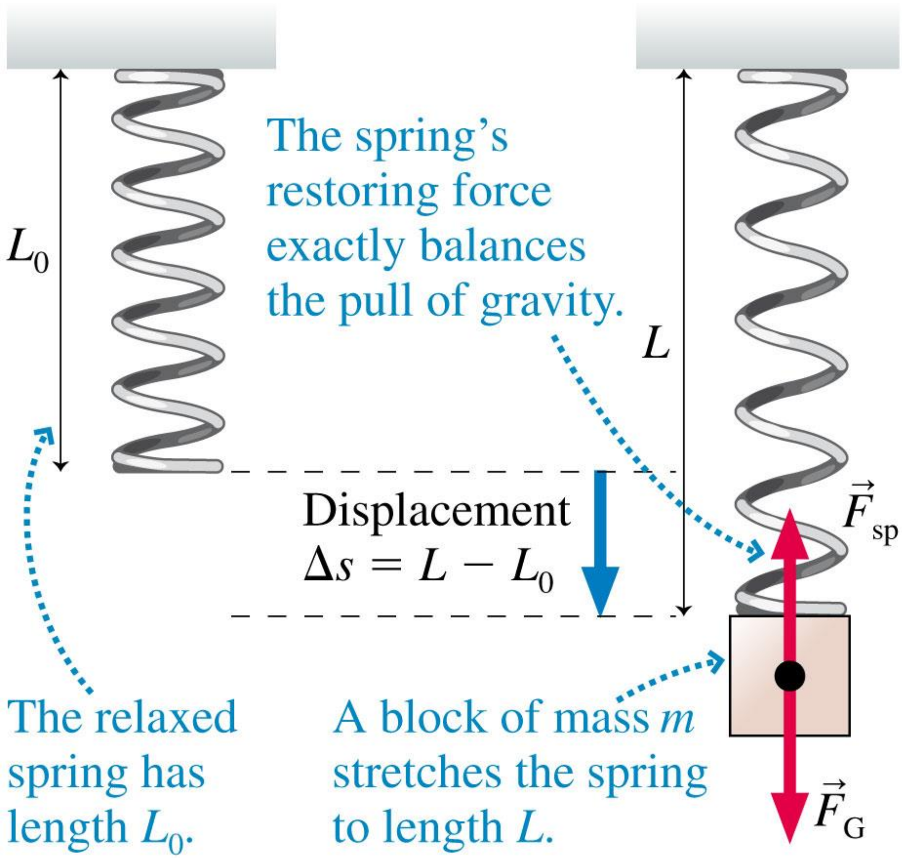
$$U_{grav} = mgh$$

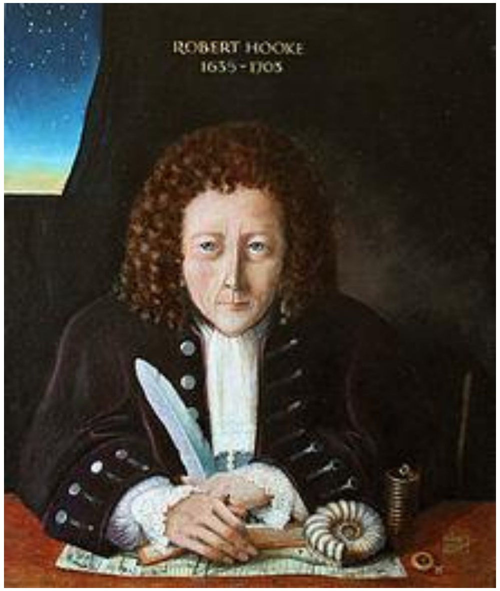
Elastic Potential Energy, U_s

- In physics, we tend to model objects in terms of springs (which can also be thought of as rubber bands).
- We tend to think of these springs as being able to store a potential energy that we refer to as **elastic potential energy, U_s**.
- To understand how springs, or rubber bands, store this elastic potential energy, we must look at an important idea called **Hooke's law**.

2. Hooke’s Law

Hooke's Law





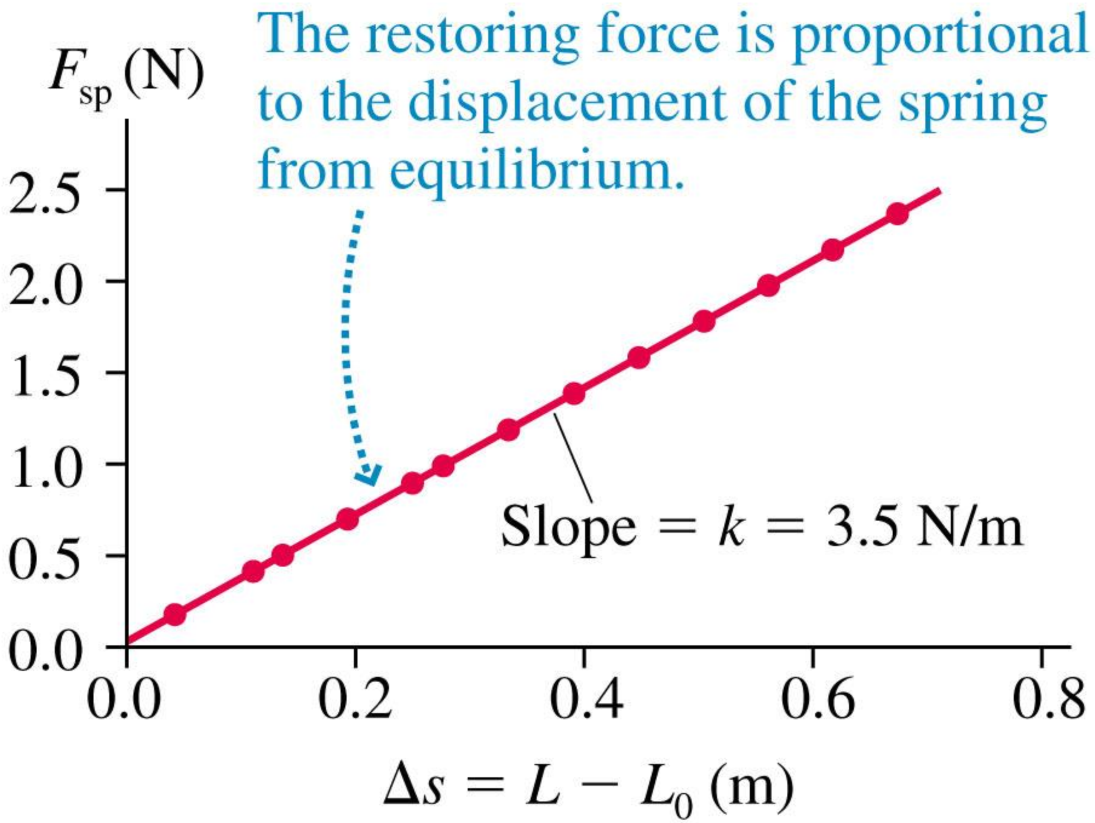
Robert Hooke

1635-1703

- The figure above shows how a hanging mass stretches a spring of equilibrium length L_0 to a new length L.
- The mass hangs in static equilibrium, so the upward spring force balances the downward gravitational force.

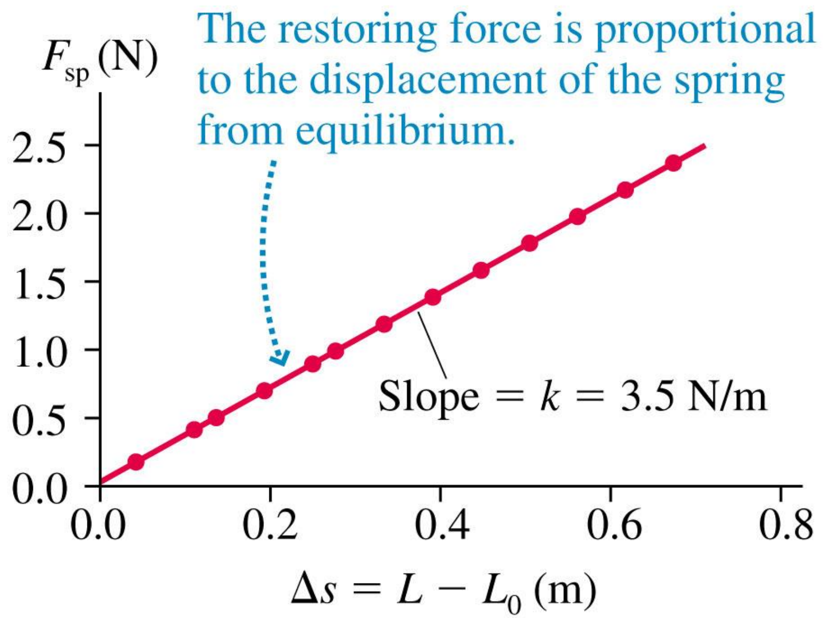
$$F_{sp} = F_G = mg$$

Hooke's Law



- The figure shows measured data for the restoring force of a real spring.
- Δs is the **displacement from equilibrium**. $$F_{sp} = k\Delta s$$
- The proportionality constant k is called the **spring constant**.
- The units of k are N/m.

Hooke's Law

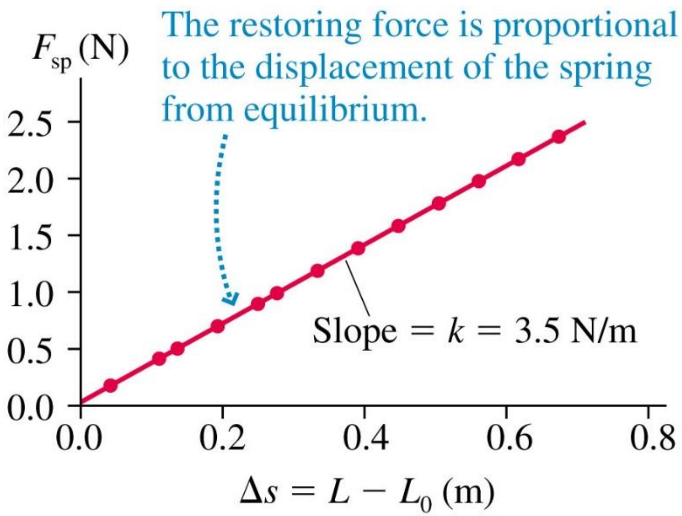


$$(F_{sp})_s = -k\Delta s$$

Q. So what's so special about Hooke's law? What does it imply?

- Well, we can see that the restoring force in the spring is proportional to the displacement that the spring undergoes.
- Furthermore, the physical meaning of the integral for the above graph is energy, energy that becomes stored in the spring as we stretch it or compress it, and energy that we refer to as **elastic potential energy, U_s**; so let's now return to this form of energy.

Elastic Potential Energy, U_s



$$(F_{sp})_s = -k\Delta s$$

- Ignoring the direction of the restoring force, the average restoring force F is given by $F = \frac{ks+0}{2}$

- Thus, the work done in stretching or compressing a spring from rest to a distance s is given by $w = Fs = \frac{ks^2}{2}$

- We can refer to this work, which is now stored in the spring, in the form of a restoring force, as **elastic potential energy**.

Elastic Potential Energy, U_s

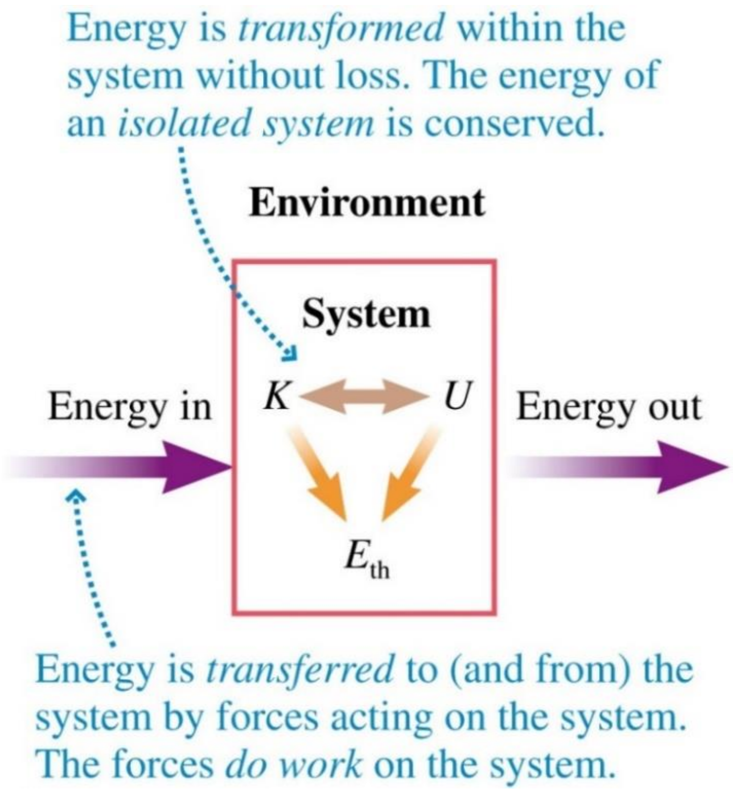
- Frequently in physics, we are interested in how energy is transformed from one form to another. In the next couple of slides, we are going to analyse an object attached to a spring as its energy transforms from elastic potential energy into kinetic energy.
- To this, we must briefly look at the **basic energy model** in physics.
- Before we do so, however, we must consider one other form of energy, **thermal energy**.

Thermal Energy, E_{th}

- Thermal energy is the sum of the microscopic kinetic and potential energies of all the atoms and bonds that make-up the object.
- This thermal energy occurs as a result of friction, which can be either useful or not useful, depending on the situation.

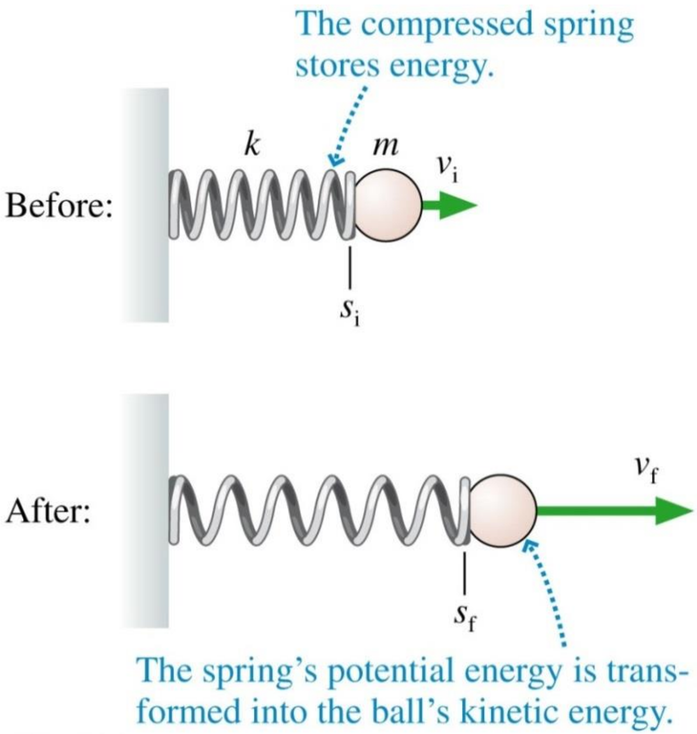
3. The Basic Energy Model

The Basic Energy Model



- Within an isolated system, energy can be transformed from one type to another.
- If we just focus on kinetic energy transforming itself into potential energy, and vice-versa, and assume there is no energy lost due to thermal energy, we are looking at the **law of conservation of mechanical energy.**

The Basic Energy Model



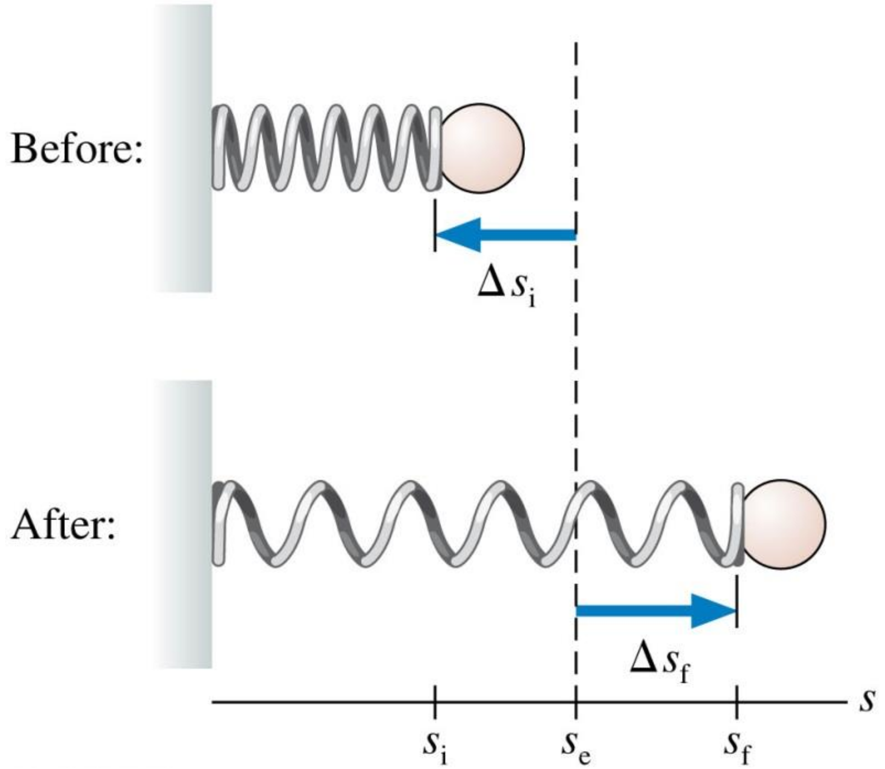
$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

- In the above **conservation-of-mechanical-energy** expression, we can replace gravitational potential energy (U_{gf}) with elastic potential energy to give us:

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(\Delta s_f)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(\Delta s_i)^2$$

The Basic Energy Model

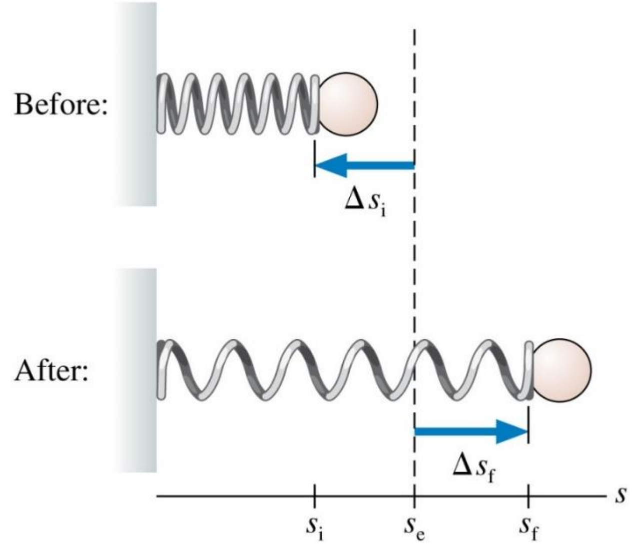


- An object moving without friction on an **ideal spring** obeys:

$$K_f + U_{sf} = K_i + U_{si}$$

where, $$U_s = \frac{1}{2}k(\Delta s)^2 \qquad \text{(elastic potential energy)}$$

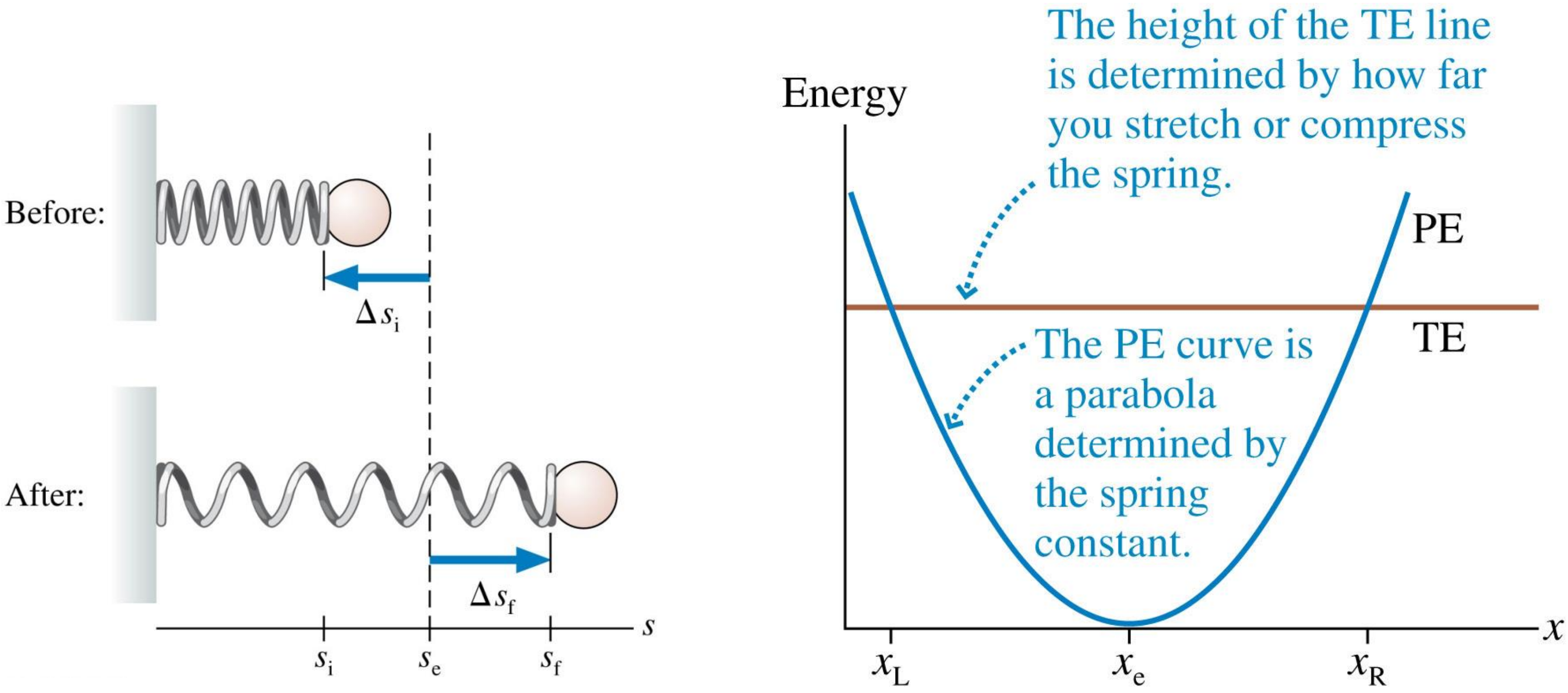
The Basic Energy Model



$$U_s = \frac{1}{2}k(\Delta s)^2 \qquad \text{(elastic potential energy)}$$

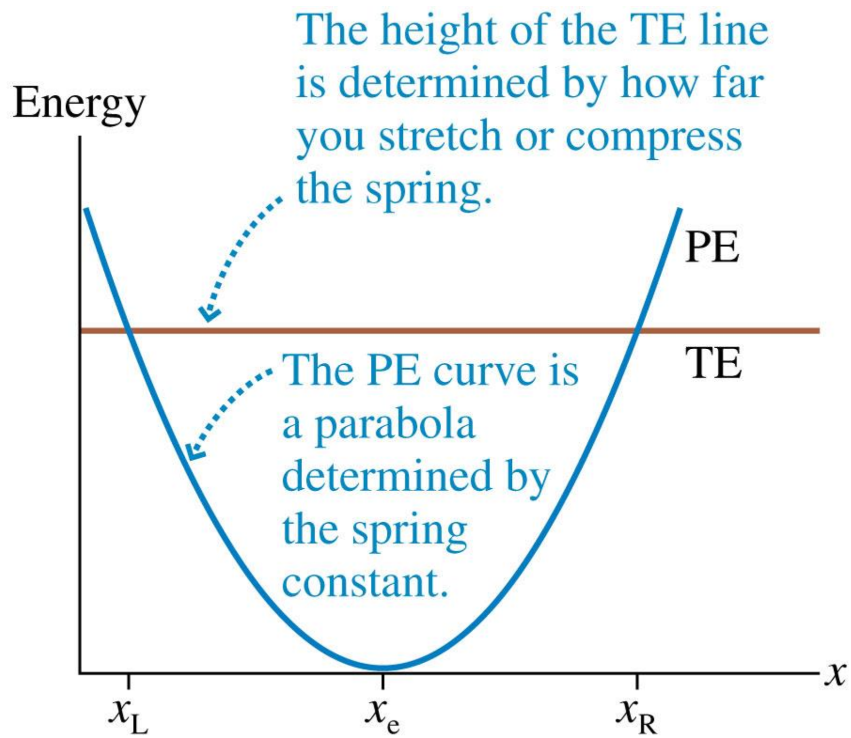
- Because Δs is squared, U_s is positive for a spring that is either stretched or compressed.
- Using a graph and bar charts, let's try to represent the energy changes that take place; we call such representations **energy diagrams**.

Energy Diagrams



- The above energy diagram is for a mass on a horizontal spring.
- You can set the total energy (TE) to any height you wish simply by stretching the spring to the proper length at the beginning of the motion.

Energy Diagrams

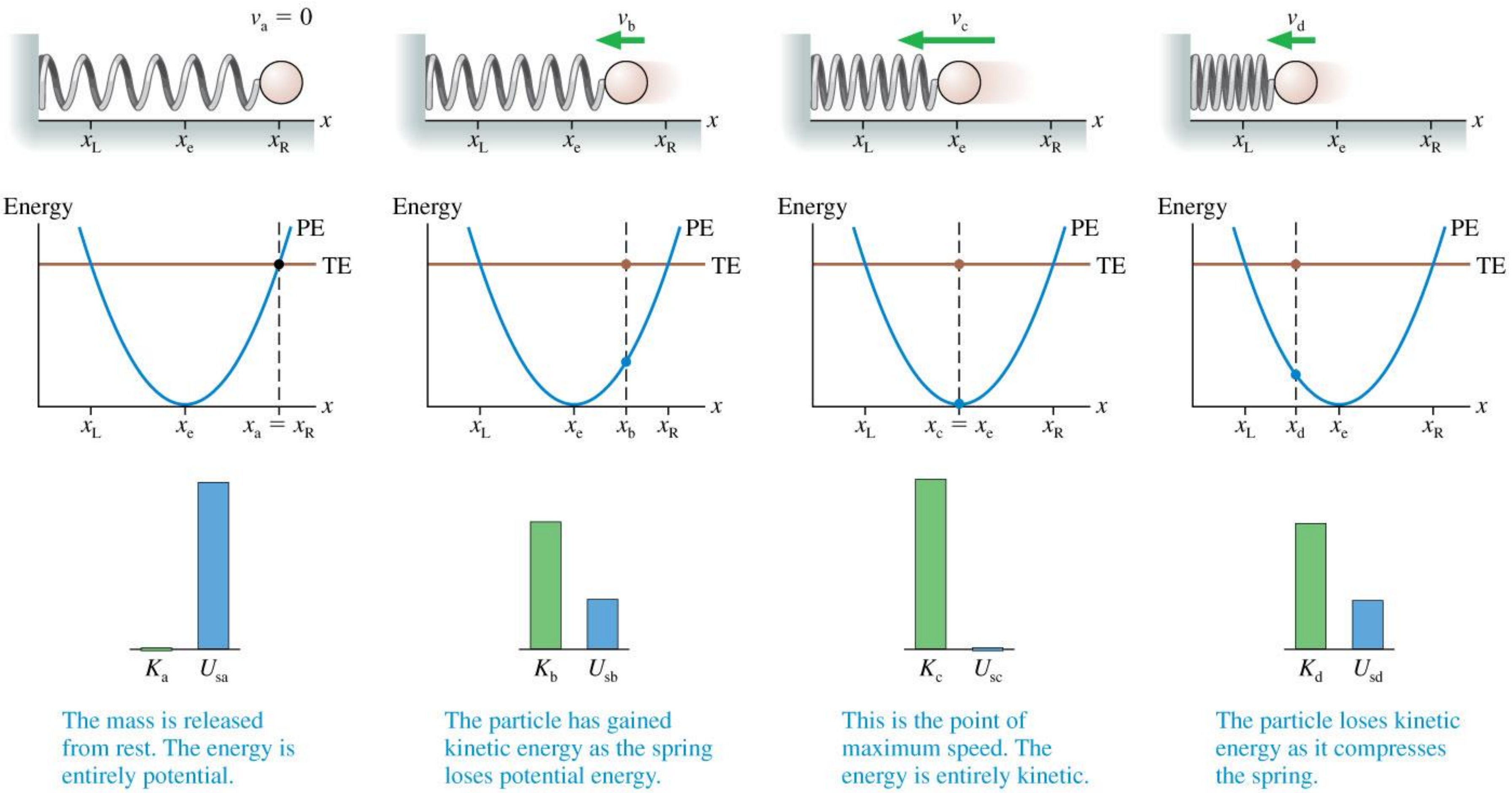


- The elastic potential energy (PE) is given by the parabola:

$$U_s = \frac{1}{2}k(x - x_e)^2$$

- The elastic potential energy (PE) depends on the spring constant k, which is unique for each material.

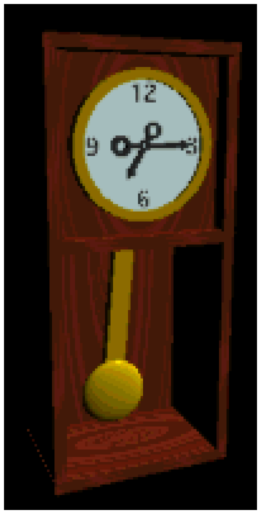
Energy Diagrams

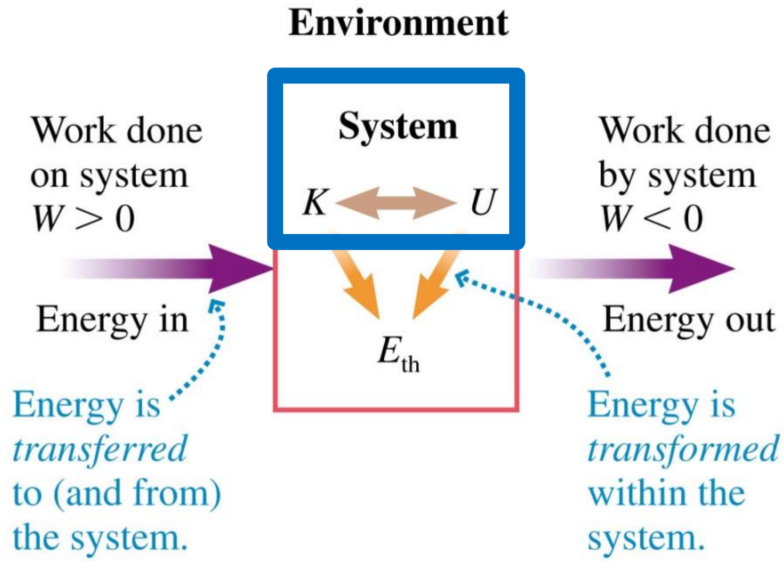


- It's important not to forget that we make the assumption that no energy is lost to friction; mechanical energy is conserved in other words.

4. Conservative and non-conservative systems

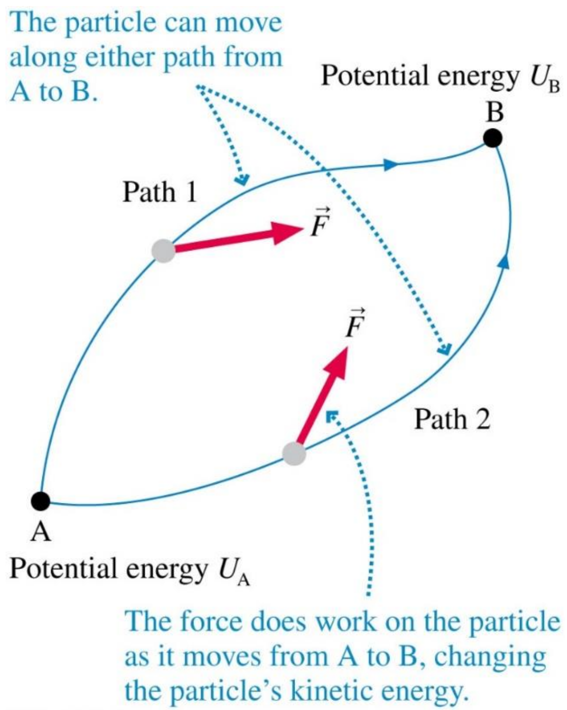
Conservative Systems

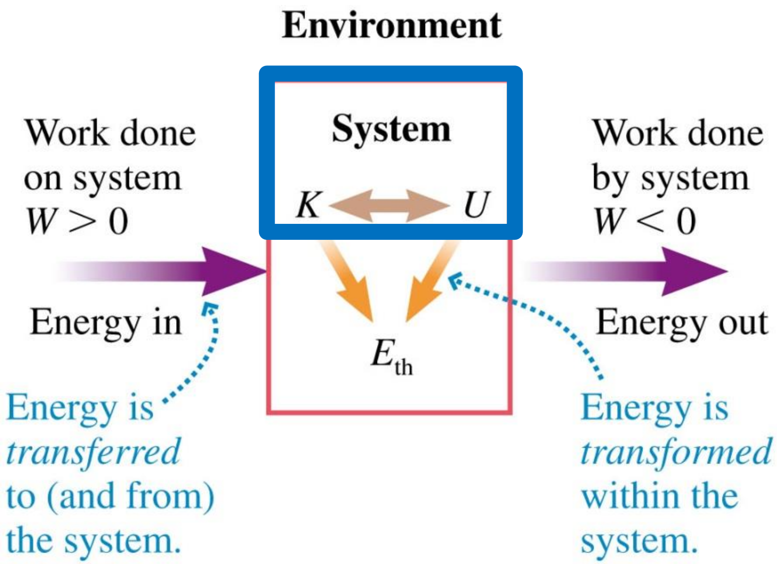




- A system in which the total mechanical energy, kinetic and potential, is constant, (assuming other forces such as friction are negligible) is called a **conservative system**.
- The forces that conserve this mechanical energy are called **conservative forces**; in the case of the pendulum, the conservative force is gravity.

Conservative Forces

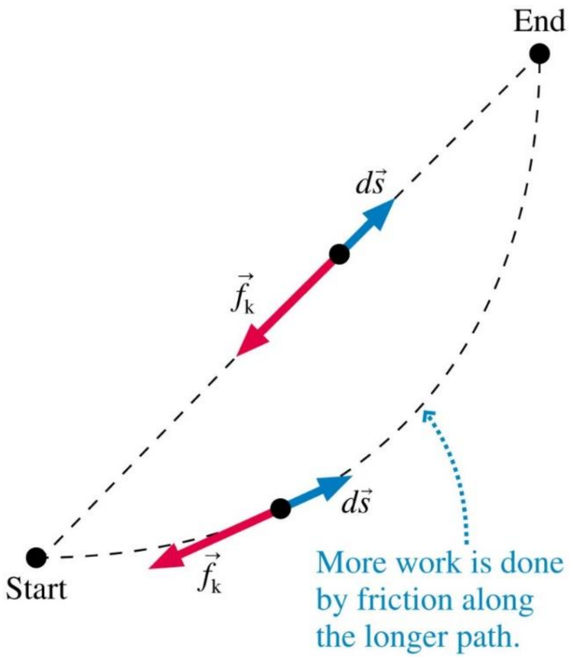


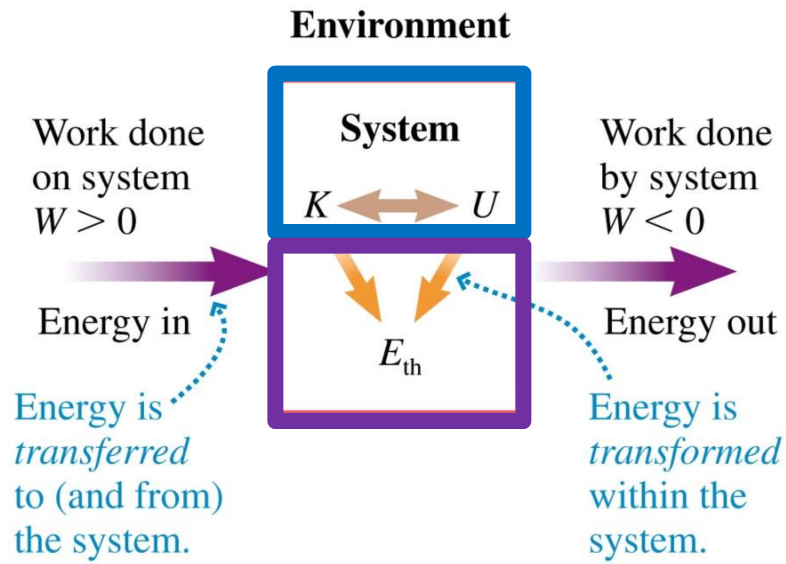


- The figure shows a particle that can move from A to B along either path 1 or path 2 while a force $\vec{F}$ is exerted on it.
- If there is a potential energy associated with the force, this is called a **conservative force**.
- The work done by $\vec{F}$ as the particle moves from A to B is independent of the path followed

$$\Delta U = U_f - U_i = -W_c(i \rightarrow f)$$

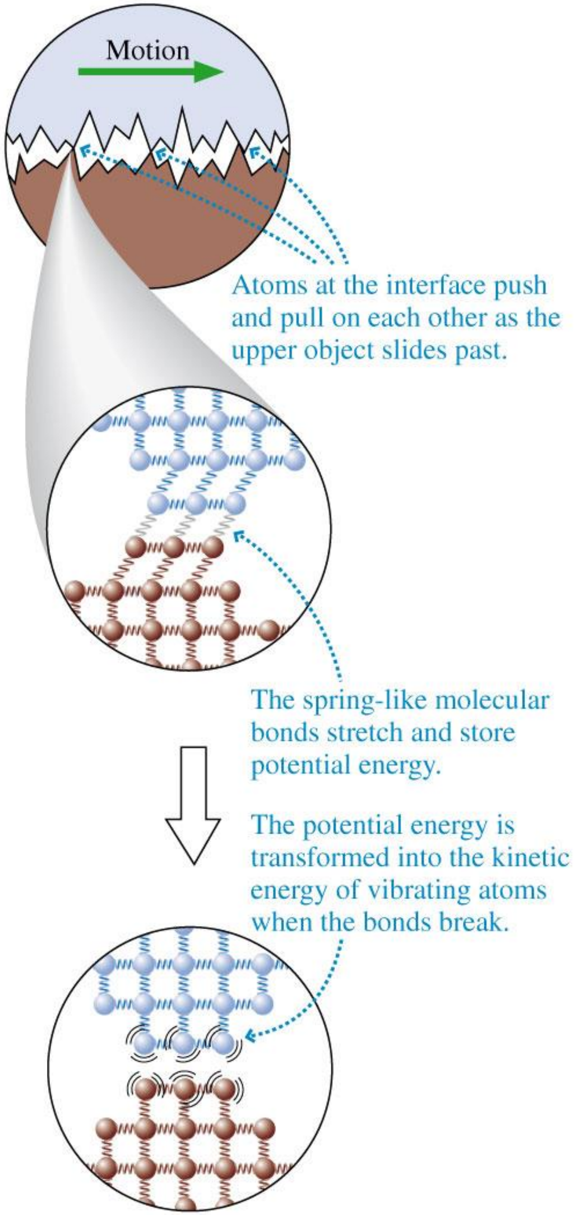
Non-Conservative Forces





- The friction force does negative work on the objects: $W_{fric} = -\mu_k mg\Delta s$, work which depends on Δs.
- A force for which the work is not independent of the path is called a **non-conservative force**.
- Taking our pendulum, for example, the non-conservative force, friction due to air resistance, eventually slows the pendulum down. In other words, it does not conserve the mechanical energy of the system.

Non-Conservative Forces

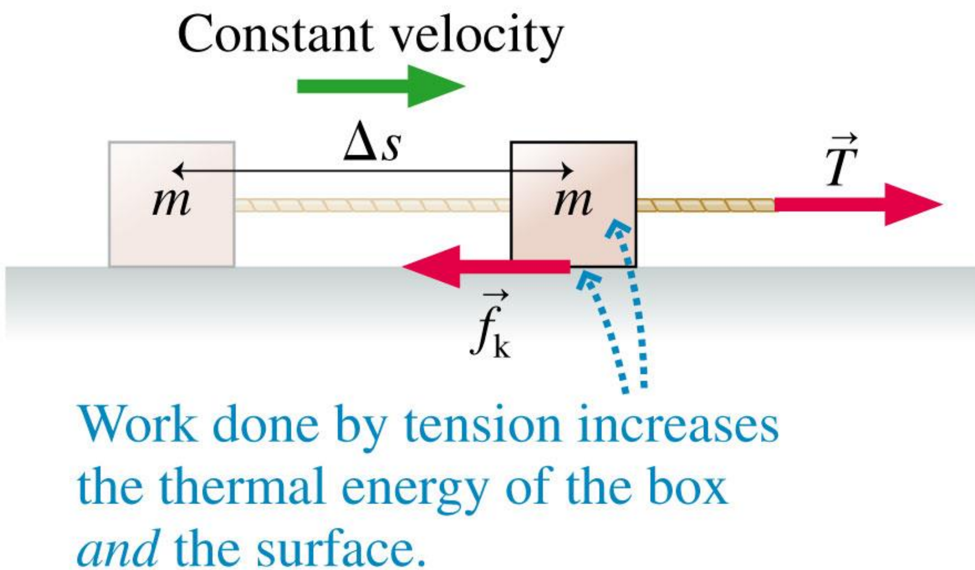


- As two objects slide against each other, atomic interactions at the boundary transform the kinetic energy, K_{macro}, into thermal energy in both objects.

$$K \rightarrow E_{th}$$

- This kinetic friction is a non-conservative force, sometimes referred to as a **dissipative force**.

Dissipative Forces



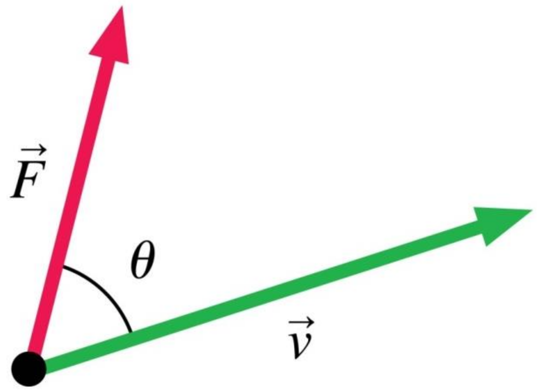
- Dissipative forces always increase the thermal energy; they never decrease it.

$$\Delta E_{th} = f_k \Delta s \text{ (increased thermal energy due to friction)}$$

- Let's now look at the rate at which work is transferred to or from a system: such a rate is called **power**.

5. Power

Power



- When energy is transferred by a force doing work, we call the rate at which it is transferred **power, $P = dW/dt$**.

- If an object moves at velocity $\vec{v}$ while acted on by force $\vec{F}$, the power delivered to the object is:

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

- The SI unit for power is the Watt, which is defined as:

$$1\,Watt = 1\,W = 1\,J/s$$

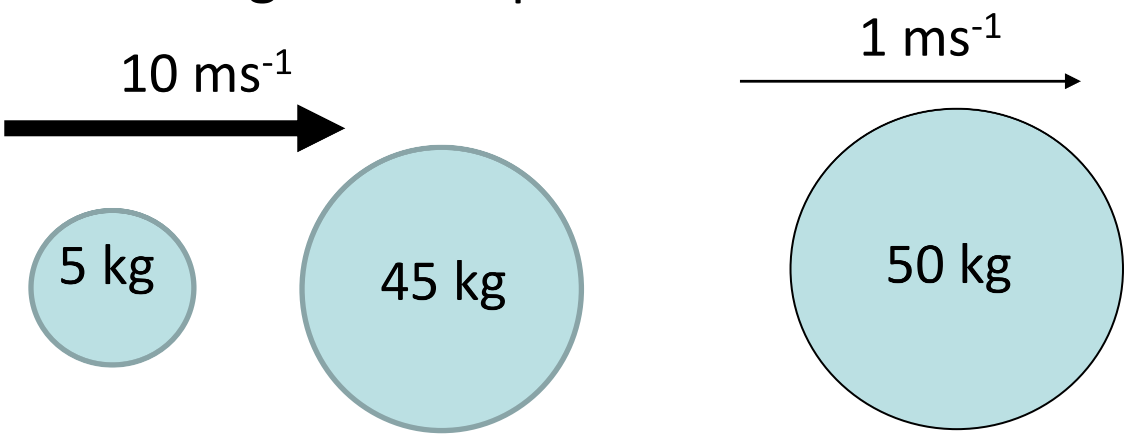
6. Momentum and Conservation of Momentum

Momentum

- Momentum is a term that was introduced by the French scientist and philosopher, Descartes, before Newton.
- What Descartes meant by the use of the term momentum is best understood by considering an example:

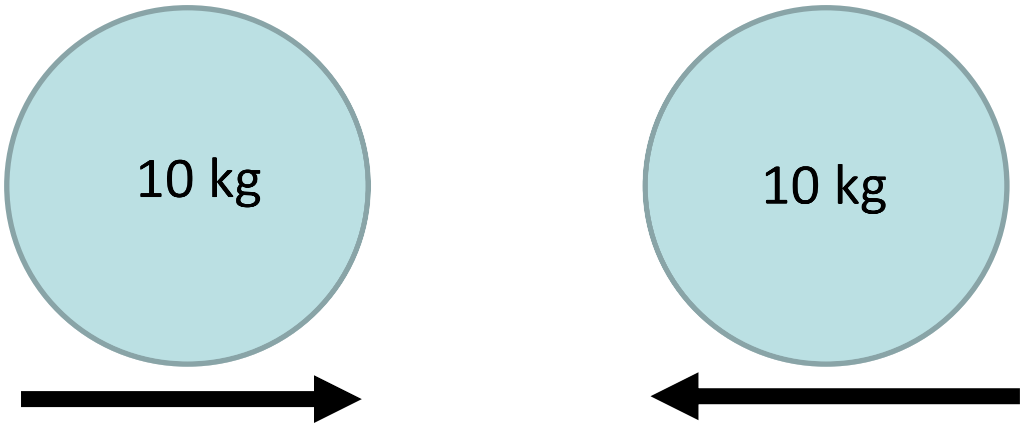
Rene Descartes

1596-1650



- Think of a ball (with a mass of $45\ \mathrm{kg}$) that is motionless.
- A $5\ \mathrm{kg}$ ball is moving towards it, with a speed of $5\ \mathrm{m/s}$, and is quite close to the object, so that we can assume the ball's flight to be close to horizontal. New speed 10 times less.

Momentum



$$momentum = p = m \times speed$$

Christian Huygens

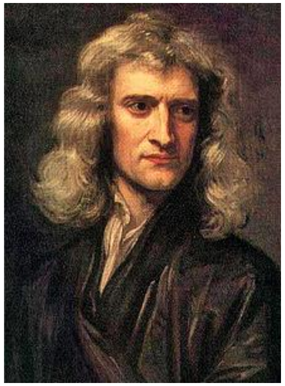
1596-16

- There was a problem, however, with Descartes's definition. For example, when two objects of equal mass come directly towards each other with equal, but opposite velocities, when they meet, they come to a complete stop. Need to replace speed with velocity.

$$momentum = p = \ mass \times velocity$$

Momentum and Its Relation to Force

$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$



1642-1726

- A force is required to change the momentum of an object, whether it is to increase the momentum, to decrease it, or to change its direction.

- Newton originally stated his second law in terms of momentum (although he called the product mv the 'quantity of motion')

Momentum and Its Relation to Force

- Newton's statement of the second law of motion, translated into modern language, is as follows:

The rate of change of momentum of an object is equal to the net force applied to it, and can be written as an equation,

$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$

where $\sum \vec{F}$ is the net force applied to the object (the vector sum of all the forces acting on it).

Momentum and Its Relation to Force

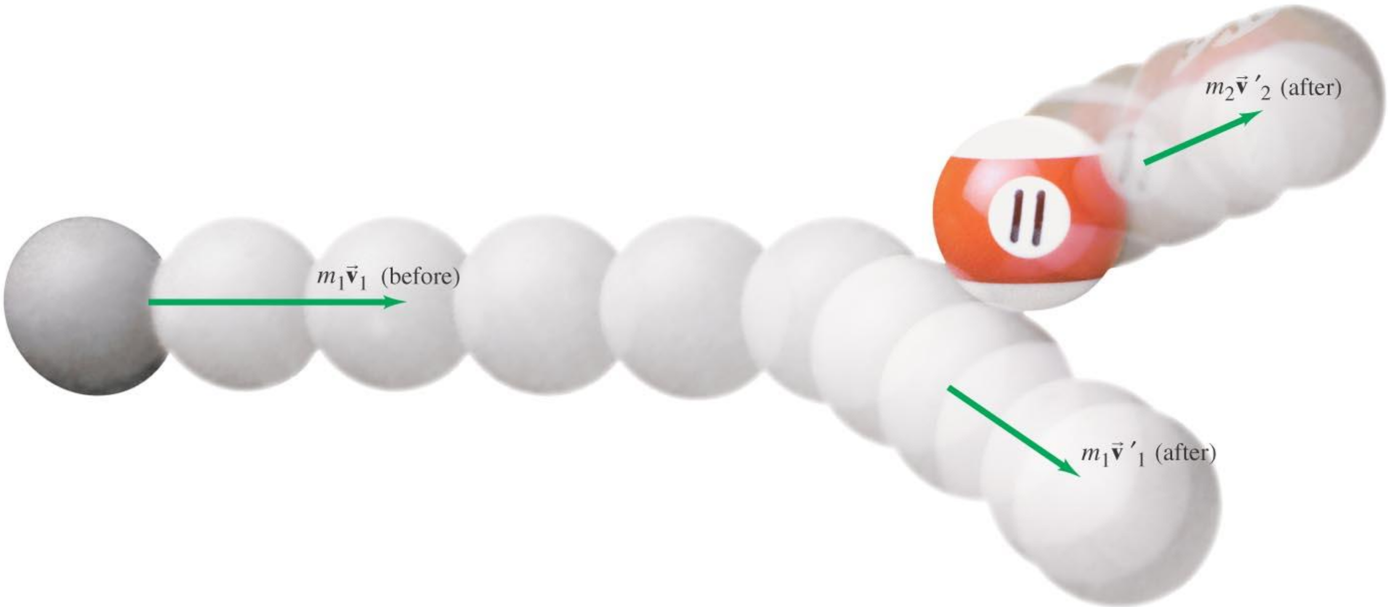
$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$

- We can derive the familiar form of the second law, $\sum \vec{F} = m\vec{a}$, from the above equation for the case of constant mass.

- If $\vec{v}$ is the velocity of an object at any moment, then the above equation gives

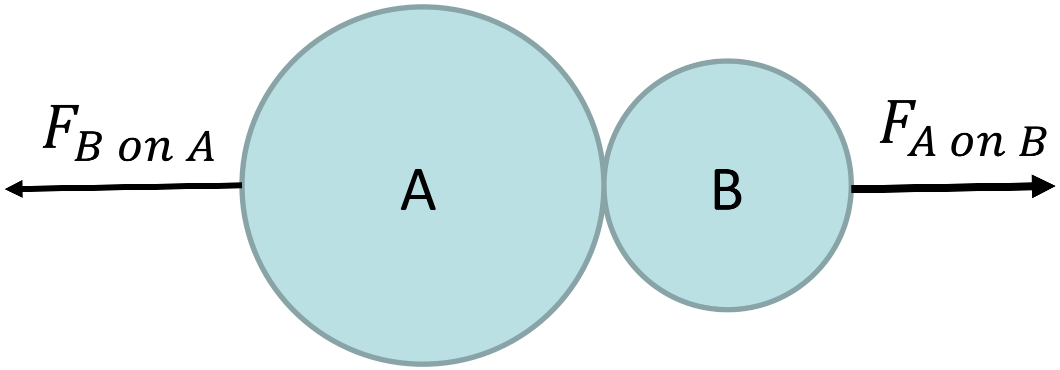
$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Conservation of Momentum



- The law of the conservation of momentum is essentially a reworking of Newton's laws.
- We use the laws of conservation of linear momentum and of energy to analyse collisions.

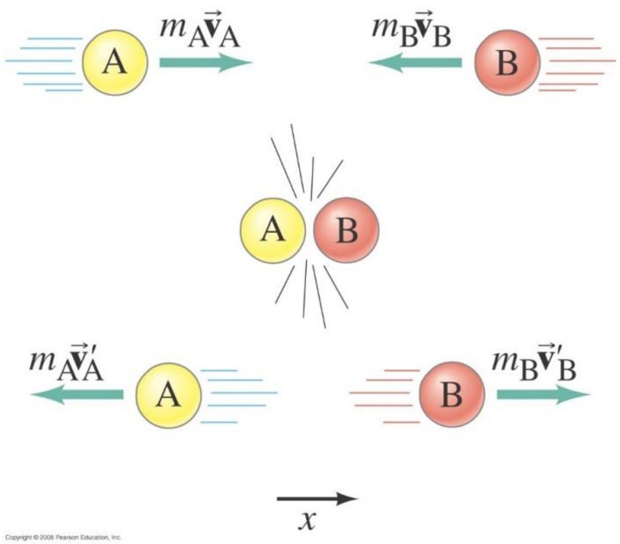
Conservation of Momentum



- Let's think of any interaction, between two objects, A and B. From Newton's 3rd Law, the force A feels from B is of equal magnitude to the force B feels from A, but in an opposite direction.

- Since force equals rate of change of momentum, it follows that throughout the interaction process, the rate of change of momentum of A is exactly opposite to the rate of change of momentum of B.

Conservation of Momentum

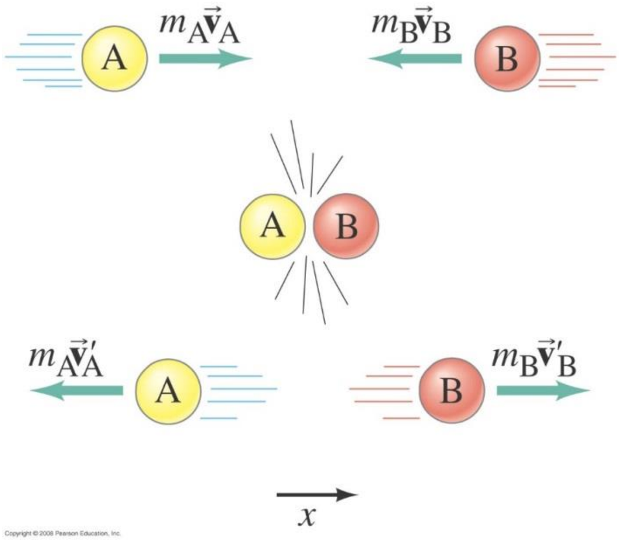


$$\text{momentum before} = \text{momentum after}$$

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

- If no external force acts on a system, the total momentum of the system is a conserved quantity.
- If we consider, for example, the head-on collision of two billiard balls, we can assume that the net external force on this system of two balls is zero– that is, the only significant forces during the collision are the forces that each ball exerts on the other.

Conservation of Momentum



$$\text{momentum before} = \text{momentum after}$$

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

- Our derivation of the conservation of momentum can be extended to include any number of interacting objects.
- Let $\vec{P}$ represent the total momentum of a system of n interacting objects which we number from 1 to n:

$$\vec{P} = m_1\vec{v}_1 + \ m_2\vec{v}_2 + \cdots + m_n\vec{v}_n = \sum \vec{p}_i$$

Conservation of Momentum

- Let $\vec{P}$ represent the total momentum of a system of n interacting objects which we number from 1 to n:

$$\vec{P} = m_1\vec{v}_1 + \ m_2\vec{v}_2 + \cdots + m_n\vec{v}_n = \sum \vec{p}_i$$

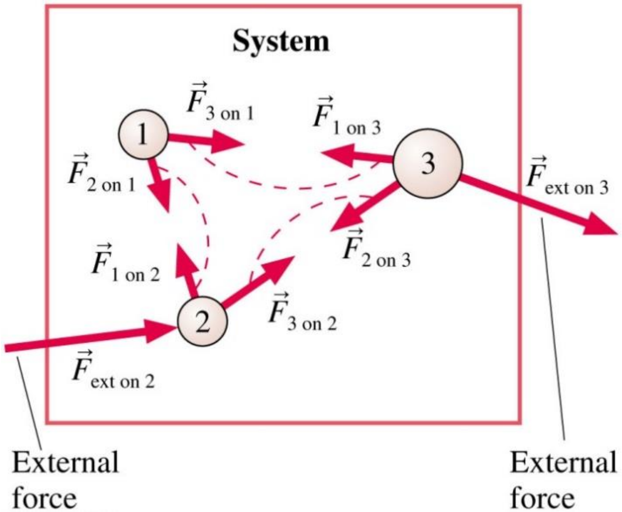
- We can differentiate with respect to time:

$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$

where $\vec{F}_i$ represents the net force on the i^{th}object.

Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$



where $\vec{F}_i$ represents the net force on the i^{th} object.

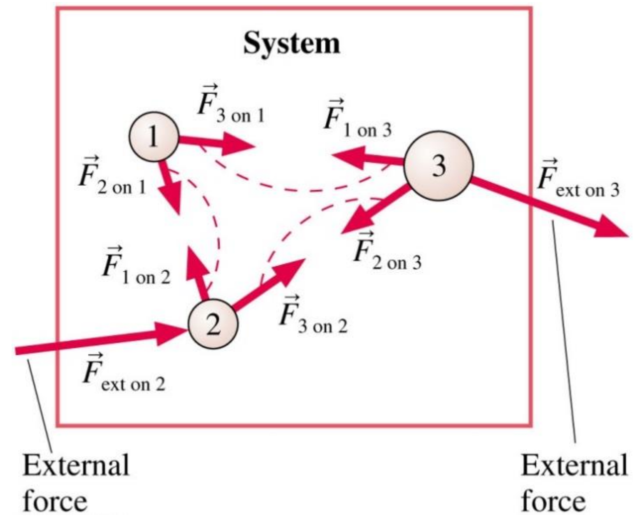
- The forces acting on the objects can be of two types:

(1) Internal forces that objects within the system exert on other objects in the system; or

(2) External forces on objects of the system, exerted by objects outside the system.

Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$



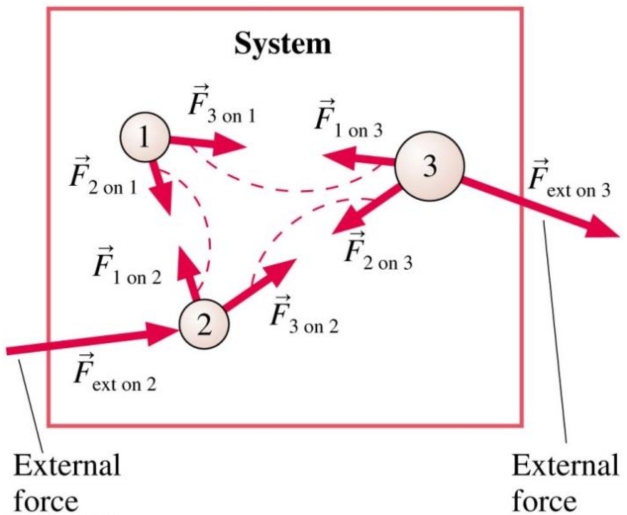
(1) Internal forces that objects within the system exert on other objects in the system.

- By Newton's third law, the internal forces occur in pairs: if one object exerts a force on a second object, the second exerts an equal and opposite force on the first object.

- Thus, all the internal forces cancel each other in pairs, meaning that **a change in momentum can only occur as a result of external forces.**

Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$



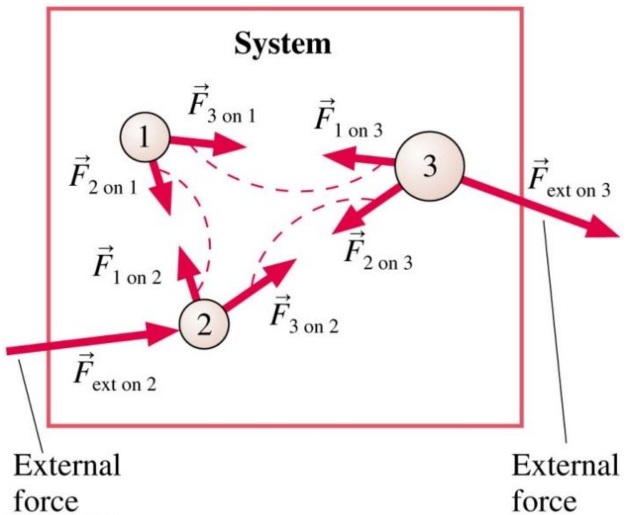
- Thus, we have

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$

- When the net external force on a system of objects is zero, the total momentum of the system remains constant. This is the law of conservation of momentum.

Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$



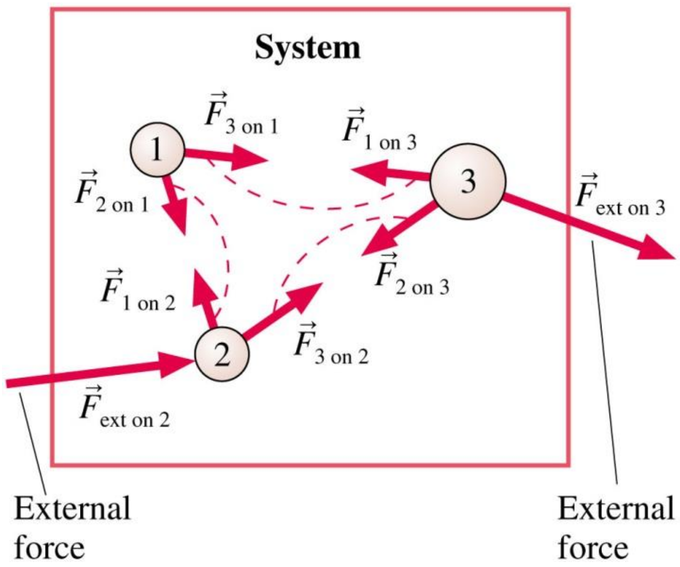
- We can also state the law as

The total momentum of an isolated system of objects remains constant,

where an isolated system means one in which no external forces act—the only forces acting are those between objects of the system.

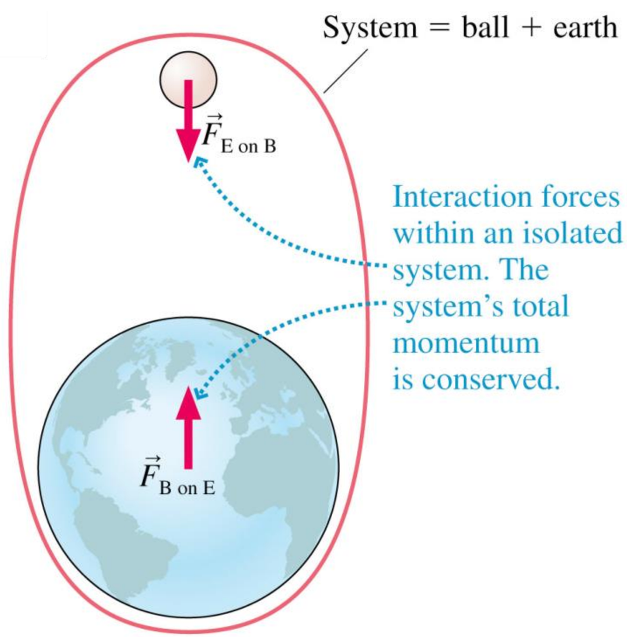
Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$



- If a net external force acts on a system, then the law of conservation of momentum will not apply. However, if the 'system' can be redefined so as to include the other objects exerting these forces, then the conservation of momentum principle can apply. Let's look at an example.

Conservation of Momentum: Defining the System



- A rubber ball is dropped, and falls toward Earth.
- Now let's define the ball + Earth as the system.
- The gravitational forces are interactions within the system.
- This is now an isolated system, so the total momentum is conserved.

$$\vec{p} = \vec{p}_{ball} + \vec{p}_{Earth}$$

Conservation of Momentum

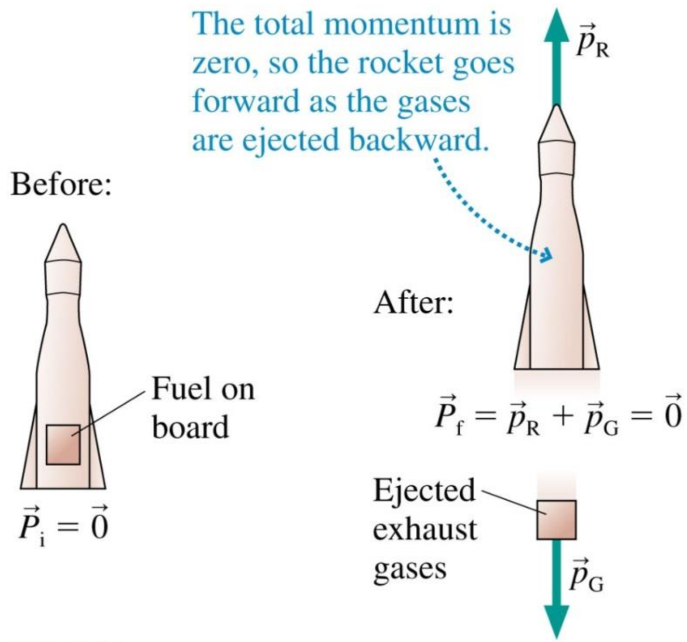
$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$

- The law of conservation of momentum is particularly useful when we are dealing with fairly simple systems such as colliding objects and certain types of explosions.

- For example, rocket propulsion can be understood on the basis of action and reaction, and can also be explained on the basis of the conservation of momentum.

- We can consider the rocket and fuel as an isolated system if it is far out in space (no external forces).

Conservation of Momentum



- The figure shows a rocket with a parcel of fuel on board.
- If we choose the rocket + gases to be the system, the burning and expulsion are both internal forces.
- The exhaust gases gain backward momentum as they are shot-out the back.
- The rockets gains equal momentum, but in the opposite direction.
- Therefore, the total momentum is zero.

Summary Thus Far

$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$

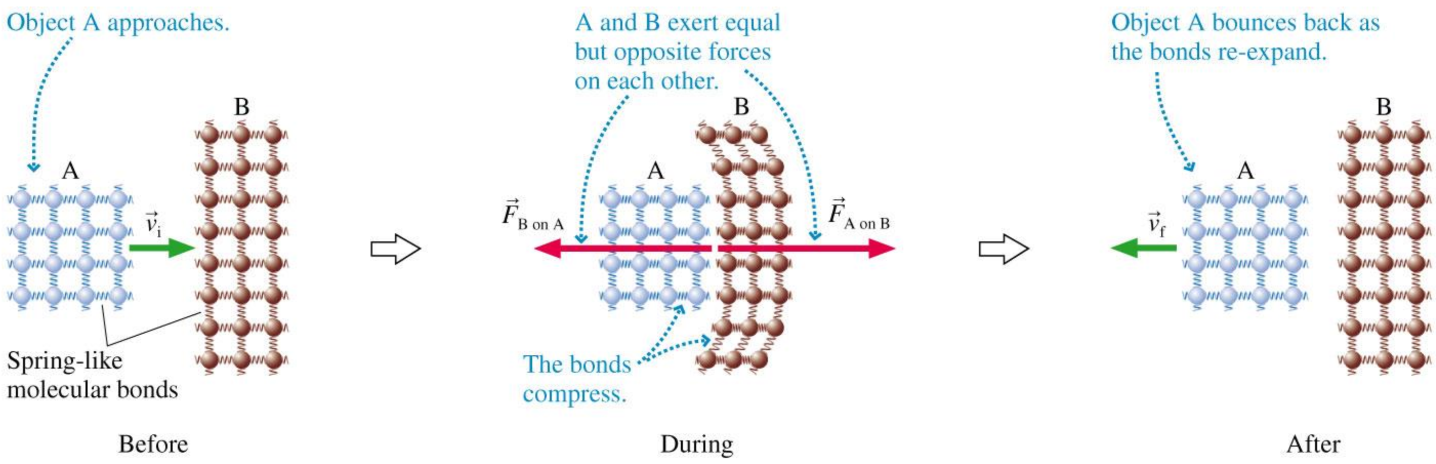
- Thus far, we have seen that the rate of change of momentum of an object is equal to the net force applied to it, and can be described using the above equation.
- Clearly, this rate of change of momentum occurs when an object interacts with another object, each object exerting an equal and opposite force on each other, and each object affecting each other's rate of change of momentum.
- If, however, we manipulate the above equation into the following,

$$\sum \vec{F} dt = d\vec{p}$$

we see that each object's change in momentum depends on the force as well as the length of time the force acts on each object; we call this **impulse**; let's study it in more detail.

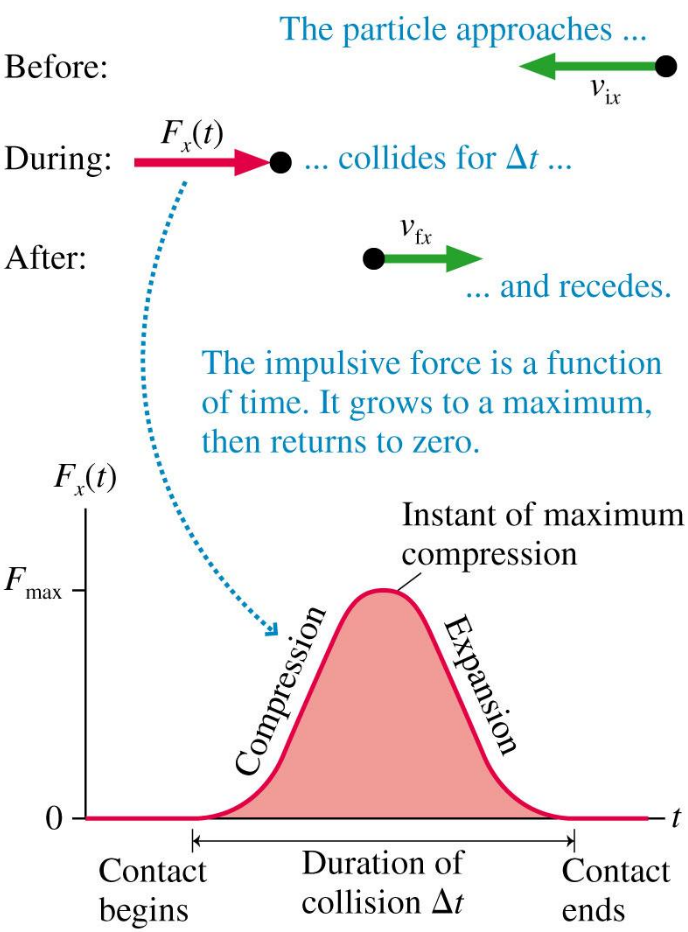
7. Impulse

Impulse



- During a collision of two ordinary objects, both objects act on each other, causing molecular bonds to compress/deform, if we view these molecular bonds as like springs.
- If the material of each object is different, then the compression (deformation) for each object will be different.
- However, during the collision, the force that each object exerts on the other will be the same, but the acceleration of each object will be different if they have a different mass and/or are made from different materials.

Impulse

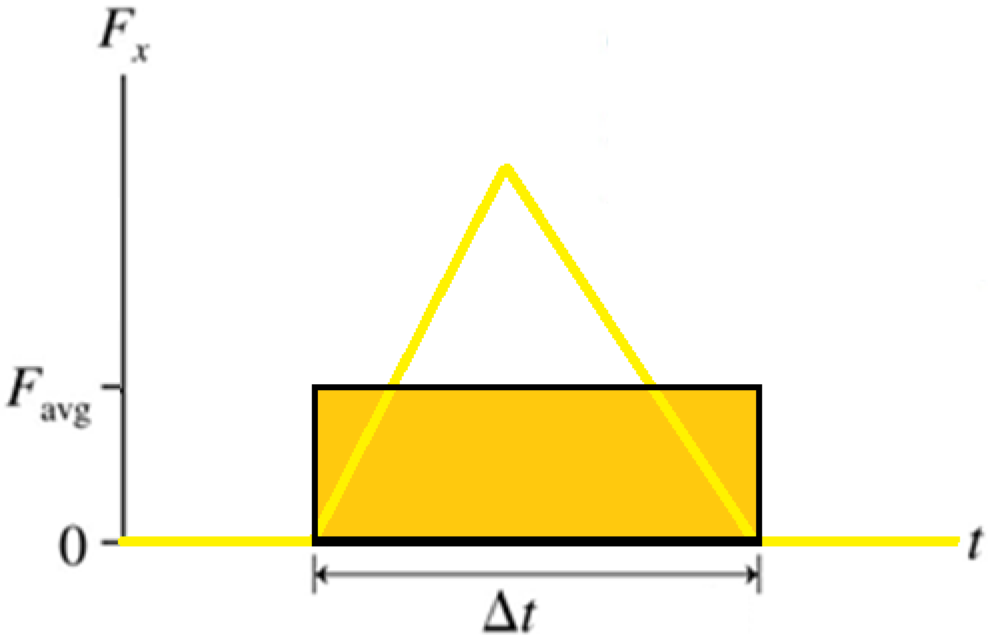


- The force usually jumps from zero at the moment of contact to a very large value within a short period of time, and then quickly returns to zero again, something like what is shown above.

Crumple Zones

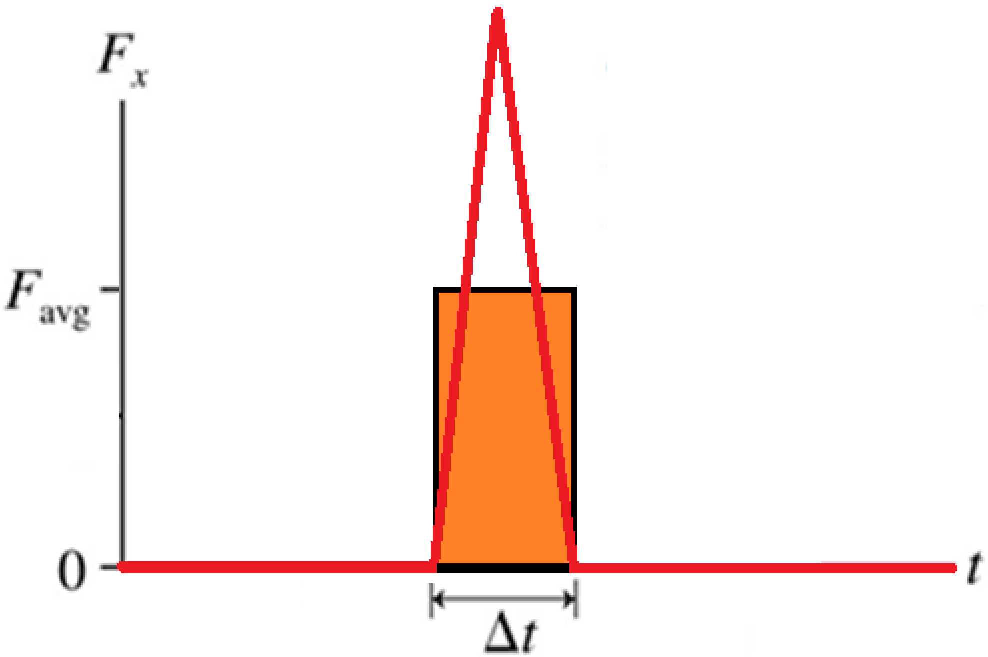
- Crumple zones make use of the idea of impulse. In the above, the yellow metal is weaker than the red metal.

Crumple Zones



- Think of the atoms of metal as being like springs: if the car hits a wall, the yellow metal springs are more easily compressed (small spring constant), taking longer to exert an equal and opposite force. This lessens the average impact force, also absorbing some of the energy of the collision.

Crumple Zones



- Think of the atoms of metal as being like springs: on the other hand, for the **red metal**, you can think of the springs as having a higher spring constant, not compressing as much (or absorbing the collision much). This can be a good thing, as you don't want this region of the car to crumple.

Home Work

Do not forget to read the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.

Lecture 5: Optional Reading

- **Ch. 7.1**, Work Done by a Constant Force; p.194-197
- **Ch. 7.2**, Work Done by a Varying Force; p.198-200
- **Ch. 7.3**, Work Done by a Spring Force; p.200-201
- **Ch. 7.4**, Kinetic Energy and the Work-Energy Principle; p.202-206
- **Ch. 8.1**, Conservative and Non-conservative Forces; p.218-219
- **Ch. 8.2**, Potential Energy; p.221-222
- **Ch. 8.4**, Conservation of Mechanical Energy; p.224-229
- **Ch. 8.6**, Energy Conservation with Dissipative Forces; p.230-232
- **Ch. 9.1**, Momentum and Its Relation to Force; p.253-254
- **Ch. 9.2**, Conservation of Momentum; p.255-258
- **Ch. 9.3**, Collisions and Impulse; p.258-259