The University of Nottingham
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Science A Physics

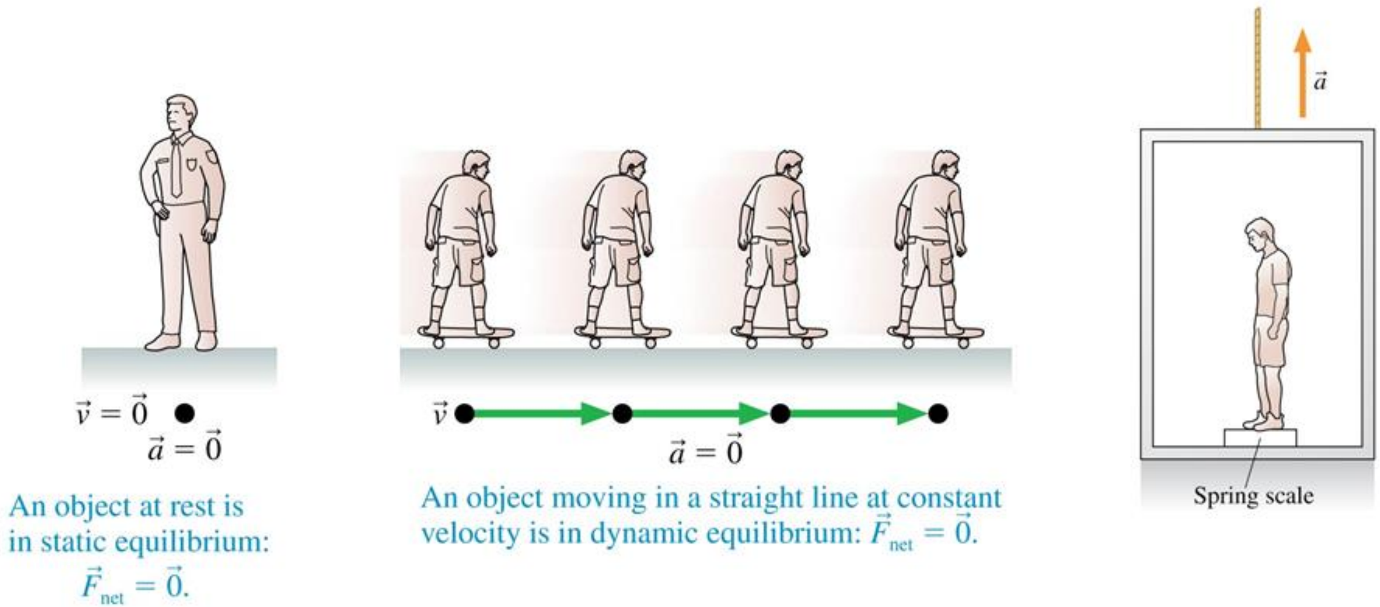
Lecture 4:

Free-body Diagrams

Aims of today's lecture

1. Free-body diagrams
2. Weight
3. Static friction
4. Kinetic friction
5. Modelling joined objects

Analysing the Forces Acting on an Object

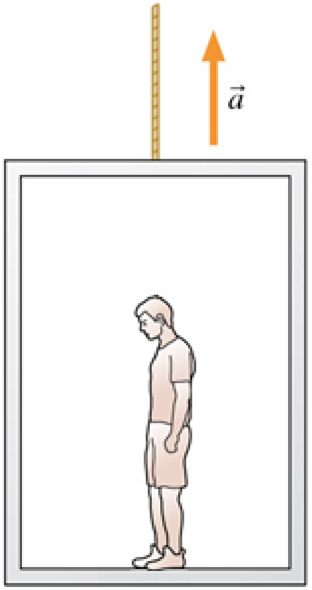


- In our first three lectures, we seen how we can describe motion using terms such as ‘displacement’, ‘velocity’ and ‘acceleration’. We also have seen how we can describe this motion in terms of equations (kinematics), and what it is that causes motion, namely force.

- In this lecture, we look at how we analyse the forces acting on an object. To do so, we use a technique called **free-body diagrams**.

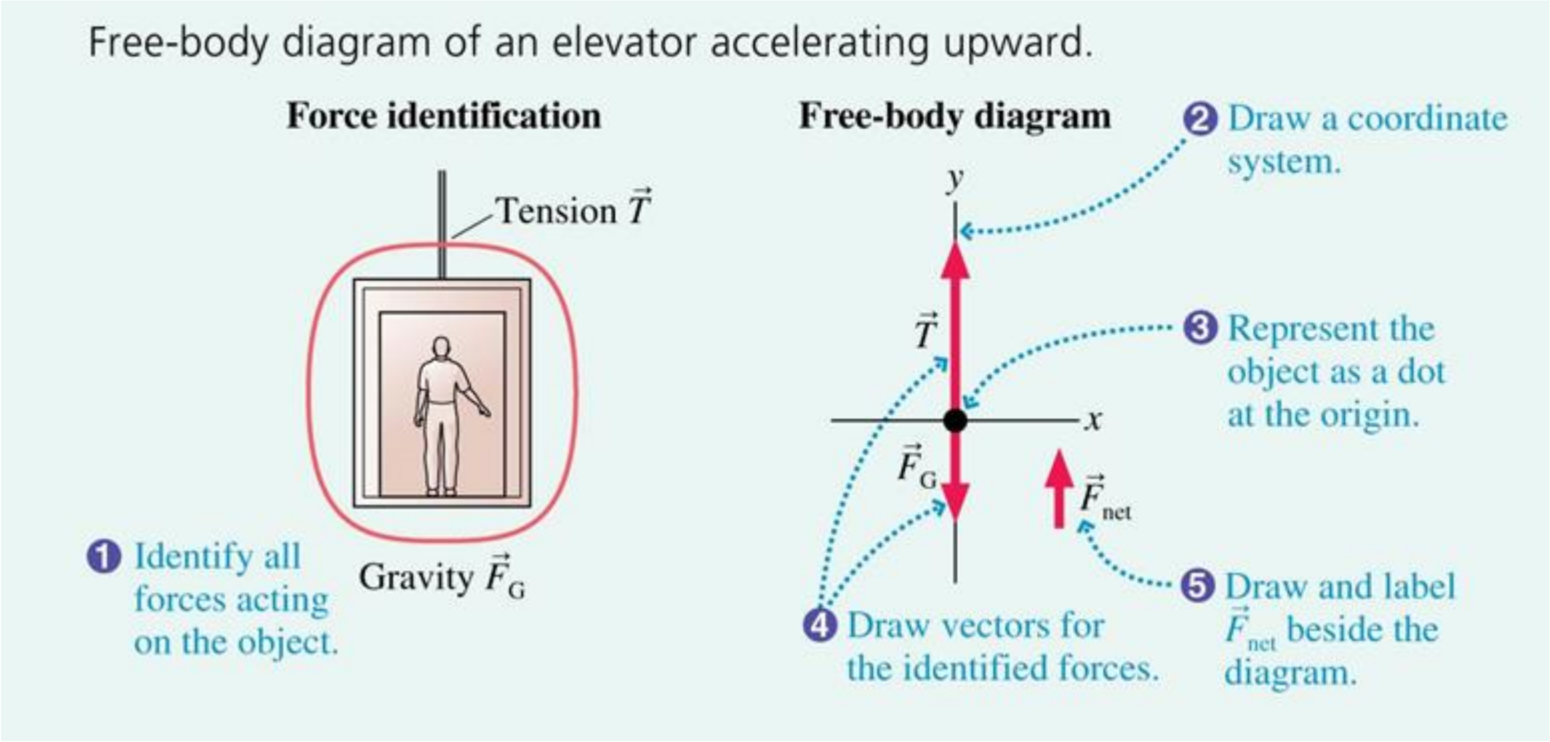
1. Free-body diagrams

Free-body diagrams



- Consider an elevator suspended by a cable, speeding up as it moves upward from the ground floor. How do we identify the forces and draw a free-body diagram of the elevator?

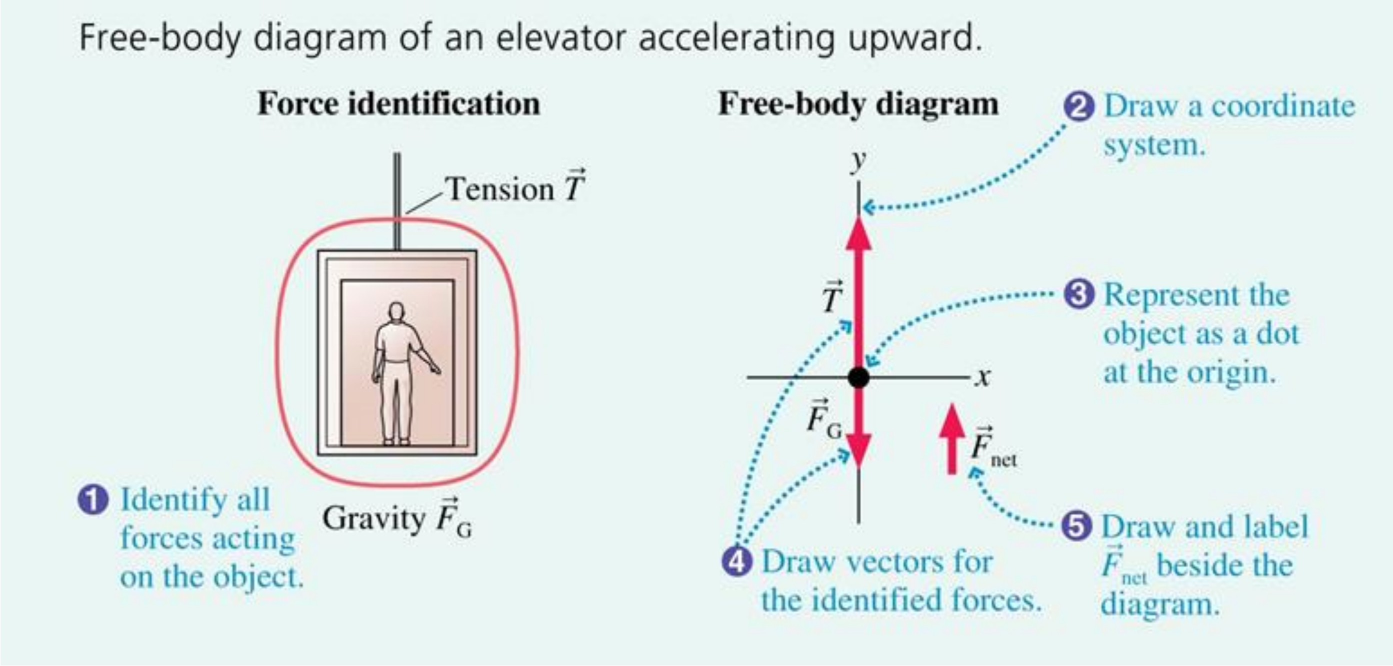
Free-body diagrams



MODEL:

Treat the elevator as a particle.

Free-body diagrams

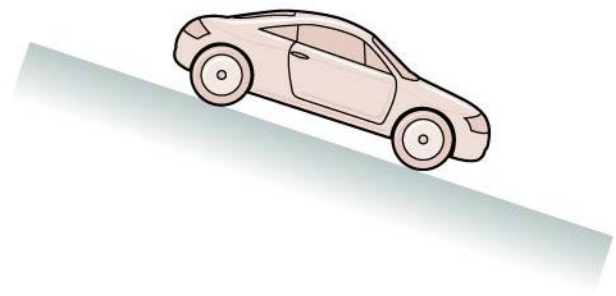


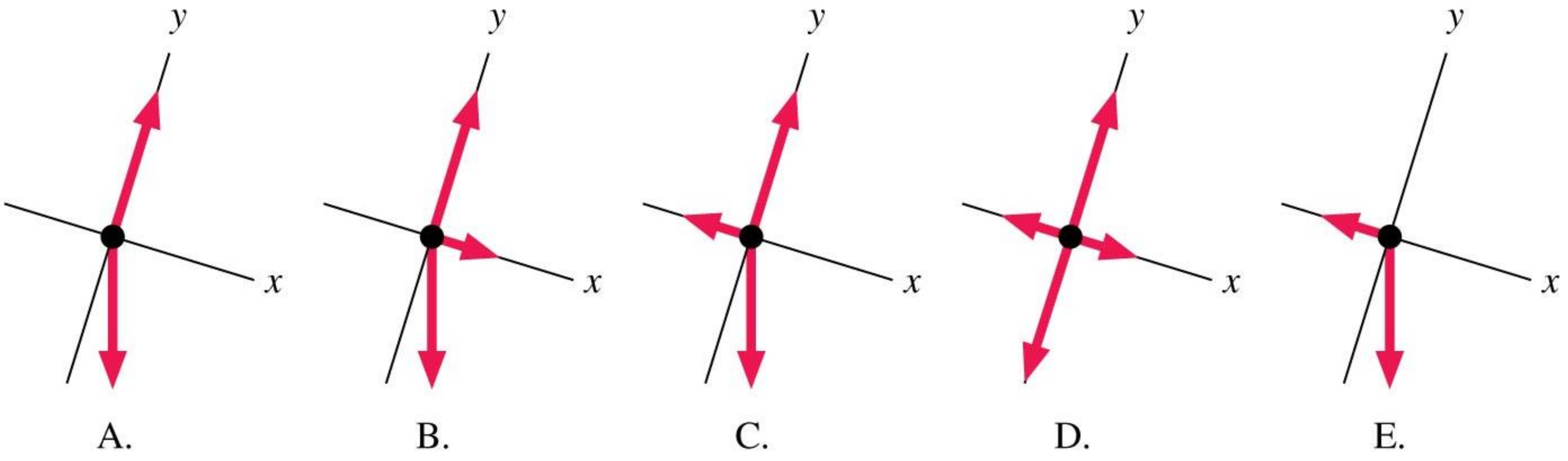
ASSESS:

The coordinate axes, with a vertical y-axis, are the ones we would use in a pictorial representation of the motion. The elevator is accelerating upward, so $\vec{F}_{net}$ must point upward. For this to be true, the magnitude of $\vec{T}$ must be larger than the magnitude of $\vec{F}_G$.

Have a Think: A Static Equilibrium Problem

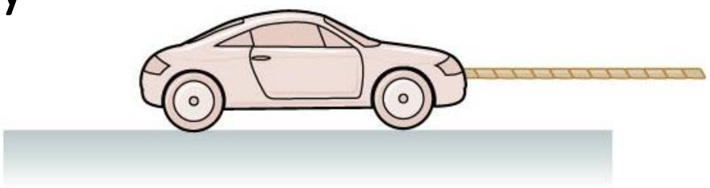
Q.1 A car is parked on a hill. Which is the correct free-body diagram?

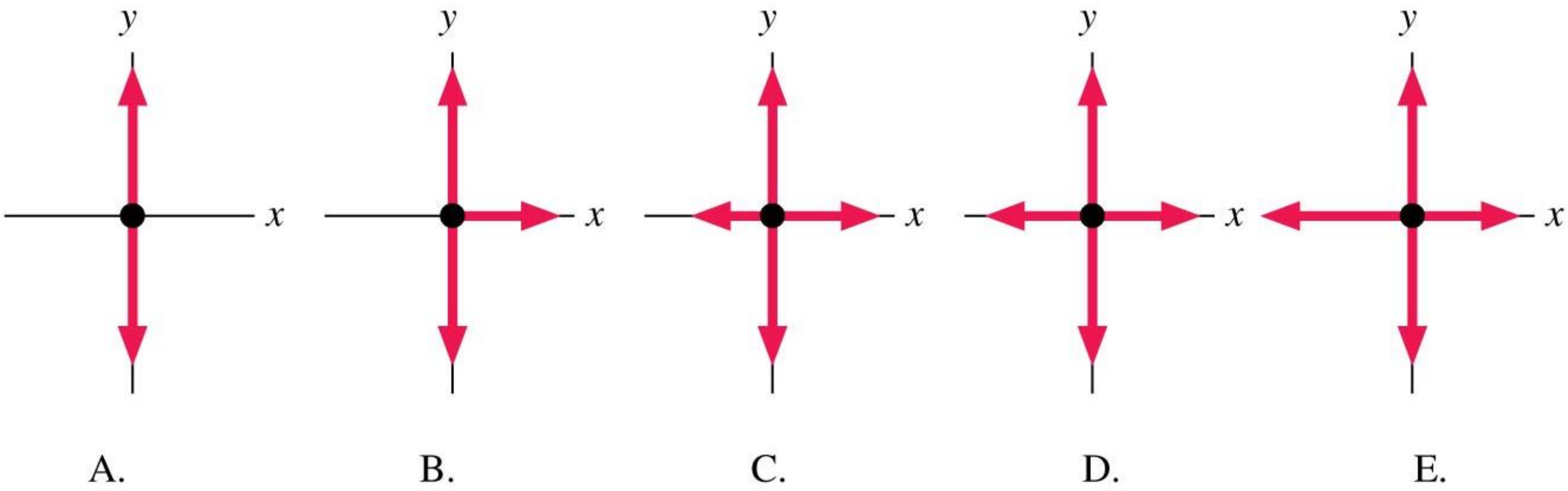




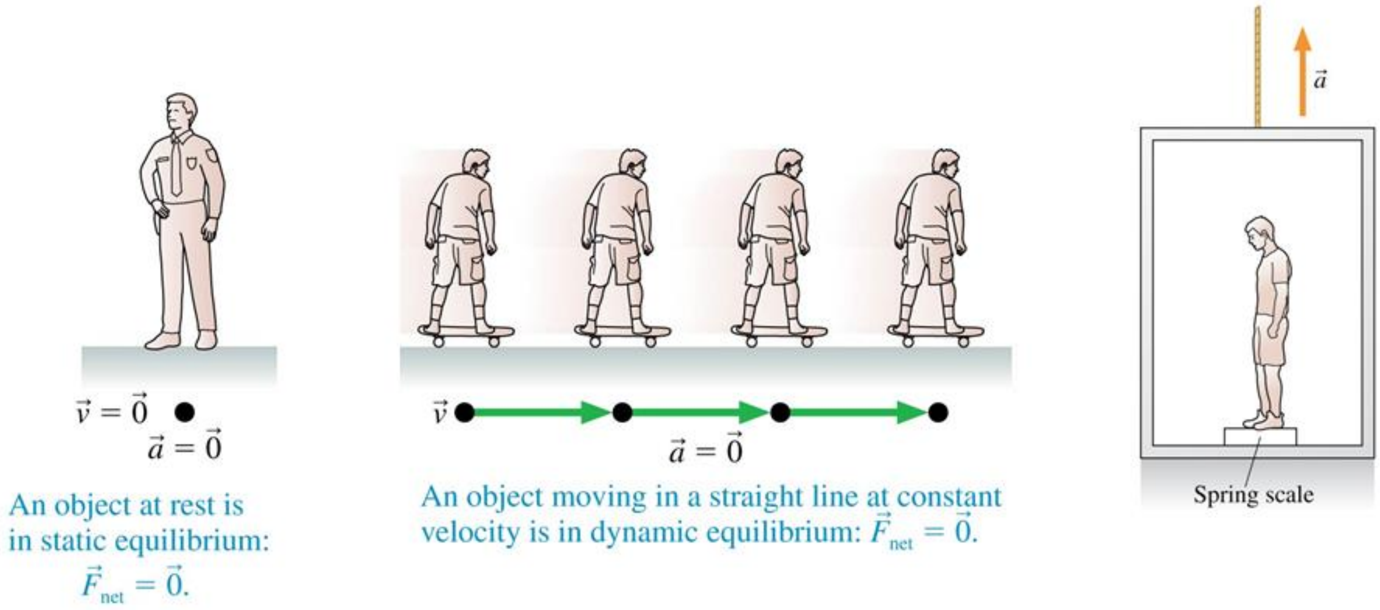
Have a Think: A Dynamics Equilibrium Problem

Q.2 A car is towed to the right at constant speed. Which is the correct free-body diagram?



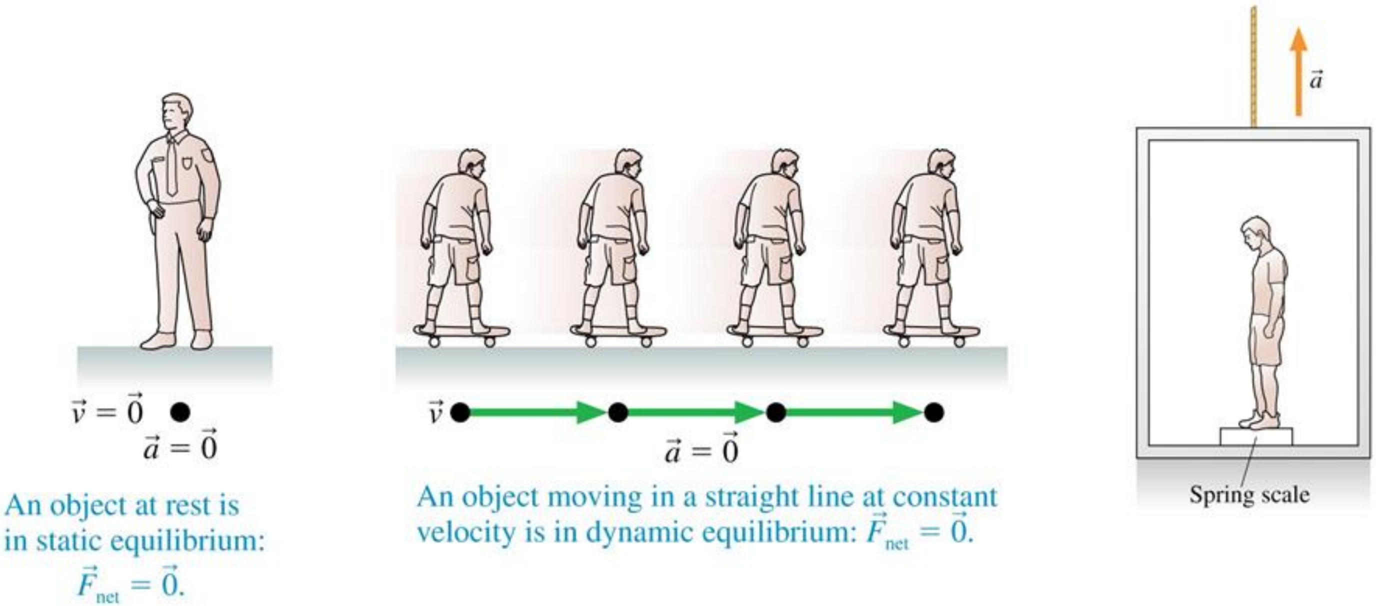


Analysing the Forces Acting on an Object



- Having now looked at how we 'free' an object from its surroundings to analyse the forces acting on it, let's now look at some more contexts in which we can use free-body diagrams.

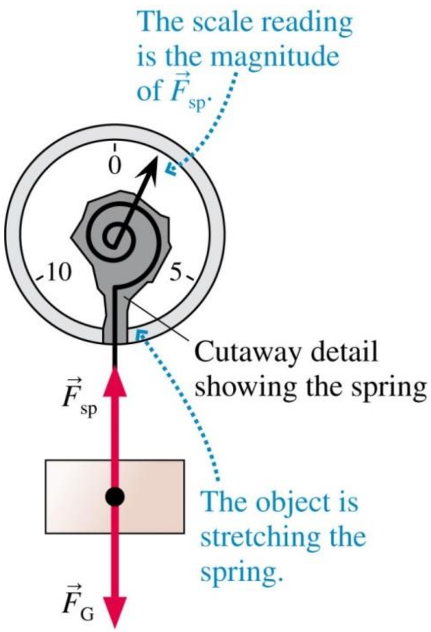
Analysing the Forces Acting on an Object



- Many of these contexts involve the idea of **weight**, so let's look at weight in a bit more detail. We will see that when we talk about weighing an object, we are really determining the force that gravity exerts on the object.

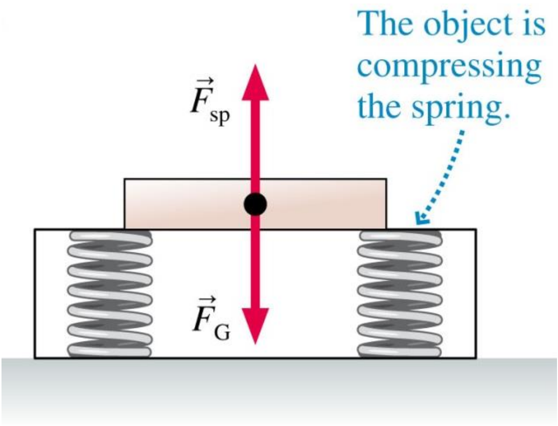
2. Weight

Weight



- You can weigh apples in a grocery store by placing them on a pan which stretches a spring.
- The reading on the spring scale is the magnitude of F_{sp}.
- We define the **weight** of an object as the reading for F_{sp} when this force (measured in Newtons) is balanced by the force of gravity; in other words, when the object is stationary.

Weight

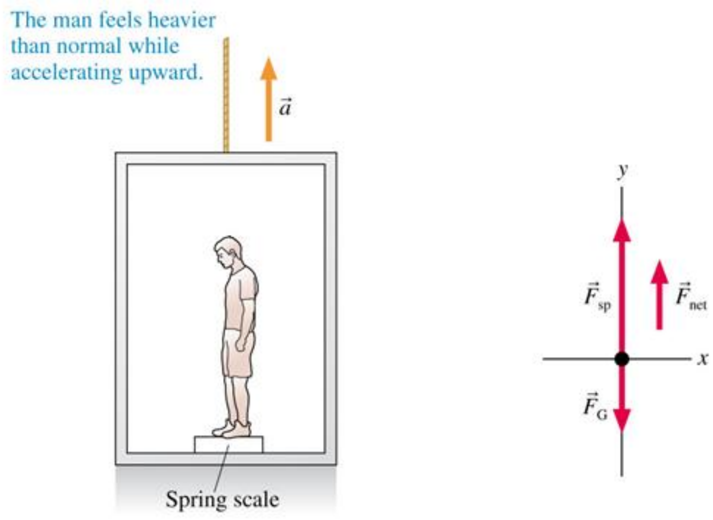


- Similarly, a bathroom scale uses compressed springs attached to a calibrated scale to give us a measurement for the weight of an object.
- When the object is stationary, the upward spring force exactly balances the downward gravitational force of magnitude mg:

$$F_{sp} = F_G = mg$$

- Let's now consider putting a weighing scale inside an elevator.

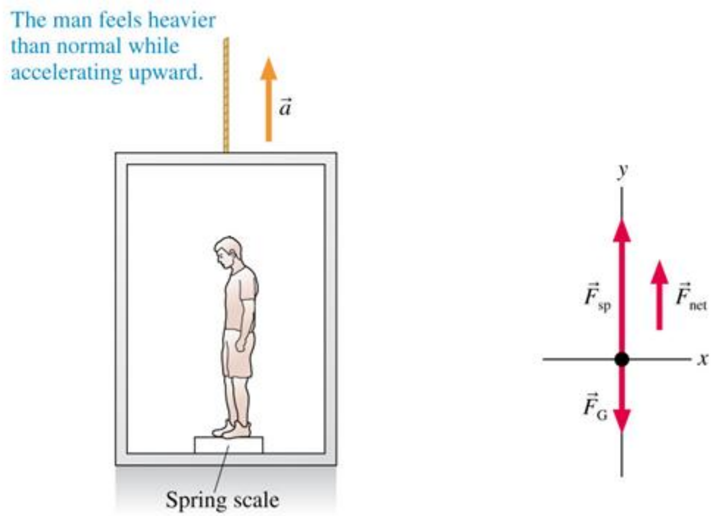
Weight



- The figure shows a man weighing himself in an accelerating elevator.
- Looking at the free-body diagram, the y-component of Newton's second law is:

$$(F_{net})_y = (F_{sp})_y + (F_G)_y = F_{sp} - mg = ma_y$$

Weight



- The man's weight as he accelerates vertically is:

$$w = \text{scale reading for } F_{sp} = mg + ma_y = mg\left(1 + \frac{a_y}{g}\right)$$

=> You weigh **more** as an elevator accelerates upward!

Weightlessness

- If an object is accelerating downward with $a_y = -g$, then $w = 0$.

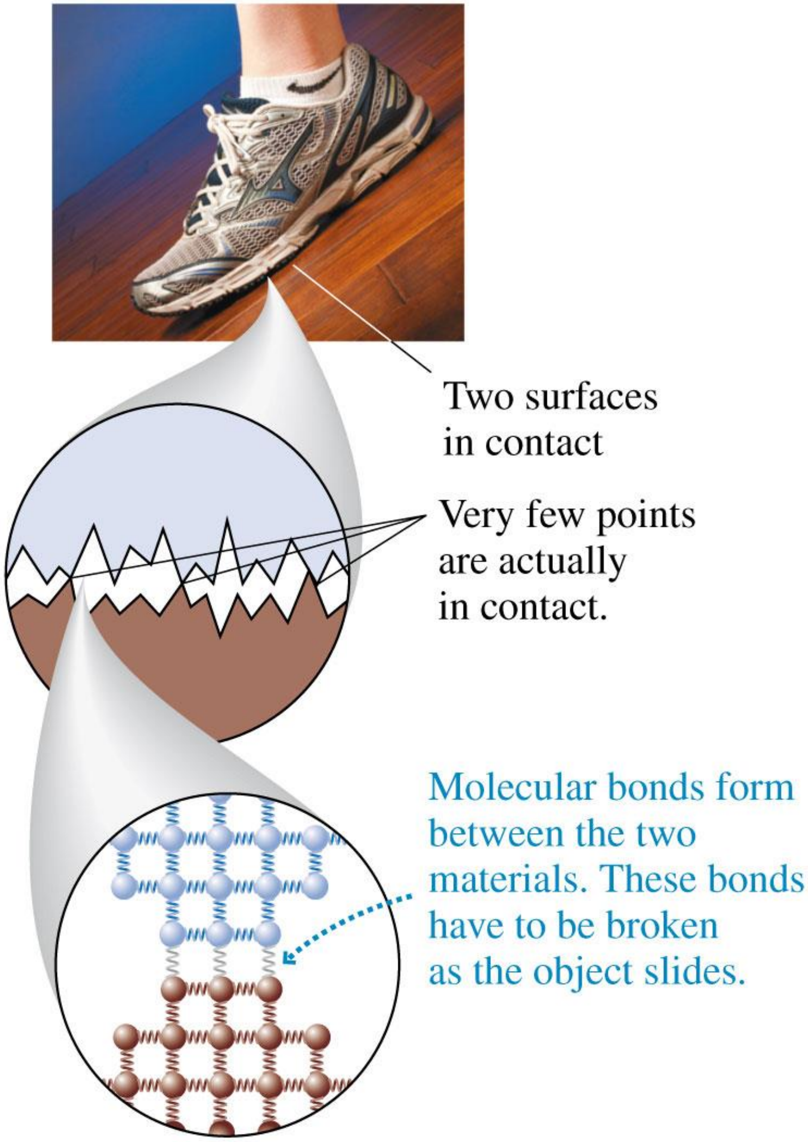
=> An object in free-fall has no weight!

$$w = \text{scale reading for } F_{sp} = mg + ma_y = mg\left(1 + \frac{a_y}{g}\right)$$

- The next context that we consider applying free-body diagrams to is that of **static friction**.

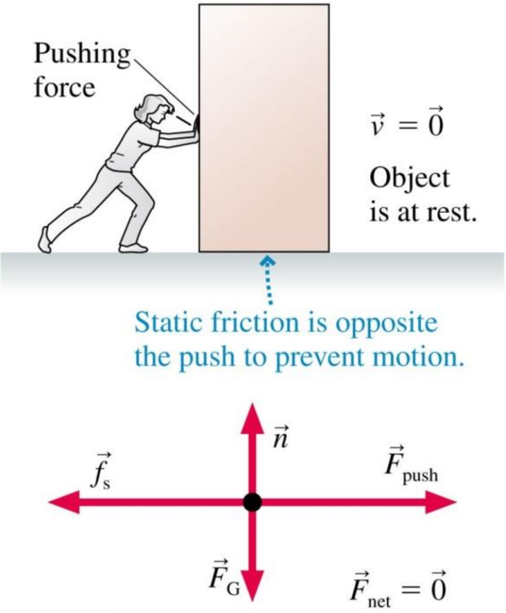
3. Static Friction

Static Friction



- All surfaces are very rough on a microscopic scale.
- When two surfaces are pressed together, the high points on each side come into contact and form molecular bonds.
- These bonds can produce a force **tangent** to the surface, called the **static friction** force.

Static Friction



- The figure shows a person pushing on a box that, due to static friction, isn't moving.
- Because of Newton's first law, the static friction force must exactly balance the pushing force:

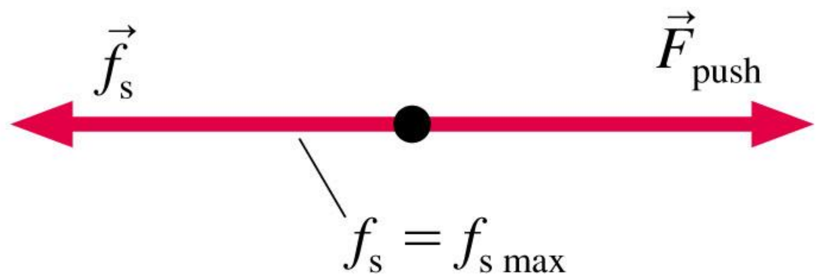
$$f_s = F_{push}$$

- $\vec{f}_s$ points in the direction opposite to the way the object would move if there was no static friction.

Static friction acts in response to an applied force.

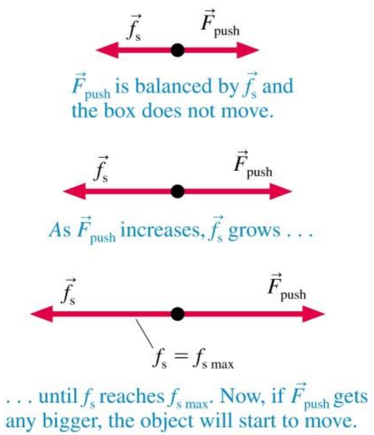
$\vec{F}_{push}$ is balanced by $\vec{f}_s$ and the box does not move.

As $\vec{F}_{push}$ increases, $\vec{f}_s$ grows . . .



. . . until f_s reaches $f_{s\,max}$. Now, if $\vec{F}_{push}$ gets any bigger, the object will start to move.

Static Friction

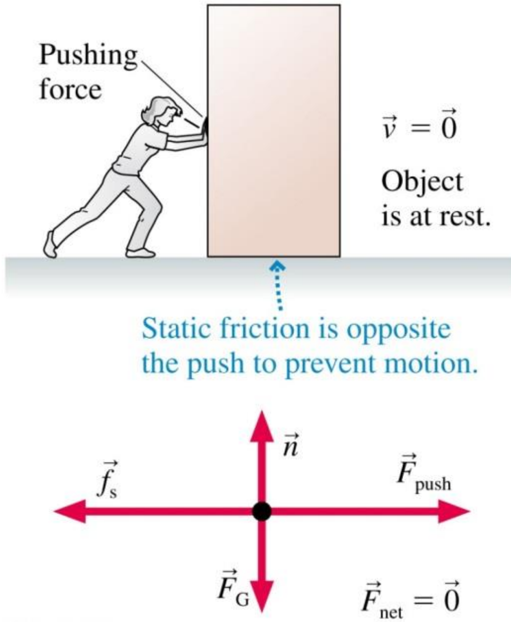


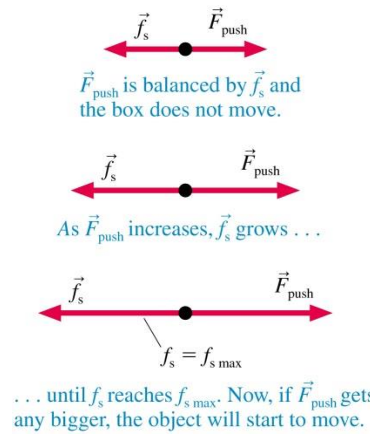
- Static friction force has a maximum possible size $f_{s\ \mathrm{max}}$.
- An object remains at rest as long as $f_s < f_{s,\ \mathrm{max}}$.
- The object just begins to slip when $f_s = f_{s,\,max}$.
- A static friction force $f_s > f_{s,\,max}$ is not physically possible.

$$f_{s,\,max} = \mu_s n$$

where the proportionality constant μ_s is called the **coefficient of static friction**.

Static Friction

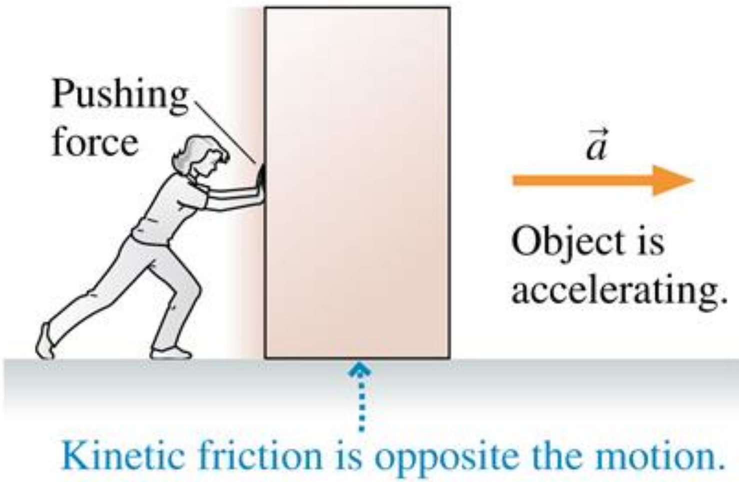


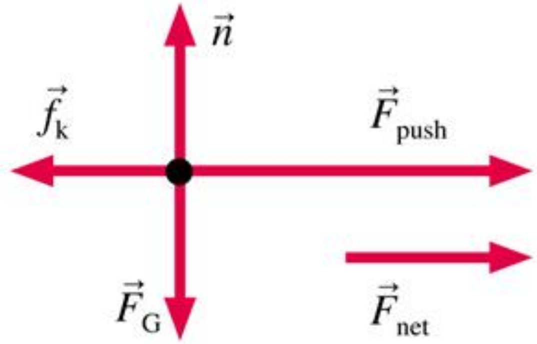


- Once we overcome the static friction, the object will begin to move, but does the friction between the object and the ground disappear? Not quite.
- There is still friction, albeit it is less; we call such friction **kinetic friction**.

4. Kinetic Friction

Kinetic Friction



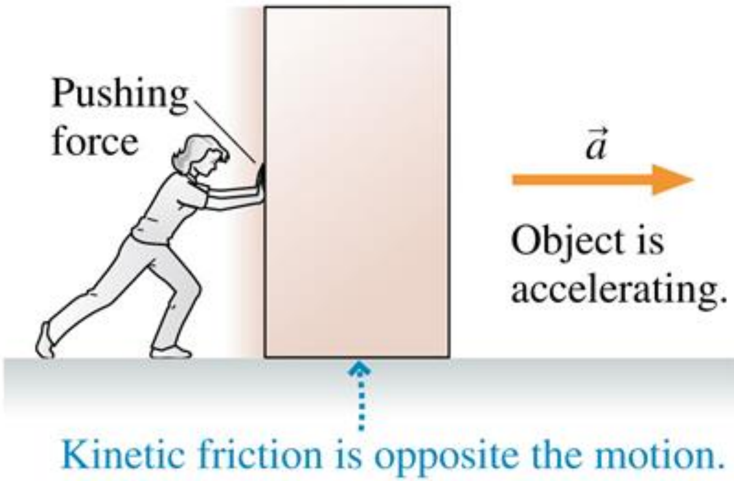


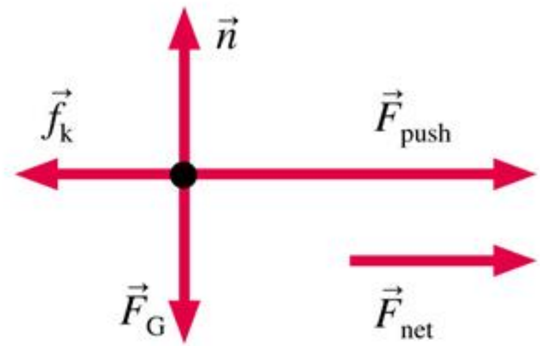
- The **kinetic friction** force is proportional to the magnitude of the normal force:

$$f_k = \mu_k n$$

where the proportionality constant μ_k is called the **coefficient of kinetic friction**.

Kinetic Friction





- The kinetic friction direction is opposite to the velocity of the object relative to the surface.

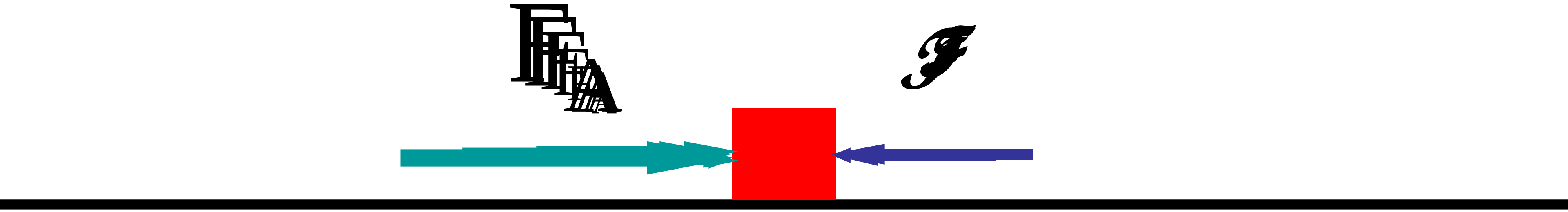
- For any particular pair of surfaces, $\mu_k < \mu_s$.

Coefficients of Static friction vs Coefficients of Kinetic Friction

Materials	Static μ_s	Kinetic μ_k
Rubber on concrete	1.00	0.80
Steel on steel (dry)	0.80	0.60
Steel on steel (lubricated)	0.10	0.05
Wood on wood	0.50	0.20
Wood on snow	0.12	0.06
Ice on ice	0.10	0.03

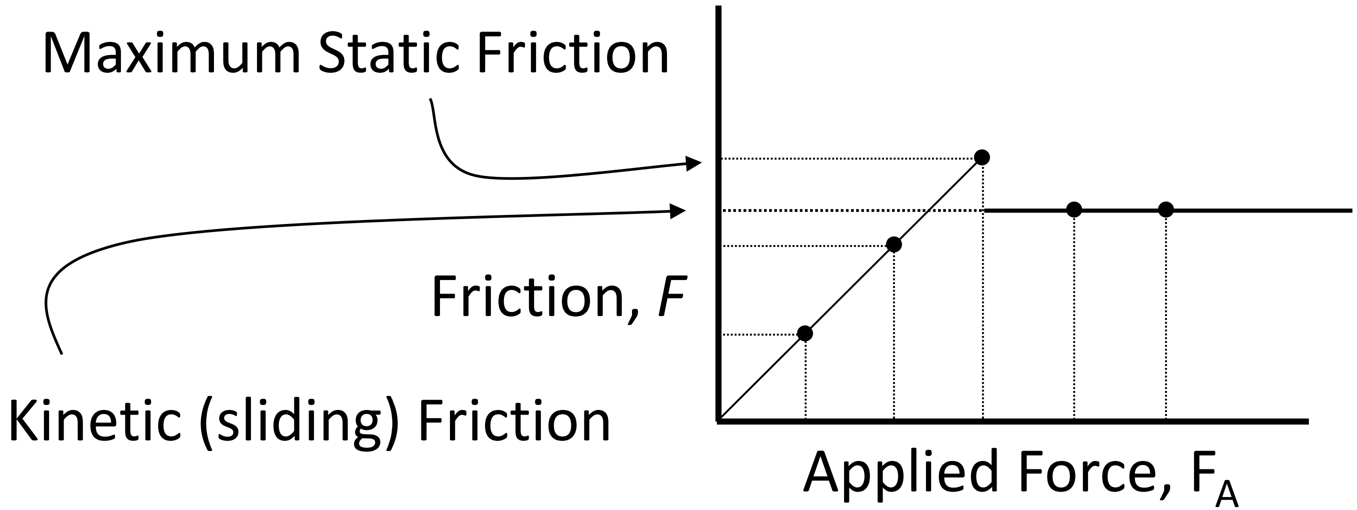
- As you can see for different surfaces, the coefficient of kinetic friction is always less than the coefficient of static friction.
- Let's now summarise, by way of a graph, the friction force response to an increasing applied force on an object.

The Friction Force Response to an Increasing Applied Force

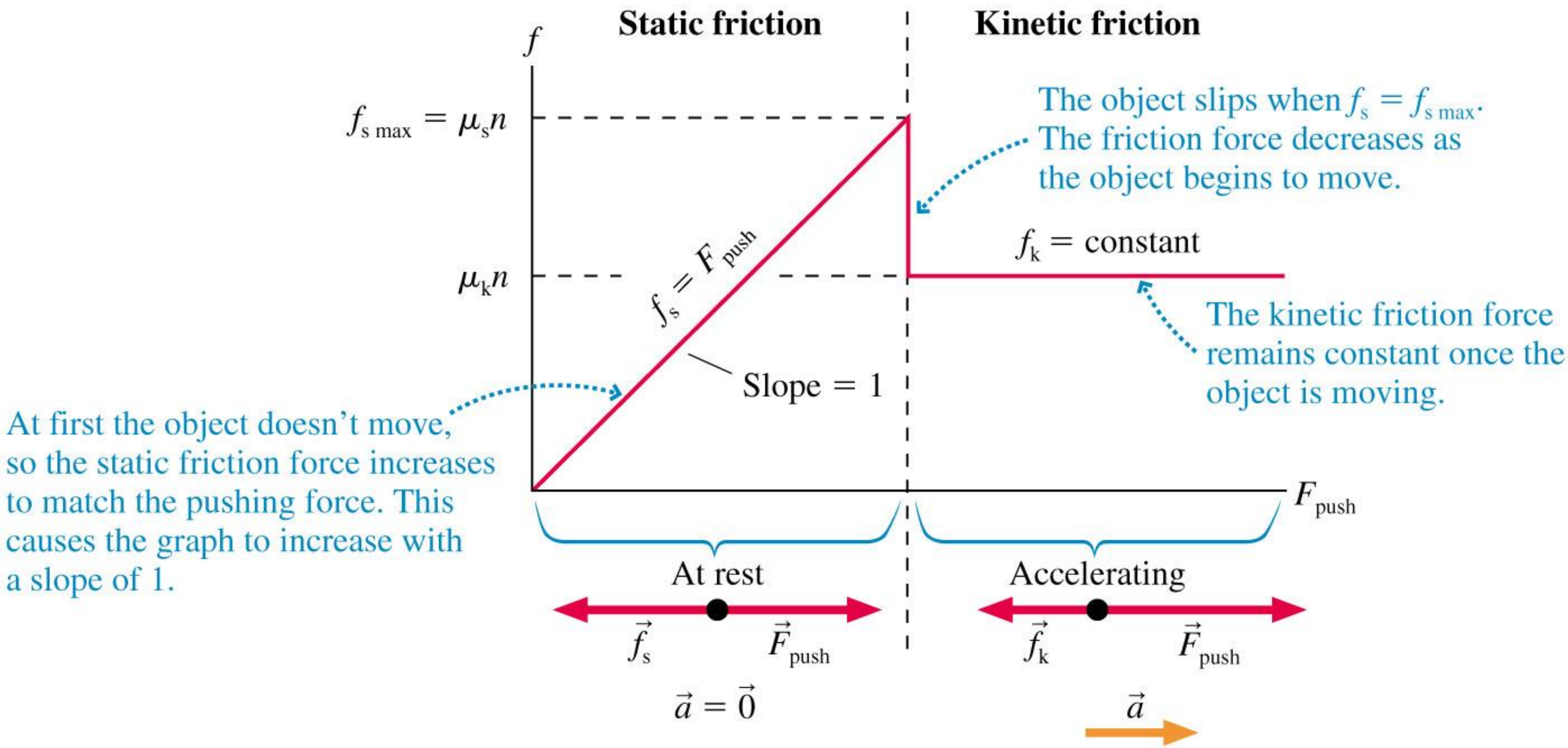


On the verge of slipping

Sliding



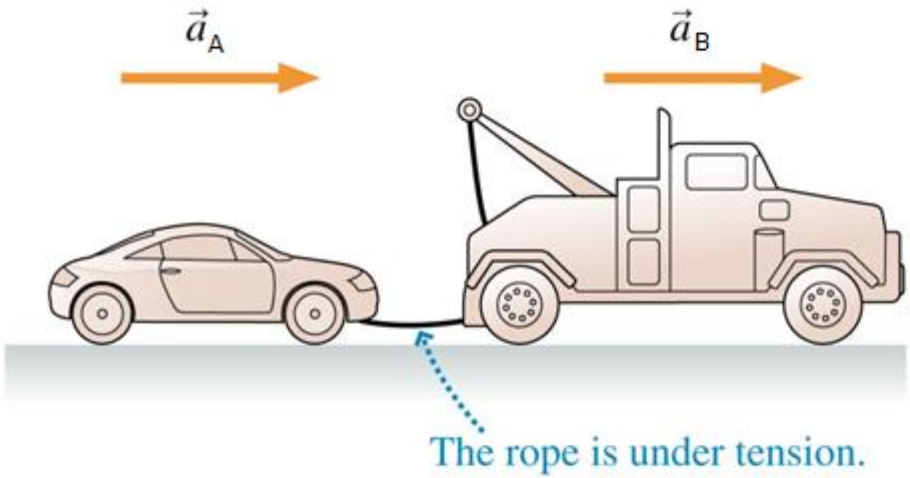
The Friction Force Response to an Increasing Applied Force



- We are now going to talk about how we model (using free-body diagrams) objects that are joined together (via ropes or strings) and are moving.

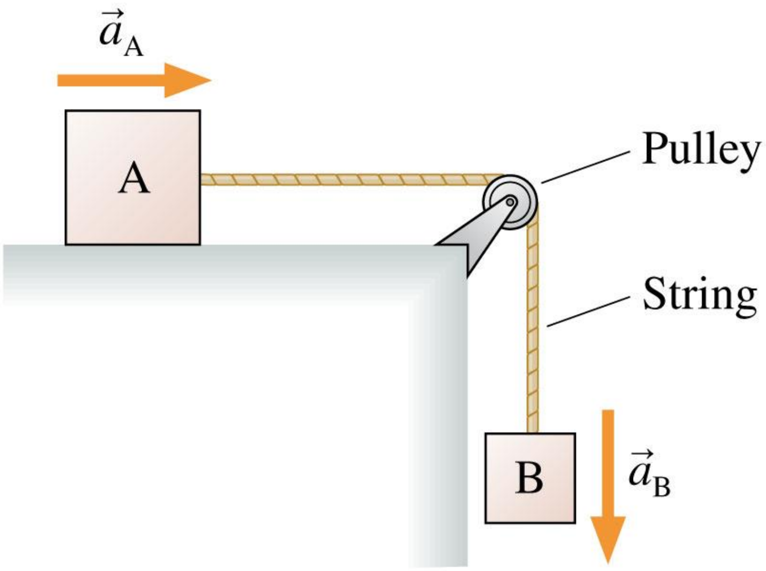
5. Modelling Joined Objects

Modelling Joined Objects



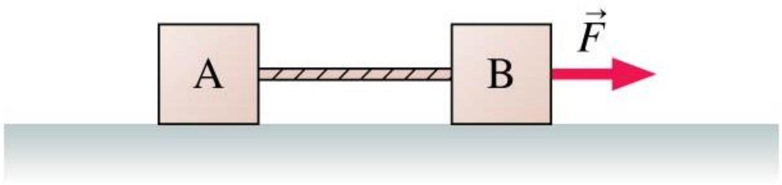
- If two objects, such as a car (A) and a truck (B) are joined to each other and accelerate together, we say that both objects have the same acceleration: $a_A = a_B$.
- Because the accelerations of both objects are equal, we can drop the subscripts A and B, and call both of them a_x .

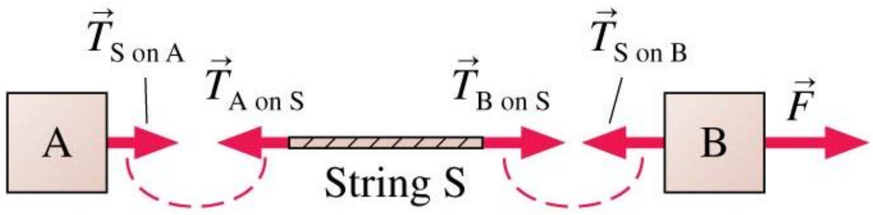
Modelling Joined Objects



- Sometimes the acceleration of A and B may have different signs.
- For the two blocks A and B, because they are joined by a string, they both experience the same acceleration when acted on by gravity.
- But, as A moves to the right in the $+x$ direction, B moves down in the $-y$ direction.
- In this case, the acceleration is $a_{Ax} = -a_{By}$.

Modelling Joined Objects

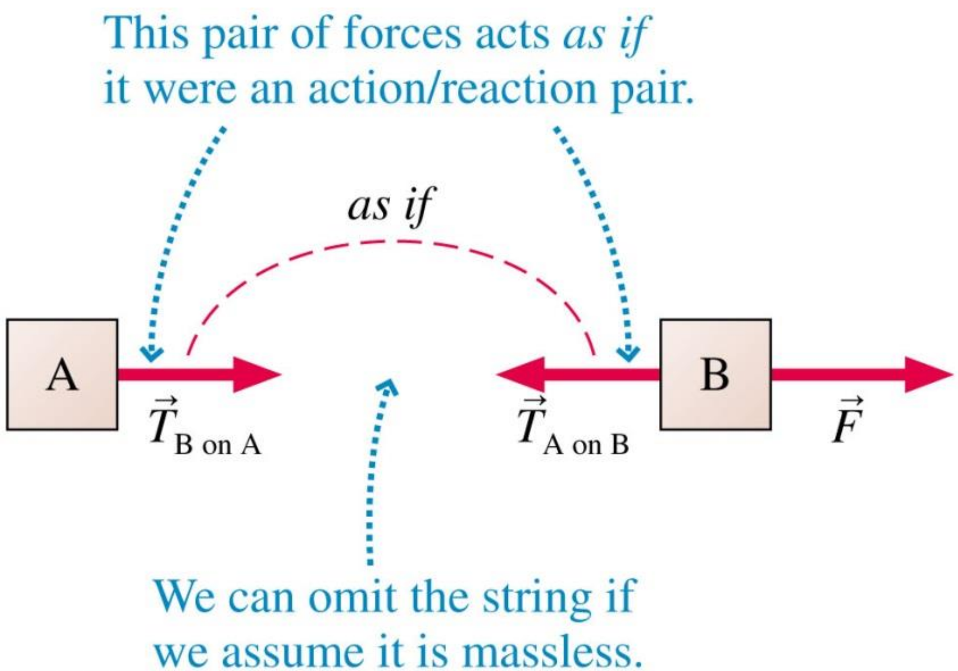




- Often in problems, the mass of the string or rope is much less than the masses of the objects that it connects.
- In such cases, we can adopt the following **massless string approximation**:

$$T_{\text{B on S}} = T_{\text{A on S}} \qquad \text{(massless string approximation)}$$

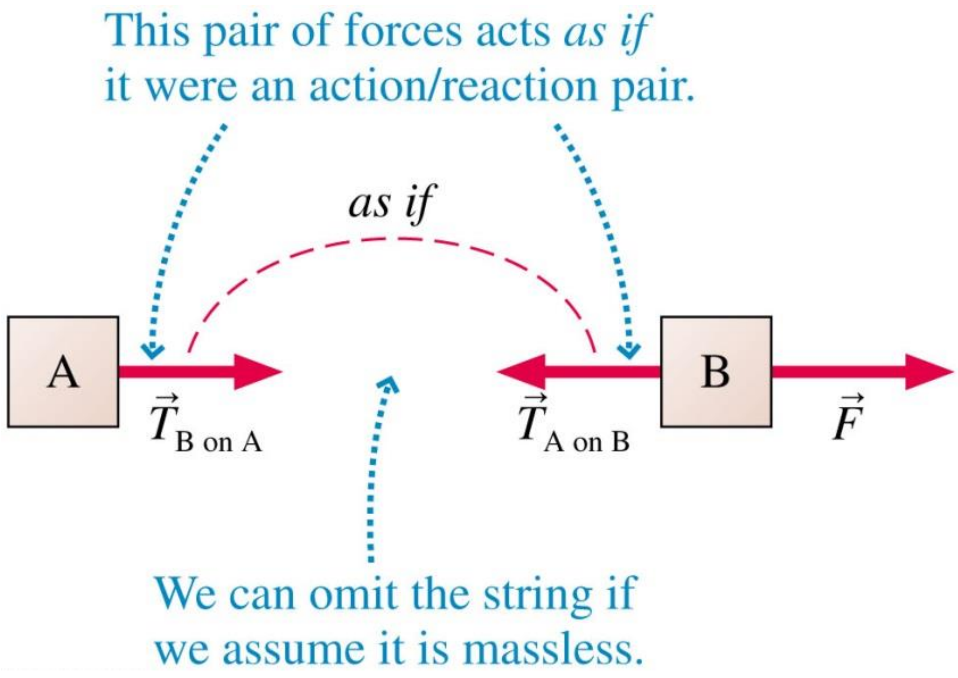
Modelling Joined Objects



- Two blocks are connected by a massless string, as block B is pulled to the right.
- Forces $\vec{T}_{S\ on\ A}$ and $\vec{T}_{S\ on\ B}$ act ***as if*** they are an action/reaction pair:

$$\vec{T}_{S\ on\ A} = -\vec{T}_{S\ on\ B}$$

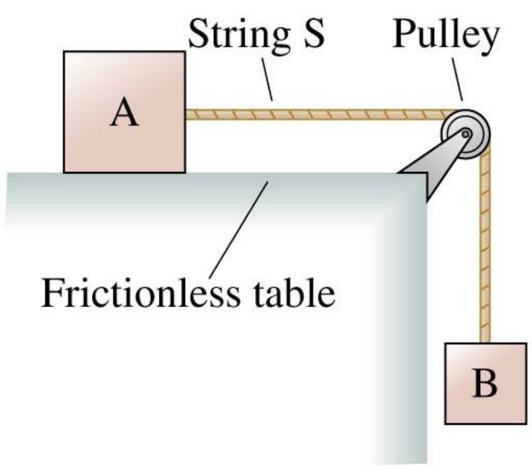
Modelling Joined Objects

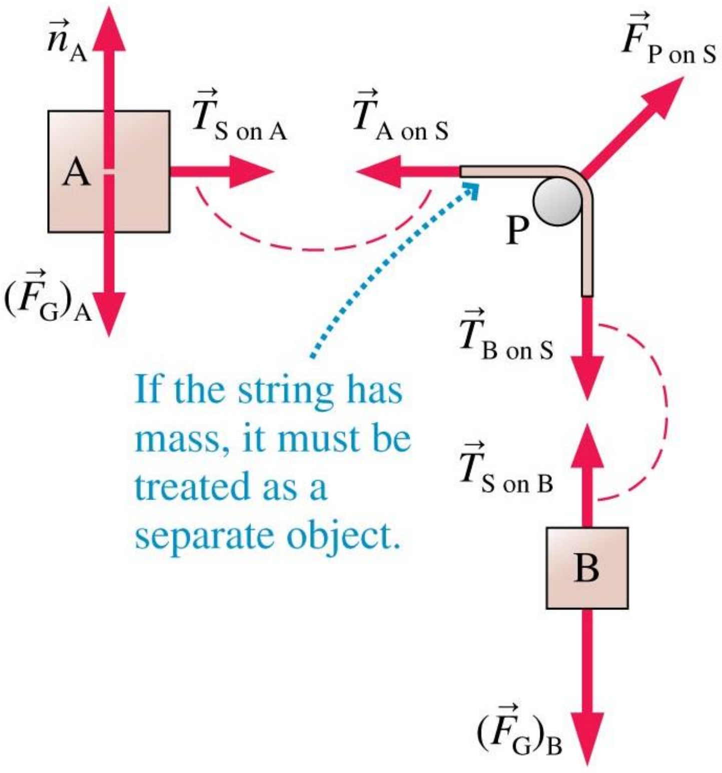


$$\vec{T}_{S\ on\ A} = -\vec{T}_{S\ on\ B}$$

- All a massless string does is transmit a force from A to B without changing the magnitude of that force.
- For problems in this module, you can assume that any strings or ropes are massless unless it explicitly states otherwise.

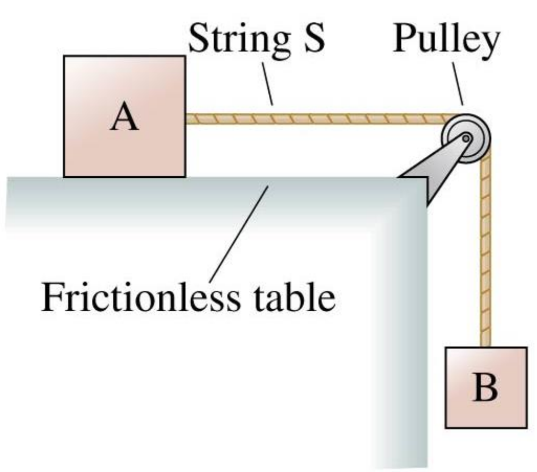
Modelling Joined Objects

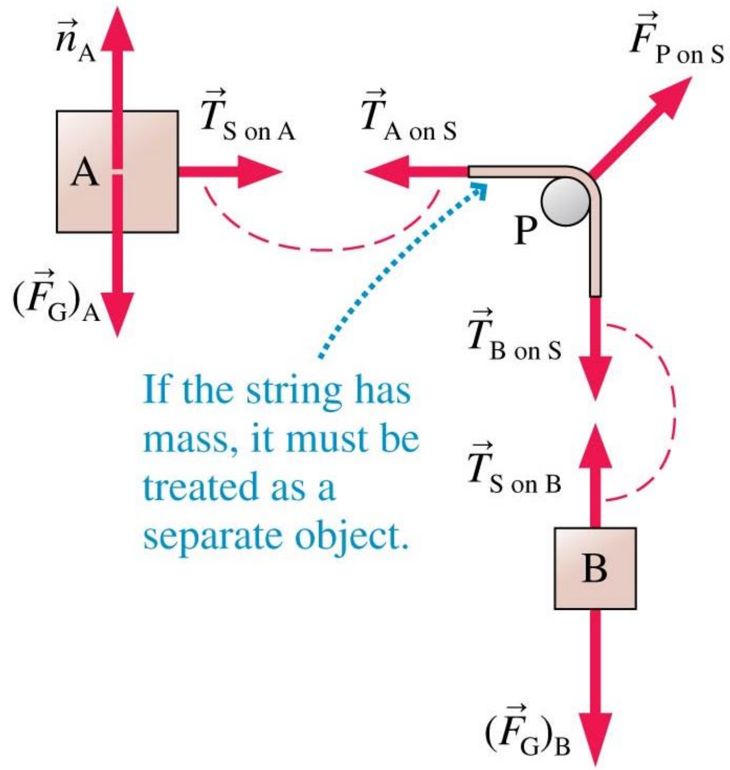




- Block B drags block A across a frictionless table as it falls.
- We assume that the string and the pulley are both massless.

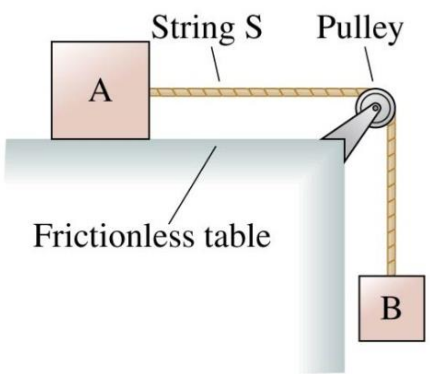
Modelling Joined Objects

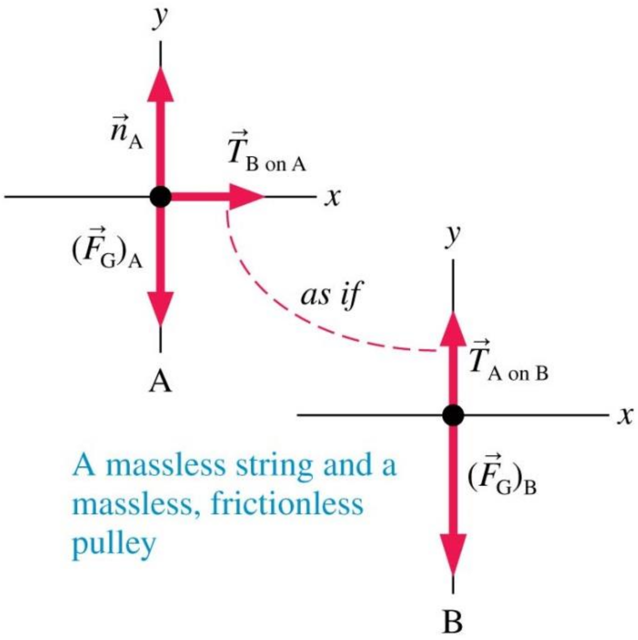




- We also assume that there is no friction where the pulley turns on its axle.
- Therefore, $\vec{T}_{A\ on\ S} = \vec{T}_{B\ on\ S}$.

Modelling Joined Objects





- Since $\vec{T}_{A\ on\ B} = \vec{T}_{B\ on\ A}$, we can draw the simplified free-body diagram as shown above.
- Forces $\vec{T}_{A\ on\ B}$ and $\vec{T}_{B\ on\ A}$ act ***as if*** they are in an action/reaction pair, even though they are not opposite in direction because the tension force gets 'turned' by the pulley.

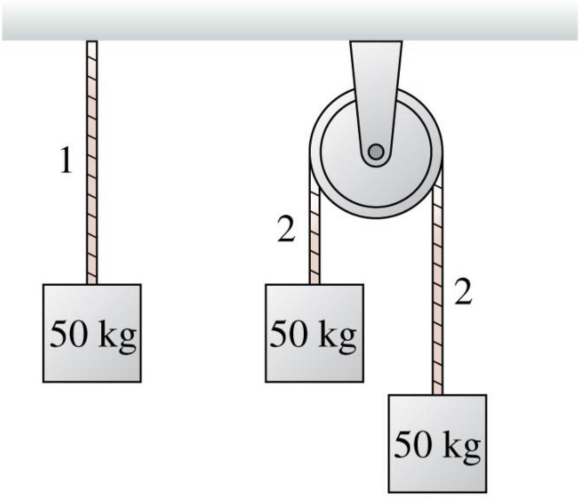
Have a Think: Modelling Joined Objects

Q.3 All three 50-kg blocks are at rest. The tension in rope 2 is

a) greater than the tension in rope 1.

b) equal to the tension in rope 1.

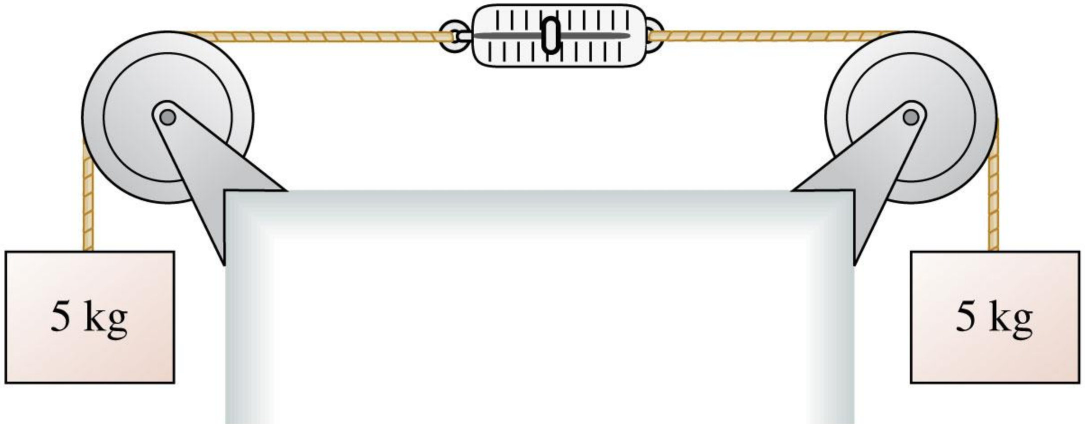
c) less than the tension in rope 1.



Have a Think: Modelling Joined Objects

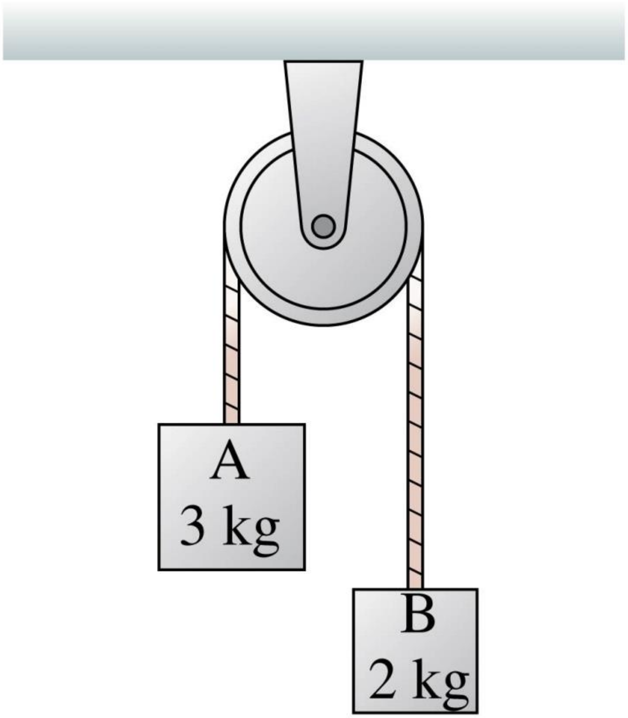
Q.4 The two masses are at rest. The pulleys are frictionless. The scale is in kg. The scale reads

a) 0 kg.
b) 5 kg.
c) 10 kg.



Have a Think: Modelling Joined Objects

Q.5 The acceleration constraint here is



a) $a_{Ay} = a_{By}$.

b) $-a_{Ay} = -a_{By}$.

c) $a_{Ay} = -a_{By}$.

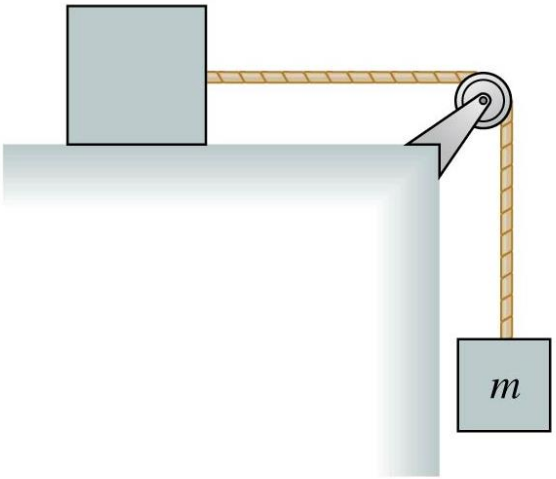
d) $a_{By} = -a_{Ay}$.

e) Either C or D.

Have a Think: Modelling Joined Objects

Q.6 The top block is accelerated across a frictionless table by the falling mass m. The string is massless, and the pulley is both massless and frictionless. The tension in the string is

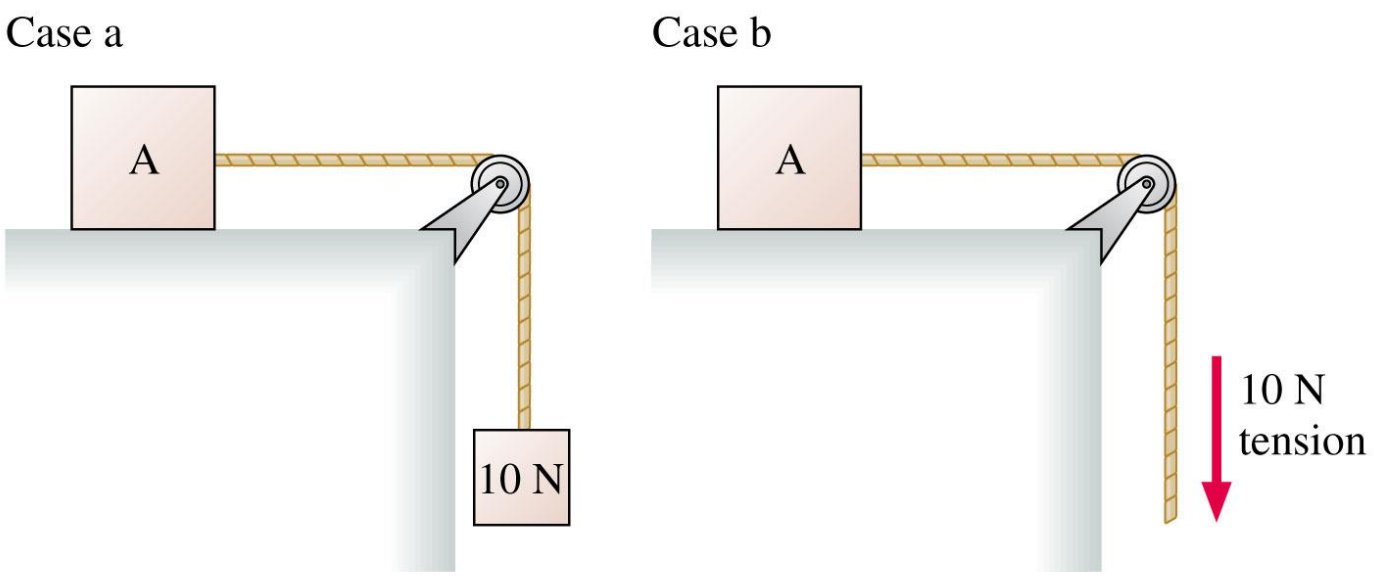
a) $T < mg$.
b) $T = mg$.
c) $T > mg$.



Have a Think: Modelling Joined Objects

Q.7 Block A is accelerated across a frictionless table. The string is massless, and the pulley is both massless and frictionless. Which is true?

a) Block A accelerates faster in case **a** than in case **b**.
b) Block A has the same acceleration in case **a** and case **b**.
c) Block A accelerates slower in case **a** than in case **b**.



Summary of today's Lecture

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1. Free-body diagrams
2. Weight
3. Static friction
4. Kinetic friction
5. Modelling joined objects

Lecture 4: Optional Reading

- **Ch. 4.5**, Newton's 3rd law of motion; p.107-109
- **Ch. 4.6**, Weight; p.110-112
- **Ch. 4.7**, Solving problems with Newton's Laws; free-body diagrams p.110-112
- **Ch. 4.8**, Problem solving—a general approach; p.120

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.