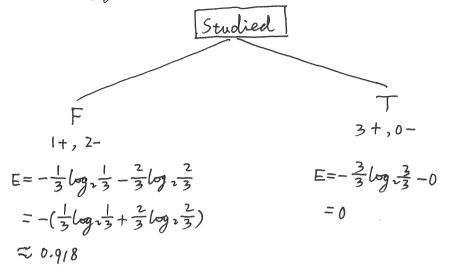We want to split first on the variable which maximizes the information gain

$$\text{Entropy (Pass)} = -\frac{4}{6}\log_2\frac{4}{6} - \frac{2}{6}\log_2\frac{2}{6} \approx 0.918$$

GPA	Studied	Pass
L	F	F
L	T	T
M	F	F
M	T	T
H	F	T
H	T	T

Studied

F
1+, 2−

$$E = -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3}$$

$$= -\left(\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}\right)$$

$$\approx 0.918$$

T
3+, 0−

$$E = -\frac{3}{3}\log_2\frac{3}{3} - 0$$

$$= 0$$

$$\text{Gain (S, Studied)} = 0.918 - \frac{3}{6} \times 0.918 - \frac{3}{6} \times 0 = 0.459$$

GPA

L
1+, 1−

$$E = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}$$

$$= -\log_2\frac{1}{2}$$

$$= 1$$

M
1+, 1−

$$E = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}$$

$$= -\log_2\frac{1}{2}$$

$$= 1$$

H
2+, 0−

$$E = -\frac{2}{2}\log_2\frac{2}{2} - 0$$

$$= 0$$

$$\text{Gain (S, GPA)} = 0.918 - \frac{2}{6} \times 1 - \frac{2}{6} \times 1 - \frac{2}{6} \times 0 = 0.918 - \frac{4}{6} = 0.918 - 0.667 = 0.251$$

Studied

F   T

GPA   T

L   M   H

F   F   T