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COMP2054 Tutorial Session 5: Master Theorem

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Session outcomes

- Identify which recurrences can the Master Theorem be applied to.
- Prove runtime complexities of recurrences using the M.T.

Master Theorem

Master Theorem cases and complexity proofs

Master Theorem "Cheat Sheet"

For a given recurrence of the form $T(n) = a \cdot T(n/b) + f(n)$ the M.T. can tell us the growth rate of $T(n)$ according to three cases:

Case 1: Recurrence dominates (plus special case that $f(n) = 0$)

IF $f(n)$ is $O(n^c)$ with $c < \log_b a$ THEN $T(n)$ is $\Theta\left(n^{\log_b a}\right)$

Case 2: Neither term dominates

IF $f(n)$ is $\Theta\left(n^c (\log n)^k\right)$ with $c = \log_b a$ and $k \geq 0$ THEN $T(n)$ is $\Theta\left(n^c (\log n)^{k+1}\right)$

Case 3: f(n) dominates

IF $f(n)$ is $\Omega(n^c)$ with $c > \log_b a$ THEN $T(n)$ is $\Theta\left(f(n)\right)$

In this module we assume the regularity condition for case 3 holds but you should be aware of it!

Q1. $T(n) = 2 \cdot T(n/2)$ and $T(1) = 1$

Q2. $T(n) = 2 \cdot T(n/2)$+n and $T(1) = 1$

Q3. $T(n) = 2 \cdot T(n/4)$+n and $T(1) = 1$

Q4. $T(n) = T(n-1) + 1$ and $T(1) = 1$

Identify which case of the Master Theorem each applies to (if any) and find the scaling behaviour

With the base case $T(1) = 1$:

- Q5. $T(n) = 2 \cdot T(n/4) + 1$
- Q6. $T(n) = 4 \cdot T(n/2) + n^2$
- Q7. $T(n) = 2 \cdot T(n - 1)$
- Q8. $T(n) = 3 \cdot T(n/3) + n \log n$
- Q9. $T(n) = 2 \cdot \mathrm{T}(\mathrm{n}/2) + 2n^2$
- Q10. $T(n) = 2 \cdot T(n/2) + n(\log n)^2$

Additional Practice Questions

If you would like some additional practice with the Master Theorem, check the [MT Additional Practice Questions] document on Moodle.

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Thank you