



Foundation Algebra (CELEN036)

Problem Sheet 3

Topics: Trigonometry I

Topic 1: Trigonometric Identities

1. Prove the following trigonometric identities:

$$(i) \quad \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 2 \sec x.$$

$$(ii) \quad \frac{\cos \theta}{1 + \cot \theta} = \frac{\sin \theta}{1 + \tan \theta}.$$

$$(iii) \quad (1 + \tan^2 \theta)(1 - \sin^2 \theta) = 1.$$

$$(iv) \quad \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} - \frac{1}{\tan^2 x} - \frac{1}{\cot^2 x} - \frac{1}{\sec^2 x} - \frac{1}{\csc^2 x} = 1.$$

$$(v) \quad (\csc^2 \theta - 1) = \cos^2 \theta \csc^2 \theta.$$

$$(vi) \quad \tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta.$$

$$(vii) \quad \cot^2 x - \cos^2 x = \cot^2 x \cos^2 x.$$

$$(viii) \quad \cos^4 \theta + \sin^4 \theta = 1 - 2 \cos^2 \theta \sin^2 \theta.$$

$$(ix) \quad \cos^6 \theta + \sin^6 \theta = 1 - 3 \cos^2 \theta \sin^2 \theta.$$

$$(x) \quad \cos^8 \theta - \sin^8 \theta = (\cos^2 \theta - \sin^2 \theta)(1 - 2 \sin^2 \theta \cos^2 \theta).$$

$$(xi) \quad \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}} = \sin \theta.$$

$$(xii) \quad \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \sec \theta + \tan \theta.$$

$$(xiii) \quad (\sec \theta + \tan \theta)^2 = \frac{\csc \theta + 1}{\csc \theta - 1}.$$

$$(xiv) (\csc \theta + \cot \theta)^2 = \frac{\sec \theta + 1}{\sec \theta - 1}.$$

$$(xv) (1 + \sin \theta)(1 + \cos \theta)(\sec \theta - 1)(\csc \theta - 1)(\tan \theta + \cot \theta) = 1.$$

$$(xvi) \sqrt{\frac{1 + \sin^2 \theta \sec^2 \theta}{1 + \cos^2 \theta \csc^2 \theta}} = \tan \theta.$$

2. Given $p = \sec \theta - \tan \theta$ and $q = \sec \theta + \tan \theta$, show that $p = \frac{1}{q}$.

Topic 2: Conversion formulae

3. Convert the following degrees to radians or vice-versa:

$$(i) -160^\circ$$

$$(ii) 720^\circ$$

$$(iii) 255^\circ$$

$$(iv) -85^\circ$$

$$(v) 345^\circ$$

$$(vi) 145^\circ$$

$$(vii) -\frac{16\pi}{7}$$

$$(viii) \frac{71\pi}{35}$$

$$(ix) \frac{6\pi}{13}$$

$$(x) \frac{4\pi}{3}$$

$$(xi) -\frac{15\pi}{8}$$

$$(xii) \frac{\pi}{9}$$

Topic 3: Range and period of trigonometric functions

4. Find the range and period of the following functions:

$$(i) 3 \sin 2x$$

$$(ii) 9 \cos(3x + 4) + 5$$

$$(iii) 3 \csc(3 - 4x)$$

$$(iv) 2 \tan(3x - 7)$$

$$(v) 7 \sin \left(-\frac{3\pi}{4}x - \frac{\pi}{4} \right) + 6$$

$$(vi) -5 \cos \left(\frac{\pi}{8}x \right) + 3$$

$$(vii) -3 \cos(\pi x + 2) - 6$$

$$(viii) -4 \cos(5x - 9) - 7$$

Topic 4: Find values of trigonometric function

5. Given $\cos \theta = -\frac{4}{5}; \frac{\pi}{2} < \theta < \pi$. Find the value of $\csc \theta + \cot \theta$.

6. Given $\cot \theta = -\frac{15}{8}$; $\frac{3\pi}{2} < \theta < 2\pi$. Find the value of $\sin \theta + \cos \theta$.
7. Given that $\sec \theta = -3$, and θ is obtuse. Find the values of $\tan \theta$ and $\csc \theta$.
8. If $\tan \theta + \sec \theta = \frac{3}{2}$; $0 < \theta < \frac{\pi}{2}$, show that $\sec \theta - \tan \theta = \frac{2}{3}$. Hence find the values of $\cos \theta$ and $2 \sin \theta + \tan \theta - \cos \theta$

Topic 5: Solving trigonometric equations

9. Solve the following equations for θ in given interval:

(i) $\sin \theta = \frac{1}{\sqrt{2}}$; $\theta \in [0, \pi]$

(ii) $\tan \theta = \frac{1}{\sqrt{3}}$; $\theta \in [0, 2\pi]$

(iii) $\sin 2\theta = 0.5$; $\theta \in [0, 2\pi]$

(iv) $\cos 3\theta = -0.5$; $\theta \in [0, 180^\circ]$

10. Solve $\tan^2 \theta = \sec 2\theta - 1$; $\theta \in [0, 180^\circ]$.

Answers

3. (i) $-\frac{8}{9}\pi$ (ii) 4π (iii) $\frac{17}{12}\pi$

(iv) $-\frac{17}{36}\pi$ (v) $\frac{23}{12}\pi$ (vi) $\frac{29}{36}\pi$

(vii) $-\frac{2880^\circ}{7}$ (viii) $\frac{2556^\circ}{7}$ (ix) $\frac{1080^\circ}{13}$

(x) 240° (xi) -337.5° (xii) 20°

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
4. Range	$[-3, 3]$	$[-4, 14]$	$\mathbb{R} - (-3, 3)$	\mathbb{R}	$[-1, 13]$	$[-2, 8]$	$[-9, -3]$	$[-11, -3]$
Period	π	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{8}{3}$	16	2	$\frac{2\pi}{5}$

5. $\frac{1}{3}$

6. $\frac{7}{17}$

7. $-\frac{2\sqrt{2}}{3}$ and $\frac{3}{2\sqrt{2}}$

8. $\frac{12}{13}$ and $\frac{41}{156}$

9. (i) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (ii) $\frac{\pi}{6}$ or $\frac{7\pi}{6}$ (iii) $\frac{\pi}{12}$ or $\frac{5\pi}{12}$ or $\frac{13\pi}{12}$ or $\frac{17\pi}{12}$

(iv) 40° or 80° or 160°

10. 0° or 180°