The University of Nottingham Ningbo China

Centre for English Language Education

Semester One, 2015-2016

FOUNDATION ALGEBRA FOR PHYSICAL SCIENCES & ENGINEERING

Time allowed 1 Hour 30 Minutes

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

This paper contains EIGHT questions which carry equal marks.

An indication is given of the weighting of each subsection of a question by means of a figure enclosed by square brackets, eg. [5], *immediately following that subsection.*

Only silent, self-contained calculators with a Single-line Display or Dual-line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

Do NOT turn examination paper over until instructed to do so.

ADDITIONAL MATERIAL: *Formula Sheet (attached to the back of the question paper)*

INFORMATION FOR INVIGILATORS:

1. *Please collect Answer Booklets, Question Papers, and Formula Sheet at the end of the exam.*
2. *Please give a 15 minute warning.*

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1 (a) Given quadratic function $f(x) = x^2 + (k+4)x + (k+7)$.

(i) Find $k > 0$ if the roots of the equation $f(x) = 0$ are equal.

(ii) Solve the equation $f(x) = 0$ for the value of k obtained above. [3]

(b) Given function $f : \mathbb{R}^+ \to \mathbb{R}$ defined by

$$f(x) = \ln x + 3.$$

Find the inverse function, $f^{-1}(x)$. [1]

(c) Solve the logarithmic equation for $x > 5$

$$\log_2(x-3) + \log_2(x-5) = 3.$$ [3]

(d) Solve for $x \in \mathbb{R}$, the inequality $\mid 3x - 4 \mid \geq 5$. [3]

2 (a) Prove the identity $\left(\dfrac{1+\sin x}{1-\sin x}\right) - \left(\dfrac{1-\sin x}{1+\sin x}\right) = 4 \sec x \tan x$. [2]

(b) Use the identity $\sin^2\theta = 1 - \cos^2\theta$ to solve the trigonometric equation

$$10\,\sin^2\theta - 3\,\cos\theta = 6 \qquad \text{where } \theta \in \left(0\,,\frac{\pi}{2}\right).$$ [3]

(c) (i) Express $\cos x - 2\sin x$ in the form $R\cos(x-\theta)$, where $R > 0$ and $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(ii) Hence, sketch the curve $y = \cos x - 2\sin x$. [3]

(d) Prove that $\dfrac{\tan 70^\circ + \tan 50^\circ}{1 - \tan 70^\circ \cdot \tan 50^\circ} + \sqrt{3} = 0$. [2]

3 (a) Given $p(x) = 2x^3 + 7x^2 - 7x - 12$.

(i) Use the method of synthetic division to show that $(x + 4)$ is a factor of $p(x)$.

(ii) Hence, express $p(x)$ completely as a product of linear factors.

[4]

(b) Given polynomial function $f(x) = 3x^2 + ax + b$.

Find the constants of a and b if the remainder when $f(x)$ is divided by $(x - 1)$ is 2, and the remainder when $f(x)$ is divided by $(x + 1)$ is 6.

[3]

(c) Express $\dfrac{13}{(3x-2)(2x+3)}$ as a sum of partial fractions.

[3]

4 Consider solving numerically the equation

$$f(x) = x^3 - 4x - 5 = 0. \tag{4.1}$$

(a) Apply the Intermediate Value Theorem to show that a root α of the equation (4.1) lies in the interval $(2, 3)$. [2]

(b) Show that the equation (4.1) can be rearranged to obtain the iterative formula

$$x_{n+1} = \sqrt{4 + \frac{5}{x_n}}. \tag{4.2}$$

[2]

(c) Starting with $x_0 = 2.5$, use the iterative formula (4.2) to find the root of the equation (4.1), correct to 5 decimal places. Show the steps of calculation for the approximation x_1.

[3]

(d) Given $x_0 = 2.5$, use the Bisection method to find approximations x_1 and x_2, correct to 2 decimal places. Write the steps involved in the calculations.

[3]

5 (a) Use the Binomial Theorem to find the term with x^6 in the expansion of $\left(2 - \dfrac{3x^2}{5}\right)^8$. [2]

(b) In the expansion of $(2 - ax)^5$, the coefficient of x^2 is 20. Find $a > 0$. [2]

(c) (i) Use the general Binomial expansion formula to expand

$$(1 + 2\,x)^{1/3} \qquad ; \quad |\,x\,| < \frac{1}{2} \qquad \text{up to and including the term in } x^3.$$

(ii) By substituting $x = -\dfrac{1}{100}$, find the approximate value of $\sqrt[3]{0.98}$, correct to 3 decimal places. [3]

(d) The area of an equilateral triangle with sides having length l units is given by $A = \dfrac{\sqrt{3}}{4}\, l^2$.

If there is an error of 1.2% in the measured value of l, use the approximation

$$(1 + x)^n \;\approx\; 1 + nx + \frac{n\,(n-1)}{2}\,x^2$$

to find the resulting error, δA, in the calculated area. [3]

6 (a) Given matrices $A = \begin{pmatrix} 1 & -4 \\ 4 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -2 & 5 \end{pmatrix}$, and $C = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$.

Find the matrix $5C^T - 4AB$. [3]

(b) Consider solving the system of simultaneous linear equations using the matrix method.

$$\left.\begin{aligned} 7x - 8y - 7 &= 0 \\ 4x - 5y - 1 &= 0 \end{aligned}\right\} \qquad (6.1)$$

(i) Express the system of equations (6.1) in the standard matrix form $AX = B$.

(ii) Find the inverse matrix, A^{-1}.

(iii) Use $X = A^{-1}B$ to solve the system of equations (6.1). [5]

(c) Find k if the matrix $A = \begin{pmatrix} -2 & k-2 \\ k+3 & 3 \end{pmatrix}$ does not have an inverse. [2]

7 (a) Express the complex number $z = 1 - i^2 - i^3 + i^4 - i^5$ in the form $a + i\,b \quad (a, b \in \mathbb{R})$. [1]

(b) (i) Solve for $x, y \in \mathbb{R}$, the complex equation $(2 - i)\,x - (1 + 3i)\,y - 7 = 0$.

(ii) Evaluate $|\, x + i\,y \,|$ for the values of x and y obtained above. [3]

(c) Given complex numbers $z_1 = 4 - 3\,i$ and $z_2 = 3 + 4\,i$.

(i) Express $z = z_1 \cdot z_2$ in the form $a + i\,b$, where $a, b \in \mathbb{R}$.

(ii) Find $|\, z \,|$ and $\arg(z)$.

(iii) Hence express z in the polar form $z = r\,(\cos\theta + i\,\sin\theta) \quad ; \quad -\pi < \theta \leq \pi$. [3]

(d) Given complex numbers $z_1 = 1 + i$, $z_2 = 2 - i$, and $z_3 = 4 + 3\,i$.

Use properties of modulus to evaluate $\left| \dfrac{z_1 \cdot \overline{z_3}}{z_2^2} \right|$. [3]

8 (a) The tenth term of an Arithmetic sequence is 15. Use the formula $S_n = \dfrac{n}{2}\,[\,2a + (n-1)d\,]$ to find the sum of its first 19 terms. [2]

(b) Given Geometric progression (G.P.): $162,\ 54,\ 18,\ 6,\ ....$

(i) Show that the n^{th} term of the G.P. is $a_n = 486 \cdot (3)^{-n} \quad (n \in \mathbb{N})$.

(ii) Use the formula $S_n = \dfrac{a\,(1 - r^n)}{1 - r}$ to find the sum of the first five terms of the G.P. [3]

(c) Given $f(n) = 6n^2 + 4n - 1$.

(i) Use the formulae for $\sum n$ and $\sum n^2$ to show that

$$\sum_{1}^{n} f(n) \;=\; n\,(n+2)\,(2n+1).$$

(ii) Find the sums: $\displaystyle\sum_{1}^{10} f(n)$ and $\displaystyle\sum_{1}^{20} f(n)$.

(iii) Hence find the sum: $\displaystyle\sum_{11}^{20} (6n^2 + 4n - 1)$. [5]