The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Science A Physics

Lectures 1-3:

Additional Problems: Describing Motion, Kinematics and Causing Motion

General Problem Solving Skills

Problem Solving

MODEL Make simplifying assumptions.

VISUALIZE Use:

- **Pictorial representation**
- **Graphical representation**

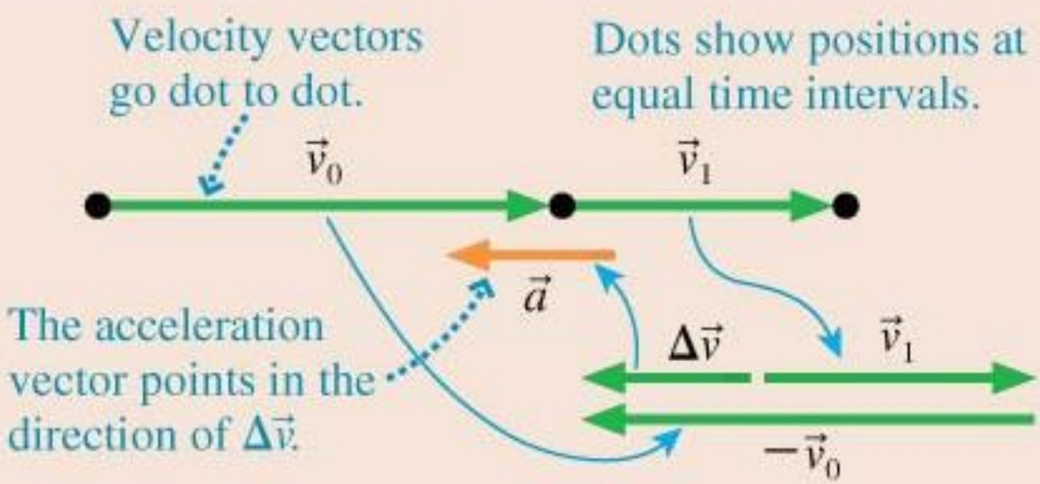
SOLVE Use a **mathematical representation** to find numerical answers.

ASSESS Does the answer have the proper units? Does it make sense?

General Problem Solving Skills

Motion Diagrams

- Help visualize motion.
- Provide a tool for finding acceleration vectors.



▶ These are the average velocity and the average acceleration vectors.

General Problem Solving Skills

The particle model represents a moving object as if all its mass were concentrated at a single point.

Position locates an object with respect to a chosen coordinate system. Change in position is called displacement.

Velocity is the rate of change of the position vector $\vec{r}$.

Acceleration is the rate of change of the velocity vector $\vec{v}$.

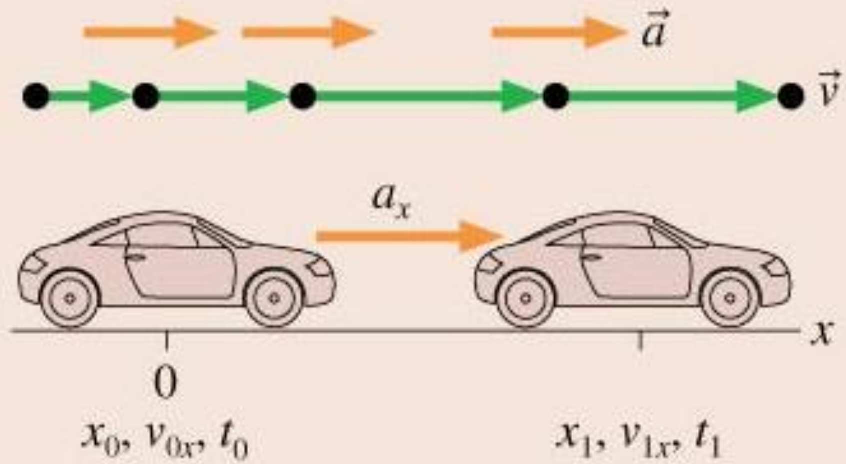
An object has an acceleration if it

- Changes speed and/or
- Changes direction.

General Problem Solving Skills

Pictorial Representation

1. Draw a motion diagram.
2. Establish coordinates.
3. Sketch the situation.
4. Define symbols.
5. List knowns.
6. Identify desired unknown.



Known

$x_0 = v_{0x} = t_0 = 0$

$a_x = 2.0\ \text{m/s}^2 \quad t_1 = 2.0\ \text{s}$

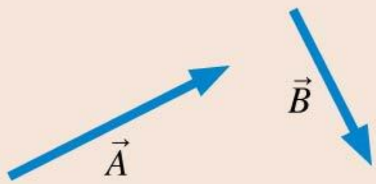
Find

x_1

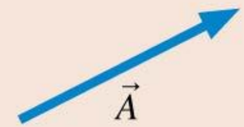
Vector Addition

TACTICS BOX 1.1 **Vector addition**

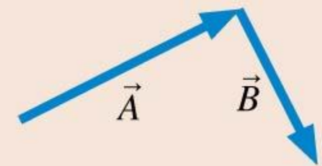
To add $\vec{B}$ to $\vec{A}$:



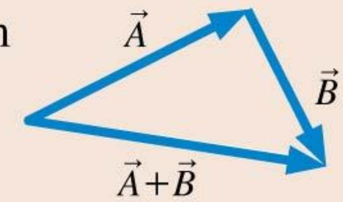
1. Draw $\vec{A}$.



2. Place the tail of $\vec{B}$ at the tip of $\vec{A}$.



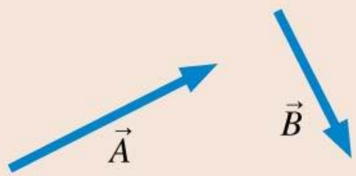
3. Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A} + \vec{B}$.



Vector Subtraction

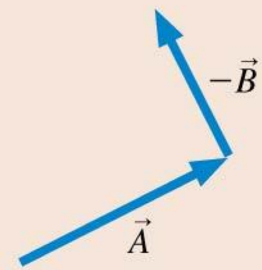
TACTICS BOX 1.2 **Vector subtraction**

To subtract $\vec{B}$ from $\vec{A}$:

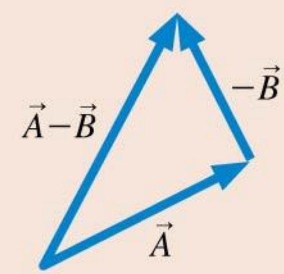


❶ Draw $\vec{A}$.

❷ Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.



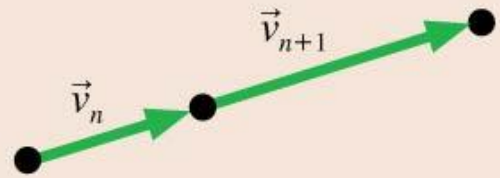
❸ Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is vector $\vec{A} - \vec{B}$.



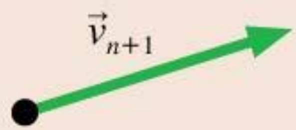
The Acceleration Vector

TACTICS BOX 1.3 **Finding the acceleration vector**

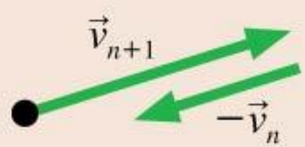
To find the acceleration as the velocity changes from $\vec{v}_n$ to $\vec{v}_{n+1}$, we must determine the *change* of velocity $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$.



❶ Draw the velocity vector $\vec{v}_{n+1}$.

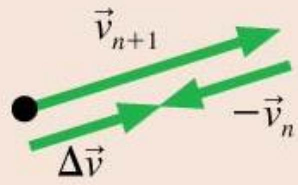


❷ Draw $-\vec{v}_n$ at the tip of $\vec{v}_{n+1}$.

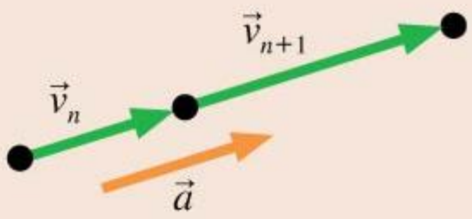


The Acceleration Vector

❸ Draw $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$
$= \vec{v}_{n+1} + (-\vec{v}_n)$
This is the direction of $\vec{a}$.



❹ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_n$ and $\vec{v}_{n+1}$.



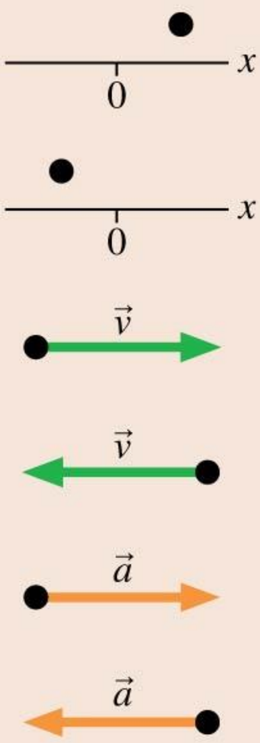
- Notice that the acceleration vectors goes beside the dots, not beside the velocity vectors.
- That is because each acceleration vector is the difference between two velocity vectors on either side of a dot.

Skiing through the Woods

Q.1 A skier glides along smooth, horizontal snow at constant speed, then speeds up going down a hill. Draw the skier's motion diagram.

Determining the Sign of the Position, Velocity and Acceleration

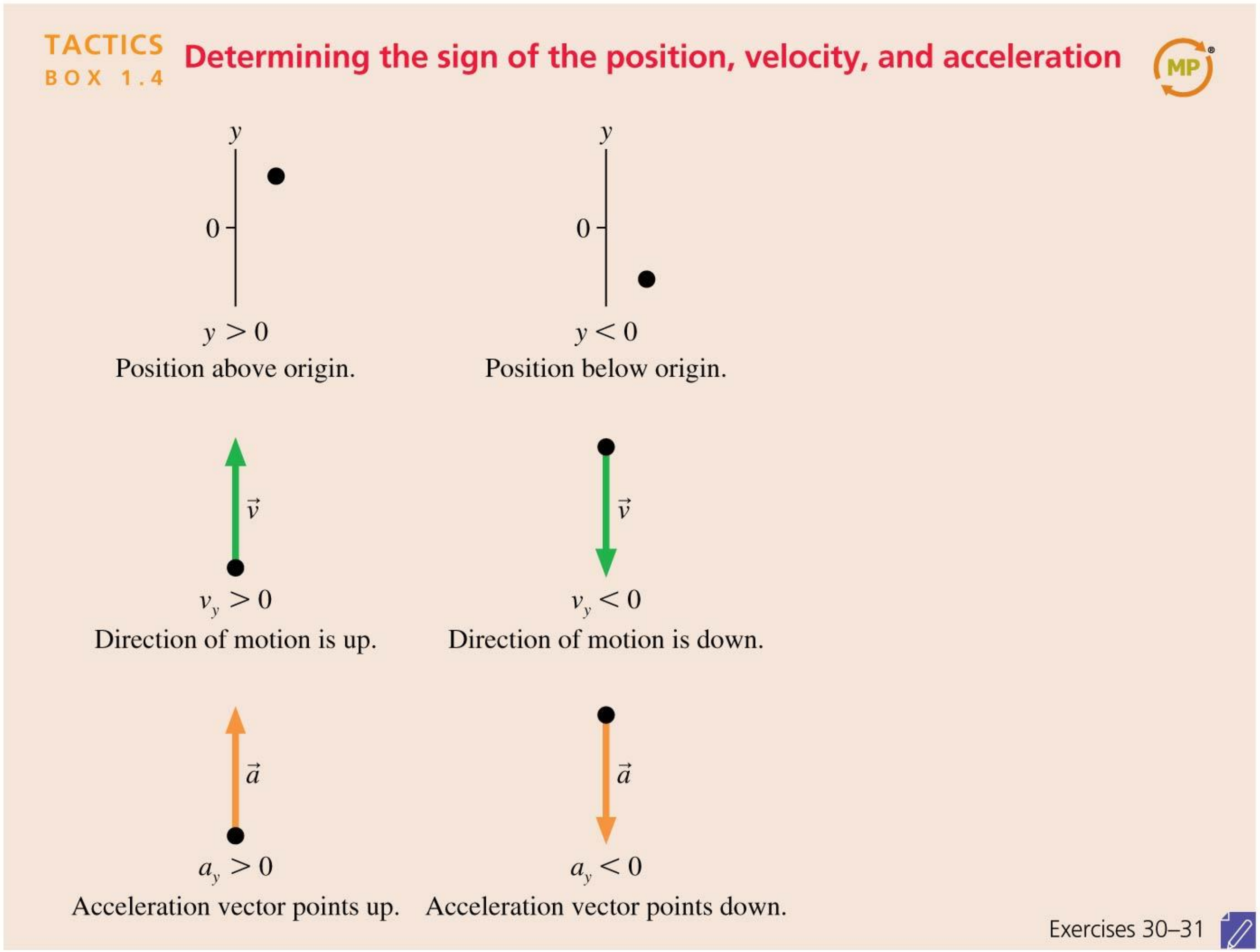
TACTICS BOX 1.4 **Determining the sign of the position, velocity, and acceleration**



$x > 0$	Position to right of origin.
$x < 0$	Position to left of origin.
$v_x > 0$	Direction of motion is to the right.
$v_x < 0$	Direction of motion is to the left.
$a_x > 0$	Acceleration vector points to the right.
$a_x < 0$	Acceleration vector points to the left.

Exercises 30–31

Determining the Sign of the Position, Velocity and Acceleration



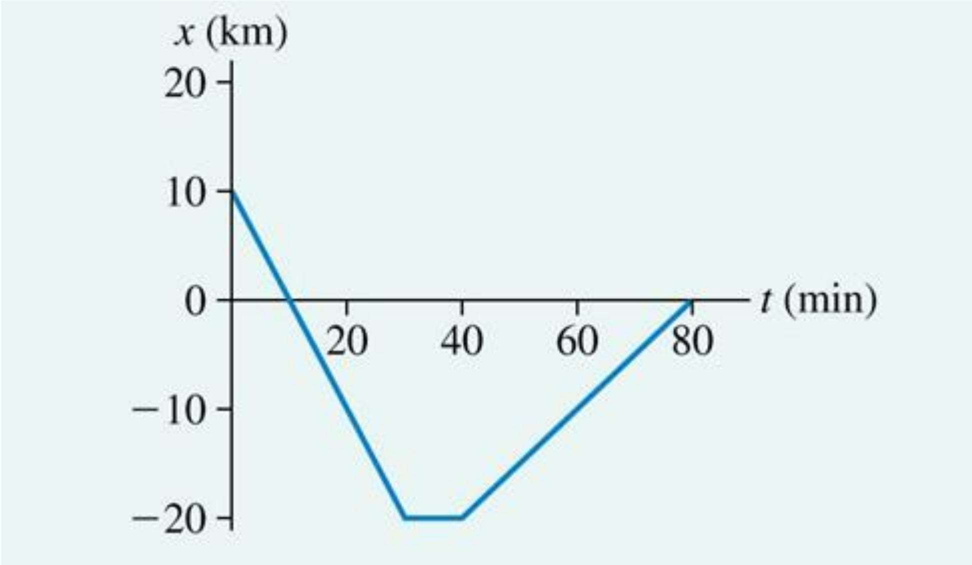
Determining the Sign of the Position, Velocity and Acceleration

TACTICS BOX 1.4 **Determining the sign of the position, velocity, and acceleration**

- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.

Exercises 30–31

Interpreting a Position Graph



Q.2 The graph in the figure represents the motion of a car along a straight road. Describe the motion of the car.

Using Graphical Addition to Find a Displacement

Q.3 A bird flies 100 m due east from a tree, then 200 m northwest (that is, $45°$ north of west).
What is the bird's net displacement?

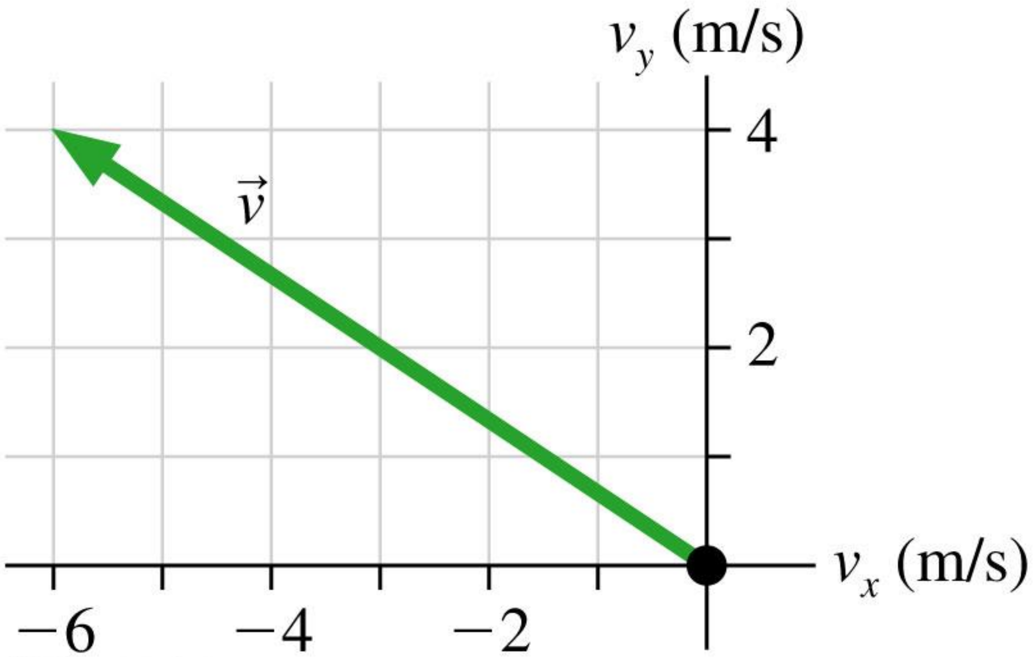
Vector Components

TACTICS BOX 3.1 **Determining the components of a vector**

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive x-direction, negative if $\vec{A}_x$ points in the negative x-direction.
3. The y-component A_y is determined similarly.

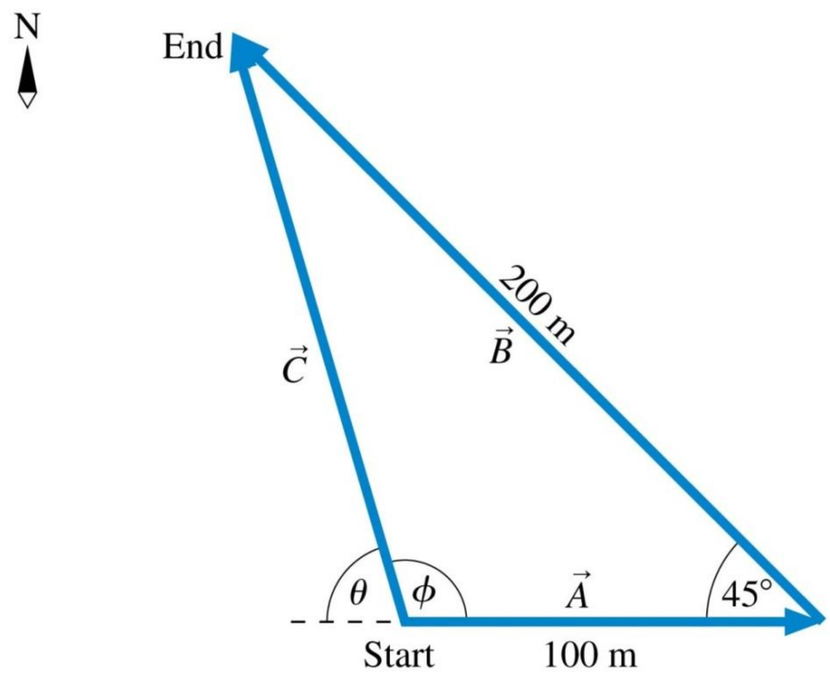
Exercises 10–18

Finding the Direction of Motion



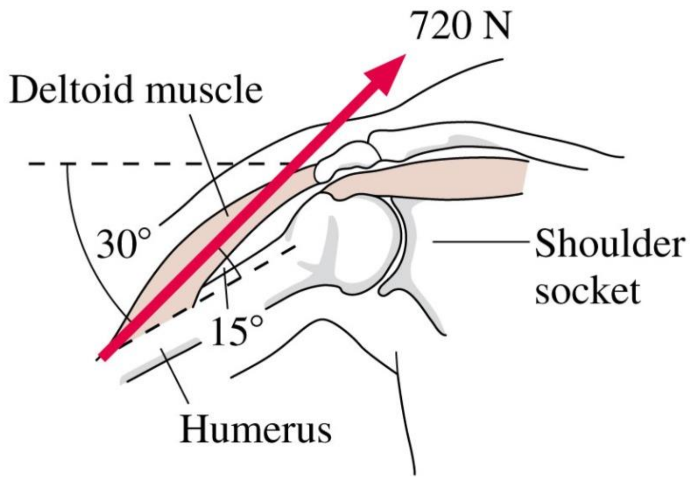
Q.4 The figure above shows a car's velocity vector $\vec{v}$.
Determine the car's speed and direction of motion.

Using Algebraic Addition to Find a Displacement



Q.5 In question 3, we considered a bird that flew 100 m to the east, and then 200 m to the northwest. In this question, we use the algebraic addition of vectors to find the bird's net displacement.

Muscle and Bone



Q.6

The deltoid – the round muscle across the top of your upper arm – allows you to lift your arm away from your side. It does so by pulling on an attachment point on the humerus, the upper arm bone, at an angle of 15° with respect to the humerus.

If you hold your arm at an angle of 30° below the horizontal, the deltoid must pull with a force of 720 N to support the weight of your arm, as shown in the figure.

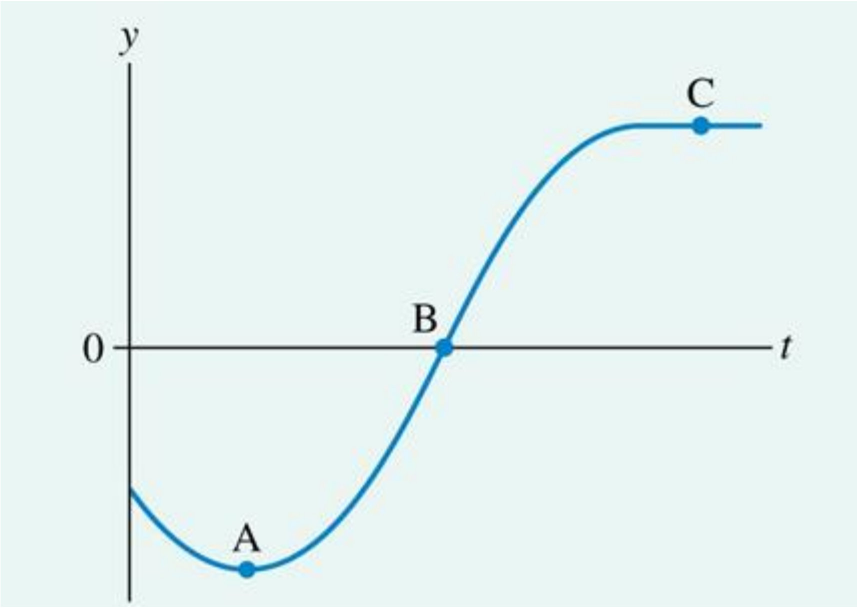
What are the components of the muscle force parallel to and perpendicular to the bone?

TACTICS BOX 2.1 Interpreting position-versus-time graphs

1. Steeper slopes correspond to faster speeds.
2. Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).
3. The slope is a ratio of intervals, $\Delta x/\Delta t$, not a ratio of coordinates. That is, the slope is *not* simply x/t.
4. We are distinguishing between the *actual* slope and the *physically meaningful* slope. If you were to use a ruler to measure the rise and the run of the graph, you could compute the actual slope of the line as drawn on the page. That is not the slope to which we are referring when we equate the velocity with the slope of the line. Instead, we find the *physically meaningful* slope by measuring the rise and run using the scales along the axes. The "rise" Δx is some number of meters; the "run" Δt is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope.

Exercises 1–3

Finding Velocity from Position Graphically



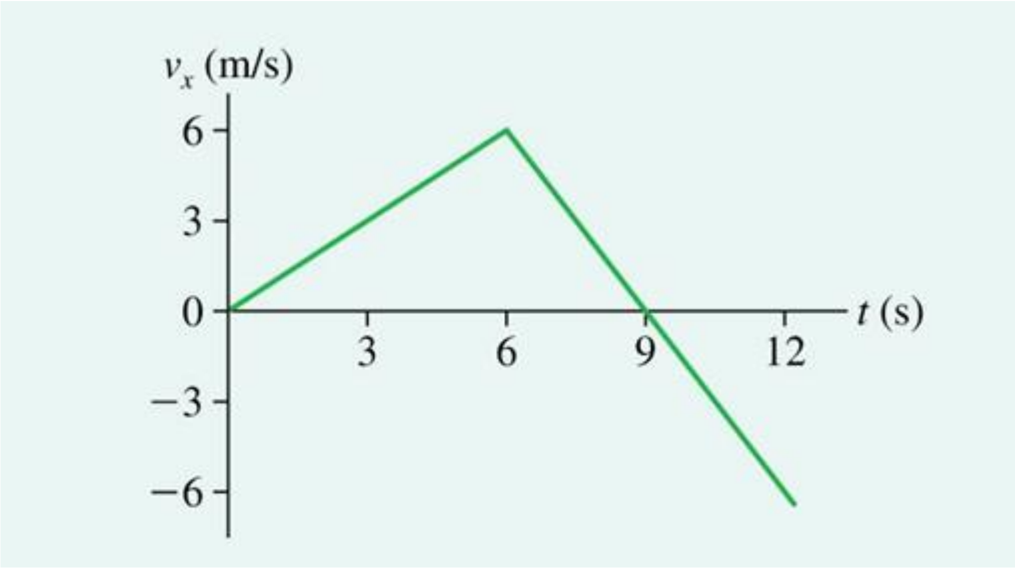
Q.7 The figure shows the position-versus-time graph of an elevator.

a. At which labeled point or points does the elevator have the least speed?

b. At which point or points does the elevator have the maximum velocity?

c. Sketch an approximate position-versus-time graph for the elevator.

Running the Court



Q.8 A basket player starts at the left end of the court and moves with the velocity shown in the figure below. Draw a motion diagram and an acceleration-versus-time graph for the basket player.

PROBLEM-SOLVING STRATEGY 2.1

Kinematics with constant acceleration

MODEL Use the particle model. Make simplifying assumptions.

VISUALIZE Use different representations of the information in the problem.

- Draw a *pictorial representation.* This helps you assess the information you are given and starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these two representations as needed.

SOLVE The mathematical representation is based on the three kinematic equations

$$v_{fs} = v_{is} + a_s \Delta t$$

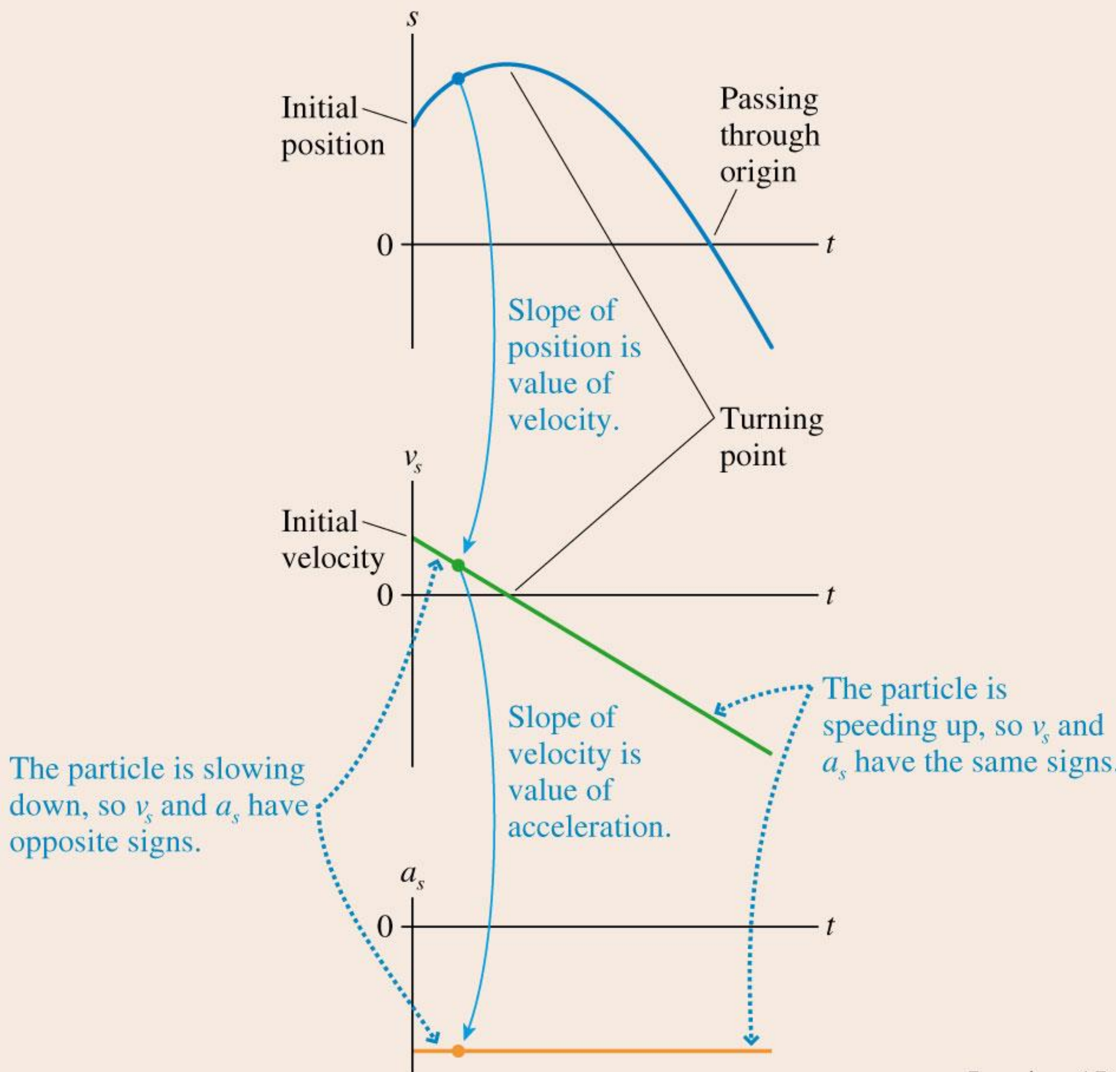
$$s_f = s_i + v_{is} \Delta t + \tfrac{1}{2} a_s (\Delta t)^2$$

$$v_{fs}^2 = v_{is}^2 + 2a_s \Delta s$$

- Use x or y, as appropriate to the problem, rather than the generic s.
- Replace i and f with numerical subscripts defined in the pictorial representation.
- Uniform motion with constant velocity has $a_s = 0$.

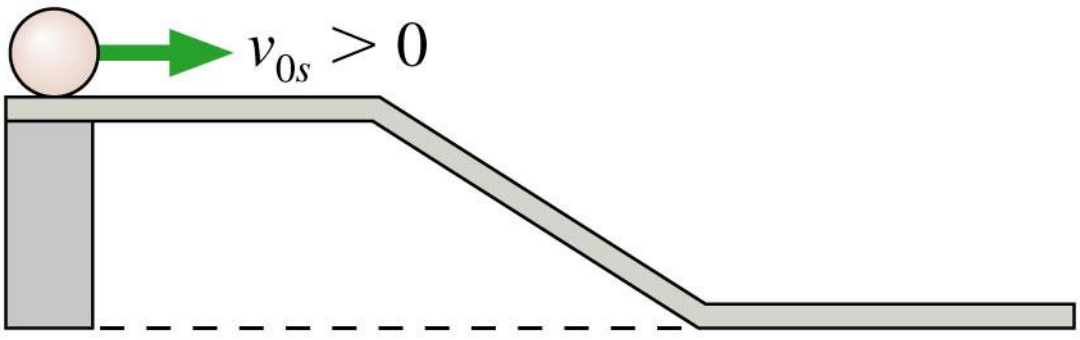
ASSESS Is your result believable? Does it have proper units? Does it make sense?

TACTICS BOX 2.2 Interpreting graphical representations of motion



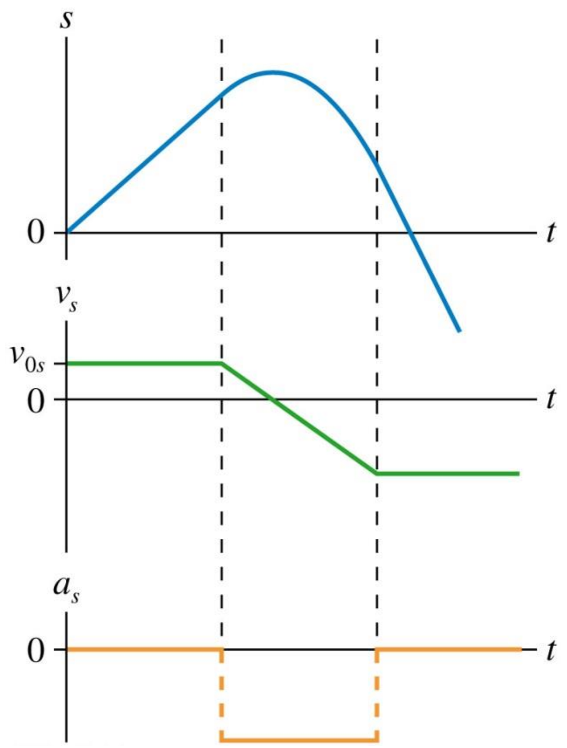
Exercises 15, 16, 22

From Track to Graphs



Q.9 Draw the position, velocity, and acceleration graphs for the ball on the frictionless track in the above figure.

From Graphs to Track

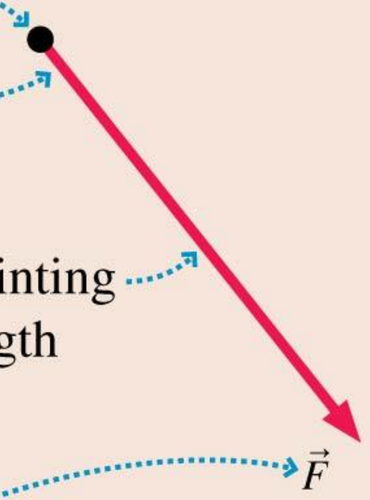


Q.10 The above figure shows a set of motion graphs for a ball moving on a track. Draw a picture of the track and describe the ball's initial condition. Each segment of the track is straight, but the segments may be tilted.

Drawing Force Vectors

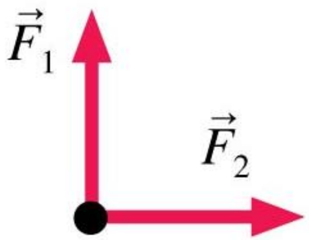
TACTICS BOX 5.1 Drawing force vectors

1. Represent the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.

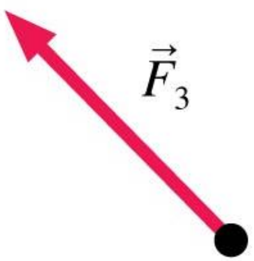


Combining Forces

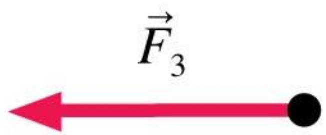
Q.11 The net force on an object points to the left. Two of three forces are shown. Which is the missing third force?



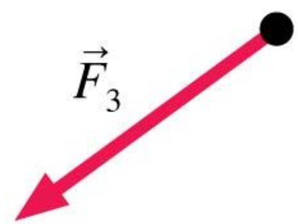
Two of the three forces exerted on an object



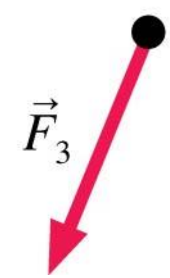
A.



B.



C.



D.

Identifying Forces

TACTICS BOX 5.2 Identifying forces

1. **Identify the object of interest.** This is the object whose motion you wish to study.
2. **Draw a picture of the situation.** Show the object of interest and all other objects—such as ropes, springs, or surfaces—that touch it.
3. **Draw a closed curve around the object.** Only the object of interest is inside the curve; everything else is outside.

Exercises 3–8

Identifying Forces

TACTICS BOX 5.2 **Identifying forces**

❹ **Locate every point on the boundary of this curve where other objects touch the object of interest.** These are the points where *contact forces* are exerted on the object.

❺ **Name and label each contact force acting on the object.** There is at least one force at each point of contact; there may be more than one. When necessary, use subscripts to distinguish forces of the same type.

❻ **Name and label each long-range force acting on the object.** For now, the only long-range force is the gravitational force.

Exercises 3–8

Forces on a Bungee Jumper

Q.12 A bungee jumper has leapt off a bridge, and is nearing the bottom of her fall. What forces are being exerted on the jumper?

Forces on a Skier

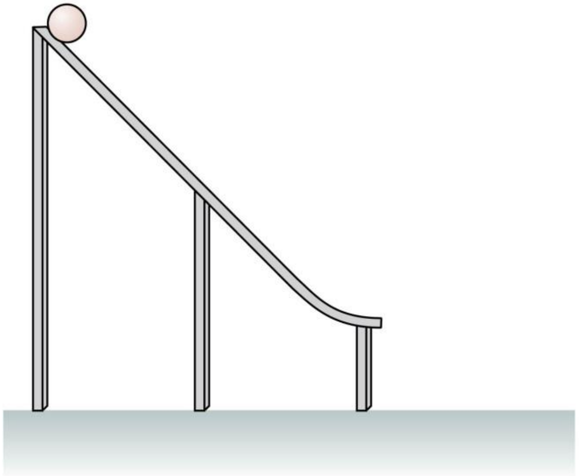
Q.13 A skier is being towed up a snow-covered hill by a tow rope. What forces are being exerted on the skier?

Forces on a Rocket

Q.14 A rocket is being launched to place a new satellite in orbit. Air resistance is not negligible. What forces are being exerted on the rocket?

Forces

Q.15 A ball rolls down an incline and off a horizontal ramp. Ignoring air resistance, what force or forces act on the ball as it moves through the air just after leaving the horizontal ramp?



a) The weight of the ball acting vertically down.

b) A horizontal force that maintains the motion.

c) A force whose direction changes as the direction of motion changes.

d) The weight of the ball and a horizontal force.

e) The weight of the ball and a force in the direction of motion.

Drawing a Free-body Diagram

TACTICS BOX 5.3 **Drawing a free-body diagram**

1. **Identify all forces acting on the object.** This step was described in Tactics Box 5.2.
2. **Draw a coordinate system.** Use the axes defined in your pictorial representation.
3. **Represent the object as a dot at the origin of the coordinate axes.** This is the particle model.

Exercises 24–29

Drawing a Free-body Diagram

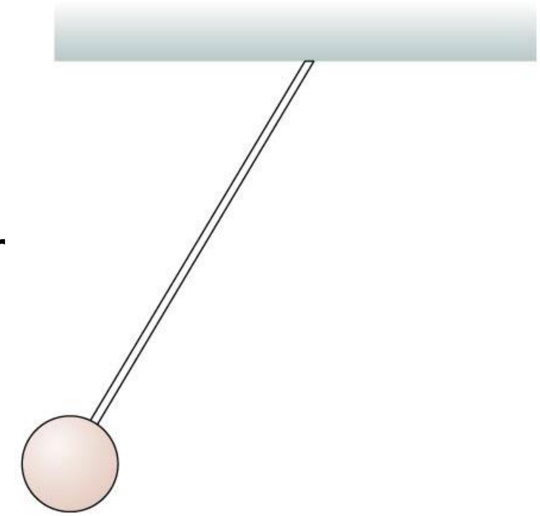
TACTICS BOX 5.3 **Drawing a free-body diagram**

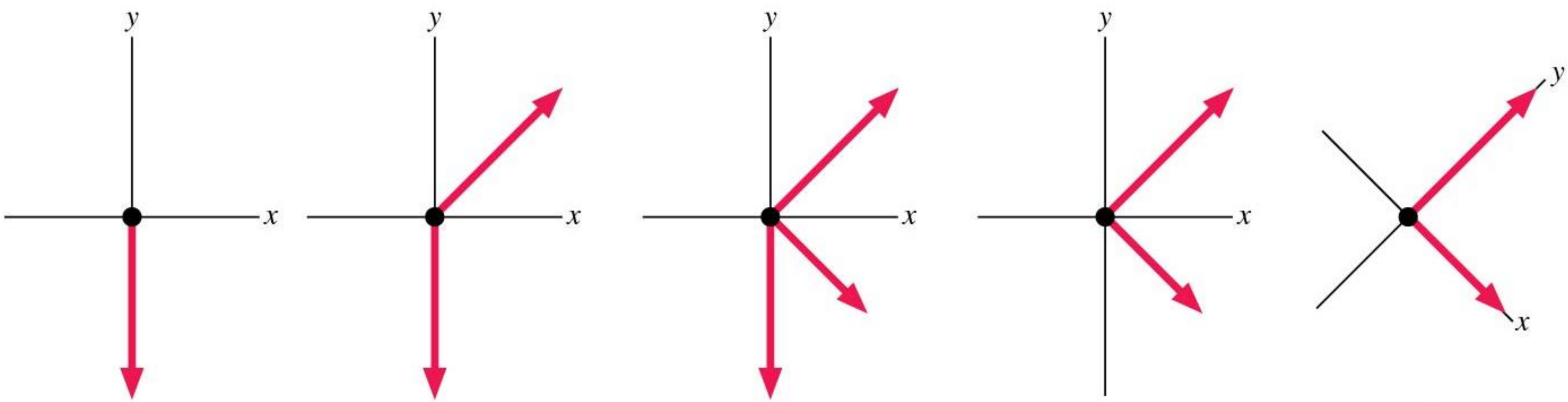
4. **Draw vectors representing each of the identified forces.** This was described in Tactics Box 5.1. Be sure to label each force vector.
5. **Draw and label the *net force* vector $\vec{F}_{net}$.** Draw this vector beside the diagram, not on the particle. Or, if appropriate, write $\vec{F}_{net} = \vec{0}$. Then check that $\vec{F}_{net}$ points in the same direction as the acceleration vector $\vec{a}$ on your motion diagram.

Exercises 24–29

Free-body Diagrams

Q.16 A ball, hanging from the ceiling by a string, is pulled back and released. Which is the correct free-body diagram just after its release?





A. **B.** **C.** **D.** **E.**

Free-body Diagrams

Q.17 A tow rope pulls a skier up a snow-covered hill at a constant speed. Draw a pictorial representation of the skier.

PROBLEM-SOLVING STRATEGY 6.1

Equilibrium problems

MODEL Make simplifying assumptions. When appropriate, represent the object as a particle.

VISUALIZE

- Establish a coordinate system, define symbols, and identify what the problem is asking you to find. This is the process of translating words into symbols.
- Identify *all* forces acting on the object and show them on a free-body diagram.
- These elements form the **pictorial representation** of the problem.

PROBLEM-SOLVING STRATEGY 6.1 **Equilibrium problems**

SOLVE The mathematical representation is based on Newton's first law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = \vec{0}$$

The vector sum of the forces is found directly from the free-body diagram.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Towing a Car up a Hill

Q.18 A car with a weight of 15,000 N is being towed up a 20° slope at constant velocity. Friction is negligible. The tow rope is rated at 6000 N maximum tension. Will it break?

PROBLEM-SOLVING STRATEGY 6.2

Dynamics problems

MODEL Make simplifying assumptions.

VISUALIZE Draw a **pictorial representation.**

- Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying to find.
- Use a motion diagram to determine the object's acceleration vector $\vec{a}$.
- Identify all forces acting on the object *at this instant* and show them on a free-body diagram.
- It's OK to go back and forth between these steps as you visualize the situation.

PROBLEM-SOLVING STRATEGY 6.2

Dynamics problems

SOLVE The mathematical representation is based on Newton's second law:

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

The vector sum of the forces is found directly from the free-body diagram. Depending on the problem, either

- Solve for the acceleration, then use kinematics to find velocities and positions; or
- Use kinematics to determine the acceleration, then solve for unknown forces.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 22

Speed of a Towed Car

Q.19 A 1500 kg car is pulled by a tow truck. The tension in the tow rope is 2500 N, and a 200 N friction force opposes the motion. If the car starts from rest, what is its speed after 5.0 seconds?

Make Sure the Cargo Doesn't Slide

Q.20 A 100 kg box of dimensions 50 cm x 50 cm x 50cm is in the back of a flatbed truck. The coefficients of friction between the box and the bed of the truck are $\mu_s = 0.40$ and $\mu_k = 0.20$. What is the maximum acceleration the truck can have without the box slipping?

TACTICS BOX 7.1 **Analyzing interacting objects**

❶ **Represent each object as a circle.** Place each in the correct position relative to other objects.

- Give each a name and a label.
- The surface of the earth (contact forces) and the entire earth (long-range forces) should be considered separate objects. Label the **e**ntire **e**arth EE.
- Ropes and pulleys often need to be considered objects.

TACTICS BOX 7.1 Analyzing interacting objects

❷ **Identify interactions.** Draw connecting lines between the circles to represent interactions.

- Draw *one* line for each interaction. Label it with the type of force.
- Every interaction line connects two and only two objects.
- There can be at most two interactions at a surface: a force parallel to the surface (e.g., friction) and a force perpendicular to the surface (e.g., a normal force).
- The entire earth interacts only by the long-range gravitational force.

TACTICS BOX 7.1

Analyzing interacting objects

MP

3. **Identify the system.** Identify the objects of interest; draw and label a box enclosing them. This completes the interaction diagram.
4. **Draw a free-body diagram for each object in the system.** Include only the forces acting *on* each object, not forces exerted by the object.

- Every interaction line crossing the system boundary is one external force acting on the object. The usual force symbols, such as $\vec{n}$ and $\vec{T}$, can be used.
- Every interaction line within the system represents an action/reaction pair of forces. There is one force vector on *each* of the objects, and these forces always point in opposite directions. Use labels like $\vec{F}_{\text{A on B}}$ and $\vec{F}_{\text{B on A}}$.
- Connect the two action/reaction forces—which must be on *different* free-body diagrams—with a dashed line.

Exercises 1–7

Newton's 3rd Law

Q.21 A mosquito runs head-on into a truck. Splat! Which is true during the collision?

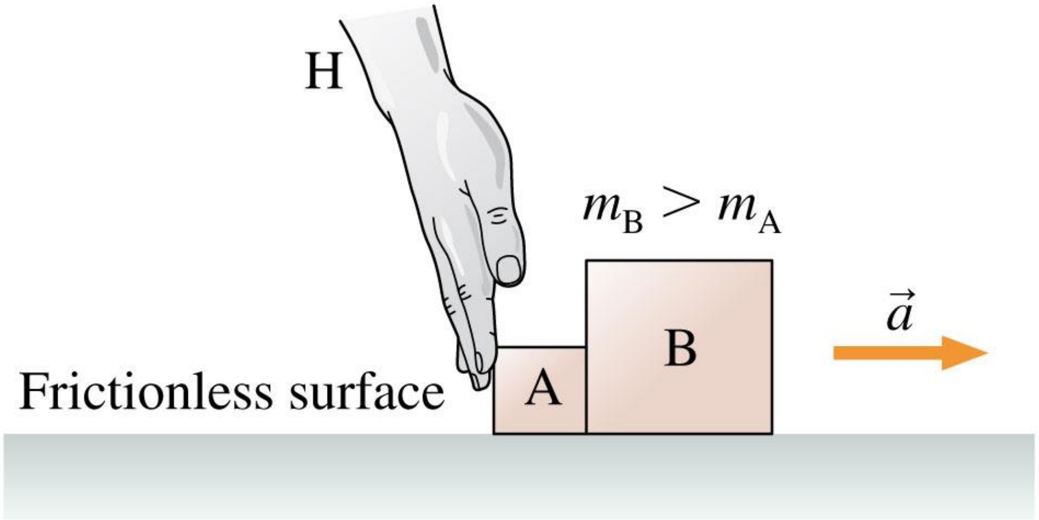
a) The mosquito exerts more force on the truck than the truck exerts on the mosquito.
b) The truck exerts more force on the mosquito than the mosquito exerts on the truck.
c) The mosquito exerts the same force on the truck as the truck exerts on the mosquito.
d) The truck exerts a force on the mosquito but the mosquito does not exert a force on the truck.
e) The mosquito exerts a force on the truck but the truck does not exert a force on the mosquito.

Newton's 3rd Law

Q.22 The same mosquito runs head-on into a truck. Which is also true during the collision?

a) The magnitude of the mosquito's acceleration is larger than that of the truck.
b) The magnitude of the truck's acceleration is larger than that of the mosquito.
c) The magnitude of the mosquito's acceleration is the same as that of the truck.
d) The truck accelerates but the mosquito does not.
e) The mosquito accelerates but the truck does not.

The Force on Accelerating Boxes



Q.23 The hand shown in the figure above pushes boxes A and B to the right across a frictionless table. The mass of B is larger than the mass of A.

a. Draw free-body diagrams of A, B, and the hand H, showing only the horizontal forces. Connect action/reaction pairs with dashed lines.

b. Rank in order, from largest to smallest, the horizontal forces shown on your free-body diagrams.

PROBLEM-SOLVING STRATEGY 7.1

Interacting-objects problems

MODEL Identify which objects are part of the system and which are part of the environment. Make simplifying assumptions.

VISUALIZE Draw a pictorial representation.

- Show important points in the motion with a sketch. You may want to give each object a separate coordinate system. Define symbols and identify what the problem is trying to find.
- Identify acceleration constraints.
- Draw an interaction diagram to identify the forces on each object and all action/reaction pairs.
- Draw a *separate* free-body diagram for each object. Each shows only the forces acting *on* that object, not forces exerted by the object.
- Connect the force vectors of action/reaction pairs with dashed lines. Use subscript labels to distinguish forces that act independently on more than one object.

PROBLEM-SOLVING STRATEGY 7.1

Interacting-objects problems

MP

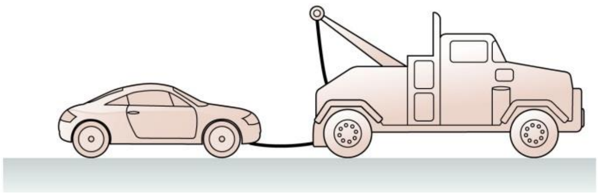
SOLVE Use Newton's second and third laws.

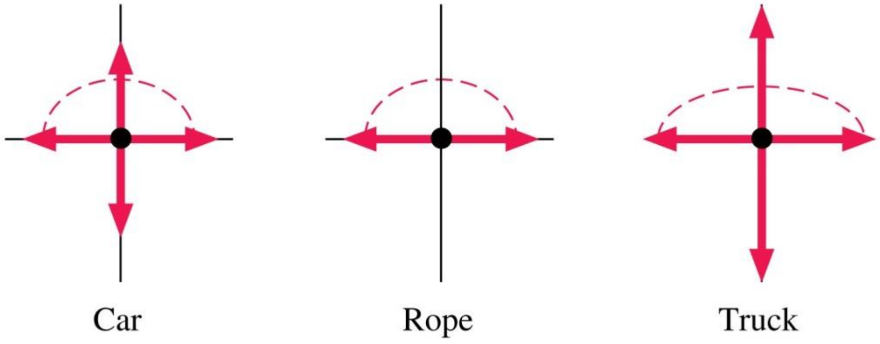
- Write the equations of Newton's second law for *each* object, using the force information from the free-body diagrams.
- Equate the magnitudes of action/reaction pairs.
- Include the acceleration constraints, the friction model, and other quantitative information relevant to the problem.
- Solve for the acceleration, then use kinematics to find velocities and positions.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Newton's 3rd Law

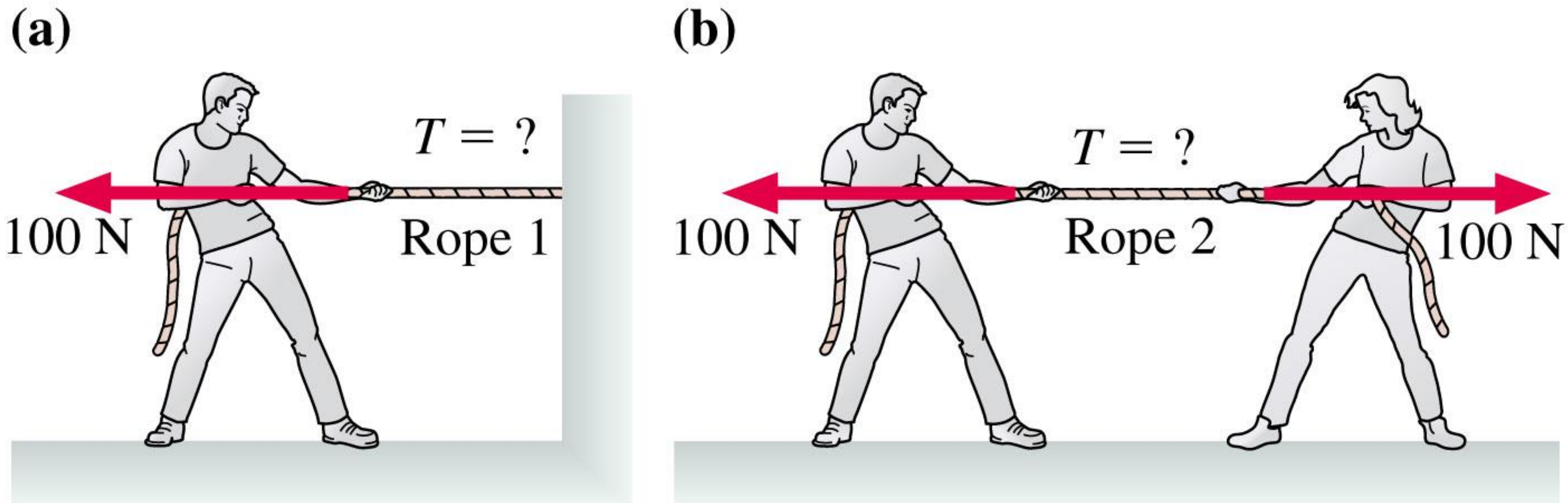
Q.24 What, if anything, is wrong with these free-body diagrams for a truck towing a car at steady speed? The truck is heavier than the car and the rope is massless.





a) Nothing is wrong.
b) One or more forces have the wrong length.
c) One of more forces have the wrong direction.
d) One or more action/reaction pairs are wrong.
e) Both B and D.

Pulling a Rope

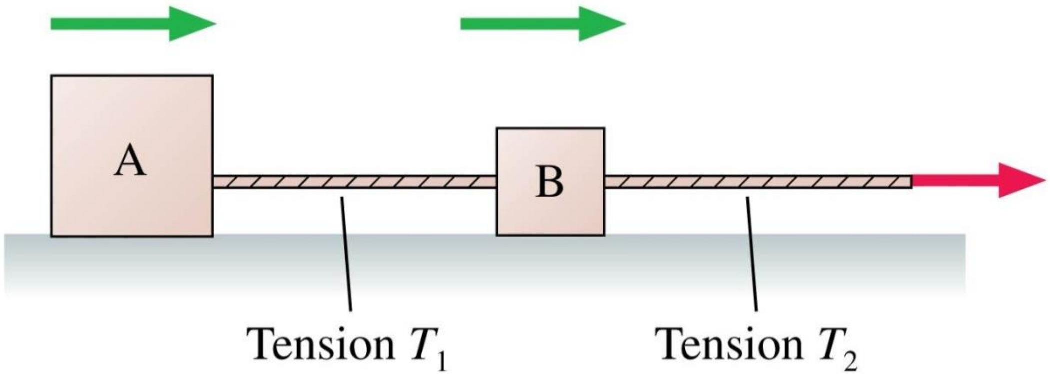


Q.25 Figure (a) shows a student pulling horizontally with a 100 N force on a rope that is attached to a wall. In figure (b) two students in a tug-of-war pull on opposite ends of a rope with 100 N each. Is the tension in the second rope larger than, smaller than, or the same as that in the first rope?

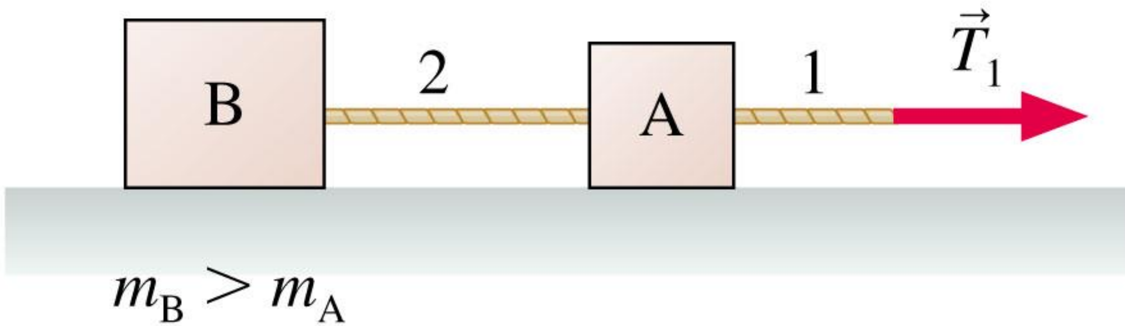
Tension Forces

Q.26 Boxes A and B are being pulled to the right on a frictionless surface. Box A has a larger mass than B. How do the two tension forces compare?

a) $T_1 > T_2$
b) $T_1 = T_2$
c) $T_1 < T_2$
d) Not enough information to tell.



Comparing Two Tensions



Q.27 Blocks A and B in the figure below are connected by massless string 2 and pulled across a frictionless table by massless string 1. B has a larger mass than A. Is the tension in string 2 larger than, smaller than, or equal to the tension in string 1?