University of Nottingham
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Foundation Calculus & Mathematical Techniques — CELEN037

Weekly Worksheet-10 — **Topics:** Differential Equation and its applications

Type 1: Solving Variable-Separable form ordinary differential equations (ODEs):

1. Solving the following Variable-Separable form ODEs:

(i)	$\dfrac{dy}{dx} = -\dfrac{x}{y}$	(ii)	$\dfrac{dy}{dx} = \dfrac{y}{x}$
(iii)	$\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$	(iv)	$\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$
(v)	$\dfrac{dy}{dx} = \dfrac{y^2}{x+1}$	(vi)	$x^2 \dfrac{dy}{dx} = y+1$
(vii)	$\dfrac{dy}{dx} = x^2\,(1+y^2)$	$(viii)$	$x\dfrac{dy}{dx} = y + xy$
(ix)	$(1+y^2)\dfrac{dy}{dx} = y\,e^x$	(x)	$y\dfrac{dy}{dx} = (1+y^2)\,\tan x$
(xi)	$\dfrac{dy}{dx} = \tan x\,\tan y$	(xii)	$\ln(\sin x)\dfrac{dy}{dx} = \cot x$
$(xiii)$	$\dfrac{dy}{dx} + y = y\,\sec^2 x$	(xiv)	$\dfrac{dy}{dx} = \dfrac{\tan y}{x\,\sec^2 y}$
(xv)	$(e^x + e^{-x})\dfrac{dy}{dx} = e^x - e^{-x}$	(xvi)	$\dfrac{dy}{dx} = \dfrac{x\,e^x}{y\sqrt{1+y^2}}$
$(xvii)$	$\dfrac{dy}{dx} = e^{x-y} + x^2\,e^{-y}$	$(xviii)$	$\dfrac{dy}{dx} = \dfrac{y\,\cos x}{1+\sin x}$
(xix)	$x\,dy = (2x^2+1)\,dx$	(xx)	$y\,(1+e^x)\,dy = (y+1)\,e^x\,dx$
(xxi)	$xy\,(y+1)\,dy = (x^2+1)\,dx$	$(xxii)$	$y\,\ln y\,dx = x\,dy$
$(xxiii)$	$\operatorname{cosec} y\,dx + \cos^2 x\,dy = 0$	$(xxiv)$	$(x+1)\,dy - (y-1)\,dx = 0.$

Type 2: Solving initial value problems (IVPs) of Variable-Separable form:

2. Solving the following IVPs of Variable-Separable form:

(i)	$\dfrac{dy}{dx} = \dfrac{x^2}{y^2}\quad;\quad y(0)=2$	(ii)	$\dfrac{dy}{dx} + 4xy^2 = 0\quad;\quad y(0)=1$
(iii)	$\dfrac{dy}{dx} = \dfrac{y\,\sin x}{1+y^2}\quad;\quad y(0)=1$	(iv)	$\dfrac{dy}{dx} = y\,\tan x\quad;\quad y(0)=1$

(v) $\quad \dfrac{dy}{dx} = \dfrac{(\sec x \, \tan x) \cdot (y+1)}{\sec x + 1} \quad ; \quad y(0) = 1$

(vi) $\quad x + 2y\sqrt{x^2+1}\,\dfrac{dy}{dx} = 0 \quad ; \quad y(0) = 1$

(vii) $\quad x \sin y \, dx + (x^2+1) \cos y \, dy = 0 \quad ; \quad y(1) = \dfrac{\pi}{2}$

$(viii)$ $\quad e^{\frac{dy}{dx}} = x + 1 \quad (x > -1) \quad ; \quad y(0) = 3.$

Type 3: Solution of differential equations:

3. (i) Show that $y = C_1 \sin 4x + C_2 \cos 4x$ is a solution of the ODE $\dfrac{d^2y}{dx^2} + 16\,y = 0$, where C_1 and C_2 are arbitrary constants.

   (ii) Show that $y = C_1 e^{2x} + C_2 e^{3x}$ is a solution of the ODE $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$, where C_1 and C_2 are arbitrary constants.

   (iii) Show that $y = C_1 e^{-2x} + C_2 e^{x}$ is a solution of the ODE $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 0$, where C_1 and C_2 are arbitrary constants.

   (iv) Show that $y = a \cos^{-1} x + b$ $\quad (a,\, b$ are arbitrary constants) is a solution of the differential equation $(1-x^2)\,\dfrac{d^2y}{dx^2} - x\,\dfrac{dy}{dx} = 0.$

   (v) Show that $y = e^{-x} + ax + b$ is a solution of the ODE $e^{x}\left(\dfrac{d^2y}{dx^2}\right) - 1 = 0$, where a and b are arbitrary constants.

   (vi) Show that $y = \dfrac{a}{x} + b$, where a and b are arbitrary constants, is a solution of the ODE $\dfrac{d^2y}{dx^2} + \dfrac{2}{x}\,\dfrac{dy}{dx} = 0.$

Type 4: Applications of ODE of Variable-Separable form:

4. (i) The rate of increase of population of a country is proportional to the population (P) at that time. Formulate a differential equation to show that the population at time t is $P = P_0 \cdot e^{kt}$, where $k > 0$ is constant and P_0 is the initial population.

   (ii) The rate of decay of a radio active material is proportional to the amount (m) of material present at that time.

   (a) Formulate a differential equation to show that the amount of material at time t is $m = m_0 \cdot e^{kt}$ where $k < 0$ is constant and m_0 is the initial amount.

   (b) If it takes 2000 years for half the original amount to decay, find the percentage of the original amount that remains after 400 years.

   (iii) The population of a city increases at the rate of 2% per year. How many year will it take for the population to double.