CELEN037 Seminar 2

Topics

- Chain Rule for Differentiation
- The Fast-Track Chain Rule Method
- Logarithmic Differentiation
- Implicit Differentiation
- Derivatives of Inverse Functions

Chain Rule for Differentiation

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example: Given $y = \ln(\sec x)$, find $\frac{dy}{dx}$ using Chain Rule

Chain Rule for Differentiation

University of Nottingham
UK | CHINA | MALAYSIA

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example: Given $y = \ln(\sec x)$, find $\dfrac{dy}{dx}$ using Chain Rule

Solution:

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example: Given $y = \ln(\sec x)$, find $\dfrac{dy}{dx}$ using Chain Rule

Solution:

Let $u = \sec x$, so that $y = \ln u$.

University of Nottingham
UK | CHINA | MALAYSIA

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example: Given $y = \ln(\sec x)$, find $\frac{dy}{dx}$ using Chain Rule

Solution:

Let $u = \sec x$, so that $y = \ln u$.

Then $\frac{du}{dx} = \sec x \cdot \tan x$, and $\frac{dy}{du} = \frac{1}{u}$.

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example: Given $y = \ln(\sec x)$, find $\dfrac{dy}{dx}$ using Chain Rule

Solution:

Let $u = \sec x$, so that $y = \ln u$.

Then $\dfrac{du}{dx} = \sec x \cdot \tan x$, and $\dfrac{dy}{du} = \dfrac{1}{u}$.

Hence $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{u} \cdot \sec x \cdot \tan x$

University of Nottingham
UK | CHINA | MALAYSIA

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example: Given $y = \ln(\sec x)$, find $\frac{dy}{dx}$ using Chain Rule

Solution:

Let $u = \sec x$, so that $y = \ln u$.

Then $\frac{du}{dx} = \sec x \cdot \tan x$, and $\frac{dy}{du} = \frac{1}{u}$.

$$\text{Hence } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \sec x \cdot \tan x$$

$$= \frac{1}{\sec x} \cdot \sec x \cdot \tan x$$

University of Nottingham
UK | CHINA | MALAYSIA

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example: Given $y = \ln(\sec x)$, find $\dfrac{dy}{dx}$ using Chain Rule

Solution:

Let $u = \sec x$, so that $y = \ln u$.

Then $\dfrac{du}{dx} = \sec x \cdot \tan x$, and $\dfrac{dy}{du} = \dfrac{1}{u}$.

$$\text{Hence } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \sec x \cdot \tan x$$
$$= \frac{1}{\sec x} \cdot \sec x \cdot \tan x$$
$$= \tan x$$

Practice Problems on Worksheet:

1: Q1(ii)

2: Q1(vii)

3: Q1(viii)

4: Q1(ix)

Practice Problems on Worksheet:

1: Q1(ii)

2: Q1(vii)

3: Q1(viii)

4: Q1(ix)

Answers:

1: $-\dfrac{\cos(\cos(\ln x)) \cdot \sin(\ln x)}{x}$

2: $-\tan x \cdot \cos(\ln(\cos x))$

3: $-\dfrac{\sec^2(\cos(\sqrt{x})) \cdot \sin(\sqrt{x})}{2\sqrt{x}}$

4: $e^x \cdot \cot(e^x)$

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$$

University of Nottingham
UK | CHINA | MALAYSIA

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$$

Example: Given $y = \sqrt{\sin(e^{\cos x})}$, find $\frac{dy}{dx}$ using Chain Rule

University of Nottingham
UK | CHINA | MALAYSIA

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$$

Example: Given $y = \sqrt{\sin(e^{\cos x})}$, find $\dfrac{dy}{dx}$ using Chain Rule

Solution:

University of Nottingham UK | CHINA | MALAYSIA

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$$

Example: Given $y = \sqrt{\sin(e^{\cos x})}$, find $\frac{dy}{dx}$ using Chain Rule

Solution:

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\sqrt{\sin(e^{\cos x})}} \cdot \frac{d}{dx}\left(\sin(e^{\cos x})\right)$$

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$$

Example: Given $y = \sqrt{\sin(e^{\cos x})}$, find $\dfrac{dy}{dx}$ using Chain Rule

Solution:

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} \cdot \frac{1}{\sqrt{\sin(e^{\cos x})}} \cdot \frac{d}{dx}\left(\sin(e^{\cos x})\right) \\ &= \frac{1}{2\sqrt{\sin(e^{\cos x})}} \cdot \cos\left(e^{\cos x}\right) \cdot \frac{d}{dx}\left(e^{\cos x}\right)\end{aligned}$$

University of Nottingham
UK | CHINA | MALAYSIA

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$$

Example: Given $y = \sqrt{\sin(e^{\cos x})}$, find $\frac{dy}{dx}$ using Chain Rule

Solution:

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} \cdot \frac{1}{\sqrt{\sin(e^{\cos x})}} \cdot \frac{d}{dx}\left(\sin(e^{\cos x})\right) \\ &= \frac{1}{2\sqrt{\sin(e^{\cos x})}} \cdot \cos\left(e^{\cos x}\right) \cdot \frac{d}{dx}\left(e^{\cos x}\right) \\ &= \frac{\cos\left(e^{\cos x}\right)}{2\sqrt{\sin(e^{\cos x})}} \cdot e^{\cos x} \cdot \frac{d}{dx}(\cos x)\end{aligned}$$

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$$

Example: Given $y = \sqrt{\sin(e^{\cos x})}$, find $\dfrac{dy}{dx}$ using Chain Rule

Solution:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{1}{2} \cdot \frac{1}{\sqrt{\sin(e^{\cos x})}} \cdot \frac{d}{dx}\left(\sin(e^{\cos x})\right) \\
&= \frac{1}{2\sqrt{\sin(e^{\cos x})}} \cdot \cos\left(e^{\cos x}\right) \cdot \frac{d}{dx}\left(e^{\cos x}\right) \\
&= \frac{\cos\left(e^{\cos x}\right)}{2\sqrt{\sin(e^{\cos x})}} \cdot e^{\cos x} \cdot \frac{d}{dx}(\cos x) \\
&= -\frac{\cos\left(e^{\cos x}\right) \cdot e^{\cos x} \cdot \sin x}{2\sqrt{\sin(e^{\cos x})}}
\end{aligned}$$

University of Nottingham
UK | CHINA | MALAYSIA

Practice Problems on Worksheet:

1: Q2(ii)
2: Q2(iii)
3: Q2(iv)
4: Q2(v)

Practice Problems on Worksheet:

1: Q2(ii)

2: Q2(iii)

3: Q2(iv)

4: Q2(v)

Answers:

1: $-\dfrac{\cos(\cos(\ln x)) \cdot \sin(\ln x)}{x}$

2: $-\tan x \cdot \cos(\ln(\cos x))$

3: $-\dfrac{\sec^2(\cos(\sqrt{x})) \cdot \sin(\sqrt{x})}{2\sqrt{x}}$

4: $e^x \cdot \cot(e^x)$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\ln y = \ln\left((\tan x)^{\sin x}\right)$$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\begin{aligned}\ln y &= \ln\left((\tan x)^{\sin x}\right)\\ &= \sin x \cdot \ln(\tan x)\end{aligned}$$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\begin{aligned}\ln y &= \ln\left((\tan x)^{\sin x}\right)\\ &= \sin x \cdot \ln(\tan x)\end{aligned}$$

Then $\dfrac{d}{dx}(\ln y) = \dfrac{d}{dx}(\sin x \cdot \ln(\tan x))$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\begin{aligned}\ln y &= \ln\left((\tan x)^{\sin x}\right)\\ &= \sin x \cdot \ln(\tan x)\end{aligned}$$

Then $\dfrac{d}{dx}(\ln y) = \dfrac{d}{dx}(\sin x \cdot \ln(\tan x))$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \sin x \cdot \frac{1}{\tan x}\cdot \sec^2 x + \ln(\tan x)\cdot\cos x$$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\begin{aligned}\ln y &= \ln\left((\tan x)^{\sin x}\right)\\ &= \sin x \cdot \ln(\tan x)\end{aligned}$$

Then $\dfrac{d}{dx}(\ln y) = \dfrac{d}{dx}(\sin x \cdot \ln(\tan x))$

$$\begin{aligned}\frac{1}{y} \cdot \frac{dy}{dx} &= \sin x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot \cos x\\ &= \sec x + \ln(\tan x) \cdot \cos x\end{aligned}$$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\begin{aligned}\ln y &= \ln\left((\tan x)^{\sin x}\right)\\ &= \sin x \cdot \ln(\tan x)\end{aligned}$$

Then $\dfrac{d}{dx}(\ln y) = \dfrac{d}{dx}(\sin x \cdot \ln(\tan x))$

$$\begin{aligned}\frac{1}{y} \cdot \frac{dy}{dx} &= \sin x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot \cos x\\ &= \sec x + \ln(\tan x) \cdot \cos x\end{aligned}$$

Hence $\dfrac{dy}{dx} = y \cdot (\sec x + \ln(\tan x) \cdot \cos x)$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\begin{aligned} \ln y &= \ln\left((\tan x)^{\sin x}\right) \\ &= \sin x \cdot \ln(\tan x) \end{aligned}$$

Then $$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sin x \cdot \ln(\tan x))$$

$$\begin{aligned} \frac{1}{y} \cdot \frac{dy}{dx} &= \sin x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot \cos x \\ &= \sec x + \ln(\tan x) \cdot \cos x \end{aligned}$$

Hence $$\begin{aligned} \frac{dy}{dx} &= y \cdot (\sec x + \ln(\tan x) \cdot \cos x) \\ &= (\tan x)^{\sin x} \cdot (\sec x + \ln(\tan x) \cdot \cos x) \end{aligned}$$

Example: Given $y = (\tan x)^{\sin x}$, find $\dfrac{dy}{dx}$

Solution:

$$\ln y = \ln\left((\tan x)^{\sin x}\right)$$
$$= \sin x \cdot \ln(\tan x)$$

Then $$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sin x \cdot \ln(\tan x))$$
$$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot \cos x$$
$$= \sec x + \ln(\tan x) \cdot \cos x$$

Hence $$\frac{dy}{dx} = y \cdot (\sec x + \ln(\tan x) \cdot \cos x)$$
$$= (\tan x)^{\sin x} \cdot (\sec x + \ln(\tan x) \cdot \cos x)$$

Steps

$\Leftarrow$ 1. Take Logarithm on both sides

$\Leftarrow$ 2. Apply rules of logarithms

$\Leftarrow$ 3. Differentiate both sides w.r.t x

Practice Problems on Worksheet:

1: Q3(vii)
2: Q3(viii)
3: Q3(ix)
4: Q3(x)

Practice Problems on Worksheet:

1: Q3(vii)

2: Q3(viii)

3: Q3(ix)

4: Q3(x)

Answers:

1: $x^x(\ln x + 1)$

2: $\dfrac{\sqrt[x]{x}(1 - \ln x)}{x^2}$

3: $\cos(x^x)\, x^x(\ln x + 1)$

4: $\dfrac{\sqrt[3]{x}\tan^4 x}{\cos(e^x)}\left(\dfrac{1}{3x} + \dfrac{4\sec^2 x}{\tan x} + \tan(e^x)\cdot e^x\right)$

Example: Given $\ln(x+y) = \ln(xy) + 1$, find $\dfrac{dy}{dx}$

Example: Given $\ln(x+y) = \ln(xy) + 1$, find $\dfrac{dy}{dx}$

Solution: Differentiate both sides w.r.t. x:

Example: Given $\ln(x+y) = \ln(xy) + 1$, find $\dfrac{dy}{dx}$

Solution: Differentiate both sides w.r.t. x:

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = \frac{1}{xy}\left(y + x\cdot\frac{dy}{dx}\right) + 0$$

Example: Given $\ln(x+y) = \ln(xy) + 1$, find $\dfrac{dy}{dx}$

Solution: Differentiate both sides w.r.t. x:

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = \frac{1}{xy}\left(y + x\cdot\frac{dy}{dx}\right) + 0$$

$$\frac{dy}{dx}\left(\frac{1}{x+y} - \frac{1}{y}\right) = \frac{1}{x} - \frac{1}{x+y}$$

Example: Given $\ln(x+y) = \ln(xy) + 1$, find $\dfrac{dy}{dx}$

Solution: Differentiate both sides w.r.t. x:

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = \frac{1}{xy}\left(y + x\cdot\frac{dy}{dx}\right) + 0$$

$$\frac{dy}{dx}\left(\frac{1}{x+y} - \frac{1}{y}\right) = \frac{1}{x} - \frac{1}{x+y}$$

$$\frac{dy}{dx}\cdot\frac{-x}{y(x+y)} = \frac{y}{x(x+y)}$$

Example: Given $\ln(x+y) = \ln(xy) + 1$, find $\dfrac{dy}{dx}$

Solution: Differentiate both sides w.r.t. x:

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = \frac{1}{xy}\left(y + x\cdot\frac{dy}{dx}\right) + 0$$

$$\frac{dy}{dx}\left(\frac{1}{x+y} - \frac{1}{y}\right) = \frac{1}{x} - \frac{1}{x+y}$$

$$\frac{dy}{dx}\cdot\frac{-x}{y(x+y)} = \frac{y}{x(x+y)}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^2}{x^2}$$

Practice Problems on Worksheet:

1: Q4(vii)

2: Q4(viii)

3: Q4(ix)

4: Q4(x)

Practice Problems on Worksheet:

1: Q4(vii)

2: Q4(viii)

3: Q4(ix)

4: Q4(x)

Answers:

1: $\frac{dy}{dx} = \frac{-x-2xy+2y^2}{x^2-4xy+y}, \quad \left.\frac{dy}{dx}\right|_{(1,1)} = \frac{1}{2}$

2: $\frac{dy}{dx} = 1, \quad \left.\frac{dy}{dx}\right|_{(0,0)} = 1$

3: $\frac{dy}{dx} = \frac{3x^2-2xy-y^2}{x^2+2xy-3y^2}, \quad \left.\frac{dy}{dx}\right|_{(1,-1)} = -1$

4: $\frac{dy}{dx} = \frac{-2xy^3+5y^2+3y-2}{3x^2y^2-3x-10xy}, \quad \left.\frac{dy}{dx}\right|_{(2,1)} = -\frac{1}{7}$

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

University of Nottingham UK | CHINA | MALAYSIA

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

Example: Given $y = \sec^{-1}\left(\sqrt{1+x^2}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $x > 0$, find $\frac{dy}{dx}$

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

Example: Given $y=\sec^{-1}\left(\sqrt{1+x^2}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $x>0$, find $\frac{dy}{dx}$

Solution: Using $\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$, we can get

University of Nottingham
UK | CHINA | MALAYSIA

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

Example: Given $y=\sec^{-1}\left(\sqrt{1+x^2}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $x>0$, find $\frac{dy}{dx}$

Solution: Using $\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$, we can get

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+x^2}}\right)^2}}\cdot\frac{d}{dx}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$$

Example: Given $y = \sec^{-1}\left(\sqrt{1+x^2}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $x > 0$, find $\frac{dy}{dx}$

Solution: Using $\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$, we can get

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+x^2}}\right)^2}} \cdot \frac{d}{dx}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \frac{-1}{\sqrt{1-\frac{1}{1+x^2}}} \cdot \left(-\frac{1}{2}\right)\frac{1}{\sqrt{(1+x^2)^3}} \cdot 2x$$

University of Nottingham
UK | CHINA | MALAYSIA

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

Example: Given $y=\sec^{-1}\left(\sqrt{1+x^2}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $x>0$, find $\frac{dy}{dx}$

Solution: Using $\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$, we can get

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+x^2}}\right)^2}}\cdot\frac{d}{dx}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$=\frac{-1}{\sqrt{1-\frac{1}{1+x^2}}}\cdot\left(-\frac{1}{2}\right)\frac{1}{\sqrt{(1+x^2)^3}}\cdot 2x$$

$$=\frac{x}{\sqrt{\frac{x^2}{1+x^2}}\cdot\sqrt{(1+x^2)^3}}$$

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

Example: Given $y=\sec^{-1}\left(\sqrt{1+x^2}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $x>0$, find $\frac{dy}{dx}$

Solution: Using $\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$, we can get

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+x^2}}\right)^2}}\cdot\frac{d}{dx}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$=\frac{-1}{\sqrt{1-\frac{1}{1+x^2}}}\cdot\left(-\frac{1}{2}\right)\frac{1}{\sqrt{(1+x^2)^3}}\cdot 2x$$

$$=\frac{x}{\sqrt{\frac{x^2}{1+x^2}}\cdot\sqrt{(1+x^2)^3}}$$

$$=\underbrace{\frac{x}{\sqrt{x^2(1+x^2)^2}}=\frac{x}{x(1+x^2)}}_{x>0}$$

Derivatives of Inverse Trigonometirc Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}},\quad \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

Example: Given $y=\sec^{-1}\left(\sqrt{1+x^2}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $x>0$, find $\frac{dy}{dx}$

Solution: Using $\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$, we can get

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+x^2}}\right)^2}}\cdot\frac{d}{dx}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$=\frac{-1}{\sqrt{1-\frac{1}{1+x^2}}}\cdot\left(-\frac{1}{2}\right)\frac{1}{\sqrt{(1+x^2)^3}}\cdot 2x$$

$$=\frac{x}{\sqrt{\frac{x^2}{1+x^2}}\cdot\sqrt{(1+x^2)^3}}$$

$$=\underbrace{\frac{x}{\sqrt{x^2(1+x^2)^2}}=\frac{x}{x(1+x^2)}}_{x>0}$$

$$=\frac{1}{1+x^2}$$

Practice Problems on Worksheet:

1: Q5(iii)
2: Q5(iv)
3: Q5(v)
4: Q5(vi)

University of Nottingham
UK | CHINA | MALAYSIA

Practice Problems on Worksheet:

1: Q5(iii)

2: Q5(iv)

3: Q5(v)

4: Q5(vi)

Answers:

1: $\dfrac{1}{2(1+x^2)}$

2: $\dfrac{1}{2}$

3: $\dfrac{2}{1+x^2}$

4: $\dfrac{2(1-x^2)}{x^4+6x^2+1}$

Office Hours

Day	Time	Venue
Tuesday	12:00 to 14:00	Trent 314a
	13:00 to 14:00	Trent 322
Thursday	16:30 to 17:30	TB 417
	17:00 to 18:00	IAMET 315
Friday	14:00 to 15:00	PB 330
	17:00 to 18:00	TB 417