FOUNDATION SCIENCE A

SEMINAR 2: NEWTON'S LAW, WORK, MOMENTUM AND IMPULSE

LEARNING OUTCOMES

Students are expected to understand the concepts and resolve the following mechanical problems regarding:

- Newton’s Law
- Work, Conservation of Force and Energy
- Momentum and Impulse

USING NEWTON'S LAW

QUESTION 1:

How much tension must a rope withstand if it is used to accelerate a 1210 kg car horizontally along a frictionless surface at 1.20 m.s^{-2}?

Answer:

Use Newton's second law to calculate the tension.

$$\sum F = F_T = m.a$$

$$= 1210 \text{ kg} \times 1.20 \text{ m.s}^{-2}$$

$$= 1452 \text{ N}$$

USING NEWTON'S LAW

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QUESTION 2:

a) What is the acceleration of two falling sky divers (mass = 132 kg including parachute) when the upward force of air resistance is equal to one-fourth of their weight?

b) After popping open the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute?

USING NEWTON'S LAW

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Answer:

a) There will be two forces on the skydivers – their combined weight, and the upward force of air resistance F_A. Choosing upwards to be the positive direction. Write Newton's second law for the skydivers.

$\sum F = F_A$ - m.g = m.a   Therefore, $0.25 \times m \times g - m \times g = m.a$

$a = -0.75 \times g = -0.75 \times 9.80\ m.s^{-2}$

$= -7.35\ m.s^{-2}$

Due to the sign of the result, the direction of the **acceleration is down**.

USING NEWTON'S LAW

b) After popping open the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute?

Answer:

b) If they are descending at constant speed, then the net force on them must be zero, and so the force of air resistance must be equal to their weight.

$$F_A = m \times g$$

$$= 132 \times 9.80\ m.s^{-2}$$

$$= 1294\ N$$

USING NEWTON'S LAW

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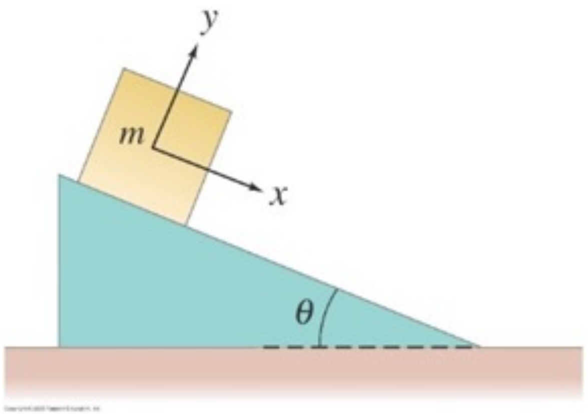
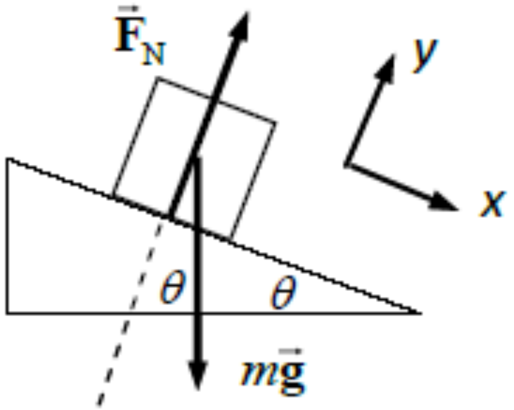
QUESTION 3:



The block shown in the figure below has a mass, m = 7.0 kg and lies on a fixed smooth frictionless plane tilted at an angle, θ = 22.0° to the horizontal.

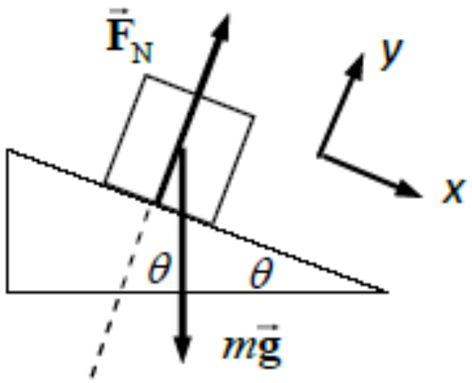
a) Determine the acceleration of the block as it slides down the plane.

b) If the block starts from rest 12.0 m up the plane from its base, what will be the block's speed when it reaches the bottom of the incline?



USING NEWTON'S LAW

Answer:



a) Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the y direction. Use Newton's second law for the x direction to find the acceleration.

$$\sum F_x = \text{m.g.sin}\,\theta = \text{ma}$$

$$\text{a} = \text{g.sin}\,\theta \quad = 9.80\ \text{m.s}^{-2} \times \sin 22° \qquad \text{a} = 3.67\ \text{m.s}^{-2}$$

b) Use the equation below, with $v_0 = 0\ \text{m.s}^{-1}$ to calculate the final speed.

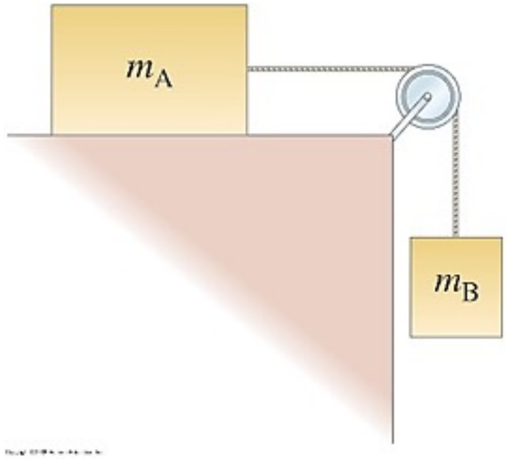
$$v^2 - v_0^2 = 2\text{a}(x - x_\text{p})$$

$$v = \sqrt{2\text{a}(x - x_0)} \qquad v = \sqrt{2 \times 3.67\ \text{m.s}^{-2} \times 12\ \text{m}} \qquad v = 9.39\ \text{m.s}^{-2}$$

USING NEWTON'S LAW

QUESTION 4:

a) If m_A = 13.0 kg and m_B = 5.0 kg, in the figure below, determine the acceleration of each block.



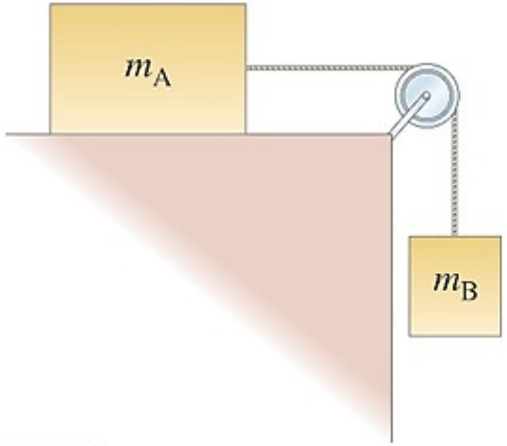
b) If initially m_A is at rest 1.25 m from the edge of the table, how long does it take to reach the edge of the table if the system is allowed to move freely?

c) If m_B = 1.0 kg, how large must m_A be if the acceleration of the system is to be kept at 0.01 g?

USING NEWTON'S LAW

a) If m_A = 13.0 kg and m_B = 5.0 kg, determine the acceleration of each block.

b) If initially m_A is at rest 1.25 m from the edge of the table, how long does it take to reach the edge of the table if the system is allowed to move freely?



Answer:

a) Both blocks have the same acceleration.

$$a = g.\frac{m_B}{m_A + m_B} \qquad a = 9.8\ \text{m.s}^{-2}.\ \frac{5.0\ \text{kg}}{5.0\ \text{kg} + 13\ \text{kg}} \qquad a = 2.722\ \text{m.s}^{-2}$$

b) Use the equation below to find the time.

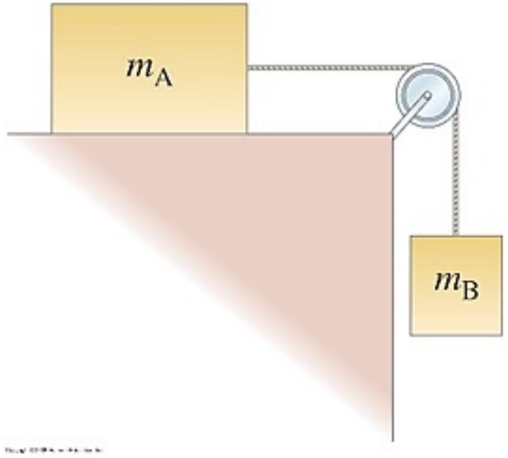
$$x - x_0 = v_0 t + \frac{1}{2}\ at^2 \qquad t = \sqrt{\frac{2(x - x_0)}{a}} \qquad t = \sqrt{\frac{2(1.25\ \text{m})}{2.722\ \text{m.s}^{-2}}} \qquad t = 0.96\ \text{s}$$

USING NEWTON'S LAW

c) If m_B = 1.0 kg, how large must m_A be if the acceleration of the system is to be kept at 0.01 g?

Answer:

c) Again use the acceleration equation from part a)



$$a = g.\frac{m_B}{m_A + m_B} = 0.01\, g$$

$$\frac{m_B}{m_A + m_B} = 0.01\, g$$

$$m_A = 99\, m_B$$

Therefore, m_A = 99 kg

FRICTION & NEWTON'S LAW

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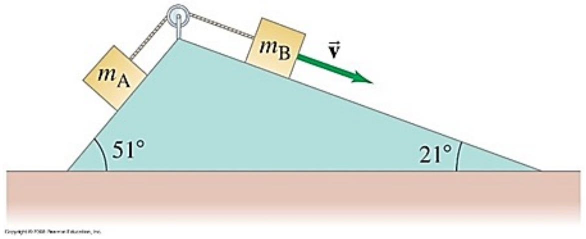
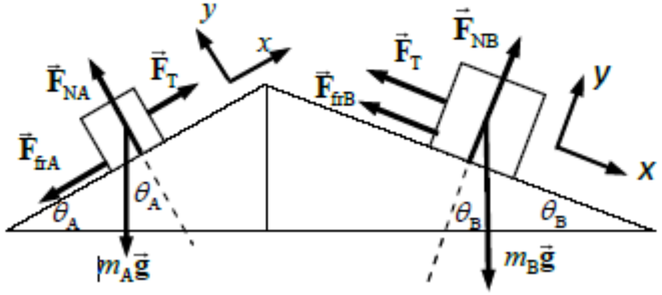
QUESTION 5:



Two masses m_A = 2.0 kg and m_B = 5.0 kg are on inclines and are connected together by a string.

The coefficient of kinetic friction (μ_k) between each mass and its incline, μ_k = 0.30.

If m_A moves up, and m_B moves down, determine their acceleration.



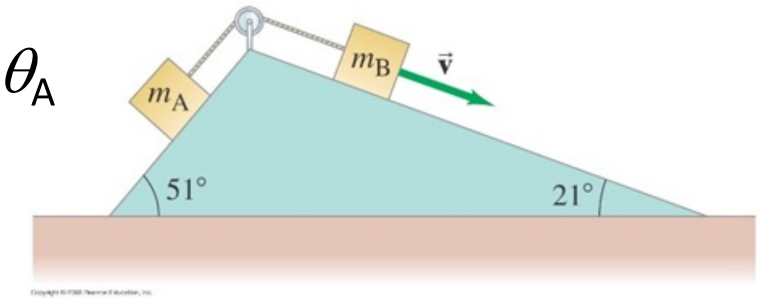
FRICTION & NEWTON'S LAW

Answer:

Block A:

$$\sum F = F_{yA} = F_{NA} - m_A.g.\cos\theta_A = 0 \quad F_{NA} = m_A.g.\cos\theta_A$$

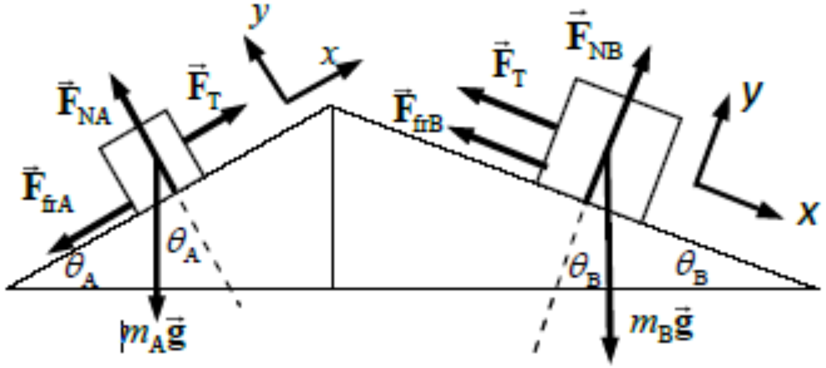
$$\sum F = F_{xA} = F_T - m_A.g.\sin\theta_A - F_{frA} = m_A.\,a$$



Block B:

$$\sum F = F_{yB} = F_{NB} - m_B.g.\cos\theta_B = 0 \quad F_{NB} = m_B.g.\cos\theta_B$$

$$\sum F = F_{xB} = m_B.g.\sin\theta_B - F_{frB} = m_B.\,a$$



FRICTION & NEWTON'S LAW

Add the final equations together from both analyses and solve for the acceleration, noting that in both cases the friction force is found as $F_{fr} = \mu F_N$.

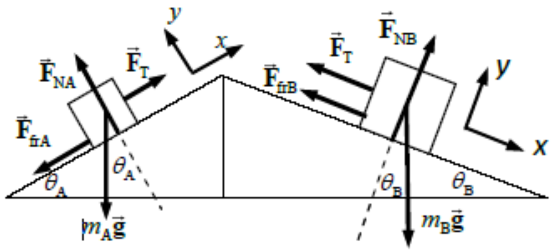
$$m_A.a = F_T - m_A.g.\sin\theta_A - \mu_A.m_A.g.\cos\theta_A \qquad m_B.a = m_B.g.\sin\theta_B - \mu_B.m_B.g.\cos\theta_B - F_T$$

$$m_A.a + m_B.a = F_T - m_A.g.\sin\theta_A - \mu_A.m_A.g.\cos\theta_A + m_B.g.\sin\theta_B - \mu_B.m_B.g.\cos\theta_B - F_T$$

$$a = g\left[\frac{-m_A(\sin\theta_A + \mu_A.\cos\theta_A) + m_B(\sin\theta_B - \mu_B.\cos\theta_B)}{m_A + m_B}\right]$$

$$= 9.80\ \mathrm{m.s^{-2}}\left[\frac{-2\mathrm{kg}(\sin 51^\circ + 0.3.\cos 51^\circ) + 5\mathrm{kg}(\sin 21^\circ - 0.3.\cos 21^\circ)}{m_A + m_B}\right]$$

$$a = -2.2\ \mathrm{m.s^{-2}}$$



WORK DONE BY A CONSTANT FORCE

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QUESTION 6:

Assuming zero air resistance, how high will a 1.85 kg rock go if thrown straight up by someone who does 80.0 J of work on it?

Answer:

The rock will rise until gravity does –80.0 J of work on the rock. The displacement is upwards, but the force is downwards, so the angle between them is 180°. Using the equation below

$$w_G = m.g.d.\cos\theta$$

$$d = \frac{w_G}{m.g.\cos\theta}$$

$$= \frac{-80\ \mathrm{J}}{1.85\ \mathrm{kg}\ (9.80\ \mathrm{m.s^{-2}}) \times -1}$$

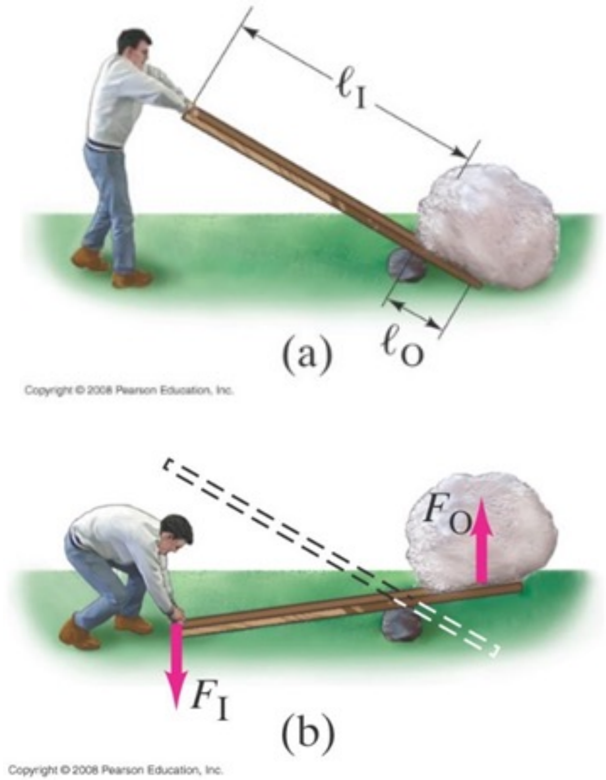
$d = 4.41$ m

WORK DONE BY A CONSTANT FORCE

QUESTION 7:

A lever such as that shown in the figure below can be used to lift objects, which might not otherwise be lifted. Show that the ratio of the output force, F_0, to input force, F_I, is related to the lengths l_I and l_0 from the pivot by $\frac{F_0}{F_I} = \frac{l_I}{l_0}$.

Assumptions: Ignore friction and the mass of the lever, and assume the work output equals work input.



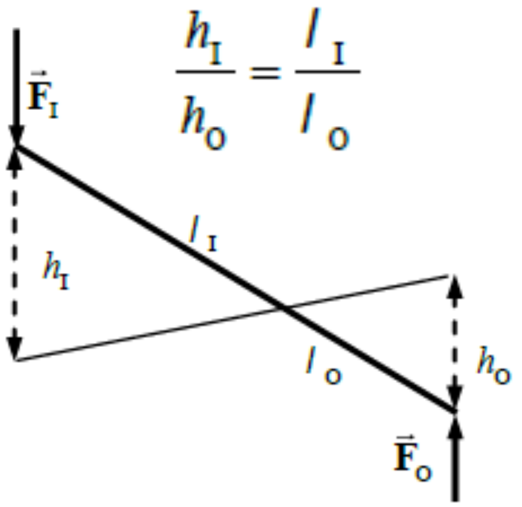
WORK DONE BY A CONSTANT FORCE

Answer:

Consider the diagram shown. If we assume that the person pushes straight down on the end of the lever, then the work done (the "input" work) is given by $W_I = F_I h_I$

The object moves a shorter distance, as seen from the diagram, and so $W_O = F_O h_O$.

Equating the two amounts of work.



$$W_O = W_I \quad \therefore \quad F_O h_O = F_I h_I$$

$$\& \quad \frac{h_I}{h_o} = \frac{I_I}{Io} \qquad \text{So,} \quad \frac{F_O}{F_I} = \frac{h_O}{h_I}$$

WORK DONE BY A VARYING FORCE

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QUESTION 8:

In pedaling a bicycle uphill, a cyclist exerts a downward force of 450 N during each stroke. If the diameter of the circle traced by each pedal is 36 cm, calculate how much work is done in each stroke.

Answer:

The downward force is 450 N, and the downward displacement would be a diameter of the pedal circle.

Using the following equation

$$w = F.d.\cos\theta \quad = 450\text{ N} \times 0.36\text{ m} \times \cos 0° \quad = 160\text{ J}$$

THE WORK-ENERGY PRINCIPLE

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QUESTION 9:

How much work is required to stop an electron (m = 9.11 x10^{-31} kg) which is moving with a speed of 1.40 x10^{6} m.s^{-1}?

Answer:

The work done on the electron is equal to the change in its kinetic energy.

$$w = \Delta K$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= 0 - \frac{1}{2} \times 9.11\ \text{x}10{-}31\ \text{kg} \times (1.40\ \text{x}106\ \text{m.s}^{-1})^2$$

$$= -8.93\ \text{x}10^{-19}\ \text{J}$$

Note that the work is negative since the electron is slowing down.

CONSERVATIVE FORCE & POTENTIAL ENERGY

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QUESTION 10:

A 1200 kg car rolling on a horizontal surface has velocity, v, = 75 km.h^{-1} when it strikes a horizontal coiled spring and is brought to rest in a distance of 2.2 m. What is the spring stiffness constant of the spring?

Answer:

Assume that all of the kinetic energy of the car becomes potential energy of the compressed spring.

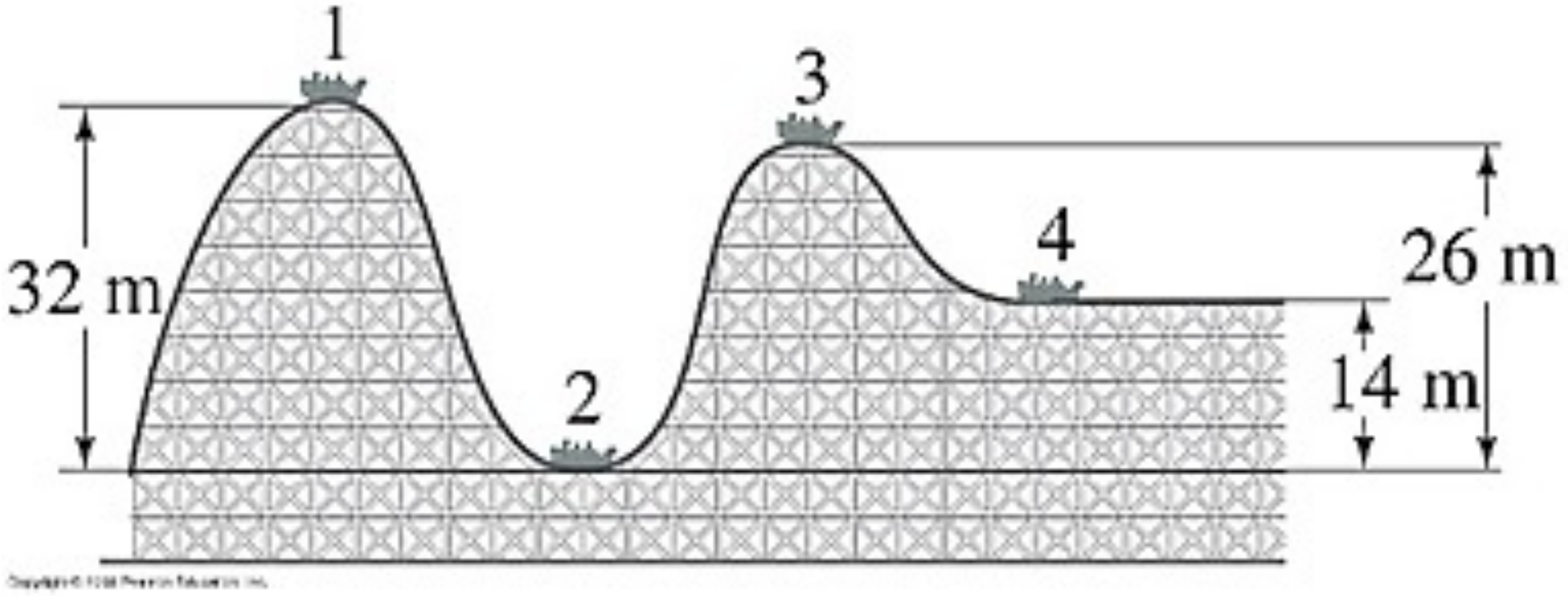
$$= \frac{1}{2}\mathrm{m}v_0^{\,2} \quad = \frac{1}{2}kx_f^{\,2}$$

$$k = \frac{mv_0^2}{x_f^{\,2}} \quad = \frac{1200\ \mathrm{kg}\left[\left(75\ \mathrm{km.h}^{-1}\right).\left(\frac{1\ \mathrm{m.s}^{-1}}{3.6\ \mathrm{km.h}^{-1}}\right)\right]}{(2.2\ \mathrm{m})^2} \qquad k = 1.1\ \mathrm{x}10^5\ \mathrm{N.m}^{-1}$$

CONSERVATION OF MECHANICAL ENERGY

QUESTION 11:

A roller-coaster car shown in the figure below is pulled up to point 1 where it is released from rest. Assuming no friction, calculate the speed at points 2, 3 and 4.



CONSERVATION OF MECHANICAL ENERGY

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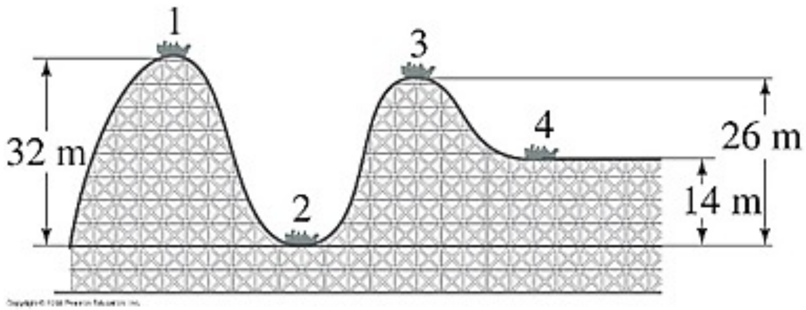
Answer:

Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational potential energy. Let $v_1 = 0$ m.s^{-1} and $y_1 = 32$ m.



At Point 2,
$y_2 = 0$ m

$$\frac{1}{2}\text{m}.v_1^2 + \text{m}.g.y_1 = \frac{1}{2}\text{m}.v_2^2 + \text{m}.g.y_2$$

y = 0, so

$$\text{m}.g.y_1 = \frac{1}{2}\text{m}.v_2^2$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{m.s}^{-1})(32\text{ m})}$$

$v_2 = 25$ m.s^{-1}

CONSERVATION OF MECHANICAL ENERGY

Answer:

At Point 3,
y_3 = 26 m

$$\frac{1}{2}\mathrm{m}.v_1^{\,2} + \mathrm{m}.g.\mathrm{y}_1 = \frac{1}{2}\mathrm{m}.v_3^{\,2} + \mathrm{m}.g.\mathrm{y}_3 \qquad \mathrm{m}.g.\mathrm{y}_1 = \frac{1}{2}\mathrm{m}.v_3^{\,2} + \mathrm{m}.g.\mathrm{y}_3$$

$$v_3 = \sqrt{2g(y_1 - y_3)}$$

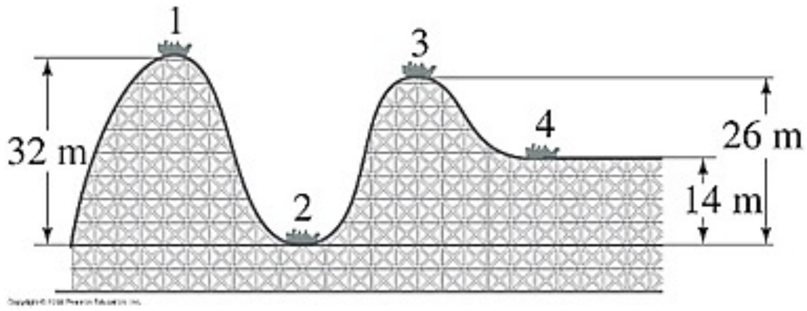
$$v_3 = \sqrt{2(9.80\mathrm{m.s}^{-1})(6\ \mathrm{m})} \qquad v_3 = 11\ \mathrm{m.s}^{-1}$$

At Point 4,
y_4 = 14 m

$$\frac{1}{2}\mathrm{m}.v_1^{\,2} + \mathrm{m}.g.\mathrm{y}_1 = \frac{1}{2}\mathrm{m}.v_4^{\,2} + \mathrm{m}.g.\mathrm{y}_4 \qquad \mathrm{m}.g.\mathrm{y}_1 = \frac{1}{2}\mathrm{m}.v_4^{\,2} + \mathrm{m}.g.\mathrm{y}_4$$

$$v_4 = \sqrt{2g(y_1 - y_4)}$$

$$v_4 = \sqrt{2(9.80\mathrm{m.s}^{-1})(14\ \mathrm{m})} \qquad v_4 = 19\ \mathrm{m.s}^{-1}$$



MOMENTUM

QUESTION 12:

Air in a 120 km.h^{-1} wind strikes head-on the face of a building 45 m wide × 65 m high and is brought to rest.

If air has a density of 1.3 kg.m^{-3}, determine the average force of the wind on the building.

Answer:

- The air is moving with an initial speed of 120 km.h^{-1} so, $\left(\frac{1\ \mathrm{m.s^{-1}}}{3.6\ \mathrm{km.h^{-1}}}\right)$ = 33.33 m.s^{-1}.
- Thus, in 1 second, a volume of air measuring 45 m x 65 m x 33.33 m will be brought to rest.

MOMENTUM

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Answer:

- By Newton's third law, the average force on the building will be equal in magnitude to the force causing the change in momentum of the air.

- The mass of the stopped air is its volume times its density.

$$F = \frac{\Delta P}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{V\rho\Delta v}{\Delta t}$$

$$= \frac{(45\text{ m})(65\text{ m})(33.33\text{ m})(1.3\text{ kg.m}^{-3})(33.33 - 0\text{ m.s}^{-1})}{1\text{ s}}$$

$$F = 4.2 \times 10^{6}\text{ N}$$

CONSERVATION OF MOMENTUM

QUESTION 13:

A 9150 kg railroad car travels alone on a level frictionless track with a constant speed of 15.0 m.s^{-1}. A 4350 kg load, initially at rest, is dropped onto the railroad car.
What will be the car's new speed?

Answer:

Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let A represent the car and B represent the load. The positive direction is the direction of the original motion of the car.

$$p_{\text{initial}} = p_{\text{final}} \qquad m_A.v_\text{A} + m_B.v_\text{B} = v'(m_A + m_B)$$

$$v' = \frac{m_A.v_\text{A} + m_B.v_\text{B}}{m_A + m_B} = \frac{(9150\ \text{kg})(15.0\ \text{m.s}^{-1})}{9150\ \text{kg} + 4350\ \text{kg}} \qquad v' = 10.2\ \text{m.s}^{-1}$$

CONSERVATION OF MOMENTUM

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QUESTION 14:

A 22 g bullet traveling 210 m.s^{-1} penetrates a 2.0 kg block of wood and emerges going 150 m.s^{-1}. If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?

Answer:

Consider the motion in one dimension with the positive direction being the direction of motion of the bullet. Let A represent the bullet and B represent the block. Since there is no net force outside of the block-bullet system (like friction with the table), the momentum of the block and bullet combination is conserved. Note that v_B= 0 m.s^{-1}

$$p_{\text{initial}} = p_{\text{final}} \qquad m_A.v_A + m_B.v_B = m_A.v'_A + m_B.v'_B \qquad v'_B = \frac{m_A(v_A - v'_A)}{m_B}$$

$$v'_B = \frac{(0.022\ \text{kg})(210 - 150\ \text{m.s}^{-1}}{2.0\ kg} \qquad v'_B = 0.66\ \text{m.s}^{-1}$$

COLLISION & IMPULSE

QUESTION 15:

A 12 kg hammer strikes a nail at a velocity of 8.5 m.s^{-1} and comes to rest in a time interval of 8.0 ms.

a) What is the impulse given to the nail?

b) What is the average force acting on the nail?

Answer:

a) The impulse given to the nail is the opposite of the impulse given to the hammer. This is the change in momentum. Call the direction of the initial velocity of the hammer the +ve direction.

$$\Delta p_{\text{nail}} = -\Delta p_{\text{hammer}} \quad = mv_i - mv_f \quad = (12\text{ kg})(8.5\text{ m.s}^{-1}) - 0 \quad = 1.0 \times 10^2\text{ kg.m.s}^{-1}$$

b) The average force is the impulse divided by the time of contact.

$$F_{avg} = \frac{\Delta P}{\Delta t} \quad = \frac{1.0 \times 10^2\text{ kg.m.s}^{-1}}{8.0 \times 10^{-3}\text{ s}} \quad = 1.3 \times 10^4\text{ N}$$

COLLISION & IMPULSE

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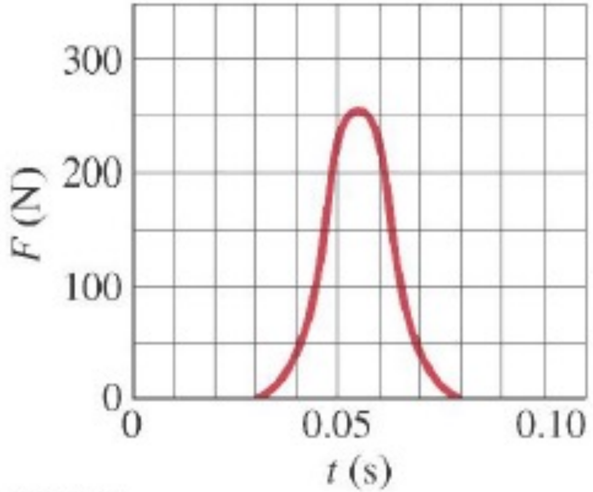
QUESTION 16:

Suppose the force acting on a tennis ball (m = 0.060 kg) points in the $+x$ directior and is given by the graph of the figure, as a function of time. Use graphical methods to estimate

a) the total impulse given to the ball, and

b) the velocity of the ball after being struck, assuming the ball is being served so it is nearly at rest initially.



Answer:

a) The impulse given the ball is the area under the *F* vs *t* graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.04 sec. A width slightly smaller than the base was chosen to compensate for the "inward" concavity of the force graph. $\Delta p = \frac{1}{2}(250\text{ N})(0.04\text{ s})$ $\quad \Delta p = 5\text{ N.s}$

b) The velocity can be found from the change in momentum. Call the +ve direction the direction of the ball's travel after being served.

$$\Delta p = m\Delta v \quad = m(v_f - v_i) \quad v_f = v_i + \frac{\Delta p}{m} \quad = 0 + \frac{5\text{ N.s}}{6.0\times10^{-2}\text{ kg}} \quad v_f = 80\text{ m.s}^{-1}$$

Q&A?
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