University of Nottingham
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Foundation Calculus and Mathematical Techniques (CELEN037)

Problem Sheet 2 **Topics: Differentiation**

Topic 1: Chain Rule

1. Find the derivative $\frac{dy}{dx}$ of the following functions

   (i) $y = \sqrt{\sin x}$ (ii) $y = e^{-3x^2}$

   (iii) $y = \ln(\cot 3x)$ (iv) $y = \frac{1 + \ln x}{1 - \ln x}$

2. Use the Chain Rule to find the derivative $\frac{dy}{dx}$ of $y = \sin(\sec(\ln(3^x)))$.

Topic 2: Logarithmic Differentiation

3. Use logarithmic differentiation to find the derivative $\frac{dy}{dx}$ of the following functions

   (i) $y = \left(\frac{1}{x}\right)^x$ (ii) $y = e^{x^2 \cdot \cos x}$

   (iii) $y = \sqrt{x} \cdot e^{x^2} \cdot \left(x^2 + 1\right)^3$ (iv) $y = \frac{\sqrt{x^2 - 1} \cdot \sin x}{(2x + 3)^4}$

4. Use logarithmic differentiation to find the derivative $\frac{dy}{dx}$ of $y = \sec(x^{\ln x})$.

Topic 3: Implicit Differentiation

5. Use implicit differentiation to find the derivative $\frac{dy}{dx}$ of the following functions

   (i) $e^y \cdot \cos x = 1 + \sin(xy)$ (ii) $y^2 = \ln(x^2 + y^2)$

   (iii) $\ln(xy) + 2x - y = 1$ (iv) $\tan(x - y) = \frac{y}{1 + x^2}$

6. Find the gradient of $x^2 + 2xy - 2y^2 + x = 2$ at the point $(4, -1)$.

Topic 4: Derivatives of Inverse Functions

7. Find the derivative $\frac{dy}{dx}$ of the following functions

   (i) $y = \tan^{-1}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$ (ii) $y = \cos^{-1}\left(e^{2x}\right)$

   (iii) $y = \sin^{-1}(e^{\sin x})$ (iv) $y = \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$

8. Use the definition of the derivative of an inverse function to find $\frac{dy}{dx}$ for $x = \cos^{-1}(\sqrt{1-y^2})$.
   Hint: $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.

Answers

1. (i) $\frac{\cos x}{2\sqrt{\sin x}}$ (ii) $-6x \cdot e^{-3x^2}$ (iii) $-\frac{3}{\sin 3x \cdot \cos 3x}$ (iv) $\frac{2}{x(1-\ln x)^2}$
2. $\cos(\sec(\ln(3^x))) \cdot \sec(\ln(3^x)) \cdot \tan(\ln(3^x)) \cdot \ln 3$
3. (i) $-x^{-x}(1+\ln x)$
   (ii) $x \cdot e^{x^2 \cdot \cos x} \cdot (2\cos x - x \sin x)$
   (iii) $\sqrt{x} \cdot e^{x^2} \left(x^2+1\right)^3 \left(\frac{1}{2x} + 2x + \frac{6x}{x^2+1}\right)$
   (iv) $\frac{\sqrt{x^2-1} \cdot \sin x}{(2x+3)^4} \cdot \left(\frac{x}{x^2-1} + \cot x - \frac{8}{2x+3}\right)$
4. $\frac{2\sec(x^{\ln x}) \cdot \tan(x^{\ln x}) \cdot x^{\ln x} \cdot \ln x}{x}$
5. (i) $\frac{e^y \sin x + y\cos(xy)}{e^y \cos x - x\cos(xy)}$ (ii) $\frac{x}{(x^2+y^2-1)y}$
   (iii) $\frac{y(1+2x)}{x(y-1)}$ (iv) $\frac{\sec^2(x-y)\left(1+x^2\right)^2 + 2xy}{\sec^2(x-y)\left(1+x^2\right)^2 + 1 + x^2}$
6. $-\frac{7}{12}$
7. (i) $\left(1 + \frac{2}{e^{2x}+e^{-2x}}\right) \cdot \ln(e^x + e^{-x})$ (ii) $\frac{-2e^{2x}}{\sqrt{1-e^{4x}}}$
   (iii) $\frac{e^{\sin x} \cdot \cos x}{\sqrt{1-e^{2\sin x}}}$ (iv) 0
8. $\pm\sqrt{1-y^2}$