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Foundation Calculus & Mathematical Techniques — CELEN037

Weekly Worksheet-9 **Topics:** Applications of (Definite) Integration, Numerical Integration

Type 1: Area calculation using definite integrals:

1. (i) Find the area of the region bounded by the curve $y = x^{3/2}$, lines $x = 1$, $x = 3$, and the X-axis.

   (ii) Find the area of the region bounded by the curves $x^2 = y$ and $x = y - 2$

   (iii) Find the area of the region bounded by the curves $y = x^2$ and $y = 2x - x^2$.

Type 2: Volume calculation using definite integrals:

Type 2A: Volume of solid of revolution:

(1) If the region bounded by the curve $y = f(x)$, lines $x = a$, $x = b$, and the X-axis is revolved about the X-axis, then the volume of the solid of revolution is:

$$V = \pi \int_a^b y^2 \ dx = \pi \int_a^b [\ f(x)\ ]^2 \ dx$$

(2) If the region bounded by the curve $x = g(y)$, lines $y = c$, $y = d$, and the Y-axis is revolved about the Y-axis, then the volume of the solid of revolution is:

$$V = \pi \int_c^d x^2 \ dy = \pi \int_c^d [\ g(y)\ ]^2 \ dy$$

2. (i) Find the volume of solid of revolution when region bounded by the curve $y = x$, lines $x = 0$ and $x = 4$ is revolved about X-axis.

   (ii) Find the volume of solid of revolution when region bounded by the curve $y = \sin x$, lines $x = 0$ and $x = \pi$ is revolved about X-axis.

   (iii) Find the volume of solid of revolution when region bounded by the curve $y = (x + 2)^2$, lines $x = 0$ and $x = 1$ is revolved about X-axis.

   (iv) Find the volume of solid of revolution when region bounded by the curve $x = y^2$, lines $y = 0$ and $y = 2$ is revolved about Y-axis.

   (v) Find the volume of solid of revolution when region bounded by the curve $x = e^y$, lines $y = 0$ and $y = 1$ is revolved about Y-axis.

Type 2B: Volume of solid of revolution:

(3) If the region bounded by two curves $y = f_1(x)$ and $y = f_2(x)$ between the points (of intersection) $x = a,\ x = b$ is revolved about the X-axis, then the volume of the solid of revolution is:

$$V = \pi \left| \int_a^b \left[f_1(x) \right]^2 - \left[f_2(x) \right]^2 \ dx \right|$$

(4) If the region bounded by two curves $x = g_1(y)$ and $x = g_2(y)$ between the points (of intersection) $y = c,\ y = d$ is revolved about the Y-axis, then the volume of the solid of revolution is:

$$V = \pi \left| \int_c^d \left[g_1(y) \right]^2 - \left[g_2(y) \right]^2 \ dy \right|$$

3. (i) Find the volume of solid of revolution when region bounded by curves $x = y^2$ and $x = y + 2$ is revolved about Y-axis.

(ii) The region bounded by curves $y = \sin x,\ y = \cos x$ and lines $x = 0,\ x = \pi/4$ is revolved about X-axis. Find the volume of solid of revolution.

(iii) Find the volume of solid of revolution when region bounded by curves $y = x^2$ and $x = y^2$ is revolved about Y-axis.

Numerical Integration

Type 3: Trapezoidal rule (Trapezium rule):

$$\int_a^b f(x)\ dx \approx \frac{h}{2} \cdot \left[f_0 + 2\left(f_1 + f_2 + \ldots\ldots + f_{n-1} \right) + f_n \right]$$

4. (i) Use Trapezoidal rule with 5 sub-intervals of equal width to evaluate $\displaystyle\int_4^9 \frac{1}{\sqrt{x}}\ dx.$

Give approximation to 3 d.p.

(ii) Use Trapezoidal rule with 8 sub-intervals of equal width to evaluate $\displaystyle\int_0^8 \sqrt{x+1}\ dx.$

Give approximation to 3 d.p.

Type 4: Simpson's rule:

$$\int_a^b f(x)\ dx \approx \frac{h}{3} \cdot \left[f_0 + 4\left(f_1 + f_3 + f_5 + \ldots. + f_{n-1} \right) + 2\left(f_2 + f_4 + \ldots. + f_{n-2} \right) + f_n \right]$$

5. (i) Use Simpson's rule with 6 sub-intervals of equal width to evaluate $\displaystyle\int_0^1 x \cdot \sin\left(x^2\right)\ dx.$

Give approximation to 3 d.p.

(ii) Use Simpson's rule with 4 sub-intervals of equal width to evaluate $\displaystyle\int_0^2 e^{x^2}\ dx.$

Give approximation to 3 d.p.

(iii) Use Simpson's rule with 8 sub-intervals of equal width to evaluate $\displaystyle\int_{-1}^3 x \cdot \sqrt{2 + x^3}\ dx.$

Give approximation to 3 d.p.