UNITED KINGDOM · CHINA · MALAYSIA

Science A Physics

Lectures 7-12:

Answers to Additional Problems: Simple Harmonic Motion, Fluids and Light

Frequency and Period of a Loudspeaker Cone

Q.1 What is the oscillator period of a loudspeaker cone that vibrates back and forth 5000 times per second?

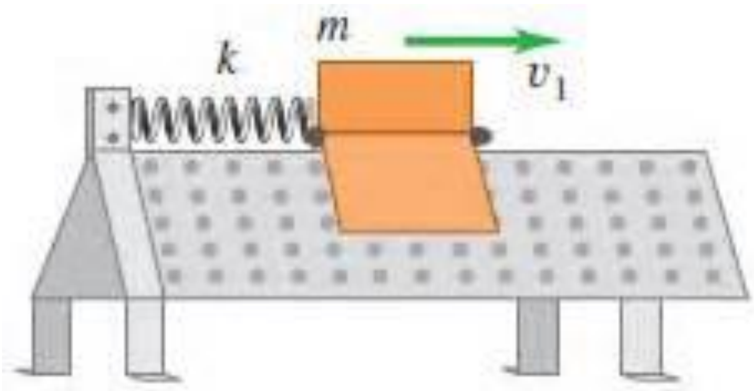
Frequency and Period of a Loudspeaker Cone

SOLVE:

The oscillation frequency is f = 5000 cycles/s = 5000 Hz = 5.0 kHz. The period is the inverse of the frequency; hence

$$T = \frac{1}{f} = \frac{1}{5000\ \text{Hz}} = 2.0\times10^{-4}\ \text{s} = 200\ \mu\text{s}$$

A System in Simple Harmonic Motion



Q.2 An air-track glider is attached to a spring, pulled 20.0 cm to the right, and released at $t = 0$ s. It makes 15 oscillations in 10.0 s.

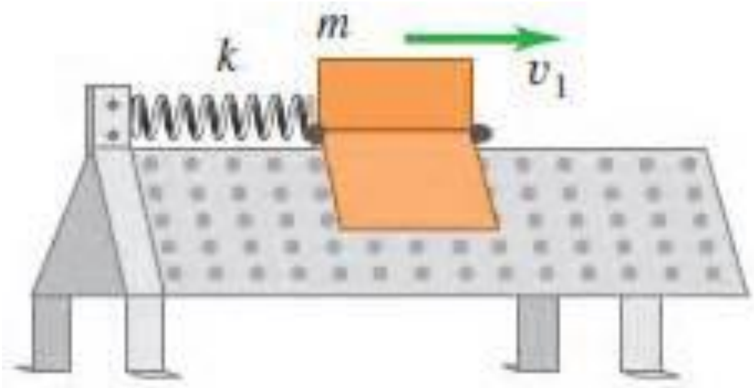
a. What is the period of oscillation?

b. What is the object's maximum speed?

c. What are the position and velocity at t = 0.800 s?

MODEL: An object oscillating on a spring is in SHM.

A System in Simple Harmonic Motion



SOLVE:

a. The oscillation frequency is

$$f = \frac{15\ oscillations}{10.0\ s} = 1.50\frac{oscillations}{s} = 1.50\ Hz$$

Thus, the period is $T = 1/f = 0.667$ s.

b. The oscillation amplitude is $A = 0.200$ m. Thus

$$v_{\max} = \frac{2\pi A}{T} = \frac{2\pi(0.200\ \text{m})}{0.667\ \text{s}} = 1.88\ \text{m/s}$$

c. The object starts at x = +A at t = 0 s. The position at t = 0.800 s is

$$x = A\cos\left(\frac{2\pi t}{T}\right) = (0.200\ \mathrm{m})\cos\left(\frac{2\pi(0.800\ \mathrm{s})}{0.667\ \mathrm{s}}\right)$$
$$= (0.200\ \mathrm{m})\cos(7.54\ \mathrm{rad}) = 0.0625\ \mathrm{m} = 6.25\ \mathrm{cm}$$

The velocity at this instant of time is

$$v_x = -v_{\max}\sin\left(\frac{2\pi t}{T}\right) = -\left(1.88\,\frac{\mathrm{m}}{\mathrm{s}}\right)\sin\left(\frac{2\pi(0.800\ \mathrm{s})}{0.667\ \mathrm{s}}\right)$$
$$= -\left(1.88\,\frac{\mathrm{m}}{\mathrm{s}}\right)\sin(7.54\ \mathrm{rad}) = -1.79\,\frac{\mathrm{m}}{\mathrm{s}} = -179\ \mathrm{cm/s}$$

At t = 0.800 s, which is slightly more than one period, the object is 6.25 cm to the right of equilibrium and moving to the left at 179 cm/s. Notice the use of radians in the calculations.

A mass oscillating in simple harmonic motion starts at x = A and has period T. At what time, as a friction of T, does the object first pass through x = ½ A?

SOLVE: This is one-quarter of the total distance in one-quarter of a period. You might expect it to take 1/8T to reach ½ A, but this is not the case because the SHM graph is not linear between $x = A$ and $x = 0$. We need to use $x(t) = A\cos(2\pi t/T)$. First, we write the equation with x = ½ A:

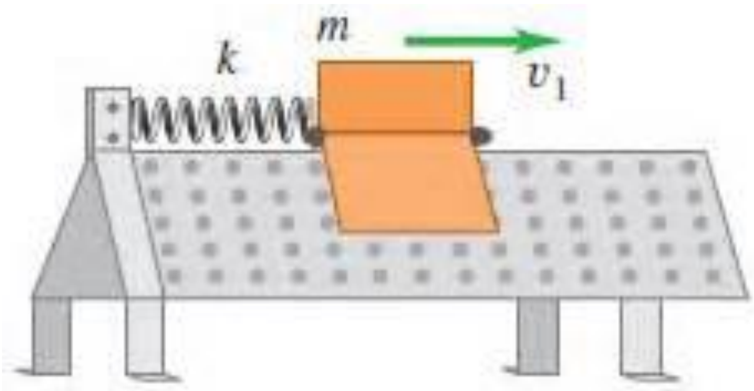
$$x = \frac{A}{2} = A\cos\left(\frac{2\pi t}{T}\right)$$

Then we solve for the time at which this position is reached:

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{1}{2}\right) = \frac{T}{2\pi}\frac{\pi}{3} = \frac{T}{6}$$

ASSESS: The motion is slow at the beginning and then speeds up, so it takes longer to move from $x = A$ to $x = ½\,A$ than it does to move from $x = ½\,A$ to $x = 0$. Notice that the answer is independent of the amplitude A.

Using the Initial Conditions



Q.3 An object on a spring oscillates with a period of 0.80 s and an amplitude of 10 cm. At t = 0 s, it is 5.0 cm to the left of equilibrium and moving to the left. What are its position and direction of motion at t = 2.0 s?

MODEL:

An object oscillating on a spring is in simple harmonic motion.

Using the Initial Conditions

SOLVE: We can find the phase constant φ_0 from the initial condition x_0 = -5.0 cm = A cos φ_0. This condition gives

$$\varphi_0 = \cos^{-1}\left(\frac{x_0}{A}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \pm\frac{2}{3}\pi \text{ rad} = \pm 120°$$

Because the oscillator is moving to the left at t = 0, it is in the upper half of the circular-motion diagram and must have a phase constant between 0 and π rad. Thus, φ_0 is $\frac{2}{3}\pi$ rad. The angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80 \text{ s}} = 7.85 \text{ rad/s}$$

Using the Initial Conditions

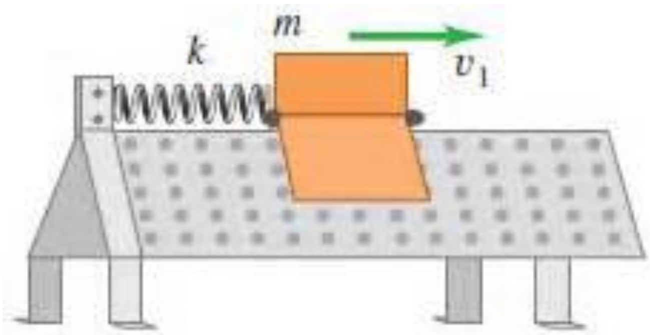
Thus the object's position at time t = 2.0 s is

$$x(t) = A\cos(\omega t + \varphi_0)$$
$$= (10\ cm)cos\left((7.85\frac{rad}{s})(2.0\ s) + \frac{2}{3}\pi \right)$$
$$= (10\ cm)cos(17.8\ rad) = 5.0\ cm$$

The object is now 5.0 cm to the right of equilibrium. But which way is it moving? There are two ways to find out. The direct way is to calculate the velocity at t = 2.0 s:

$$v_x = -\omega A\sin(\omega t + \emptyset_0) = +68\ \mathrm{cm/s}$$

Using the Initial Conditions

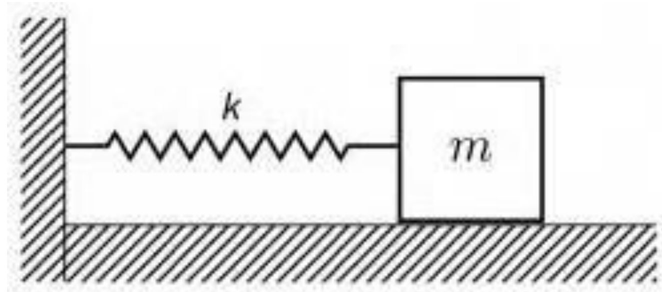


The velocity is positive, so the motion is to the right. Alternatively, we could note that the phase at t = 2.0 s is φ = 17.8 rad. Dividing by π, you can see

$$\varphi = 17.8\,rad = 5.67\pi\,rad = (4\pi + 1.67\pi)\,rad$$

The 4π rad represents two complete revolutions. The ‘extra’ phase of 1.67π rad falls between π and 2π rad, so the particle in the circular-motion diagram is in the lower half of the circle and moving to the right.

Using Conservation of Energy



Q.4 A 500 g block on a spring is pulled a distance of 20 cm and released. The subsequent oscillations are measured to have a period of 0.80 s.

a. At what position or positions is the block's speed 1.0 m/s?

b. What is the spring constant?

MODEL: The motion is SHM. Energy is conserved.

Using Conservation of Energy

SOLVE: a. The block starts from the point of maximum displacement, where $E = U = \frac{1}{2} kA^2$. At a later time, when the position is x and the speed is v, energy conservation requires

$\frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2$

Solving for x, we find

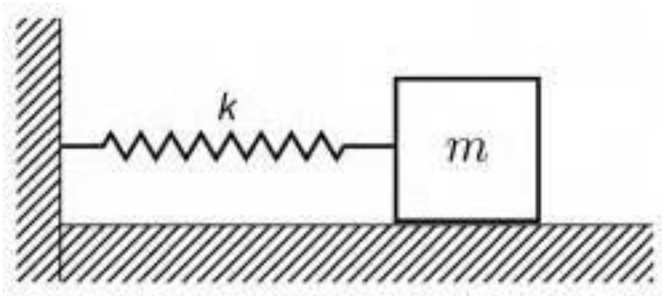
$$x = \sqrt{A^2 - \frac{mv^2}{k}} = \sqrt{A^2 - \left(\frac{v}{\omega}\right)^2}$$

Where we used $k/m = \omega^2$. The angular frequency is easily found from the period: $\omega = 2\pi/T = 7.85$ rad/s. Thus

$$x = \sqrt{(0.20\ \mathrm{m})^2 - \left(\frac{1.0\ \mathrm{m/s}}{7.85\ \mathrm{rad/s}}\right)^2} = \pm 0.15\ \mathrm{m} = \pm 15\ \mathrm{cm}$$

There are two positions because the block has this speed on either side of equilibrium.

Using Conservation of Energy

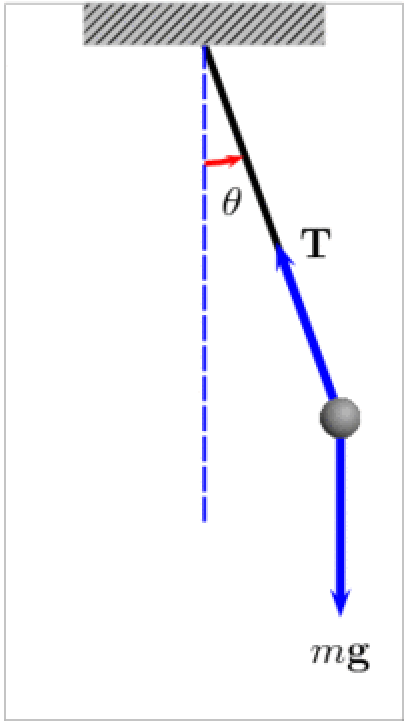


b. Although part a did not require that we know the spring constant, it is straightforward to find:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.50\text{ kg})}{(0.80\text{ s})^2} = 31\text{ N/m}$$

The Maximum Angle of a Pendulum



Q.5 A 300 g mass on a 30-cm-long string oscillates as a pendulum. It has a speed of 0.25 m/s as it passes through the lowest point. What maximum angle does the pendulum reach?

MODEL: Assume that the angle remains small, in which case the motion is simple harmonic motion.

SOLVE :The angular frequency of the pendulum is

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\ \mathrm{m/s^2}}{0.30\ \mathrm{m}}} = 5.72\ \mathrm{rad/s}$$

The speed at the lowest point is $v_{max} = \omega A$, so the amplitude is

$$A = s_{max} = \frac{v_{max}}{\omega} = \frac{0.25\ \mathrm{m/s}}{5.72\ \mathrm{rad/s}} = 0.0437\mathrm{m}$$

The maximum angle, at the maximum are length s_{max}, is

$$\theta = \frac{s_{max}}{L} = \frac{0.0437\ \mathrm{m}}{0.30\ \mathrm{m}} = 0.146\, rad = 8.3°$$

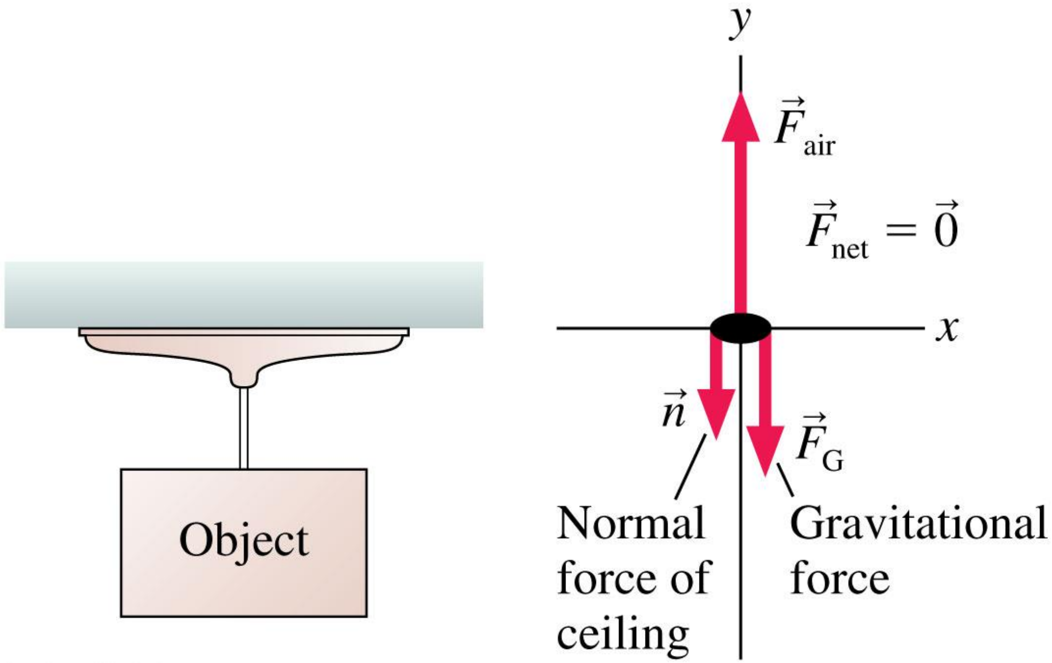
ASSESS: Because the maximum angle is less than 10°, our analysis based on the small-angle approximation is reasonable.

A Suction Cup

Q.6 A 10.0-cm-diameter suction cup is pushed against a smooth ceiling. What is the maximum mass of an object that can be suspended from the suction cup without pulling it off the ceiling? The mass of the suction cup is negligible.

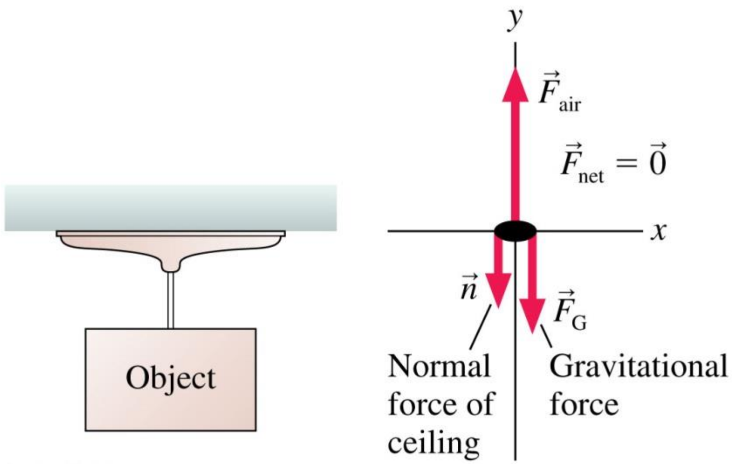
MODEL: Pushing the suction cup against the ceiling pushes the air out. We'll assume that the volume enclosed between the suction cup and the ceiling is a perfect vacuum with p = 0 Pa. We'll also assume that the pressure in the room is 1 atm.

A Suction Cup



VISUALISE: The figure above shows a free-body diagram of the suction cup stuck to the ceiling. The downward normal force of the ceiling is distributed around the rim of the suction cup, but in the particle model we can show this as a single force vector.

A Suction Cup



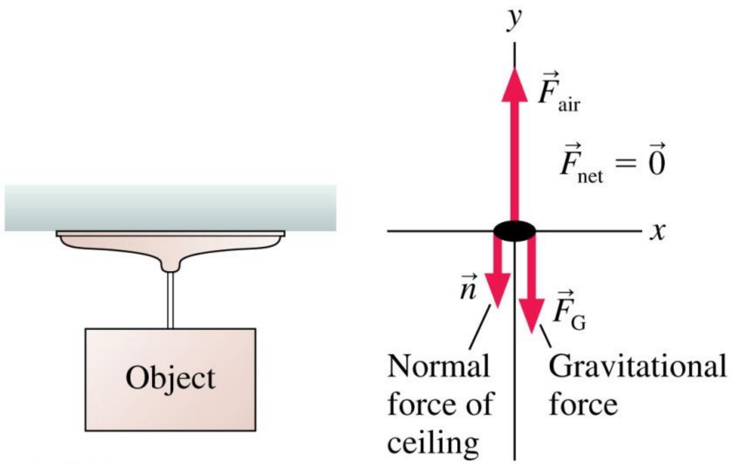
SOLVE: The suction cup remains stuck to the ceiling, in static equilibrium, as long as $F_{air} = n + F_G$. The magnitude of the upward force exerted by the air is

$$F_{air} = pA = p\pi r^2 = (101{,}300\ \text{Pa})\pi(0.050\ \text{m})^2 = 796\ \text{N}$$

There is no downward force from the air in this case because there is no air inside the cup. Increasing the hanging mass decreases the normal force n by an equal amount. The maximum weight has been reached when n is reached to zero. Thus

$$(F_G)_{max} = mg = F_{air} = 796\ \text{N}$$

A Suction Cup



ASSESS:

The suction cup can support a mass of up to 81 kg if all the air is pushed out, leaving a perfect vacuum inside. A real suction cup won't achieve a perfect vacuum, but suction cups can hold substantial weight.

TACTICS BOX 15.1 Hydrostatics

❶ **Draw a picture.** Show open surfaces, pistons, boundaries, and other features that affect pressure. Include height and area measurements and fluid densities. Identify the points at which you need to find the pressure.

❷ **Determine the pressure at surfaces.**

- **Surface open to the air:** $p_0 = p_{atmos}$, usually 1 atm.
- **Surface covered by a gas:** $p_0 = p_{gas}$.
- **Closed surface:** $p = F/A$, where F is the force the surface, such as a piston, exerts on the fluid.

❸ **Use horizontal lines.** Pressure in a connected fluid is the same at any point along a horizontal line.

❹ **Allow for gauge pressure.** Pressure gauges read $p_g = p - 1$ atm.

❺ **Use the hydrostatic pressure equation.** $p = p_0 + \rho g d$.

Exercises 4–13

Pressure on a Submarine

Q.7 A submarine cruises at a depth of 300 m. What is the pressure at this depth? Give the answer in both Pascals and atmospheres.

Pressure on a Submarine

SOLVE:

The density of seawater is $\rho = 1030 \text{ kg/m}^3$. The pressure at depth $d = 300$ m is found to be

$$p = p_0 + \rho g d = 1.013 \times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(300 \text{ m})$$

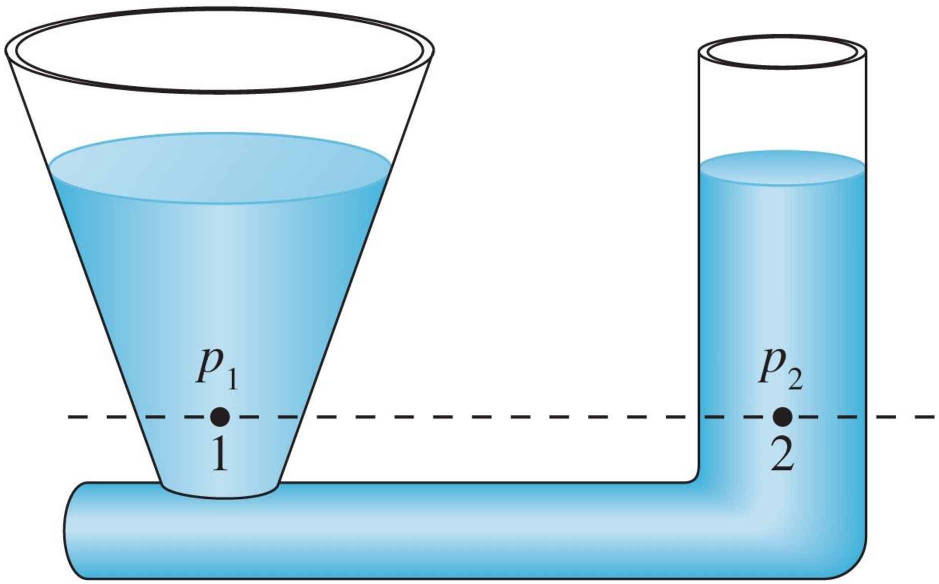
$$= 3.13 \times 10^6 \text{ Pa}$$

Converting the answer to atmospheres gives

$$p = 3.13 \times 10^6 \text{ Pa} \times \frac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}} = 30.9 \text{ atm}$$

Liquids in Hydrostatic Equilibrium

Q.8 What can you say about the pressures at points 1 and 2?



a) $p_1 > p_2$.
b) $p_1 = p_2$.
c) $p_1 < p_2$.

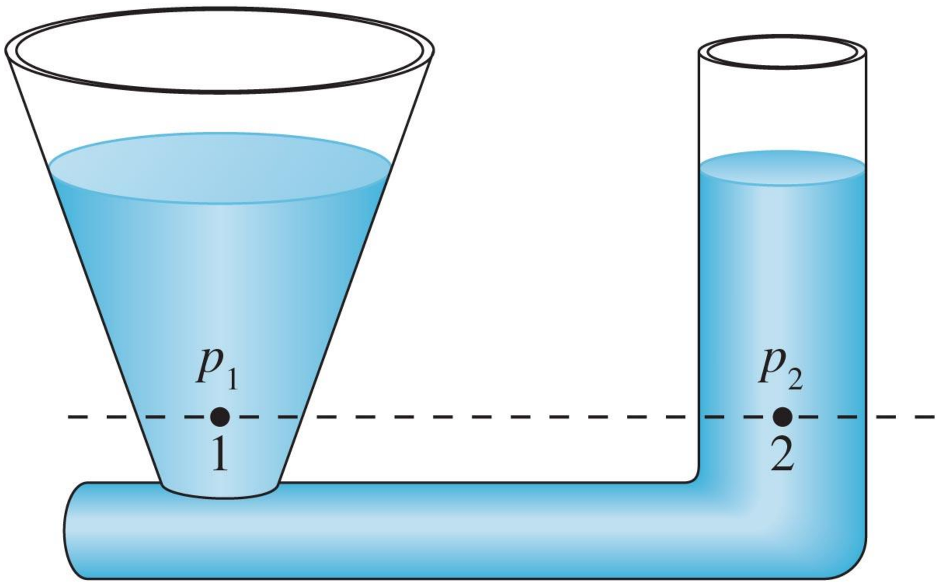
Liquids in Hydrostatic Equilibrium

Q.8 What can you say about the pressures at points 1 and 2?

a) $p_1 > p_2$.

b) $p_1 = p_2$.

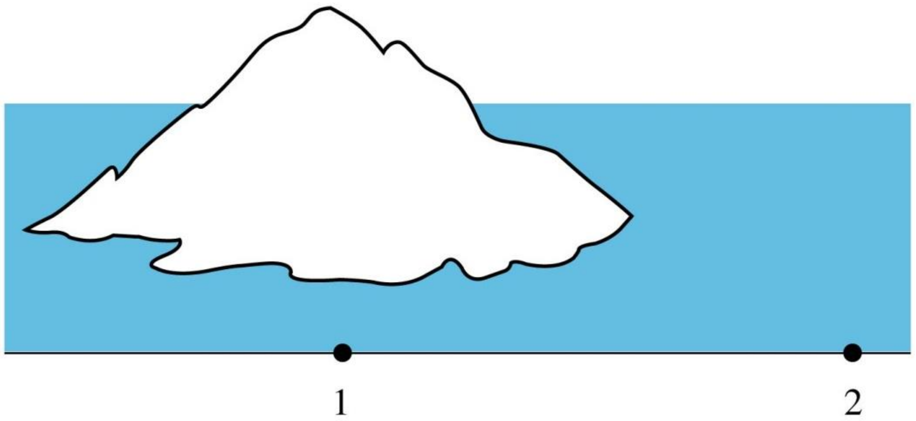
c) $p_1 < p_2$.



Hydrostatic pressure is the same at all points on a horizontal line through a connected fluid.

Liquids in Hydrostatic Equilibrium

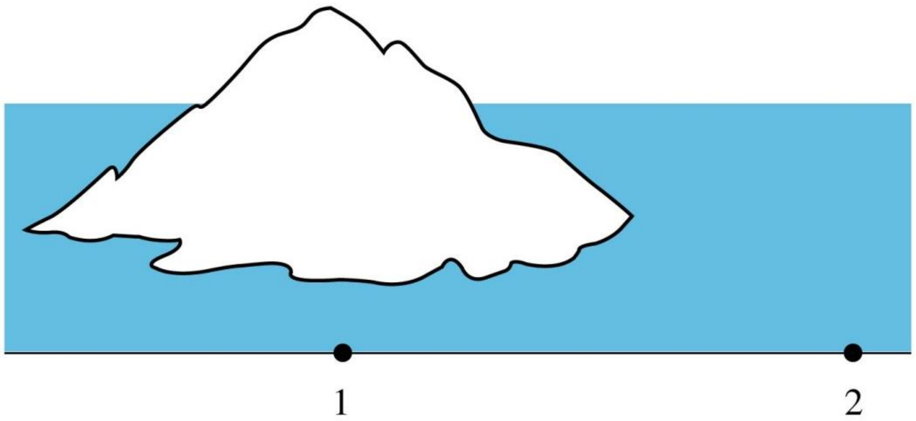
Q.9 An iceberg floats in a shallow sea. What can you say about the pressures at points 1 and 2?



a) $p_1 > p_2$.
b) $p_1 = p_2$.
c) $p_1 < p_2$.

Liquids in Hydrostatic Equilibrium

Q.9 An iceberg floats in a shallow sea. What can you say about the pressures at points 1 and 2?



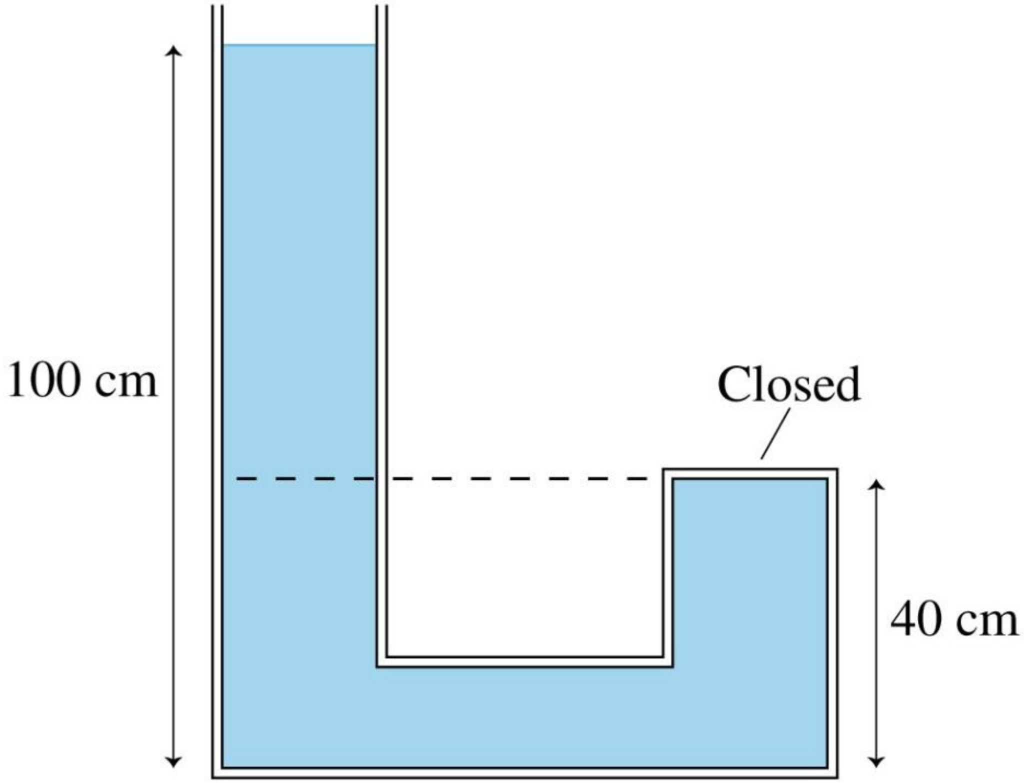
a) $p_1 > p_2$.

b) $\boldsymbol{p_1 = p_2}$.

c) $p_1 < p_2$.

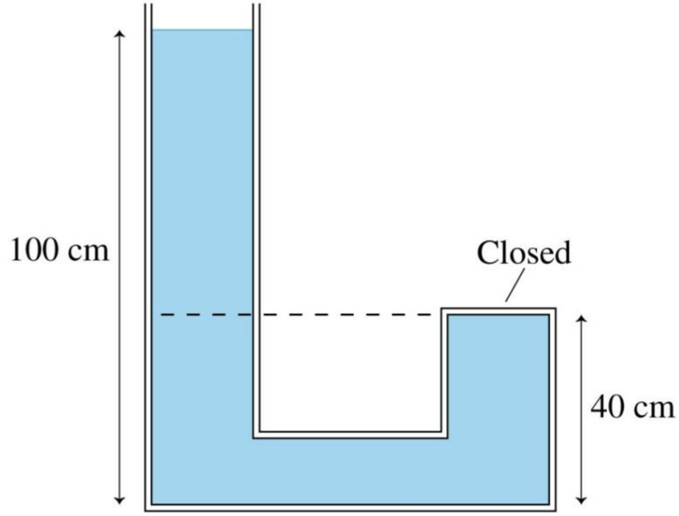
Hydrostatic pressure is the same at all points on a horizontal line through a connected fluid.

Pressure in a Closed Tube



Q.10 Water fills the tube shown in the above figure. What is the pressure at the top of the closed tube?

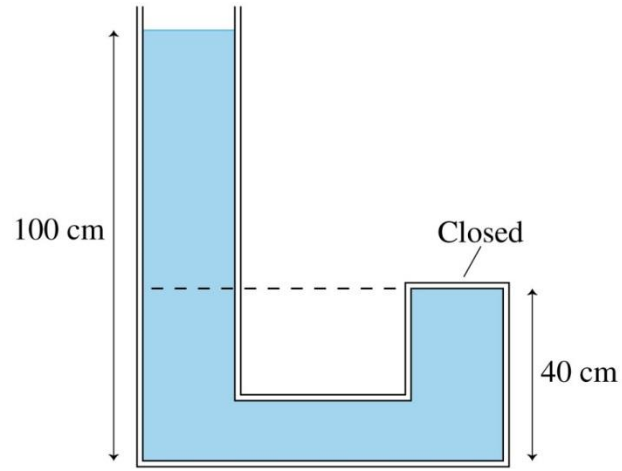
Pressure in a Closed Tube



MODEL:

This is a liquid in hydrostatic equilibrium. The closed tube is not an open region of the container, so the water cannot rise to an equal height. Nevertheless, the pressure is still the same at all points on a horizontal line. In particular, the pressure at the top of the closed tube equals the pressure in the open tube at the height of the dashed line. Assume $p_0 = 1.00$ atm.

Pressure in a Closed Tube



SOLVE: A point 40 cm above the bottom of the open tube is at a depth of 60 cm. The pressure at this depth is

$$p = p_0 + \rho g d$$

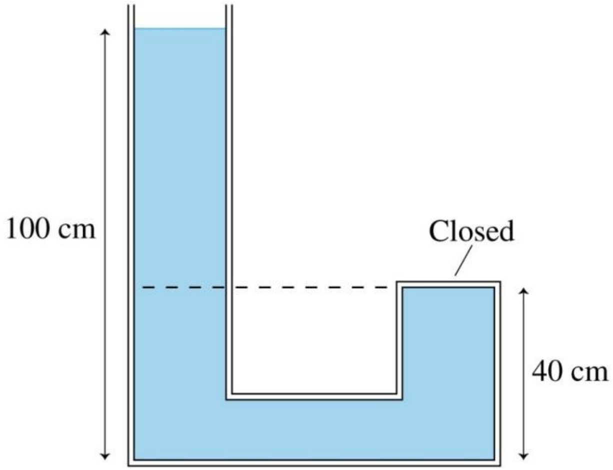
$$= 1.013\times10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.60 \text{ m})$$

$$= 1.072\times10^6 \text{ Pa}$$

$$= 1.06 \text{ atm}$$

This is the pressure at the top of the closed tube.

Pressure in a Closed Tube



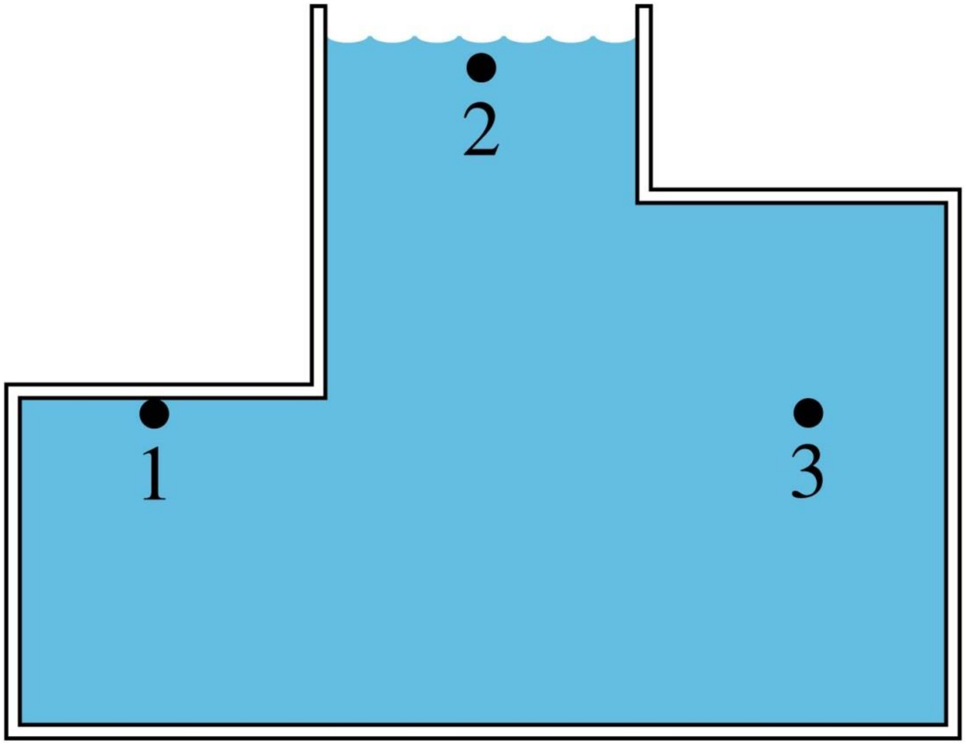
ASSESS:

The water in the open tube **pushes** the water in the closed tube up against the top of the tube, which is why the pressure is greater than 1 atm.

Pressure in a Closed Tube

Q.11 What can you say about the pressures at points 1, 2, and 3?

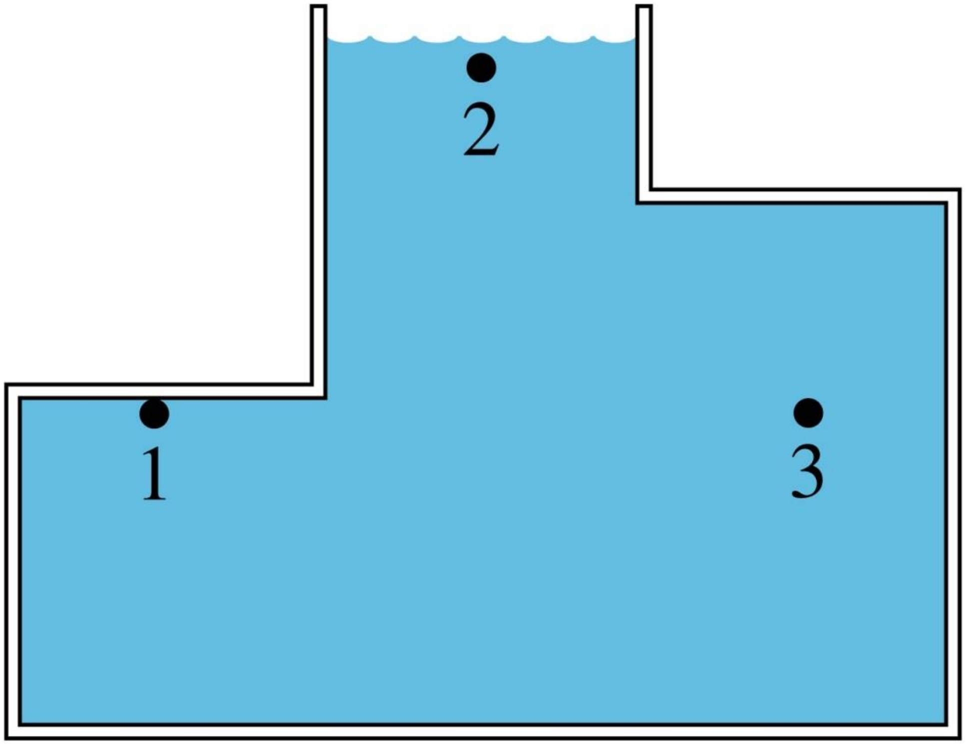
a) $p_1 = p_2 = p_3$.
b) $p_1 = p_2 > p_3$.
c) $p_3 > p_1 = p_2$.
d) $p_3 > p_1 > p_2$.
e) $p_1 = p_3 > p_2$.



Pressure in a Closed Tube

Q.11 What can you say about the pressures at points 1, 2, and 3?

a) $p_1 = p_2 = p_3$.
b) $p_1 = p_2 > p_3$.
c) $p_3 > p_1 = p_2$.
d) $p_3 > p_1 > p_2$.
e) $\boldsymbol{p_1 = p_3 > p_2}$.



Hydrostatic pressure is the same at all points on a horizontal line through a connected fluid.

Lifting a Car

Q.12 The hydraulic lift at a car repair shop is filled with oil. The car rests on a 25-cm-diameter piston. To lift the car, compressed air is used to push down on a 6.0-cm-diameter piston. What does the pressure gauge read when a 1300 kg car is 2.0 m above the compressed air piston?

MODEL: Assume that the oil is incompressible. Its density, is 900 kg/m^3.

Lifting a Car

SOLVE:

The pressure applied to the fluid by the compressed-air piston is

$$p_1 = \frac{F_1}{A_1} = \frac{782\ N}{0.00283\ m^2} \cdot 2.76 \times 10^5\ Pa = 2.7\ atm$$

This is the pressure in excess of atmospheric pressure, which is what a pressure gauge measures, so the gauge reads, depending on it units, 276 kPa or 2.7 atm.

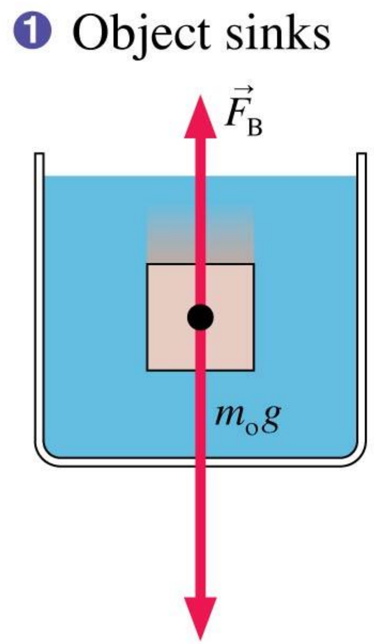
Lifting a Car

ASSESS:

782 N is roughly the weight of an average adult man. The multiplication factor $A_2/A_1 = 17$ makes it quite easy for this much force to lift the car.

Will an Object Float or Sink?

Finding whether an object floats or sinks



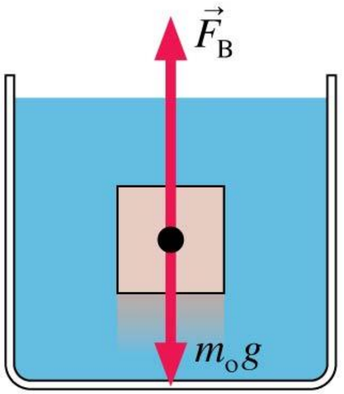
An object sinks if it weighs more than the fluid it displaces—that is, if its average density is greater than the density of the fluid:

$$\rho_{avg} > \rho_f$$

Will an Object Float or Sink?

Finding whether an object floats or sinks

❷ Object floats

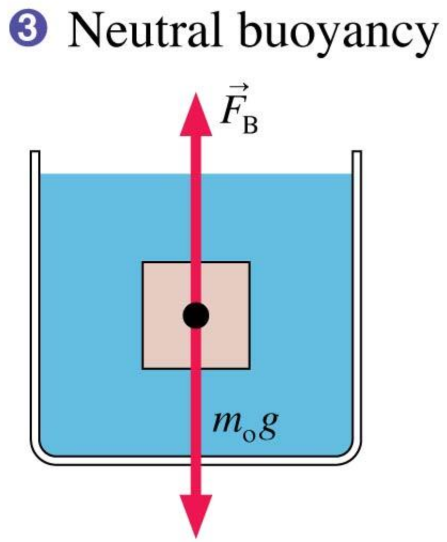


An object floats on the surface if it weighs less than the fluid it displaces—that is, if its average density is less than the density of the fluid:

$$\rho_{avg} < \rho_f$$

Will an Object Float or Sink?

Finding whether an object floats or sinks



An object hangs motionless if it weighs exactly the same as the fluid it displaces—that is, if its average density equals the density of the fluid:

$$\rho_{avg} = \rho_f$$

Buoyancy

Q.13 A heavy lead block and a light aluminum block of equal sizes are both submerged in water. Upon which is the buoyant force greater?

a) On the lead block.
b) On the aluminum block.
c) They both experience the same buoyant force.

Buoyancy

Q.13 A heavy lead block and a light aluminum block of equal sizes are both submerged in water. Upon which is the buoyant force greater?

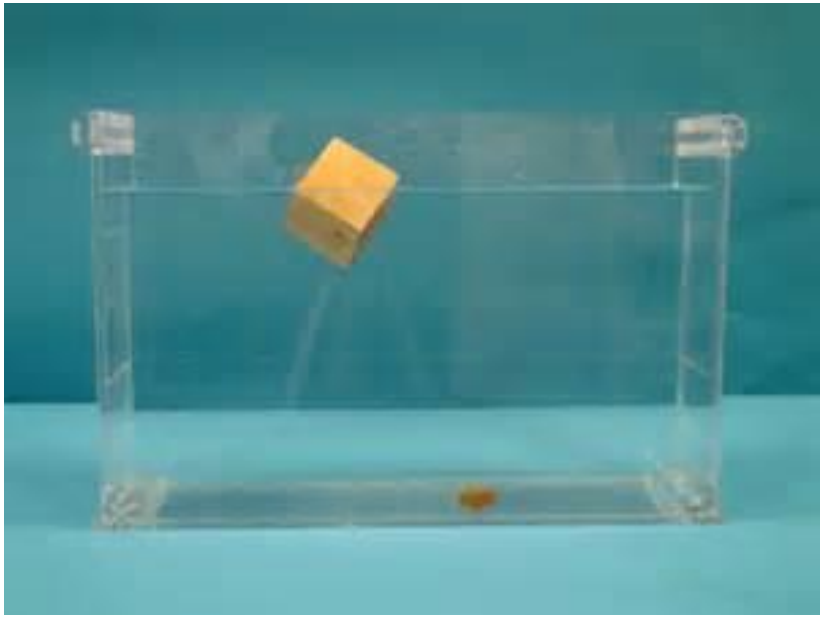
a) On the lead block.

b) On the aluminum block.

c) They both experience the same buoyant force.

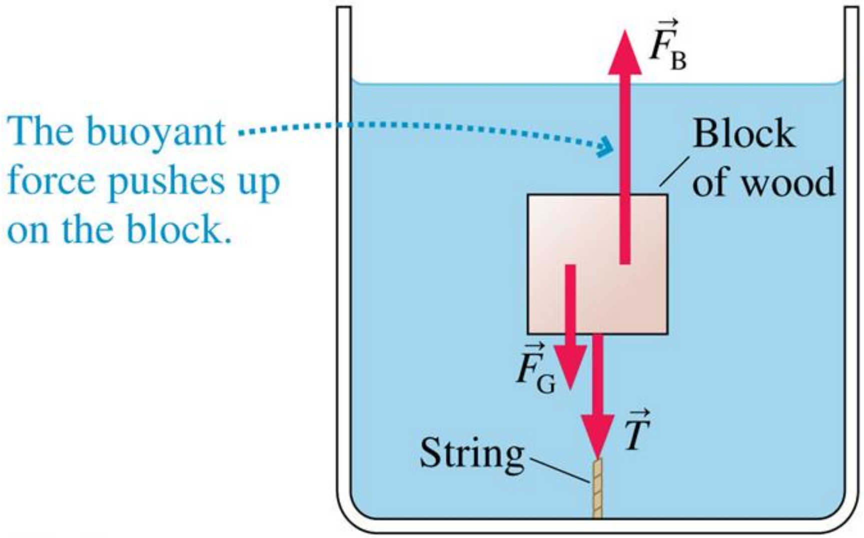
Same size $\Rightarrow$ both displace the same volume and weight of water.

Holding a Block of Wood Underwater



Q.14 A 10 cm × 10 cm × 10 cm block of wood with density 700 kg/m^3 is held underwater by a string tied to the bottom of the container. What is the tension in the string?

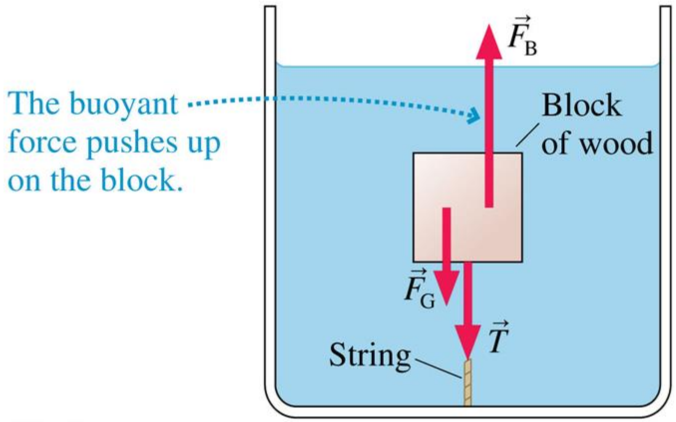
Holding a Block of Wood Underwater



Q.14 A 10 cm × 10 cm × 10 cm block of wood with density 700 kg/m^3 is held underwater by a string tied to the bottom of the container. What is the tension in the string?

MODEL: The buoyant force is given by Archimedes' principle.

Holding a Block of Wood Underwater



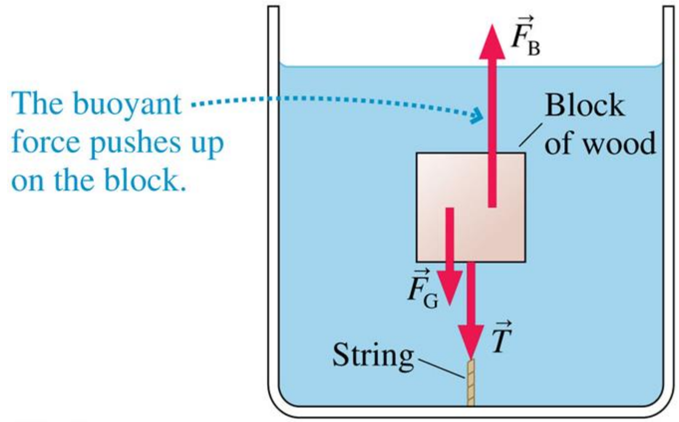
SOLVE: The block is in static equilibrium, so

$$\sum F_y = F_B - T - m_o g = 0$$

Thus, the tension is $T = F_B - m_o g$. The mass of the block is $m_o = \rho V_o$, and the buoyant force is $F_B = \rho_f V_f g$. Thus

$$T = \rho_f V_f g - \rho_o V_o g = (\rho_f - \rho_o) V_o g$$

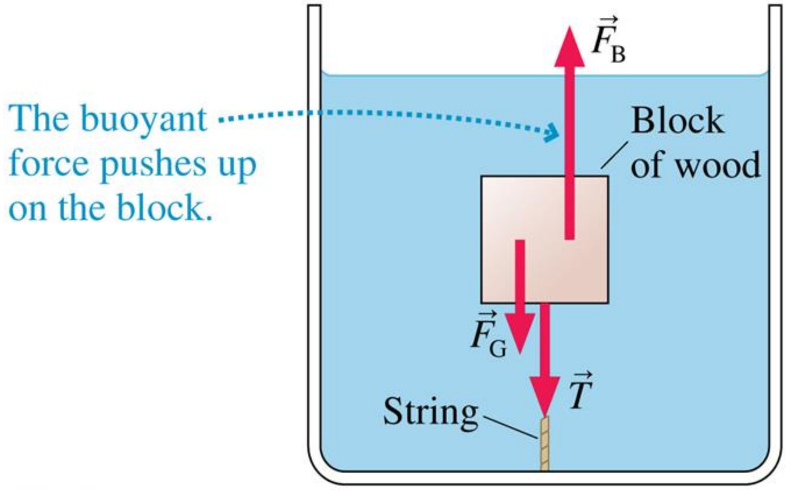
Holding a Block of Wood Underwater



Where we've used the fact that $V_f = V_o$ for a completely submerged object. The volume is $V_o = 1000\ \mathrm{cm}^3 = 1.0 \times 10^{-3}\mathrm{m}^3$, and hence the tension in the string is

$$T = \left(\left(1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right) - \left(700\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\right)\left(1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 2.9\ \mathrm{N}$$

Holding a Block of Wood Underwater



ASSESS:

The tension depends on the **difference** in densities. The tension would vanish if the wood density matched the water density.

Buoyancy

Q.15 Two blocks are of identical size. One is made of lead and sits on the bottom of a pond; the other is of wood and floats on top. Upon which is the buoyant force greater?

a) On the lead block.
b) On the wood block.
c) They both experience the same buoyant force.

Buoyancy

Q.15 Two blocks are of identical size. One is made of lead and sits on the bottom of a pond; the other is of wood and floats on top. Upon which is the buoyant force greater?

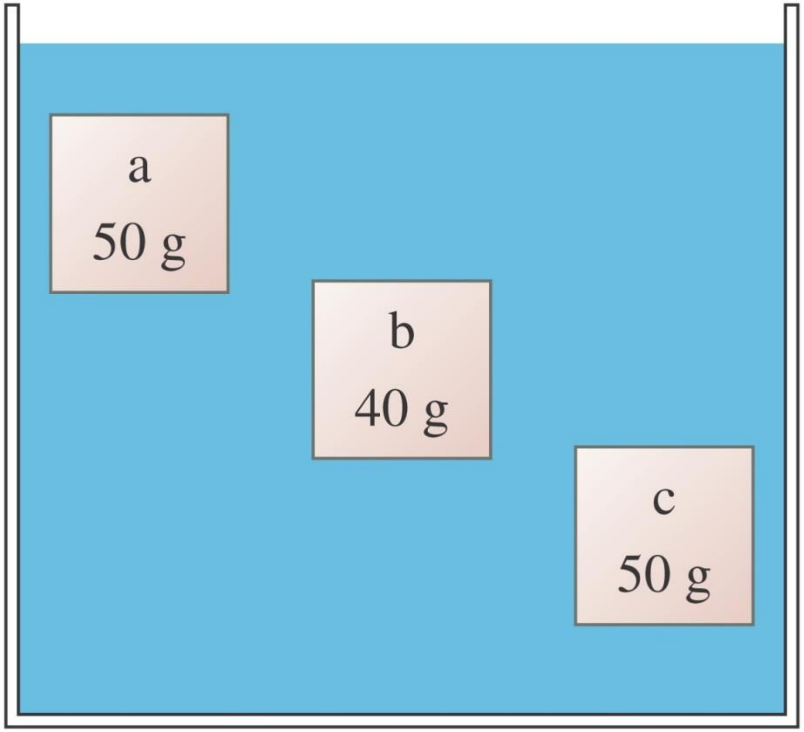
a) On the lead block.
b) On the wood block.
c) They both experience the same buoyant force.

The fully submerged lead block displaces more much water than the wood block.

Buoyancy

Q.16 Blocks a, b, and c are all the same size. Which experiences the largest buoyant force?

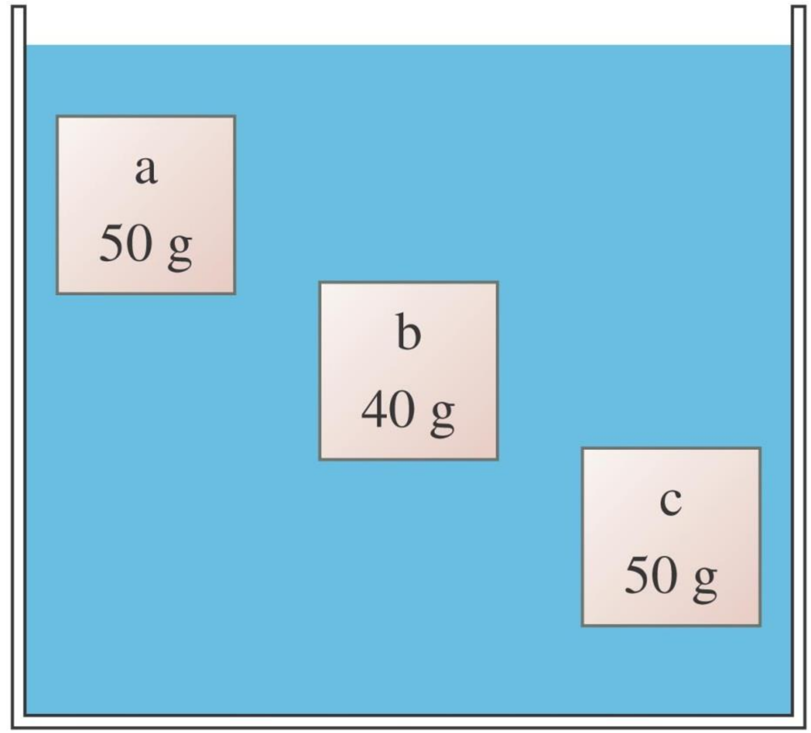
a) Block a.
b) Block b.
c) Block c.
d) All have the same buoyant force.
e) Blocks a and c have the same buoyant force, but the buoyant force on block b is different.



Buoyancy

Q.16 Blocks a, b, and c are all the same size. Which experiences the largest buoyant force?

a) Block a.
b) Block b.
c) Block c.
d) All have the same buoyant force.
e) Blocks a and c have the same buoyant force, but the buoyant force on block b is different.



Bernoulli's Equation

The energy equation for fluid in a flow tube is:

$$p_1 + \frac{1}{2}\rho v_1^{\,2} + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^{\,2} + \rho g y_2$$

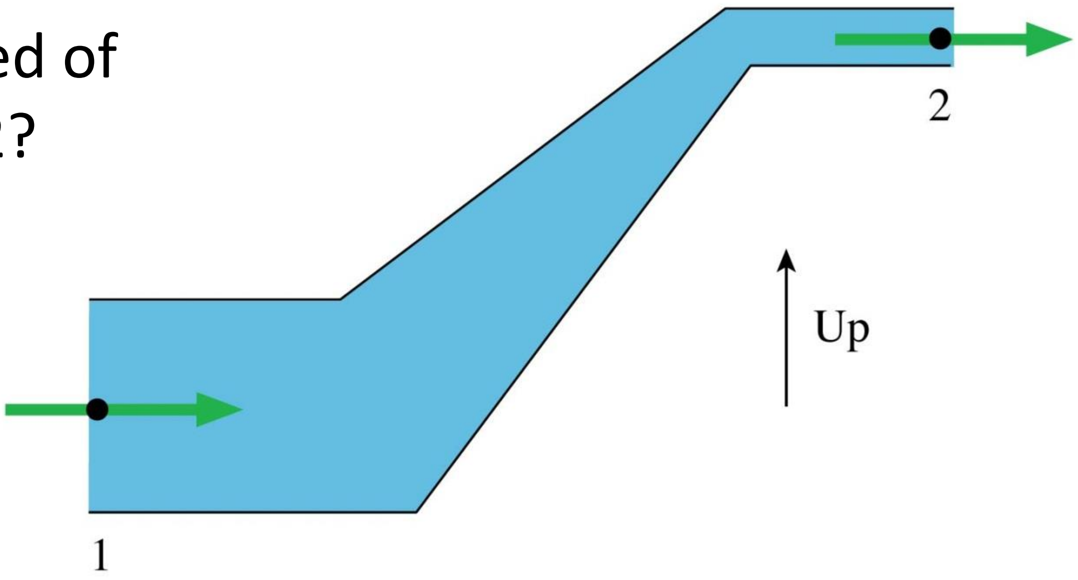
An alternative form of **Bernoulli's equation** is:

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Fluid Dynamics

Q.17 Water flows from left to right through this pipe. What can you say about the speed of the water at points 1 and 2?

a) $v_1 > v_2$.
b) $v_1 = v_2$.
c) $v_1 < v_2$.

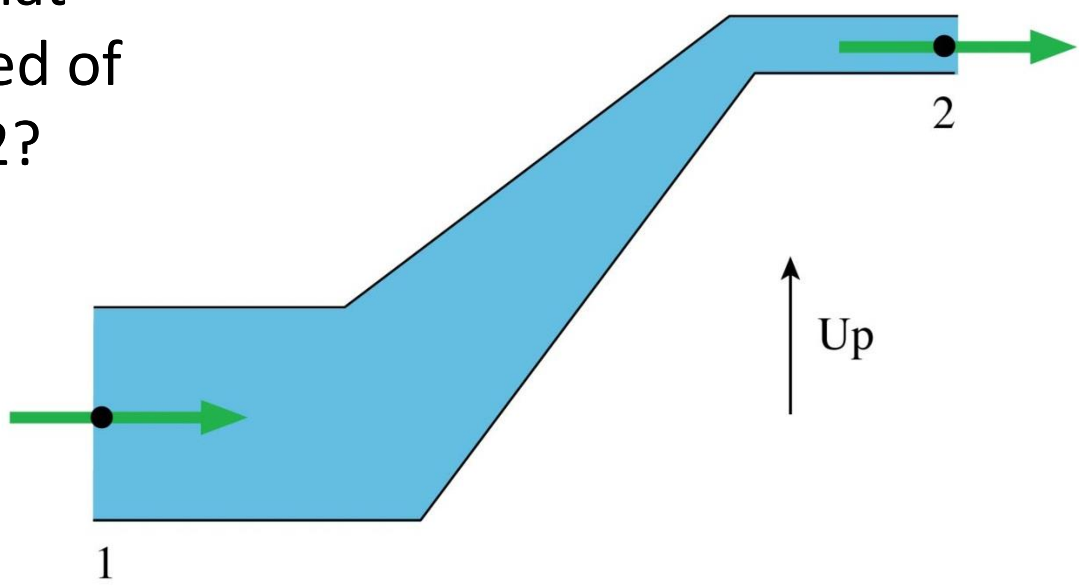


Fluid Dynamics

Q.17 Water flows from left to right through this pipe. What can you say about the speed of the water at points 1 and 2?

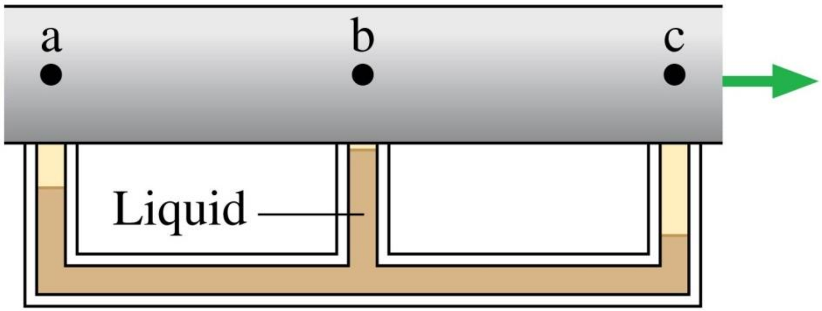
a) $v_1 > v_2$.
b) $v_1 = v_2$.
c) $v_1 < v_2$.

Continuity: $v_1 A_1 = v_2 A_2$



Bernoulli's Equation

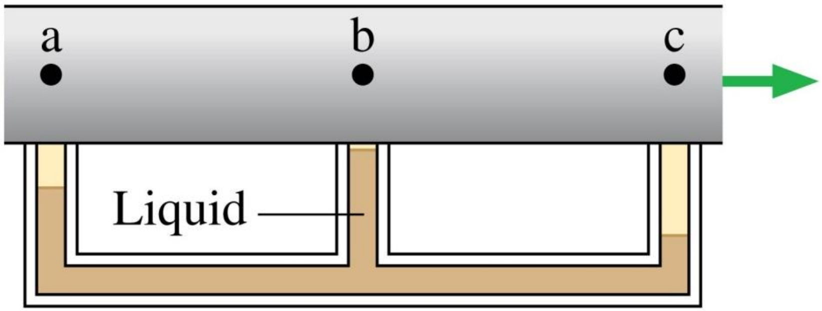
Q.18 Gas flows from left to right through this pipe, whose interior is hidden. At which point does the pipe have the smallest inner diameter?



a) Point a.
b) Point b.
c) Point c.
d) The diameter doesn't change.
e) Not enough information to tell.

Bernoulli's Equation

Q.18 Gas flows from left to right through this pipe, whose interior is hidden. At which point does the pipe have the smallest inner diameter?



a) Point a.

b) Point b.

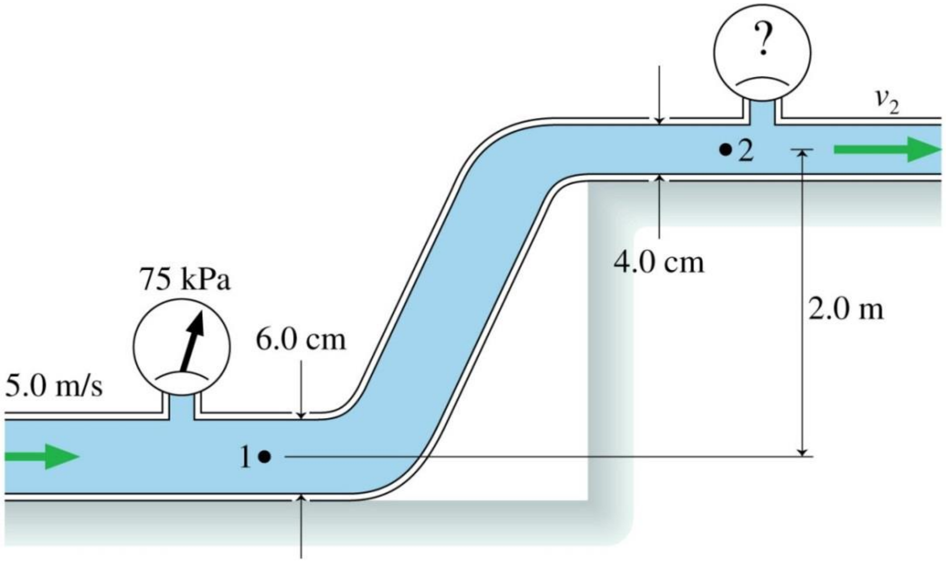
c) Point c.

d) The diameter doesn't change.

e) Not enough information to tell.

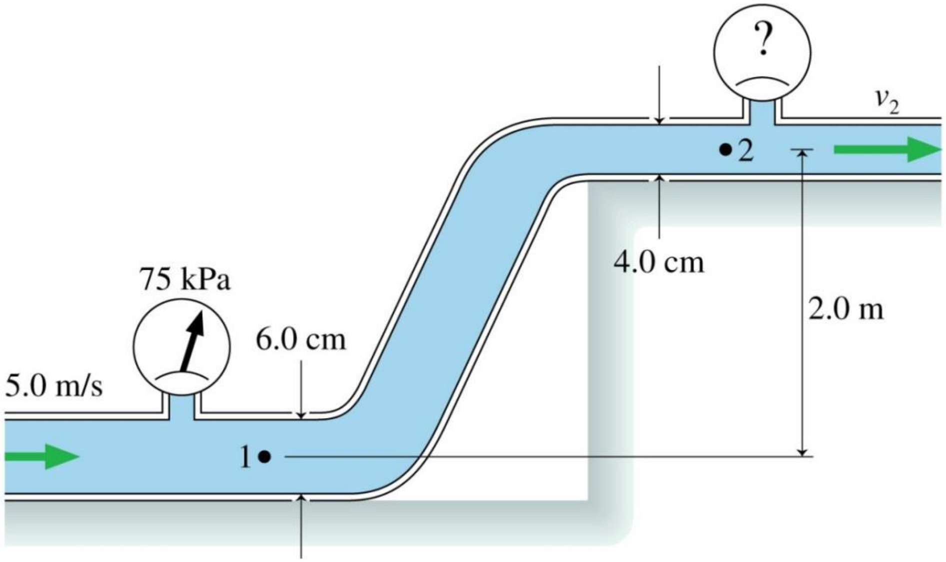
Smallest pressure $\Rightarrow$ fastest speed $\Rightarrow$ smallest diameter

An Irrigation System



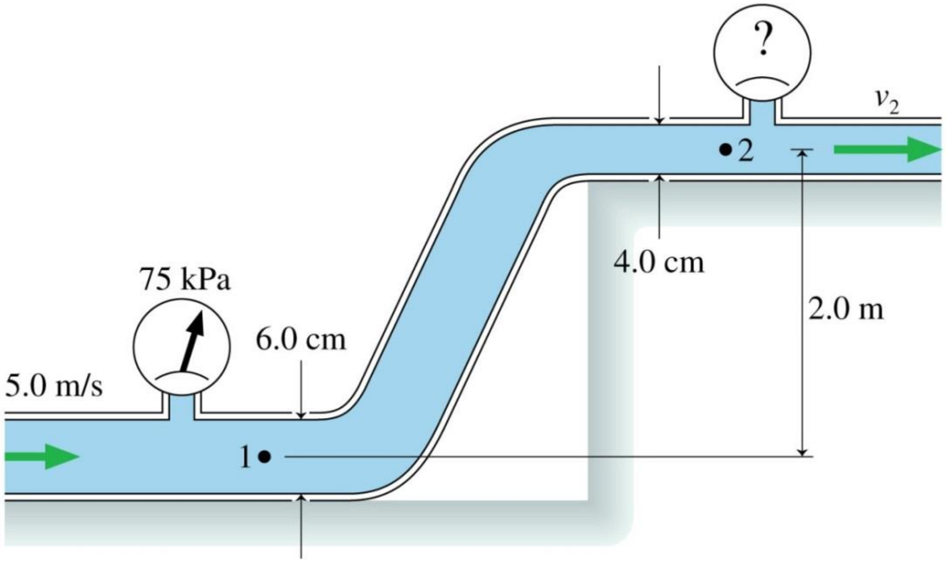
Q.19 Water flows through the pipes shown in the figure. The water's speed through the lower pipe is 5.0 m/s, and a pressure gauge reads 75 kPa. What is the reading of the pressure gauge on the upper pipe?

An Irrigation System



MODEL: Treat the water as an ideal fluid obeying Bernoulli's equation. Consider a streamline connecting point 1 in the lower pipe with point 2 in the upper pipe.

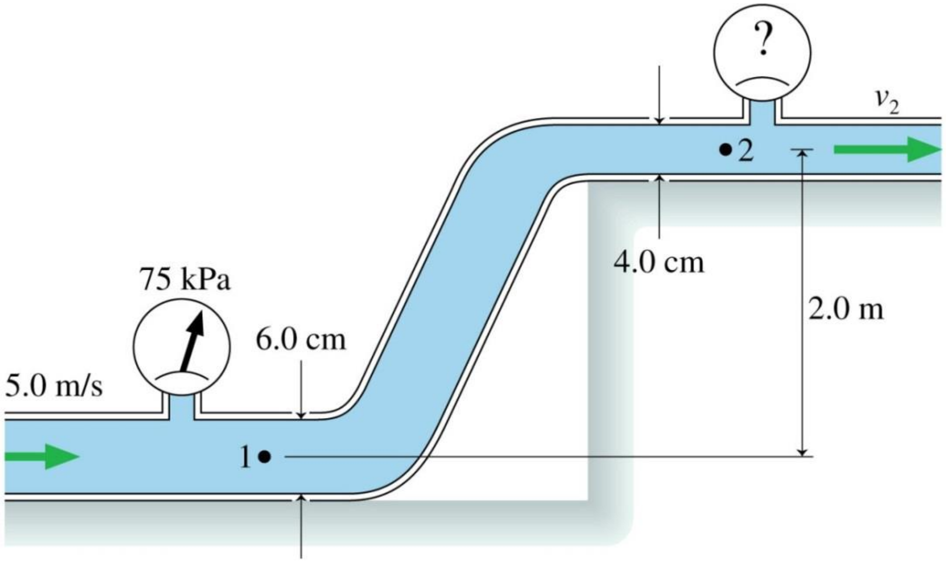
An Irrigation System



SOLVE Bernoulli's equation relates the pressure fluid speed, and heights at points 1 and 2. It is easily solved for the pressure p_2 at point 2:

$$p_2 = p_2 + \frac{1}{2}\rho {v_1}^2 - \frac{1}{2}\rho {v_2}^2 + \rho g y_1 - \rho g y_2$$
$$= p_2 + \frac{1}{2}\rho({v_1}^2 - {v_2}^2) + \rho g(y_1 - y_2)$$

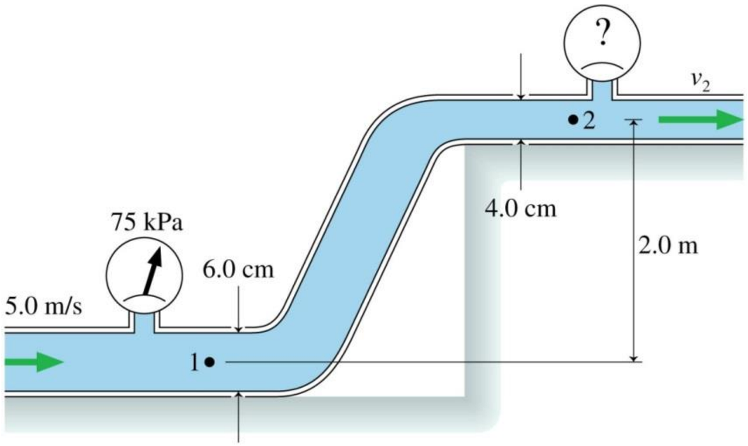
An Irrigation System



All quantities on the right are known except v_2, and that is where the equation of continuity will be useful. The cross-sectional areas and water speeds at points 1 and 2 are related by

$$v_1 A_1 = v_2 A_2$$

An Irrigation System



$$v_1 A_1 = v_2 A_2$$

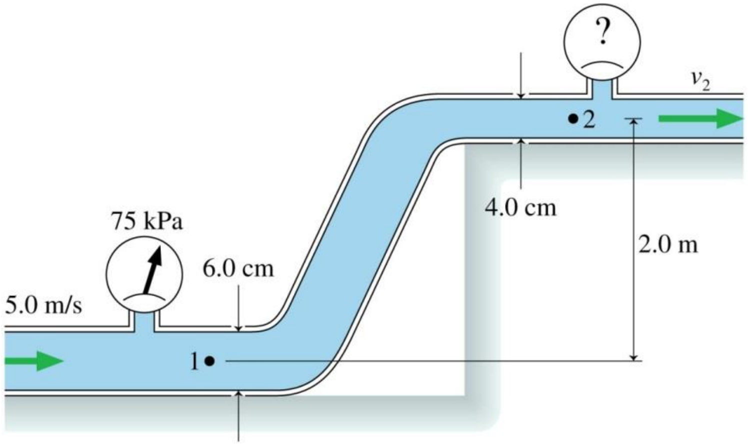
From which we find

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{{r_1}^2}{{r_2}^2} v_1 = \frac{(0.030\ \mathrm{m})^2}{(0.020\ \mathrm{m})^2} (5.0\ \mathrm{m/s}) = 11.25\mathrm{m/s}$$

The pressure at point 1 is p_1 = 75 kPa + 1 atm = 176,300 Pa. We can now use the above expression for p_2 to calculate p_2 = 105,900 Pa. This is the absolute pressure; the pressure gauge on the upper pipe will read

p_2 = 105,900 Pa – 1 atm = 4.6 kPa

An Irrigation System



ASSESS:

Reducing the pipe size decreases the pressure because it makes $v_2 > v_1$. Gaining elevation also reduces the pressure.

Light Reflecting from a Mirror

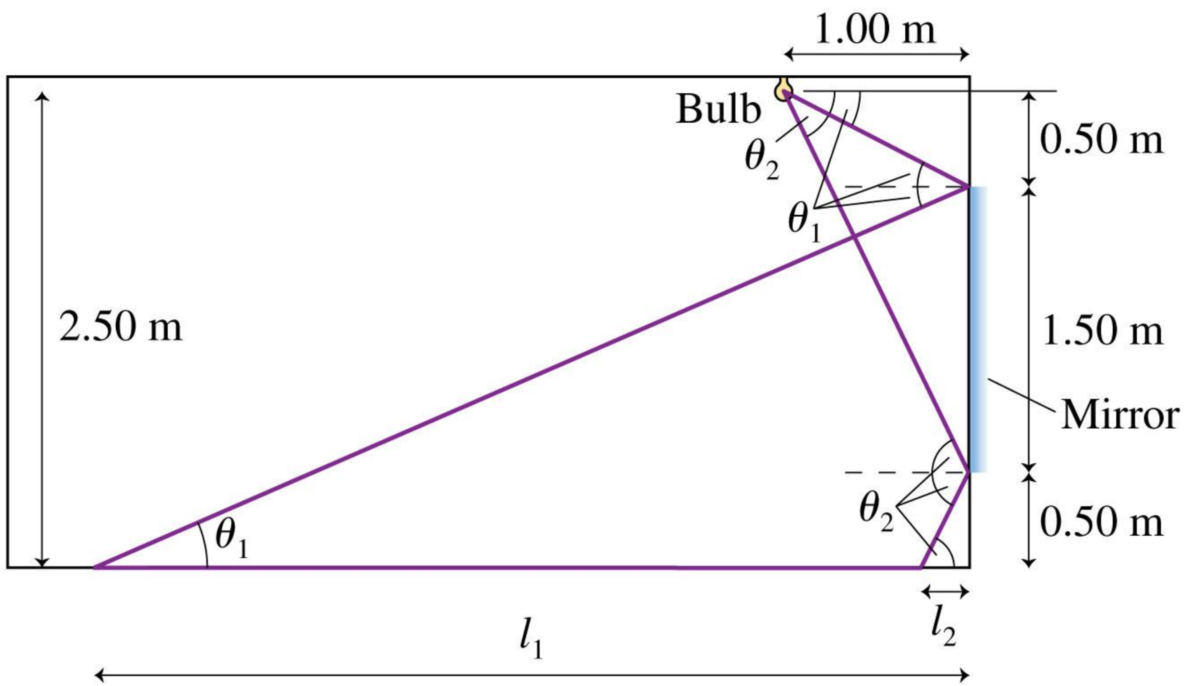
Q.20 A dressing mirror on a closet door is 1.5 m tall. The bottom is 0.50 m above the floor. A bare lightbulb hangs 1.00 m from the closet door, 2.50 m above the floor. How long is the streak of reflected light across the floor?

Light Reflecting from a Mirror

MODEL:

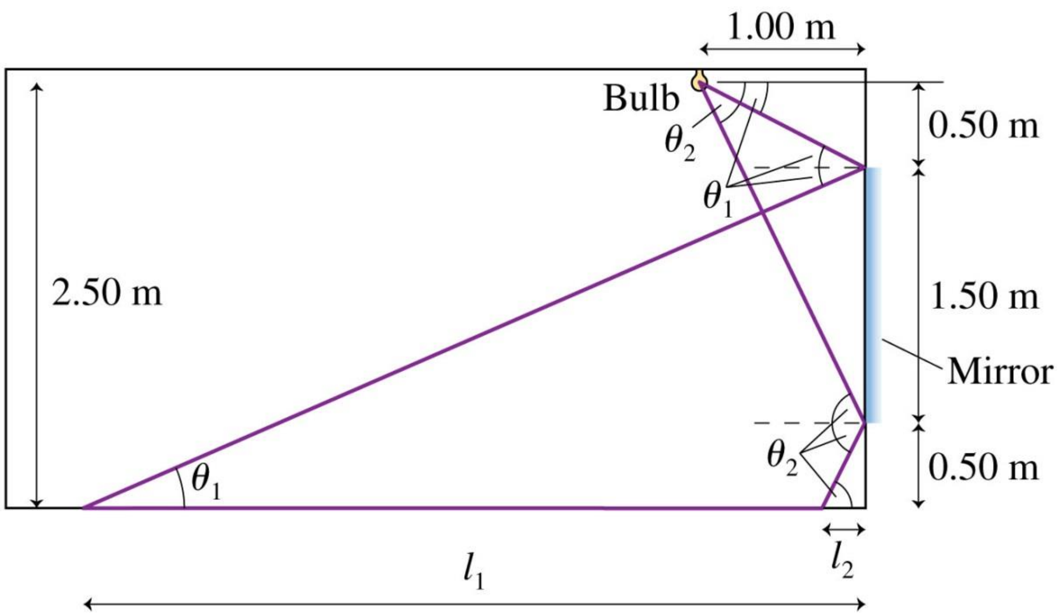
Treat the lightbulb as a point source and use the ray model of light.

Light Reflecting from a Mirror



VISUALISE: The figure above is a pictorial representation of the light rays. We need to consider only the two rays that strike the edges of the mirror. All other reflected rays will fall between these two.

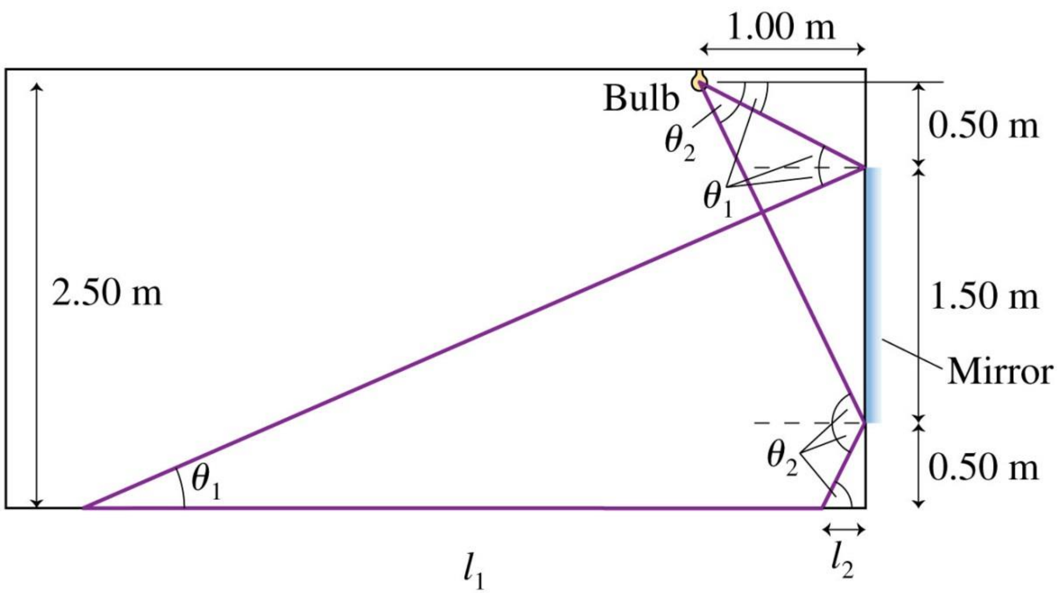
Light Reflecting from a Mirror



SOLVE: The figure above uses the law of reflection to set the angles of reflection equals to the angles of incidence. Other angles are identified with simple geometry. The two angles of incidence are

$$\theta_1 = \tan^{-1}\left(\frac{0.50\ \text{m}}{1.00\ \text{m}}\right) = 26.6°$$

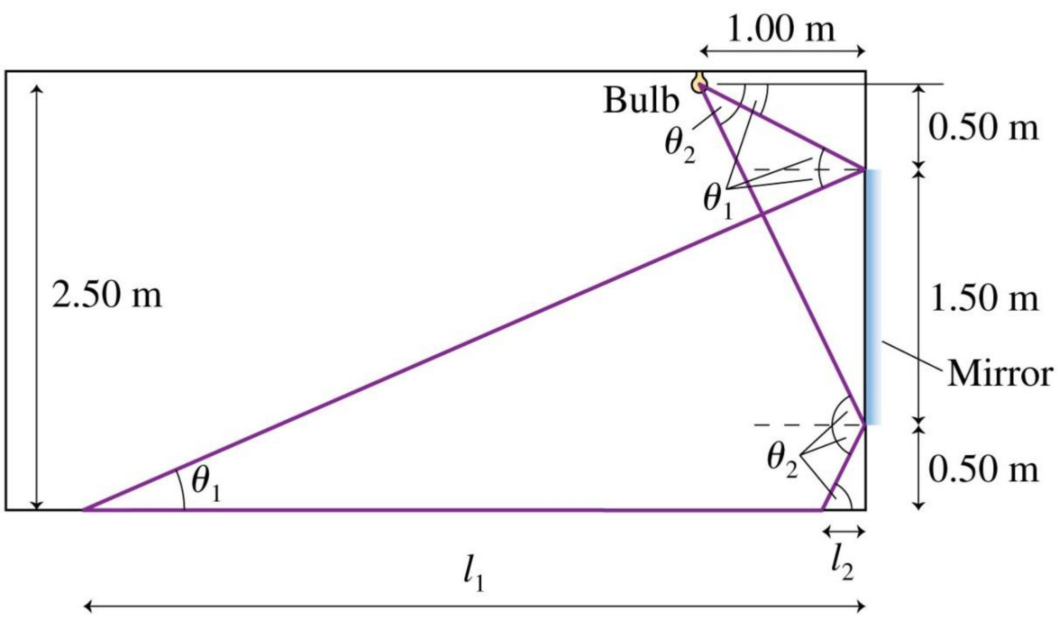
Light Reflecting from a Mirror



SOLVE: The figure above uses the law of reflection to set the angles of reflection equals to the angles of incidence. Other angles are identified with simple geometry. The two angles of incidence are

$$\theta_2 = \tan^{-1}\left(\frac{2.00\ \text{m}}{1.00\ \text{m}}\right) = 63.4°$$

Light Reflecting from a Mirror



SOLVE: The distance to the points where the rays strike the floor are then

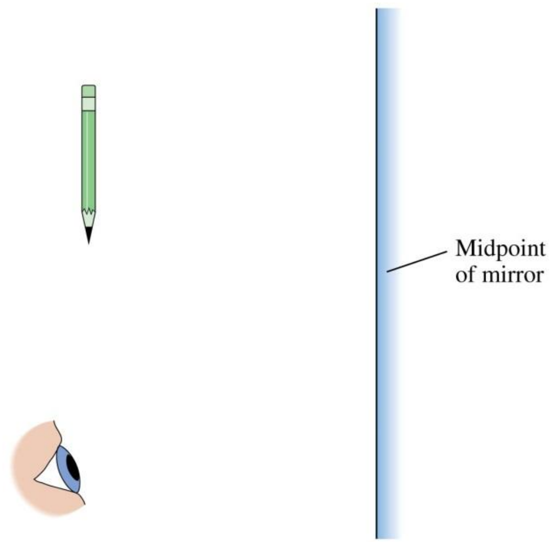
$$l_1 = \frac{2.00\text{ m}}{\tan\theta_1} = 4.00\text{ m}$$

$$l_2 = \frac{0.50\text{ m}}{\tan\theta_2} = 0.25\text{ m}$$

Thus, the length of the light streak is $l_1 - l_2 = 3.75$ m.

The Plane Mirror

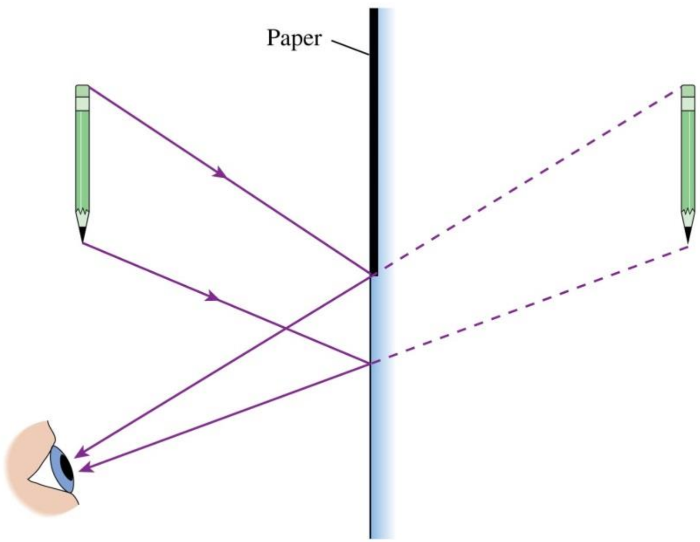
Q.21 You are looking at the image of a pencil in a mirror. What do you see in the mirror if the top half of the mirror is covered with a piece of dark paper?



a) The full image of the pencil.
b) The top half only of the pencil.
c) The bottom half only of the pencil.
d) No pencil, only the paper.

The Plane Mirror

Q.21 You are looking at the image of a pencil in a mirror. What do you see in the mirror if the top half of the mirror is covered with a piece of dark paper?



a) **The full image of the pencil.**

b) The top half only of the pencil.

c) The bottom half only of the pencil.

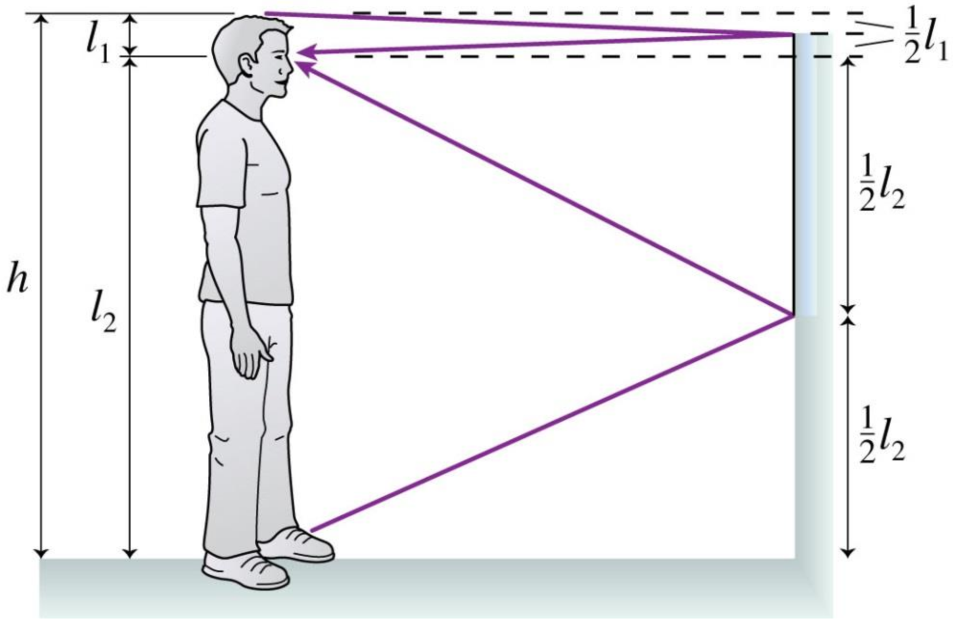
d) No pencil, only the paper.

How High is the Mirror?

Q.22 If your height is h, what is the shortest mirror on the wall in which you can see your full image? Where must the top of the mirror be hung?

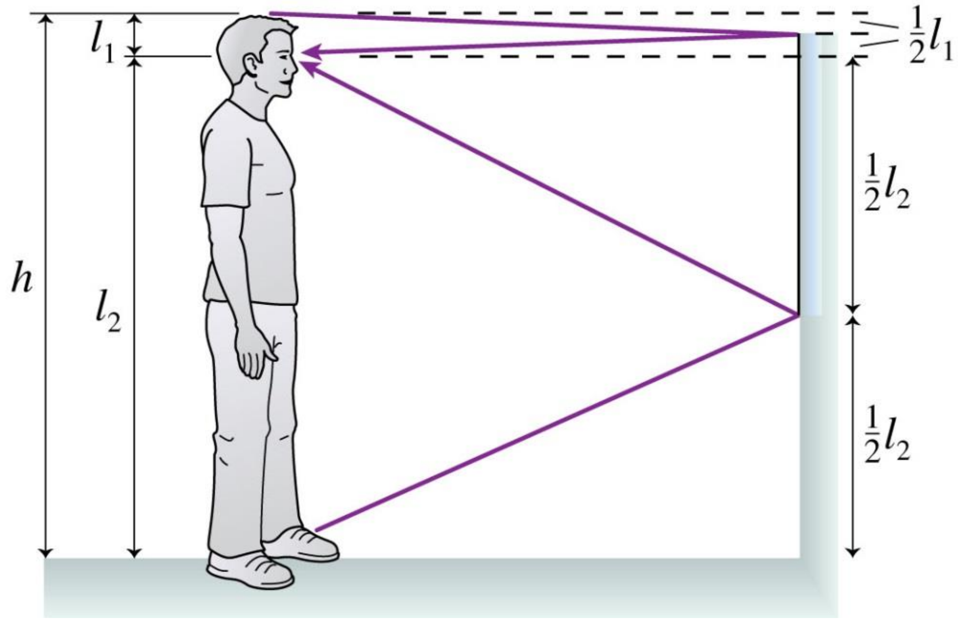
MODEL: Use the ray model of light.

How High is the Mirror?



VISUALISE: The figure above is a pictorial representation of the light rays. We need to consider only the two rays that leave your head and feet, and reflect into your eye.

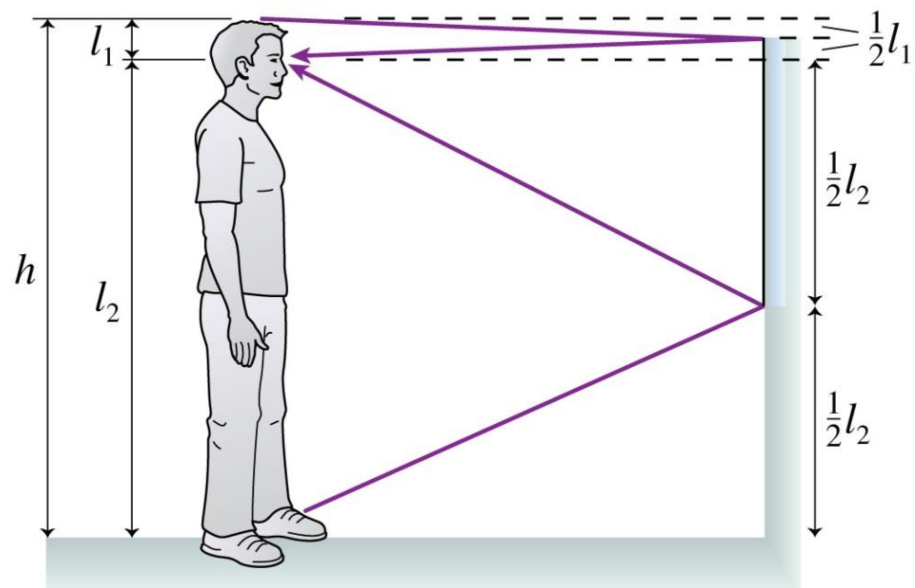
How High is the Mirror?



SOLVE:

Let the distance from your eyes to the top of your head be l_1 and the distance to your feet be l_2. Your height is $h = l_1 + l_2$.

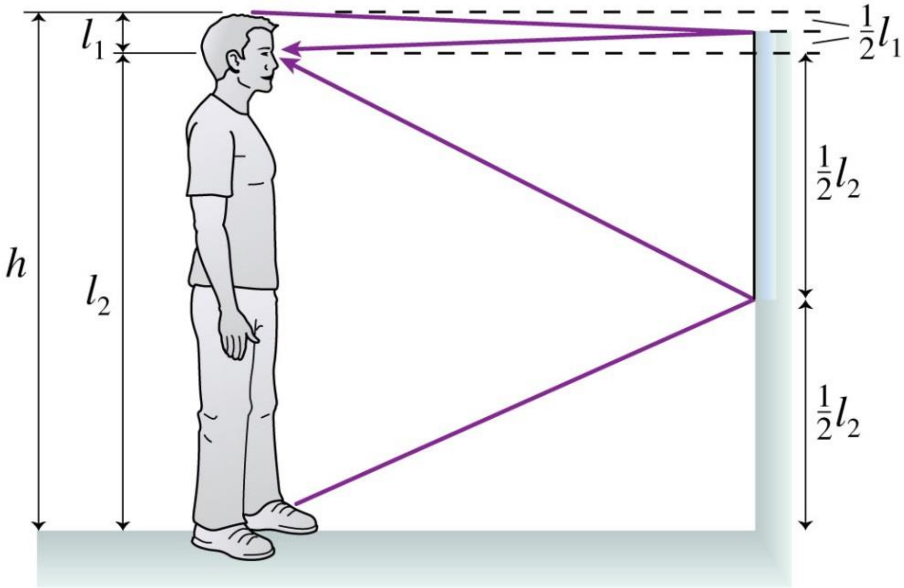
How High is the Mirror?



SOLVE:

- A light ray from the top of your head that reflects from the mirror at $\theta_t = \theta_i$ and enters your eye must, by congruent triangles, strike the mirror a distance ½ l_1 above your eyes.
- Similarly, a ray from your foot to your eye strike the mirror a distance ½ l_2 below your eyes.

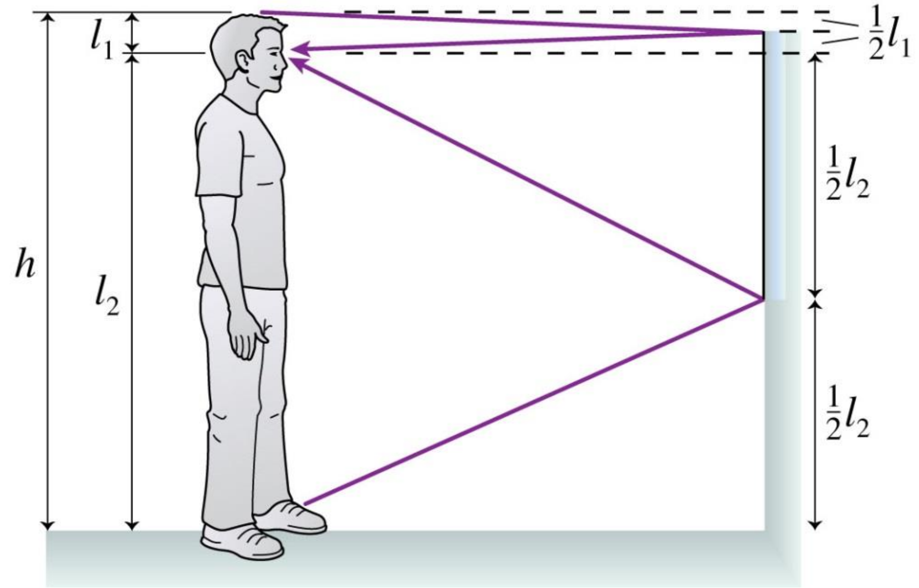
How High is the Mirror?



SOLVE:

- The distance between these two points on the mirror is ½ l_1 + ½ l_2 = ½ h. A ray from anywhere else on your body will reach your eye if it strikes the mirror between these two points.
- Pieces of the mirror outside these two points are irrelevant, not because rays don't strike them but because the reflected rays don't reach your eye.

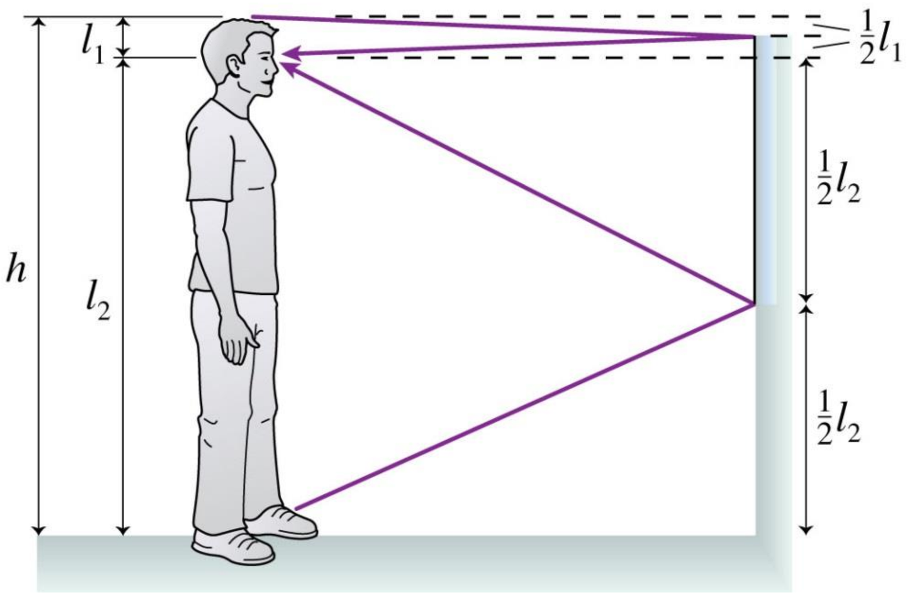
How High is the Mirror?



SOLVE:

- Thus, the shortest mirror in which you can see your full reflection is ½ *h*. But this will work only if the top of the mirror is hung midway between your eyes and the top of your head.

How High is the Mirror?



ASSESS:

- It is interesting that the answer does not depend on how far you are from the mirror.

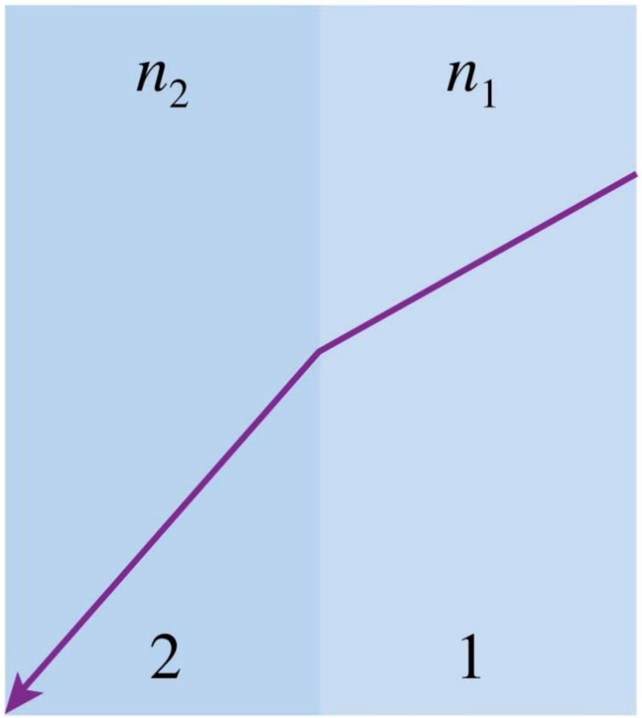
TACTICS BOX 23.1 **Analyzing refraction**

1. **Draw a ray diagram.** Represent the light beam with one ray.
2. **Draw a line normal to the boundary.** Do this at each point where the ray intersects a boundary.
3. **Show the ray bending in the correct direction.** The angle is larger on the side with the smaller index of refraction. This is the qualitative application of Snell's law.
4. **Label angles of incidence and refraction.** Measure all angles from the normal.
5. **Use Snell's law.** Calculate the unknown angle or unknown index of refraction.

Exercises 11–15

Refraction

Q.23 A laser beam passing from medium 1 to medium 2 is refracted as shown. Which is true?



a) $n_1 < n_2$.

b) $n_1 > n_2$.

c) There's not enough information to compare n_1 and n_2.

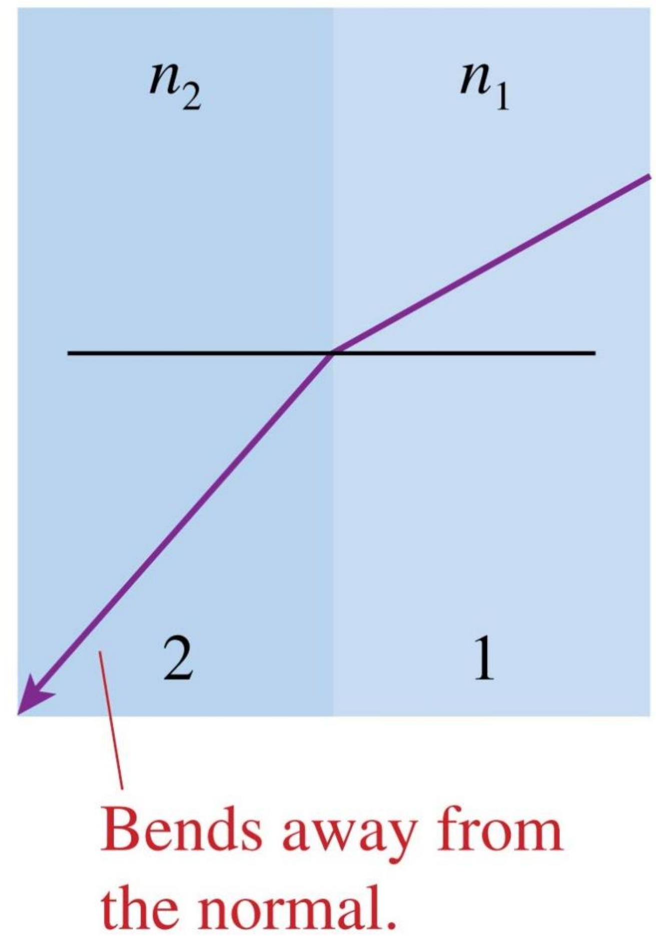
Refraction

Q.23 A laser beam passing from medium 1 to medium 2 is refracted as shown. Which is true?

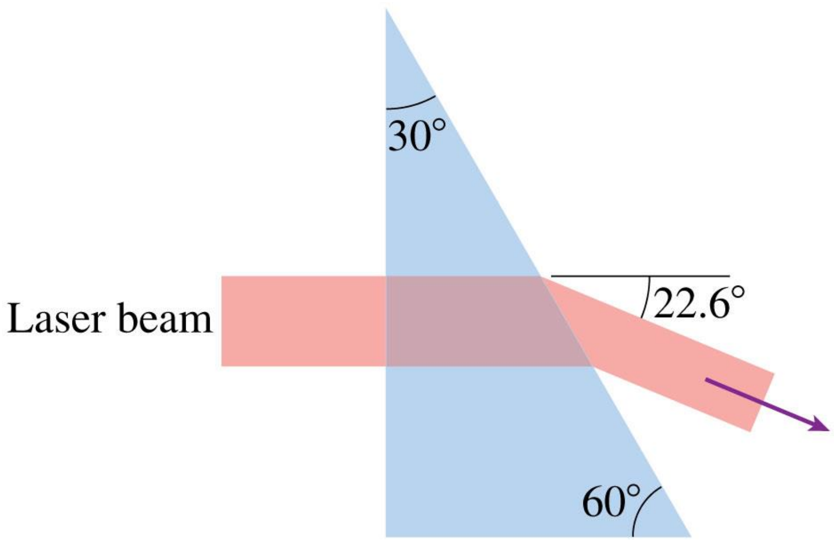
a) $n_1 < n_2$.

b) $n_1 > n_2$.

c) There's not enough information to compare n_1 and n_2.

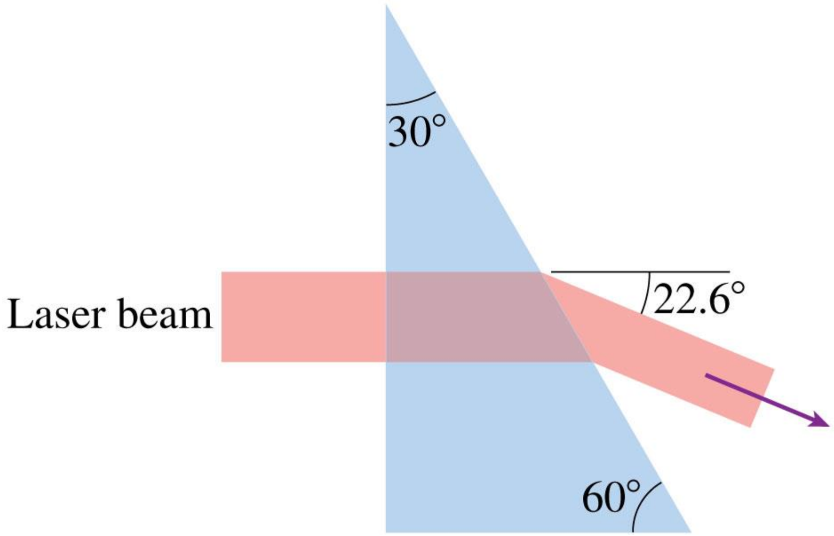


Measuring the Index of Refraction



Q.24 The figure above shows a laser beam deflected by a prism. What is the prism's index of refraction?

Measuring the Index of Refraction

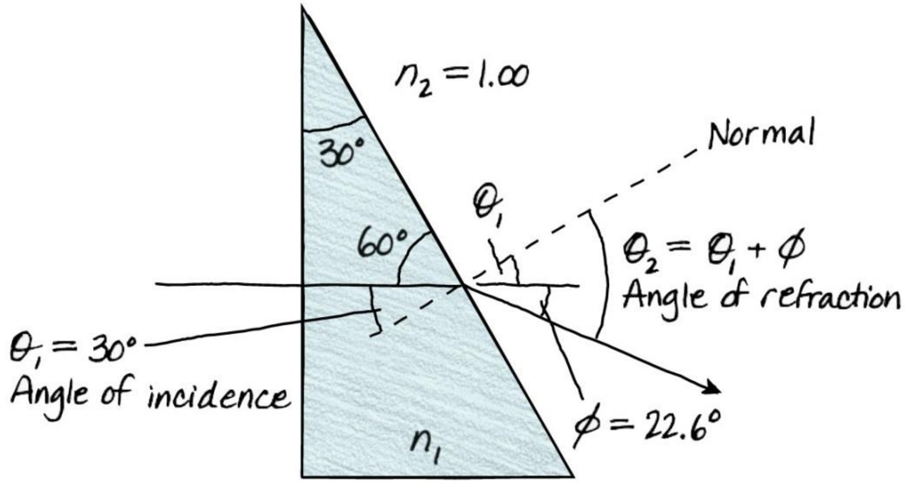


Q.24 The figure above shows a laser beam deflected by a prism. What is the prism's index of refraction?

MODEL:

Represent the laser beam with a single ray and use the ray model of light.

Measuring the Index of Refraction

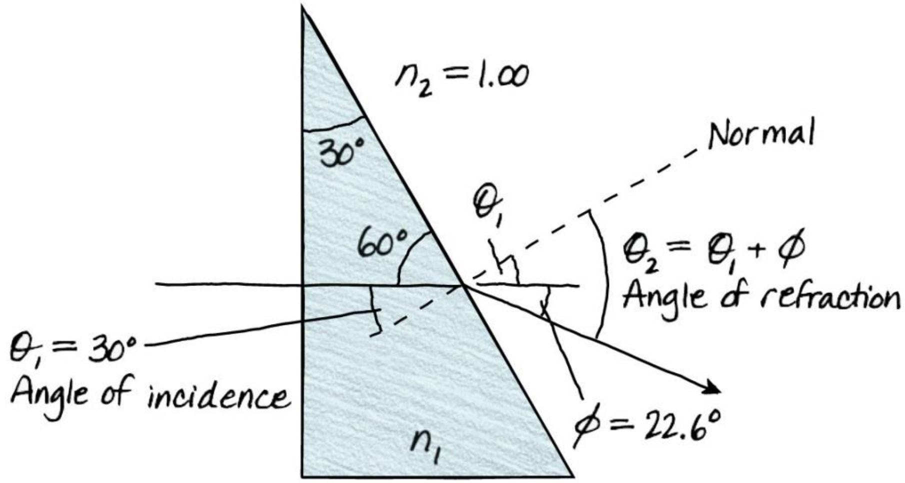


θ_1 and θ_2 are measured from the normal.

VISUALISE:

- The ray is incident perpendicular to the front face of the prism ($\theta_{incident} = 0°$). Thus, it is transmitted through the first boundary without deflection.
- At the second boundary, it is especially important to draw the normal to the surface at the point of incidence and to measure angles from the normal.

Measuring the Index of Refraction

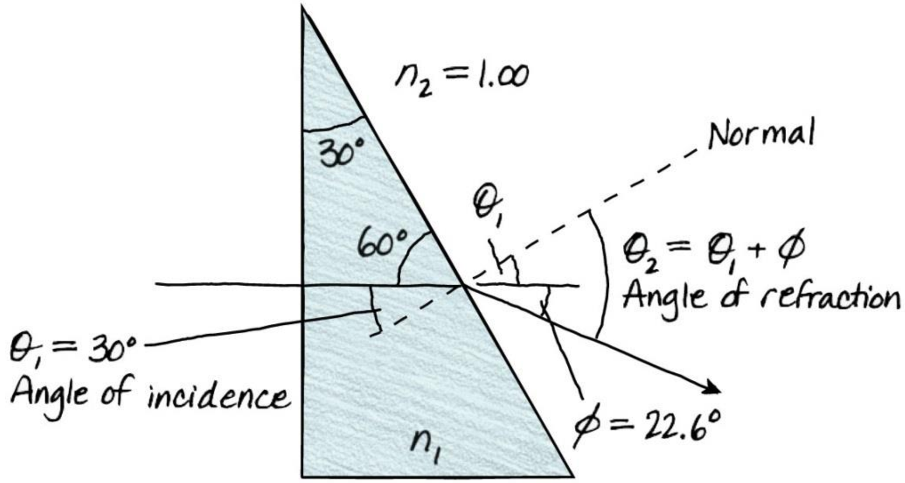


θ_1 and θ_2 are measured from the normal.

SOLVE:

- From the geometry of the triangle, you can find that the laser's angle of incidence on the hypotenuse of the prism is θ_1 = 30°, the same as the apex angle of the prism.

Measuring the Index of Refraction



θ_1 and θ_2 are measured from the normal.

SOLVE:

- The ray exists the prism at angle θ_2 such that the deflection is $\phi = \theta_2 - \theta_1 = 22.6°$. Thus, $\theta_2 = 52.6°$. Knowing both angles and $n_2 = 1.00$ for air, we can use Snell's law to find n_1:

$$n_1 = \frac{n_1 \sin\theta_2}{\sin\theta_1} = \frac{1.00\sin 52.6°}{\sin 30°} = 1.59$$

Measuring the Index of Refraction

$n_1 = 1.59$

ASSESS:

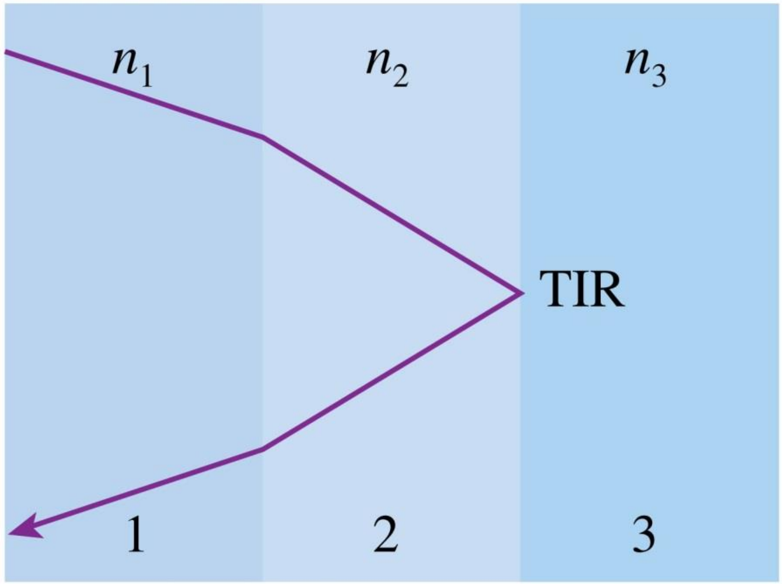
Referring to the indices of refraction in Table 23.1, we see that the prism is made of plastic.

TABLE 23.1 Indices of refraction

Medium	n
Vacuum	1.00 exactly
Air (actual)	1.0003
Air (accepted)	1.00
Water	1.33
Ethyl alcohol	1.36
Oil	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.41
Silicon (infrared)	3.50

Total Internal Reflection

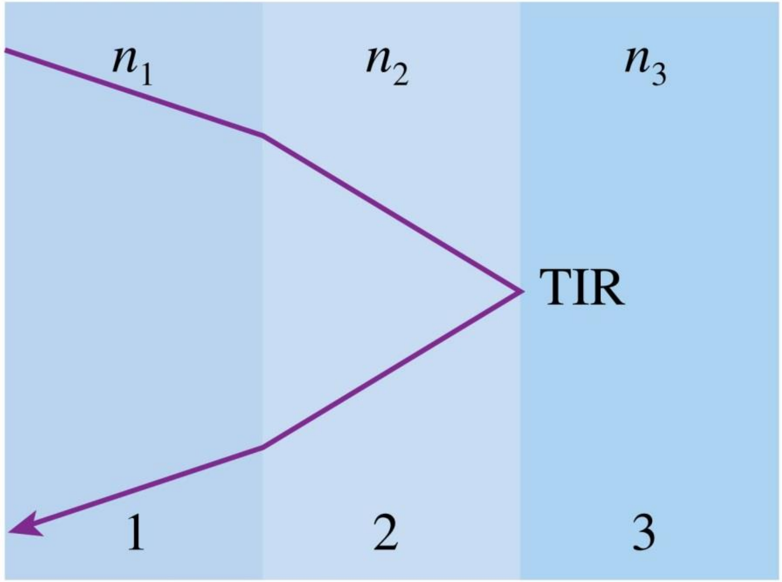
Q.25 A laser beam undergoes two refractions plus total internal reflection at the interface between medium 2 and medium 3. Which is true?



a) $n_1 < n_3$.
b) $n_1 > n_3$.
c) There's not enough information to compare n_1 and n_3.

Total Internal Reflection

Q.25 A laser beam undergoes two refractions plus total internal reflection at the interface between medium 2 and medium 3. Which is true?



a) $n_1 < n_3$.

b) $n_1 > n_3$.

c) There's not enough information to compare n_1 and n_3.