The University of Nottingham Ningbo China

Centre for English Language Education

MID-SEMESTER Exam: SAMPLE PAPER

FOUNDATION CALCULUS FOR PHYSICAL SCIENCES & ENGINEERING

Time allowed 60 Minutes

Candidates may complete the information required on the front page of this booklet but must NOT write anything else until the start of the examination period is announced.

This paper comprises TWENTY questions.

Answers must be written (with necessary steps) in this booklet.

Figures enclosed by square brackets, eg. [3], indicate marks for that question.

Only silent, self-contained calculators with a Single-line Display

or Dual-line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

Do NOT turn examination paper over until instructed to do so.

ADDITIONAL MATERIAL: *Useful formulae (on Page 2 of this booklet).*

INFORMATION FOR INVIGILATORS: *Please collect this booklet at the end of the exam.*

Student ID: ____________________ **Seminar Group**: __________ *(e.g. A23 or B13 or C17)*

Marks obtained: _____________

Useful formulae:

- **Differentiation: Useful results**

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(u \cdot v \cdot w) = u\,v \cdot \frac{dw}{dx} + v\,w \cdot \frac{du}{dx} + u\,w \cdot \frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,\frac{du}{dx} - u\,\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \qquad \textbf{and} \qquad \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

- **Derivatives of standard functions**

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\,\cot x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

- **Maclaurin's series**

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}\,f''(0) + \frac{x^3}{3!}\,f'''(0) + \ldots\ldots + \frac{x^n}{n!}\,f^{(n)}(0) + \ldots\ldots$$

- **Integration**

$$\int x^n \; dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \cos x \; dx = \sin x + C$$

$$\int \sin x \; dx = -\cos x + C$$

$$\int \sec^2 x \; dx = \tan x + C$$

$$\int \operatorname{cosec}^2 x \; dx = -\cot x + C$$

$$\int \sec x \,\tan x \; dx = \sec x + C$$

$$\int \operatorname{cosec} x \,\cot x \; dx = -\operatorname{cosec} x + C$$

$$\int e^x \; dx = e^x + C$$

$$\int a^x\, dx = \frac{a^x}{\ln a} + C \qquad (a > 0)$$

$$\int \frac{1}{x} \; dx = \ln|\,x\,| + C$$

$$\int \frac{1}{x^2+a^2} \; dx = \frac{1}{a}\,\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \; dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{|\,x\,|\sqrt{x^2-a^2}} = \frac{1}{a}\,\sec^{-1}\left|\,\frac{x}{a}\,\right| + C$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} \; dx = \ln\,|\,x + \sqrt{x^2 \pm a^2}\,| + C$$

$$\int \frac{1}{x^2-a^2} \; dx = \frac{1}{2a}\,\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + C$$

- **Trigonometry**

$$\left[\begin{array}{l} \cos^2\theta + \sin^2\theta = 1 \\ \tan^2\theta + 1 = \sec^2\theta \\ \cot^2\theta + 1 = \operatorname{cosec}^2\theta \end{array}\right.$$

$$\left[\begin{array}{lcl} 2\sin A\cos B & = & \sin(A+B) + \sin(A-B) \\ 2\cos A\sin B & = & \sin(A+B) - \sin(A-B) \\ 2\cos A\cos B & = & \cos(A+B) + \cos(A-B) \\ -2\sin A\sin B & = & \cos(A+B) - \cos(A-B) \end{array}\right.$$

1 Use the first principle to prove that $\frac{d}{dx} cos3x = -3sin3x$. [2]

2 Given $y = \left(x + \frac{1}{x}\right) \cdot lnx$, use product rule to find $\frac{dy}{dx}$. [3]

3 Given $y = \frac{lnx}{x^2}$, by using quotient rule for derivative prove that $x^3 \frac{dy}{dx} + 2lnx = 1$. [3]

4 Given $y = e^{\cos(3x+2)}$, use chain rule for derivatives to find $\frac{dy}{dx}$. [3]

5 For the given implicit function $x^2 + y^2 = 7xy$, find $\left.\frac{dy}{dx}\right|_{(1,0)}$. [3]

6 Given $y = \frac{\sin^3 2x}{\sqrt[5]{x}\, tanx}$, use logarithmic differentiation to find $\frac{dy}{dx}$. [4]

7 Use the method of parametric differentiation to find $\frac{dy}{dx}$ [3]

if $x = a(1 - cos2\theta)$ and $y = b(\theta + sin2\theta)$, (a, b are constants).

8 Obtain the equation of tangent line to the curve $e^x y^2 + x^2 = 25$ at (0, 5). [3]

9 Find and classify the stationary points for the curve [4]

$f(x) = 2x^3 - 9x^2 - 24x + 6$.

10 Sketch the curve given in Q. 9. [1]

11 Determine the rate of gaining altitude of an aircraft having speed 500 km/h if it is climbing at a 60^0 angle with the horizontal. [2]

12 Given $y = tanx$, differentiate up to third order. [2]

13 Obtain Maclaurin's series expansion of $f(x) = \ln(1 - x)$. Hence find the expansion of $\ln(1 + x)$. [4]

14 Use the Newton-Raphson method to approximate the value of $\sqrt[3]{3}$, correct to 7 d.p., by starting with $x_0 = 1$. [4]

15 Evaluate the integral: $\int \left(\frac{x^4-27x}{3-x}\right) dx$. [3]

16 Evaluate the integral $\int \frac{sin3x}{(1+2cos3x)^3}\, dx$, using the substitution $1 + 2cos3x = t$. [3]

17 Use appropriate substitution and evaluate the integral $\int \frac{cosx}{\sqrt[3]{4+sinx}}\, dx$. [3]

18 By completing square in denominator, evaluate $\int \frac{1}{\sqrt{4x^2+4x+3}}\, dx$. [4]

19 Evaluate the integral $\int sin^8x\, cos^3x\, dx$. [3]

20 Evaluate the integral $\int cos7x\, cos3x\, dx$. [3]