The University of Nottingham Ningbo China

Centre for English Language Education

Semester One, 2017-2018

FOUNDATION ALGEBRA FOR PHYSICAL SCIENCES & ENGINEERING

Time allowed 1 Hour 30 Minutes

Candidates may complete the front cover of their answer book and sign their attendance card but must NOT write anything else until the start of the examination period is announced.

This paper contains EIGHT questions which carry equal marks.

An indication is given of the weighting of each subsection of a question by means of a figure enclosed by square brackets, eg. [5], *immediately following that subsection.*

Only silent, self-contained calculators with a Single-line Display or Dual-line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

Do NOT turn examination paper over until instructed to do so.

ADDITIONAL MATERIAL: *Formula Sheet (attached to the back of the question paper)*

INFORMATION FOR INVIGILATORS:

1. *Please give a 15 minute warning.*
2. *Please collect Answer Booklets, Question Papers and Formula Sheet at the end of the exam.*

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1 (a) Given $f(x) = x^2 - 8x + 25$.

(i) Use the method of completing the square to express $f(x)$ in the form $(x-a)^2 + b$, where a and b are to be determined.

(ii) Sketch the curve $y = f(x)$. [2]

(b) Given $f(x) = x + 3$. Find a linear function $g(x) = ax + b \ \ (a, b \in \mathbb{R},\ a \neq 0)$ such that

$$(f \circ g)(2) = 4 \quad \text{and} \quad g^{-1}(3) = 1.$$ [4]

(c) Solve for $x \in \mathbb{R}$, the exponential equation $e^{2x} - e^x - 6 = 0$. [2]

(d) Is it possible to have $\ln(x+2) + \ln(x-2) = \ln(2x-5)$ for $x \in \mathbb{R}$?

Justify your answer. [2]

2 (a) Use the formula for $\sin(A+B)$ and multi-angle formulae to show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta\,.$$ [1]

(b) Given $A + B + C = 2\,\pi$.

Show that $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$. [2]

(c) Given $\cos\theta = \dfrac{3}{5}$; $0 < \theta < \dfrac{\pi}{2}\cdot$ Using multi-angle formulae or otherwise, calculate

(i) $\cos 2\theta$;

(ii) $\sin 4\theta$. [3]

(d) Given $f(x) = 2\cos x - \sin x$.

(i) Express $f(x)$ in the form $R\cos(x+\theta)$, where $R > 0$ and $\theta \in \left(0\,, \dfrac{\pi}{2}\right)$ are to be determined.

(ii) Find the range and period of f.

(iii) Sketch the curve for $y = f(x)$. [4]

3 (a) Given $p(x) = 6\,x^3 - 17\,x^2 - 30\,x + 56$.

(i) Use the method of synthetic division to show that $(x+2)$ is a factor of $p(x)$.

(ii) Hence, express $p(x)$ completely as a product of linear factors.

(iii) Solve for $x \in \mathbb{R}$, the polynomial equation $p(x) = 0$.

[4]

(b) Given polynomial function $p(x) = 2\,x^3 + a\,x^2 - 9\,x + b$.

Find the constants of a and b if $(x-2)$ and $(x+3)$ are factors of $p(x)$.

[3]

(c) Express $\dfrac{2}{(x-1)\,(x^2+1)}$ as a sum of partial fractions.

[3]

4 Consider solving numerically the equation

$$f(x) = x^3 + 3\cos x - 4 = 0. \tag{4.1}$$

(a) Apply the Intermediate Value Theorem to show that a root of the equation (4.1) lies in the interval $(1, 2)$. [1]

(b) Show that the equation (4.1) can be rearranged to obtain the iterative formula

$$x_{n+1} = \sqrt{\frac{4 - 3\cos x_n}{x_n}}\,. \tag{4.2}$$

[2]

(c) Starting with $x_0 = 1.5$, the iterative formula (4.2) is used to calculate the root x^* of the equation (4.1). If the desired accuracy is for 5 decimal places, write the values of approximations x_i $(i = 1, 2, 3, 4)$. Also find the root x^*.

[4]

(d) Starting with $x_0 = 1.5$, use the Bisection method to find approximations x_1 and x_2, correct to 2 decimal places. Write the steps involved in the calculations.
(<u>You do not need to find the root.</u>)

[3]

5 (a) Use the Binomial Theorem to expand $(2\,x + 3\,y)^5$. [2]

(b) (i) Use the Generalized Binomial Theorem to expand

$$\frac{1}{(1-3\,x)^4} \quad ; \quad |\,x\,| < \frac{1}{3}\,.$$

(ii) Hence find the coefficient of x^4 in the above expansion. [3]

(c) Find the term independent of x in the expansion of $\left(x - \dfrac{5}{x}\right)^8$. [2]

(d) (i) Use the approximation

$$(1+x)^n \approx 1 + n\,x + \frac{n\,(n-1)}{2\,!}\,x^2$$

to approximate the value of $(1.025)^3$.

(ii) The radius r of a sphere is measured with an error $\delta r = 2.5\,\%$ of r.

Use the result in 5(d)(i) to estimate the resulting error δV in its volume. [3]

6 (a) Given matrices $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$. Find matrix $C = (AB)^T - B^{-1}$. [3]

(b) Consider solving the system of simultaneous linear equations using the matrix method.

$$\left.\begin{aligned} 3\,x + 4\,y - 13 &= 0 \\ 9\,x - 7\,y + 37 &= 0 \end{aligned}\right\} \qquad (6.1)$$

(i) Write the system of equations (6.1) in matrix form

$$AX = B \qquad (6.2)$$

clearly stating what A, B, and X are.

(ii) Find the inverse matrix, A^{-1}.

(iii) Starting with (6.2), show that the solution matrix X is given by

$$X = A^{-1}B. \qquad (6.3)$$

(iv) Use (6.3) to solve the system of equations (6.1).

(No marks will be given for any solution obtained by another method). [5]

(c) Given $A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$. Show that $A^2 = I$ and hence find the inverse matrix A^{-1}. [2]

7 (a) Given $\left(1+i+2\,i^2\right)\cdot(x+i\,y)=3\,i-5$. Find $x, y \in \mathbb{R}$. [3]

(b) (i) Solve for $z \in \mathbb{C}$, the cubic equation $z^3+8=0$.

(ii) Show the three roots obtained in 7(b)(i) on the Argand diagram. [3]

(c) (i) Given complex numbers $z_1 = 12-5\,i$, $z_2 = 3+4\,i$, and $z_3 = 6-8\,i$.

Use properties of modulus to evaluate $\left|\dfrac{\overline{z_1}^{\,2}}{\overline{z_3}}\cdot\left(\dfrac{z_2\cdot z_3}{z_1}\right)\right|$.

(ii) Express complex number z_1 defined in 7(c)(i) in polar form

$r\,(\cos\theta + i\,\sin\theta\,)$, where $r>0$ and $-\pi<\theta\le\pi$. [4]

8 (a) The fourth term of an Arithmetic sequence is 10 and its twelfth term is 66. Find

(i) the 20 term of the sequence;

(ii) the sum of its first 20 terms. [3]

(b) For a sequence in Geometric progression (G.P.), the first term is 10 and the sum of its first three terms is 310. Find the G.P. [2]

(c) Find the sum of the infinite geometric series:

$$\frac{1}{2}+\frac{1}{10}+\frac{1}{50}+\frac{1}{250}+\cdots.$$ [1]

(d) Given $f(n)=\dfrac{1}{n\,(n+1)}$.

(i) Simplify $f(n)-f(n+1)$.

(ii) Use the result in 8 (d) (i) to show that

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots\ldots \text{ (up to } n \text{ terms)} = \frac{n\,(n+3)}{4\,(n+1)\,(n+2)}\,.$$

(iii) Hence, find the sum: $\displaystyle\sum_{1}^{\infty}\frac{1}{n\,(n+1)\,(n+2)}$. [4]