Answer to Exercise 4.1

Note: these are not necessarily the only possibilities, nor necessarily the "simplest". But they are all fairly simple, and your answers should not be much more complicated.

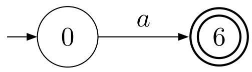
1. $(\mathbf{b}+\mathbf{c})^*\mathbf{a}(\mathbf{b}+\mathbf{c})^*$
2. $(\mathbf{a}+\mathbf{c})^*\mathbf{b}(\mathbf{a}+\mathbf{c})^*\mathbf{b}(\mathbf{a}+\mathbf{b}+\mathbf{c})^*$
3. $(\mathbf{a}+\mathbf{b})^*(\epsilon+\mathbf{c})(\mathbf{a}+\mathbf{b})^*(\epsilon+\mathbf{c})(\mathbf{a}+\mathbf{b})^*$
4. $(\mathbf{a}+\mathbf{b})^*(\mathbf{a}+\mathbf{c})^*$
5. $\mathbf{a}^*(\mathbf{b}\mathbf{a}^*\mathbf{c}+\mathbf{c}\mathbf{a}^*\mathbf{b})\mathbf{a}^*$
6. $\mathbf{c}^*(\mathbf{a}+\mathbf{b})((\mathbf{a}+\mathbf{b})\mathbf{c}^*(\mathbf{a}+\mathbf{b})+\mathbf{c})^*$
7. $(\mathbf{a}+\mathbf{b}+\mathbf{c})^*\mathbf{abba}(\mathbf{a}+\mathbf{b}+\mathbf{c})^*$

Answer to Exercise 4.2

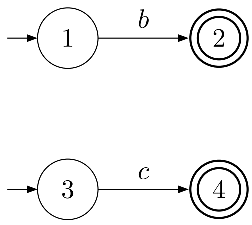
$$
\begin{array}{lll}
 & L((\mathbf{aa}+\epsilon\mathbf{b}^*\emptyset)(\mathbf{b}+\mathbf{c})) & \\
= & & \{L(EF)=L(E)L(F)\} \\
 & L(\mathbf{aa}+\epsilon\mathbf{b}^*\emptyset)L(\mathbf{b}+\mathbf{c}) & \\
= & & \{L(E+F)=L(E)\cup L(F)\}\text{ (twice)} \\
 & (L(\mathbf{aa})\cup L(\epsilon\mathbf{b}^*\emptyset))(L(\mathbf{b})\cup L(\mathbf{c})) & \\
= & & \{L(EF)=L(E)L(F)\}\text{ (three times)} \\
 & (L(\mathbf{a})L(\mathbf{a})\cup L(\epsilon)L(\mathbf{b}^*)L(\emptyset))(L(\mathbf{b})\cup L(\mathbf{c})) & \\
= & & \{L(E^*)=(L(E))^*\} \\
 & (L(\mathbf{a})L(\mathbf{a})\cup L(\epsilon)L(\mathbf{b})^*L(\emptyset))(L(\mathbf{b})\cup L(\mathbf{c})) & \\
= & & \{L(\mathbf{x})=\{x\},\ L(\emptyset)=\emptyset,\ L(\epsilon)=\{\epsilon\}\} \\
 & (\{a\}\{a\}\cup\{\epsilon\}\{b\}^*\emptyset)(\{b\}\cup\{c\}) & \\
= & & \{L\emptyset=\emptyset\text{ (twice)}\} \\
 & (\{a\}\{a\}\cup\emptyset)(\{b\}\cup\{c\}) & \\
= & & \{\text{Set union}\} \\
 & \{a\}\{a\}\{b,c\} & \\
= & & \{\text{Concatenation of languages}\} \\
 & \{aab,aac\} &
\end{array}
$$

Answer to Exercise 4.3

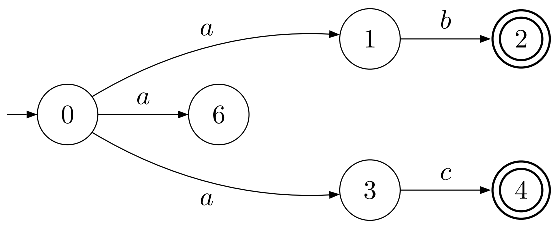
Construct an NFA A for $(\mathbf{a}(\mathbf{b}+\mathbf{c}))^*$ according to the lecture notes. Start with the innermost subexpressions and then join the NFAs together step by step. (I have named the states according to how they will be named in the final NFA to make it easier to follow the derivation. It is OK to leave states unnamed to the end.) NFA for $\mathbf{a}$:



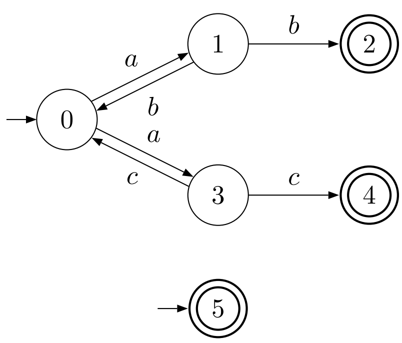
NFA for $\mathbf{b}+\mathbf{c}$



Join the above two NFAs to obtain an NFA for $\mathbf{a}(\mathbf{b}+\mathbf{c})$:



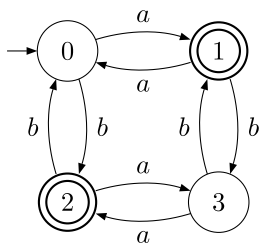
The last step is to carry out the construction corresponding to the $*$-operator. States 1 and 3 both immediately precede a final state, and we should thus add corresponding transition edges from those back to all start states. But there is only one start state, 0, so only one edge from each. Additionally, we must not forget to add an extra start state which is also final (here state 5) to ensure the NFA accepts ϵ. Finally, state 6 is manifestly now a "dead end" and can thus be eliminated:



Note that the isolated state 5 also is part of the same automaton.

Answer to Exercise 5.1

Recall this was the transition diagram:



In the BASE step of the table-filling algorithm, we mark every pair of states (q, q') in the DFA where one of q, q' is final and the other not. Thus we get:

	0	1	2
3		x	x
2	x		
1	x		

Next, for each remaining pair (q, q'), we see if there is any letter $c \in \Sigma$ such that $(\delta(q, c), \delta(q', c))$ is marked in the table. There is no such pair. Thus the algorithm is complete.

Therefore we combine states 0 and 3, and combine states 1 and 2. We therefore get the minimised DFA: