The University of Nottingham

UNITED KINGDOM · CHINA · MALAYSIA

Science A Physics

Lecture 1:

Describing Motion

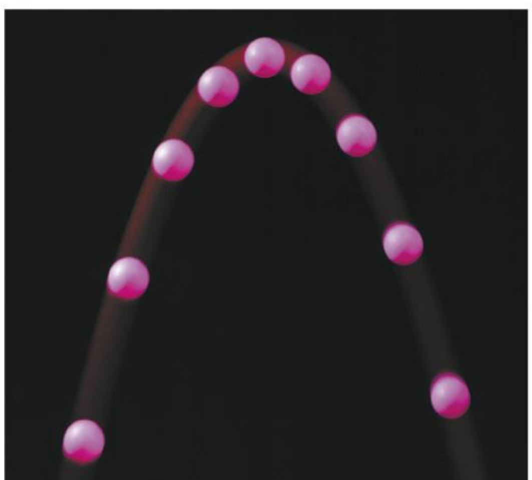
Aims of today's lecture

1. Motion diagrams
2. Vectors
3. Displacement
4. Average speed and average velocity
5. Acceleration
6. Vector components

Types of Motion

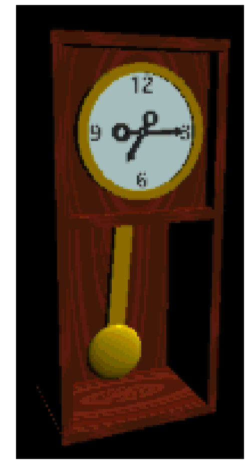
Linear motion

Circular motion



Projectile motion

Rotational motion

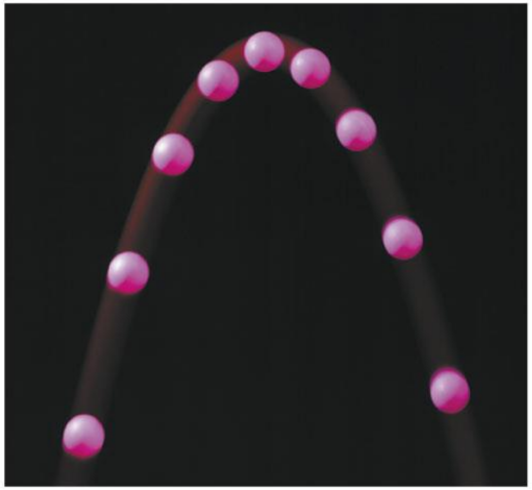


Periodic
(or oscillatory motion)

Types of Motion

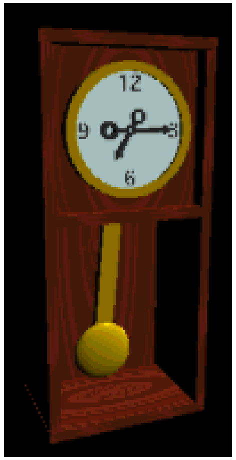
Linear motion

Circular motion



Projectile motion

Rotational motion



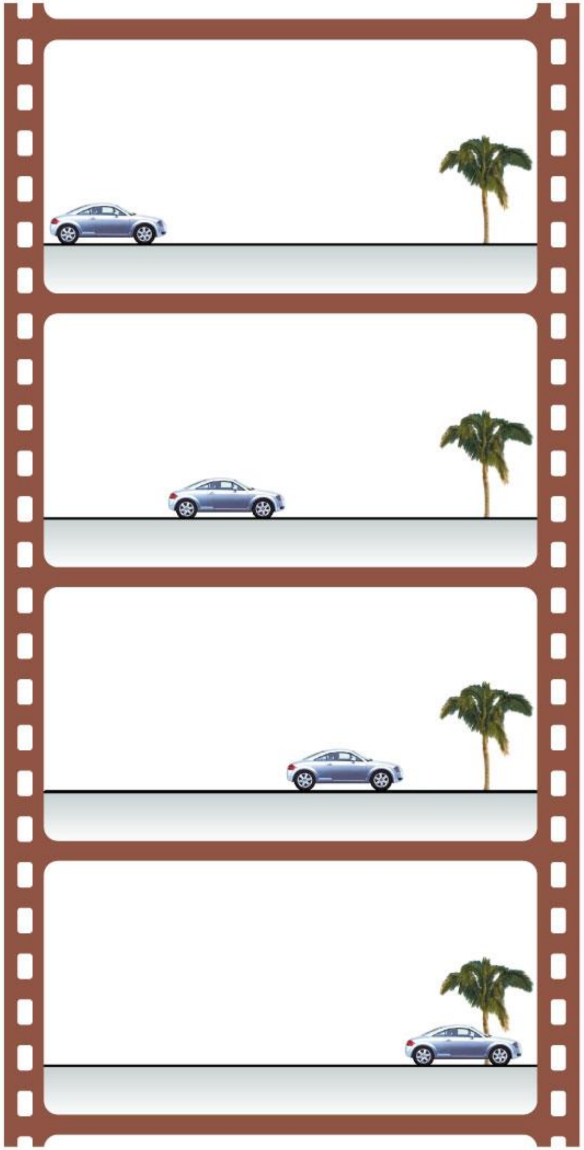
Periodic
(or oscillatory motion)

- Our focus for this course.

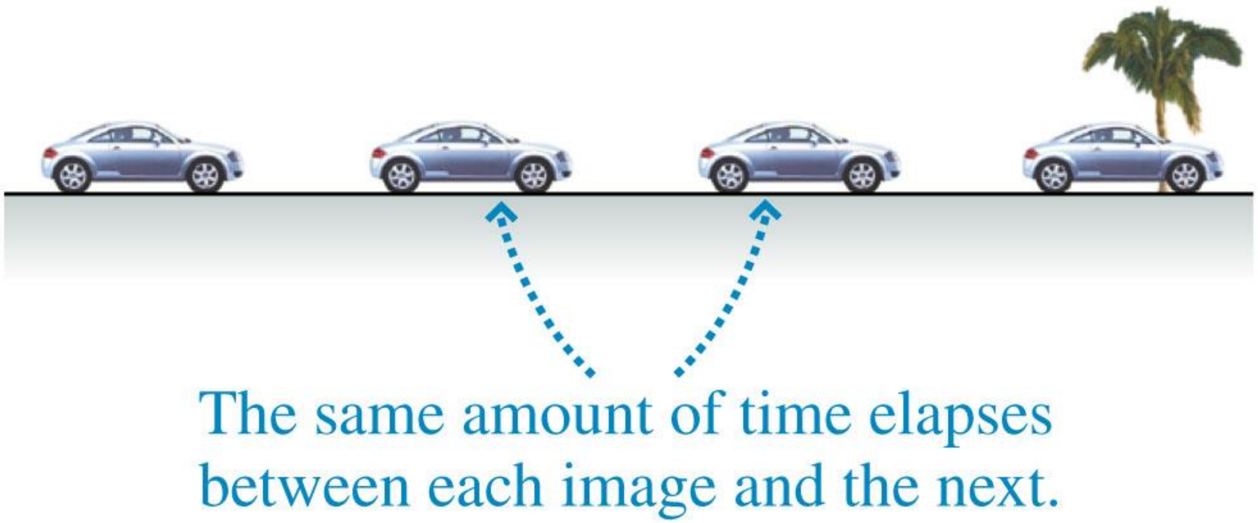
1. Motion Diagrams

Motion Diagrams

- Consider a movie of a moving object.
- A movie camera takes photographs at a fixed rate (i.e., 30 photographs every second).
- Each separate photo is called a **frame**.
- The car is in a different position in each frame.
- Shown are four frames in a **filmstrip**.



Motion Diagrams

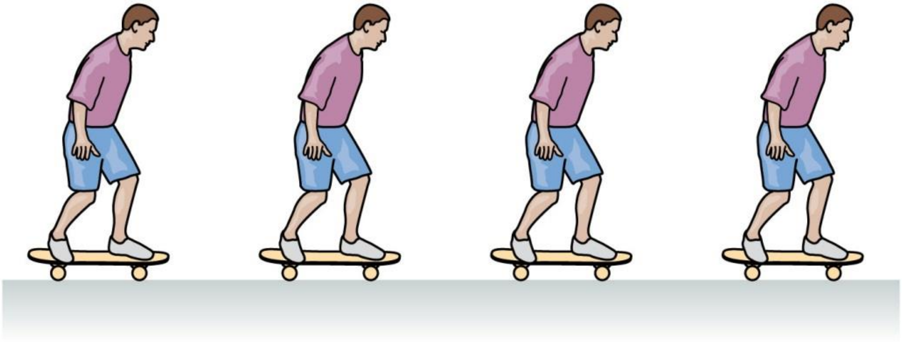


- Cut individual frames of the filmstrip apart.
- Stack them next to each other.
- This composite photo shows an object’s position at several equally spaced instants of time.
- This is called a **motion diagram**.

Motion Diagrams

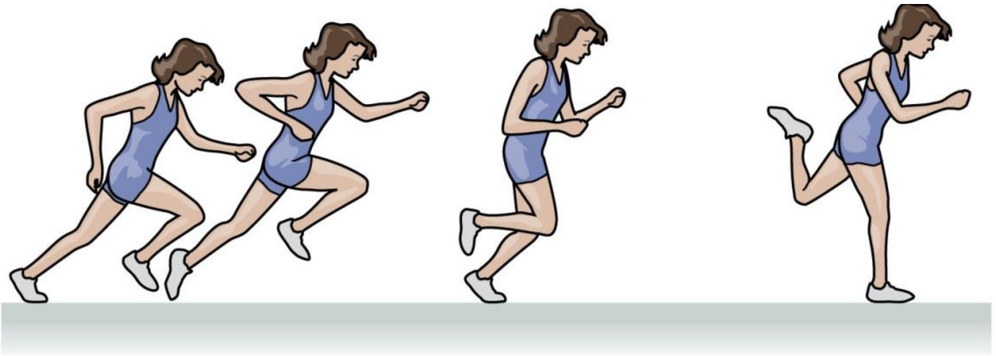
- An object that has a single position in a motion diagram is at rest.
- E.g. **a stationary ball on the ground**.

- An object with images that are equally spaced is moving with constant speed.
- E.g. **a skateboarder rolling down the sidewalk**.



Motion Diagrams

- An object with images that have increasing distance between them is speeding up.
- E.g. **a sprinter starting the 100 metre dash**.



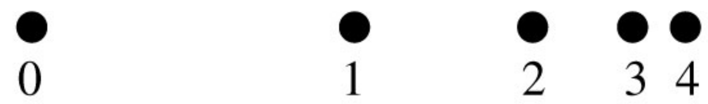
- An object with images that have decreasing distance between them is slowing down.
- E.g. **a car stopping for a red light**.

Motion Diagrams

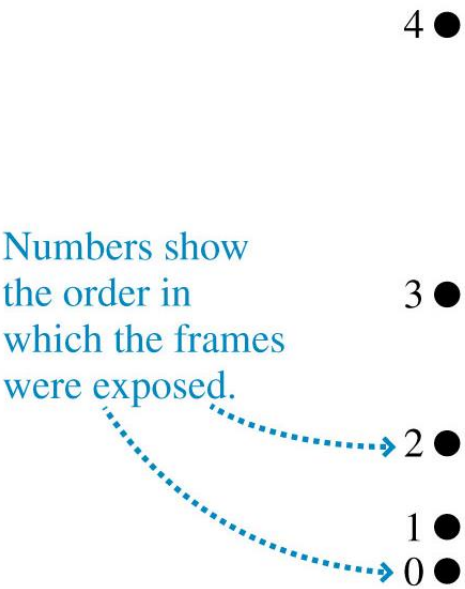
- A motion diagram can show more complex motion in two dimensions.
- E.g. **a jump shot from centre court**.
- In this case, the ball is slowing down as it rises, and speeding up as it falls.

Motion Diagrams

- Often, motion of the object as a whole is not influenced by details of the object's size and shape.
- We only need to keep track of a single point on the object.
- So we can treat the object as if all its mass were concentrated into a single point, called a **particle**.
- Below is a motion diagram of a car stopping, using what we call the **particle model**.

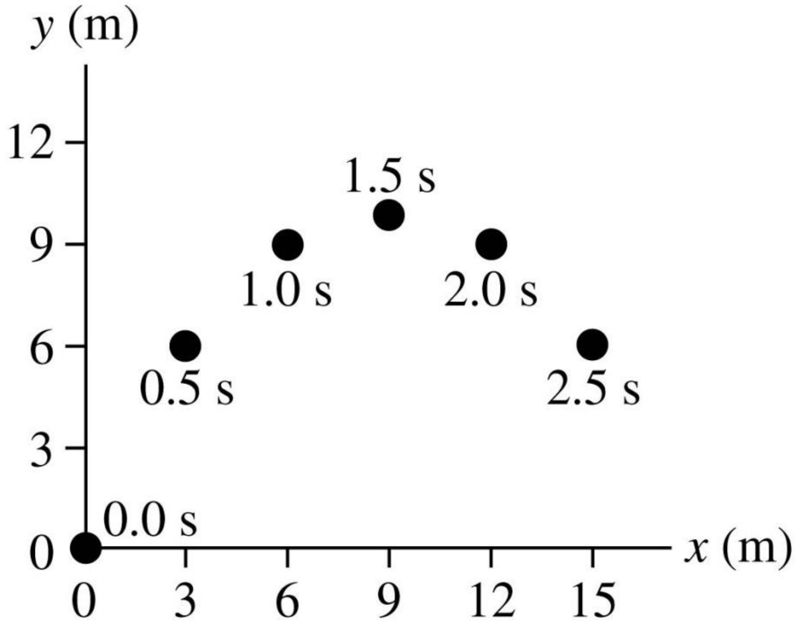


A Motion Diagram in Which the Object is Represented as a Particle



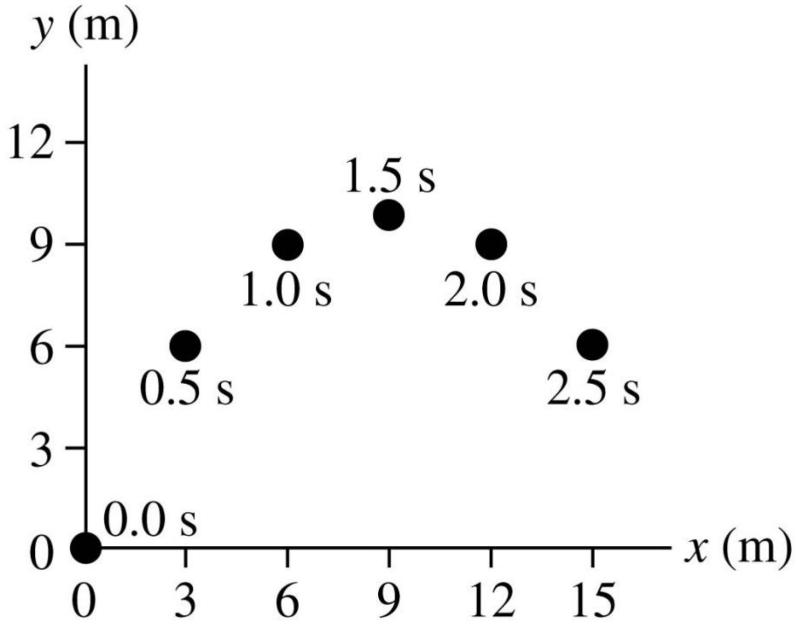
Go Long March, Go!

Moving from Motion Diagrams to Graphs



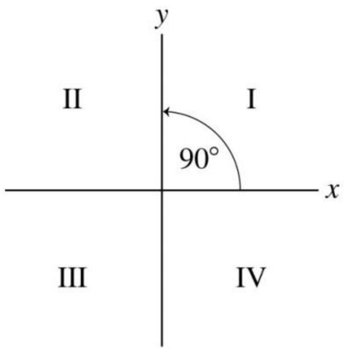
- In a motion diagram, it is useful to add numbers to specify where the object is and when the object was at that position, relative to an agreed origin.
- Shown is the motion diagram of a basketball, with 0.5 s intervals between frames.

Moving from Motion Diagrams to Graphs



- A **coordinate system** has been added to show (x, y).
- The frame at $t = 0$ is frame 0, when the ball is at the origin.
- The ball's position in frame 4 can be specified with coordinates $(x_4, y_4) = (12\text{ m}, 9\text{ m})$ at time $t_4 = 2.0\text{ s}$.

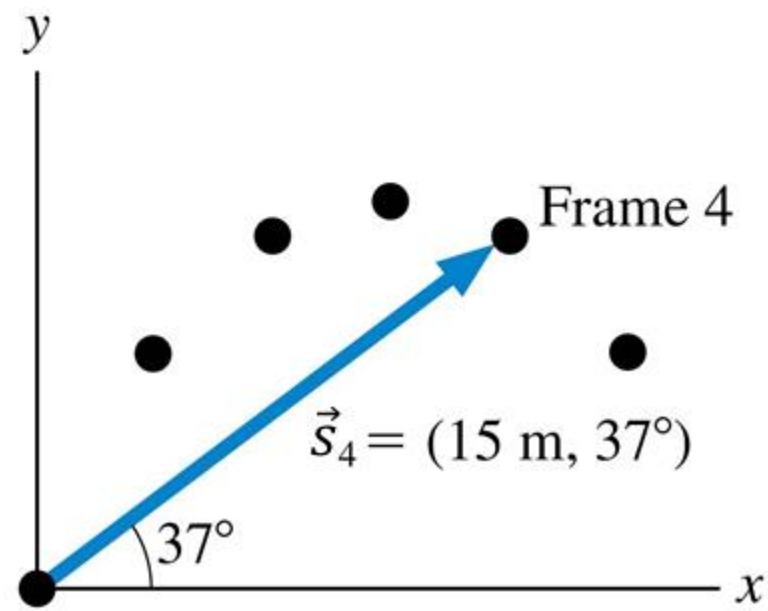
Moving from Motion Diagrams to Graphs



- A **coordinate system** is an artificially imposed grid that you place on a problem.
- You are free to choose:
  - where to place the origin, and
  - how to orient the axes.
- The above is a conventional xy-coordinate system and the four **quadrants** I through IV.
- Rather than use graphs to describe motion, we can also use vectors, which is an even more efficient technique in certain contexts.

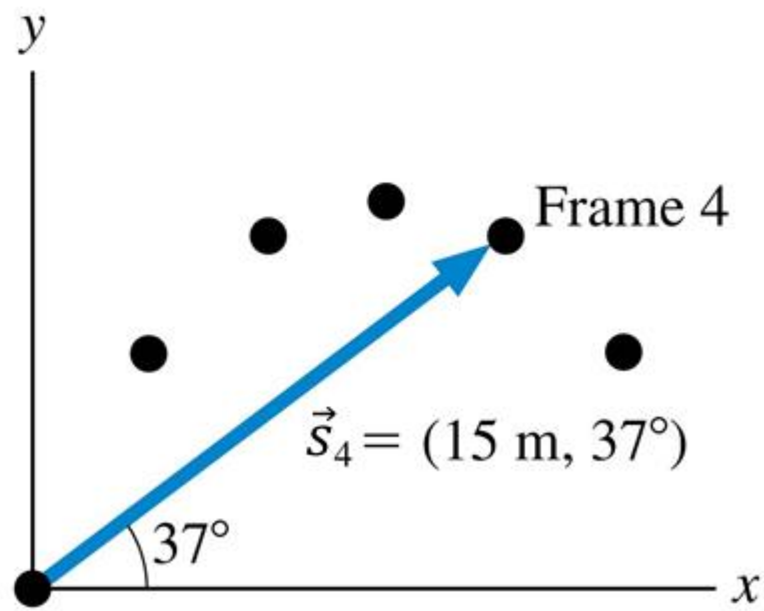
2. Vectors

Vectors



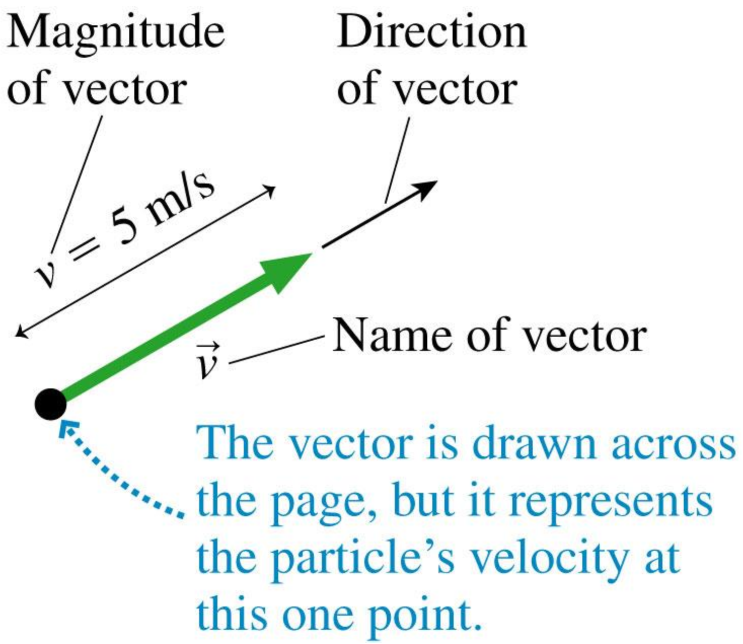
- Another way to locate an object such as a ball is to draw an arrow from the origin to the point representing the object.
- You can then specify the length and direction of the arrow.
- In this context, this arrow is called the **position vector $\vec{s}$** for the object.

Vectors



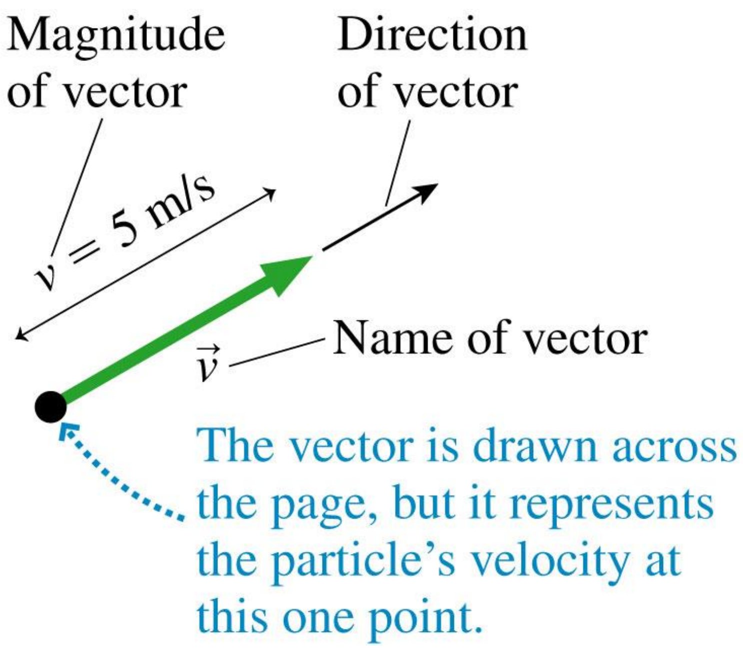
- The **position vector $\vec{s}$** is an alternative form of specifying position.
- It does not tell us anything different from what the coordinates (x, y) tell us.
- Also, as you will see in your textbook, the symbol for the position vector can also be $\vec{x}$.

Vectors



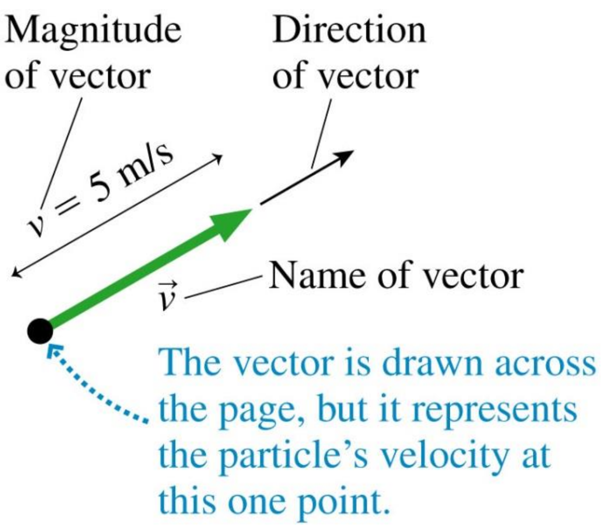
- A quantity that is fully described by a single number is called a **scalar quantity** (i.e., mass, temperature, volume).
- A quantity having both a magnitude and a direction is called a **vector quantity**.

Vectors



- The **geometric representation** of a vector is an arrow with the tail of the arrow placed at the point where the measurement is made.

Vectors

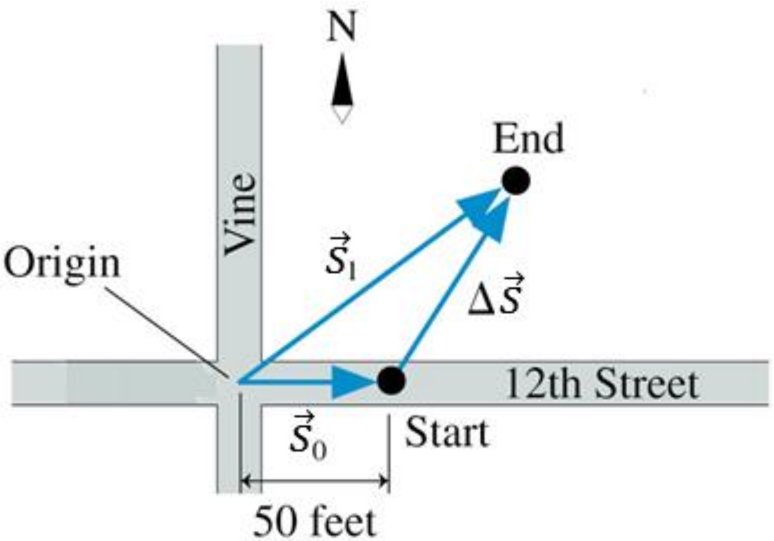


- We label vectors by drawing a small arrow over the letter that represents the vector, i.e., $\vec{s}$ (or $\vec{x}$) for position, $\vec{v}$ for velocity, and $\vec{a}$ for acceleration.
- When describing motion, we frequently use the terms 'displacement', 'velocity' and 'acceleration'. Let's look at the first of these terms.

3. Displacement

Displacement

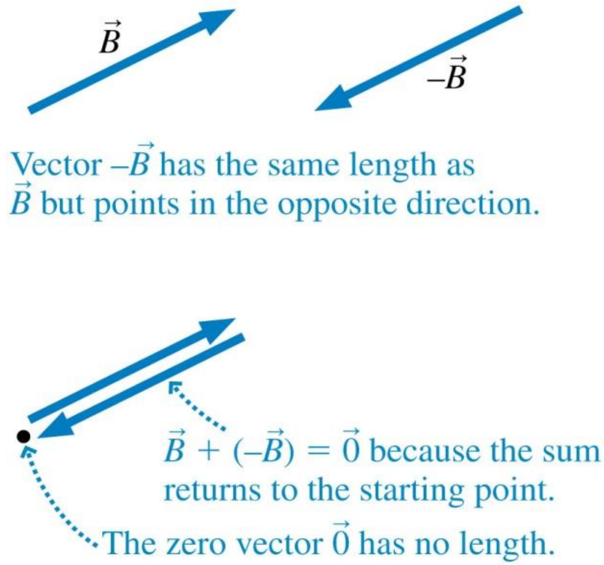
- Sam's initial position is the vector $\vec{s}_0$.
- Vector $\vec{s}_1$ is his position after he finishes walking.
- Sam has changed position, and a change in position is called a **displacement**.
- His displacement is the vector labeled $\Delta\vec{s}$.



$$s_1 = \vec{s}_0 + \Delta\vec{s}$$

Displacement

The negative of a vector.



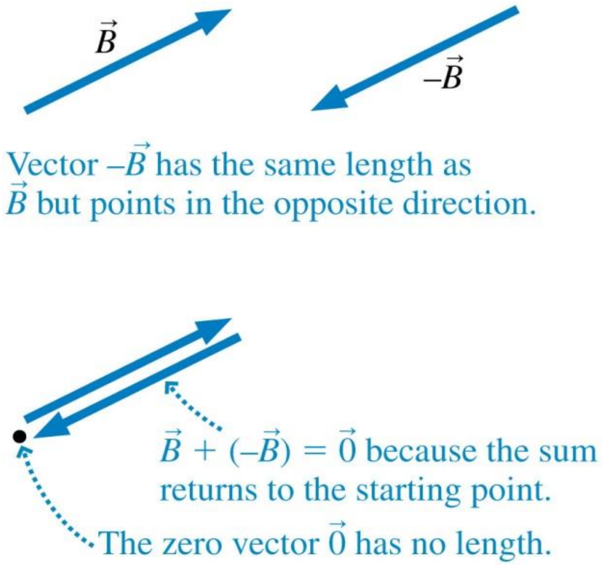
- The displacement $\Delta\vec{s}$ of an object as it moves from an initial position $\vec{s}_i$ to a final position $\vec{s}_f$ is

$$\Delta\vec{s} = s_f - \vec{s}_i$$

- The definition of $\Delta\vec{s}$ involves vector subtraction.

Displacement

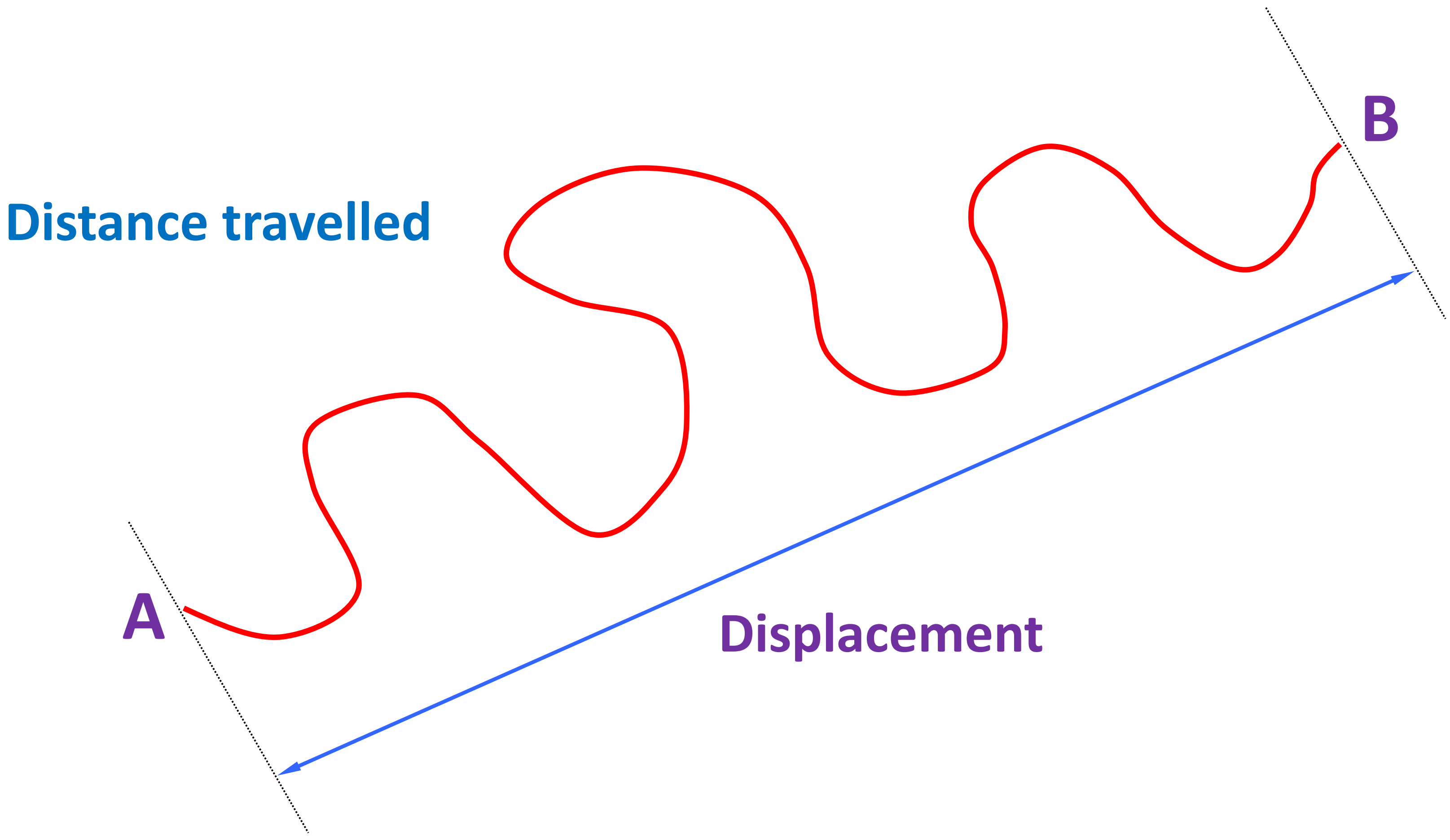
The negative of a vector.



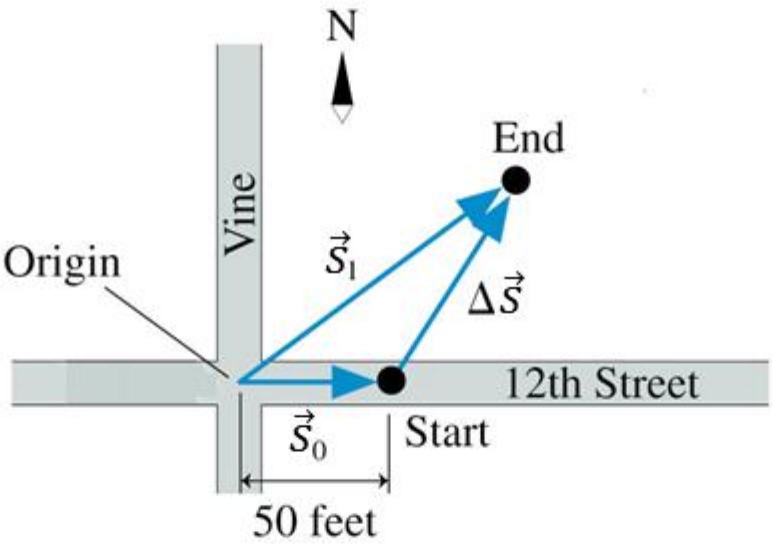
- With numbers, subtraction is the same as the addition of a negative number.
- Similarly, with vectors

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Distance versus Displacement



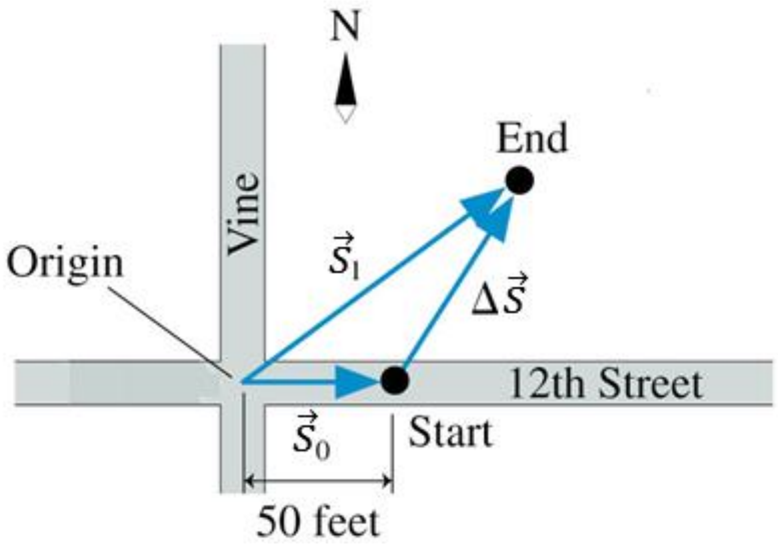
Moving from Displacement to Average Velocity



- An object may move from an initial position $\vec{s}_0$ at time t_i to a final position $\vec{s}_1$ at time t_f.
- This change in displacement for a given change in time is called **average velocity**, which is different from **average speed**.

4. Average Speed & Average Velocity

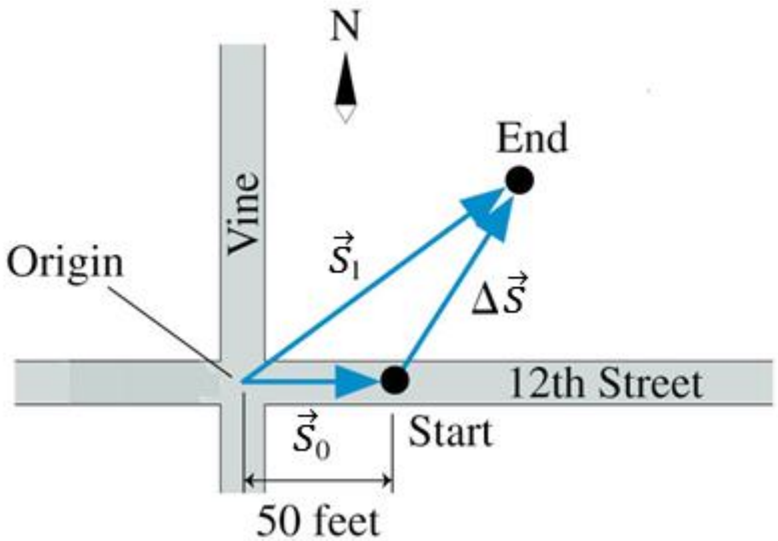
Average Speed versus Average Velocity

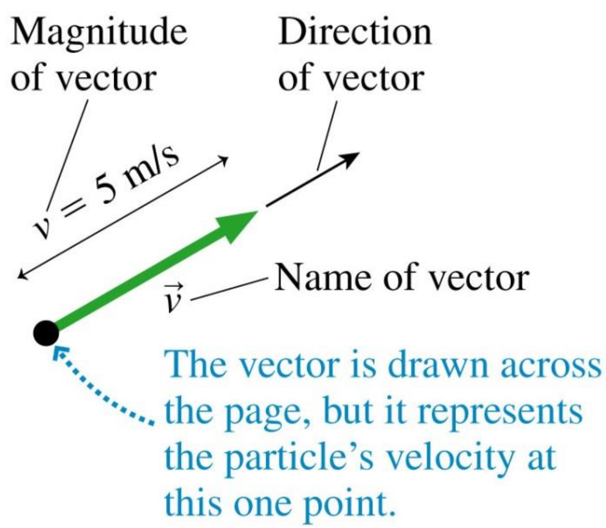


- **Average speed** does not include information about the direction of motion. It is simply:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time spent traveling}} = \frac{s}{\Delta t}$$

Average Speed versus Average Velocity

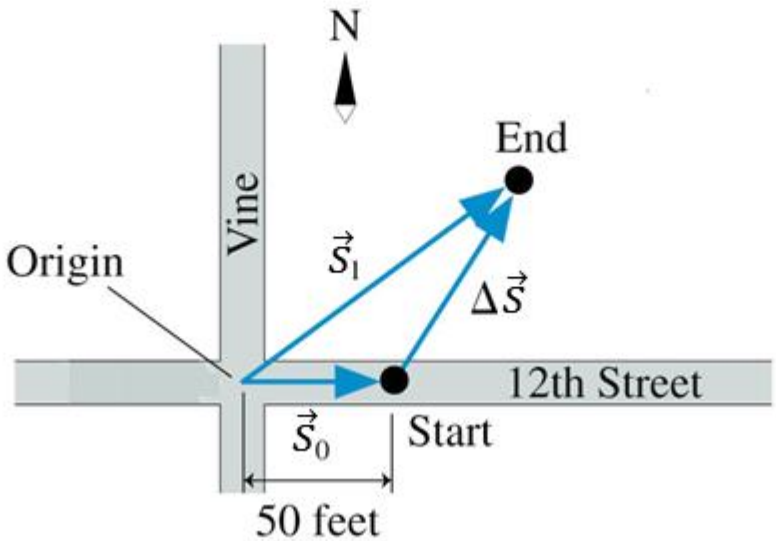


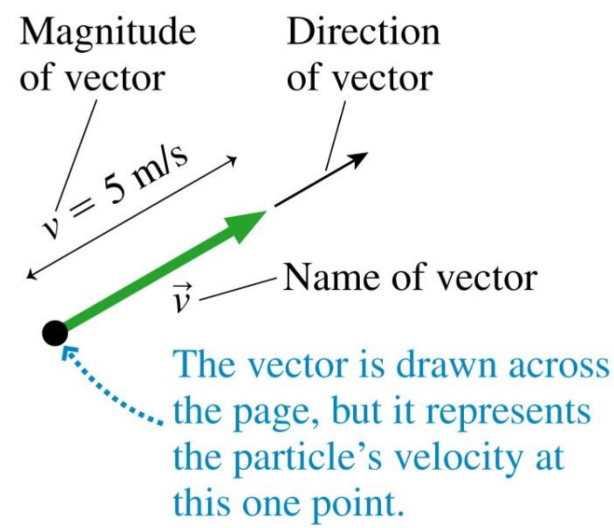


- **Average velocity**, on the other hand, does include information about the direction of motion. It is a vector which describes the average displacement of an object within a given time.

$$\vec{v}_{avg} = \frac{\Delta\vec{s}}{\Delta t}$$

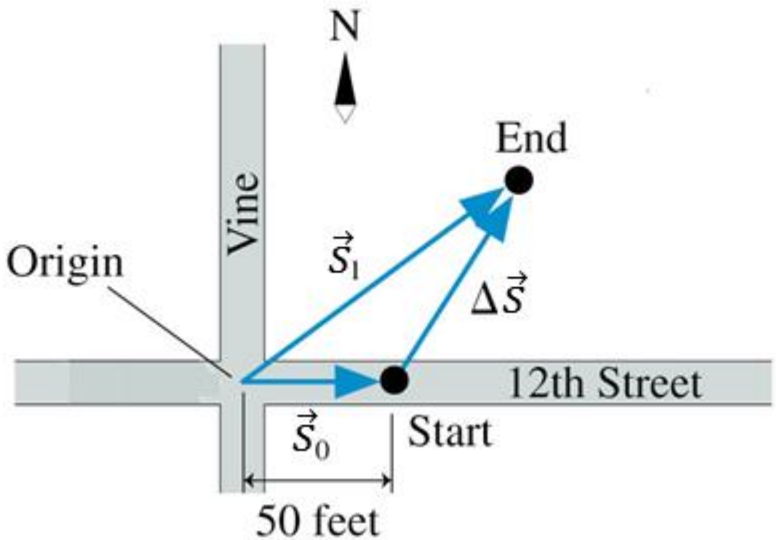
Average Velocity versus Instantaneous Velocity

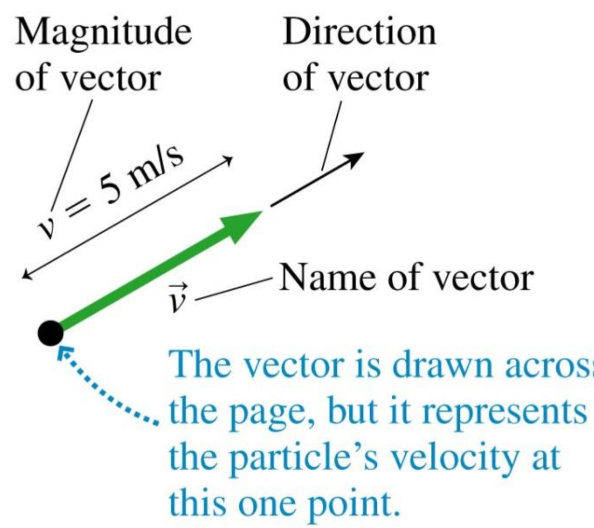




- When we speak of velocity vectors, we can speak of an average velocity vector (sometimes symbolised by $\vec{v}_{avg}$ or $\vec{v}$) or an instantaneous velocity vector (symbolised by $\vec{v}$).
- An average velocity vector is equal to an instantaneous velocity vector when an object's change in displacement for a unit of time does not vary.

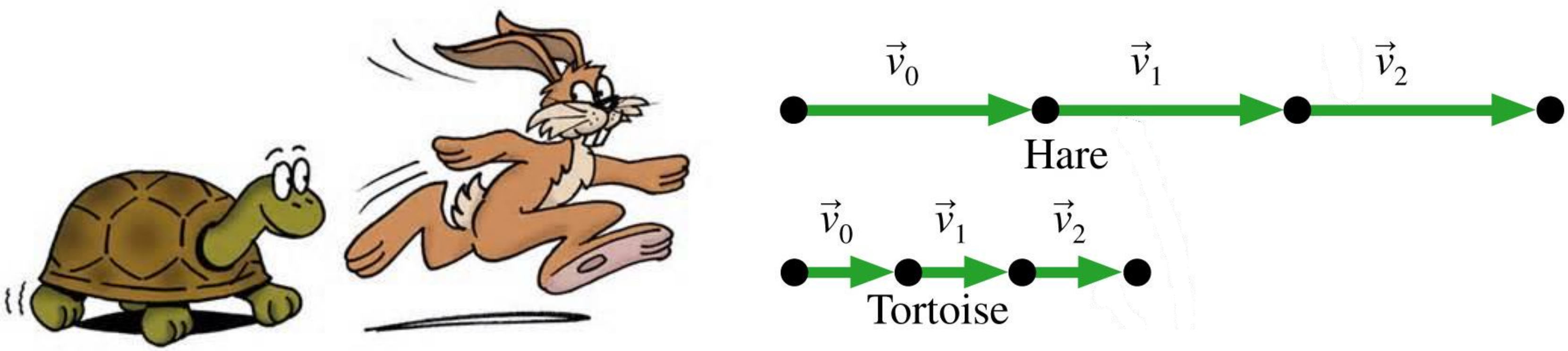
Average Velocity versus Instantaneous Velocity





- It usually clear from the context of the problem whether we are talking about average velocity or instantaneous velocity.
- The average velocity vector is in the same direction as the displacement $\Delta\vec{s}$ for a given unit of time (Δt).
- The instantaneous velocity vector is in the same direction as an infinitesimal displacement $d\vec{s}$ for an infinitesimal unit of time (dt).

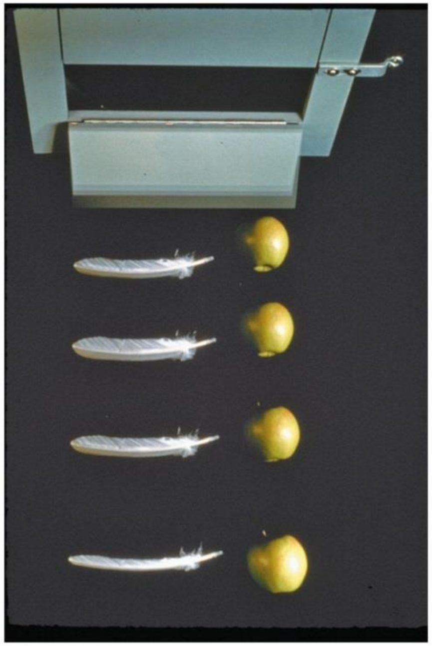
Combining Motion Diagrams with Velocity Vectors



- Shown above is a motion diagram for a tortoise racing a hare.
- The arrows are average velocity vectors.
- The length of each arrow also represents the average speed.
- Often, we are interested in how the velocity of an object changes with time; we call this **acceleration**.

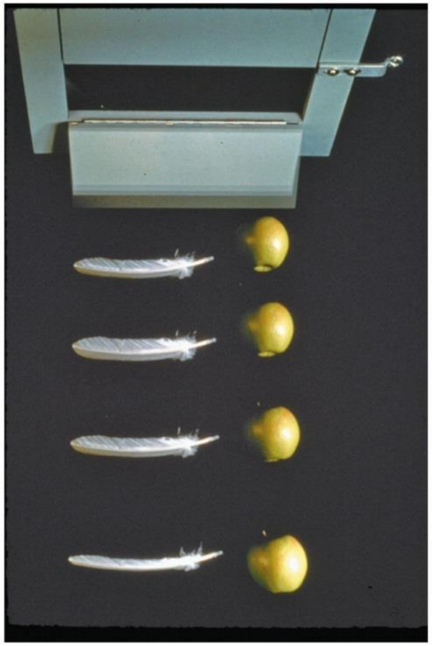
5. Acceleration

Acceleration



- Sometimes an object's velocity is constant as it moves.
- More often, an object's velocity changes as it moves.
- Acceleration describes this **change in velocity for a given change in time**.
- Like velocity, we can talk of an average acceleration and an instantaneous acceleration.

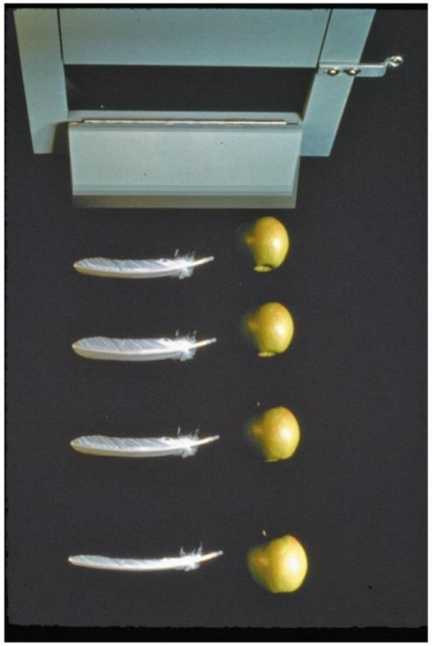
Acceleration



- Consider an object whose velocity changes from $\vec{v}_1$ to $\vec{v}_2$ during the time interval Δt.
- The quantity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ is the change in velocity.
- The **rate of change of velocity** is called the **average acceleration**:

$$a_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Free-fall Acceleration



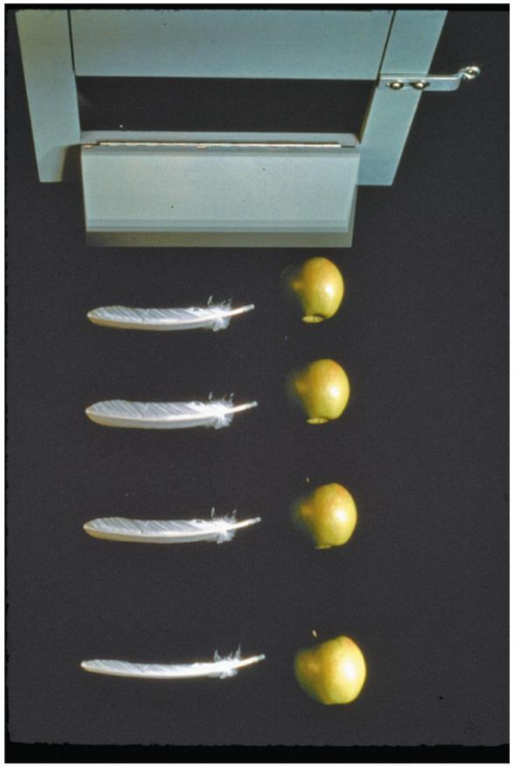
- In the above situation, in the absence of air resistance, the above two objects fall at the same rate, and hit the ground at the same time.
- The apple and feather are seen here falling in a vacuum.

Free-fall Acceleration

- The motion of an object moving under the influence of gravity only, and no other forces, is called **free-fall**.

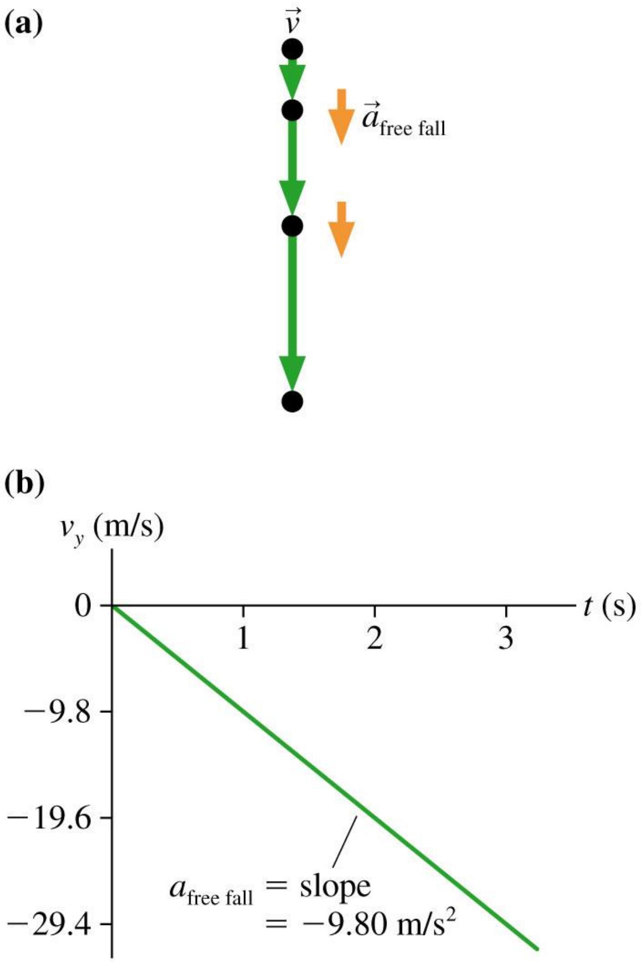
- Two objects dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed.

- Consequently, any two objects in free-fall, regardless of their mass, have the same acceleration:



$\vec{a}_{free\ fall} = 9.80\ m/s^2$, vertically downward

Free-fall Acceleration

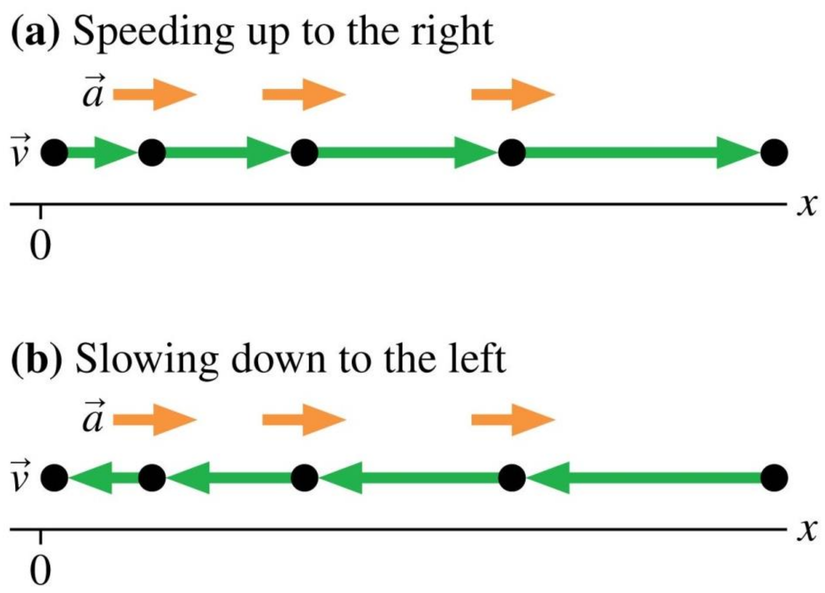


- The velocity graph is a straight line with a slope:

$$a_y = a_{free\ fall} = -g$$

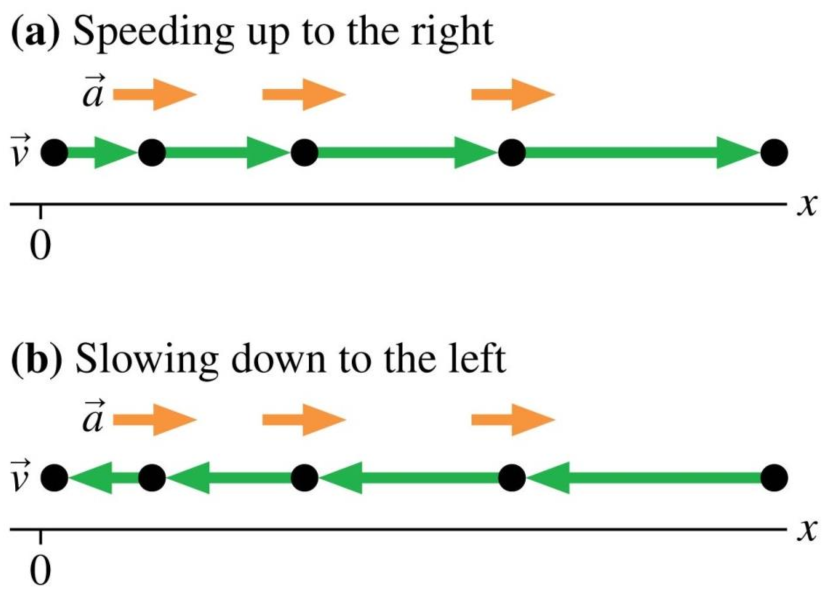
- Where g is a positive number which is equal to 9.80 m/s^2 on the surface of the earth.
- Other planets have different values of g.

Acceleration



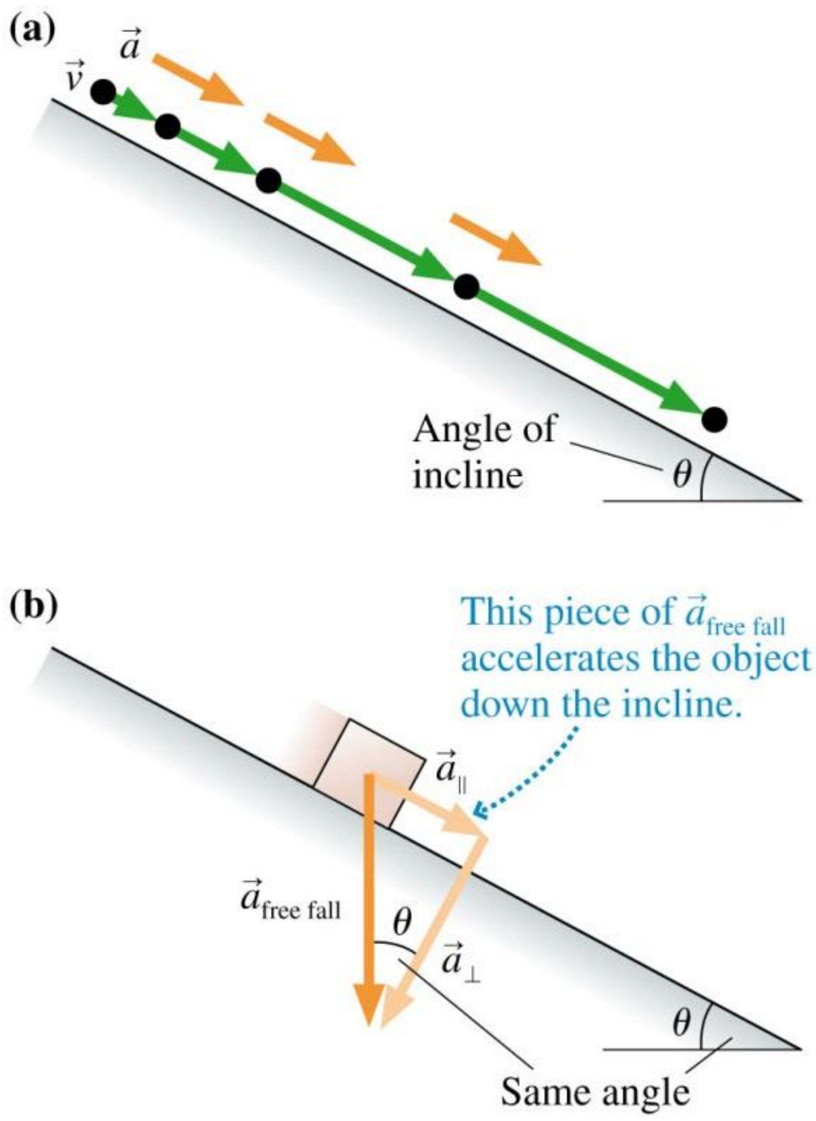
- When an object is speeding up, the acceleration and velocity vectors point in the **same direction**.
- When an object is slowing down, the acceleration and velocity vectors point in **opposite directions**.
- An object's velocity is constant if and only if its acceleration is zero.

Acceleration



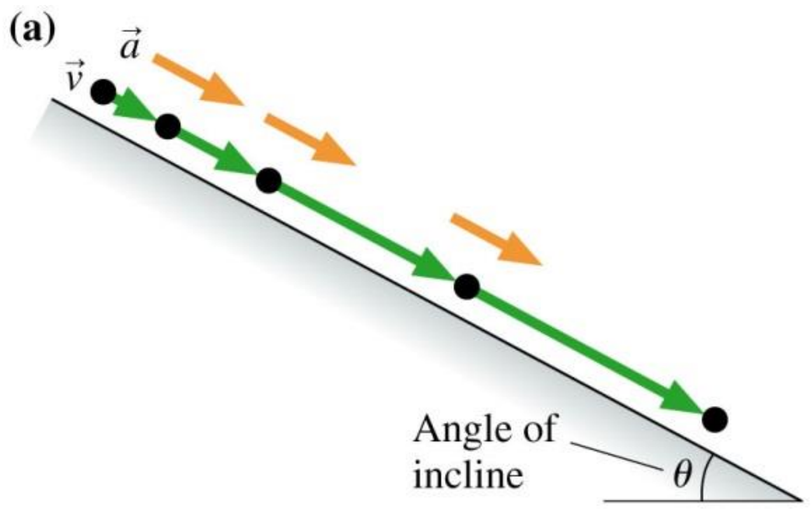
N.B. Notice how in the above diagrams, one object is speeding up and the other is slowing down, but that both objects have acceleration vectors toward the right.

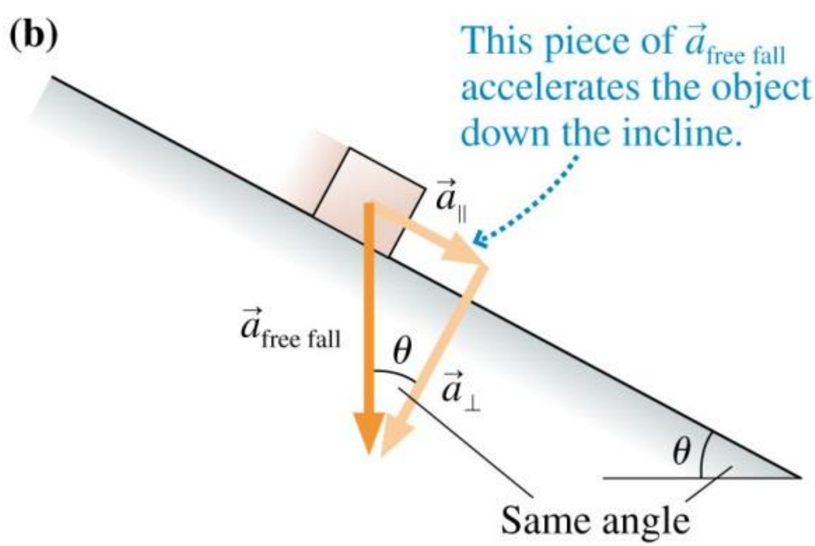
Acceleration on an Inclined Plane



- Figure (a) shows the motion diagram of an object sliding down a straight, frictionless inclined plane.
- Figure (b) shows the free-fall acceleration the object would have if the incline suddenly vanished.

Acceleration on an Inclined Plane



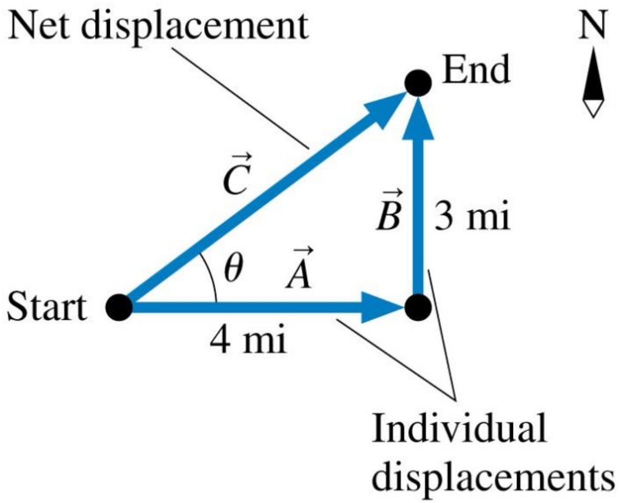


- You can see in Figure (b) that we can 'resolve' the free-fall acceleration vector into two components: $\vec{a}_{\parallel}$ and $\vec{a}_{\perp}$

- **Resolving vectors** into **vector components** is an idea that comes from the addition of vectors to produce what we call **resultant vectors**. Let's look at this in more detail.

6. Vector Components

Vector Addition



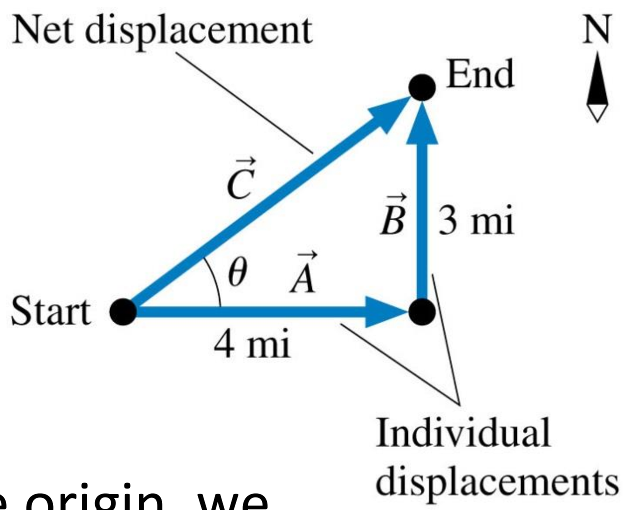
- A hiker's displacement is 4 miles to the east, then 3 miles to the north, as shown.
- Vector $\vec{C}$ is the net displacement:

$$\vec{C} = \vec{A} + \vec{B}$$

- Because $\vec{A}$ and $\vec{B}$ are at right angles, the magnitude of $\vec{C}$ is given by the Pythagorean theorem:

$$C = \sqrt{A^2 + B^2} = \sqrt{(4\text{ mi})^2 + (3\text{ mi})^2} = 5\text{ mi}$$

Vector Addition



- A hiker's displacement is 4 miles to the east, then 3 miles to the north, as shown.

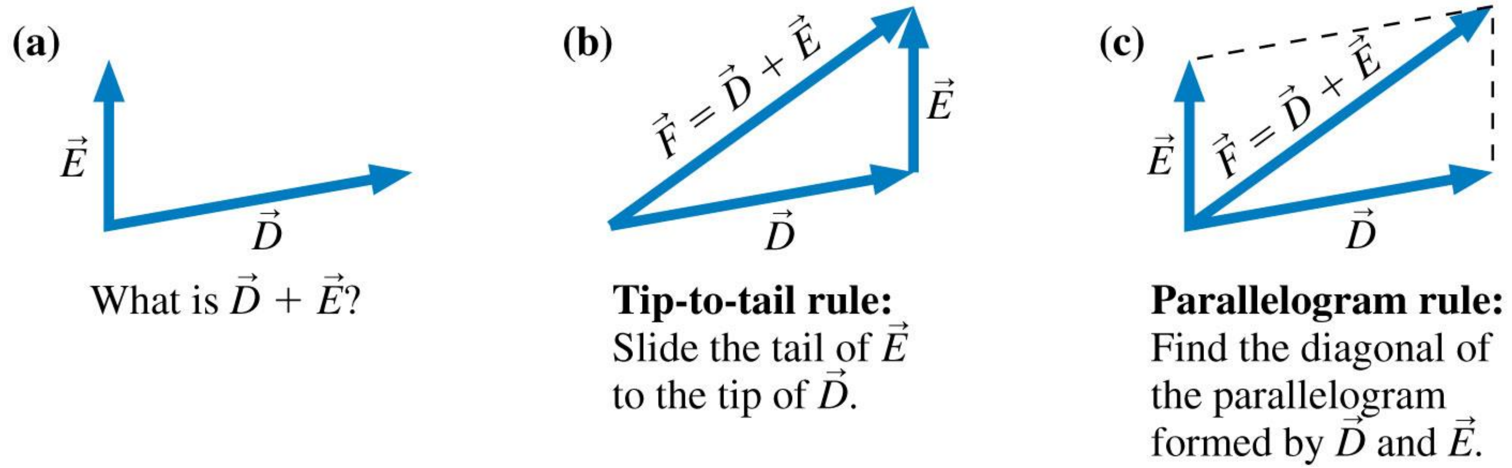
- To describe the direction of $\vec{C}$ relative to the origin, we must find the angle, using trigonometry:

$$\theta = tan^{-1}\left(\frac{B}{A}\right) = tan^{-1}\left(\frac{3\text{ mi}}{4\text{ mi}}\right) = 37°$$

- Altogether, the hiker's net displacement is:

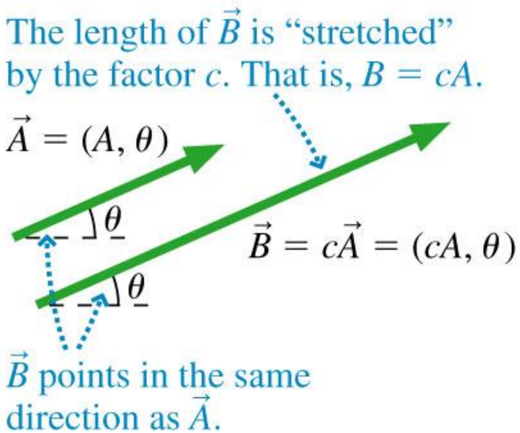
$$\vec{C} = \vec{A} + \vec{B} = (5\text{ mi}, 37°\text{ north of east})$$

Parallelogram Rule for Vector Addition

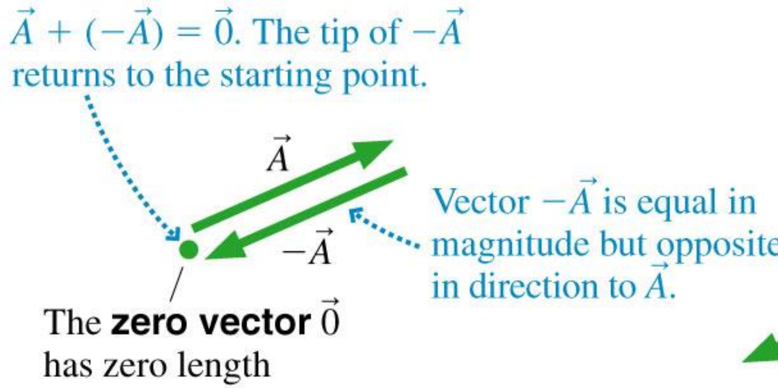


- It is often convenient to draw two vectors with their tails together, as shown in (a) above.
- To evaluate $\vec{F} = \vec{D} + \vec{E}$, you could move $\vec{E}$ over and use the tip-to-tail rule, as shown in (b) above.
- Alternatively, $\vec{F} = \vec{D} + \vec{E}$ can be found as the diagonal of the parallelogram defined by $\vec{D}$ and $\vec{E}$, as shown in (c) above.

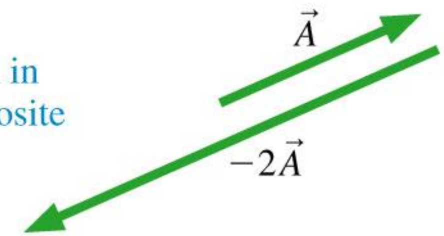
For You to Read: More Vector Mathematics



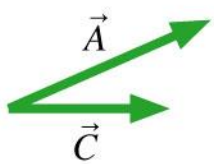
Multiplication by a scalar



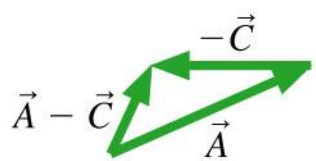
The negative of a vector



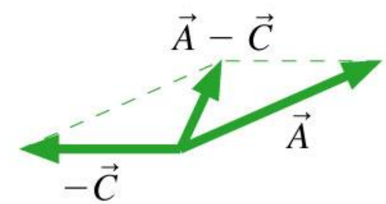
Multiplication by a negative scalar



Vector subtraction: What is $\vec{A} - \vec{C}$?
Write it as $\vec{A} + (-\vec{C})$ and add!

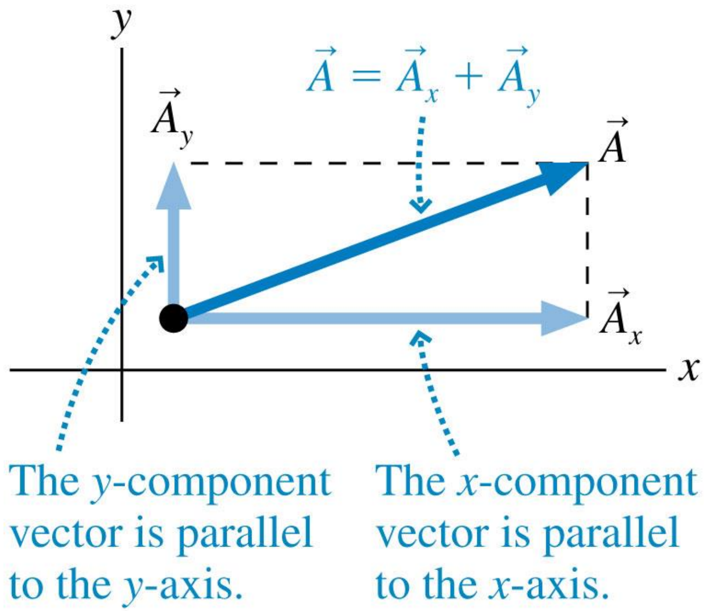


Tip-to-tail method using $-\vec{C}$



Parallelogram method using $-\vec{C}$

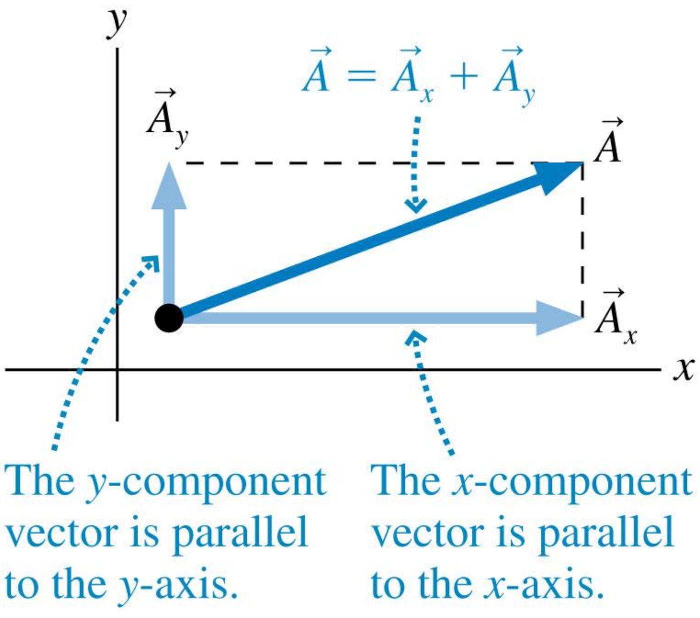
Vector Components



- The figure shows a vector A and an xy-coordinate system that we've chosen.
- We can define two new vectors parallel to the axes that we call the **component vectors** of A, such that:

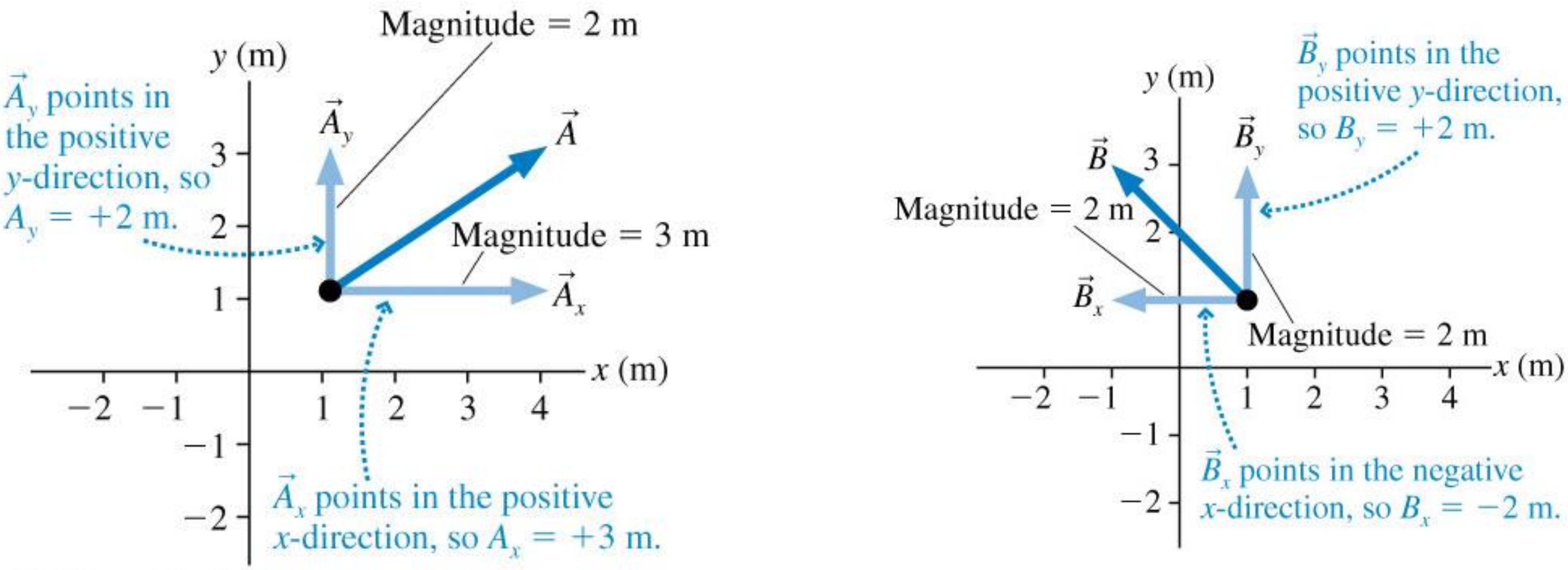
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Vector Components



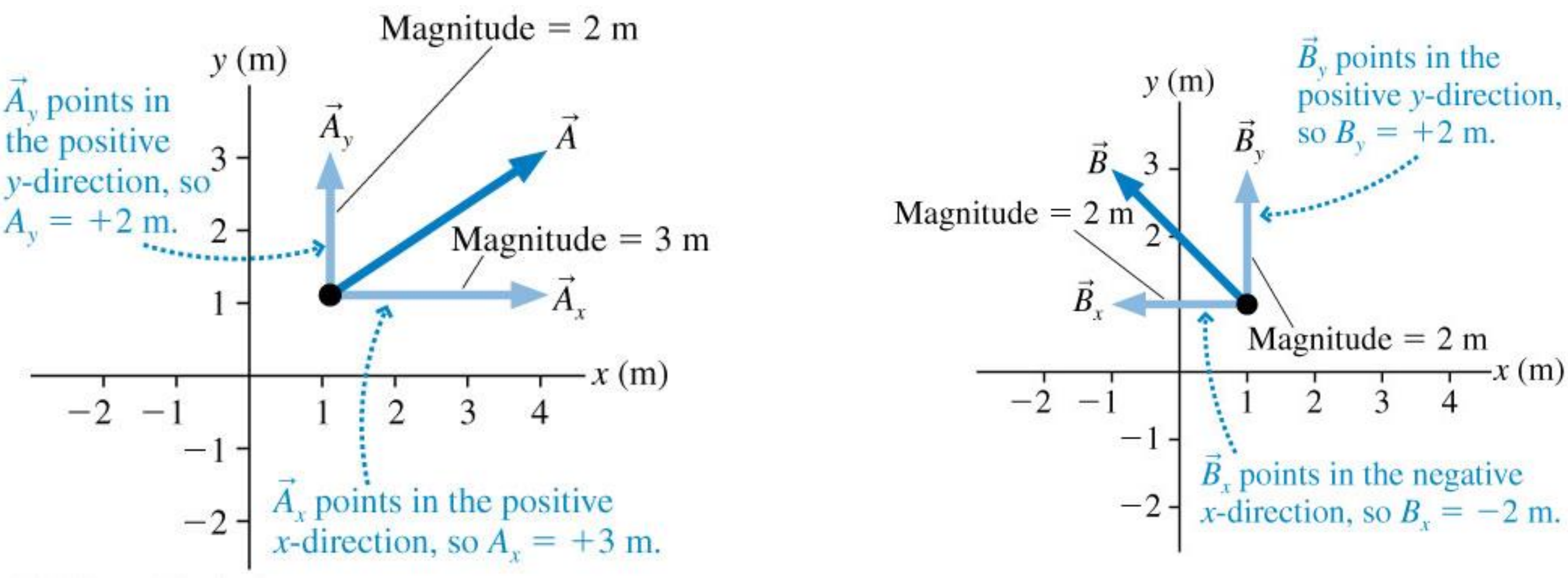
- We have broken $\vec{A}$ into two perpendicular vectors that are parallel to the coordinate axes.
- This is called the **decomposition (or the resolving)** of $\vec{A}$ into its component vectors.

Vector Components



- Suppose a vector $\vec{A}$ has been decomposed into component vectors $\vec{A}_x$ and $\vec{A}_y$ parallel to the coordinate axes.
- We can describe each component vector with a single number called the component.

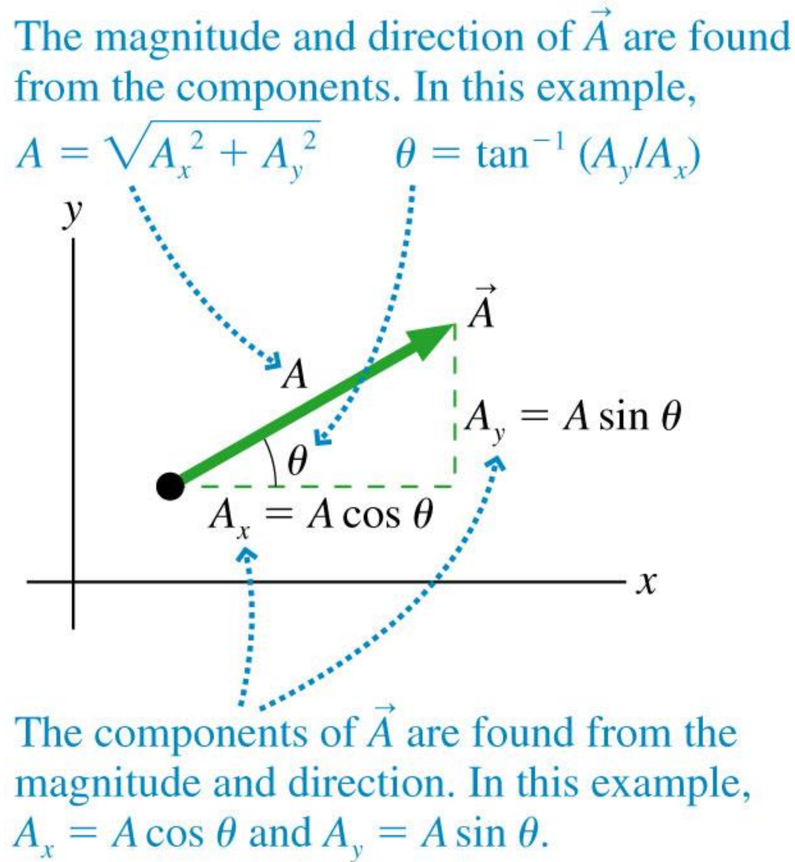
Vector Components



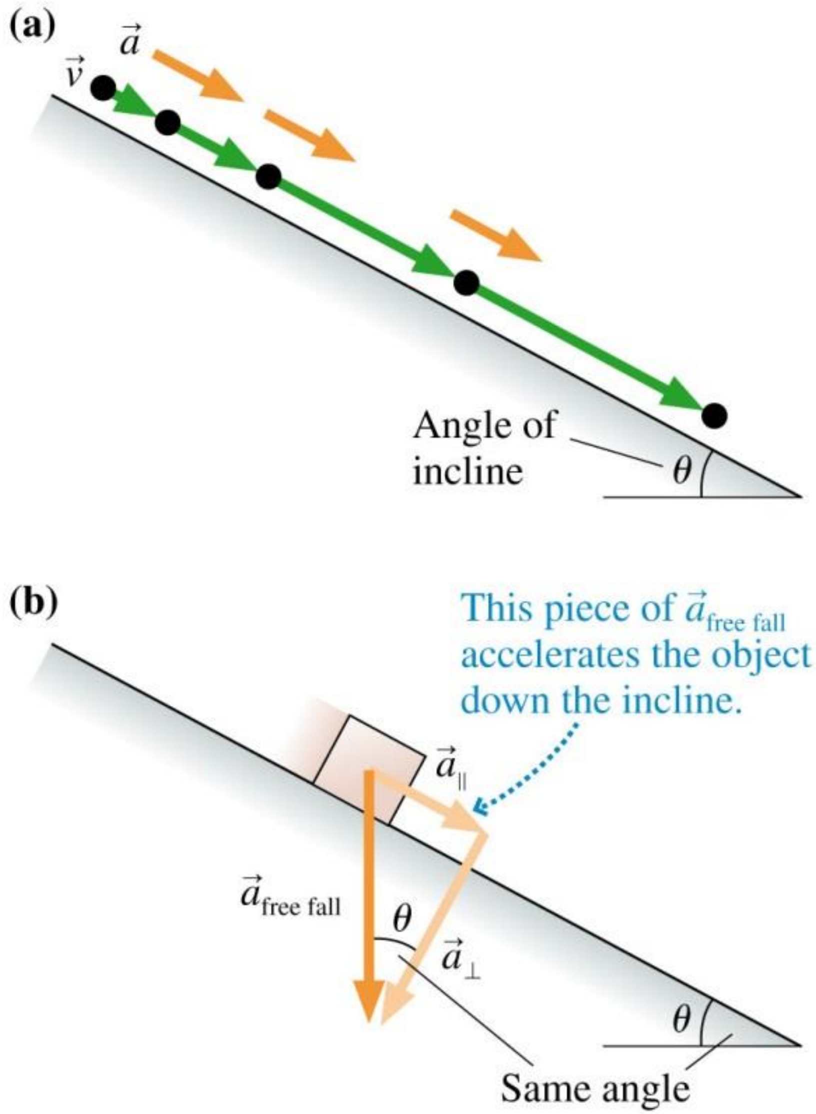
- The component tells us how big the component vector is, and, with its sign, which ends of the axis the component vector points toward.
- Shown above are two examples of determining the components of a vector.

Moving Between the Geometric Representation and the Component Representation

- We will frequently need to decompose a vector into its components.
- We will also need to ‘reassemble’ a vector from its components.
- The figure to the right shows how to move back-and-forth between the geometric and component representations of a vector.



Acceleration on an Inclined Plane



- The reaction force between the block and the incline in Figure (b) 'cancels' the acceleration component that is perpendicular to the block.

Q. See if you can now use our discussion of vector components to work-out the acceleration of the block down the incline.

Summary of today's Lecture

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1. Motion diagrams
2. Vectors
3. Displacement
4. Average speed and average velocity
5. Acceleration
6. Vector components

Lecture 1: Optional Reading

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- **Ch. 2.5**, Motion at constant acceleration; p.38-39.
- **Ch. 2.6**, Solving problems; p.40-43.
- **Ch. 2.8**, Variable acceleration; p.49-50.
- **Ch. 2.9**, Graphical analysis and numerical integration; p.50-52
- **Ch. 3.1-3.4**, Vectors (various topics); p.66-72.

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Moodle** to complete your assignments.