University of Nottingham
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Foundation Algebra (CELEN036)

Problem Sheet 8 **Topics: Matrices**

Topic 1: Algebra of matrices

1. Find the values of x, y, z, and w if the equation holds: $\begin{pmatrix} x+1 & 2y+3 \\ 3z+4 & 4w+1 \end{pmatrix} = \begin{pmatrix} 1 & 9 \\ 10 & 5 \end{pmatrix}$.

2. Given matrices $A = \begin{pmatrix} 1 & a & -3 \\ x+z & 0 & y \\ 9 & b & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a+b & 4 & -3 \\ 7 & x & 2 \\ b+c & -3 & w \end{pmatrix}$, if $A = B$, find the values of a, b, c, x, y, z, and w.

3. Given matrices $A = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 8 & 9 & 10 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 2 \\ 5 & 6 & 7 \\ 6 & 7 & 8 \end{pmatrix}$, find

   (i) $A + B$ (ii) $B - A$ (iii) $2A - 3B$.

4. Given matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, verify that

   (i) $3(A + B) = 3A + 3B$ (ii) $(A - B)^T = A^T - B^T$.

5. Find the value of the constant k, if any, for which the matrix $C = A + B$ is symmetric, where $A = \begin{pmatrix} 1 & 5 & 7 \\ k^2 & 4 & 0 \\ 0 & 2 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 7 & 1 \\ 3 & 1 & -k \\ 8 & k+4 & -1 \end{pmatrix}$.

6. Given matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$, find AB and BA.

7. Given matrices $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, show that $AB = O$.

8. Given matrices $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, show that $(A + B)^2 = \begin{pmatrix} 121 & 99 \\ 88 & 88 \end{pmatrix}$.

9. Given matrices $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, verify that $(AB)^T = B^T A^T$.

10. Given matrices $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$, show that $AB = I$, where I is the identity matrix of the same order as A and B.

11. Find the following matrices (if they exist):

    (i) $3A$ (ii) AB (iii) BA (iv) $(A+B)(A-B)$ (v) $A^2 - B^2$,

    where

    (a) $A = \begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -5 \\ 0 & 6 \end{pmatrix}$; (b) $A = \begin{pmatrix} 3 & 5 \\ 1 & -4 \\ 2 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \end{pmatrix}$.

12. Given matrices $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$, and $C = \begin{pmatrix} 0 & -1 \\ 5 & 2 \end{pmatrix}$,

    (i) find $(AB)C$ and $A(BC)$;

    (ii) verify that $A(B+C) = AB + AC$;

    (iii) find the matrix F such that $B^3 - 2C^2 + I + 3F = O$.

Topic 2: Inverse matrices

13. Evaluate the following determinants:

    (i) $\begin{vmatrix} 2 & 4 \\ 6 & 7 \end{vmatrix}$ (ii) $\begin{vmatrix} -2 & 1 \\ 5 & -3 \end{vmatrix}$ (iii) $\begin{vmatrix} -2 & -3 \\ -4 & -5 \end{vmatrix}$ (iv) $\begin{vmatrix} 6 & 45 \\ -2 & 15 \end{vmatrix}$

14. Find the inverse of the following matrices, if it exists:

    (i) $\begin{pmatrix} 2 & 4 \\ 6 & 7 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} -1 & 3 \\ 2 & 2 \end{pmatrix}$ (iv) $\begin{pmatrix} 8 & 4 \\ 2 & 1 \end{pmatrix}$

    (v) $\begin{pmatrix} 6 & 45 \\ -2 & 15 \end{pmatrix}$ (vi) $\begin{pmatrix} 9 & 2 \\ 4 & 1 \end{pmatrix}$ (vii) $\begin{pmatrix} -2 & -3 \\ -4 & -5 \end{pmatrix}$ (viii) $\begin{pmatrix} a & b \\ a^2 & ab \end{pmatrix}$

15. Show that the inverse matrix of $A = \begin{pmatrix} -1 & 1 \\ -2 & 0 \end{pmatrix}$ is $B = \begin{pmatrix} 0 & -0.5 \\ 1 & -0.5 \end{pmatrix}$.

Topic 3: Solving 2×2 systems of equations using matrix method

16. Solve the following systems of linear equations using matrix method:

(i) $\begin{cases} x + 2y = 13 \\ 2x - 5y = 8 \end{cases}$ (ii) $\begin{cases} 3x + 2y = -3 \\ 5x + 3y = -4 \end{cases}$ (iii) $\begin{cases} x + y = 17 \\ 2x - y = 10 \end{cases}$

(iv) $\begin{cases} 2x - y = 10 \\ x + 3y = -2 \end{cases}$ (v) $\begin{cases} x + 2y = 15 \\ 3x - y = 10 \end{cases}$ (vi) $\begin{cases} 3x - y = 3 \\ x + y = 5 \end{cases}$

(vii) $\begin{cases} x + 6y = 10 \\ 2x + 5y = 6 \end{cases}$ (viii) $\begin{cases} 2x + 3y = 10 \\ x - 2y = -9 \end{cases}$ (ix) $\begin{cases} 7x + 8y = 56 \\ x - y = 8 \end{cases}$

(x) $\begin{cases} 2x - 3y = 1 \\ x + 2y = -3 \end{cases}$ (xi) $\begin{cases} 2x - y = 17 \\ x + 3y = 12 \end{cases}$ (xiii) $\begin{cases} 4x - 2y = 2 \\ 3x - y = 4 \end{cases}$

17. Express the following system of equations into the matrix form

$$\begin{cases} 3x + 2y = 4 \\ 6x + my = 8 \end{cases}$$

where m is a constant.

(i) Determine the value of m for which the system does not have a unique solution.

(ii) Use the matrix method to find the solution to the system when $m = 1$.

Answers

1. $x = 0,\ y = 3,\ z = 2,\ w = 1$

2. $a = 4,\ b = -3,\ c = 12,\ x = 0,\ y = 2,\ z = 7,\ w = 4$

3. (i) $\begin{pmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{pmatrix}$ (ii) $\begin{pmatrix} -2 & -2 & -2 \\ 2 & 2 & 2 \\ -2 & -2 & -2 \end{pmatrix}$ (iii) $\begin{pmatrix} 4 & 3 & 2 \\ -9 & -10 & -11 \\ -2 & -3 & -4 \end{pmatrix}$

5. $k = -3$

6. $AB = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}, \qquad BA = \begin{pmatrix} aw + cx & bw + dx \\ ay + cz & by + dz \end{pmatrix}$

11. (a):

(i) $\begin{pmatrix} -3 & 3 \\ 3 & 6 \end{pmatrix}$ (ii) $\begin{pmatrix} -3 & 11 \\ 3 & 7 \end{pmatrix}$ (iii) $\begin{pmatrix} -8 & -7 \\ 6 & 12 \end{pmatrix}$ (iv) $\begin{pmatrix} -12 & 28 \\ 4 & -26 \end{pmatrix}$ (v) $\begin{pmatrix} -7 & 46 \\ 1 & -31 \end{pmatrix}$

(b):

(i) $\begin{pmatrix} 9 & 15 \\ 3 & -12 \\ 6 & -6 \end{pmatrix}$ (ii) $\begin{pmatrix} 13 & 0 & 8 \\ -7 & 0 & -3 \\ -2 & 0 & 0 \end{pmatrix}$ (iii) $\begin{pmatrix} 5 & 3 \\ 8 & 8 \end{pmatrix}$ (iv) Not defined (v) Not defined

12. (i) $(AB)C = A(BC) = \begin{pmatrix} 0 & -1 \\ 15 & 7 \\ 30 & 11 \end{pmatrix}$ (iii) $F = \begin{pmatrix} -4 & -\frac{4}{3} \\ 11 & -10 \end{pmatrix}$

13. (i) -10 (ii) 1 (iii) -2 (iv) 180

14. (i) $\begin{pmatrix} -\frac{7}{10} & \frac{2}{5} \\ \frac{3}{5} & -\frac{1}{5} \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} -\frac{1}{4} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{8} \end{pmatrix}$ (iv) A^{-1} does not exist.

(v) $\begin{pmatrix} \frac{1}{12} & -\frac{1}{4} \\ \frac{1}{90} & \frac{1}{30} \end{pmatrix}$ (vi) $\begin{pmatrix} 1 & -2 \\ -4 & 9 \end{pmatrix}$ (vii) $\begin{pmatrix} \frac{5}{2} & -\frac{3}{2} \\ -2 & 1 \end{pmatrix}$ (viii) A^{-1} does not exist.

16. (i) $x = 9,\ y = 2$ (ii) $x = 1,\ y = -3$ (iii) $x = 9,\ y = 8$ (iv) $x = 4,\ y = -2$
(v) $x = 5,\ y = 5$ (vi) $x = 2,\ y = 3$ (vii) $x = -2,\ y = 2$ (viii) $x = -1,\ y = 4$
(ix) $x = 8,\ y = 0$ (x) $x = -1,\ y = -1$ (xi) $x = 9,\ y = 1$ (xii) $x = 3,\ y = 5$

17. (i) $m = 4$ (ii) $x = \frac{4}{3}, \quad y = 0$