CSC 307: DISCRETE STRUCTURES

MR FELIX

- A **set** is a collection of distinct objects or elements. These elements can be anything, such as numbers, letters, or even other sets.
- Sets are denoted using capital letters, often from the beginning of the alphabet (e.g., A, B, C).
- The elements of a set are enclosed within curly braces. For example, if we have a set of natural numbers less than 10, it can be represented as {1, 2, 3, 4, 5, 6, 7, 8, 9}.
- Sets are defined by their elements, and the order in which elements are listed does not matter. For example, {1, 2, 3} is the same set as {3, 2, 1}.

- **Elements, Membership, and Notation**
- An element is an individual object within a set. For example, in the set of natural numbers less than 10, 3 is an element.
- We use the symbol "∈" to denote that an element belongs to a particular set. For example, "$3 \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$".
- To indicate that an element is not part of a set, we use the symbol "∉". For example, "$10 \notin \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$".

Set Equality

Two sets are equal if and only if they have exactly the same elements.

For example, if set $A = \{1, 2, 3\}$ and set $B = \{3, 2, 1\}$, then $A = B$ because they have the same elements.

Subsets and Supersets

A set A is considered a **subset** of another set B if every element in A is also an element in B. This is denoted as $A \subseteq B$.

Conversely, if A is a subset of B, then B is a **superset** of A, denoted as $B \supseteq A$.

A set is always considered a subset of itself, i.e., $A \subseteq A$.

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Union and Intersection

- The **union** of two sets, denoted by $A \cup B$, contains all the elements that are in either set A or set B or in both.

For example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$.

- The **intersection** of two sets, denoted by $A \cap B$, contains all the elements that are common to both set A and set B.
- Using the same sets A and B, $A \cap B = \{3\}$.

Set Difference and Complement

The **set difference** between two sets, A - B (or A \ B), contains all the elements that are in set A but not in set B.

For example, if A = {1, 2, 3} and B = {3, 4, 5}, then A - B = {1, 2}.

The **complement** of a set A with respect to a universal set U is the set of all elements in U that are not in A. It is denoted as A'.

Disjoint Sets

Two sets A and B are **disjoint** if they have no elements in common, i.e., $A \cap B = \emptyset$.

For example, if $A = \{1, 2\}$ and $B = \{3, 4\}$, then A and B are disjoint sets.

De Morgan's Laws

De Morgan's Laws are important principles in set theory:

First De Morgan's Law: The complement of the union of two sets is equal to the intersection of their complements.

$$(A \cup B)' = A' \cap B'$$

Second De Morgan's Law: The complement of the intersection of two sets is equal to the union of their complements.

$$(A \cap B)' = A' \cup B'$$

These laws are valuable for simplifying complex set expressions and logical statements.

Symmetric Difference

The **symmetric difference** of two sets, denoted by $A \Delta B$, contains all the elements that are in either set A or set B but not in both.

$$A \Delta B = (A \cup B) - (A \cap B)$$

Let's consider the same sets A and B:

$A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

First, find the union of A and B: $A \cup B = \{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

Now, find the complement of the union: $(A \cup B)' = \{1, 2, 3, 4, 5\}' =$ {complement of $\{1, 2, 3, 4, 5\}$ with respect to the universal set}

If the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then $(A \cup B)' = \{6, 7, 8\}$

So, according to the First De Morgan's Law, $(A \cup B)' = A' \cap B'$:

$(A \cup B)' = \{6, 7, 8\}$ $A' \cap B' = (\{1, 2, 3, 4, 5\})' \cap (\{1, 2, 3, 4, 5\})' = \{6, 7, 8\}$

Example 2: Second De Morgan's Law - Complement of Intersection

Let's consider the same sets A and B:

$A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

Now, we want to find the complement of the intersection of A and B,

$(A \cap B)'$.

First, find the intersection of A and B: $A \cap B = \{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$

Now, find the complement of the intersection: $(A \cap B)' = \{3\}' =$ {complement of {3} with respect to the universal set}

Using the same universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then $(A \cap B)' = \{1, 2, 4, 5, 6, 7, 8\}$So, according to the Second De Morgan's Law, $(A \cap B)' = A' \cup B'$:

$(A \cap B)' = \{1, 2, 4, 5, 6, 7, 8\}$ $A' \cup B' = (\{1, 2, 3, 4, 5\})' \cup (\{3, 4, 5\})' = \{1, 2, 4, 5, 6, 7, 8\}$

Example 3: Symmetric Difference

Now, let's find the symmetric difference of sets A and B:

$A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

Symmetric Difference ($A \Delta B$) contains elements that are in A or in B, but not in both.

First, find the union of A and B: $A \cup B = \{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

Then, find the intersection of A and B: $A \cap B = \{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$

Finally, calculate the symmetric difference: $A \Delta B = (A \cup B) - (A \cap B) = \{1, 2, 3, 4, 5\} - \{3\} = \{1, 2, 4, 5\}$

So, $A \Delta B = \{1, 2, 4, 5\}$, which includes elements that are in either A or B but not in both.