CSC 333: Computational Science and Numerical Method.

Mr. Enoch. OJ-Blaze

WELCOME /COURSE HIGHLIGHTS

- **Foundational Knowledge.** Dive deep into the principles of computational science, understanding how mathematical models are developed, analyzed, and solved using digital computation.
- **Practical Experience.** Engage in hands-on exercises and projects that allow you to apply theoretical knowledge, leveraging state-of-the-art computational tools and software.
- **Collaborative Learning/projects.** Share ideas, discuss problems, and collaborate on projects with your peers, fostering a supportive and interactive learning environment.
- **Cutting-Edge Research**: Stay updated with the latest advancements in computational methodologies, ensuring that your knowledge remains relevant and forward-thinking.

COURSE DIVISION

- MODULE 1: Operation research –optimization
- MODULE 2: Numerical computing & simulation
- MODULE 3: High performance computing -HPC

WHAT TO EXPECT

- This course will challenge you, inspire you, and equip you with skills highly sought after in a variety of industries.
- From understanding operation research with wide real-life application ,error analysis and stability of numerical algorithms to diving into the intricacies of high-performance computing and simulation.

you're about to experience a dynamic blend of theory and practical application.

TIPS FOR SUCCESS – “A something excited ”

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- Engage Actively: Actively participate in discussions, group projects, and labs. Your engagement will amplify your learning experience.
- Seek Help When Needed: If you're struggling with a topic, don't hesitate to reach out to peers, or myself. Remember, every question is a gateway to deeper understanding.
- Practice Regularly: Regular practice is the key to mastering computational techniques. Utilize the resources provided, work on additional problems, and explore real-world applications.

MODULE 1: OPERATIONS RESEARCH - OR

WHAT IS OR

- Operation Research (OR), is a multidisciplinary field that applies advanced analytical methods to help make better decisions.
- It encompasses a variety of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency
- Decision science

Concept of decision making and OR

- Many a time we speak of the word decision, as if we know much about decision.
- But what is decision?
- What does it consists of?
- What are its characteristics?

Concept of decision making and OR

- A decision is the conclusion of a process designed to weigh the relative uses or utilities of a set of alternatives on hand, so that decision maker selects the best alternative which is best to his problem or situation and implement it.
- Decision Making involves all activities and thinking that are necessary to identify the most **optimal(best)** or preferred choice among the **available alternatives(feasible solutions).**

Why decision making and OR

- **Scarcity or resources.** The necessity of making decisions arises because of our existence in the world with various needs ,ambitions and goals, whose resources are limited and sometimes scarce. Every one of us competes to use these resources to fulfill our goals.
- **Multi-facet needs**. Our needs can be biological, physical, financial, social, ego or higher-level self-actualization needs.
- **Conflict of needs.** One peculiar characteristics of decision-making is the inherent conflict that desists among various goals relevant to any decision situation (for example, a student thinking of studying for first class and spending time on Netflix, social media, without attending classes. A man wants to have lot of leisure in his life at the same time earn more etc.).
- **Not making a decision is also a decision, but the question remains is it optimal ?**

REQUIREMENT FOR DECISION MAKING

The basic requirements for decision-making are

- (i) **A set of goals or objectives,**
- **(ii) Methods of evaluating alternatives in an objective manner,**
- **(iii) A system of choice criteria and a method of projecting the repercussions of alternative choices of courses of action.**
- The evaluation of consequences of each course of action is important due to sequential nature of decisions.

DECISION MAKING

The process of decision-making consists of two phases.

- The first phase consists of formulation of goals and objectives, enumeration of environmental constraints, identification and evaluation of alternatives.
- The second stage deals with selection of optimal course of action for a given set of constraints

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OPERATION RESEARCH

- Operations Research, is concerned with how to choose optimal strategy under specified set of assumptions, including all available strategies and their associated payoffs.
- As the name indicates, the word Operations is used to refer to the problems of military and the word Research is use for inventing new method. As this method of solving the problem was invented during the war period, the subject is given the name ‘OPERATIONS RESEARCH’ and abbreviated as ‘O.R.’

HISTORY OF OR – WW II

- **The formal discipline of OR originated during World War II.**
- The military faced various strategic and operational challenges and sought assistance from scientists to address these challenges. Teams of mathematicians, physicists, and engineers in the UK and the US were assembled to solve strategic and tactical problems related to radar, logistics, and other wartime challenges.
- **The problem attained importance because at that time the resources available with England was very limited and the objective was to win the war with available meager resources.**
- The **resources such as food, medicines, ammunition, manpower etc**., were required to manage war and for the use of the population of the country. It was necessary to decide upon the most effective utilization of the available **resources to achieve the objective.**
- These specialists had a brain storming session and came out with a method of solving the problem, which they coined the name **"Linear Programming".**
- As this method of solving the problem was invented during the war period, the subject was given the name 'OPERATIONS RESEARCH' and abbreviated as 'O.R.'

HISTORY OF OR – POST WWII

- Industries started realizing the potential of OR in optimizing production, logistics, and other operations.
- The shift from military to civilian applications marked the rapid expansion of OR into areas like transportation, finance, and manufacturing. etc
- Linear programming, introduced by George Dantzig in the late 1940s, became one of the cornerstones. Later, methods such as integer programming, dynamic programming, queuing theory, and network analysis further enriched the OR toolkit.

HISTORY OF OR – MODERN ERA

- Today, OR is a key component in various industries, aiding in supply chain management, financial services, healthcare, and many others.
- The integration of OR with other disciplines like computer science has led to the advent of fields like data analytics and machine learning, which utilize OR principles for complex problem-solving.

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SCOPE AND APPLICATIONS OF OR

- The scope of Operations Research (OR) is vast and continually evolving, thanks to its foundational principle of applying analytical methods to decision-making.

APPLICATIONS OF OR - BUSINESS & INDUSTRY

- Supply Chain Management Inventory optimization, logistics, distribution strategies.
- Production Planning Efficient machinery scheduling, workforce allocation, process optimization.
- Financial Engineering Portfolio optimization, risk assessment, stock market prediction.
- HR Management Optimal hiring strategies, task assignment, workforce scheduling.
- Marketing: Sales forecasting, market segmentation, dynamic pricing strategies.

APPLICATIONS OF OR - TRANSPORTATION

- Route Optimization Best routes for delivery, airlines, shipping.
- Fleet Management Vehicle maintenance planning, capacity optimization.
- Public Transportation Bus and train scheduling, route planning, fare optimization.
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APPLICATIONS OF OR – HEALTH CARE

- Hospital Management Efficient patient scheduling, resource allocation, staff rostering.
- Drug Development Predictive modeling for drug interactions, clinical trial optimization.
- Medical Decision Making Treatment recommendation, diagnosis support through predictive modeling.

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APPLICATIONS OF OR – MILTARY & DEFENSE

- Strategic Planning: Resource allocation, mission planning, strategy optimization.
- Logistics: Efficient movement of troops, equipment deployment.
- Surveillance: Drone route planning, reconnaissance strategy.

APPLICATIONS OF OR

ENERGY & UTILITIES	- Power Grid Management: Electricity distribution optimization, load shedding strategies.
	- Oil and Gas: Pipeline optimization, exploration strategies, refining process enhancement.
AGRICULTURE	- Crop Yield Prediction: -driven yield forecasting, pest control strategies.
	- Resource Allocation: Optimal distribution of water, fertilizers, and machinery.
SPORTS	- Game Strategies: Play analysis, player performance optimization, injury prevention.
	- Team Management: Player selection strategies, trade optimization, training regimen planning.

APPLICATIONS OF OR

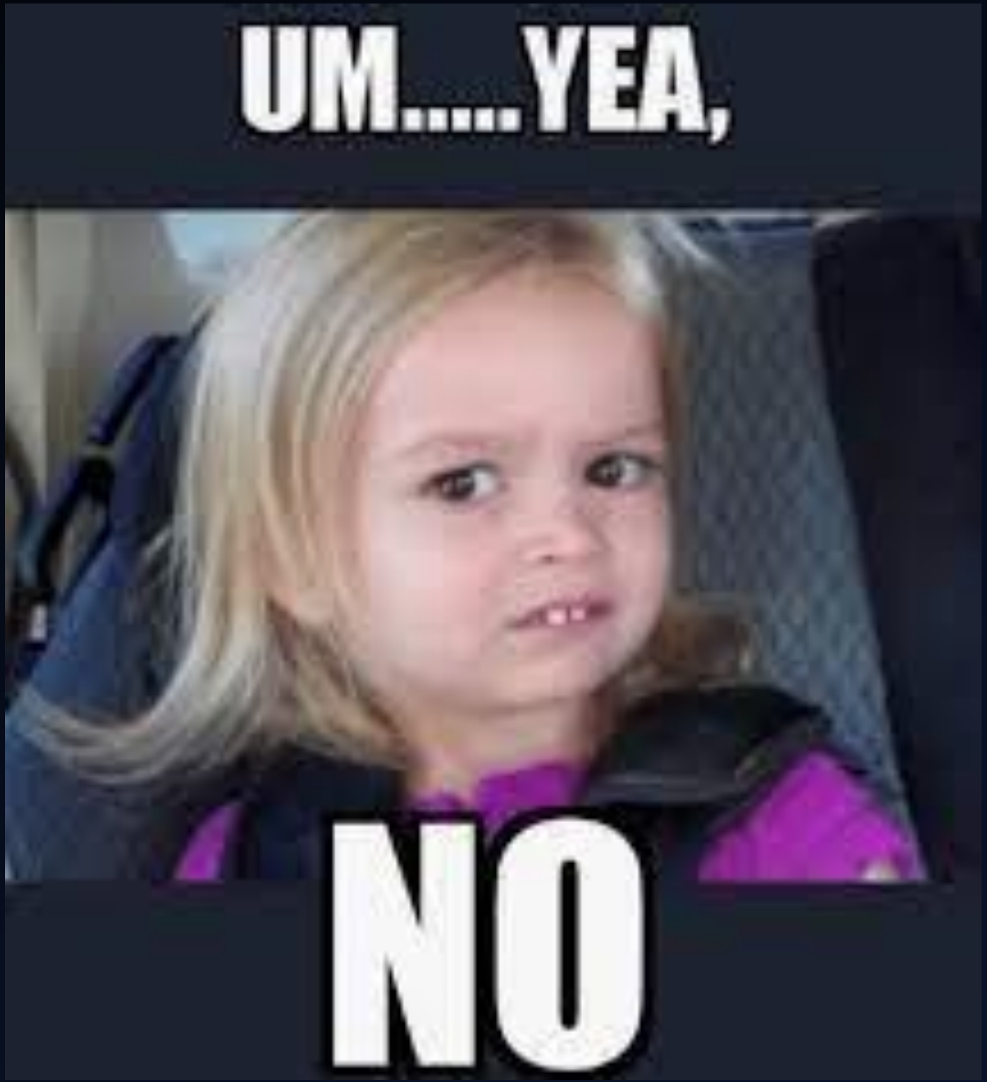
EDUCATION	- Timetabling: Efficient scheduling of classes, exam timetabling, faculty allocation.
	- Resource Allocation: Optimal distribution of labs, books, technology resources.
RETAIL & E-COMMERCE	- Demand Forecasting: Predictive analytics for inventory management, sales prediction.
	- Price Optimization: Dynamic pricing, discount strategies, sales promotion optimization.
R&D	- Project Selection: ROI prediction, resource allocation, feasibility analysis.
	- Optimal Experimentation: Efficient design of experiments, results analysis optimization.

APPLICATIONS OF OR

TELECOMMUNICATIONS	- Network Design: Optimal layout for communication networks, tower placement.
	- Traffic Management Data flow optimization, bottleneck prevention, quality of service maintenance.

APPLICATION OF OR IN GOVERNMENT AND PUBLIC POLICY ???

Most of you



Meanwhile Others about Nigeria

sudden realization about Nigeria

Meanwhile over thinkers about Nigeria

Pained finalst

APPLICATIONS OF OR

Public Policy & Govt.	- Urban Planning: Traffic optimization, infrastructure development, utilities management.
	- Disaster Management: Optimal resource deployment, relief strategy planning.
	- Environmental Policy: Ecosystem modeling, pollution control strategies, resource management.

CHARACTERISTICS / FEATURES OF OR

- **It is Interdisciplinary in Nature**: OR is not confined to a single discipline. Instead, it borrows principles, methods, and tools from various fields.
- **Employs Analytical Methodology** :OR relies on a structured analytical approach.it Ensures that decisions are based on data and logical reasoning rather than intuition, leading to more reliable and effective solutions
- **It is Problem-Oriented**: it focuses on solving specific problems. It starts with understanding and defining the problem. approach ensures practical, actionable solutions that cater to real-world challenges.
- **It uses Models**: It often uses mathematical model to represent complex systems and scenarios. These models capture the essential features of the problem and allow for simulations and analyses
- **Objective-Oriented approach is employed:** Every OR study has a clear objective, whether it's maximizing profit, minimizing cost, or achieving a specific target.

CHARACTERISTICS / FEATURES OF OR

- **It’s an Iterative Process:** OR solutions often undergo multiple iterations. Initial solutions are refined based on feedback, further analysis, or changing conditions
- **Decision-Making**: By supporting decisions with rigorous analysis, OR reduces uncertainty and enhances the quality of decisions.
- **Scientific Approach is employed** : OR uses systematic, methodical techniques grounded in science and logic. A scientific approach ensures that solutions are evidence-based, reproducible, and consistent.
- **Focuses on Systems Perspective:** Instead of looking at problems in isolation, OR takes a holistic view, seeing the entire system and its interaction. A systems perspective captures the interdependencies within a problem, ensuring solutions don't inadvertently create issues elsewhere.

PHASES / STEPS IN SOLVING OPERATIONS RESEARCH PROBLEMS

PHASE 1 - Problem Definition

- This is the initial phase where the real-world problem is clearly identified and understood. It involves recognizing the need for an OR study and articulating the goals and challenges.
- In defining the problem, it is important that the whole system be examined critically in order to recognize all the areas that could be affected by any decision taking.
- It is essential to examine the symptoms and true causes of the problem when defining the issue.
- **Tasks :** Understand the problem context, identify stakeholders, clarify objectives, and recognize constraints. Essentially, transform the broad problem into a precise statement.

PHASE 2 - Data Collection

- To make the model meaningful and applicable, it's necessary to populate it with relevant data.
- Tasks : Collect quantitative and qualitative data from various sources, ensuring accuracy. This can involve surveys, historical data analysis, observations, or accessing established databases.

PHASE 3 - Model Formulation

- Based on the defined problem, a conceptual representation, often mathematical, is formulated. This model represents the essential aspects of the problem and becomes the foundation for further analysis.
- Tasks : Decide on the decision variables (factors that can be controlled), establish the objective function (which describes the goal, such as maximizing profit or minimizing cost), and set the constraints (limitations or restrictions).

PHASE 4 - Model Formulation

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PHASE 5 - Model Solution

- This involves the manipulation of the model to arrive at the best (optimal) solution to the problem.
- It may require solving some mathematical equations for optimal decisions as in calculus or linear programming models. It may also be a logical approach or a functional approach which does not require solving a mathematical equation, such as in queuing theory.
- The optimal solution is then determined by some criteria. This phase finds a solution that achieves the objective within the set constraints.
- Tasks : Utilize appropriate OR techniques or algorithms, such as linear programming, dynamic programming, simulation, etc. Often, specialized software tools assist in this phase

PHASE 6 - Testing and Validation of the solution

- This involves determining the accuracy or the completeness of the data used in the model because inaccurate data leads to inaccurate solutions. If the model can adequately predict the effect of the changes in the system, however simple it may be, it is acceptable.
- Before implementing the solution, it's crucial to validate the model's outputs to ensure they are realistic and reliable.
- Tasks: Compare the model's predictions or recommendations with real-world outcomes, expert opinions, or intuitive expectations. If discrepancies arise, refining the model or re-examining the data might be necessary.

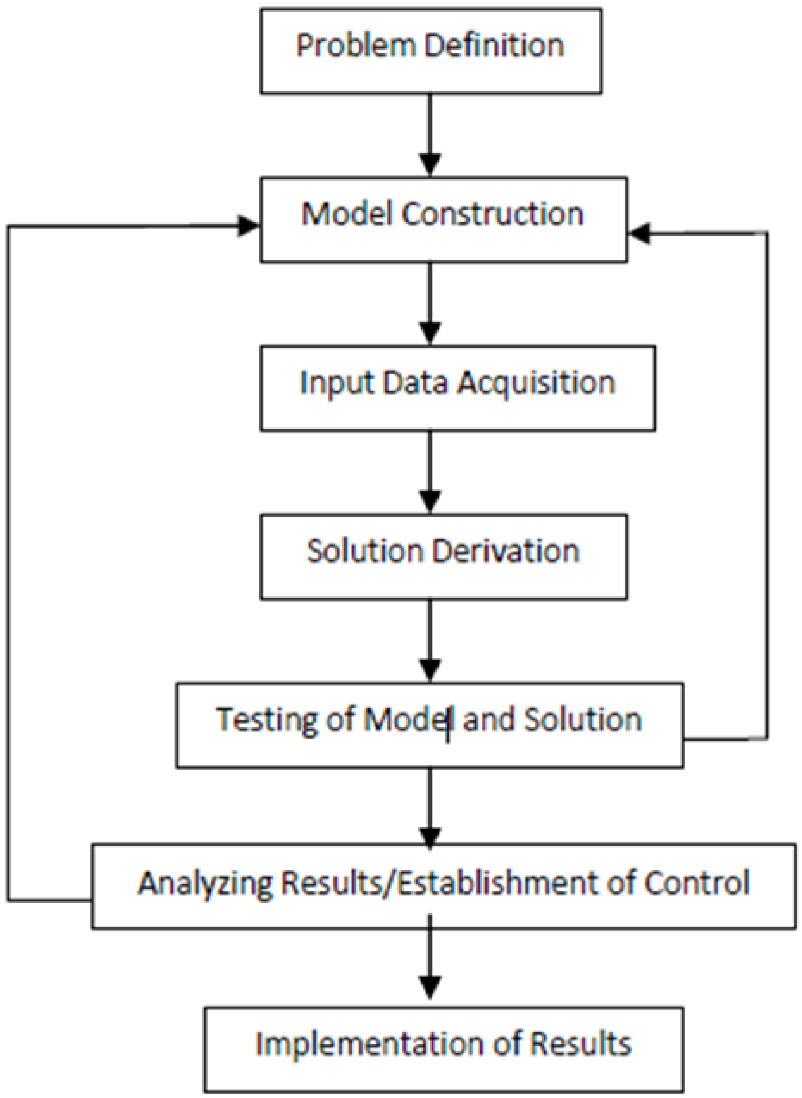
PHASE 7 - Implementation

- This phase involves applying the recommended solution from the model to the real-world problem.
- Tasks: Engage with decision-makers and stakeholders, ensure they understand and support the proposed solution, address practical challenges that might arise, and monitor the implementation closely.

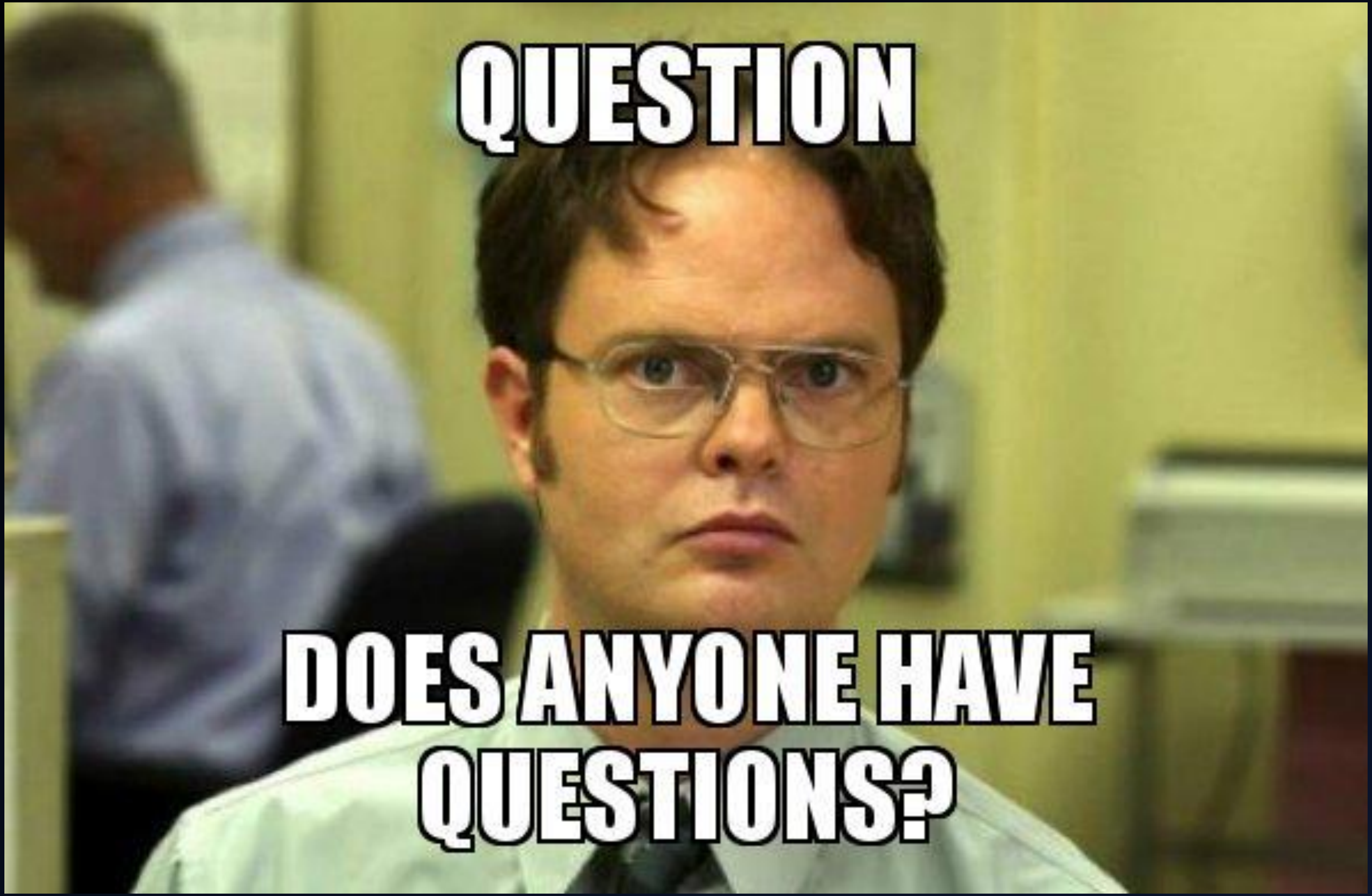
PHASE 8 - Follow-up and refinement

- Follow-up : After the solution has been implemented, it's essential to monitor its effectiveness and outcomes.
- Tasks : Assess results, gather feedback, compare actual outcomes with model predictions, and evaluate the overall effectiveness of the solution. This phase might reveal areas for further improvement or refinement.
- Iterative Refinement (if necessary): OR is often an iterative process. Feedback from the post-implementation review or changes in the environment might necessitate returning to earlier phases.
- Tasks : Adjust the model or solution approach based on new insights, data, or feedback, and reapply the OR techniques.

Flow Chart Showing the Stages in Operations Research




THANKS FOR YOUR ATTENTION
38
CLAP AND ASK NO QUESTIONS


QUESTION
DOES ANYONE HAVE QUESTIONS?

MODELLING IN OPERATION RESEARCH

- modeling refers to the process of representing a real-world system or problem using mathematical expressions, symbols, and relationships.
- This is usually a mathematical process
- The primary purpose of modeling in OR is to analyze complex systems and make decisions based on the insights derived from these models
- Operations research is simply a scientific approach to decision making that seeks to best design and operate a system, usually under conditions requiring the allocation of scarce resources. By a system, we mean an organization of interdependent components that work together to accomplish the goal of the system that can be modelled

Probabilistic and Deterministic model

- **Deterministic models** are models when the change of one variable has a certain or defined change in the outcome. These are models in which every detail is specified or determined. There's no randomness involved. Linear and nonlinear models can be deterministic if they don't include any random variables.
- **Probabilistic Models** are models that incorporate randomness or uncertainty. This model incorporate uncertainty in terms of probabilities of future event occurring.
- **Deterministic models predict the exact outcome of a situation because it is based on certain known laws.**
- **Probabilistic models deal with situation that are random in character and can predict the outcome within a stated or known degree of accuracy**

Probabilistic and Deterministic model

- **Deterministic model** does not include elements of randomness. Every time you run the model with the same initial conditions you will get the same results
- **probabilistic model** does include elements of randomness. Every time you run the model, you are likely to get different results, even with the same initial conditions.

Probabilistic vs Deterministic model

Feature	Deterministic Model	Probabilistic Model (Stochastic)
Nature	Fixed outcomes for given inputs	Variable outcomes for given inputs
Assumption	All parameters are known with certainty	Some parameters have inherent randomness
Use Cases	Systems with predictable outcomes	Systems with inherent uncertainty or randomness
Advantage	Simplicity; easier to understand & solve	Realism; captures real-world uncertainties
Limitation	Might not reflect real-world uncertainties	Typically, more complex to formulate and solve
Certainty	Operates under certainty	Embraces and represents uncertainty
Complexity	Generally, less complex	Generally, more complex due to randomness
Realism	Might be less realistic in uncertain scenarios	Often more realistic due to inclusion of randomness
Solution Techniques	Direct mathematical methods	Statistical techniques or simulations
Application Areas	Predictable, well-defined processes	Areas with key components of uncertainty or randomness

Probabilistic model examples

- **Stock Prices**: The daily price of a stock is influenced by countless unpredictable factors, making its exact future price random.
- **Radioactive Decay**: While we can predict the half-life of a radioactive substance, the exact time at which a particular atom will decay is random.
- **Genetic Mutation**: While there are factors that can increase the mutation rate, the exact occurrence and location of a mutation in DNA during replication are random.
- **Dice Roll**: The outcome of rolling a fair six-sided dice is stochastic; each of the six outcomes has an equal probability of 1/6.

Deterministic model examples

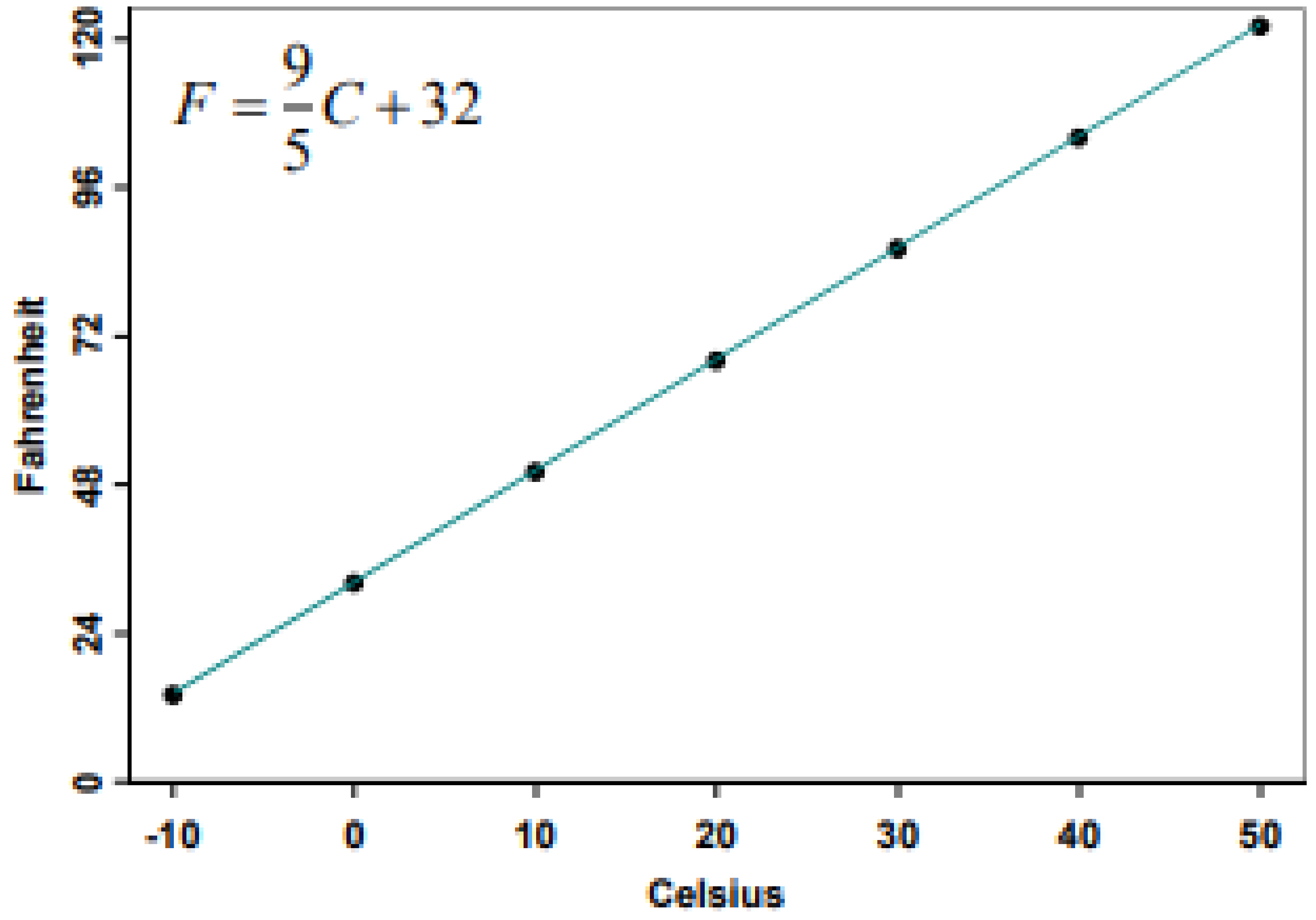
- **Mathematical Equations**: $y=2x+5$ will always give the same value of *y* for a given *x*.
- **Mechanical Systems**: A clock's pendulum swings at a predictable rate if not subjected to external disturbances.
- **Computer Algorithms**: A sorting algorithm, like bubble sort, will always take the same steps and produce the same output order for a given input list.
- **Planetary Motion**: Given the current position and velocity of a planet, its future position can be predicted precisely using Newton's laws (ignoring other minor perturbative effects).

DETERMINISTIC AND PROBABILISTIC MODEL

- A *deterministic model* does not include elements of randomness.
- Every time you run the model with the same initial conditions you will get the *same* results.
- Mathematical equation describing the linear relation is generally written as
  $$\mathrm{Y_i} = \mathrm{a} + \mathrm{bX_i}\ ; \qquad (i = 1,2, \dots n)$$
- E.g.,
  i. Relation between Celsius and Fahrenheit scales : $F = 32 + \frac{9}{5}C$
  ii. Relation between area and radius of circle: $A = \pi r^2$

- A *probabilistic model* includes elements of randomness.
- Every time you run the model, you are likely to get *different* results, even with the same initial conditions.
- Mathematical equation describing the inexact relation is generally written as
  $$\mathrm{Y_i} = \mathrm{a} + \mathrm{bX_i} + e_i\ ; \quad (i = 1,2, \dots n)$$
- E.g.,
  i. Production of rice

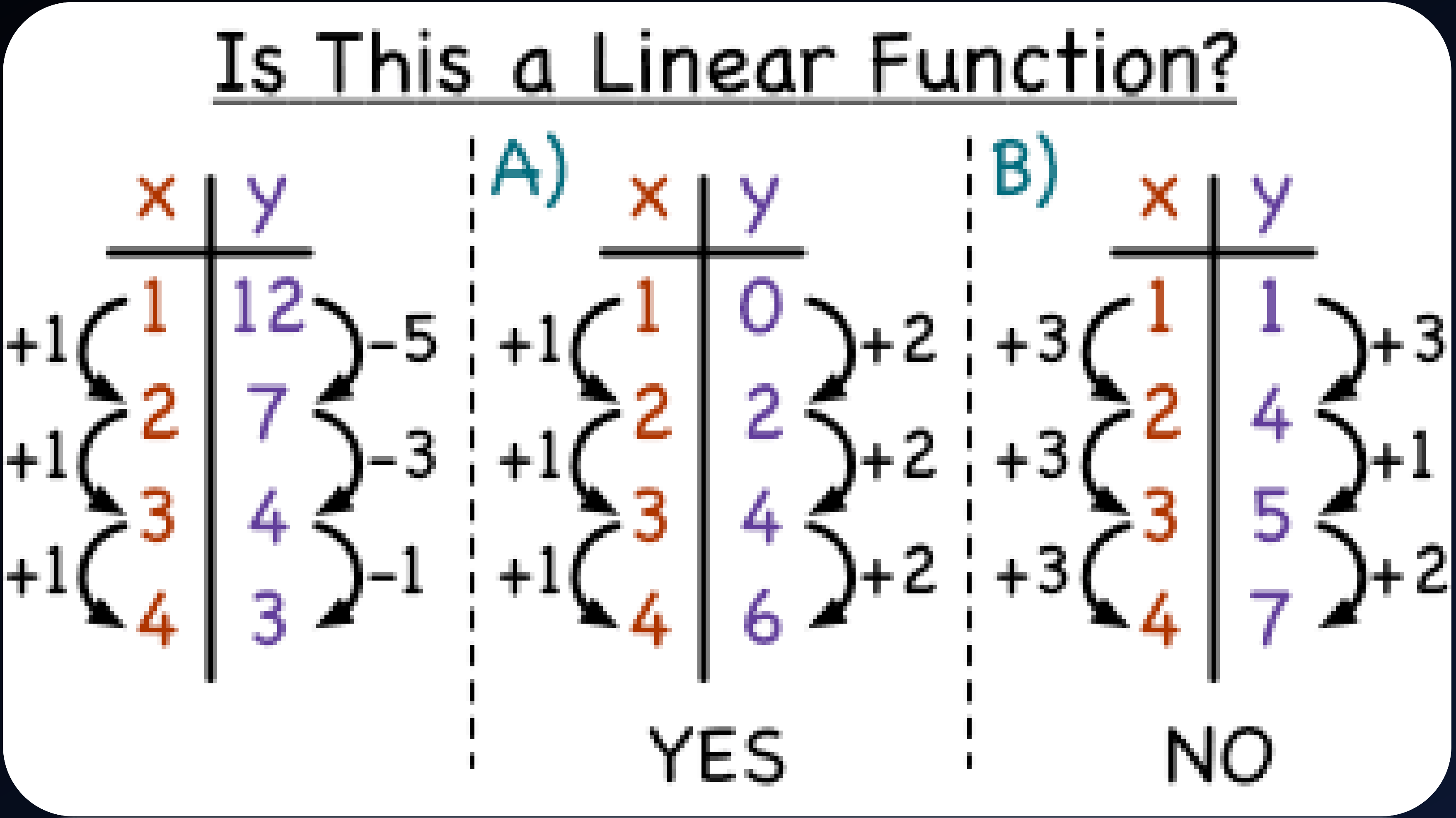
Syed Awais Ali


$F = \frac{9}{5}C + 32$
Fahrenheit
0
24
48
72
96
120
-10
0
10
20
30
40
50
Celsius

Linear versus Non-linear

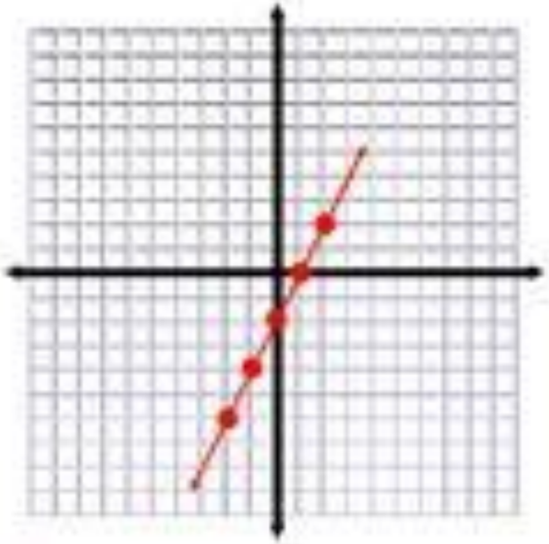
- **Linear models -** model is linear if the relationship between the variables can be represented as a linear equation (or a system of linear equations). Linear programming is a common technique used in OR to find the best outcome in a given mathematical model.
- **Non-linear models .** Model is nonlinear if the relationship between the variables cannot be represented by a linear equation. The relationship might be polynomial, exponential, logarithmic, etc
- **models are linear when changes in the independent or input variables result in constant proportional changes in the dependent or output variables**

Linear versus Non-linear



equation $y = 2x - 2$

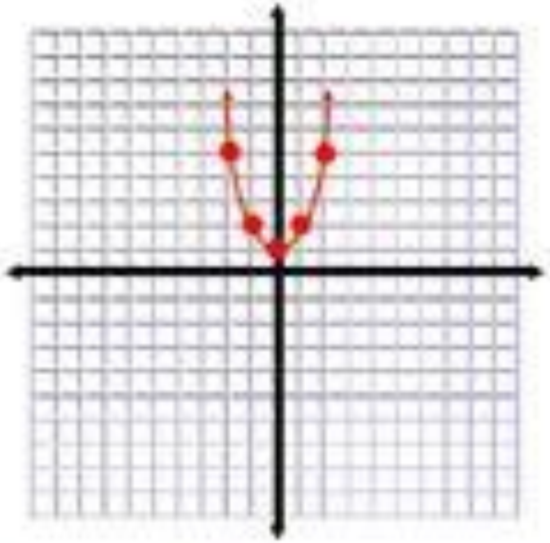
x	$2x - 2$	y	(x, y)
-2	2(-2) - 2	-6	(-2, -6)
-1	2(-1) - 2	-4	(-1, -4)
0	2(0) - 2	-2	(0, -2)
1	2(1) - 2	0	(1, 0)
2	2(2) - 2	2	(2, 2)



Linear

equation $y = x^2 + 1$

x	$x^2 + 1$	y	(x, y)
-2	$(-2)^2 + 1$	5	(-2, 5)
-1	$(-1)^2 + 1$	2	(-1, 2)
0	$(0)^2 + 1$	1	(0, 1)
1	$(1)^2 + 1$	2	(1, 2)
2	$(2)^2 + 1$	5	(2, 5)



Nonlinear

Linear or Nonlinear

For any interval, we need the $\frac{\Delta y}{\Delta x}$ to be equal

nonlinear

X	Y
-7	10
-3	9
1	8
7	7

+4 +4 6

−1 −1 −1

$m = -1/4$ $-1/6$

X	Y
-5	10
-3	9
-1	8
1	7

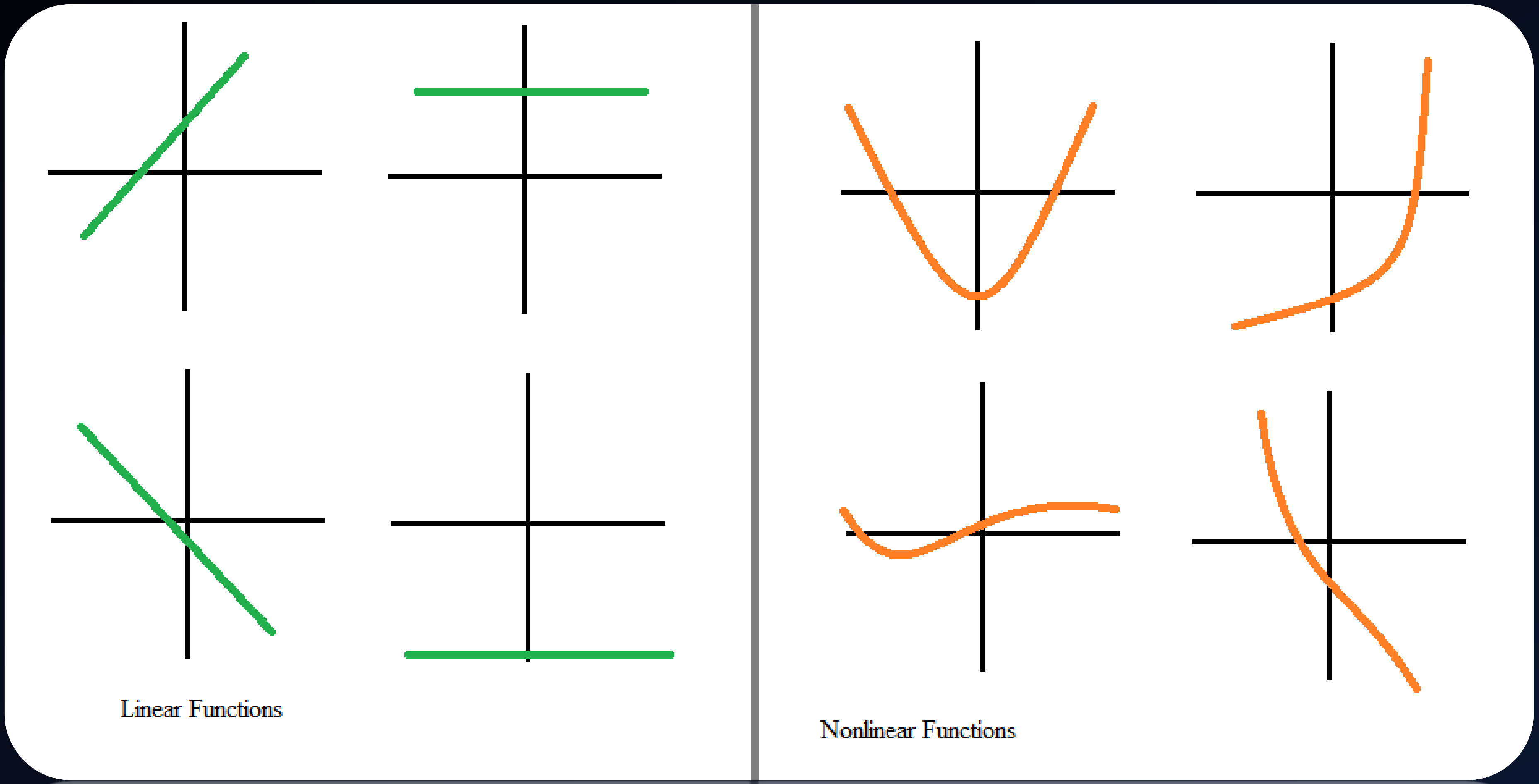
Linear model examples

- **Mathematics**: The equation $y=mx+b$, where m is the slope and b is the y-intercept.
- **Physics**: Ohm's law in electricity which states $V=IR$, where V is the voltage, I is the current, and R is the resistance. The relationship between V and I is linear.
- **Economics**: A simple model of demand might state that the quantity demanded decreases linearly as the price increases, given by $Q=a-bP$, where Q is the quantity and P is the price.
- **Biology**: A dose-response relationship where a proportional increase in dose results in a proportional response, at least within a certain range.

Non-Linear model examples

- **Mathematics**: The equation $\boldsymbol{y=ax2+bx+c}$ is a quadratic (and thus nonlinear) model.
- **Physics**: The force exerted by a spring according to Hooke's Law is $\boldsymbol{F=-kx}$ when the spring is within its elastic limit. However, if the spring is stretched beyond this limit, the relationship becomes nonlinear.
- **Economics**: The Cobb-Douglas production function given by $Q=AL\alpha K\beta$, where Q is output, L is labor, K is capital, and A,α,β are parameters, is a nonlinear model.
- **Biolog**: The growth of populations can often be modeled by the logistic growth equation, given by $\boldsymbol{dt/dP=rP(1-K/P)}$, where P is the population, r is the growth rate, and K is the carrying capacity. This equation represents nonlinear growth.

Linear and non linear models graphs



Analytic versus Simulation

- **An analytic model** is one that represents the relationship between the variables in the form of formulas. Linear programming model is an example.
- **A simulation model,** - describes the process involved in a simulation, indicating the mathematical relationships that exist at each stage In this sense, simulation is an imitation of the step-by-step process involved in the build-up of system relationships. We shall return to the subject of simulation later.

STEPS IN MODELING OPERATIONS RESEARCH PROBLEMS

STEPS IN MODELLING OR PROBLEMS

- 1. Problem Definition
- The operations researcher first defines the organization’s problem.
- Defining the problem includes specifying the organization’s objectives and the parts of the organization that must be studied before the problem can be solved
- Clearly identify and define the problem and understand the objectives, requirements, and constraints of the decision-makers.

STEPS IN MODELLING OR PROBLEMS

- 2. Observation and Data Collection:
- Gather relevant data and information about the problem.
- Data can be historical, experimental, or forecasted. In most cases, by far the harder task is collecting the data.
- This task is can be time consuming and prone to errors. Most models, built to analyze real-world problems, require lots of data.
- In some instances, data is available through databases. In other instances, data may not be readily available, and once it is found, it is seldom in a form directly suitable for your purposes. In this case data must

STEPS IN MODELLING OR PROBLEMS

- 3. Formulation:
- This phase requires you to convert the real-world problem into a mathematical model.
- Identify decision variables, objective function, and constraints. Incorporate any parameters and interrelationships.
- It is not usually self-evident which particular type of model formulation is the most appropriate. There are no clear rules for making a choice, the choices of other modelers in similar situations can give suggestions, and your own experience and knowledge will always influence your eventual choice

STEPS IN MODELLING OR PROBLEMS

- **4. Selection of a Solution Technique:**
- Choose an appropriate method or algorithm to solve the model. This could involve linear programming, simulation, queuing theory, network analysis, etc., depending on the nature of the problem.

- **5. Solution of the Model:**
- Use the selected technique to find the solution to the mathematical model. This could result in an optimal strategy, a set of recommended decisions, or insights about the system's behavior.

STEPS IN MODELLING OR PROBLEMS

- 6. Validation and Testing:
- Validate the model to ensure it accurately represents the real-world system. This might involve comparing the model's predictions against actual outcomes or using expert judgment. Make necessary refinements or calibrations to the model based on this validation.
- 7. Implementation:
- Translate the model's solution into actionable decisions in the real-world scenario. Implement these decisions and monitor the results.
- .

STEPS IN MODELLING OR PROBLEMS

- 8. Review and Feedback:
- Continuously monitor the results of the implementation. Obtain feedback and use it to adjust and refine the model, its parameters, or the solution strategy
- 9. Iteration As new data becomes available, or as the problem environment changes, the model might need updates. This is an iterative process where steps might be revisited based on feedback, changing conditions, or the need for further precision.

COMPONENTS OF AN OR MODEL

An OR model is a representation of a system or a problem, typically expressed in mathematical terms, to facilitate analysis and derive insights. The main components of an OR model is:

- 1. **Decision Variables**: These are the variables that the decision-maker can control or decide upon. Examples: How much of a product to produce, how many employees to hire, etc.
- 2. **Objective Function**: This represents the goal of the model. It's usually something to be maximized (like profit) or minimized (like cost). It's a mathematical expression involving decision variables.

COMPONENTS OF OR MODEL

- 3.Constraints: These are the restrictions or limitations on the possible solutions to the problem. Constraints can arise from limited resources, time restrictions, capacity limitations, etc.
- They are also typically represented mathematically, often in the form of inequalities or equations involving the decision variables.
- 4.Parameters: These are constants that are part of the model but are not under the control of the decision-maker. Examples: Cost of raw materials, demand for a product, etc.
- Parameters can sometimes be deterministic (fixed values) or probabilistic (random with a known distribution

COMPONENTS OF OR MODEL

- 5. **Environment**: The set of external factors or conditions that the model operates within.It might include market conditions, regulations, technology used, etc.
- 6. **Interrelationships**: These represent the connections or dependencies between various parts of the model. For example, how a change in production might affect cost

LINEAR PROGRAMMING MODEL/PROBLEM

LP MODEL

- Linear programming (LP) is a mathematical method used to find the best possible outcome or solution from a given set of linear relationships or conditions. It's a specific case of mathematical programming and is used to solve optimization problems where the objective function and the constraints are all linear.

- A model, which is used for optimum allocation of scarce or limited resources to competing products or activities under such assumptions as certainty, linearity, fixed technology, and constant profit per unit, is linear programming.

LP MODEL

- As a decision-making tool, it has demonstrated its value in various fields such as production, finance, marketing, research and development and personnel management.
- Determination of optimal product mix (a combination of products, which gives maximum profit), transportation schedules, Assignment problem and many more.

GENERAL FORM OF LP MODEL

$Z = c_1x_1 + c_2 \; x_2 + \ldots c_n \, x_n$ subjects to the conditions, $\longrightarrow$ OBJECTIVE FUNCTION

$a_{11}x_1 + a_{12} \, x_2 + a_{13} \, x_3 + \ldots + a_{1j} \, x_j + \ldots + .. a_{1n} \, x_n \; (\geq, =, \leq) \; b_1$

$a_{21} \, x_1 + a_{22} \, x_2 + a_{23} \, x_3 + \ldots\ldots\ldots\ldots + a_{2j} \, x_j + \ldots\ldots + a_{2n}x_n \; (\geq, =, \leq) \; b_2$

....

....

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mj} \, x_j \ldots + a_{mn} \, x_n \; (\geq, =, \leq) \; b_m$

Structural Constraints

and all x_j are = 0 $\longrightarrow$ NON NEGETIVITY CONSTRINT.

Where $j = 1, 2, 3, \ldots n$

COMPONENTS OF LP MODEL

- 1. Decision Variables:
- These are the variables that decide the output. For instance, in a production problem, they could represent the quantity of each product to be produced.
- 2. Objective Function:
- It represents the main goal of the model. It could be a function that needs to be maximized (like profit) or minimized (like cost). The objective function is a linear equation representing the sum of weighted decision variables.
- For instance, if x and y are two products with profits of $100 and $150 respectively, the profit function might be:
- $P = 100x + 150y$

COMPONENTS OF LP MODEL

3. Constraints:

- These are the set of linear inequalities or equations that restrict the feasible solutions. They could represent limitations in resources like manpower, material, money, time, etc.
- For example, if x and y need 2 and 3 units of a particular material and only 12 units are available, a constraint would be:

$$2x + 3y \leq 12.$$

COMPONENTS OF LP MODEL

4. Non-Negativity Restrictions:

In many LP problems, the decision variables represent quantities that cannot be negative (e.g., the number of items produced, hours worked). Hence, the variables have a non-negativity restriction, usually represented as:

$x \geq 0x \geq 0$ and $y \geq 0$ $y \geq 0$.

PROPERTIES OF LP MODEL

i. All LP model must have an objective function that seek to maximize or minimize some quantity, usually profit or costs.

ii. All LP models must have structural constraints or limitations that limit the degree to which the objective can be pursued (e.g. deciding how many units of product in a product line to be produced is restricted to the manpower and machinery available).

iii. There must be alternative course of action to choose from (e.g. if there are 4 different products, management may decide (using LP) how to allocate limited resources among them).

iv. The relationship between variables and constraints must be linear. Objectives and constraints in LP model must be expressed in linear equations and inequalities

v. The model must have non-negativity constraint

FORMULATING LP MODELS

FORMULATING LP MODEL

The following steps can be used to formulate the model of any optimization problem.

- Step 1

Choose variables and notations that will be used to form the objective and constraints functions.

- Step 2

Identify the objective function to either maximize or minimize (e.g. that which maximizes profit or minimizes cost).

- Step 3

Develop mathematical relationships to describe objective and constraints

EX1 : LP MODEL – MAXIMIZATION

Example 1 : Allocation of Resources in Production

A farmer has 100 acres on which to plant two crops: corn and wheat. To produce these crops, there are certain expenses as shown below:

EX1 : LP MODEL – MAXIMIZATION

Item	Cost per Acre (#)
Corn Seed Fertilizer Planting/care/harvesting	12 58 50
Total	120
Wheat Seed Fertilizer Planting/care/harvesting	40 80 90
Total	210

EX1 : LP MODEL – MAXIMIZATION CONT.

After the harvest, the farmer must store the crops awaiting proper market conditions. Each acre yields an average of 110 bushels of corn or 30 bushels of wheat. The limitations of resources are as follows:

- Available capital: #15,000
- Available storage facilities: 4,000 bushels

If net profit (the profit after all expenses have been subtracted) per bushel of corn is #1.30 and for wheat is #2.00

QUESTION: how should the farmer plant the 100 acres to maximize the profits?

EX1 : LP MODEL – MAXIMIZATION CONT

FORMULATING THE LP MODEL

STEP 1: DEFINE/CHOOSE THE VARIABLE

To formulate a mathematical model, begin by letting:

- x = number of acres to be planted in corn.
- y = number of acres to be planted in wheat.

PROFIT

Profit from corn/wheat = profit per corn/wheat * storage capacity(bushels)

1.30 * 110 = 143

2.00 * 30 = 60

EX1 : LP MODEL – MAXIMIZATION CONT

CREATE TABULAR DESC.FOR THE LP MODE

Products	Constraints			Profit per acres
	Capital	Storage capacity (bushels)	Acres Land	
X (corn)	120	110		143 (1.30 * 110)
Y (wheat)	210	30		60(2.00 * 30)
Max.Available resources	15,000	4,000	100	

EX1 : LP MODEL – MAXIMIZATION CONT

FORMULATING THE LP MODEL

STEP 2: SET UP THE CONSTRAINT INEQUALITIES EQN

The amount of available land is 100 acres.

$x + y \leq 100$.

Why not $x + y = 100$?

it might be more profitable for the farmer to leave some land out of production i.e. it is not necessary to plant on the whole parcel of land

EX1 : LP MODEL – MAXIMIZATION CONT

FORMULATING THE LP MODEL

STEP 2: SET UP THE CONSTRAINT INEQUALITIES EQN

Expenses constraint

We also know that:

expenses for planting the corn = $120x$

expenses for planting the wheat = $210y$.

The total expenses cannot exceed #15,000.

This is the available capital.

$$120x + 210y \leq 15000$$

EX1 : LP MODEL – MAXIMIZATION CONT

FORMULATING THE LP MODEL

STEP 2: SET UP THE CONSTRAINT INEQUALITIES EQN

Yield constraints

The yields are:

Yield of acreage planted in corn = $110x$

Yield of acreage planted wheat = $30y$

The total yield cannot exceed the storage capacity of 4,000 bushels:

$$110x + 30y \leq 4000.$$

EX1 : LP MODEL – MAXIMIZATION CONT

FORMULATING THE LP MODEL

STEP 3: STEP 3: NON- NEGATIVITY CONSTRAINTS

There are certain limitations or constraints.

The number of acres planted cannot be negative, so

$x \geq 0$

$y \geq 0$

- X, Y >= 0

EX1 : LP MODEL – MAXIMIZATION CONT

THE LINEAR PROG.MODEL

The linear programming model is stated as follows:

Max: $P = 143x + 60y$ [objective function]

Subject to constraints:

$x + y \le 100$	[Available land]
$120x + 210y \le 15000$	[Available capital]
$110x + 30y \le 4000$	[Storage capacity]
$x \ge 0, y \ge 0$	[non-negativity]

EX 2 :   LP MODEL – MAXIMIZATION

STEP 1: Decision variables

Let

x = number of units of product P

y = number of units of product Q

EX2 : LP MODEL – MAXIMIZATION CONT

CREATE TABULAR DESC.FOR THE LP MODE

Products	Constraints		Profit per unit
	Machine shop	Assembly shop	
X	2	4	3
Y	3	2	4
Max. Available hours per day	16	22	

EX 2 : LP MODEL – MAXIMIZATION

STEP 2: DEFINE Constraints

Machine shop

- $2x$ = Number of hours available for product P.
- $3y$ = Number of hours available for product Q
- The total hours cannot exceed 16

This is the maximum available: $2x + 3y \leq 16$

Assembly shop

- $4x$ = Number of hours available for product P.
- $2y$ = Number of hours available for product Q
- The total hours cannot exceed 22

This is the maximum available: $4x + 2y \leq 22$

EX 2 : LP MODEL – MAXIMIZATION

linear programming model is:

Objective function

Maximize $Z = 3x + 4y$

Subject to constraint:

$$2x + 3y \le 16$$

$$4x + 2y \le 22$$

$$x,y >= 0$$

EX 3 : LP MODEL – MINIMIZATION

Diet Problem

A convalescent hospital wishes to provide at a minimum cost, a diet that has a minimum of 200g of carbohydrates, 100g of protein and 120g of fats per day. These requirements can be met with two foods:

Food	Carbohydrates	Protein	Fats
A	10g	2g	3g
B	5g	5g	4g

EX 3 : LP MODEL – MINIMIZATION

Diet Problem cont.

If food A cost 29k per ounce and food B cost 15k per ounce, how many ounces of each food should be purchased for each patient per day in order to meet the minimum requirements at the lowest cost? formulate the LP model.

EX 3 : LP MODEL – MINIMIZATION

Diet Problem

Decision variables

Let

- X= Number of ounces of food A.
- y = Number of ounces of food B.

EX 3 : LP MODEL – MINIMIZATION

Diet Problem

Products (food)	Constraints			Cost per ounce
	Carbs	Protein	Fats	
X	10	2	3	29
Y	5	5	4	15
Min.required ounces /day.	200	100	120	

EX 3 : LP MODEL – MINIMIZATION

Constraints

Carbohydrate

$10x$ = Number of ounces of carbohydrate in food A

$5y$ = Number of ounces of carbohydrate in food B

The total minimum ounce of carbs required is 200g

This is constraint on carbohydrate:

$$10x + 5y \geq 200$$

EX 3 : LP MODEL – MINIMIZATION

Constraints

Carbohydrate

$10x$ = Number of ounces of carbohydrate in food A

$5y$ = Number of ounces of carbohydrate in food B

The total minimum ounce of carbs required is 200g

This is constraint on carbohydrate:

$$10x + 5y \geq 200$$

EX 3 : LP MODEL – MINIMIZATION

Protein

$2x$ = Number of ounces of protein in food A

$5y$ = Number of ounces of protein in food B

The total minimum ounce of protein required 100

This is constraint on protein: $2x + 5y \geq 100$

Fat

$3x$ = Number of ounces of fat in food A

$4y$ = Number of ounces of fat in food B

The total minimum ounce of fat required is 120

This is constraint on fat: $2x + 5y \geq 120$

EX 3 : LP MODEL – MINIMIZATION

linear programming model is:

Minimize $Z = 29x + 15y$

Subject to constraint:

$$10x + 5y \geq 200$$

$$2x + 5y \geq 100$$

$$2x + 5y \geq 120$$

$$x, y >= 0$$

CLASS ACCTIVITY : LP MODEL –

Three nutrient components namely, thiamine, phosphorus and iron are found in a diet of two food items A and B. The amount of each nutrient (in milligrams per ounce i.e mg/oz) is given below

	A	B
Thiamine	0.15 mg/oz	0.10 mg/oz
Phosphorus	0.75 mg/oz	1.70 mg/oz
Iron	1.30 mg/oz	1.10 mg/oz

The cost of food A and B are Rs. 2 per oz and Rs 1.70 per oz respectively. The minimum daily requirements of these nutrients are at least 1,00 mg of thiamine, 7.50 mg of phosphorus and 10.00 mg of iron. Write the problem in the linear programming form