Discrete Mathematics Exercise 1

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1. ***Solution:***

- $p \wedge q$
- $p \wedge (\neg q)$
- $(\neg p) \wedge (\neg q)$
- $p \vee q$

2. ***Solution:***

The truth table of a), b), e), f) and g) is as follows:

p	q	$(\neg p \vee q) \wedge (\neg q \vee p)$	$(\neg p \vee q) \vee (\neg q \vee p)$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	**T**	**T**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**T**	**F**
F	**T**	**F**	**T**	**T**	**T**	**F**
F	**F**	**T**	**T**	**T**	**T**	**T**

The truth table of c) and d) is as follows:

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	**T**	**T**	**T**	**T**
T	**T**	**F**	**T**	**T**
T	**F**	**T**	**T**	**T**
T	**F**	**F**	**F**	**F**
F	**T**	**T**	**F**	**F**
F	**T**	**F**	**F**	**F**
F	**F**	**T**	**F**	**F**
F	**F**	**F**	**F**	**F**

It can be clearly seen that a) ≡ g), c) ≡ d) and e) ≡ f).

3. $\boldsymbol{a)\ (p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)}$

Proof:

Proposition a) is true under a truth assignment $\mathcal{J}$ that p, q and r are all true. In other words, a) is satisfiable.

$\boldsymbol{b)\ \neg((p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p))}$

Proof:

Proposition b) is true under a truth assignment $\mathcal{J}$ that q and r are true while p is false. In other words, b) is satisfiable.