Exercise Sheet 8

Discrete Mathematics, 2020.10.21

1. ([R], Page 581, Exercise 6(c)) Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if
   c) $x - y$ is a rational number.

2. ([R], Page 581, Exercise 7(b)(f)) Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if
   b) $xy \geq 1$
   f) x and y are both negative or both nonnegative.

3. ([R], Page 582, Exercise 36) Exercises 36 deal with these relations on the set of real numbers:
   $R_1 = \{(a, b) \in \mathbb{R}^2 | a > b\}$, the "greater than " relation,
   $R_2 = \{(a, b) \in \mathbb{R}^2 | a \geq b\}$, the "greater than or equal to" relation,
   $R_3 = \{(a, b) \in \mathbb{R}^2 | a < b\}$, the "less than" relation,
   $R_4 = \{(a, b) \in \mathbb{R}^2 | a \leq b\}$, the "less than or equal to" relation
   $R_5 = \{(a, b) \in \mathbb{R}^2 | a = b\}$, the "equal to " relation,
   $R_6 = \{(a, b) \in \mathbb{R}^2 | a \neq b\}$, the "unequal to " relation and
   $\mathbb{R} \times \mathbb{R}$, all pairs of real numbers. Find:
   a) $R_1 \circ R_1$
   b) $R_1 \circ R_2$
   c) $R_1 \circ R_3$
   e) $R_1 \circ R_5$
   f) $R_1 \circ R_6$
   g) $R_2 \circ R_3$
   h) $R_3 \circ R_3$

4. Prove that the composition operator $\circ$ is associative over relations.

5. Prove that a relation R on a set A is transitive iff. $R \circ R \subseteq R$.

6. Prove that a relation R on a set A is antisymmetric iff. $R \cap R^{-1} \subseteq I_A$, where $I_A := \{(a, a) \mid a \in A\}$.

7. Prove or disprove that, if both R_1 and R_2 are two equivalence relations on A, then $R_1 \cup R_2$ is also an equivalence relation on A.

8. Prove or disprove that, if both R_1 and R_2 are two equivalence relations on A, then $R_1 \cap R_2$ is also an equivalence relation on A.