Probability Theory and Mathematical Statistics **概率统计**

Homework 1025-1028

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3-25. 解: $X=0,1,\cdots,5 \quad Y=0,1,\cdots,3 \Rightarrow Z=0,1,\cdots 8$ ($\because P(X=0,Y=0)=0$)

$M=1,\cdots 5$ ($\because P(X=0,Y=0)=0$) $\quad N=0,1,\cdots 3$

$Z=X+Y$ 的分布律如下:

	0	1	2	3	4	5	6	7	8
Z	0	0.02	0.06	0.13	0.19	0.24	0.19	0.12	0.05

□

$M=\max\{X,Y\}$ 分布律如下:

	0	1	2	3	4	5
M	0	0.04	0.16	0.28	0.24	0.28

$N=\min\{X,Y\}$ 分布律如下:

	0	1	2	3
N	0.28	0.3	0.25	0.17

□□

3-29. 解: $P(Z=1) = p\cdot p+(1-p)(1-p) = 2p^2-2p+1$

$P(Z=0) = 2p(1-p) = 2p-2p^2$

相互独立时. $P(Z=0, X=0) = P(Z=0)P(X=0)$

即 $\quad p\cdot(1-p) = (2p-2p^2)\cdot(1-p) \Rightarrow 1-p=0$ 或 $p=0$

或 $1-p=\frac{1}{2}$

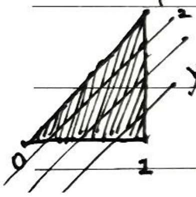
X和Y是随机变量 $\Rightarrow p\neq 0.\ p\neq 1$, 代入验算知 $p=\frac{1}{2}$ □

3-31. 解: $f_Z(z)=\int_{-\infty}^{+\infty} f_X(x)\, f_Y(z-x)\, dx$ ($\because X,Y$相互独立)

$$= \int_{z-b}^{z+b} f_X(x)\cdot\frac{1}{2b}\, dx = \frac{1}{2b}\int_{z-b}^{z+b}\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

$$= \frac{1}{2b}\left(\Phi\left(\frac{z+b-\mu}{\sigma}\right)-\Phi\left(\frac{z-b-\mu}{\sigma}\right)\right)$$ □

3-32. 解: $F_Z(z) = \int_{-\infty}^{+\infty} dx \int_{2x-z}^{+\infty} f(x,y)\, dy$



$= \int_{-\infty}^{+\infty} dx \int_{-\infty}^{z} f(x, 2x-z)\, dz = \int_{-\infty}^{z} dz \int_{-\infty}^{+\infty} f(x, 2x-z)\, dx$

$\therefore f_Z(z) = \int_{-\infty}^{+\infty} f(x, 2x-z)\, dx$

① 当 $z \geqslant 2$ 时或 $z \leqslant 0$ 时. $f_Z(z) = 0$

② $0 < z < 2$ 时. $f_Z(z) = \int_{\frac{z}{2}}^{1} f(x, 2x-z)\, dx = 1 - \frac{z}{2}$

$\therefore f_Z(z) = \begin{cases} 1 - \frac{z}{2}, & 0 < z < 2 \\ 0, & \text{其它} \end{cases}$ □

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3-34. 解: $f_Z(z) = \int_{-\infty}^{+\infty} f(yz, y)\, |y|\, dy = \int_{-\infty}^{+\infty} f_X(yz)\, f_Y(y)\, |y|\, dy$ ($\because X, Y$ 相互独立)

● $\because X \in (10, +\infty)$, $Y \in (10, +\infty)$ $\therefore Z > 0$

① $z \leqslant 0$ 时. $f_Z(z) = 0$

② $0 < z < 1$ 时. $f_Z(z) = \int_{10/z}^{+\infty} \frac{10}{y^2 z^2} \cdot \frac{10}{y^2} |y|\, dy = \frac{1}{z^2} \int_{10/z}^{+\infty} \frac{100}{y^3}\, dy = \frac{1}{2}$

③ $z \geqslant 1$ 时. $f_Z(z) = \int_{10}^{+\infty} \frac{10}{y^2 z^2} \cdot \frac{10}{y^2} |y|\, dy = \frac{1}{z^2} \int_{10}^{+\infty} \frac{100}{y^3}\, dy = \frac{1}{2z^2}$

综上. $f_Z(z) = \begin{cases} \frac{1}{2}, & 0 < z < 1 \\ \frac{1}{2z^2}, & z \geqslant 1 \\ 0, & \text{其它} \end{cases}$ □

3-36. 解: $X_i \geqslant 0$ $(\forall i \in \{1, 2, \cdots, 5\}) \Rightarrow Z \geqslant 0$

(1) $P(Z \leqslant z) = P(X_1 \leqslant z, X_2 \leqslant z, \cdots X_5 \leqslant z) = P(X_1 \leqslant z) P(X_2 \leqslant z) \cdots P(X_5 \leqslant z)$

($\because X_1, \dots X_5$ 相互独立)

$= [F_R(z)]^5 = \left[\int_{-\infty}^{z} f_R(z)\right]^5$ 其中 F_R, f_R 表示参数为 $\sigma=2$ 的 Rayleigh 分布的分布函数与概率密度

① $z < 0$ 时. $P(Z \leqslant z) = 0^5 = 0$.

② $z \geq 0$ 时. $P(Z \leq z) = \left[\int_0^z \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx\right]^5$

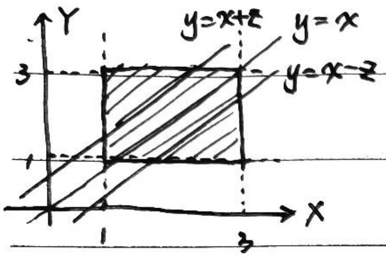
$$= \left[\int_0^z e^{-\frac{x^2}{2\sigma^2}} d\left(\frac{x^2}{2\sigma^2}\right)\right]^5$$

$$= \left(\int_0^{z^2/2\sigma^2} e^{-u} du\right)^5 = \left(1 - e^{-z^2/2\sigma^2}\right)^5 = \left(1 - e^{-z^2/8}\right)^5$$

$$\therefore P(Z \leq z) = \begin{cases} \left(1 - e^{-\frac{z^2}{8}}\right)^5, & z \geq 0 \\ 0, & 其它 \end{cases}$$

□

3-37. 解: $Z = |X - Y| \geq 0$. $\because X \in (1,3), Y \in (1,3)$ $\therefore Z \in [0, 2)$



当 $0 \leq z < 2$ 时,

$F_Z(z) = P(Z \leq z) = P(X - z \leq Y \leq X + z)$

$$\because f(x,y) = f_X(x) f_Y(y) = \begin{cases} \frac{1}{4} & 1 \leq x \leq 3, 1 \leq y \leq 3 \\ 0 & 其它 \end{cases}$$

$$\therefore F_Z(z) = \frac{1}{4}\left(4 - 2 \cdot \frac{1}{2}(2-z)^2\right) = \frac{1}{4}(4z - z^2) = z - \frac{1}{4}z^2$$

$$F_Z'(z) = 1 - \frac{1}{2}z$$

$$综上: f_Z(z) = \begin{cases} 1 - \frac{1}{2}z, & 0 \leq z < 2 \\ 0, & 其它 \end{cases}$$