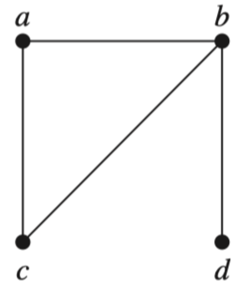
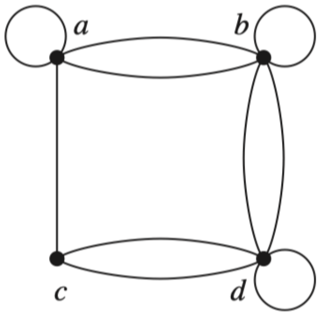
Exercise Sheet 13

Discrete Mathematics, 2020.11.6

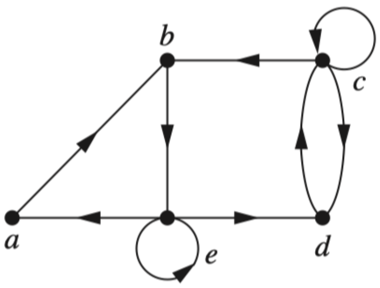
1. ([R], Page 650, Exercise 3,5,7,9) For Exercises 3–9, determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.

3. 

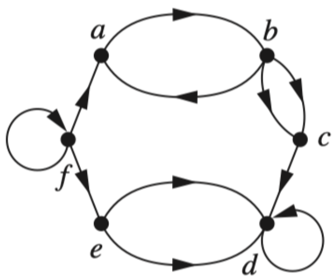
Answer:

5. 

Answer:

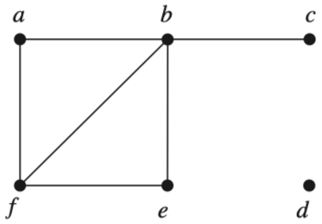
7. 

Answer:

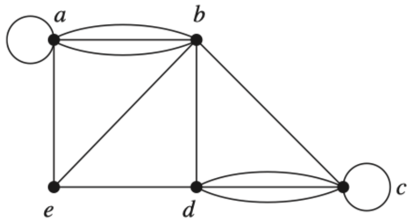
9. 

Answer:

2. ([R], Page 665, Exercise 1, 2) In Exercises 1-2 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph.

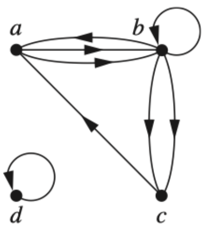
1. 

Answer:

2. 

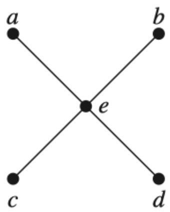
Answer:

3. ([R], Page 665, Exercise 8) In Exercises 8 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

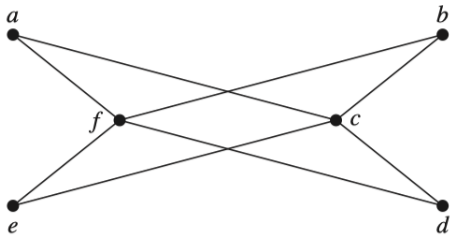


Answer:

4. ([R], Page 665, Exercise 21,24) In Exercises 21,24 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.

21. 

Answer:

24. 

Answer:

5. ([R], Page 666, Exercise 31) Suppose there is an integer k such that every man on a desert island is willing to marry exactly k of the women on the island and every woman on the island is willing to marry exactly k of the men. Also, suppose that a man is willing to marry a woman if and only if she is willing to marry him. Show that it is possible to match the men and women on the island so that everyone is matched with someone that they are willing to marry.

6. ([R], Page 666, Exercise 32) In this exercise we prove a theorem of Öystein Ore. Suppose that $G = (V, E)$ is a bipartite graph with bipartition (V_1, V_2) and that $A \subseteq V_1$. Show that the maximum number of vertices of V_1 that are the endpoints of a matching of G equals $|V_1| - \max_{A \subseteq V_1} \operatorname{def}(A)$, where $\operatorname{def}(A) = |A| - |N(A)|$. (Here, def (A) is called the deficiency of A.) [Hint: Form a larger graph by adding $\max_{A \subseteq V_1} \operatorname{def}(A)$ new vertices to V_2 and connect all of them to the vertices of V_1.]