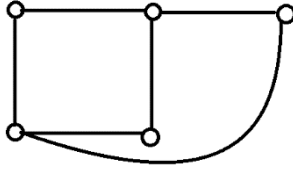# Discrete Mathematics Exercise 19
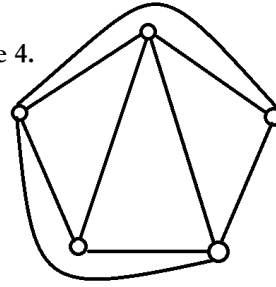
Qiu Yihang, 2020/12/09

**1.** *Solution:*

**Exercise 2.**                    **Exercise 4.**



**2.** *Proof:*

Suppose the connected planar simple graph is $G$.

Since $G$ has no simple circuits of length 4 or less, for any region $R$, $\deg(R) \geq 5$.

Therefore, $\sum_R \deg(R) \geq \sum_R 5 = 5r$.

By Euler's Formula, $r = |E| - |V| + 2$.

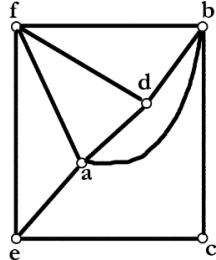Thus, $2|E| = \sum_R \deg(R) \geq 5r = 5|E| - 5|V| + 10$, **i.e.,** $3|E| \leq 5|V| - 10$.

In other words, $e \leq (5/3)v - (10/3)$.
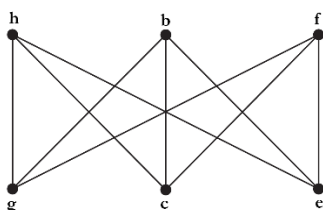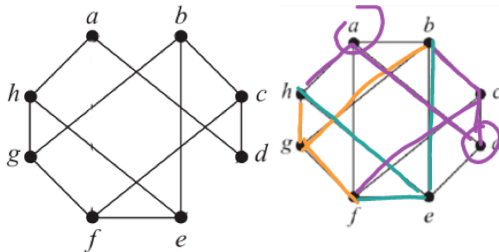
**QED**

**3.** *Solution:*

**7** is a **planar** graph.
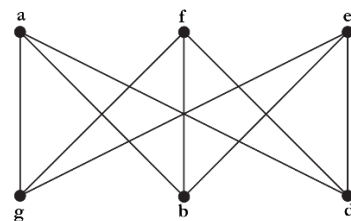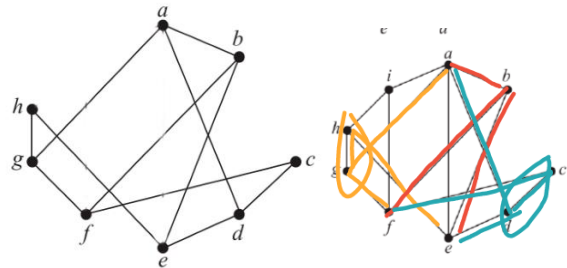
A planar representation is as follows:



**8** is a **non-planar** graph.

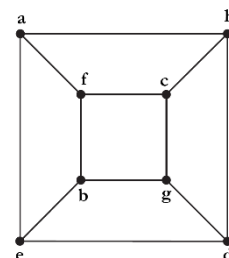A subgraph of **8** (depicted as follows) is homeomorphic to $K_{3,3}$.



**9** is a **non-planar** graph.

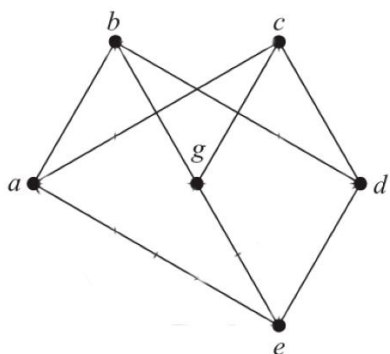A subgraph of **9** (depicted as follows) is homeomorphic to $K_{3,3}$.



**23** is a **planar** graph.

A planar representation is as follows.

**25** is a **non-planar** graph.

A subgraph of **25** (depicted as follows) is a $K_{3,3}$.



**i.e.**