Problem Set 5

May 18, 2022

Deadline: 13:00 June 2, 2022

Question 1

An image with intensities in the range $[0, 1]$ has the PDF $p_r(r)$ shown in Figure 1. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.

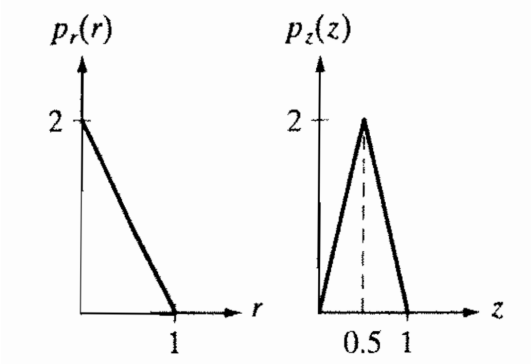

Figure 1: The original PDF $p_r(r)$ and the specified PDF $p_z(z)$.

Question 2

The correlation of a filter $w(x, y)$ of size $m \times n$ with an image $f(x, y)$ is given by the equation

$$w(x, y) \circ f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t) \tag{1}$$

in which $a = (m - 1)/2, b = (n - 1)/2$, and the convolution of $w(x, y)$ and $f(x, y)$ is given by the expression

$$w(x, y) * f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t) \tag{2}$$

(a) Suppose that you filter an image, $f(x, y)$, with a spatial filter mask, $w(x, y)$, using convolution, as defined in Equation 2, where the mask is smaller than the image in both spatial directions. Show the important property that, if the coefficients of the mask sum to zero, then the sum of all the elements in the resulting convolution array (filtered image) will be zero also (you may ignore computational inaccuracies). Also, you may assume that the border of the image has been padded with the appropriate number of zeros.

(b) Would the result to (a) be the same if the filtering is implemented using correlation, as defined in Equation 1?

Question 3

The 2-D convolution theorem is given by the expressions

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v)H(u,v) \tag{3}$$

and, conversely,

$$f(x,y)h(x,y) \Leftrightarrow F(u,v) * H(u,v) \tag{4}$$

Prove Equation 3 and Equation 4.

Question 4

Consider a 3×3 spatial mask that averages the 4 closest neighbors of a point (x, y), but excludes the point itself from the average.

(a) For the image shown in Figure 2, show your result after applying the 3×3 spatial mask to the image. Assume that the border of the image has been padded with the appropriate number of zeros.

1	0	0	0	0
1	1	0	0	0
0	1	1	0	0
0	0	1	1	1
0	1	1	0	0
1	1	0	0	0
1	0	0	0	0

Figure 2: The binary image of size 7×5.

(b) Find the equivalent filter, $H(u, v)$, in the frequency domain.

(c) Show that your result in (b) is a lowpass filter.