Problem Set 2

March 15, 2022

Deadline: 13:00 April 5, 2022

Question 1

Suppose $\tilde{x}[n]$ is a periodic sequence with period N. Then $\tilde{x}[n]$ is also periodic with period $3N$. Let $\tilde{X}[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period N, and let $\tilde{X}_3[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period $3N$.

(a) Express $\tilde{X}_3[k]$ in terms of $\tilde{X}[k]$.

(b) By explicitly calculating $\tilde{X}[k]$ and $\tilde{X}_3[k]$, verify your result in Part (a) when $\tilde{x}[n]$ is as given in Figure 1.

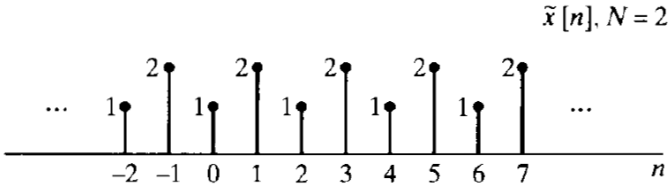

Figure 1: The periodic sequence $\tilde{x}[n]$ in Part (b).

Question 2

Consider the sequence $x[n]$ given by $x[n] = \alpha^n u[n]$. A periodic sequence $\tilde{x}[n]$ is constructed from $x[n]$ in the following way:

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n + rN]$$

(a) Determine the Fourier transform $X\left(e^{j\omega}\right)$ of $x[n]$.

(b) Determine the discrete Fourier series $\tilde{X}[k]$ of $\tilde{x}[n]$

(c) How is $\tilde{X}[k]$ related to $X\left(e^{j\omega}\right)$?