Exercise Sheet 12

Discrete Mathematics, 2020.11.3

1. We call f a choice function of S if $f : S \to \bigcup S$ and $f(X) \in X$ for any $X \in S$. Thus, the axiom of choice actually says: if $\emptyset \notin S$ then S has at least a choice function. Now, please determine whether f is a choice function of S in the following examples. (If yes, you only need to say yes. If no, briefly explain why.)

   a) $S = \{\{0\}, \{1\}, \{2\}\}$, $f = \{(\{0\}, 0), (\{1\}, 1), (\{2\}, 2)\}$

   b) $S = \{\{0\}, \{1\}, \{2\}\}$, $f = \{(0, 0), (1, 1), (2, 2)\}$

   c) $S = \emptyset$, $f = \emptyset$

   d) $S = \mathcal{P}(\mathbb{N})$, $f = \{(\{m \in \mathbb{N} \mid m \geq n\}, n) \mid n \in \mathbb{N}\}$

2. Suppose R is an equivalence relation on A. Prove that there exists an injection from $\{[a]_R \mid a \in A\}$ into A, based on the axiom of choice.