Discrete Mathematics Exercise 18

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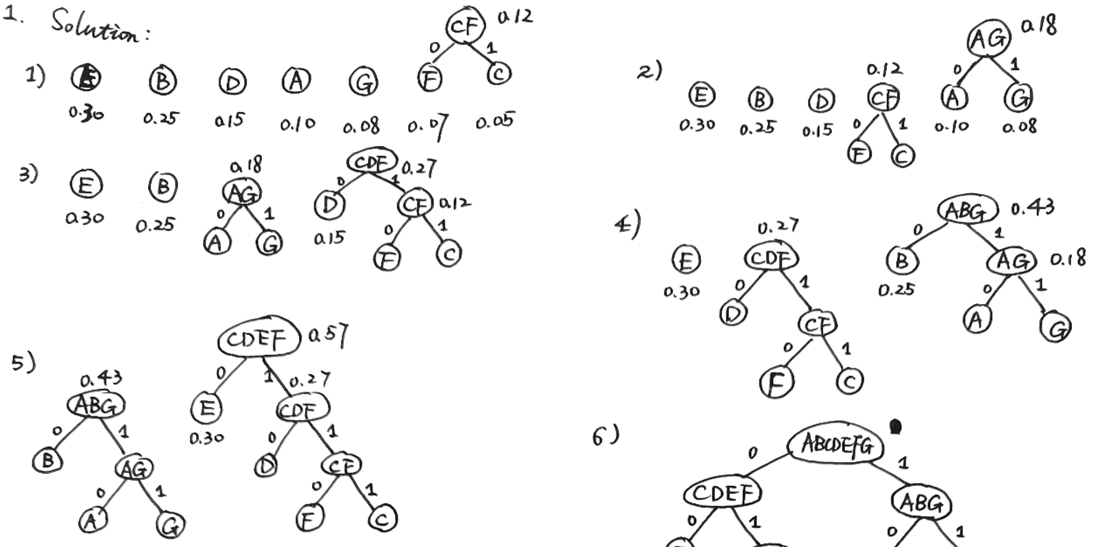
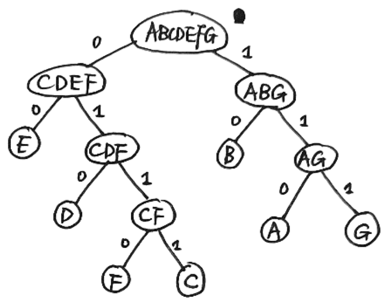
1. ***Solution:***

1. Solution:



6)



The codings of A ~ G:

A: 110
B: 10
C: 0111
D: 010
E: 00
F: 0110
G: 111

The average number of bits required to encode a symbol:

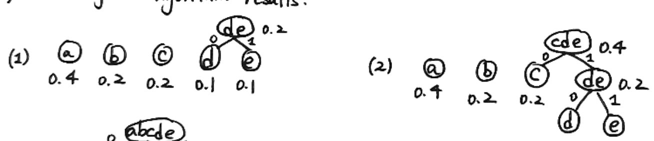
$3\times 0.10 + 2\times 0.25 + 4\times 0.05 + 3\times 0.15 + 2\times 0.30 + 4\times 0.07 + 3\times 0.08$

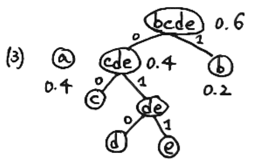
$= 2.57$ (bits).

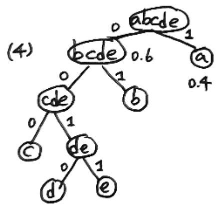
□

2. ***Solution:***

a) The first algorithm results:



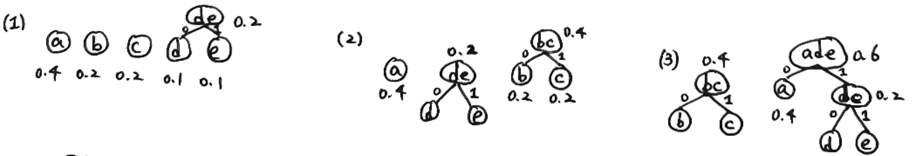


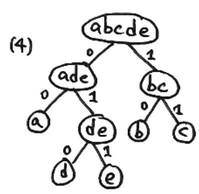


The encoding of these symbols:

"a" 1
"b" 01
"c" 000
"d" 0010
"e" 0011

The second algorithm results:





The encoding of these symbols: "a" 00 "b" 10 "c" 11 "d" 010 "e" 011

b) The average number of bits required to encode these symbols:

$0.4 \times 1 + 0.2 \times 2 + 0.2 \times 3 + 0.1 \times 4 + 0.1 \times 4 = 0.4 \times 2 + 0.2 \times 2 + 0.2 \times 2 + 0.1 \times 3 + 0.1 \times 3 = 2.2$ (bits) $\square$

3. ***Solution:***

Exercise **6** has no Euler Circuit but has Euler Paths. (since only $\deg(b)$ and $\deg(c)$ are odd).

An Euler Path: $b, a, d, e, f, d, g, i, h, a, i, d, c, i, b, c$.

Exercise **8** has Euler Circuits. (since the degrees of all vertices in it are even.)

An Euler Circuit: $a, b, d, c, b, g, h, c, j, e, d, i, j, o, n, i, h, m, n, l, m, f, k, l, g, f, a$.

4. ***Solution:***

Exercise 36 has a Hamilton Circuit. A Hamilton Circuit: $a, b, c, e, f, i, h, g, d, a$.

5. ***Solution:***

Exercise 33 has no Hamilton Paths.

It's obvious that for any vertex u in a Hamilton Path, excluding the starting point and the termination, $\deg(u) \geq 2$. Otherwise, there do not exist two different edges incident with u, **i.e.** the edge into u and out of u is the same edge, contradicting to Hamilton Path's definition.

Thus, if $\deg(u) = 1$, u must be the starting point or termination of the Hamilton Path. Therefore, if a graph has a Hamilton Path, there exists at most two vertices whose degree is 1.

In Exercise 33, $\deg(e) = \deg(f) = \deg(g) = 1$. Thus, it has no Hamilton Paths.

6. ***Proof:***

Proof by Contradiction.

Suppose $G = (V, E)$ is the bipartite graph with a bipartition $\{V_1, V_2\}$. Let $n = |V|$. Then n is an odd number.

Assume there exists a Hamilton Circuit: $x_1, x_2, \ldots \ldots, x_n, x_1$.

Let $x_1 \in V_1$. By the definition of bipartite graphs, we know every edge is incident with exactly one vertex in V_1 and exactly one vertex in V_2. Thus, $x_2 \in V_2, x_3 \in V_1, \ldots \ldots, x_n \in V_1, x_1 \in V_2$.

$V_1 \cap V_2 = \{x_1\}$. **Contradiction.** (since $\{V_1, V_2\}$ is a bipartition.)

Thus, a bipartite graph with an odd number of vertices does not have a Hamilton Circuit.

QED