Problem Set 4

April 20, 2022

Deadline: 13:00 May 11, 2022

Question 1

An LTI system is applied to a superposition of windowed sinusoids $x[n]$, shown below. Plotted in Figure 1 are the magnitude of $X(e^{j\omega})$, and the magnitude response, phase response, and group delay of the system.

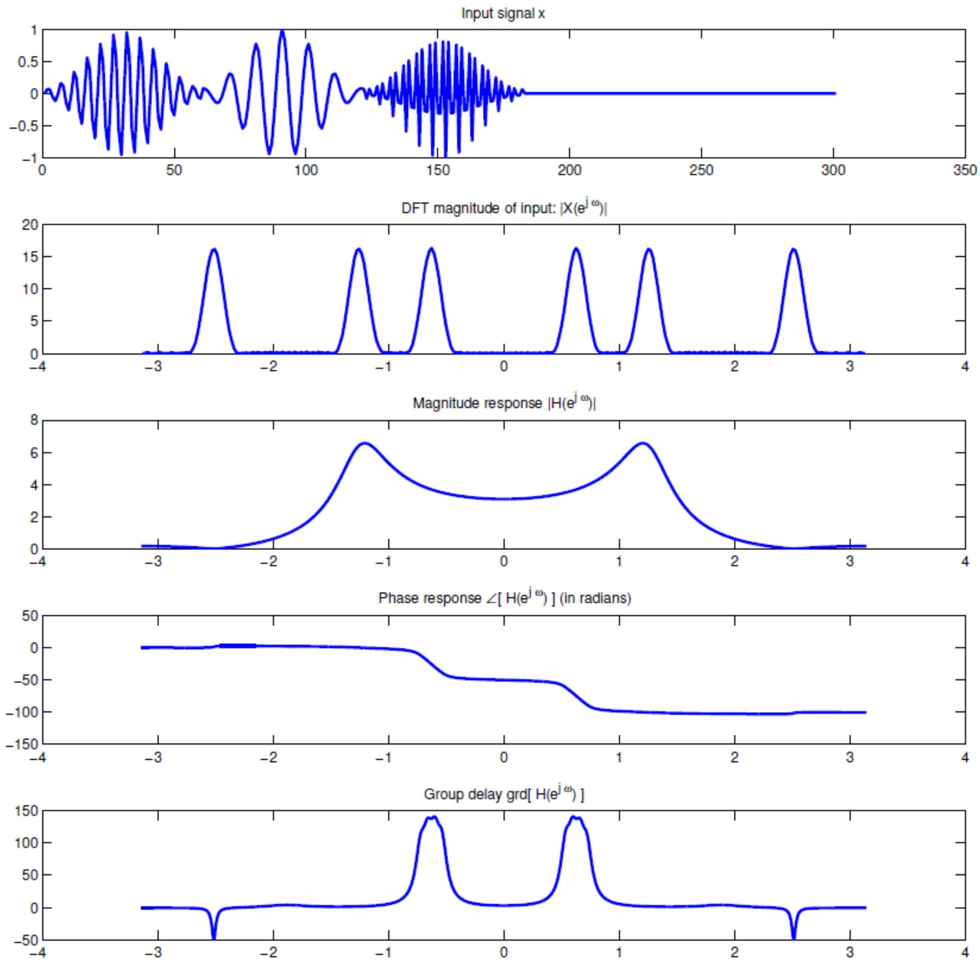

Figure 1

Sketch the output y[n]. Explain your sketch in terms of the magnitude response, phase response, and group delay.

Question 2

A discrete LSI system has impulse response function:

$$H(z) = \frac{1 - z^{-1}}{1 - \frac{5}{4}z^{-1} - \frac{3}{2}z^{-2}},$$

(a) Determine the difference equation of the system.

(b) When the system has input $x[n] = \cos(\omega n)$, the output is $y[n] = A\cos(\omega n + \phi)$. Determine the amplitude A and phase $(\omega n + \phi)$, when $\omega = \frac{\pi}{2}$.
Hint: Here recommend work graphically for **(b)**, by drawing the poles and zeros.

Question 3

Determine the expressions for the group delay of each of the LTI systems whose frequency responses are given below.

(a) $H_a(e^{j\omega}) = a + be^{-j\omega}$

(b) $H_b(e^{j\omega}) = \frac{1}{1+ce^{-j\omega}}$

(c) $H_c(e^{j\omega}) = \frac{a+be^{-j\omega}}{1+ce^{-j\omega}}, |c| < 1$

(d) $H_d(e^{j\omega}) = \frac{1}{(1+ce^{-j\omega})(1+de^{-j\omega})}, |c| < 1, |d| < 1$

Question 4

A stable system with system function $H(z)$ has the pole-diagram shown in Figure 2. It can be represented as the cascade of a stable minimum-phase system $H_{min}(z)$ and a stable all-pass system $H_{ap}(z)$.

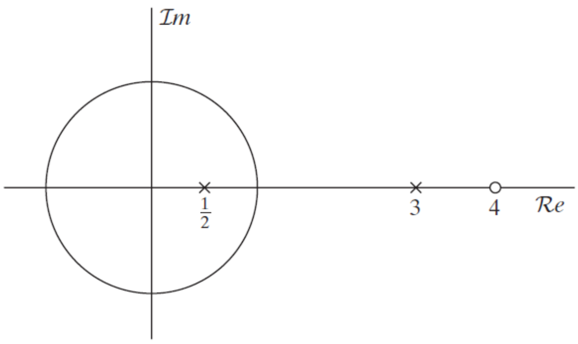

Figure 2

Determine a choice for $H_{min}(z)$ and $H_{ap}(z)$ (up to a scale factor) and draw their corresponding pole-zero plots. Indicate whether your decomposition is unique up to a scale factor.

Question 5

Consider using a first-order filter to realize all-pass system function:

$$|H(e^{j\omega})| = 1 = \alpha \cdot \frac{|v_2|}{|v_1|}$$

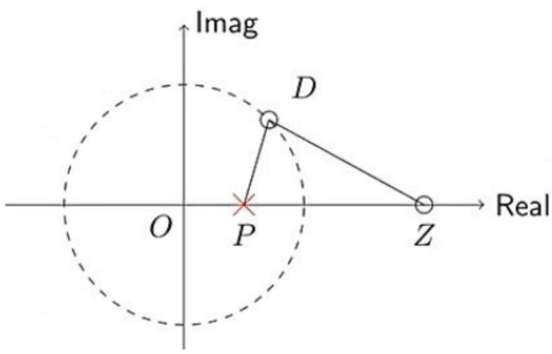

Figure 3

From Figure 3, we can see the geometric meaning of the equation above is:

$$\frac{|DZ|}{|DP|} = \frac{1}{\alpha}$$

$|DZ|$ is the distance from dynamic point D to zero point Z and $|DP|$ is the distance from D to pole point P. Since the system is all-pass, the zeros and poles will not be on the unit circle. Prove that when D moves around the unit circle with frequency ω, Z and P always satisfy $|OZ| \cdot |OP| = 1$.