

## Discrete Mathematics Exercise 4

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1. a)

**Solution:**

We figure out that under this partial assignment,  $\llbracket p_1 \vee p_5 \vee \neg p_2 \rrbracket_J = \mathbf{F}$ .

Given that  $p_2 \mapsto \mathbf{T}$  is the result of the Unit Propagation of  $p_3 \mapsto \mathbf{F}$ ,  $p_5 \mapsto \mathbf{F}$  is a pick itself and  $p_1 \mapsto \mathbf{F}$  is the result of the Unit Propagation of  $p_5 \mapsto \mathbf{F}$ , the conflict clause generated should be  $p_3 \vee p_5$ .

b)

**Solution:**

According to the process of CDCL, we can easily know that  $p_5$  and  $p_1$  will be unpicked.

c)

**Solution:**

Given that  $p_3 \mapsto \mathbf{F}$ , from  $p_3 \vee p_5$  we know that  $p_5 \mapsto \mathbf{T}$ . Then from  $p_6 \vee \neg p_5$  we know  $p_6 \mapsto \mathbf{T}$ .

Thus, the next unit propagation is  $[p_5 \mapsto \mathbf{T}, p_6 \mapsto \mathbf{T}]$ .

2. a)

**Solution:**

Since  $p_2 \mapsto \mathbf{F}$  and  $p_4 \mapsto \mathbf{F}$ , from  $p_2 \vee p_4 \vee \neg p_9$  we know that  $p_9 \mapsto \mathbf{F}$ . Then from  $\neg p_6 \vee p_9$  we know  $p_6 \mapsto \mathbf{F}$ .

Similarly, from  $p_6 \vee p_{10}$  we know  $p_{10} \mapsto \mathbf{T}$ .

Thus, the result of unit propagation is  $[p_9 \mapsto \mathbf{F}, p_6 \mapsto \mathbf{F}, p_{10} \mapsto \mathbf{T}]$ .

b)

**Solution:**

Since  $p_1 \mapsto \mathbf{F}, p_7 \mapsto \mathbf{T}$  and  $p_4 \mapsto \mathbf{F}$ , from  $p_1 \vee p_4 \vee \neg p_5 \vee \neg p_7$  we know that  $p_5 \mapsto \mathbf{F}$ .

Thus, the result of unit propagation is  $[p_5 \mapsto \mathbf{F}]$ .

c)

**Solution:**

Since  $\llbracket p_1 \vee p_5 \rrbracket_J = \mathbf{F}$ ,  $p_1 \mapsto \mathbf{F}$  is a pick itself and  $p_5 \mapsto \mathbf{F}$  is the result of  $p_4 \mapsto \mathbf{F}$ ,  $p_1 \mapsto \mathbf{F}$  and  $p_7 \mapsto \mathbf{T}$ , the conflict clause generated should be  $p_1 \vee p_4 \vee \neg p_7$ .

d)

**Solution:**

According to the process of CDCL, we can easily know that  $p_5$  and  $p_1$  will be unpicked.

**e)**

**Solution:**

Since  $p_7 \mapsto \mathbf{T}$  and  $p_4 \mapsto \mathbf{F}$ , from  $p_1 \vee p_4 \vee \neg p_7$ , we know that  $p_1 \mapsto \mathbf{T}$ .

Thus, the result of unit propagation is  $[p_1 \mapsto \mathbf{T}]$ .

**3. a) Solution:**  $\exists x(C(x) \wedge D(x) \wedge F(x))$

**b) Solution:**  $\forall x(C(x) \vee D(x) \vee F(x))$

**c) Solution:**  $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

**d) Solution:**  $\forall x (\neg(C(x) \wedge D(x) \wedge F(x)))$

**e) Solution:**  $(\exists x(C(x))) \wedge (\exists x(D(x))) \wedge (\exists x(F(x)))$