Problem Set 3

March 29, 2022

Deadline: 13:00 April 19, 2022

Question 1

Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follows:

$$\begin{aligned} x[n] &= \cos(\frac{\pi n}{2}), \quad && n = 0, 1, 2, 3. \\ h[n] &= 2^n, && n = 0, 1, 2, 3. \end{aligned}$$

(a) Calculate the four-point DFT $X[k]$.

(b) Calculate the four-point DFT $H[k]$.

(c) Calculate $y[n] = x[n] \textcircled{4} h[n]$ by doing the circular convolution directly.

(d) Calculate $y[n]$ of part **(c)** by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

Question 2

Consider the DFT defined on a centered interval with an odd number of transform points.

(a) Assuming N is even, verify the orthogonality property

$$\sum_{n=-\frac{N}{2}}^{\frac{N}{2}} W_{N+1}^{-n(m-k)} = (N+1) \sum_{r=-\infty}^{\infty} \delta[m - k - r(N+1)].$$

(b) Given this orthogonality property and the forward DFT,

$$F_k = \frac{1}{N+1} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} f_n W_{N+1}^{-nk},$$

for $k = -N/2 : N/2$, derive the corresponding inverse DFT.

(c) Verify that the sample points for this DFT, $x_n = nA/(N+1)$, do not include either endpoint of the interval $[-A/2, A/2]$. Show that $x_{\pm\frac{N}{2}}$ approach $\pm A/2$ as N increases.

Question 3

The rectangular wave on the interval $[-1/2, 1/2]$ is defined by

$$f(x) = \begin{cases} -1 & \text{for } -1/2 \leq x < 0, \\ 0 & \text{for } x = 0, \\ 1 & \text{for } 0 < x \leq 1/2. \end{cases}$$

Note that f is real and odd, and therefore its Fourier coefficients are odd and imaginary. Assume that the function is sampled at the points $x_n = n/8$, where $n = -3 : 4$, to produce the input sequence

$$\bar{f}_n = \{-1, -1, -1, 0, 1, 1, 1, 1\}.$$

Compute the eight-point DFT of $\bar{f}_n$. Is the DFT odd and imaginary? How do you explain the error? How should the input sequence be defined? Verify that when the input sequence is correctly defined, the DFT is odd and imaginary.

Question 4

Consider two finite-length sequences $x[n]$ and $h[n]$ for which $x[n] = 0$ outside the interval $0 \leq n \leq 49$ and $h[n] = 0$ outside the interval $0 \leq n \leq 9$.

(a) What is the maximum possible number of nonzero values in the linear convolution of $x[n]$ and $h[n]$?

(b) The 50-point circular convolution of $x[n]$ and $h[n]$ is

$$x[n] \, 50 \, h[n] = 10. \quad 0 \leq n \leq 49.$$

The first 5 points of the linear convolution of $x[n]$ and $h[n]$ are

$$x[n] * h[n] = 5, \quad 0 \leq n \leq 4.$$

Determine as many points as possible of the linear convolution of $x[n] * h[n]$.