Exercise Sheet 16
Discrete Mathematics, 2020.11.24

1. Given a rooted tree G, define relation $R_1 = \{(u,v) \mid u \text{ is } v\text{'s ancestor in } G\}$, and relation $R_2 = \{(u,v) \mid u\text{'s level is (strictly) smaller than } v\text{'s level in } G\}$. Prove that $R_1 \subseteq R_2$.

2. Given a rooted tree G, u, v, w are vertices in G and u, v are ancestors of w. Prove that either (i) u is v's ancestor, (ii) u is v's descendent, or (iii) $u = v$.

3. Prove that in a rooted tree G, if v is a descendant of u's, then the unique simple path from v to u only passes through u's descendants and u.

4. Suppose G is a rooted tree such that every internal vertex has at least two children. Prove that G has more leaves than internal vertices.

5. ([R], Page 797, Exercise 46) Use Exercise 43 to prove that if G is a connected, simple graph with n vertices and G does not contain a simple path of length k, then it contains at most $(k-1)n$ edges.