

Exercise Sheet 11

Discrete Mathematics, 2020.10.27

1. ([R], Page 172, Exercise 2(a)(b)(d)(e)) Determine whether each of these sets is countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.
 - a) the integers greater than 10
 - b) the odd negative integers
 - d) the real numbers between 0 and 2
 - e) the set $A \times \mathbb{Z}^+$ where $A = \{2, 3\}$
2. Prove that $[0, 1) \times [0, 1) \approx [0, 1)$.
3. Prove that for any sets A , B and C , $(A \rightarrow (B \rightarrow C)) \approx (B \rightarrow (A \rightarrow C))$.
4. Prove that for any sets A , B and C , $(A \rightarrow B \times C) \approx (A \rightarrow B) \times (A \rightarrow C)$.
5. Prove that the set of all binary relations on \mathbb{R} is equinumerous to the set of all functions from \mathbb{R} into \mathbb{R} .
6. **(Optional homework. 1 additional point.)** A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is monotonically increasing if for any $x < y$, $f(x) < f(y)$. Prove that the set of monotonically increasing functions from \mathbb{R} into \mathbb{R} is equinumerous to \mathbb{R} . (选做题可以不做)
7. **(Optional homework. 2 additional points.)** We know that $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x - y \in \mathbb{Q}\}$ is an equivalence relation on \mathbb{R} . Suppose $P = \{[a]_R \mid a \in \mathbb{R}\}$. Prove that $P \approx \mathbb{R}$. (选做题可以不做)