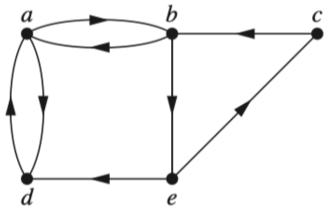
Exercise Sheet 14

Discrete Mathematics, 2020.11.10

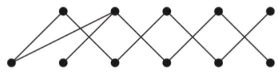
1. ([R], Page 689, Exercise 2(a)(b)(d)) Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths? （只需给出答案无需证明）
   $a)a, b, e, c, b$
   $b)a, d, a, d, a$
   $d)a, b, e, c, b, d, a$



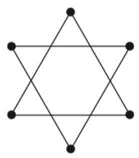
2. ([R], Page 689, 6) How many connected components does each of the graphs in Exercises 3-5 have? For each graph find each of its connected components. （只需给出答案无需证明）

3.

4.

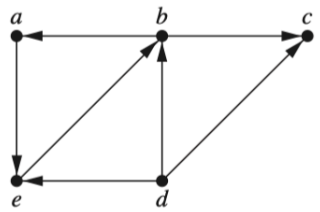


5.

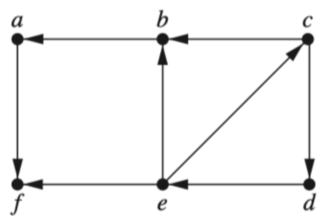


3. ([R], Page 690, 14(a)(b)(c)) Find the strongly connected components of each of these graphs. （只需给出答案无需证明）

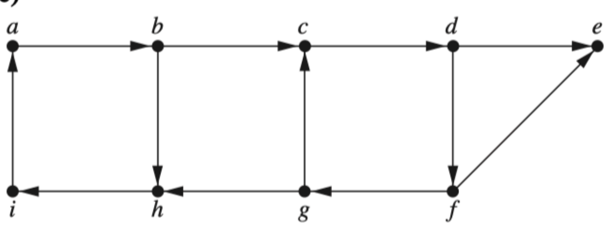
a)



b)



c)



4. ([R], Page 692, Exercise 63) Show that a simple graph G is bipartite if and only if it has no circuits with an odd number of edges.

5. Suppose $G = (V, E)$ is a subgraph of $G' = (V, E \cup \{e_0\})$ ($e_0 \notin E$ is an additional edge) and both G and G' are undirected graphs. Prove: if e_0 connects u and v but u is not connected to v in G, then $[u]_{\mathrm{conn}(G')} = [v]_{\mathrm{conn}(G')} = [u]_{\mathrm{conn}(G)} \cup [v]_{\mathrm{conn}(G)}$.

6. **(Optional Homework, 2 additional points)**

   Matriods. A finite matroid is an ordered pair $(E, \mathcal{I})$, where E is a finite set and $\mathcal{I} \subseteq 2^E$ is a collection of subsets of E such that

   - $\emptyset \in \mathcal{I}$, and
   - for any sets $A \subseteq B \subseteq E$, if $B \in \mathcal{I}$ then $A \in \mathcal{I}$, and
   - for any sets $A, B \in \mathcal{I}$, if $|A| < |B|$ then there exists $x \in B \backslash A$ such that $A \cup \{x\} \in \mathcal{I}$.

   Disjoint Paths. Let $G = (V, E)$ be a(n undirected) simple graph. Given a path $\rho = x_0, e_1, \ldots, x_{n-1}, e_n, x_n$ in G, we denote by $\mathit{Edges}(\rho)$ the set of all edges appearing in ρ, i.e., $\mathit{Edges}(\rho) := \{e_0, \ldots, e_n\}$. We say that two paths ρ, ρ' in G are disjoint if $\mathit{Edges}(\rho) \cap \mathit{Edges}(\rho') = \emptyset$.

   Independent Sets of Vertices. Let $G = (V, E)$ be a(n undirected) connected simple graph such that $|V| \geq 2$. We assume a designated vertex v^* called the source vertex. A set $U \subseteq V \backslash \{v^*\}$ of vertices is called independent if $U = \{u_1, \ldots, u_k\}$ (k can be zero) and there are k pairwise-disjoint paths $\rho_1, \ldots, \rho_k$ such that each path ρ_i connects v^* and u_i (as endpoints).

   Define $\mathcal{I}$ to be the set of all independent sets of vertices. Show that $(V, \mathcal{I})$ is a finite matroid. **(选做题可以不做)**