Exercise Sheet 17

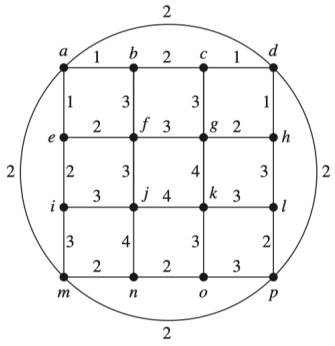
Discrete Mathematics, 2020.12.2

1. Given a connected, undirected graph $G = (V, E)$ and $u, v, w \in V$. Let T be the spanning tree generated by a DFS process of G. Prove that if u is v's ancestor in T and there exists an edge from v to w in G, then either (i) w is u's ancestor in T, (ii) w is u's descendent in T, or (iii) $w = u$.

2. Given a connected, undirected graph $G = (V, E)$ and $u, v \in V$. Let T be the spanning tree generated by a DFS process of G. Prove that if u is v's ancestor in T then the first time visiting u happens before the first time visiting v in the DFS process.

3. Given a connected, undirected graph $G = (V, E)$ and $u, v, w \in V$. In a DFS process of G, suppose that there are two distinct forward moves, one from u to v and the other from u to w. Prove that any simple path in G from v to w either passes through u or passes through at least one u's ancestor in the tree generated by this DFS process.

4. ([R], Page 802, Exercise 4) In Exercises 4 use Prim's algorithm to find a minimum spanning tree for the given weighted graph. You can pick any vertex as your starting point and you should describe: on every step, which edge is added. (Proof is not needed.)



5. ([R], Page 802, Exercise 8) Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph in Exercise 4. You should describe: on every step, which edge is added. (Proof is not needed.)

6. ([R], Page 803, Exercise 19) Show that there is a unique minimum spanning tree in a connected weighted graph if the weights of the edges are all different.