

1. Simulation – Parameter Tuning

<model 1>

$$Y_1 = \sin(\beta_1^T X) + \sin(\beta_2^T X) + 0.5\varepsilon$$

n=300, p=40

Case (1) : true beta 0 개수 78개	Sparse DR																		
Case (1) : $\beta_1 = (1, 0, \dots, 0)^T$ $\beta_2 = (0, 1, 0, \dots, 0)^T$ $\lambda_1 = 0.1, 0.05$ $\lambda_2 = 0.05$ slice 개수: 4개	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.7092513</td><td>0.12164155</td><td>2.602801</td><td>1.965467</td></tr><tr><td>sparse dr</td><td>77.16</td><td>0.8865939</td><td>0.06533786</td><td>2.749936</td><td>1.929122</td></tr></table>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.7092513	0.12164155	2.602801	1.965467	sparse dr	77.16	0.8865939	0.06533786	2.749936	1.929122
method	p	corr1	corr2	mse1	mse2														
dr	0.00	0.7092513	0.12164155	2.602801	1.965467														
sparse dr	77.16	0.8865939	0.06533786	2.749936	1.929122														
Case (1) : $\beta_1 = (1, 0, \dots, 0)^T$ $\beta_2 = (0, 1, 0, \dots, 0)^T$ $\lambda_1 = 0.1, 0.01$ $\lambda_2 = 0.01$ slice 개수: 10개	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.693193</td><td>0.1045944</td><td>2.838022</td><td>1.963565</td></tr><tr><td>sparse dr</td><td>62.33</td><td>0.892805</td><td>0.4696206</td><td>2.952623</td><td>2.441872</td></tr></table>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.693193	0.1045944	2.838022	1.963565	sparse dr	62.33	0.892805	0.4696206	2.952623	2.441872
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Case (1) –처음 결과 : $\beta_1 = (1, 0, \dots, 0)^T$ $\beta_2 = (0, 1, 0, \dots, 0)^T$ $\lambda_1 = 0.01, 0.01$ $\lambda_2 = 0.01$ slice 개수: 4개	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.7092513</td><td>0.12164155</td><td>2.602801</td><td>1.965467</td></tr><tr><td>sparse dr</td><td>59.94</td><td>0.7548557</td><td>0.07432311</td><td>2.724729</td><td>2.125324</td></tr></table>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.7092513	0.12164155	2.602801	1.965467	sparse dr	59.94	0.7548557	0.07432311	2.724729	2.125324
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Case (2) : true beta 0 개수 60개	Sparse DR																		
Case (2) : $\beta_1=(1,1,1,1,1,0.1,0.1,0.1,0.1,0.1,0,0...0,0)$ $\beta_2=(0,0...0,0,0.1,0.1,0.1,0.1,1,1,1,1,1)$ $\lambda_1 = 0.05, 0.01$ $\lambda_2 = 0.01$ slice: 10개	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.1373907</td><td>0.1228866</td><td>2.074601</td><td>1.904770</td></tr><tr><td>sparse dr</td><td>62.66</td><td>0.1829674</td><td>0.1011862</td><td>2.230346</td><td>2.137052</td></tr></table>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.1373907	0.1228866	2.074601	1.904770	sparse dr	62.66	0.1829674	0.1011862	2.230346	2.137052
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<p>Case (2)</p> <p>$\beta_1=(1,1,1,1,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0)$</p> <p>$\beta_2=(0,0,0,0,0,0,1,0,1,0,1,0,1,0,1,1,1,1,1,1)$</p> <p>$\lambda_1=0.05, 0.05$</p> <p>$\lambda_2=0.01$</p> <p>slice: 4개</p>	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0</td><td>0.1416286</td><td>0.1105666</td><td>2.086301</td><td>1.882642</td></tr><tr><td>sparse dr</td><td>78</td><td>0.1094201</td><td>0.1119920</td><td>2.069400</td><td>1.972753</td></tr></table> <p>lambda를 키웠더니 beta 0의 개수가 너무 많게 나옴</p>	method	p	corr1	corr2	mse1	mse2	dr	0	0.1416286	0.1105666	2.086301	1.882642	sparse dr	78	0.1094201	0.1119920	2.069400	1.972753
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<p>Case (2)—가장 처음 결과</p> <p>$\beta_1=(1,1,1,1,1,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0)$</p> <p>$\beta_2=(0,0,0,0,0,0,1,0,1,0,1,0,1,0,1,1,1,1,1,1)$</p> <p>$\lambda_1=0.01, 0.01$</p> <p>$\lambda_2=0.01$</p> <p>slice: 4개</p>	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.1395863</td><td>0.1152813</td><td>2.099900</td><td>1.933958</td></tr><tr><td>sparse dr</td><td>55.27</td><td>0.1792127</td><td>0.1335033</td><td>2.245751</td><td>2.133104</td></tr></table>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.1395863	0.1152813	2.099900	1.933958	sparse dr	55.27	0.1792127	0.1335033	2.245751	2.133104
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Case (3) : true beta 0 개수 60개	Sparse DR																		
Case (3) : $\beta_1=(1,1,1,1,1,1,1,1,1,1,1,0,0...0,0)$ $\beta_2=(0,0...0,0,0,1,1,1,1,1,1,1,1,1,1)$ $\lambda_1 = 0.05, 0.05$ $\lambda_2 = 0.05$ slice: 10개	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.1543846</td><td>0.1349226</td><td>2.146090</td><td>1.944538</td></tr><tr><td>sparse dr</td><td>77.96</td><td>0.1269302</td><td>0.1231912</td><td>2.113108</td><td>1.934631</td></tr></table> <p>lambda를 늘렸더니 beta 0의 개수가 너무 많게 나옴</p>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.1543846	0.1349226	2.146090	1.944538	sparse dr	77.96	0.1269302	0.1231912	2.113108	1.934631
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<model 2>

$$Y_2 = \frac{\beta_1^T X}{0.5 + (2\beta_2^T X + 0.3)^2} + 0.3\varepsilon, n=300, p=50$$

Case(1): : true beta 0 개수 98개	Sparse DR																		
<div>Case (1)</div> <div>: $\beta_1 = (1, 0, \dots, 0)^T, \beta_2 = (0, 1, 0, \dots, 0)^T$</div> <div>$\lambda_1 = 0.1, 0.1$</div> <div>$\lambda_2 = 0.1$</div> <div>slice: 10개</div>	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0</td><td>0.8344419</td><td>0.1473274</td><td>0.3430334</td><td>2.111377</td></tr><tr><td>sparse dr</td><td>98</td><td>1.0000000</td><td>0.5477459</td><td>0.0000000</td><td>1.223341</td></tr></table>	method	p	corr1	corr2	mse1	mse2	dr	0	0.8344419	0.1473274	0.3430334	2.111377	sparse dr	98	1.0000000	0.5477459	0.0000000	1.223341
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dr	0	0.8344419	0.1473274	0.3430334	2.111377														
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<div>Case (1) – 가장 처음 결과</div> <div>: $\beta_1 = (1, 0, \dots, 0)^T, \beta_2 = (0, 1, 0, \dots, 0)^T$</div> <div>$\lambda_1 = 0.01, 0.01$</div> <div>$\lambda_2 = 0.01$</div> <div>slice: 4개</div>	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.8219249</td><td>0.2578787</td><td>0.36904806</td><td>1.966568</td></tr><tr><td>sparse dr</td><td>79.11</td><td>0.9833684</td><td>0.4017083</td><td>0.03622688</td><td>2.077622</td></tr></table>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.8219249	0.2578787	0.36904806	1.966568	sparse dr	79.11	0.9833684	0.4017083	0.03622688	2.077622
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Case(2) : : true beta 0 개수 80개	Sparse DR																		
Case (2) $\beta_1=(1,1,1,1,1,0,1,0,1,0,1,0,1,0,1,0,0...0,0)$ $\beta_2=(0,0...0,0,0,1,0,1,0,1,0,1,0,1,1,1,1,1,1)$ $\lambda_1=0.05, 0.05$ $\lambda_2=0.01$ slice: 10개	<table><thead><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr></thead><tbody><tr><td>dr</td><td>0.00</td><td>0.7676112</td><td>0.1834386</td><td>3.420411</td><td>2.004947</td></tr><tr><td>sparse dr</td><td>97.84</td><td>0.5064309</td><td>0.1217545</td><td>2.915175</td><td>2.048526</td></tr></tbody></table> <p>lambda를 늘렸더니 beta 0의 개수가 너무 많게 나옴</p>	method	p	corr1	corr2	mse1	mse2	dr	0.00	0.7676112	0.1834386	3.420411	2.004947	sparse dr	97.84	0.5064309	0.1217545	2.915175	2.048526
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dr	0.00	0.7676112	0.1834386	3.420411	2.004947														
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$\lambda_2 = 0.01$ slice: 4개	
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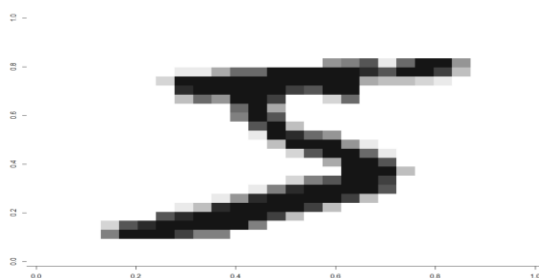
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dr	0.00	0.4743199	0.1638061	2.259702	2.030333																			
sparse dr	81.02	0.2930573	0.1406950	2.327612	2.197306																			
Case (3) – 가장 처음 결과 $\beta_1=(1,1,1,1,...,1,1,1,0,0...0,0)$ $\beta_2=(0,0...0,0,0,1,1,1,1,...,1,1)$ $\lambda_1=0.01, 0.01$ $\lambda_2=0.01$ slice: 4개	<table><tr><th>method</th><th>p</th><th>corr1</th><th>corr2</th><th>mse1</th><th>mse2</th></tr><tr><td>dr</td><td>0.00</td><td>0.3479419</td><td>0.1719531</td><td>2.255180</td><td>2.009467</td></tr><tr><td>sparse dr</td><td>73.48</td><td>0.3639681</td><td>0.1779225</td><td>2.360421</td><td>2.225217</td></tr></table>						method	p	corr1	corr2	mse1	mse2	dr	0.00	0.3479419	0.1719531	2.255180	2.009467	sparse dr	73.48	0.3639681	0.1779225	2.360421	2.225217
method	p	corr1	corr2	mse1	mse2																			
dr	0.00	0.3479419	0.1719531	2.255180	2.009467																			
sparse dr	73.48	0.3639681	0.1779225	2.360421	2.225217																			

2. Application

Data1 : mnist data

<https://github.com/jlmlville/mnist>

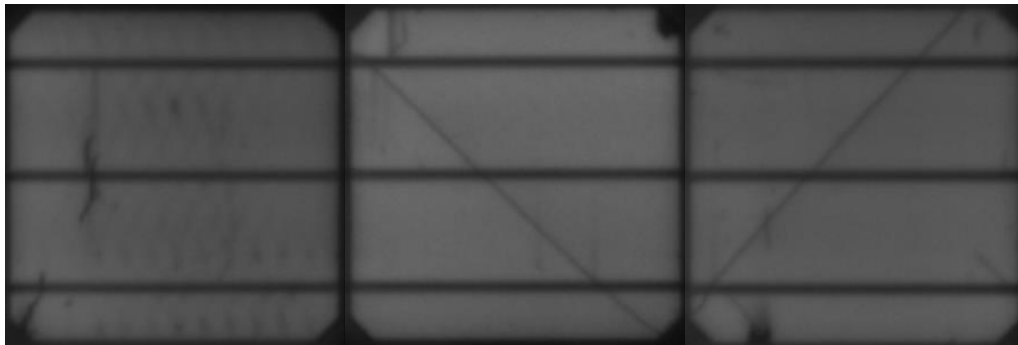
grayscale의 28*28 data (60000개 – training data, 10000개 – test data)



이미지 데이터를 불러와 70000*784 matrix로 변환하여 진행

Data2: 태양 전지 결함 데이터 – 전기 발광 이미지 (Visual Identification of Defective Solar Cells in Electroluminescence Imagery)

<https://github.com/zae-bayern/elpv-dataset> : 300*300, n=2624



cell0001.png

cell0002.png

cell0003.png

labels.csv

attribute	Description
Image url	이미지 저장 이름
Defect probability	결함 확률(0, 0.33333, 0.66666, 1) 1로 가까워질수록 결함이 있는 태양 전지이다.
Solar module type	태양 전지의 종류(mono/poly) Mono: 단일 실리콘 결정으로 만들어진 검은색 태양전지를 가지고 있으며, 일반적으로 효율이 더 높다. 가격이 비싸다. Poly: 여러 개의 실리콘 결정으로 만들어지며 파란색 셀을 가지고 있다. 효율은 다소 떨어지지만 가격이 저렴하다.