<GEP formulation>

Table 1: The GEP formulation of some SDR methods

Method	A	B
SIR	$\mathrm{cov}[E\{\mathbf{x} - E(\mathbf{x})\} \mid y]$	$\Sigma_{\mathbf{x}}$
PFC	Σ_{fit}	$\Sigma_{\mathbf{x}}$
SAVE	$\Sigma_{\mathbf{x}}^{1/2} E\left[\{I - \mathrm{cov}(\mathbf{z} \mid y)\}^2\right] \Sigma_{\mathbf{x}}^{1/2}$, where $\mathbf{z} = \Sigma_{\mathbf{x}}^{-1/2}\{\mathbf{x} - E(\mathbf{x})\}$	$\Sigma_{\mathbf{x}}$
PHD(y-based)	$\Sigma_{\mathbf{x}}^{1/2}\Sigma_{y\mathbf{zz}}\Sigma_{y\mathbf{zz}}\Sigma_{\mathbf{x}}^{1/2}$, where $\Sigma_{y\mathbf{zz}} = E\left[\{y - E(y)\}\mathbf{zz}^{\top}\right]$	$\Sigma_{\mathbf{x}}$
PHD(r-basied)	$\Sigma_{\mathbf{x}}^{1/2}\Sigma_{\mathbf{rzz}}\Sigma_{\mathbf{rzz}}\Sigma_{\mathbf{x}}^{1/2}$, where $\Sigma_{\mathbf{rzz}} = E\left[\{y - E(y) - E(y\mathbf{z}^{\intercal})\mathbf{z}\}\mathbf{zz}^{\intercal}\right]$	$\Sigma_{\mathbf{x}}$
DR	$\Sigma_{\mathbf{x}}^{1/2}\{2E\left[E^2(\mathbf{zz}^{\intercal} \mid y)\right] + 2E^2\left[E(\mathbf{z} \mid y)E(\mathbf{z}^{\intercal} \mid y)\right]$	
	$+2E[E(\mathbf{z} \mid y)E(\mathbf{z} \mid y)]E\left[E(\mathbf{z} \mid y)E(\mathbf{z}^{\intercal} \mid y)\right] - 2\mathbf{I}_p\}\Sigma_{\mathbf{x}}^{1/2}$	$\Sigma_{\mathbf{x}}$

<simulation 결과 1>

y = sign(b1'X)*log(|b2'X+5|)+0.2*e

p=20

- Case (i) : true beta의 0개수: 32

(i) $\beta_1 = (1, 1, 1, 1, 0, \ldots, 0)^{\intercal}$ and $\beta_2 = (0, \ldots, 0, 1, 1, 1, 1)^{\intercal}$;

method	p	corr1	corr2	mse1	mse2
dr	0.000	0.7698654	0.7831131	3.212332	2.104733
sparse dr	22.735	0.9446084	0.9686214	3.518986	2.545846

- Case(ii): true beta의 0개수: 32

(ii) $\beta_1 = (1, 1, 0{\cdot}1, 0{\cdot}1, 0, \ldots, 0)^{\intercal}$ and $\beta_2 = (0, \ldots, 0, 0{\cdot}1, 0{\cdot}1, 1, 1)^{\intercal}$

method	p	corr1	corr2	mse1	mse2
dr	0.0	0.8280320	0.7915093	3.198981	0.8406891
sparse dr	23.9	0.9728796	0.9764424	3.562182	0.5640522

- Case(iii): true beta의 0개수: 20

(iii) $\beta_1 = (1, \ldots, 1, 0, \ldots, 0)^{\intercal}$ and $\beta_2 = (0, \ldots, 0, 1, \ldots, 1)^{\intercal}$,

method	p	corr1	corr2	mse1	mse2
dr	0.000	0.7963774	0.8138976	3.567960	1.715391
sparse dr	16.115	0.9581149	0.9099131	3.986382	2.114001

모든 case에서 Correlation 결과가 그냥 dr일때보다 sparse dr일 때 많이 좋아졌음

<논문 s-sir simulation result>

		$\hat{\beta}_1$ NUM	$\hat{\beta}_1$ COR	$\hat{\beta}_1$ MSE	$\hat{\beta}_2$ NUM	$\hat{\beta}_2$ COR	$\hat{\beta}_2$ MSE	$(\hat{\beta}_1, \hat{\beta}_2)$ VCC
Case (i)	SIR	0·000	0·926	1·604	0·000	0·911	1·245	0·934
	S-SIR	15·16	0·975	1·352	15·38	0·974	1·026	0·946
Case (ii)	SIR	0·000	0·884	0·551	0·000	0·856	0·544	0·932
	S-SIR	17·67	0·984	0·245	17·68	0·986	0·205	0·968
Case (iii)	SIR	0·000	0·916	4·793	0·000	0·942	4·168	0·917
	S-SIR	9·220	0·877	5·006	9·630	0·908	4·329	0·816

그러나, 모든 case에서 sir과 s-sir 결과가 zero의 개수가 true값에 더 가깝다. Case3의 경우, dr과 sparse dr의 결과가 더 좋다.

<simulation 결과 2>

y = cos(2*b1'X)-cos(b2'X)+0.5*e

p=10, true beta의 0 개수: 18

$\beta_1 = (1, 0, \ldots, 0)^{\mathsf{T}}$ and $\beta_2 = (0, 1, 0, \ldots, 0)^{\mathsf{T}}$.

- n=100

method	p	corr1	corr2	mse1	mse2
dr	0.00	0.7783505	0.8714745	0.5937655	2.571295
sparse dr	8.65	0.8215246	0.9570768	0.5305380	3.022709

- n=200

method	p	corr1	corr2	mse1	mse2
dr	0.00	0.8224365	0.8978549	0.4947189	2.070118
sparse dr	10.52	0.9102280	0.9532808	0.2818287	1.984507

n=200일 때 결과가 더 좋음, dr보다 sparse dr의 결과가 훨씬 좋아짐

			$\hat{\beta}_1$			$\hat{\beta}_2$		$(\hat{\beta}_1, \hat{\beta}_2)$
		NUM	COR	MSE	NUM	COR	MSE	VCC
$n = 100$	PHD	0·000	0·793	0·437	0·000	0·726	0·406	0·651
	S-PHD	8·280	0·911	0·194	8·140	0·902	0·158	0·837
$n = 200$	PHD	0·000	0·894	0·184	0·000	0·933	0·150	0·848
	S-PHD	8·360	0·970	0·055	8·230	0·991	0·020	0·963

그러나 s-phd 결과가 훨씬 좋음