<GEP formulation>

Table 1: The GEP formulation of some SDR methods

Method	A	B
SIR	$\mathrm{cov}[E\{\mathbf{x}-E(\mathbf{x})\} \mid y]$	$\Sigma_{\mathbf{x}}$
PFC	Σ_{fit}	$\Sigma_{\mathbf{x}}$
SAVE	$\Sigma_{\mathbf{x}}^{1/2} E\left[\{I-\mathrm{cov}(\mathbf{z} \mid y)\}^2\right] \Sigma_{\mathbf{x}}^{1/2}$, where $\mathbf{z}=\Sigma_{\mathbf{x}}^{-1/2}\{\mathbf{x}-E(\mathbf{x})\}$	$\Sigma_{\mathbf{x}}$
PHD(y-based)	$\Sigma_{\mathbf{x}}^{1/2}\Sigma_{yzz}\Sigma_{yzz}\Sigma_{\mathbf{x}}^{1/2}$, where $\Sigma_{yzz}=E\left[\{y-E(y)\}\mathbf{z}\mathbf{z}^{\top}\right]$	$\Sigma_{\mathbf{x}}$
PHD(r-basied)	$\Sigma_{\mathbf{x}}^{1/2}\Sigma_{\mathbf{rzz}}\Sigma_{\mathbf{rzz}}\Sigma_{\mathbf{x}}^{1/2}$, where $\Sigma_{\mathbf{rzz}}=E\left[\{y-E(y)-E\left(y\mathbf{z}^{\intercal}\right)\mathbf{z}\}\mathbf{z}\mathbf{z}^{\intercal}\right]$	$\Sigma_{\mathbf{x}}$
DR	$\Sigma_{\mathbf{x}}^{1/2}\{2E\left[E^2\left(\mathbf{z}\mathbf{z}^{\intercal} \mid y\right)\right]+2E^2\left[E(\mathbf{z} \mid y)E\left(\mathbf{z}^{\intercal} \mid y\right)\right]$	
	$+2E[E(\mathbf{z} \mid y)E(\mathbf{z} \mid y)]E\left[E(\mathbf{z} \mid y)E\left(\mathbf{z}^{\intercal} \mid y\right)\right]-2\mathbf{I}_p\}\Sigma_{\mathbf{x}}^{1/2}$	$\Sigma_{\mathbf{x}}$

<simulation 결과 1>

y = sign(b1'X)*log(|b2'X+5|)+0.2*e

p=20

- Case (i) : true beta의 0개수: 32

method	p	corr1	corr2	mse1	mse2
dr	0.000	0.7698654	0.7831131	3.212332	2.104733
sparse dr	22.735	0.9446084	0.9686214	3.518986	2.545846

- Case(ii): true beta의 0개수: 32

method	p	corr1	corr2	mse1	mse2
dr	0.0	0.8280320	0.7915093	3.198981	0.8406891
sparse dr	23.9	0.9728796	0.9764424	3.562182	0.5640522

- Case(iii): true beta의 0개수: 20

method	p	corr1	corr2	mse1	mse2
dr	0.000	0.7963774	0.8138976	3.567960	1.715391
sparse dr	16.115	0.9581149	0.9099131	3.986382	2.114001

Correlation 결과가 그냥 dr일때보다 sparse dr일 때 많이 좋아졌음

Number of zero의 개수도 ssir보다는 더 참값과 비슷해짐

<simulation 결과 2>

y = cos(2*b1'X)-cos(b2'X)+0.5*e

p=10, true beta의 0 개수: 18

- n=100

method	p	corr1	corr2	mse1	mse2
dr	0.00	0.7783505	0.8714745	0.5937655	2.571295
sparse dr	8.65	0.8215246	0.9570768	0.5305380	3.022709

- n=200

method	p	corr1	corr2	mse1	mse2
dr	0.00	0.8224365	0.8978549	0.4947189	2.070118
sparse dr	10.52	0.9102280	0.9532808	0.2818287	1.984507