Stochastic Simulation (MIE1613H) - Homework 1

Due: Jan 30, 2018

- Submit your homework to Portal in PDF format by the deadline. Late submissions are penalized.
- At the top of your homework include your name, student number, department, program and and e-mail address.
- You may discuss the assignment with other students, but each student must solve the problems, write the code and the solutions individually.
- The simulations must be programmed in Python 2.7 unless you have permission from the instructor to use another language. You must include both the source code (including comments to make it easy to follow) and the output of the simulation.
- Full mark is given to answers that are correct and clearly explained. Correct answers without a clear explanation and poor presentation of the solutions will not receive full mark.

Question 1. Assume that X is exponentially distributed with rate $\lambda = 0.2$. Compute $E[(X-2)^+]$ using Monte Carlo simulation. (Note: $a^+ = \text{Max}(a, 0)$, i.e, if $a < 0$ then $a^+ = 0$ and if $a \geq 0$ then $a^+ = a$.)

(a) Provide a 95% confidence interval for your Monte Carlo estimate.

(b) Produce a plot that demonstrates the convergence of the Monte Carlo estimate to the exact value (3.35160) as the number of samples increases.

Question 2. Modify the TTF simulation (the maintenance example from the first class) so that it can have any number of spare components, not just 1. Assume that components can only be repaired one at a time.

(a) Run your simulation for 1000 replications and report a 95% confidence interval for the expected time to failure when there are 1 and 2 spare components available.

HINT: You do not need to modify the state.

Question 3. Develop an event-based simulation of the $M/G/1$ queue assuming that the service times are exponentially distributed. Assume that the system starts empty. Use the number of customers in the system $X(t)$ as the state. Use arrival rate $\lambda = 0.8$, mean service time $\tau = 1$ and run the simulation for $T = 100,000$ time units.

(a) Estimate the following:

$$E[\int_0^T X(t)dt/T],$$

using $n = 30$ replications. (b) Explain in words what the expected value represents.

HINT: There are two possible events: arrival of a new customer and departure of a customer in service. The next event could only be a departure if the system is not empty.