Stochastic Simulation (MIE1613H) - Homework 2

Due: Feb 19th

- Submit your homework to Portal in PDF format by the above deadline. Late submissions are penalized.
- At the top of your homework include your name, student number, department, program and and e-mail address.
- You may discuss the assignment with other students, **but each student must solve the problems, write the code and the solutions individually.**
- Code must be in Python 2.7. unless you have permission from instructor to use another language. You must include both the source code (including comments to make it easy to follow) and the output.
- Full mark is given to answers that are correct and clearly explained. **Write a brief and clear explanation of your solution for each problem.**

Problem 1. (Up-and-out call option) Another variation on a European option is to introduce an upper barrier $b > 0$. Under this variation, if we denote the stock price at time t by $X(t)$, the holder of the option receives payoff $(X(T) - K)^+$ at time T only if the stock price remains below the barrier at pre-specified times $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k = T$. Therefore, the expected (discounted) payoff is

$$C_T = E[e^{-rT}(X(T) - K)^+ \mathbf{1}\{S(\tau_1) < b, \ldots, S(\tau_k) < b\}],$$

where $\mathbf{1}\{A\}$ is the indicator function for the event A and is equal to 1 if the event occurs and 0 if the event does not occur.

Assuming that the price evolves according to a Geometric Brownian Motion (GBM) with percentage drift $\mu = r = 0.05$ and percentage volatility $\sigma = 0.4$, (a) estimate the expected payoff using simulation for $T = 1$ and $b = 70$ with $\tau_{i+1} - \tau_i = 0.1$ for $i = 0, 1, \ldots, k-1$ and assuming the initial price of $X(0) = 30$ and strike price of $K = 40$. Use 10,000 replications, and at least 32 steps when discretizing the GBM. (b) Produce a histogram of the payoff of the option and briefly discuss your observation compared to the expected payoff you've computed.

Problem 2. (Chapter 4, Exercise 4) Beginning with the VBASim event-based $M/G/1$ simulation, implement the changes necessary to make it an $M/G/s$ simulation (a single queue with any number of servers). Keeping $\lambda = 1$ and $\tau/s = 0.8$, simulate the system for $s = 1, 2, 3$ and (a) report the estimated expected number of customers in the system (including customers in the queue and service), expected system time, and expected number of busy servers in each case. (b) Compare the results and state clearly what you observe. What you're doing is comparing queues with the same service capacity, but with 1 fast server as compared to 2 or more slow servers.

HINT: The attribute "NumberOfUnits" of the Resource object returns the number of available units for any instance of the object.

Problem 3. (Chapter 4, Exercise 5) Modify the VBASim event-based simulation of the $M/G/1$ queue to simulate a $M/G/1/c$ retrial queue. This means that customers who arrive to find c customers in the system (including the customer in service) leave immediately, but arrive again after an exponentially distributed amount of time with mean MeanTR. (You do not need to report any outputs for this problem.)

HINT: The existence of retrial customers should not affect the arrival process for first-time arrivals.

Problem 4. A manufacturing system processes two types of jobs with two processing stations (machines). Class 1 jobs are high-value jobs and need to be completed quickly. Class 2 jobs are of lower value, and hence greater delays can be tolerated than with Class 1 jobs. Accordingly, the processing system has been set up as in Figure 1. Station 1 processes Class 1 jobs whenever they are available. Station 2 processes both Class 1 and Class 2 jobs according to a non-preemptive priority scheduling rule in favor of Class 1 jobs. That is, each time a job is completed at **Station 2,** the next job is a Class 1 job unless the queue for Class 1 jobs is empty, in which case the next job is a Class 2 job.

(a) Develop a simulation model of the processing network using VBASim assuming the following inputs. Class 1 and 2 jobs arrive to the system according to stationary Poisson processes with average time between arrivals MeanTBA1 and MeanTBA2, respectively. Processing times for Class 1 jobs are exponentially distributed with mean MeanPT1_1 if they are processed in Station 1 and MeanPT1_2 if they are processed in Station 2. Processing times for Class 2 jobs are also exponentially distributed with mean MeanPT2.

The output of your simulation model must be the estimated mean queue lengths (for each queue; not including the customers in service), mean system times (for each customer class; including the service times), and utilization of each server.

(b) Report the output for the following two set of input parameters by running your simulation for $n = 20$ replications of length $T = 50,000$ and a warmup period equal to 5000.

(b1) MeanTBA1 = 1.3, MeanTBA2 = 2.5, MeanPT1_1 = 1, MeanPT1_2 = 2, MeanPT2 = 1,

(b2) MeanTBA1 = 1.0, MeanTBA2 = 2.5, MeanPT1_1 = 1, MeanPT1_2 = 2, MeanPT2 = 1.

(c) Does the system reach steady-state for each of the above set of parameters? Argue using the output of your simulation.

HINT: Go over the Tandem queue example and the posted simulation code. Techniques from there will be useful here (and in general for simulating queueing networks).

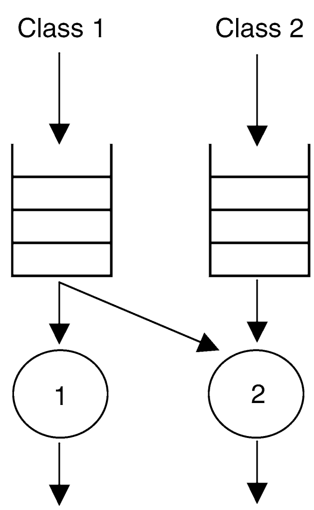

Figure 1: The processing network.