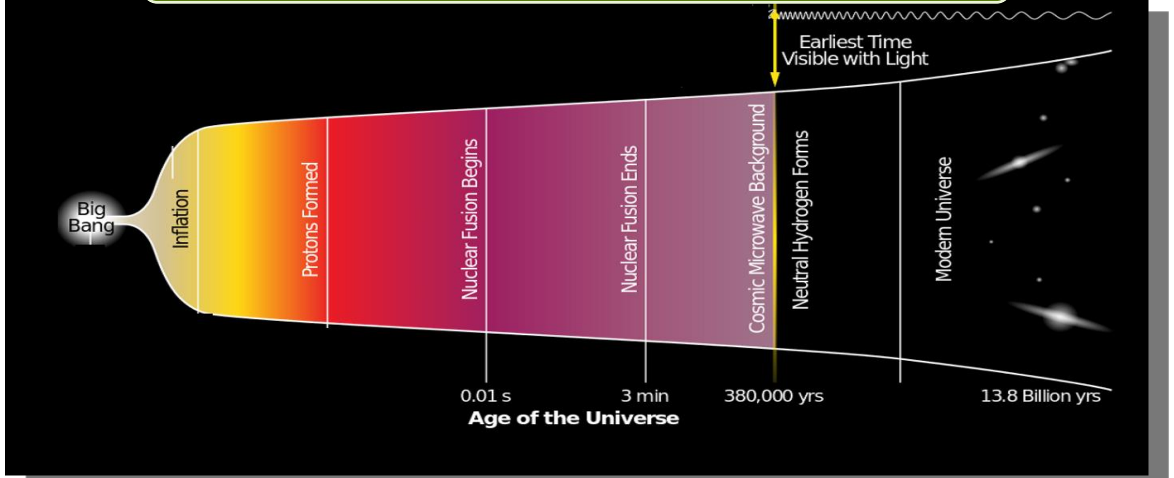
Statistical analysis of the Cosmic Microwave Background

Introduction

- In my project I have studied the Cosmic Microwave Background, also know as CMB.
- The CMB is microwave radiation that fills all space, is one of the most studied topic in Cosmology and it is a landmark evidence of the Big Bang Theory for the origin of the universe.
- The cosmic microwave background radiation is an emission of uniform, black body thermal energy coming from all parts of the sky, and according to standard cosmology, it represent the point in time when the temperature dropped enough to allow the CMB photons to travel unimpeded.

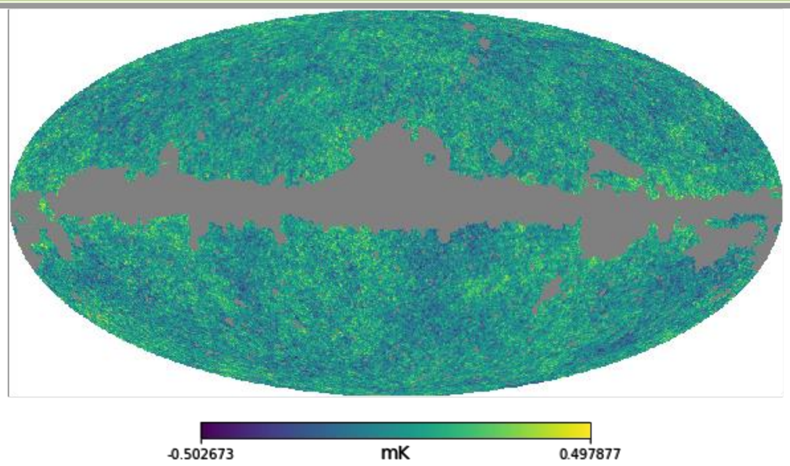
The evolution of the early universe



Data presentation

- The map that I have used for my project is obtained by the NASA spacecraft «Wilkinson Microwave Anisotropy Probe», also known as WMAP.
- It measured the temperature differences across the sky in the CMB and the temperature presented is the difference from the actual value with the mean value ($\bar{T} = 2.725$).
- The dataset can be obtained by the NASA website and can the map can be easily seen using the python library Healpy.
- From the NASA website I have dowloaded the masked map, so the map without any celestial object.

Mollweide of the masked CMB

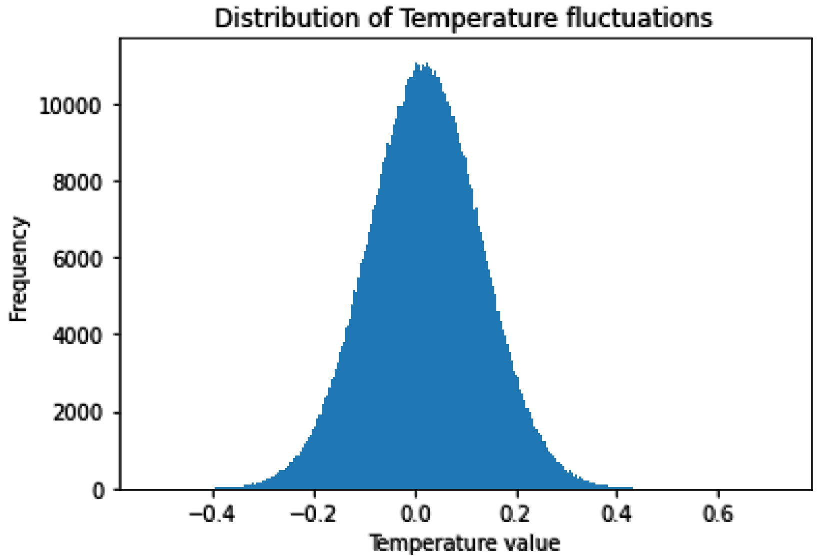


1D statistic

- As first think I have done the 1D statistics to understand if the temperature fluctuations follow a Gaussian distribution.
- So I have applied the Kolmogorov-Smirnov test with a confidence limit of $\alpha = 0.01$, and I have considered the normalized temperature.

$$T^* = \frac{\delta T - \overline{\delta T}}{std(\delta T)}$$

- The probability obtained into the test is: $p \sim 0.03$. So the null hipotesys is accepted and I can fully describe my distribution with $\overline{\delta T} \sim 0.2$ and $std(\delta T) \sim 0.1$.



2D statistic

- In the 2D analysis I have applied the Sign-Singular Measures, a measure very usefull to understand if a process have a multifractal behaviour.
- Let $A \subset X$ be an interval such that $\mu(A) \neq 0$. μ is sign singular if, for any interval A, there is an interval $B \subset A$ such that $\mu(B)$ has the opposite sign from $\mu(A)$. Therefore the measure μ changes sign on arbitrarily fine scale. As in the case of multifractals, it can be possible to introduce a quantity that characterize the sign-singular measure.
- I have used for this purpose the cancellation exponent, introduce by Edward Ott and Yunson Du.

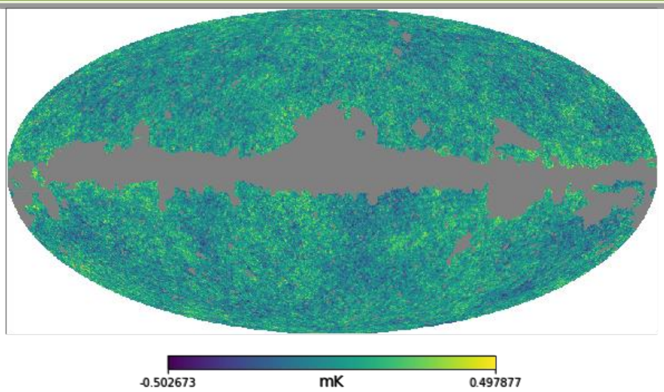
$$k = \lim_{\varepsilon \to 0} \frac{\ln \sum_i |\mu(I_i)|}{\ln \frac{1}{\varepsilon}}$$

- Where I denotes the ε-lenght interval.
- In order to have $k > 0$ I need that $\sum_i |\mu(I_i)|$ increase when ε decrease, thus it means that the cancellation of positive and negative values decrease over time. Therefore $k > 0$ is a good indicator of oscillation in sing in arbitraly fine scale.

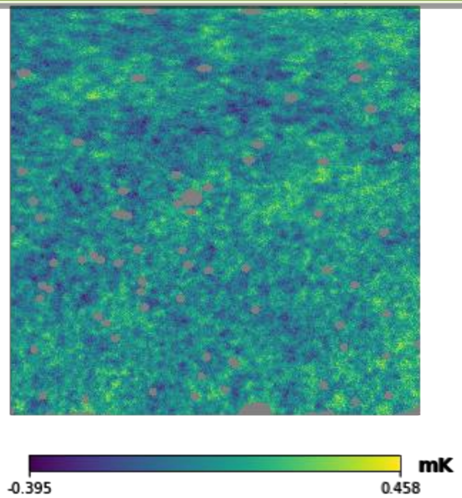
Sign-singular measures

- For semplicity I have defined I_i as a square region with side ε_i pixels.
- For this poupuse I had the need to reproject my spherical CMB map into a plane map.
- Due to the fact that the projection can modify my results I have decided to study the upper part of the CMB and the lower part of it, using firstly a cartesian projection and then a gnomonic one.

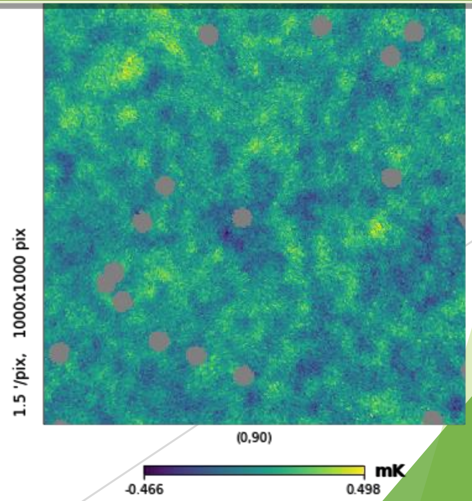
Mollweide of the masked CMB



Cartesian view centered on north pole



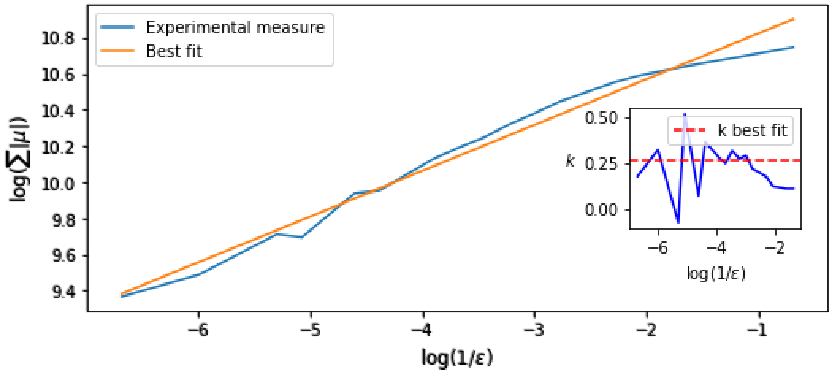
Gnomview centered on north pole



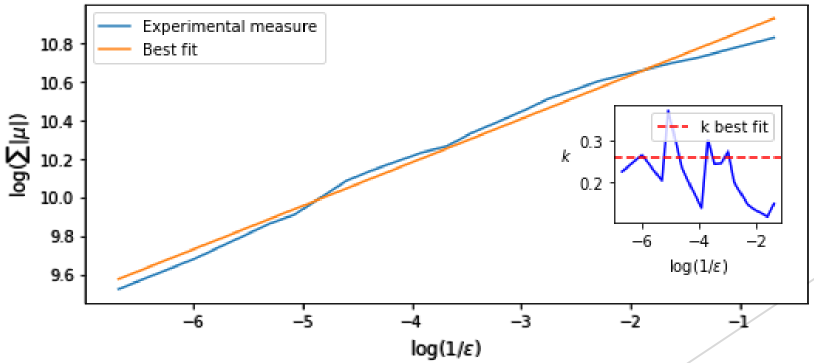
Sign-singular measures

- For the cartesian projection I studied the part near the center, I Have tried to maximize the number of point to study minimizing the possible effects due to the projection.
- As we can see the behaviour of the upper side and lower side of the CMB is pretty much similar.
- In this case I have fitted the relations between $\ln \sum_i |\mu(I_i)|$ and $\ln \frac{1}{\varepsilon}$ with a straight line.
- K is defined by the local slope.

Sign-singular measures on the upper side of the cartesian projection



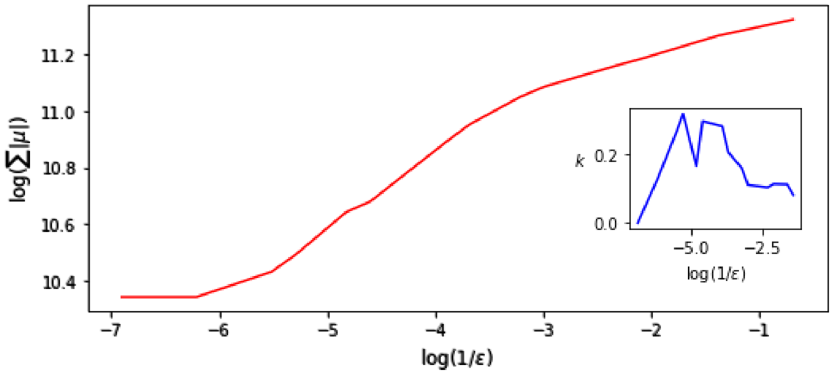
Sign-singular measures on the lower side of the cartesian projection



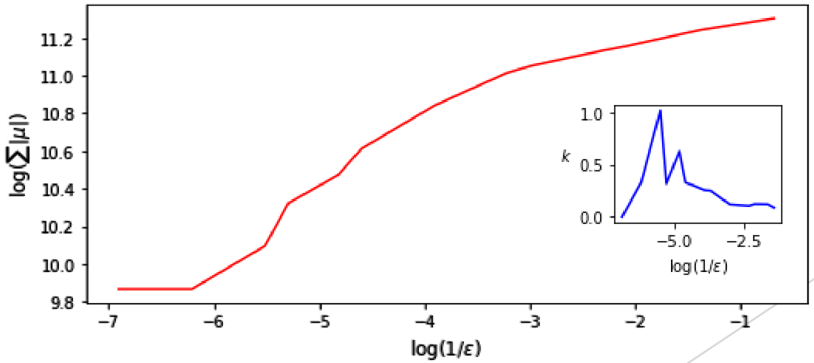
Sign-singular measures

- In the gnonomic projection I can center the projection into the two poles, and so I can study a much larger portion of the map. The problem with the gnomonic projection is that: as we go further and further away from the poles the fluctuations start to stretch.
- I have considered two regions of 1000x1000 pixels centered into the poles.
- In this case I cannot fit the relation between $\ln \sum_i |\mu(I_i)|$ and $\ln \frac{1}{\varepsilon}$ with a straight line. And we can also see a different behaviour with repect to the cartesian projection

Sign-singular measures on the upper side of the gnonomic projection



Sign-singular measures on the lower side of the gnonomic projection

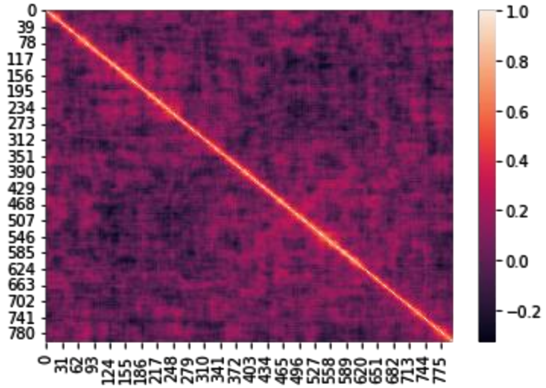


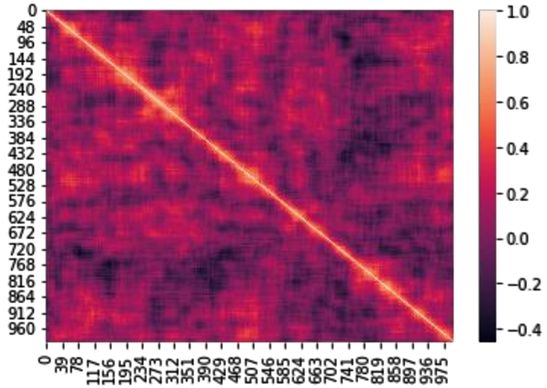
Correlation matrix

- Another things that I have studied is the correlation matrix to understand if there are some correlation of the fluctuation in temperature.
- As expected, we can see that there is a correlation only for the the values that are close to each other.

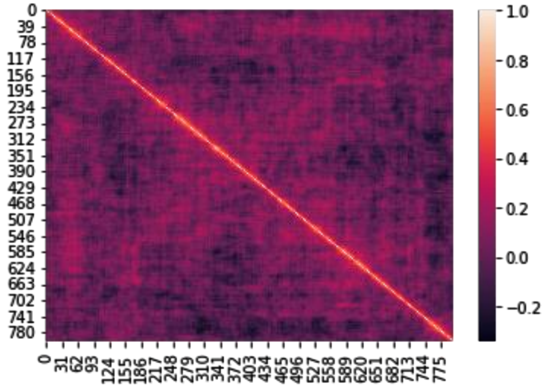
Cartesian Correlation Matrix | Gnomonic Correlation Matrix

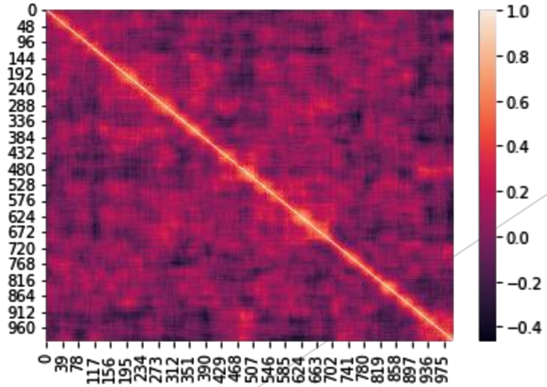
Upper side





Lower side





Conclusion

- I have done a simple 1D analysis of the temperature fluctuations of the CMB, and then in much more dettail I have done a 2D analysis to understand if the fluctuations follows a fractal or multifractal figure.
- The results are a lot inteeresting because it seems that the CMB follows a multifractal behaviour, but to be completely certain of this I need to consider I_i as a ciruclar region and I need to study the original masked map without applying any projection.
- Furthermore it will also be interesting to study other CMB map made with other spacecraft to better understand the border effectes that appear when $\varepsilon_i \to 0$.