

Computations for Greenbaum *et al.*

Generating vector field plots with basins of attraction (Fig. 2, S14)

```
In[1]:= $HistoryLength = 0;

dynamics[q10_, q20_, ss_, cc_, hh_, mm_, time_] :=
Module[{q1, q2, list1, list2, q1t, q2t}, (*two lists of the dynamics,
for q1 and q2, given starting conditions, up to time*)
q1 = q10; q2 = q20;
list1 = {q1}; list2 = {q2};
Do[
q1t = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.
\{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q1 + mm q2\};
q2t = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.
\{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q2 + mm q1\};
q1 = q1t; q2 = q2t;
AppendTo[list1, q1];
AppendTo[list2, q2];
, time];
{list1, list2}
]

eqpoints[ss_, cc_, hh_, mm_] := Module[{q1, q2, eeq1, eeq2, sols, z1, z2, z3, sn, sc},
(*Gives the equilibrium solutions for s,c,h,m*)
eeq1 = q1 == 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.
\{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q1 + mm q2\};
eeq2 = q2 == 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.
\{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q2 + mm q1\};
sols = Quiet[NSolve[{eeq1, eeq2}, {q1, q2}, Reals]];
If[Length[sols] == 7, sols = Quiet[Solve[{eeq1, eeq2}, {q1, q2}, Reals]]];$$$$$$$$

```

```

z1 = Table[
  If[
    Length[sss] ≠ 2, "N",
    If[sss[[1, 2]] < 0 - 10-5 || sss[[2, 2]] < 0 - 10-5 ||
      sss[[1, 2]] > 1. + 10-5 || sss[[2, 2]] > 1. + 10-5, "N", {sss[[1, 2]], sss[[2, 2]]}]
  ] (*Here we remove all solution that are not in the interval [0,1]
    with some small error margin due to numeric accuracy*)
  , {sss, sols}];
z2 = DeleteCases[z1, "N"];
z3 = DeleteDuplicates[Round[z2, 10-5]];
(*Due to numeric accuracy,
some solutions are counted twice sometimes. Here we remove duplicate solutions*)
N[Sort[z3]]
]

attractor[q10_, q20_, s_, c_, h_, m_, time_, th_, attrarctors_] :=
Module[{dd, dd1, distances, a1}, (*Given a list of attractors,
for a point (q10,q20) returns the closest attractor after dynamics run for time,
as long as it is within distance th. otherwise, returns 0*)
dd = dynamics[q10, q20, s, c, h, m, time];
dd1 = {Last[dd[[1]]], Last[dd[[2]]]};
distances = Table[EuclideanDistance[dd1, a], {a, attrarctors}];
a1 = Position[distances, Min[distances]][[1, 1]];
If[distances[[a1]] < th, a1, 0]
]

cff[x_] := If[x == 0, White, Lighter[colors[[x]], 0.4]]
(*Color function for attraction basin plot*)

PlotAttraction[ss_, cc_, hh_, mm_, th_, time_] := Module[{},
(*Plots a 101*101 array for q1*q2 of the equilibria and attrarction basins*)
eqs = eqpoints[ss, cc, hh, mm];
txx = ParallelTable[
  attractor[q1, q2, ss, cc, hh, mm, time, th, eqs], {q1, 0, 1, 0.01}, {q2, 0, 1, 0.01}];
colors1 = ColorData[97, "ColorList"];
colors = ReplacePart[colors1,
  {7 → colors1[[8]], 8 → colors1[[7]], 5 → colors1[[6]], 6 → colors1[[5]]}];
colors = AppendTo[colors[[1 ;; Length[eqs] - 1]], Red];
l0 = ListPlot[{{0, 0.5}}, Frame → {{True, False}, {True, False}},
  FrameTicks → {{{0, 0, {0, 0}}, {0.25, "", {0.03, 0.03}},
    {0.5, 0.5, {0.03, 0.03}}, {0.75, "", {0.03, 0.03}}, {1, 1, {0.03, 0.03}}},
    {{0, 0, {0, 0}}, {0.25, "", {0.03, 0.03}}, {0.5, 0.5, {0.03, 0.03}},
    {0.75, "", {0.03, 0.03}}, {1, 1, {0.03, 0.03}}}},
  PlotRange → {{0, 1}, {0, 1}}, AxesOrigin → {0, 0}, AspectRatio → 1,
  FrameTicksStyle → Directive[Black, 35], PlotRangeClipping → False,
  PlotRangePadding → 0.03, FrameStyle → White];
l1 = ListPlot[Partition[eqs, 1], PlotMarkers →
  Table[Graphics[{EdgeForm[{Black, Thin}], FaceForm[ccc], Disk[]}, ImageSize → 15],
    {ccc, colors}], Frame → {{True, False}, {True, False}},

```

```

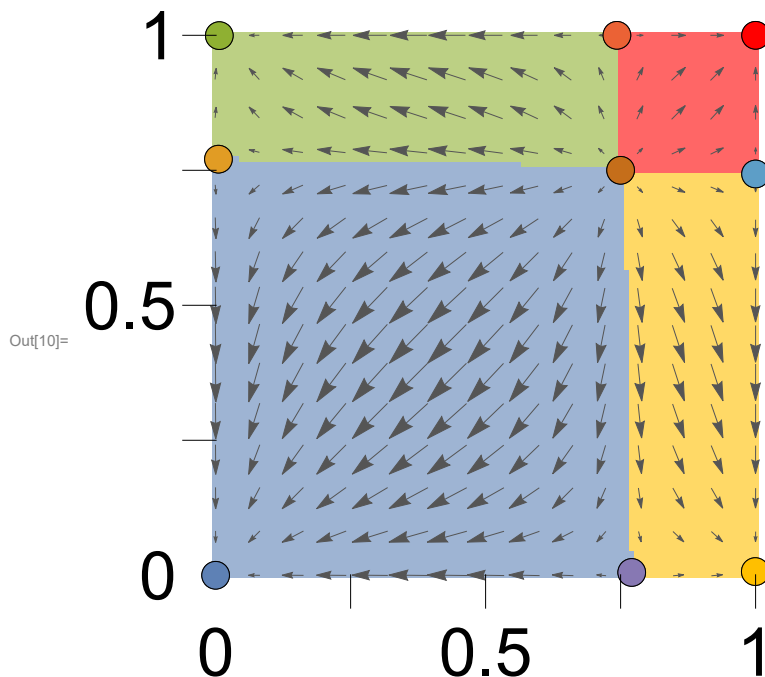
FrameTicks → {{0, 0.5, 1}, {0, 0.5, 1}}, AspectRatio → 1, PlotRangeClipping → False];
l2 = MatrixPlot[Transpose[txx], ColorFunction → cff,
  ColorFunctionScaling → False, FrameTicks → {{0, 0.5, 1}, {0, 0.5, 1}},
  DataRange → {{0, 1}, {0, 1}}, DataReversed → True];
a1 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.
  \{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q1 + mm q2\};
a2 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.
  \{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q2 + mm q1\};
l3 = VectorPlot[{a1 - q1, a2 - q2}, {q1, 0, 1}, {q2, 0, 1},
  VectorStyle → Darker[Gray], VectorPoints → Automatic];
l3 = Show[l0, l2, l3, l1]
]$$$$

```

```

In[7]:= s = 0.8; c = 1; h = 1;
m = 0.01;
DateString[]
res = PlotAttraction[s, c, h, m, 0.01, 100]
DateString[]

```



Computing m^* and q_2^* (Fig. 3, S2)

```

$HistoryLength = 0;
DTEvalid[cc_, hh_, ss_, mm_] :=

```

```

Module[{y1, y2, out, qnext1, qnext2, q, sn, sc, q1, q2, eqSolutions, eqSolutions2, outq2,
  rr2, rr3, rr4, rr5, rr6}, (*For a given c,h,s, and m, tests if a DTE exists*)
out = 1; outq2 = "Na";
If[ss == 0, out = 0,
  qnext1 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /. \{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q1 + mm q2\};$$

  (*Eq. 3*)
  qnext2 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /. \{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q2 + mm q1\};$$

  (*Eq. 3*)
  (*Next 4 code lines switch from Eq.3 to Eq. S2. Activate these lines for generating Fig. S2 (selection before migration)*)
  (*qnext1a= 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /. \{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow q1\};$$
*)
  (*qnext2b= 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /. \{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow q2\};$$
*)
  qnext1 = 
$$\frac{(1 - aa mm) qnext1a + mm qnext2b}{1 - aa mm + mm};$$

  qnext2 = 
$$\frac{(1 - mm) qnext2b + aa mm qnext1a}{1 + aa mm - mm};$$

  eqSolutions = Quiet[NSolve[qnext1 == q1 && qnext2 == q2, {q1, q2}]];
  (*Finding equilibrium solutions for Eq. 3*)
  eqSolutions2 = N[Round[Table[{e[[1, 2]], e[[2, 2]]}, {e, eqSolutions}], 10-3]];
  If[Union[Flatten[Table[{Im[e[[1]]], Im[e[[2]]]}, {e, eqSolutions2}]]] != {}, out = 0];
  (*Checks that all solutions are real*)
  y2 = If[out == 1, Union[Table[If[0 ≤ t ≤ 1, 1, 0], {t, Flatten[eqSolutions2]}]]];
  (*Checks that all solutions are between 0 and 1*)
  If[y2 != {1}, out = 0];
  If[out == 1, (*Compute q2 of the DTE*)
    jacobian = {{D[qnext1, q1], D[qnext1, q2]}, {D[qnext2, q1], D[qnext2, q2]}};
    (*The Jacobian from equation 5.*)
    rr2 = Table[Abs[Eigenvalues[jacobian /. {r[[1]], r[[2]]}]], {r, eqSolutions}]; (*Taking the absolute eigenvalues for the jacobian evaluated at the different solutions*)
    rr3 = Table[If[e[[1]] < 1 && e[[2]] < 1, True, False], {e, rr2}];
    (*Testing to see for each equilibrium point if it is stable*)
    rr4 = Round[Table[{e[[1, 2]], e[[2, 2]]}, {e, eqSolutions}], 0.00001];
    rr5 = Table[If[rr3[[i]] && (rr4[[i, 1]] > rr4[[i, 2]]), rr4[[i, 2]], Null], {i, Length[eqSolutions]}]; (*Taking all the points that are stable and q1>q2*)
    rr6 = DeleteCases[rr5, Null];
    If[rr6 == {}, out = 0]; (*If there are no DTEs, out=0*)
    outq2 = If[Length[rr6] == 1, rr6[[1]], Null]
    (*Making sure that there is only one DTE, and taking q2 of the DTE*)
  ];
];
{out, outq2} (*out=1 means DTE exists, otherwise out=0*)
]

```

```

mqstar[cc_, hh_, ss_] := Module[{maxm, jumps, t0, t1, t2, t3, outm, w1, outq},
  (*For a given c, h, and s, returns m* and q2* *)
  If[DTEvalid[cc, hh, ss, 0][[1]] == 0, outm = "Na"; outq = "Na",
    maxm = 0.13; (*Maximal m for searching m* *)
    jumps = 10-3; (*Resolution in m for searching m* *)
    t0 = Table[{N[mm], DTEvalid[cc, hh, ss, mm]}, {mm, 0, maxm, jumps}];
    t1 = Table[{t[[1]], t[[2, 1]]}, {t, t0}];
    t2 = Flatten[Take[t1, All, {2}]];
    t3 = Flatten[t1[[FirstPosition[t2, 0] - 1]][[1]];
    outm = If[NumberQ[t3], t3, "Na"];
    w1 = Table[{t[[1]], t[[2, 2]]}, {t, t0}];
    outq = Max[DeleteCases[Flatten[Take[w1, All, {2}]], "Na"]]
  ];
  {outm, outq}
]

ComputeGridmqstar[h_] := Module[{jmps, jmpc, ress, pmstar, pqstar},
  jmps = 0.01; (*resolution in s*)
  jmpc = 0.01; (*resolution in c*)
  ress = Quiet[ParallelTable[
    Table[{ccc, sss, mqstar[ccc, h, sss]}, {ccc, 0, 1, jmpc}], {sss, 0, 1, jmps}]];
  pmstar = Table[{r[[1]], r[[2]], r[[3, 1]]}, {r, Flatten[ress, 1]};
  pqstar = Table[{r[[1]], r[[2]], r[[3, 2]]}, {r, Flatten[ress, 1]};
  {pmstar, pqstar}
]

```

```

In[ ]:= h = 0;
p1 = ComputeGridmqstar[h];

```

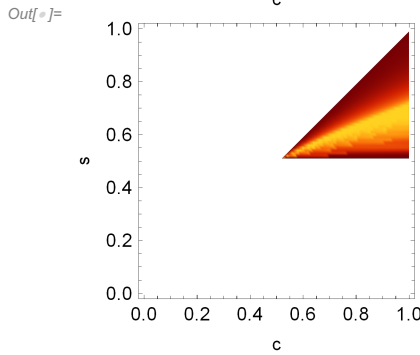
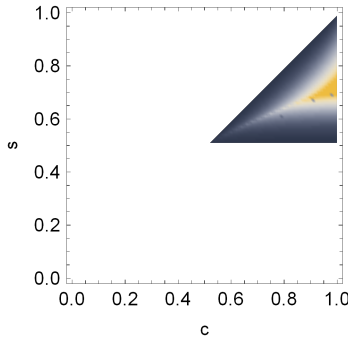
```

In[ ]:= cf1[x_] := If[! NumberQ[x], White, ColorData["GrayYellowTones"] [  $\frac{x}{0.075}$  ]]

cf2[x_] := If[! NumberQ[x], White, ColorData["SolarColors"] [  $\frac{x}{0.22}$  ]]

gm1 = ListDensityPlot[p1[[1]], PlotRange -> {{0, 1}, {0, 1}, {0, 0.3}}, ColorFunction -> cf1,
  ColorFunctionScaling -> False, FrameLabel -> {"c", "s"}, FrameTicksStyle -> 10];
gq1 = ListDensityPlot[p1[[2]], PlotRange -> {{0, 1}, {0, 1}, {0, 0.3}}, ColorFunction -> cf2,
  ColorFunctionScaling -> False, FrameLabel -> {"c", "s"}, FrameTicksStyle -> 10];
Grid[{{gm1}, {gq1}}]

```



Computing trajectories of q^* with incase in m (Fig. 4)

```

eqpointsMbS[ss_, cc_, hh_, mm_] := Module[{q1, q2, eeq1, eeq2, sols, z1, z2, z3},
  (*Gives the equilibrium solutions for s,c,h,m*)
  eeq1 = q1 ==  $\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2}$  /.
    {sn ->  $\frac{1}{2} (1 - cc) (1 - hh ss)$ , sc ->  $cc (1 - ss)$ , q ->  $(1 - mm) q1 + mm q2$ };
  eeq2 = q2 ==  $\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2}$  /.
    {sn ->  $\frac{1}{2} (1 - cc) (1 - hh ss)$ , sc ->  $cc (1 - ss)$ , q ->  $(1 - mm) q2 + mm q1$ };
  sols = Quiet[NSolve[{eeq1, eeq2}, {q1, q2}, Reals]];
  z1 = Table[
    If[

```

```

Length[ss] ≠ 2, "N",
If[ss[[1, 2]] < 0 - 10-5 || ss[[2, 2]] < 0 - 10-5 ||
  ss[[1, 2]] > 1. + 10-5 || ss[[2, 2]] > 1. + 10-5, "N", {ss[[1, 2]], ss[[2, 2]]}]
] (*Here we remove all solution that are not in the interval [0,1]
  with some small error margin due to numeric accuracy*)
, {ss, sols}];
z2 = DeleteCases[z1, "N"];
z3 = DeleteDuplicates[Round[z2, 10-5]];
(*Due to numeric accuracy,
some solutions are counted twice sometimes. Here we remove duplicate solutions*)
N[Sort[z3]]
]

dynamicsMbS[q10_, q20_, ss_, cc_, hh_, mm_, time_] :=
Module[{q1, q2, list1, list2, q1t, q2t}, (*two lists of the dynamics,
  for q1 and q2, given starting conditions, up to time*)
q1 = q10; q2 = q20;
list1 = {q1}; list2 = {q2};
Do[

$$q1t = \frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.$$


$$\{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q1 + mm q2\};$$


$$q2t = \frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.$$


$$\{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow (1 - mm) q2 + mm q1\};$$

q1 = q1t; q2 = q2t;
AppendTo[list1, q1];
AppendTo[list2, q2];
, time];
{list1, list2}
]

attractorMbS[q10_, q20_, s_, c_, h_, m_, time_, th_, attarctors_] :=
Module[{}, (*Given a list of attractors,
  for a point (q10,q20) returns the closest attractor after dynamics run for time,
  as long as it is within distance th. otherwise, returns 0*)
dd = dynamicsMbS[q10, q20, s, c, h, m, time];
dd1 = {Last[dd[[1]]], Last[dd[[2]]]};
distances = Table[EuclideanDistance[dd1, a], {a, attarctors}];
a1 = Position[distances, Min[distances]][[1, 1]];
If[distances[[a1]] < th, a1, 0]
]

mplusepsilonMbS[s_, c_, h_] := Module[{} ,
  epsilon = 0.005;

```

```

mst = mqstar[c, h, s][[1]];
atr = eqpointsMbS[s, c, h, mst + epsilon];
qq = qMbS[s, c, h, mst];
time = 1000;
thr = 0.1;
pp = attractorMbS[qq[[1]], qq[[2]], s, c, h, mst + epsilon, time, thr, atr];
atr[[pp]]
]

qeqtrajMbS[s_, c_, h_] := Module[{},
  ms = mqstar[c, h, s][[1]];
  tt = ParallelTable[qMbS[s, c, h, m], {m, 0, ms, 0.001}];
  ee = mplusepsilonMbS[s, c, h];
  {tt, ee}
]

plottraj1[list_, color_] := Module[{},
  templist = DeleteCases[list[[2, 1]], "N2"];
  dd1 = ListPlot[templist, Joined → True,
    PlotStyle → Directive[color, Arrowheads[20]], PlotRange → {{0, 1}, {0, 1}},
    Frame → True, FrameTicks → {{0, 0.2, 0.4, 0.6, 0.8, 1}, {0, 0.2, 0.4, 0.6, 0.8, 1}},
    AspectRatio → 1, FrameTicksStyle → 18] /.
    Line[x_] => {Arrowheads[{0, 0, 0, 0, If[MemberQ[{0.719}, list[[1]], 0.05, 0],
      0, If[MemberQ[{0.7, 0.75, 0.8}, list[[1]], 0.05, 0]}], Arrow[x]};
  dd2 = ListPlot[{Last[templist], list[[2, 2]]}, Joined → True,
    PlotStyle → {Dashed, color}] /.
    Line[x_] => {Arrowheads[{0, 0, If[MemberQ[{0.7, 0.719, 0.72, 0.75, 0.8}, list[[1]],
      0.05, 0], 0, 0, 0, 0, 0}], Arrow[x]};
  Show[dd1, dd2]
]

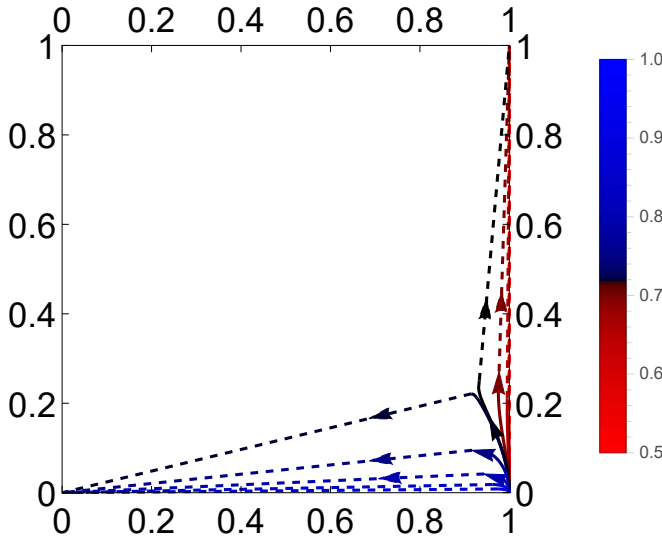
c = 1; h = 0;

svalues = {0.55, 0.6, 0.65, 0.7, 0.719, 0.720, 0.75, 0.8, 0.85, 0.89};
DateString[]
wa1 = Table[{s, qeqtrajMbS[s, c, h]}, {s, svalues}];
DateString[]

cf1[s_, smax_] :=
  If[s ≥ smax, Darker[Blue, 1 -  $\left(\frac{s - \text{smax}}{1 - \text{smax}}\right)^{0.3}$ ], Darker[Red, 1 -  $\left(\frac{\text{smax} - s}{\text{smax} - 0.5}\right)^{0.3}$ ]]

smax1 = 0.719;
wa3 = Table[plottraj1[w, cf1[w[[1]], smax2]], {w, wa1}];
g1 = Show@@wa3;
g2 = BarLegend[{cf1[#, smax1] &, {0.5, 1}}];
g3 = Grid[{g1, g2}];

```

Computing the probability of escape (Fig. 5A and S10A)

```

eqpoints[ss_, cc_, hh_, mm_] := Module[{y1, out, qnext1, qnext2, q, sn, sc, q1, q2, outq2},
  (*Gives the equilibrium solutions for s,c,h,m*)
  If[ss == 0, out = {{0, 0}, {1, 1}},
  qnext1 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.$$

    {sn →  $\frac{1}{2} (1 - cc) (1 - hh ss)$ , sc → cc (1 - ss), q → (1 - mm) q1 + mm q2};
  (*Eq. 3*)
  qnext2 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.$$

    {sn →  $\frac{1}{2} (1 - cc) (1 - hh ss)$ , sc → cc (1 - ss), q → (1 - mm) q2 + mm q1};
  (*Eq. 3*)
  (*Next 4 code lines switch from Eq.3 to Eq. S2. Activate
  these lines for generating Fig. S10 (selection before migration)*)
  (*qnext1a= 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.$$
 {sn →  $\frac{1}{2} (1 - cc) (1 - hh ss)$ , sc → cc (1 - ss), q → q1};
  qnext2b= 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.$$
 {sn →  $\frac{1}{2} (1 - cc) (1 - hh ss)$ , sc → cc (1 - ss), q → q2};
  qnext1 = 
$$\frac{(1 - aa mm) qnext1a + mm qnext2b}{1 - aa mm + mm}$$
;
  qnext2 = 
$$\frac{(1 - mm) qnext2b + aa mm qnext1a}{1 + aa mm - mm}$$
; *)
  eqSolutions = NSolve[qnext1 == q1 && qnext2 == q2, {q1, q2}];
  (*Finding equilibrium solutions for Eq. 3*)
  eqSolutions2 = N[Round[Table[{e[[1, 2]], e[[2, 2]]}, {e, eqSolutions}], 10-3]];
  eqSolutions3 = DeleteCases[
    Table[If[Im[e[[1]]] == 0 && Im[e[[2]]] == 0, e, Null], {e, eqSolutions2}], Null];
  (*Selects all the real equilibria*)

```

```

eqSolutions4 =
  DeleteCases[Table[If[0 ≤ e[[1]] ≤ 1 && 0 ≤ e[[2]] ≤ 1, e, Null], {e, eqSolutions3}], Null];
(*Selects all the equilibria in the interval [0,1] *)
out = eqSolutions4
];
out
]

dynamics[q10_, q20_, s_, c_, h_, m_, time_] :=
Module[{q1, q2, list1, list2, q1t, q2t}, (*two lists of the dynamics,
  for q1 and q2, given starting conditions (q10,q20), for "time" generations*)
  q1 = q10; q2 = q20;
  list1 = {q1}; list2 = {q2};
  sn =  $\frac{1}{2} (1 - c) (1 - h s)$ ;
  sc =  $(1 - s) c$ ;
  Do[
    q1mig =  $(1 - m) q1 + m q2$ ;
    q2mig =  $(1 - m) q2 + m q1$ ;
    meanfit1 =  $q1mig^2 (1 - s) + 2 q1mig (1 - q1mig) (2 sn + sc) + (1 - q1mig)^2$ ;
    meanfit2 =  $q2mig^2 (1 - s) + 2 q2mig (1 - q2mig) (2 sn + sc) + (1 - q2mig)^2$ ;
    q1t =  $\frac{q1mig^2 (1 - s) + q1mig (1 - q1mig) (sn + sc)}{\text{meanfit1}}$ ;
    q2t =  $\frac{q2mig^2 (1 - s) + q2mig (1 - q2mig) (sn + sc)}{\text{meanfit2}}$ ;
    (*Next 2 code lines switch from Eq.3 to Eq. S2. Activate
      these lines for generating Fig. S10 (selection before migration)*)
    (*q1t =  $(1 - m) * \frac{q1^2 (1 - s) + q1 (1 - q1) (sn + 2sc)}{q1^2 (1 - s) + 2q1 (1 - q1) (sn + sc) + (1 - q1)^2} + m * \frac{q2^2 (1 - s) + q2 (1 - q2) (sn + 2sc)}{q2^2 (1 - s) + 2q2 (1 - q2) (sn + sc) + (1 - q2)^2}$ );
    q2t =  $(1 - m) * \frac{q2^2 (1 - s) + q2 (1 - q2) (sn + 2sc)}{q2^2 (1 - s) + 2q2 (1 - q2) (sn + sc) + (1 - q2)^2} + m * \frac{q1^2 (1 - s) + q1 (1 - q1) (sn + 2sc)}{q1^2 (1 - s) + 2q1 (1 - q1) (sn + sc) + (1 - q1)^2}$ ); *)
    q1 = q1t; q2 = q2t;
    AppendTo[list1, q1];
    AppendTo[list2, q2];
    , time];
  {list1, list2}
]

attractor[q10_, q20_, s_, c_, h_, m_, time_, th_, attractors_] :=
Module[{dd, dd1, distances, a1}, (*Given a list of attractors,
  for a point (q10,q20) returns the closest attractor after dynamics run of
  length "time", as long as it is within distance "th" in allele frequency
  space (return the position of the attractor in the list of attractors). If
  there are no equilibria at distance "th", returns 0*)
  dd = dynamics[q10, q20, s, c, h, m, time];
  dd1 = {Last[dd[[1]]], Last[dd[[2]]]};
  distances = Table[EuclideanDistance[dd1, a], {a, attractors}];
  a1 = Position[distances, Min[distances]][[1, 1]];
  If[distances[[a1]] < th, a1, 0]

```

```

]

traj[s_, c_, h_, m_, Ne_, time_, q0_] :=
Module[{q1, q2, q1t, q2t, list}, (*A simulated trajectory of
  length "time" starting at q0 with genetic drift Ne(Haplod)*)
  q1 = q0[[1]]; q2 = q0[[2]];
  list = {{q1, q2}};
  sc = c * (1 - s);
  sn =  $\frac{1}{2} (1 - c) * (1 - h * s)$ ;
  Do[
    q1mig = (1 - m) q1 + m q2;
    q2mig = (1 - m) q2 + m q1;
    meanfit1 = q1mig2 (1 - s) + 2 q1mig (1 - q1mig) (2 sn + sc) + (1 - q1mig)2;
    meanfit2 = q2mig2 (1 - s) + 2 q2mig (1 - q2mig) (2 sn + sc) + (1 - q2mig)2;
    q1t =  $\frac{q1mig^2 (1 - s) + q1mig (1 - q1mig) (sn + sc)}{meanfit1}$ ;
    q2t =  $\frac{q2mig^2 (1 - s) + q2mig (1 - q2mig) (sn + sc)}{meanfit2}$ ;
    (*Next 2 code lines switch from Eq.3 to Eq. S2. Activate
      these lines for generating Fig. S10 (selection before migration)*)
    (*q1t = (1 - m) *  $\frac{q1^2 (1 - s) + q1 (1 - q1) (sn + 2sc)}{q1^2 (1 - s) + 2q1 (1 - q1) (sn + sc) + (1 - q1)^2}$  + m *  $\frac{q2^2 (1 - s) + q2 (1 - q2) (sn + 2sc)}{q2^2 (1 - s) + 2q2 (1 - q2) (sn + sc) + (1 - q2)^2}$ );
    q2t = (1 - m) *  $\frac{q2^2 (1 - s) + q2 (1 - q2) (sn + 2sc)}{q2^2 (1 - s) + 2q2 (1 - q2) (sn + sc) + (1 - q2)^2}$  + m *  $\frac{q1^2 (1 - s) + q1 (1 - q1) (sn + 2sc)}{q1^2 (1 - s) + 2q1 (1 - q1) (sn + sc) + (1 - q1)^2}$ );*)
    q1t = N[ $\frac{1}{Ne}$  RandomInteger[BinomialDistribution[Ne, q1t]]];
    q2t = N[ $\frac{1}{Ne}$  RandomInteger[BinomialDistribution[Ne, q2t]]];
    q1 = q1t; q2 = q2t;
    AppendTo[list, {q1, q2}];
    , time];
  list
]

AttractionBasins[s_, c_, h_, m_, th_, time_] :=
Module[{eeeqs}, (*Plots a 101*101 array for q1*q2 of attraction
  basins associated with each point in allele frequency space*)
  eeeqs = N[eqpoints[s, c, h, m]];
  txx = ParallelTable[
    attractor[q1, q2, s, c, h, m, time, th, eeeqs], {q1, 0, 1, 0.01}, {q2, 0, 1, 0.01}]
]

trajQ[s_, c_, h_, m_, Ne_, time2_, ab_, dte_] :=
Module[{traj, abasins}, (*For the scenario parameters (s,c,h,m,Ne),
  a grid of associated attraction basins ab, and the DTE "dte",
  run a stochastic simulated trajectory and check whether the trajectory starting at
  the DTE escapes the basin of attraction of the DTE within "time2" generations*)
  t1 = traj[s, c, h, m, Ne, time2, dte];

```

```

t2 = Round[100 t1] + 1;
t3 = Table[ab[t[[1]], t[[2]], {t, t2}];
(*A list of the attraction basins for each point in the trajectory*)
abasins = Union[t3];
If[Length[Union[t3]] == 1, 1, 0]
(*If at some point in the trajectory the frequencies are in a basin
of attraction other then the DTE's, return 0, otherwise return 1 *)
]

eq1[m_, s_, sc_, sn_] :=
(( (1 - m) q1 + m q2)^2 (1 - s) + ((1 - m) q1 + m q2) (1 - ((1 - m) q1 + m q2)) (sn + sc)) /
(( (1 - m) q1 + m q2)^2 (1 - s) +
2 ((1 - m) q1 + m q2) (1 - ((1 - m) q1 + m q2)) (2 sn + sc) + (1 - ((1 - m) q1 + m q2))^2);

eq2[m_, s_, sc_, sn_] :=
(( (1 - m) q2 + m q1)^2 (1 - s) + ((1 - m) q2 + m q1) (1 - ((1 - m) q2 + m q1)) (sn + sc)) /
(( (1 - m) q2 + m q1)^2 (1 - s) +
2 ((1 - m) q2 + m q1) (1 - ((1 - m) q2 + m q1)) (2 sn + sc) + (1 - ((1 - m) q2 + m q1))^2);
(*Next 2 code lines switch from Eq.3 to Eq. S2. Activate these lines
for generating Fig. S10 (selection before migration)*)
(*eq1[m_, s_, sc_, sn_] := (1 - m) * (q1^2 (1 - s) + q1 (1 - q1) (sn + 2sc)) / (q1^2 (1 - s) + 2q1 (1 - q1) (sn + sc) + (1 - q1)^2) + m * (q2^2 (1 - s) + q2 (1 - q2) (sn + 2sc)) / (q2^2 (1 - s) + 2q2 (1 - q2) (sn + sc) + (1 - q2)^2)
eq2[m_, s_, sc_, sn_] :=
(1 - m) * (q2^2 (1 - s) + q2 (1 - q2) (sn + 2sc)) / (q2^2 (1 - s) + 2q2 (1 - q2) (sn + sc) + (1 - q2)^2) + m * (q1^2 (1 - s) + q1 (1 - q1) (sn + 2sc)) / (q1^2 (1 - s) + 2q1 (1 - q1) (sn + sc) + (1 - q1)^2) *)
Dq1[m_, s_, sc_, sn_] := eq1[m, s, sc, sn] - q1;
Dq2[m_, s_, sc_, sn_] := eq2[m, s, sc, sn] - q2;
J[m_, s_, sc_, sn_] := {{D[Dq1[m, s, sc, sn], q1], D[Dq1[m, s, sc, sn], q2]},
{D[Dq2[m, s, sc, sn], q1], D[Dq2[m, s, sc, sn], q2]}};

stableEigenVal[m_, s_, h_, c_] := Module[{sn, sc, v1, eqPts, eigenVal, stableInd},
sc = c * (1 - s);
sn = 1/2 (1 - c) * (1 - h * s);
eqPts =
Quiet[NSolve[{Dq1[m, s, sc, sn] == 0, Dq2[m, s, sc, sn] == 0}, {q1, q2}, Reals]];
eqPts = Table[{e[[1, 1]] -> N[Round[e[[1, 2]], 10^-5]],
e[[2, 1]] -> N[Round[e[[2, 2]], 10^-5]]}, {e, eqPts}];
eigenVal = Eigenvalues[J[m, s, sc, sn]] /. eqPts;
stableInd = MapThread[And,
{Negative[Re[eigenVal[[All, 1]]], Negative[Re[eigenVal[[All, 2]]]]}];
v1 = Transpose[{Pick[eqPts, stableInd], Pick[eigenVal, stableInd]}];
Table[{v[[1, 1, 2]], v[[1, 2, 2]], v[[2]]}, {v, v1}]
]

ProbEscape[s_, c_, h_, m_, Ne_, th_, time1_, time2_, reps_] :=
Module[{eqs, ab, ev, asb, aa, r1, prob},
(*time1 = for constructing basins, time to allow for convergence;
th= distance from equilibria where convergence is assumed;

```

```

time2= length of trajectory from stable point;*)
eqs = eqpoints[s, c, h, m];
If[Length[eqs] ≠ 9, prob = "N1",
  ab = AttractionBasins[s, c, h, m, th, time1];
  If[MemberQ[Flatten[ab], 0],
    Print["time1 too low ----- s=", s, " c=", c, " h=", h, " m=", m]];
  ev = N[Round[stableEigenVal[m, s, h, c], 10-5]];
  If[Length[ev] ≠ 4, prob = "N2",
    asb = Select[ev, #[[1, 1]] ≠ #[[1, 2]] &];
    (*assymetric stable equilibria with their eigenvalues*)
    asb = Flatten[Take[asb, All, 1], 1];
    (*assymetric stable equilibria *)
    aa = asb[[1]]; (*An assymetric stable point - the DTE*)
    r1 = ParallelTable[trajQ[s, c, h, m, Ne, time2, ab, aa], {reps}];

    prob = N[ $\frac{1}{\text{reps}}$  Count[r1, 0]]
  ];
];
prob
]

```

```

c = 1;
h = 0;
th = 0.01;
time1 = 200;
Ne = 100;
time2 = 100;
reps = 1000;
maxm = 0.11;

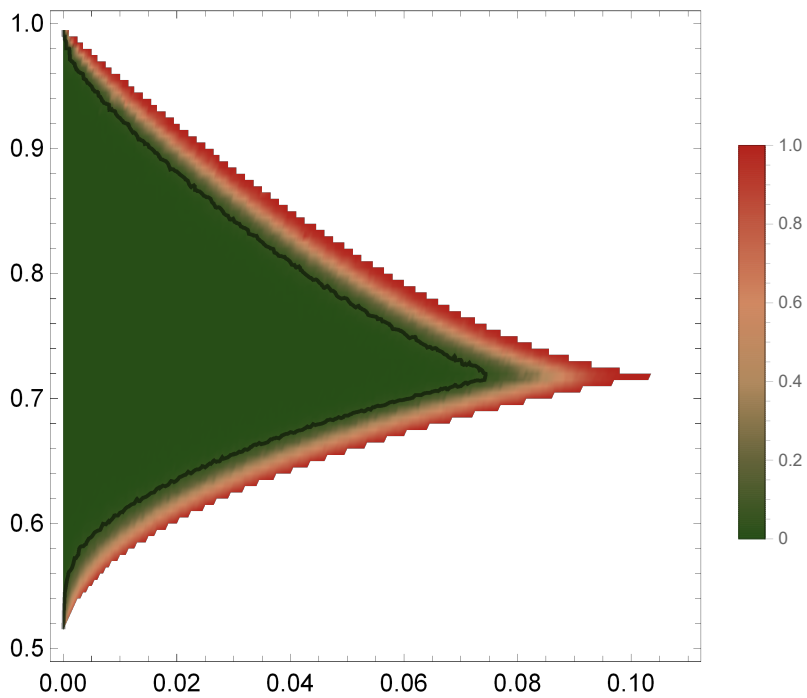
res = Table[Table[{m, s, ProbEscape[s, c, h, m, Ne, th, time1, time2, reps]}],
  {m, 0, maxm, 0.0005}], {s, 0.5, 1, 0.005}];

```

```

b0 = ListDensityPlot[res, DataRange → {{0, maxm}, {0.5, 1}},
  PlotRange → {{0, maxm}, {0.5, 1}}, PlotLegends → Automatic,
  ColorFunction → "RoseColors", FrameTicksStyle → Directive[Black, 13]];
cfff[x_] := Opacity[0, Blue]
b2 = ListContourPlot[res, DataRange → {{0, maxm}, {0.5, 1}},
  PlotRange → {{0, maxm}, {0.5, 1}}, ColorFunction → cfff, Contours → {0.05},
  ContourStyle → Directive[Black, Thick], ContourShading → True];
bbb =
  Show[
    b0,
    b2]

```



Computing the stability radius (Fig. 5B and S10B)

```

SafetyRadius[s_, c_, h_, m_, th_, time1_] := Module[{},
  eqs = eqpoints[s, c, h, m];
  If[Length[eqs] ≠ 9, radius = "N1",
  ab = AttractionBasins[s, c, h, m, th, time1];
  If[MemberQ[Flatten[ab], 0], Print["time1 to low ----- s=",
    s, " c=", c, " h=", h, " m=", m, "---", DateString[]]];
  ev = N[Round[stableEigenVal[m, s, h, c], 10-5]];
  If[Length[ev] ≠ 4, radius = "N2",
  asb = Select[ev, #[[1, 1]] ≠ #[[1, 2]] &];
  (*assmetric stable equilibria with their eigenvalues*)
  asb = Flatten[Take[asb, All, 1], 1];
  (*assmetric stable equilibria *)
  aa = asb[[1]]; (*An assymetric stable point*)
  id = ab[[Round[100 aa[[1]]] + 1, Round[100 aa[[2]]] + 1]];
  l1 = 0.01 (Position[ab, id] - 1);
  l2 = Complement[Flatten[Table[{i, j}, {i, 0, 1, 0.01}, {j, 0, 1, 0.01}], 1], l1];
  dist = Table[EuclideanDistance[aa, l], {l, l2}];
  radius = Min[dist]
  ];
];
radius
]

```

```

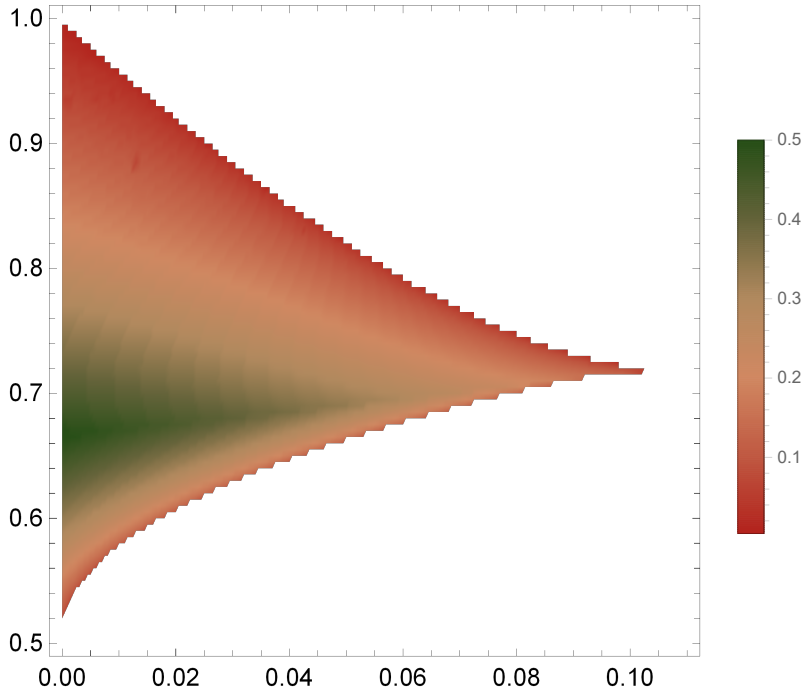
c = 1; h = 0; th = 0.01; time1 = 300;
maxm = 0.11;
res = Table[Table[{m, s, SafetyRadius[s, c, h, m, th, time1]}, {m, 0, maxm, 0.0005}],
  {s, 0.5, 1, 0.005}]

```

```

b0 = ListDensityPlot[res, DataRange → {{0, maxm}, {0.5, 1}},
  PlotRange → {{0, 0.11}, {0.5, 1}}, PlotLegends → Automatic,
  ColorFunction → ColorData[{"RoseColors", "Reverse"}],
  FrameTicksStyle → Directive[Black, 13]]

```



Computing m^* and q_2^* with asymmetric migration (Fig. 6, S12 & S13)

```

$HistoryLength = 0;
DTEvalid[cc_, hh_, ss_, mm_, aa_] :=
Module[{y1, y2, out, qnext1, qnext2, q, sn, sc, q1, q2, eqSolutions,
  eqSolutions2, outq2, rr2, rr3, rr4, rr5, rr6, jacobian}, (*For a given c,
  h,s, and m, tests if there are 9 equilibrium solutions to Eq. 3,
  and returns also the q2 of the DTE*)
out = 1; outq2 = "Na";
If[mm aa > 1, out = 0;];
If[ss == 0, out = 0,
  qnext1 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.
  \{sn \rightarrow \frac{1}{2} (1 - cc) (1 - hh ss), sc \rightarrow cc (1 - ss), q \rightarrow \frac{(1 - aa mm) q1 + mm q2}{1 - aa mm + mm}\};
  (*Eq. 3*)
  qnext2 = 
$$\frac{q^2 (1 - ss) + 2 q (1 - q) (sn + sc)}{q^2 (1 - ss) + 2 q (1 - q) (2 sn + sc) + (1 - q)^2} /.$$$$

```



```

{sn →  $\frac{1}{2} (1 - cc) (1 - hh ss)$ , sc → cc (1 - ss), q →  $\frac{(1 - mm) q2 + aa mm q1}{1 + aa mm - mm}$ };
(*Eq. 3*)
eqSolutions = Quiet[NSolve[qnext1 == q1 && qnext2 == q2, {q1, q2}]];
(*Finding equilibrium solutions for Eq. 3*)
eqSolutions2 = N[Round[Table[{e[[1, 2]], e[[2, 2]]}, {e, eqSolutions}], 10-3]];
If[Union[Flatten[Table[{Im[e[[1]]], Im[e[[2]]]}, {e, eqSolutions2}]]] ≠ {}, out = 0];
(*Checks that all solutions are real*)
y2 = If[out == 1, Union[Table[If[0 ≤ t ≤ 1, 1, 0], {t, Flatten[eqSolutions2]}]]];
(*Checks that all solutions are between 0 and 1*)
If[y2 ≠ {}, out = 0];
If[out == 1, (*Compute q2 of the DTE*)
  jacobian = {{D[qnext1, q1], D[qnext1, q2]}, {D[qnext2, q1], D[qnext2, q2]}};
  (*The Jacobian from equation 5.*)
  rr2 = Table[Abs[Eigenvalues[jacobian /. {r[[1]], r[[2]]}], {r, eqSolutions}]; (*Taking
    the absolute eigenvalues for the jacobian evaluated at the different solutions*)
  rr3 = Table[If[e[[1]] < 1 && e[[2]] < 1, True, False], {e, rr2}];
  (*Testing to see for each equilibrium point if it is stable*)
  rr4 = Round[Table[{e[[1, 2]], e[[2, 2]]}, {e, eqSolutions}], 0.00001];
  rr5 = Table[If[rr3[[i]] && (rr4[[i, 1]] > rr4[[i, 2]]), rr4[[i, 2]], Null],
    {i, Length[eqSolutions]}]; (*Taking all the points that are stable and q1>q2*)
  rr6 = DeleteCases[rr5, Null];
  If[rr6 == {}, out = 0]; (*If there are no DTEs, out=0*)
  outq2 = If[Length[rr6] == 1, rr6[[1]], Null]
  (*Making sure that there is only one DTE, and taking q2 of the DTE*)
];
];
{out, outq2} (*out=1 means DTE exists, otherwise out=0*)
]

mqstar[cc_, hh_, ss_, aa_] := Module[{maxm, jumps, t0, t1, t2, t3, outm, w1, outq},
  (*For a given c, h, and s, returns m* and q2*)
  If[DTEValid[cc, hh, ss, 0] == 0, outm = "Na"; outq = "Na",
    maxm = 0.2; (*Maximal m for searching m*)
    jumps = 10-3; (*Resolution in m for searching m*)
    t0 = Table[{N[mm], DTEValid[cc, hh, ss, mm, aa]}, {mm, 0, maxm, jumps}];
    t1 = Table[{t[[1]], t[[2, 1]]}, {t, t0}];
    t2 = Flatten[Take[t1, All, {2}]];
    t3 = Flatten[t1[[FirstPosition[t2, 0] - 1]] [[1]];
    outm = If[NumberQ[t3], t3, "Na"];
    w1 = Table[{t[[1]], t[[2, 2]]}, {t, t0}];
    outq = Max[DeleteCases[Flatten[Take[w1, All, {2}]], "Na"]]
  ];
  {outm, outq}
]

ComputeGridmqstar[h_, aa_] := Module[{jmps, jmpc, ress, pmstar, pqstar},
  jmps = 0.01; (*resolution in s*)
  jmpc = 0.01; (*resolution in c*)
  ress = Quiet[ParallelTable[

```

```

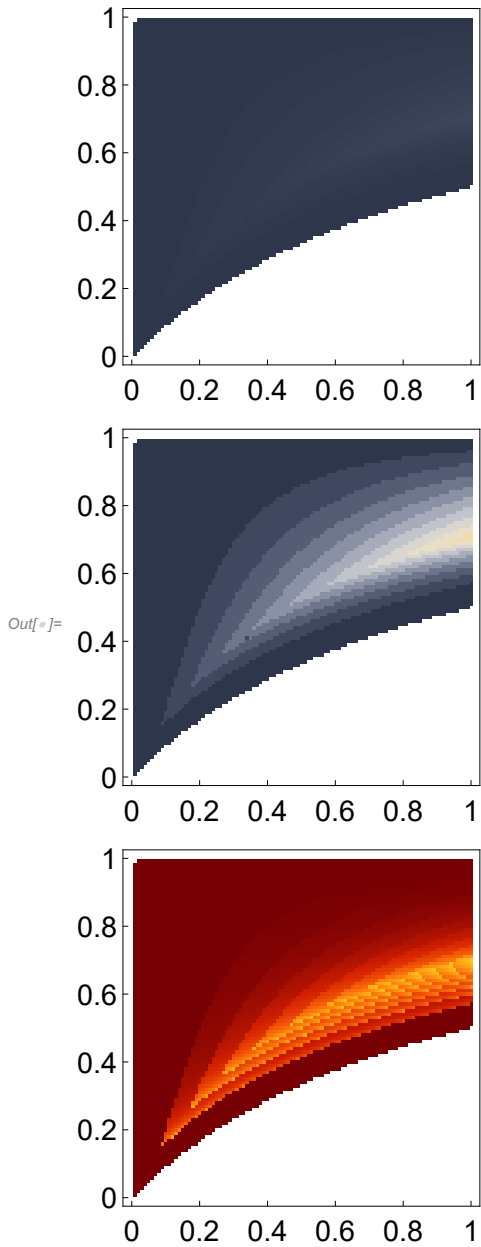
    Table[{ccc, sss, mqstar[ccc, h, sss, aa]}, {ccc, 0, 1, jmpc}], {sss, 0, 1, jmps}]]];
    pmstar = Table[{r[[1]], r[[2]], r[[3, 1]]}, {r, Flatten[ress, 1]}];
    pqstar = Table[{r[[1]], r[[2]], r[[3, 2]]}, {r, Flatten[ress, 1]}];
    {pmstar, pqstar}
]

h = 1;
a = 10;
p1 = ComputeGridmqstar[h, a];

cf1[x_] := If[! NumberQ[x], White, ColorData["GrayYellowTones"] [ $\frac{x}{0.107}$ ]]
cf2[x_] := If[! NumberQ[x], White, ColorData["SolarColors"] [ $\frac{x}{0.28}$ ]]

si = 210;
nn = Sqrt[Dimensions[p1[[1]]][[1]]];
mm1 = Partition[Flatten[Take[p1[[1]], All, {3}]], nn];
mmx1 = Table[Table[If[NumberQ[mm1[[i, j]]], 10 mm1[[i, j]], mm1[[i, j]]], {j, Length[mm1]}],
  {i, Length[mm1]}];
mmq1 = Partition[Flatten[Take[p1[[2]], All, {3}]], nn];
ticks = {0, 0.2, 0.4, 0.6, 0.8, 1};
ticksize = 15;
gmm1 = MatrixPlot[Reverse[mm1], ColorFunction -> cf1,
  ColorFunctionScaling -> False, DataRange -> {{0, 1}, {0, 1}}, FrameLabel -> None,
  FrameTicks -> ticks, FrameTicksStyle -> ticksize, ImageSize -> si];
gmmx1 = MatrixPlot[Reverse[mmx1], ColorFunction -> cf1, ColorFunctionScaling -> False,
  DataRange -> {{0, 1}, {0, 1}}, FrameLabel -> None,
  FrameTicks -> ticks, FrameTicksStyle -> ticksize, ImageSize -> si];
gmmq1 = MatrixPlot[Reverse[mmq1], ColorFunction -> cf2, ColorFunctionScaling -> False,
  DataRange -> {{0, 1}, {0, 1}}, FrameLabel -> None,
  FrameTicks -> ticks, FrameTicksStyle -> ticksize, ImageSize -> si];
ggg = Grid[{{gmm1}, {gmmx1}, {gmmq1}}]

```



Computing number of solutions (Figs. S3-S9)

```
eq1[m_, s_, sc_, sn_] :=
  (((1 - m) q1 + m q2)^2 (1 - s) + ((1 - m) q1 + m q2) (1 - ((1 - m) q1 + m q2)) (sn + 2 sc)) /
  (((1 - m) q1 + m q2)^2 (1 - s) +
    2 ((1 - m) q1 + m q2) (1 - ((1 - m) q1 + m q2)) (sn + sc) + (1 - ((1 - m) q1 + m q2))^2);

eq2[m_, s_, sc_, sn_] :=
```

```


$$\left( \left( (1-m) q_2 + m q_1 \right)^2 (1-s) + \left( (1-m) q_2 + m q_1 \right) \left( 1 - \left( (1-m) q_2 + m q_1 \right) \right) (sn + 2 sc) \right) /$$


$$\left( \left( (1-m) q_2 + m q_1 \right)^2 (1-s) + \right.$$


$$\left. 2 \left( (1-m) q_2 + m q_1 \right) \left( 1 - \left( (1-m) q_2 + m q_1 \right) \right) (sn + sc) + \left( 1 - \left( (1-m) q_2 + m q_1 \right) \right)^2 \right);$$

(*For selection-before-migration model (Eq.S2, Figs. S7-S9),
use th equations in the next two code lines instead of the pvious two lines*)
(*eq1[m_, s_, sc_, sn_] := (1-m) *  $\frac{q_1^2 (1-s) + q_1 (1-q_1) (sn+2sc)}{q_1^2 (1-s) + 2q_1 (1-q_1) (sn+sc) + (1-q_1)^2}$  + m *  $\frac{q_2^2 (1-s) + q_2 (1-q_2) (sn+2sc)}{q_2^2 (1-s) + 2q_2 (1-q_2) (sn+sc) + (1-q_2)^2}$ 
eq2[m_, s_, sc_, sn_] :=
(1-m) *  $\frac{q_2^2 (1-s) + q_2 (1-q_2) (sn+2sc)}{q_2^2 (1-s) + 2q_2 (1-q_2) (sn+sc) + (1-q_2)^2}$  + m *  $\frac{q_1^2 (1-s) + q_1 (1-q_1) (sn+2sc)}{q_1^2 (1-s) + 2q_1 (1-q_1) (sn+sc) + (1-q_1)^2}$  *)
Dq1[m_, s_, sc_, sn_] := eq1[m, s, sc, sn] - q1;
Dq2[m_, s_, sc_, sn_] := eq2[m, s, sc, sn] - q2;
J[m_, s_, sc_, sn_] := {{D[Dq1[m, s, sc, sn], q1], D[Dq1[m, s, sc, sn], q2]},
{D[Dq2[m, s, sc, sn], q1], D[Dq2[m, s, sc, sn], q2]}};

stableEigenVal[m_, s_, h_, c_] := Module[{sn, sc, v1, eqPts, eigenVal, stableInd},
sc = c * (1 - s);
sn = (1 - c) * (1 - h * s);
eqPts =
Quiet[NSolve[{Dq1[m, s, sc, sn] == 0, Dq2[m, s, sc, sn] == 0}, {q1, q2}, Reals]];
eqPts = Table[{e[[1, 1]] -> N[Round[e[[1, 2]], 10-5]],
e[[2, 1]] -> N[Round[e[[2, 2]], 10-5]]}, {e, eqPts}];
eigenVal = Eigenvalues[J[m, s, sc, sn]] /. eqPts;
stableInd = MapThread[And,
{Negative[Re[eigenVal[[All, 1]]]], Negative[Re[eigenVal[[All, 2]]]]}];
v1 = Transpose[{Pick[eqPts, stableInd], Pick[eigenVal, stableInd]}];
Table[{v[[1, 1, 2]], v[[1, 2, 2]], v[[2]], {v, v1}}
]

StabilityRadius[m_, s_, h_, c_] := Module[{}],
r1 = stableEigenVal[m, s, h, c];
r2 = Table[{r[[1, 1, 2]], r[[1, 2, 2]], Min[Abs[r[[2]]]]}, {r, r1}];
r3 = N[Round[r2, 10-8]];
r4 = Select[r3, #[[1]] == {0., 0.} || #[[1]] == {1., 1.} &];
r5 = Complement[r3, r4];
If[Length[r5] == 2 && r5[[1, 2]] == r5[[1, 2]], sr = r5[[1, 2]], Print["Error"]];
sr
]

cff2[x_] := Which[x == 7, Lighter[Blue, 0.3], x == 5, Lighter[Blue, 0.6], x == 3,
Lighter[Blue, 0.9], ! IntegerQ[x], Lighter[Red, 0.8 (1 - x)], True, White];
(*Color function for the stability grid*)

stabilitygrid[h_, m_] := Module[{kk, qq1, qq2},
kk =
ParallelTable[Table[If[Length[eqpoints[s, c, h, m]] != 9, Length[eqpoints[s, c, h, m]],
StabilityRadius[m, s, h, c]], {c, 0, 1, 0.01}], {s, 0, 1, 0.01}];
qq1 = ListPlot[{{0, 1}}, PlotRange -> {{0, 1}, {0, 1}}, Frame -> True,
FrameTicks -> {{0, 0.5, 1}, {0, 0.5, 1}}, AspectRatio -> 1];

```

```

qq2 = MatrixPlot[kk, DataReversed → True, FrameTicks → True,
  ColorFunctionScaling → False, ColorFunction → cff2, DataRange → {{0, 1}, {0, 1}}];
Show[qq1, qq2]
]

cff3[x_] := Which[
  x == 1, RGBColor["#00b359"], (*Green*)
  x == 2, RGBColor["#33d6ff"], (*Blue*)
  x == 3, RGBColor["#ff0066"], (*Red*)
  x == 4, RGBColor["#990099"], (*Purple*)
  x == 5, RGBColor["#ff7733"], (*Orange*)
  x == 6, White,
  True, White];
(*Color function for the stability grid*)

stabilitygrid2[h_, m_] := Module[{qq1, qq2},
  jumps = 0.01;
  kk = ParallelTable[Table[Neq = Length[eqpoints[s, c, h, m]];
    Nst = Length[stableEigenVal[m, s, h, c]]; Which[
      Neq == 9 && Nst == 4, 1, (*9 eq points; 2 assymetric stable*)
      Neq == 5 (*&&Nst==2*), 2, (*5 eq points; 0 assymetric stable*)
      Neq == 3 && Nst == 2, 3,
      (*3 eq points; 0 assymetric stable, 1 symetric unstable*)
      Neq == 9 && Nst == 1, 4, (*9 eq points; 0 assymetric stable*)
      Neq == 3 && Nst == 1, 5, (*3 eq points; 0 assymetric stable, 1 symetric stable*)
      Neq == 2 && Nst == 1, 6, (*2 eq points; 1 stable*)
      Neq == 2 && Nst == 2, 6, (*2 eq points; 2 stable*)
      True, 7],
    {c, 0, 1, jumps}], {s, 0, 1, jumps}];
  qq1 = ListPlot[{{0, 1}}, PlotRange → {{0, 1}, {0, 1}},
    Frame → True, FrameTicks → {{0, 0.5, 1}, {0, 0.5, 1}}, AspectRatio → 1];
  qq2 = MatrixPlot[kk, DataReversed → True, FrameTicks → True,
    ColorFunctionScaling → False, ColorFunction → cff3, DataRange → {{0, 1}, {0, 1}}];
  Show[qq1, qq2]
]

```

```

In[56]:= h = 0; m = 0.01;
DateString[]
stabilitygrid2[h, m]
DateString[]

```

