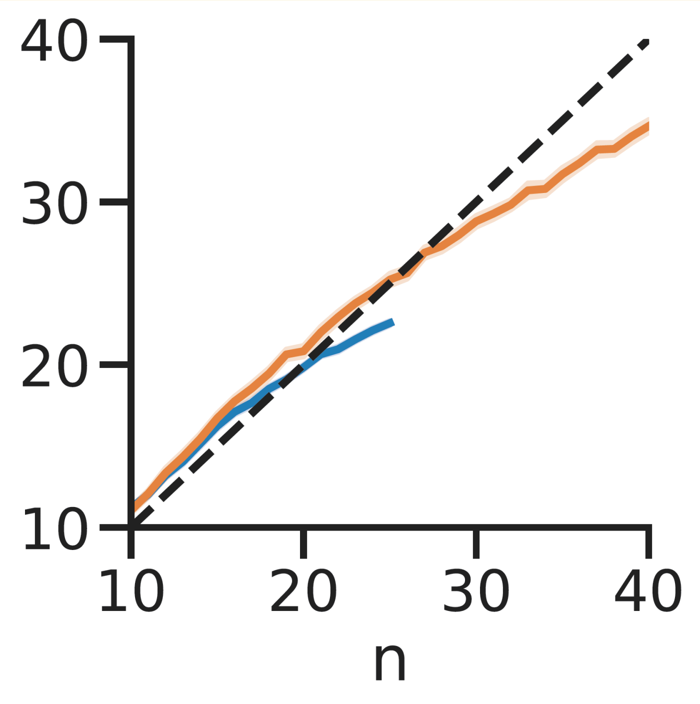
Efficient coding, efficiently coded



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Efficient Coding: Key Ideas

1. Sensory Space Representation

- Objective stimulus features x_0 encoded in 'neurocognitive' sensory space (r)
- Noise on r is, by definition, **homoscedastic** (constant variance)
- Encoding function:
  e,g., $f(x) = x^\alpha$ ($\alpha < 1$)
- Neurocognitive representation r given a objective stimulus x_0 is a *random variable.*

$$r|x_0 \sim \mathcal{N}(f(x_0), \nu^2)$$

Efficient coding: Key ideas

2. Bayesian Inference

- Estimate stimulus value of stimulus x, $\hat{x}$ using Bayesian inference:
- Posterior distribution:

$$p(x|r) = \frac{p(r|x)p(x)}{p(r)}$$

- Posterior mean is least-square estimator

$$\hat{x} = \mathbb{E}[x|r] = \int x\, p(x|r)\, dx$$

Why computational graphs?

- For model fitting we want to estimate parameters.
- Often we can not derive likelihood functions, but we can *evaluate* them for specific values.
  - Approximate integrals using grids (GPU!)
  - MCMC sampling
- Both often involve the same calculation on a very large number of variables.

Implementation Steps

1. Define the Generative Model

- **Encoding function:** $f(x) = x^\alpha$
- **Noise model:** $r \sim \mathcal{N}(f(x_0), \nu^2)$

2. Build the Likelihood Grid

- Create a grid of possible x_0 and r values
- Make a $p(r|x_0)$ for each pair

3. Bayesian Inference

- For each observed r, compute the posterior:

$$p(x|r) = \frac{p(r|x)p(x)}{p(r)}$$

- Estimate $\hat{x}$ as the expected value:

$$\hat{x} = \mathbb{E}[x|r]$$

4. Data Likelihood Function

- Define a function that returns the response distribution over $\hat{x}$ for any x_0: $p(\hat{x}|x_0)$.

Approach

Approximate (bounded) distributions using large arrays (vectorize, vectorize, vectorize).

Assignment 5: Efficient coding

- We wil now go over some code together in `notebooks/5_efficient_coding.ipynb`