Machine learning for image-based wavefront sensing

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1 Previous results

Estimation of 20 Zernike coefficients from 2 intensity measurements with convolutional neural network (CNN). The various state-of-art architectures have been explored (inception, resnet, ...). The training dataset consists in 100000 images in and out of focus ($\lambda/4$).

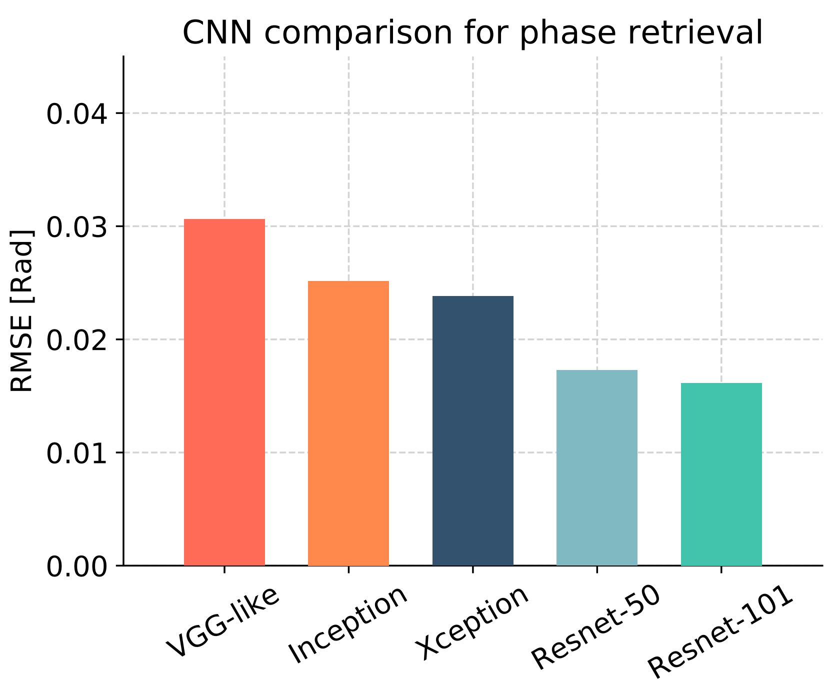
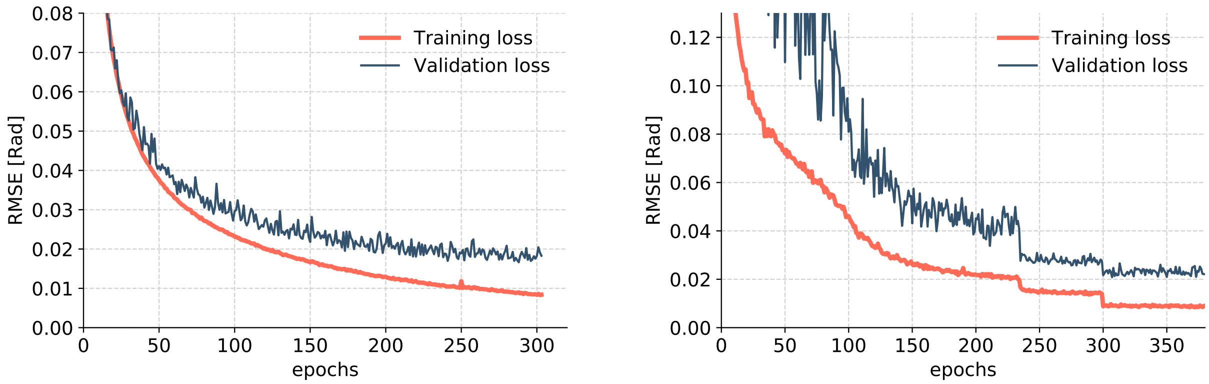


Figure 1: Root mean suqared error between the exact phase and the estimated phase (= output of the CNN)

The models remain structurally unaltered, only the last fully-connected layers have been replaced to match the outputs shape of 1×20. Also note that the last fully-connected layers does not includes activation functions in order to perform regression. The previous architectures demonstrate improved convergence starting from pretrained weights while preserving the accuracy of models trained from scratch. Here are some example of training curves for the Inception architecture and the Resnet architecture.



(a) Resnet-50 (b) Inception v3 with staircase decreasing lr.

Figure 2: Train and validation curves, $optimizer = SGD$, $lr = 0.001$.

2 Unet architecture

Reconstruction of the phase directly from intensity measurements. The Zernike basis hypothesis is no longer required.

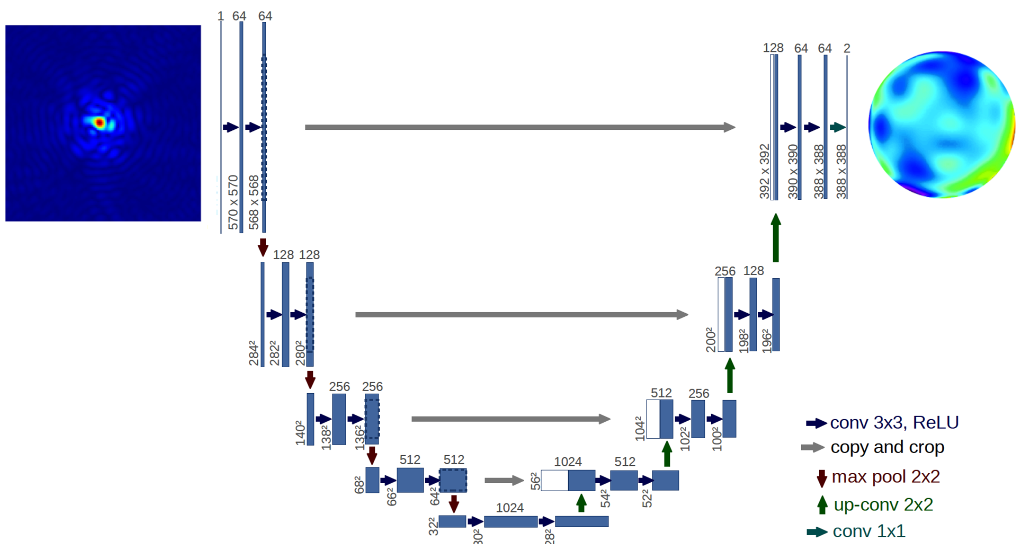
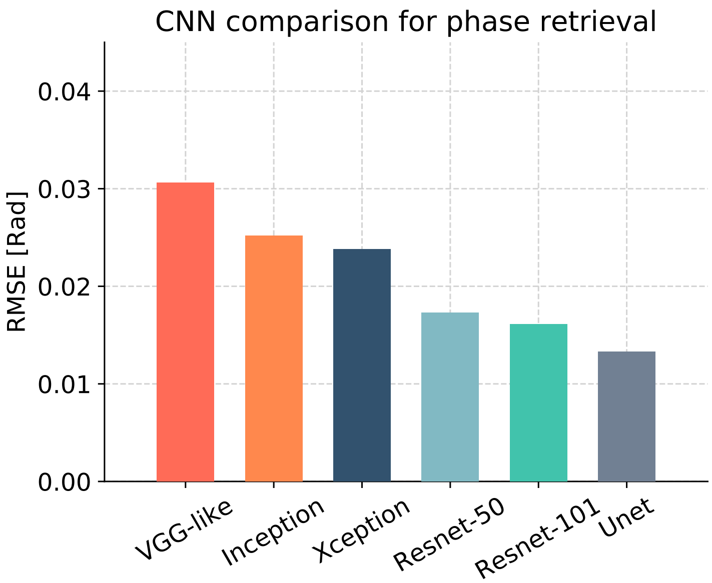


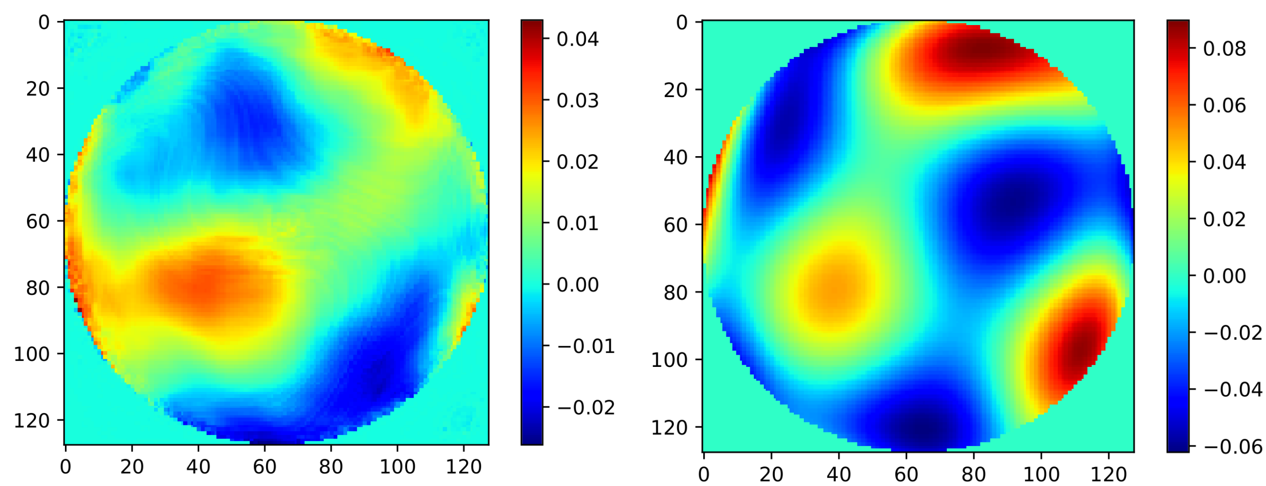
Figure 3: Unet-architecture

For the same dataset as previously, the root mean squared error associated to the Unet architecture exhibits a slightly improvement,



Model	RMSE [nm]	RMSE [rad]
VGG-like	10.72	0.0306
Inception v3	8.81	0.0251
Xception	8.34	0.0238
Resnet-50	6.05	0.0172
Resnet-101	5.65	0.0161
Unet	4.62	0.0132

Comparison of the phase estimation between the Unet and the Resnet-50 architecture. As the Resnet-50 architecture uses the Zernike basis to reconstruct the phase, the estimation is much more smooth.



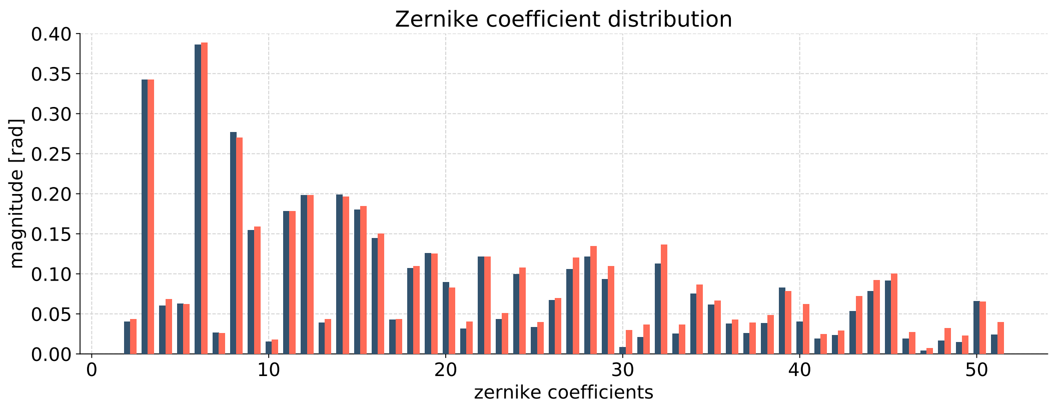
(a) Unet

(b) Resnet-50

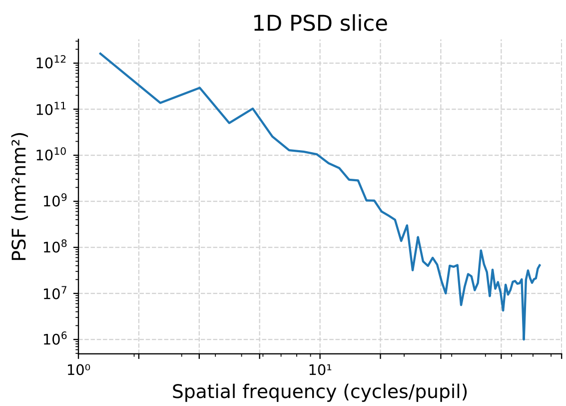
Figure 4: Residual phase error comparison for the same input.

3 Real conditions (z=100 and Noise)

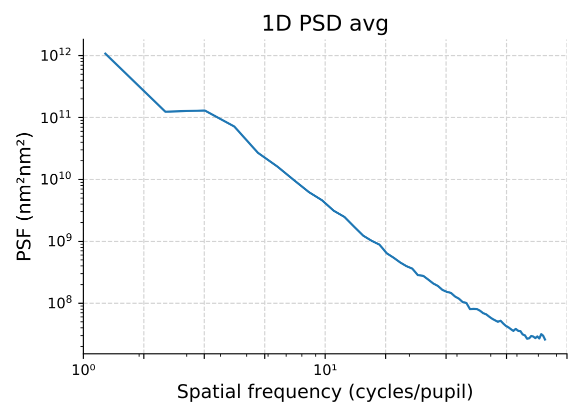
For the next section the number of Zernike coefficients has been extended to 100, the dataset is still constitued of 100000 images, in and out of focus. To synthetically create NCPA we created a phase map that reflected typical errors of conventional optics; this was achieved by using the first 100 Zernike polynomials and dividing each coefficient by its radial order. Dividing each coefficient by the radial order is an approximation of a Power Spectral Density profile, S, with $S \approx 1/f^2$ (where f is the spatial frequency).



(a) Coeffiecients distributions



(b) Slice



(c) Azimutal averaging

Figure 5: Power Spectral Density profile.

3.1 Noise

We model the number of electrons collected at a site by N_{tot} and distinguish different sources of noise,

$$N_{tot} = N + N_S + N_{DC} + N_R \tag{1}$$

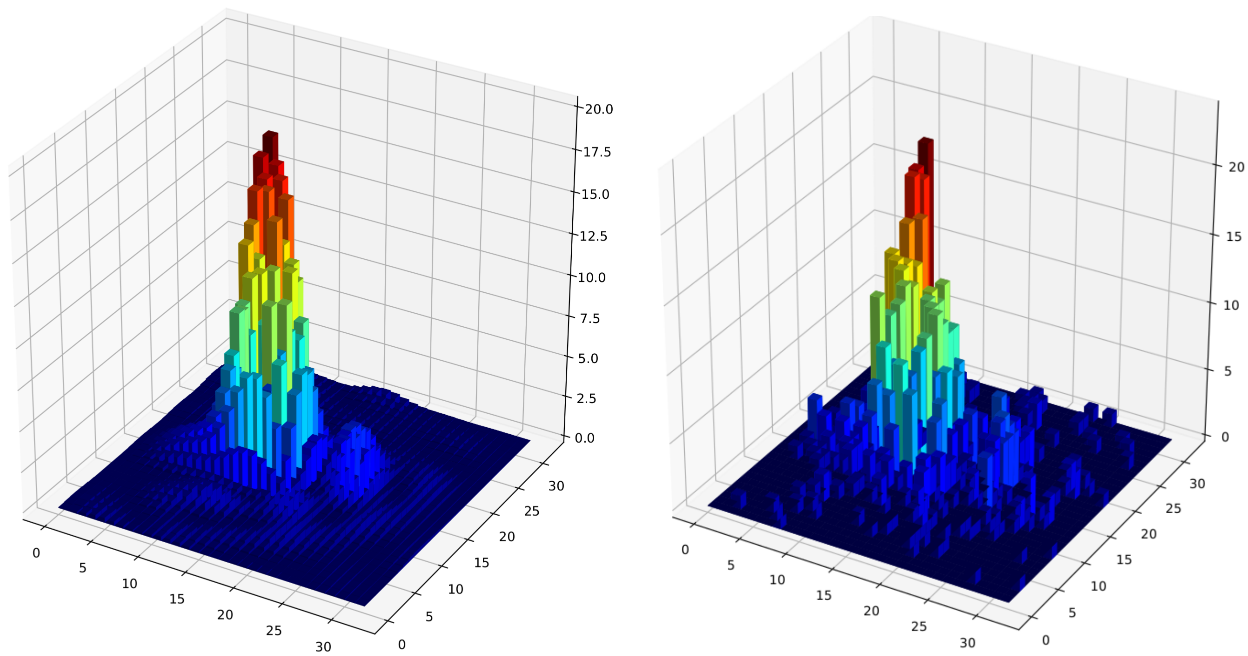
$\boldsymbol{N_S}$, Shot noise is a result of the quantum nature of light and characterizes the uncertainty in the number of electrons stored at a collection site. This number of electrons follows a Poisson distribution (P) so that its variance equals its mean (N). Shot noise is a fundamental limitation and cannot be eliminated.

$$P(k) = \frac{N^k e^{-k}}{N!} \tag{2}$$

Assuming then $N >> 1$, the Poisson distribution tends towards Gaussian distribution, with a mean of N, and a standard deviation of $\sigma = \sqrt{N}$,

$$P(k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(k-N)^2/2\sigma^2} \tag{3}$$

Therefore, if we detect N photons per second from an astronomical source, the best we can do is to say that the true mean photon rate from the star has a likely mean of N, and an uncertainty of $\sqrt{N}$.

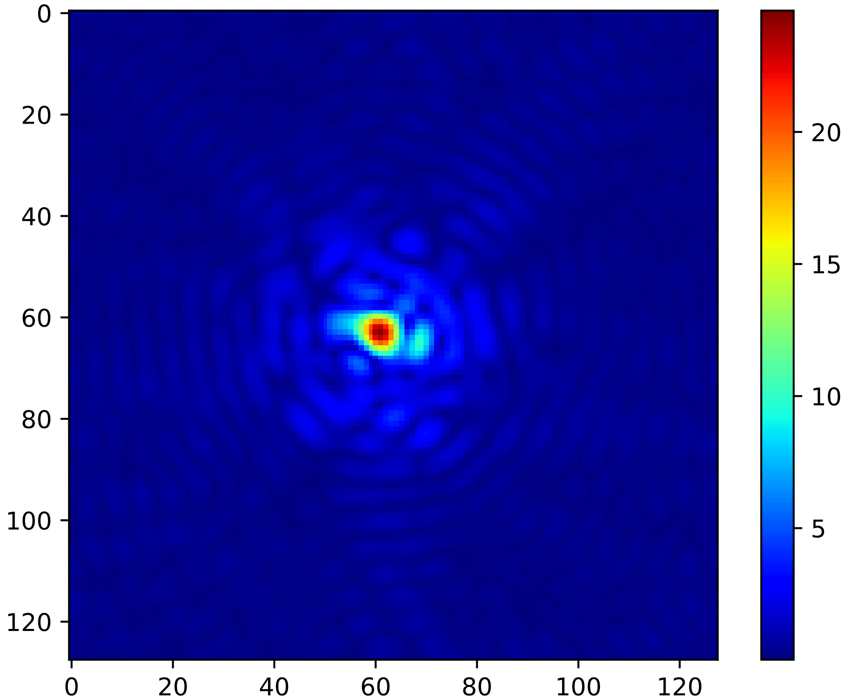


(a) (b)

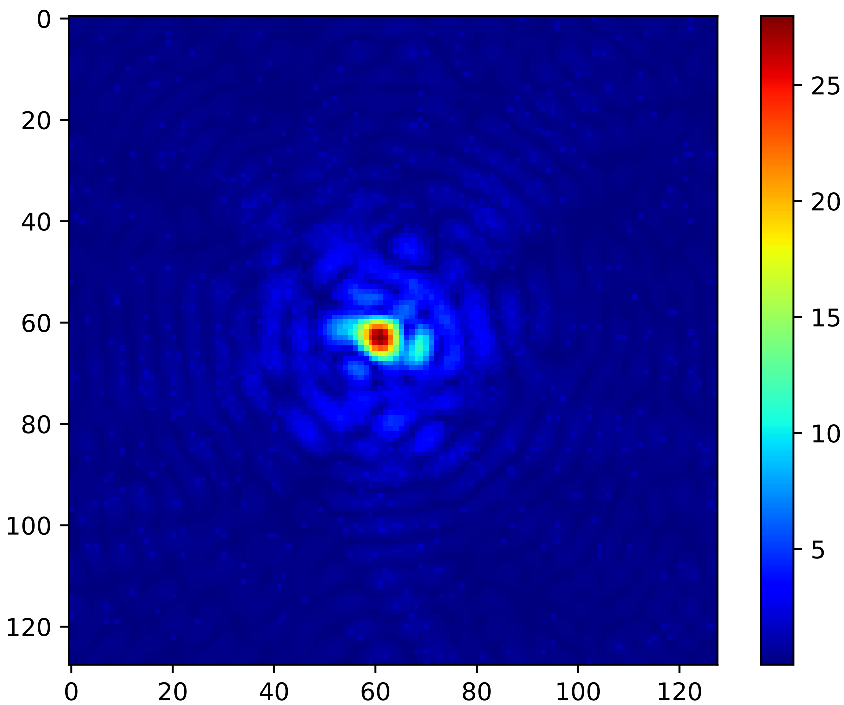
Figure 6: PSF (left) with corresponding noise (right).

The on-chip output amplifier sequentially transforms the charge collected at each site into a measurable voltage. The amplifier generates zero mean read noise $\boldsymbol{N_R}$ that is independent of the number of collected electrons. The read noise is modeled by a normal (guassian) distribution of random numbers. $\boldsymbol{N_{DC}}$, Not simulated.

$$SNR \approx 100 \tag{4}$$

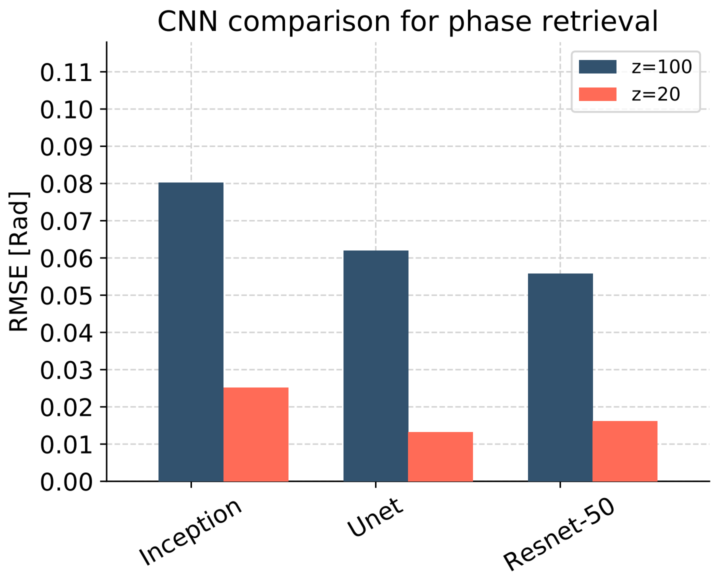


(a) No noise



(b) Noise

3.2 Simulations



Model	RMSE [nm]	RMSE [rad]
Inception v3	22.134	0.0631
Resnet-50	19.551	0.0558
Unet	21.714	0.0620

- Resnet-50: Average strehl after correction = 0.973 (In = 0.509723, Out = 0.092578)

Unet optimization procedure:

(a) (b)

Figure 8: SGD nesterov, momentum=0.9

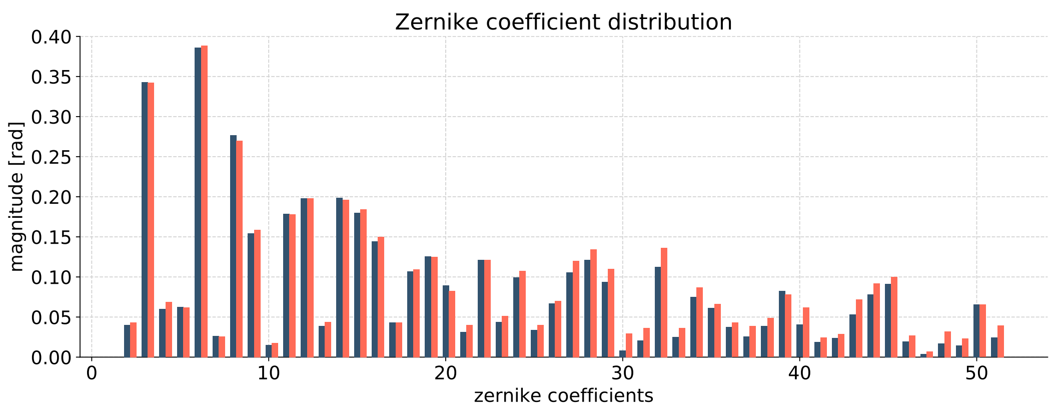
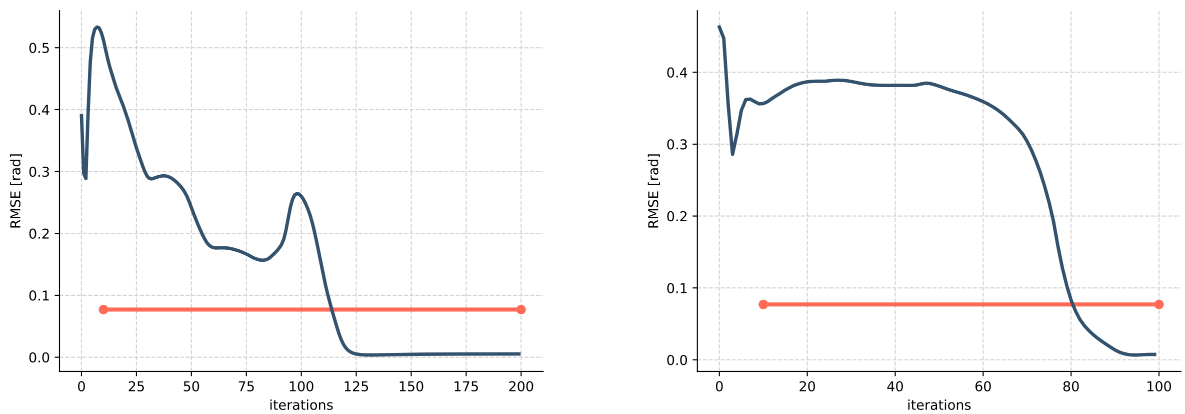


Figure 9: Exact (blue) and estimation (red) of Zernike coefficients by the Resnet-50

3.3 Algorithms

- Gerchberg-Saxton
- Error-reduction
- Input-Output (Output-Output)
- Gradient Descent (WIP)



(a) Randomly initialized (b) Zero initialized

Figure 10: Input-Output (blue) vs Resnet (red)

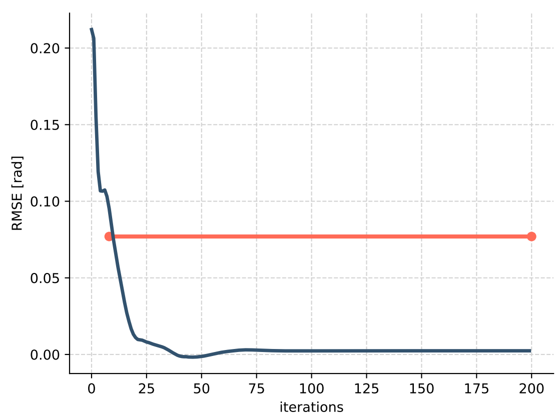


Figure 11: Fine-tuned initializing