Advanced Discrete Optimization 2022/23
Homework 1 - Due date: 12 March 2023

- You are expected to work with your team members and every team member is expected to be able solve each problem. You can discuss with other groups, but the solutions of each group should be original; this also means that you should not copy the solutions of other groups or any other source.
- You need to send your jupyter notebooks by email in a single zip file with subject "Homework1 - Table xx"
- If you encounter problems in typesetting formulas and/or producing figures, you can write them on paper and then add the scan (but please write clearly).
- **Homeworks submitted after the due date will not be accepted.**

Question 1:

In wireless networks, the main installation costs are given by the devices that must be installed at the relay nodes. A basic optimization problem in wireless network design is known as *Minimum Relay Problem.*

Let us consider an undirected graph $G = (V, E)$, where each node represents a possible relay location, and each edge represents a communication link that must be served/covered by the wireless network. The cost c_i of installing a relay at location $i \in V$ is given. The *Minimum Relay Problem* consists in selecting a set of locations with overall minimum installation cost in such a way that every communication link is covered by at least one installed relay.

The problem can be modeled as follows:

$$\min \sum_{i \in V} c_i x_i \tag{1}$$

$$x_i + x_j \geq 1 \quad \forall e = \{i, j\} \in E \tag{2}$$

$$x_i \in \{0, 1\} \quad \forall i \in V \tag{3}$$

1. Write in Pyomo/Python the following 2 heuristics:
   - ***Randomized rounding.***
     (a) Solve the Linear relaxation of the problem and obtain the fractional solution $\bar{x}$.
     (b) Build a set of vertices C as follows:
       i. Put in C all vertices $i \in V$ with $\bar{x}_i = 1$
       ii. Until the set of edges E is entirely covered by $C \subseteq V$ (that is every edge has at least one endpoint in C):
       Select randomly a vertex $i \in V$ such that $0 < \bar{x}_i < 1$, and add i to C with probability $\bar{x}_i$.

- ***Very Large Scale Neighborhood (VLSN).*** Starting from an heuristics solution (the one found with the randomized rounding), at each iteration explore the neighborhood solving the following ILP problem:

$$\min \sum_{i \in V} c_i x_i \tag{4}$$

$$x_i + x_j \geq 1 \quad \forall e = \{i, j\} \in E \tag{5}$$

$$\sum_{i \in V \setminus S} x_i + \sum_{i \in S} (1 - x_i) \leq k \tag{6}$$

$$x_i \in \{0, 1\} \quad \forall i \in V \tag{7}$$

  where S is the set of vertices whose variable is equal to 1 in the current solution and k is the neighborhood size.

2. Using the two heuristics, solve the following instances: `MRP_10.dat`, `MRP_20.dat`, `MRP_40.dat`, `MRP_80.dat`, `MRP_160.dat`.

3. Compare the heuristic solutions between them and with the bound provided by the continuous relaxation (LB) (in terms of value and computational time)