Cost function: $C(\quad) = Tr(\rho H) = Tr\left[U(\theta)\rho U^\dagger(\theta) \sum f_k \hat{P}_k\right]$

$$C(\theta,\phi) = \sum_k f_k(\phi)\, Tr\left[\hat{P}_k U(\theta)\, \rho_{ref}\, U^\dagger(\theta)\right]$$

algo:

$$H = \sum_k f_k(\phi)\, \hat{P}_k$$

$$\boxed{\theta \to \arg\min_\theta C(\theta,\phi)} \rightleftarrows \boxed{\phi \to \arg\min_\phi C(\theta,\phi)}$$

$\Sigma \cdot N_\theta \cdot O(n)$ parameter shift rule + gradient descent

eg: $U_\ell(\theta_\ell) = \prod_m e^{-i\theta_m \sigma_m} W_m$ 且 $\boxed{f_k = 1}$

则有 $\theta_+ = \theta + \Delta\theta$, $\theta_- = \theta - \Delta\theta$

$$\boxed{\frac{\partial C}{\partial \theta_\ell}} = \sum_k \frac{1}{2\sin\alpha}\left(Tr\left[\hat{P}_k U^\dagger(\theta_+)\rho\, U(\theta_+)\right] - Tr\left[\hat{P}_k U^\dagger(\theta_-)\rho\, U(\theta_-)\right]\right)$$

若更复杂，则使用chain rule

[ ] Variational Quantum Algorithms

$$\frac{\partial C(\theta,\phi)}{\partial f_k} = Tr\left[\hat{P}_k U^\dagger(\theta)\, \rho_0\, U(\theta)\right]$$

$$H^{fermi}(\phi) = \sum_k f_k^{fermi}(\phi)\, \hat{P}_k = \boxed{U(\phi)}\, \boxed{H_0}\, \boxed{U^\dagger(\phi)}$$

① effectiveness of fermi net work

每个收缩指标对应一个虚的 state

② unitary tensor network

每个 element 都 unitary → 可使 unitary

每1 element 都 unitary → 整体 unitary

③ unitary + Fermi

straightforward

④ optimization algorithm ?

⑤ 用什么 network ?

$$C_2C_2^\dagger \; C_3C_3^\dagger \; C_1^\dagger C_1 |0\rangle$$

$$\| \quad 0$$

$$C_1^\dagger C_1 |0\rangle$$

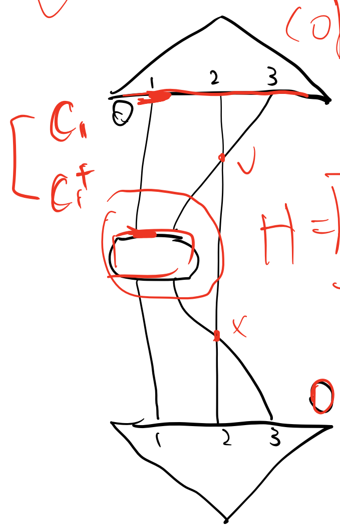
$$0$$

$$\langle 110 | \, t\hat{c}_1^\dagger \hat{c}_3 \, | 011 \rangle$$

$$= \langle 0 | (\hat{c}_1^\dagger \hat{c}_2^\dagger)^\dagger \; t\, \hat{c}_1^\dagger \hat{c}_3 \;\; c_2^\dagger c_3^\dagger \, |0\rangle$$

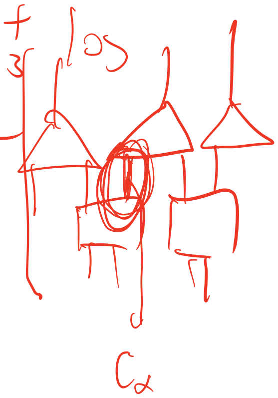
$$= \langle 0 | \hat{c}_2 \hat{c}_1 \;\; t\, \hat{c}_1^\dagger \hat{c}_3 \;\; c_2^\dagger c_3^\dagger \, |0\rangle$$

$$\langle 0 | \pm t\, \hat{c}_1 c_1^\dagger \, c_2 c_2^\dagger \, c_3 c_3^\dagger \, |0\rangle$$



$$H = \langle 0 | C_1 C_1 C_2^\dagger C_2^\dagger |0\rangle \;\; \pm 1$$

$$= 0$$



$$C_1 \quad C_1^\dagger$$

$$\to -C_1^\dagger C_1 + 1 \qquad \times$$

$$C_1 \cdots C_1 \cdots C_1^\dagger \cdots C_1^\dagger \qquad \times$$

Huang: $O(n)$ 所有的 $T_r(\rho P_k)$

Shadow:

$$\mathrm{Tr}\left(\rho \sum f P_k\right) = \underline{\mathrm{Tr}(\rho H)}$$

$O(n)$