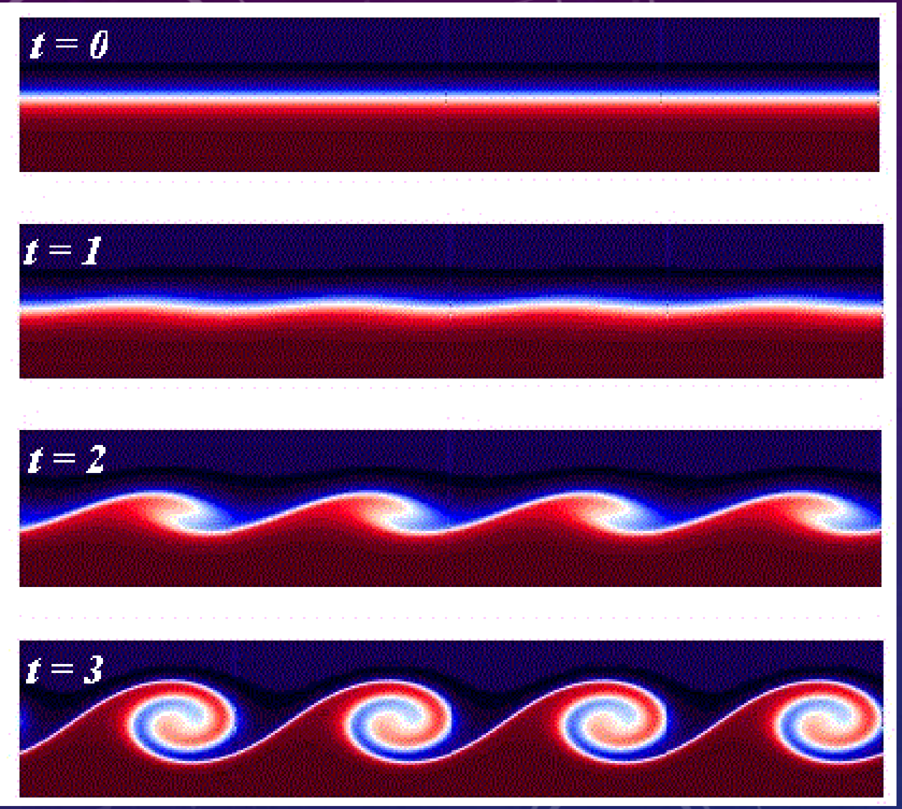


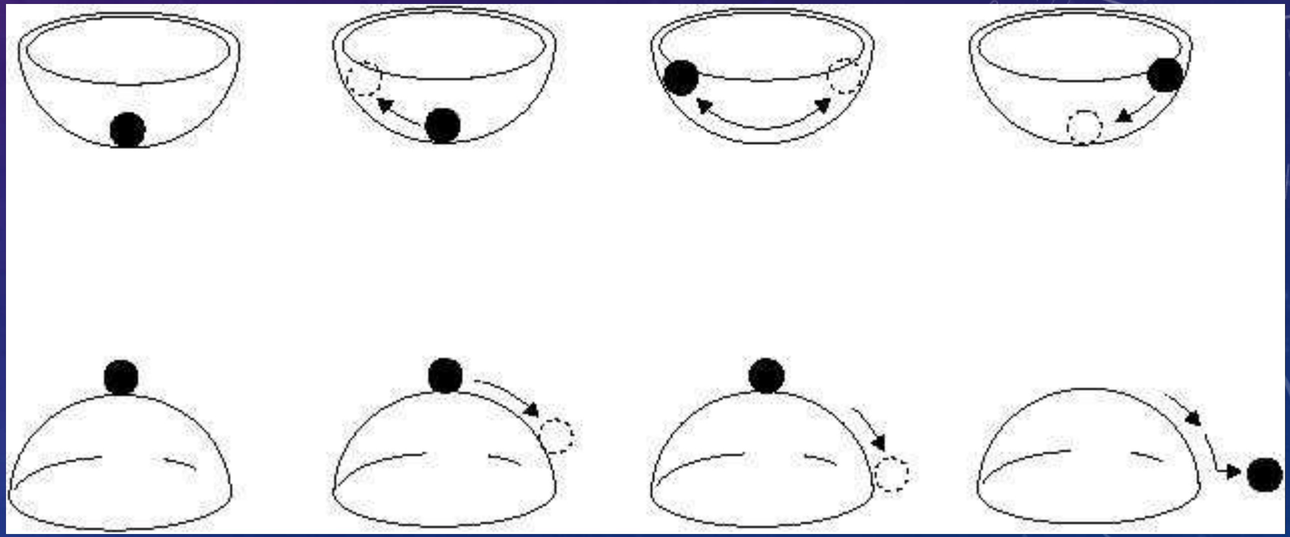
THE KELVIN HELMHOLTZ INSTABILITY: IN SPACE!

BY CHRIS GILBERT

4/18/2017

WHAT IS STABILITY?

- We can study a variable as a function of time to see how it behaves.
  - Perhaps this is the amplitude of a wave, and we want to see if it grows or dies.
  - Or, it could be the temperature or pressure, etc., of a system.
- Stability happens when a perturbation causes a restoring force that cancels the perturbation.
  - Guitar String
  - Ball in a bowl
  - Ocean Waves
  - Alfvén Waves (Sometimes)
- Instability happens when a perturbation causes a force that reinforces that perturbation.
  - Ball rolling off a hill
  - Kelvin-Helmholtz
  - Rayleigh Taylor
  - Magnetorotational

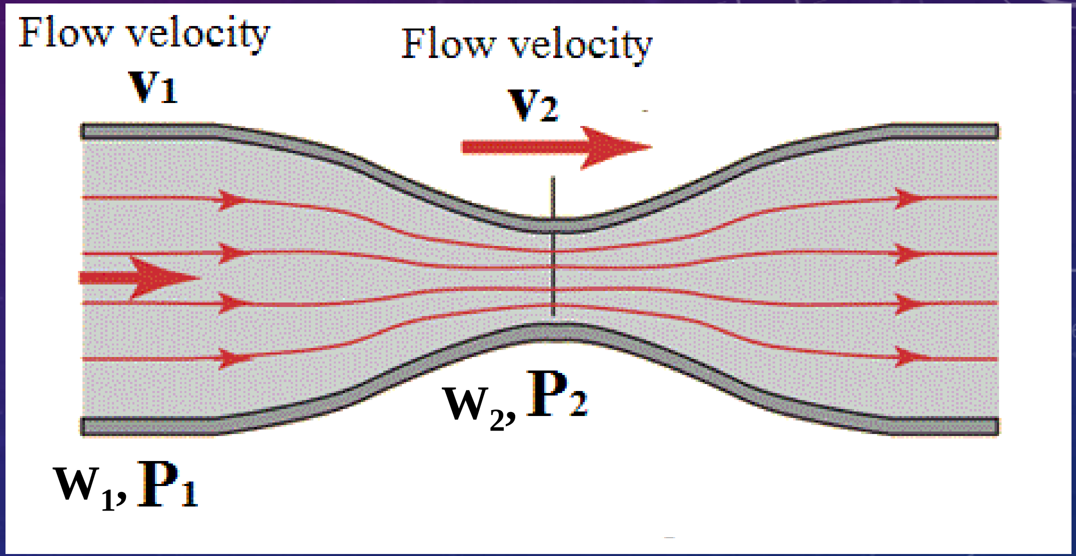


CLOUD COVERAGE EXAMPLE

- Lets Perturb the Temperature:
  - Increased temperature leads to increased evaporation of water:
    - …which is a green house gas, increasing the temperature.
    - …which forms more clouds, which reflect light, reducing the temperature.
  - The relative strengths of these two effects determine stability.

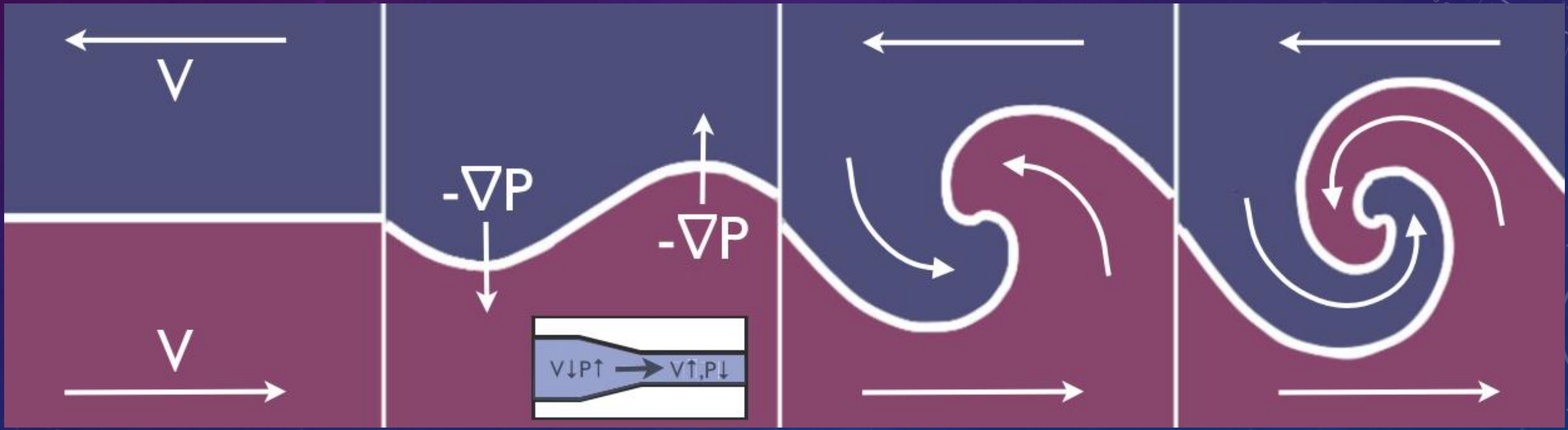
KEY EQUATIONS FOR KELVIN-HELMHOLTZ

- *Continuity Equation*
  - $\rho_1 v_1 W_1 = \rho_2 v_2 W_2$
    - Assume $\rho_1 = \rho_2$:
    - $v_2 = v_1 \frac{W_1}{W_2}$
    - As channel width decreases, velocity increases.
- *Bernoulli Equation*
  - $P + \rho g h + \frac{1}{2}\rho v^2 = P_0 = constant$
  - Call g = 0 :
    - $P = P_0 - \frac{1}{2}\rho v^2$
    - As velocity increases, pressure decreases.



Together, this says that
a constriction in the flow will decrease the pressure.

STAGES OF THE INSTABILITY



a constriction in the flow will decrease the pressure

STEPS FOR STABILITY ANALYSIS

$$\frac{\partial u_x}{\partial t} + U\frac{\partial u_x}{\partial y} = -\frac{\partial p}{\partial x}$$
$$\frac{\partial u_y}{\partial t} + U\frac{\partial u_y}{\partial y} + u_x\frac{\partial U}{\partial x} = -\frac{\partial p}{\partial y}$$
$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

- Start with the Navier-Stokes and/or the MHD equations, with terms relevant to your system.
- Linearize the equations, keeping only first order terms
  - $\rho \to \rho_0 + \rho_1$, $\rho \cdot B \to (\rho_0 + \rho_1)(\mathrm{B}_0 + B_1) \to \rho_0 B_0 + \rho_0 B_1 + \rho_1 B_0 + \cancel{\rho_1 B_1}$
- Apply the Spectral Ansatz
  - $\rho_1 = \rho_{10} e^{i(kz-\omega t)}$, $\frac{d\rho_1}{dt} = -i\omega\rho_1$, $\frac{d\rho_1}{dz} = ik\rho_1$
- Put in equations in matrix form and find the determinant
- Find roots of determinant = 0
  - Dispersion Relation
  - $\omega = \omega(k)$
  - Instability occurs when $IM\{\omega\} > 0$

$$\begin{pmatrix} ikU+\sigma & 0 & \partial_x \\ U' & ikU+\sigma & ik \\ \partial_x & ik & 0 \end{pmatrix} \begin{pmatrix} \hat{u}_x \\ \hat{u}_y \\ \hat{p} \end{pmatrix} = 0.$$

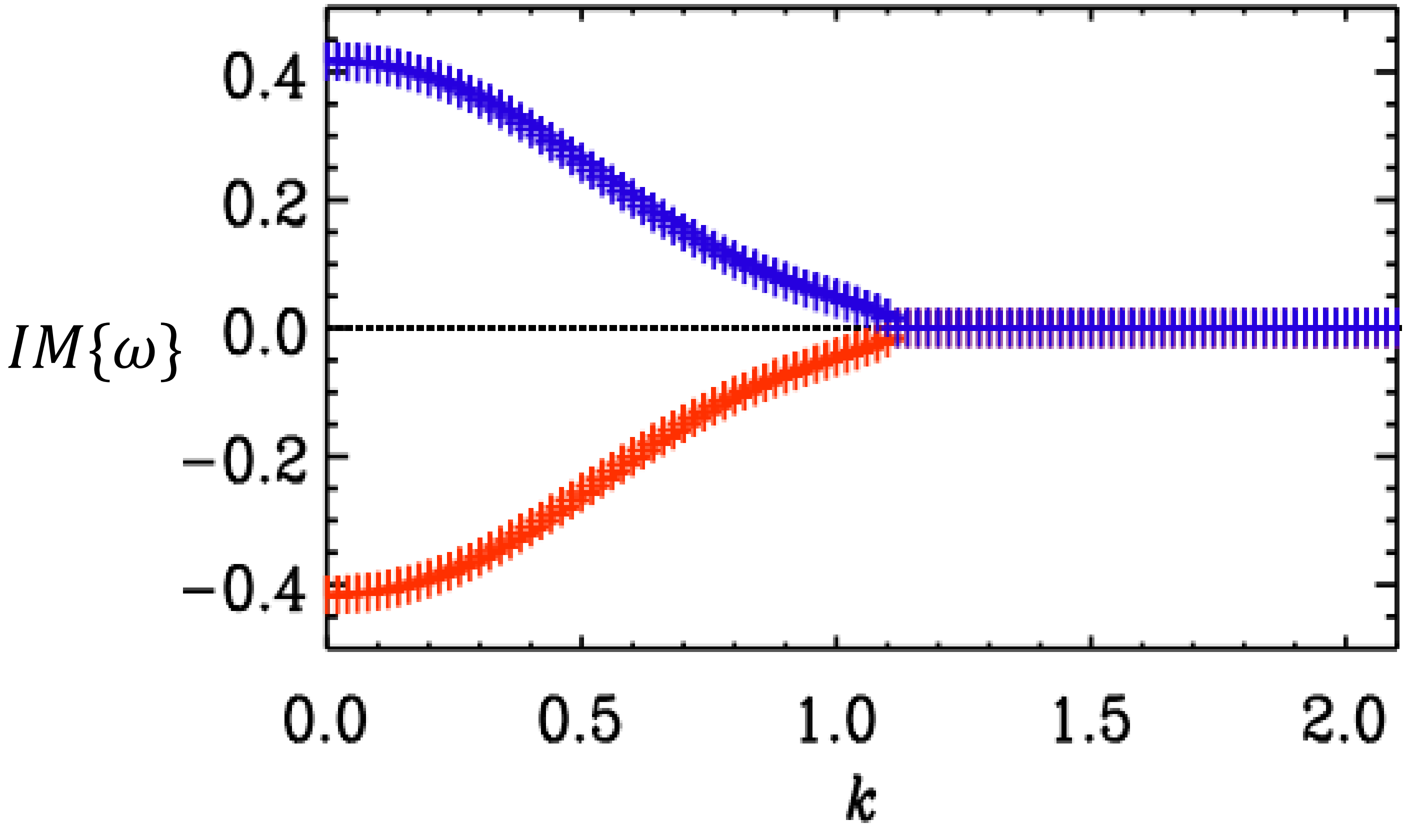
$$\sigma \equiv i\omega$$

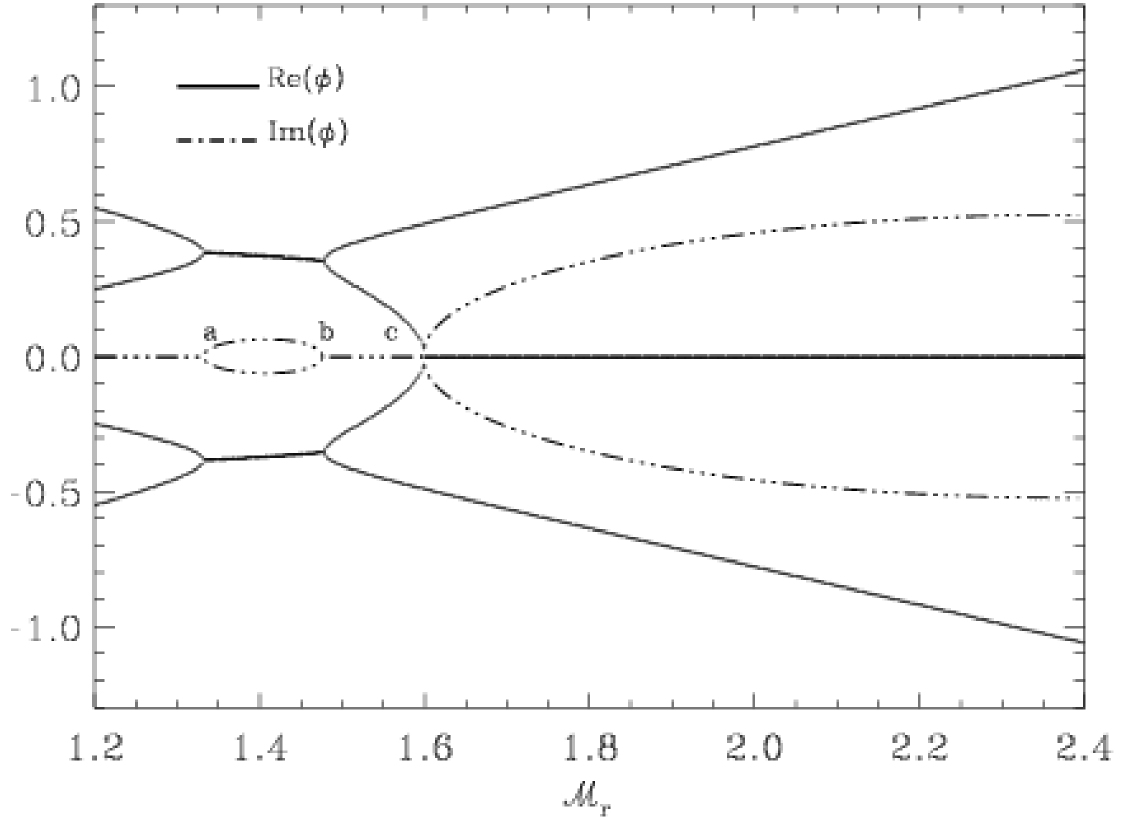
$$\omega = \frac{\mathbf{k}\cdot(\rho_1 \mathbf{V_1} + \rho_2 \mathbf{V_2})}{\rho_1 + \rho_2} \pm i\sqrt{\rho_1\rho_2\left([\mathbf{k}\cdot(\mathbf{V_1} - \mathbf{V_2})]^2 - \frac{(\mathbf{k}\cdot\mathbf{B_1})^2 + (\mathbf{k}\cdot\mathbf{B_2})^2}{4\pi\rho_{12}}\right)}$$

THE DISPERSION RELATION

$$\rho_1 = \rho_{10} e^{i(kz-\omega t)}$$

$$\omega = \frac{\mathbf{k}\cdot(\rho_1\mathbf{V_1} + \rho_2\mathbf{V_2})}{\rho_1+\rho_2} \pm i\sqrt{\rho_1\rho_2\left([\mathbf{k}\cdot(\mathbf{V_1}-\mathbf{V_2})]^2 - \frac{(\mathbf{k}\cdot\mathbf{B_1})^2 + (\mathbf{k}\cdot\mathbf{B_2})^2}{4\pi\rho_{12}}\right)}$$



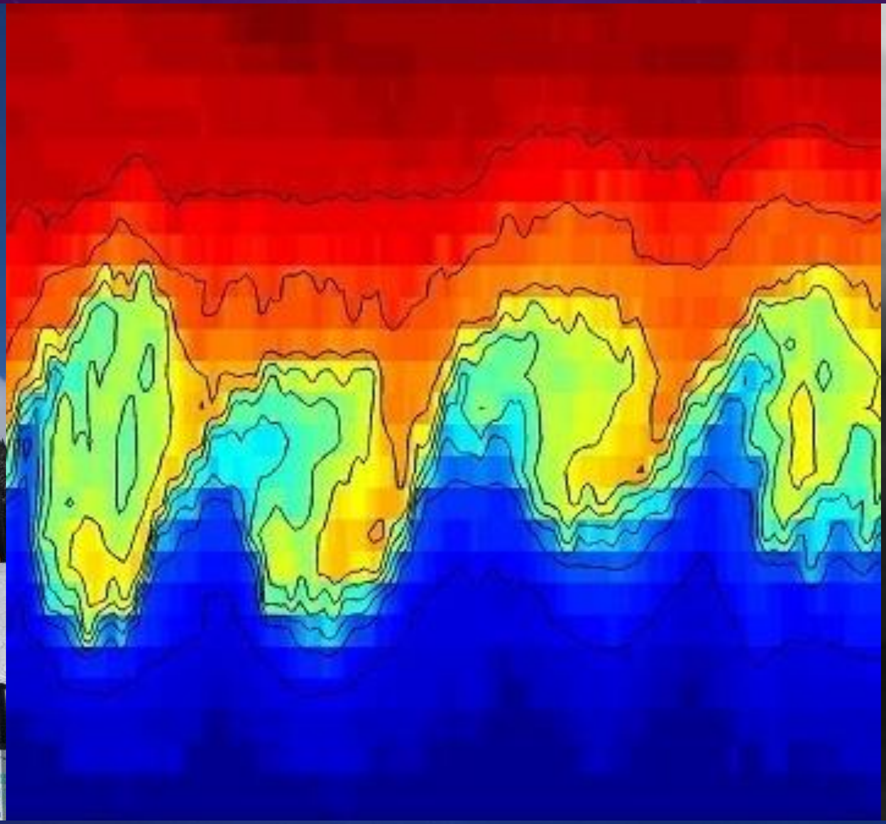


This plot describes stratifed, unmagnetized shear flow.
Figure from Fluids II, 2016, Axel Brandenburg.

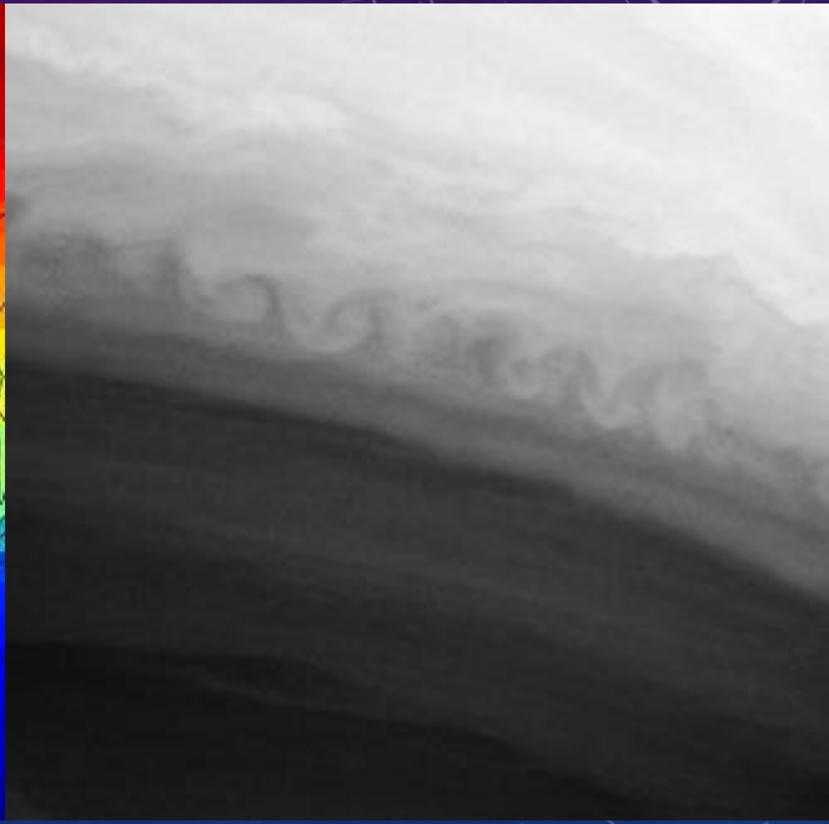
This plot describes relativistic, magnetized shear flow.
Figure from Z. Osmanov et al. 2008

KH IS EVERYWHERE!

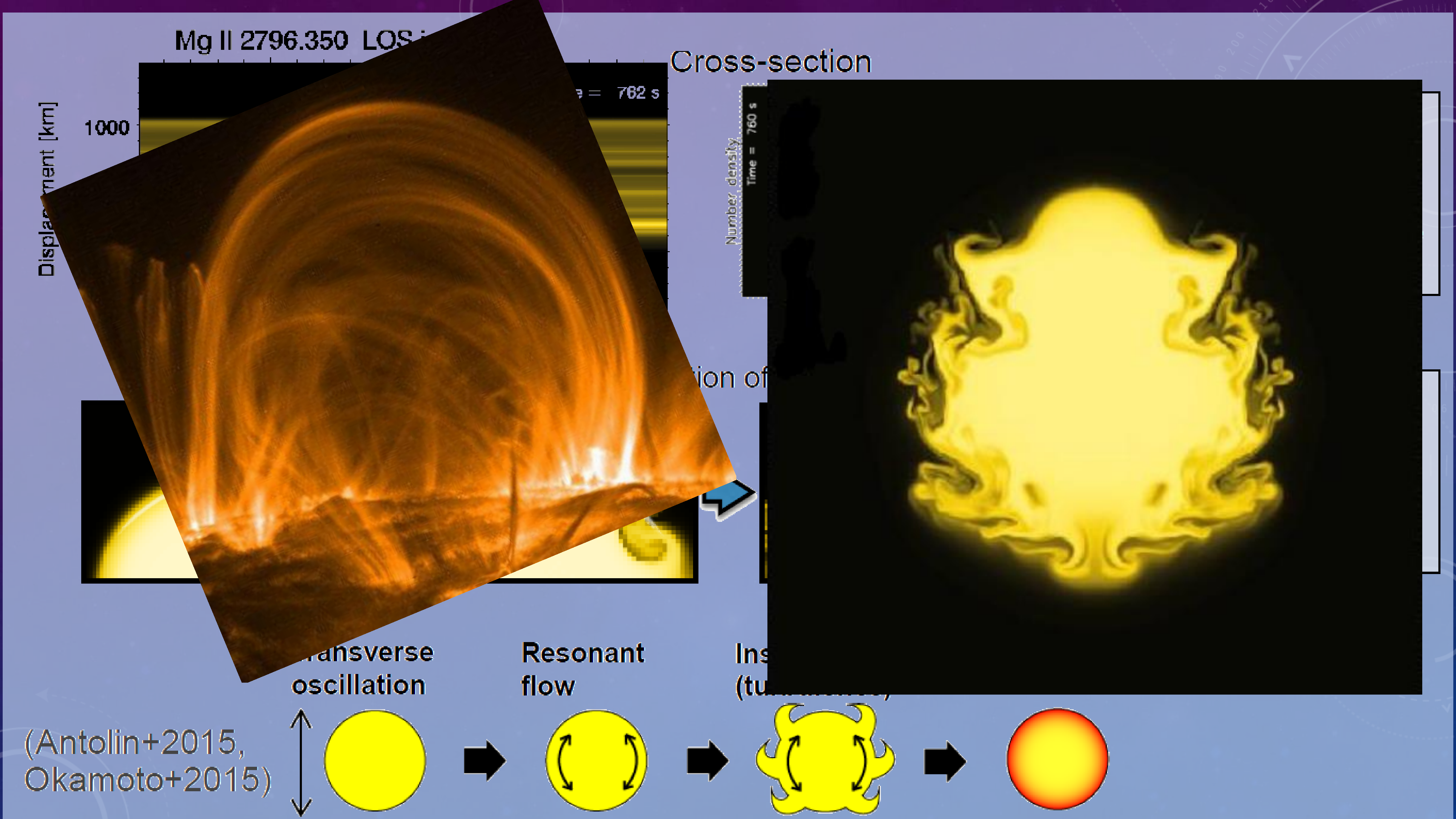
Clouds Over the Mountains



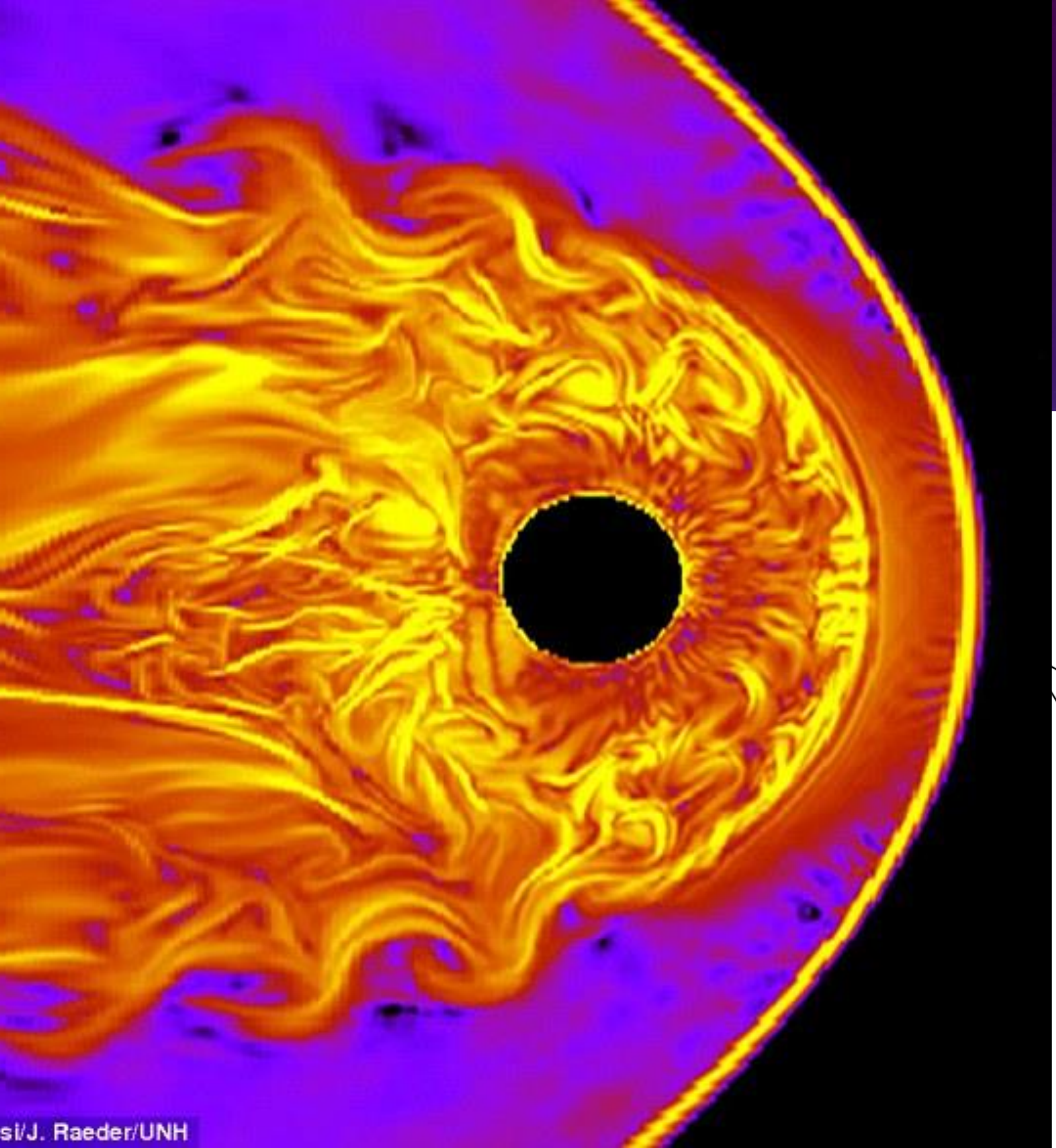
Deep Ocean Waves



Saturn’s Atmosphere

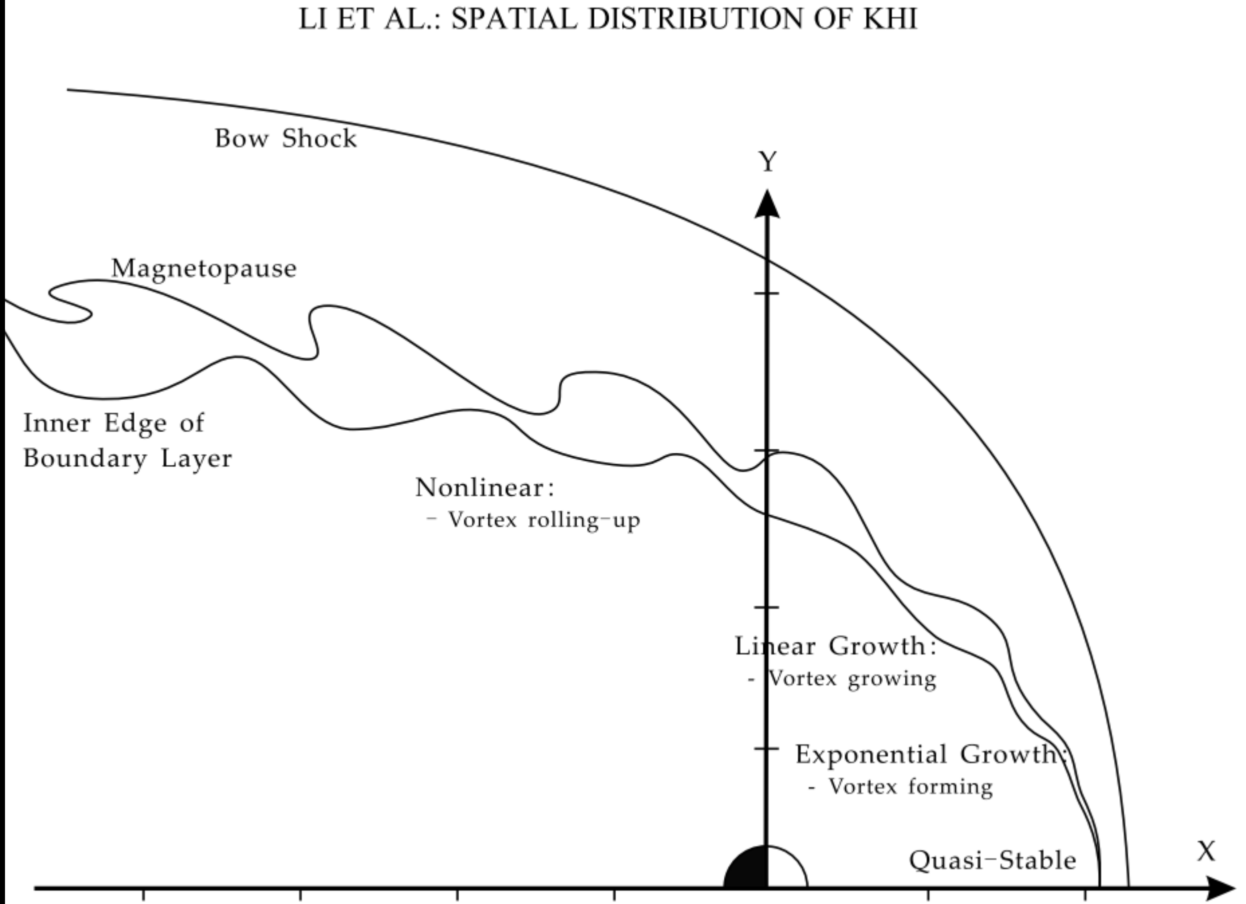


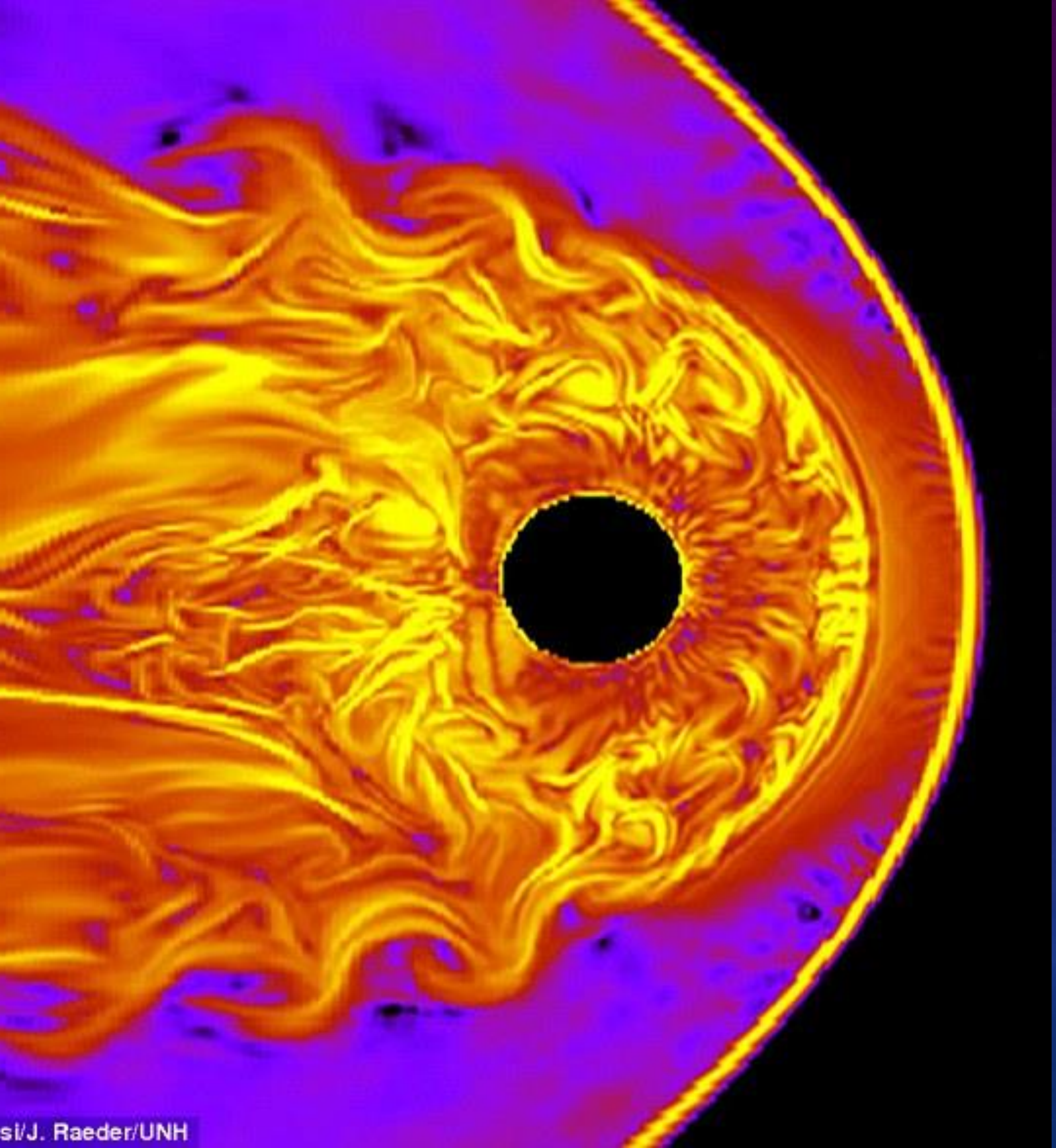
This slide taken from “Solar Prominence Science with ALMA” by Labrosse et Al, 03/2016



EARTH'S MAGNETOSPHERE: THE LOW LATITUDE BOUNDARY LAYER

Conditions are favorable for the KHI when the SW has a northward IMF

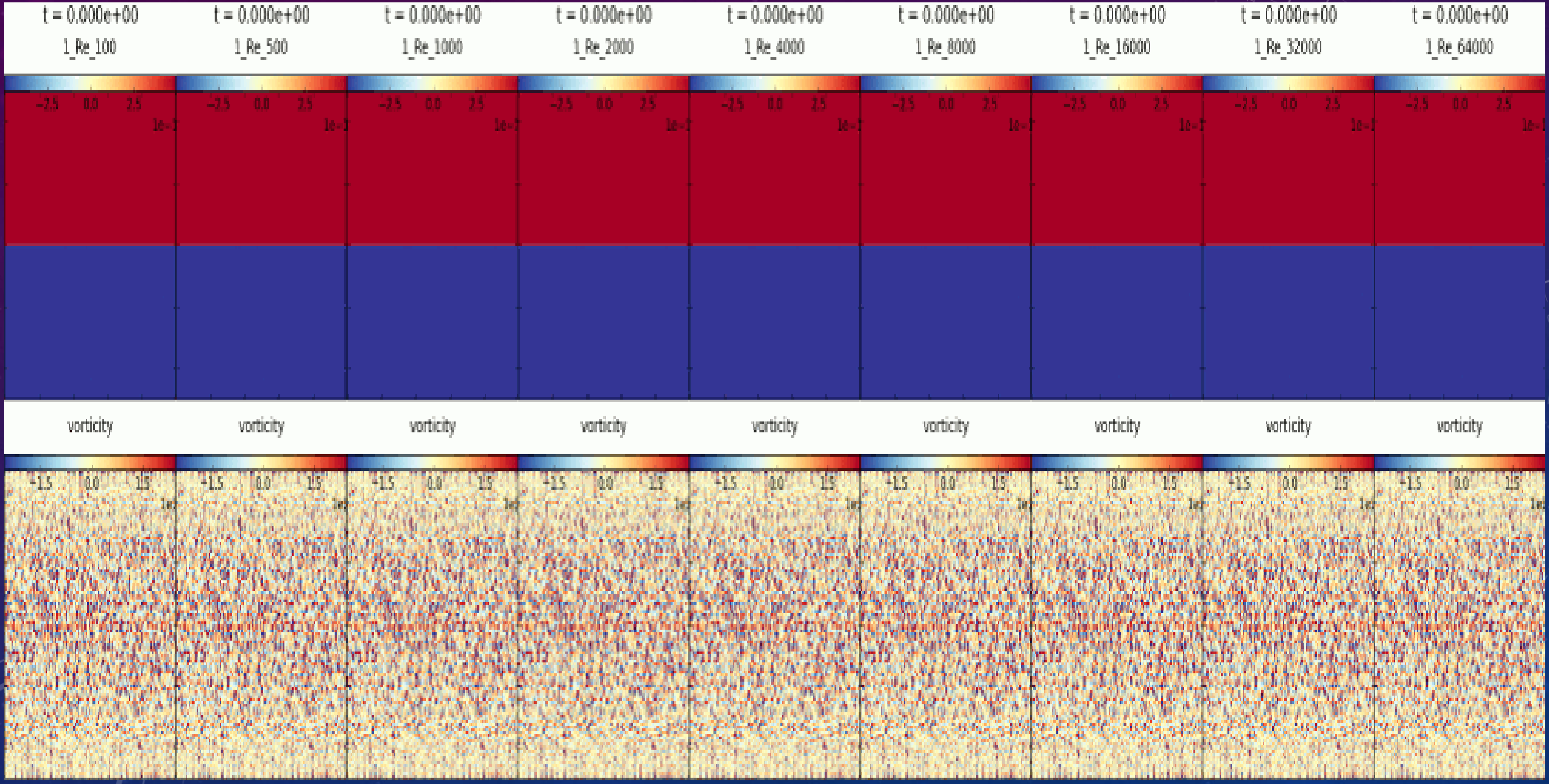


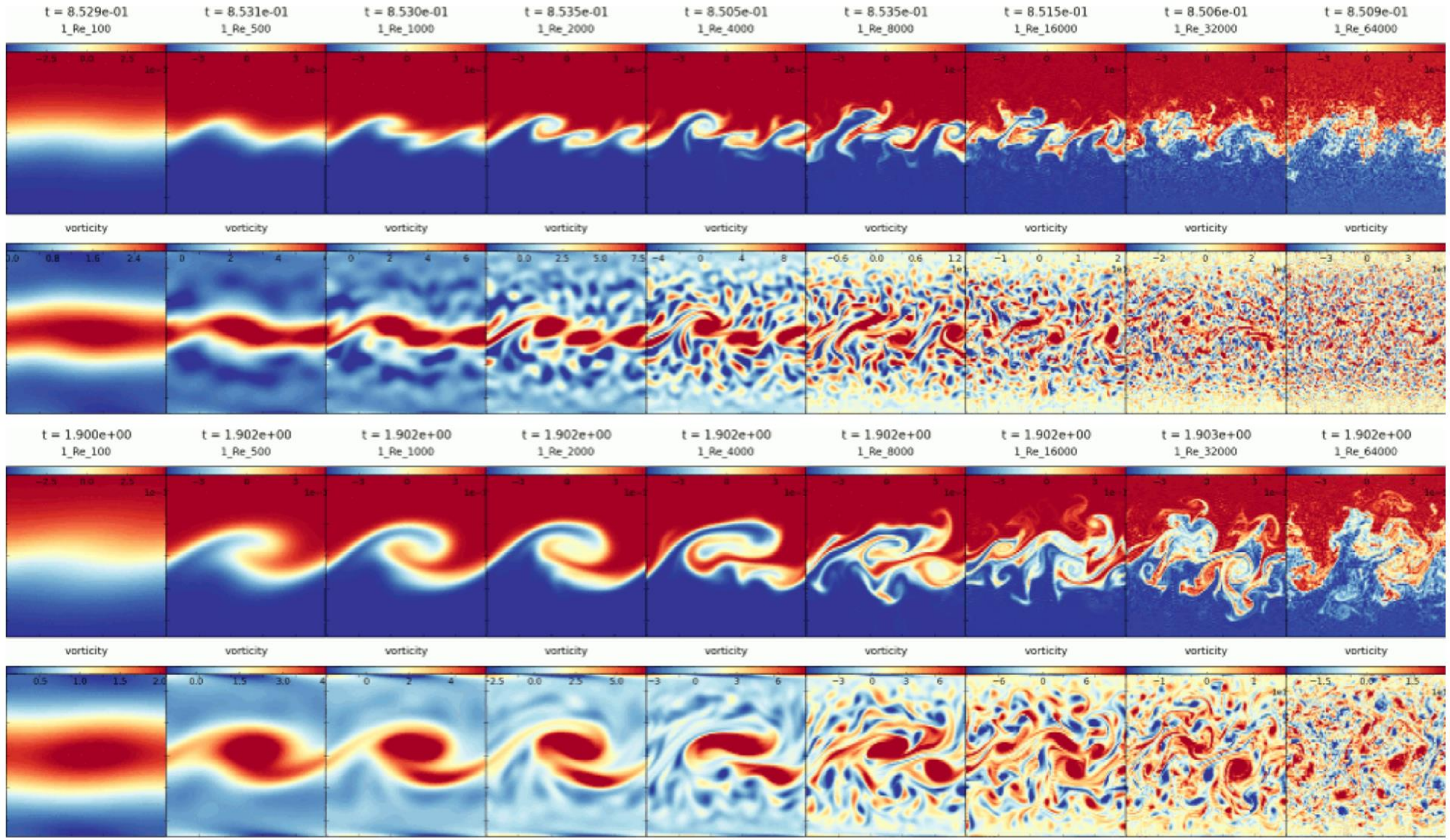


WHY IS IT IMPORTANT?

- The KHI provides a mechanism to allow plasma to cross magnetospheric boundaries.
  - MASS:
    - It enables highly efficient ion mixing across a boundary (Fujimoto & Terasawa 1994)
  - ENERGY:
    - It can generate ULF waves that can accelerate electrons in the radiation belts (Atkinson & Watanabe 1966)
    - It drives turbulent boundary layers, causing turbulent dissipation of energy. (Johnson et. al. 2014)
  - MOMENTUM:
    - It might drive large scale convection at the magnetopause, explaining the “anomalous diffusion” of momentum from the solar wind into the magnetosphere (Miura 1984)

SINGLE TANGENTIAL DISCONTINUITY

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1_Re_1000
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t = 8.505e-01
1_Re_4000
t = 8.535e-01
1_Re_8000
t = 8.515e-01
1_Re_16000
t = 8.506e-01
1_Re_32000
t = 8.509e-01
1_Re_64000
vorticity
t = 1.900e+00
1_Re_100
t = 1.902e+00
1_Re_500
t = 1.902e+00
1_Re_1000
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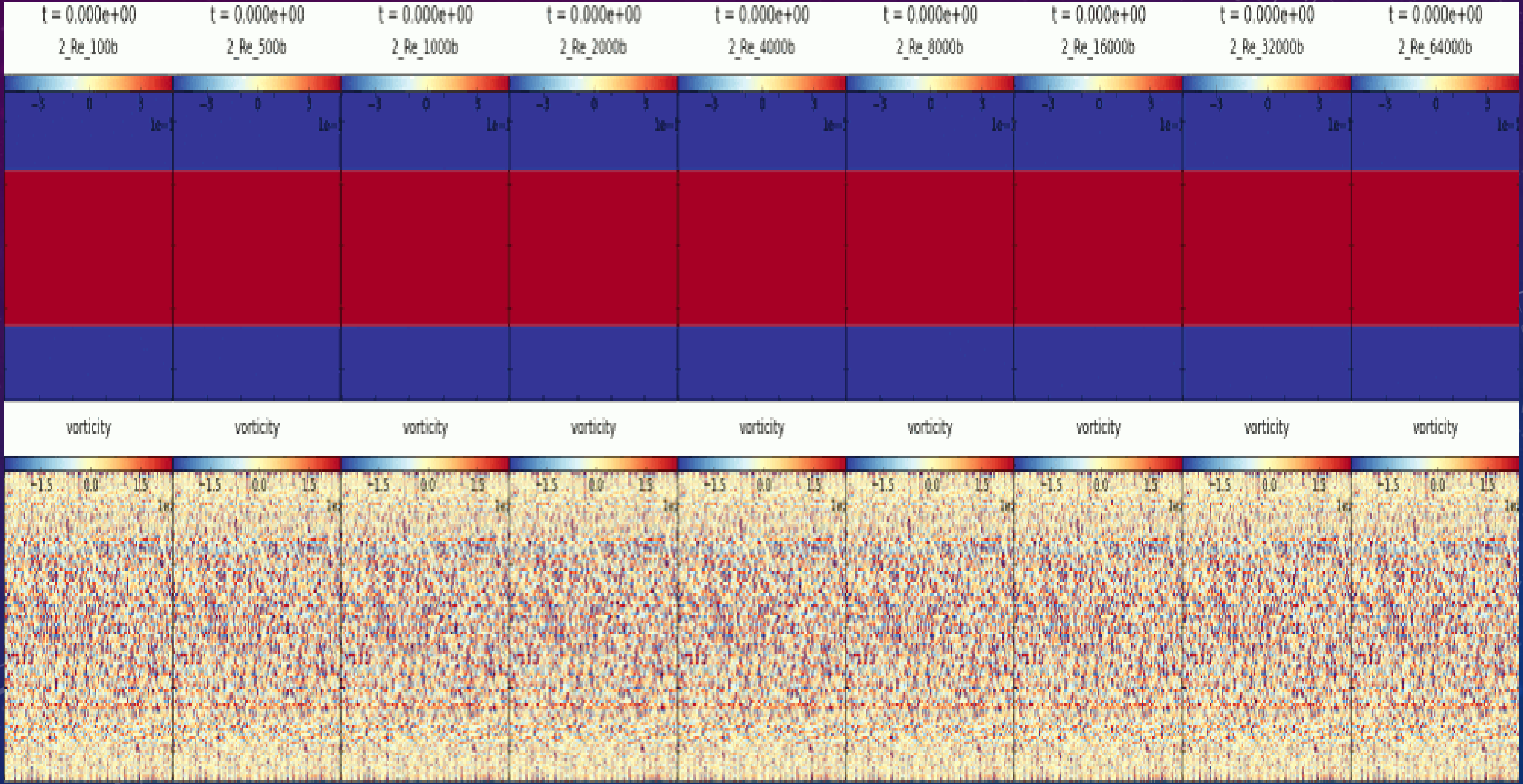
t = 1.902e+00
1_Re_4000
t = 1.902e+00
1_Re_8000
t = 1.902e+00
1_Re_16000
t = 1.903e+00
1_Re_32000
t = 1.902e+00
1_Re_64000
vorticity

DOUBLE TANGENTIAL DISCONTINUITIES



SUMMARY

- Instabilities occur when perturbations are reinforced rather than opposed.
- They can be studied by looking at the dispersion relation to see under what conditions instability occurs.
- The KH Instability occurs in shear flows, and can be explained by continuity and Bernoulli arguments.
- The KHI is present at many scales and in many locations across the solar system.
- It can act to diffuse mass, energy, and momentum across an otherwise impenetrable barrier, such as the magnetopause.

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