Roundoff Error in Approximations Assignment

The Main Question for Math 4610 at USU

The following is a list of tasks to complete for this module.

- Show that the second derivative approximation

$$f''(x) \approx \frac{2\ U_i - 5\ U_{i-1} + 4U_{i-2} - U_{i-3}}{\Delta x^2}$$

  converges as $\Delta x \to 0$.

- For the approximation, determine the order of accuracy using Taylor series expansions.
- Determine where the approximation starts to fail due to roundoff error. This means you will need to write a code that implements the difference approximation. Use $f(x) = e^{-\pi x}$ at $x = 0$ to test the result.
- The following list of ordered pairs defines the error in an approximation of some algorithim. Determine if the approximation converges using a least squares approach. It would be a good idea to plot the data to get a feel for the behavior of the data set. You can use Matplotlib for the plot.
- Use linear regression to determine the rate of convergence. You will need to write a code that computes the least squares fit to the given data.

The data:

(.05, 0.72736334), (.025, 0.51876322), (.0125, 0.36998741), (.00625, 0.26387894), (.003125, 0.18820125), (.0015625, 0.13422712), (.00078125, 0.09573220), (.000390625, 0.06827721), (.0001953125, 0.04869603)

- Write a few sentances to summarize what you learned in this module.