Содержание

1 <Неопределенный Интеграл>

1.1 <Непосредственное интегрирование>

1°. Основные правила интегрирования.

1) Если $F'(x) = f(x)$, то

$$\int f(x)dx = F(x) + C.$$

где C - произвольная постоянная.

2) $\int Af(x)dx = A\int f(x)dx$, где A - постоянная величина.

3) $\int [f_1(x) \pm f_2(x)]dx = \int f_1(x)dx \pm \int f_2(x)dx$

4) Если $\int f(x)dx = F(x) + C$ и $u = \varphi(x)$, то

$$\int f(u)du = F(u) = C.$$

В частности,

$$\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C \quad (a \neq 0).$$

2° Таблица простейших интегралов.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \tag{I}$$

$$\int \frac{dx}{x} = \ln|x| + C \tag{II}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C = -\frac{1}{a}\arctan\frac{x}{a} + C_1 \quad (a \neq 0). \tag{III}$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C \quad (a \neq 0) \tag{IV}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c \quad (a \neq 0)$$

$$\int \frac{dx}{\sqrt{x^2+a}} = \ln|x+\sqrt{x^2+a}| + C \quad (a \neq 0) \tag{V}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C = -\arccos\frac{x}{a} + C_1 \quad (a > 0) \tag{VI}$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0); \qquad \int e^x dx = e^x + C \tag{VII}$$

$$\int \sin x dx = -\cos x + C \tag{VIII}$$

$$\int \cos x dx = \sin x + C \tag{XI}$$

$$\int \frac{1}{\cos^2 x} = tgx + C \tag{X}$$

$$\int \frac{1}{\sin^2 x} = -ctgx + C \tag{XI}$$

$$\int \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| + C = \ln|cosecx - ctgx| + C \tag{XII}$$

$$\int \frac{dx}{\cos x} = \ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{2}\right)\right| + C = \ln|\operatorname{tg} x + secx| + C \tag{XIII}$$

$$\int \operatorname{sh} x dx = \operatorname{ch} x + C \tag{XIV}$$

$$\int \operatorname{ch} x dx = \operatorname{sh} x + C \tag{XV}$$

$$\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C \tag{XVI}$$

$$\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C \tag{XVII}$$

`Пример 1.`

$$\int (ax^2 + bx + c)dx = \int ax^2 dx + \int bx dx + \int c dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$$

1031.

$$\int 5a^2x^6 dx = 5a^2 \int x^6 dx = 5a^2 \cdot \frac{x^7}{7} = \frac{5a^2x^7}{7}$$

1032.

$$\int (6x^2 + 8x + 3)dx = 6\int x^2 dx + 8\int x dx + 3\int dx = 6\frac{x^3}{3} + 8\frac{x^2}{2} + 3x = 2x^3 + 4x^2 + 3x$$

1033.

$$\int x(x+a)(x+b)dx = \int (x^3 + bx^2 + ax^2 + abx)dx = \frac{x^4}{4} + \frac{bx^3}{3} + \frac{ax^3}{3} + \frac{abx^2}{2} = \frac{x^4}{4} + \frac{a+b}{3}x^3 + \frac{ab}{2}x^2$$

1034.

$$\int (a + bx^3)^2 dx = \int b^2x^6 dx + \int 2abx^3 dx + \int a^2 dx = \frac{b^2x^7}{7} + \frac{2abx^4}{4} + a^2x$$

1035.

$$\int \sqrt{2px}dx = \sqrt{2p}\int \sqrt{x}dx = \sqrt{2p}\frac{x\sqrt{x}}{\frac{3}{2}} = \frac{x\sqrt{x}p2\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}\sqrt{x^3}$$

1036.

$$\int \frac{dx}{\sqrt[n]{x}} = \int x^{-\frac{1}{n}} dx = \frac{x^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} = \frac{n}{n-1}\sqrt[n]{x^{n-1}}$$

1037.

$$\int (nx)^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \frac{x^{\frac{1-n}{n}+1}}{\frac{1-n}{n}} = n^{\frac{1-n}{n}+1}\frac{x^{\frac{1-n+n}{n}}}{1-n} = \sqrt[n]{n}\frac{\sqrt[n]{x}}{1-n} =$$

1038.

$$\int \left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)^3 dx = \int (a^2 - 3a^{\frac{4}{3}}x^{\frac{2}{3}} + 3a^{\frac{2}{3}}x^{\frac{4}{3}} - x^2)dx = a^2x - 3\sqrt[3]{a^4}\frac{\sqrt[3]{x^5}}{\frac{5}{3}} + 3\sqrt[3]{a^2}\frac{\sqrt[3]{x^7}}{\frac{7}{3}} - \frac{x^3}{3} =$$

$$= a^2x - \frac{9\sqrt[3]{a^4}}{5}\sqrt[3]{x^5} + \frac{9\sqrt[3]{a^2}}{7}\sqrt[3]{x^7} - \frac{x^3}{3}$$

1039.

$$\int(\sqrt{x}+1)(x-\sqrt{x}+1)dx=\int(x^{\frac{3}{2}}-x+\sqrt{x}+x-\sqrt{x}+1)dx=\int(x^{\frac{3}{2}}+1)dx=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}=\frac{2}{5}\sqrt[2]{x^5}$$

1040.

$$\int\frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}}dx=\int\frac{x^4-x^2-2}{\sqrt[3]{x^2}}dx=\int x^{\frac{10}{3}}dx-\int x^{\frac{4}{3}}dx-2\int x^{-\frac{2}{3}}dx=\frac{3\sqrt[3]{x^{13}}}{13}-\frac{3\sqrt[3]{x^7}}{7}-6\sqrt[3]{x}$$

1041.

$$\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx=\int\frac{x^{2m}-2x^{m-n}+x^{2n}}{\sqrt{x}}dx=\int(x^{2m-\frac{1}{2}}-2x^{m-n-\frac{1}{2}}+x^{2n-\frac{1}{2}})dx=\frac{x^{2m+\frac{1}{2}}}{2m+\frac{1}{2}}-\frac{2x^{m-n+\frac{1}{2}}}{m-n+\frac{1}{2}}+$$

$$+\frac{x^{2n+\frac{1}{2}}}{2n+\frac{1}{2}}=\frac{2\sqrt{x^{4m+1}}}{4m+1}-\frac{2\sqrt{x^{2m-2n+1}}}{2m-2n+1}+\frac{2\sqrt{x^{4n+1}}}{4n+1}$$

1042.

$$\int\frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{a}x}dx=\int\frac{a^2-4\sqrt{a^3}\sqrt{x}+6ax-4\sqrt{a}\sqrt{x^3}+x^2}{\sqrt{a}x}dx=a^{\frac{3}{2}}\int\frac{1}{x}dx-4a\int x^{-\frac{1}{2}}dx+6\sqrt{a}\int dx-$$

$$-4\int\sqrt{x}dx+\frac{1}{\sqrt{a}}\int xdx=\sqrt{a^3}\ln|x|-8a\sqrt{x}+6\sqrt{a}x-\frac{8}{3}\sqrt{x^3}+\frac{1}{2\sqrt{a}}x^2$$

1043.

$$\int\frac{dx}{x^2+7}=\int\frac{dx}{x^2+(\sqrt{7})^2}=\frac{1}{\sqrt{7}}\arctan\frac{x}{\sqrt{7}}$$

1044.

$$\frac{dx}{x^2-10}=\frac{dx}{x^2-(\sqrt{10})^2}=\frac{1}{2a}\ln\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right|$$

1045.

$$\int\frac{dx}{\sqrt{x^2+4}}=\int\frac{dx}{\sqrt{x^2+2^2}}=\ln|x+\sqrt{x^2+2}|$$

1046.

$$\int\frac{dx}{\sqrt{8-x^2}}=\int\frac{dx}{\sqrt{(2\sqrt{2})^2-x^2}}=\arcsin\frac{x}{2\sqrt{2}}$$

1047.

$$\int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx=\int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{(2-x^2)(2+x^2)}}dx=\int\frac{dx}{\sqrt{2-x^2}}-\int\frac{dx}{\sqrt{2+x^2}}=\arcsin\frac{x}{\sqrt{2}}-\ln|x+\sqrt{2+x^2}|$$

1048. а)

$$\int tg^2xdx=\int\frac{\sin^2 x}{\cos^2 x}dx=\int\frac{1-\cos^2 x}{\cos^2 x}dx=\int\frac{dx}{\cos^2 x}-\int dx=tgx-x$$

1048. б)

$$\int th^2xdx=\int\frac{sh^2x}{ch^2x}dx=\int\frac{ch^2x-1}{ch^2x}dx=\int dx-\int\frac{dx}{ch^2x}=x-thx$$

1049. а)

$$\int \operatorname{ctg}^2 x dx = \int \frac{\cos^2 x}{\sin^2 x} = \int \frac{1-\sin^2 x}{\sin^2 x}dx = \int \frac{dx}{\sin^2 x} - \int dx = -\operatorname{ctg} x - x$$

1049. б)

$$\int \operatorname{cth}^2 x = \int \frac{cth^2 x}{sh^2 x}dx = \int \frac{1+sh^2 x}{sh^2 x}dx = \int \frac{1}{sh^2 x} + \int dx = \operatorname{cth}^2 x + x$$

1050.

$$\int 3^x e^x dx = \int (3e)^x dx = \frac{(3e)^x}{\ln(3e)}$$

3°. `Интегрирование путем подведения под знак дифференциала.`

`Пример 2.` $\displaystyle\int \frac{dx}{\sqrt{5x-2}} = \frac{1}{5}\int (5x-2)^{-\frac{1}{2}} d(5x-2) = \frac{1}{5}\int u^{-\frac{1}{2}}du = \frac{2}{5}\sqrt{u} = \frac{2}{5}\sqrt{5x-2}$

где было положено $u = x^2$, причем применялись правило (1.1) и табличный интеграл (I).

`Пример 3.` $\displaystyle\int \frac{xdx}{\sqrt{1+x^4}} = \frac{1}{2}\int \frac{d(x^2)}{\sqrt{1+x^2}} = \frac{1}{2}\ln(x^2+\sqrt{1+x^4})$

Неявно подразумевалось $u = x^2$, причем применялись правило (1.1) и табличный интеграл (V).

1051.

$$\int \frac{adx}{a-x} \quad \Longrightarrow (u = a-x) \Longrightarrow \quad -a\int \frac{du}{u} = -a\ln|u| = -a\ln|a-x|$$

1052.

$$\int \frac{2x+3}{2x+1}dx = \int dx + \int \frac{2dx}{2x+1} \quad \Longrightarrow (u = 2x+1) \Longrightarrow \quad = x + \frac{du}{u} = x + \ln|u| = x + \ln|2x+1|$$

1053.

$$\int \frac{1-3x}{3+2x}dx = \quad \Longrightarrow (u = 3+2x) \Longrightarrow \quad = \int \frac{1-\frac{3}{2}u+1}{2u}du = \int \frac{du}{u} - \int \frac{3}{4}du = \ln|u| - \frac{3}{4}u = \ln|3+2x| - \frac{9}{4} - \frac{3x}{2}$$

1054.

$$\int \frac{xdx}{a+bx} = \quad \Longrightarrow (u = bx+a) \Longrightarrow \quad = \frac{1}{b}\int \frac{\frac{u-a}{b}}{u}du = \frac{1}{b^2}\int \frac{u-a}{u}du = \frac{1}{b^2}u - \frac{a}{b^2}\ln|u| = \frac{a+bx}{b^2} - \frac{a}{b^2}\ln|a+bx|$$

1055.

$$\int \frac{ax+b}{\alpha x+\beta}dx = \quad \Longrightarrow (u = \alpha x+\beta) \Longrightarrow \quad = \frac{1}{\alpha^2}\int \frac{u-\beta+\alpha b}{u}du = \frac{u}{\alpha^2} + \frac{\alpha b-\beta}{\alpha^2}\ln|u| = \frac{\alpha x+\beta}{\alpha^2} + \frac{\alpha b-\beta}{\alpha^2}\ln|\alpha x+\beta|$$

1056.

$$\int \frac{x^2+1}{x-1}dx = \quad \Longrightarrow (u = x-1) \Longrightarrow \quad = \int \frac{u^2+2u+2}{u}du = \frac{u^2}{2} + 2u + 2\ln|u| = \frac{x^2-3}{2} + x + 2\ln|x-1|$$

1057.

$$\int \frac{x^2+5x+7}{x+3}dx = \quad \Longrightarrow (u = x+3) \Longrightarrow \quad = \int \frac{u^2-6u+9+5u-15+7}{u}du = \int \frac{u^2-u+1}{u}du =$$

$$= \frac{u^2}{2} - u + \ln|u| = \frac{x^2+6x+9}{2} - x - 3 + \ln|x+3| = \frac{x^2}{2} + 2x + \frac{3}{2} + \ln|x+3|$$

1059.

$$\int \left(a + \frac{b}{x-a}\right)^2 dx = \quad \Longrightarrow (u = x - a) \Longrightarrow \quad = \int \left(a + \frac{b}{u}\right)^2 du = \int a^2 du + \int 2\frac{ab}{u} du + \int \frac{b^2}{u^2} du = a^2 u +$$

$$+2ab\ln|u| + b^2 \frac{u^{-1}}{-1} = a^2(x-a) + 2ab\ln|x-a| - \frac{b^2}{x-a} =$$

1060.

$$\int \frac{x}{(x+1)^2} dx = \quad \Longrightarrow (u = x+1) \Longrightarrow \quad = \int \frac{u-1}{u^2} du = \ln|u| - \frac{u^{-1}}{-1} = \ln|x+1| + \frac{1}{x+1}$$

1061.

$$\int \frac{bdy}{\sqrt{1-y}} = \quad \Longrightarrow (u = 1-y) \Longrightarrow \quad = -b\int \frac{du}{\sqrt{u}} = -2b\sqrt{u} = -2b\sqrt{1-y}$$

1062.

$$\int \sqrt{a-bx}dx = \quad \Longrightarrow (u = a - bx) \Longrightarrow \quad = -\frac{1}{b}\int \sqrt{u}du = -\frac{2}{3b}\sqrt{u^3}$$

1063.

$$\int \frac{x}{\sqrt{x^2+1}} dx = \quad \Longrightarrow (u = x^2+1) \Longrightarrow \quad = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{x^2+1}$$

1064.

$$\int \frac{\sqrt{x} + \ln x}{x} dx = \int \frac{dx}{\sqrt{x}} + \int \ln x d(\ln x) = 2\sqrt{x} + \frac{\ln^2 x}{2}$$

1065.

$$\int \frac{dx}{3x^2+5} = \frac{1}{3}\int \frac{dx}{x^2+\frac{5}{3}} = \frac{\sqrt{3}}{3\sqrt{5}} \arctan \frac{x\sqrt{3}}{\sqrt{5}} = \frac{1}{\sqrt{15}} \arctan x\sqrt{\frac{3}{5}}$$

1066.

$$\int \frac{dx}{7x^2-8} = \frac{1}{7}\int \frac{dx}{x^2-\frac{8}{7}} = \frac{\sqrt{14}}{28} \ln\left|\frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}}\right| = \frac{\sqrt{14}}{28} \ln\left|\frac{\sqrt{7}x-\sqrt{8}}{\sqrt{7}x+\sqrt{8}}\right|$$

1067.

$$\int \frac{dx}{(a+b)-(a-b)x^2} \quad (0 < b < a) = \frac{1}{(a-b)}\int \frac{dx}{\frac{a+b}{a-b} - x^2} = \frac{1}{2\sqrt{(a-b)(a+b)}} \ln\left|\frac{\sqrt{\frac{a+b}{a-b}}+x}{\sqrt{\frac{a+b}{a-b}}-x}\right| =$$

$$= \frac{1}{2\sqrt{a^2-b^2}} \ln\left|\frac{\sqrt{a+b}+x\sqrt{a-b}}{\sqrt{a+b}-x\sqrt{a-b}}\right|$$

1068.

$$\int \frac{x^2}{x^2+2} dx = \int \frac{x^2+2-2}{x^2+2} dx = \int dx - 2\int \frac{dx}{x^2+2} = x - \frac{2}{\sqrt{2}} \arctan \frac{x}{\sqrt{2}} = -\sqrt{2}\arctan \frac{x}{\sqrt{2}}$$

1069.

$$\int \frac{x^3}{a^2-x^2} dx = \int \frac{x^3 - a^2x + a^2x}{a^2-x^2} dx = a^2\int \frac{xdx}{a^2-x^2} - \int xdx = -\frac{a^2}{2}\int \frac{d(a^2-x^2)}{a^2-x^2} - \frac{x^2}{2} =$$

$$-\frac{a^2}{2}\ln|a^2-x^2| - \frac{x^2}{2}$$

1070.

$$\int \frac{x^2-5x+6}{x^2+4}dx = \int dx - 5\int \frac{x}{x^2+4}dx + 2\int \frac{dx}{x^2+4} = x - \frac{5}{2}\int \frac{d(x^2+4)}{x^2+4} + \frac{2}{2}\arctan\frac{x}{2} =$$

$$= x - \frac{5}{2}\ln|x^2+4| + \arctan\frac{x}{2}$$

1071.

$$\int \frac{dx}{\sqrt{7+8x^2}} = \frac{1}{2\sqrt{2}}\int \frac{dx}{\sqrt{x^2+\frac{7}{8}}} = \frac{1}{2\sqrt{2}}\ln\left|x+\sqrt{x^2+\frac{7}{8}}\right| = \frac{\sqrt{2}}{4}\ln|2\sqrt{2}x+\sqrt{8x^2+7}|$$

1072.

$$\int \frac{dx}{\sqrt{7-5x^2}} = \frac{1}{\sqrt{5}}\int \frac{dx}{\sqrt{\frac{7}{5}-x^2}} = \frac{1}{\sqrt{5}}\arcsin x\sqrt{\frac{5}{7}}$$

1073.

$$\int \frac{2x-5}{3x^2-2}dx = \frac{1}{3}\int \frac{d(x^2)}{x^2-\frac{2}{3}} - \frac{5}{3}\int \frac{dx}{x^2-\frac{2}{3}} = \frac{1}{3}\ln\left|x^2-\frac{2}{3}\right| - \frac{5\sqrt{3}}{6\sqrt{2}}\ln\left|\frac{x-\sqrt{\frac{2}{3}}}{x+\sqrt{\frac{2}{3}}}\right| = \frac{1}{3}\ln\left|x^2-\frac{2}{3}\right| - \frac{5}{\sqrt{24}}\ln\left|\frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}}\right|$$

1074.

$$\int \frac{3-2x}{5x^2+7}dx = \frac{3}{5}\int \frac{dx}{x^2+\frac{7}{5}} - \frac{1}{5}\int \frac{2x}{x^2+\frac{7}{5}}dx = \frac{3}{\sqrt{35}}\arctan x\sqrt{\frac{5}{7}} - \frac{1}{5}\int \frac{d(x^2+\frac{7}{5})}{x^2+\frac{7}{5}} = \frac{3}{\sqrt{35}}\arctan x\sqrt{\frac{5}{7}} -$$

$$-\frac{1}{5}\ln\left|x^2+\frac{7}{5}\right|$$

1075.

$$\int \frac{3x+1}{\sqrt{5x^2+1}}dx = \frac{3}{\sqrt{5}}\int \frac{x}{\sqrt{x^2+\frac{1}{5}}}dx + \frac{1}{\sqrt{5}}\int \frac{dx}{\sqrt{x^2+\frac{1}{5}}} = \frac{3}{2\sqrt{5}}\int \frac{d(x^2+\frac{1}{5})}{\sqrt{x^2+\frac{1}{5}}} + \frac{1}{\sqrt{5}}\int \frac{dx}{\sqrt{x^2+\frac{1}{5}}} =$$

$$= \frac{1}{\sqrt{5}}\ln|x+\sqrt{x^2+\tfrac{1}{5}}| + \frac{3}{\sqrt{5}}\sqrt{x^2+\tfrac{1}{5}} = \frac{1}{\sqrt{5}}\ln|x+\sqrt{x^2+\tfrac{1}{5}}| + \frac{3}{5}\sqrt{5x^2+1}$$

1076.

$$\int \frac{x+3}{\sqrt{x^2-4}}dx = 3\int \frac{dx}{\sqrt{x^2-4}} + \frac{1}{2}\int \frac{d(x^2-4)}{\sqrt{x^2-4}} = 3\ln|x+\sqrt{x^2-4}| + \sqrt{x^2-4}$$

1077.

$$\int \frac{xdx}{x^2-5} = \frac{1}{2}\int \frac{d(x^2-5)}{x^2-5} = \frac{1}{2}\ln|x^2-5|$$

1078.

$$\int \frac{xdx}{2x^2+3} = \frac{1}{4}\int \frac{d(2x^2+3)}{2x^2+3} = \frac{1}{4}\ln|2x^2+3|$$

1079.

$$\int \frac{ax+b}{a^2x^2+b^2}dx = \frac{1}{2a}\int \frac{d(a^2x^2+b)}{a^2x^2+b^2} + +\frac{b}{a^2}\int \frac{dx}{x^2+\frac{b^2}{a^2}} = \frac{1}{2a}\ln|a^2x^2+b| + \frac{1}{a}\arctan\frac{ax}{b}$$

1080.

$$\int \frac{xdx}{\sqrt{a^4-x^4}} = \frac{1}{2}\int \frac{d(x^2)}{\sqrt{a^4-(x^2)^2}} = \frac{1}{2}\arcsin\frac{x^2}{a^2}$$

1081.

$$\int \frac{x^2}{x^6+1}dx = \frac{1}{3}\int \frac{d(x^3)}{(x^3)^2+1} = \frac{1}{3}\arctan x^3$$

1082.

$$\int \frac{x^2}{\sqrt{x^6-1}}dx = \frac{1}{3}\int \frac{d(x^3)}{\sqrt{(x^3)^2-1}} = \frac{1}{3}\ln|x^3+\sqrt{x^6-1}|$$

1083.

$$\int \sqrt{\frac{\arcsin x}{1-x^2}}dx = \sqrt{\arcsin x}d(\arcsin x) = \frac{2}{3}\sqrt{\arcsin^3 x}$$

1084.

$$\int \frac{\arctan\frac{x}{2}}{x^2+4}dx = \frac{1}{2}\int \frac{\arctan\frac{x}{2}}{\frac{x^2}{4}+1}d\left(\frac{x}{2}\right) = \frac{1}{2}\int \arctan\frac{x}{2}d(\arctan\left(\frac{x}{2}\right)) = \frac{\arctan^2\frac{x}{2}}{4}$$

1085.

$$\int \frac{x-\sqrt{\arctan 2x}}{1+4x^2}dx = \frac{1}{4}\int \frac{x}{x^2+\frac{1}{4}}dx - \frac{1}{2}\int \frac{\sqrt{\arctan 2x}}{1+(2x)^2}d(2x) = \frac{1}{8}\int \frac{d(x^2+\frac{1}{4})}{x^2+\frac{1}{4}} - \frac{1}{2}\int \sqrt{\arctan 2x}d(\arctan 2x) =$$

$$= \frac{1}{8}\ln|x^2+\tfrac{1}{4}| - \frac{1}{3}\sqrt{\arctan^3 2x}$$

1086.

$$\int \frac{dx}{\sqrt{(1+x^2)\ln(x+\sqrt{1+x^2})}}dx = \int \frac{d(\ln(x+\sqrt{1+x^2}))}{\sqrt{\ln(x+\sqrt{1+x^2})}} = 2\sqrt{\ln(x+\sqrt{1+x^2})}$$

1087.

$$\int ae^{-mx}dx = -\frac{a}{m}\int e^{-mx}d(-mx) = -\frac{a}{m}e^{-mx}$$

1088.

$$\int 4^{2-3x}dx = -\frac{1}{3}\int 4^{4-3x}d(2-3x) = -\frac{4^{2-3x}}{3\ln 4}$$

1089.

$$\int (e^t - e^{-t})dt = \int e^t dt - \int e^{-t}dt = e^t + \int e^{-t}d(-t) = e^t + e^{-t}$$

1090.

$$\int (e^{\frac{x}{a}} + e^{-\frac{x}{a}})^2 dx = \int e^{\frac{2x}{a}}dx + 2\int e^{\frac{x}{a}-\frac{x}{a}}dx + \int e^{-\frac{2x}{a}}dx = \frac{a}{2}\int e^{\frac{2x}{a}}d\left(\frac{2x}{a}\right) + 2\int e^0 dx - \frac{a}{2}\int e^{-\frac{2x}{a}}d\left(-\frac{2x}{a}\right) =$$

$$= \frac{a}{2}e^{\frac{2x}{a}} + 2x - \frac{a}{2}e^{-\frac{2x}{a}}$$

1091.

$$\int \frac{(a^x-b^x)^2}{a^xb^x}dx = \int \frac{a^{2x}-2a^xb^x+b^{2x}}{a^xb^x}dx = \int \left(\frac{a}{b}\right)^x dx - 2\int dx + \int \left(\frac{b}{a}\right)^x dx = \left(\frac{a}{b}\right)^x \frac{1}{\ln a - \ln b} -$$

$$-2x + \left(\frac{b}{a}\right)^x \frac{1}{\ln b - \ln a} = \frac{1}{\ln a - \ln b}\left(\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x\right) - 2x$$

1092.

$$\int \frac{a^{2x}-1}{\sqrt{a^x}}dx = \frac{1}{\ln a}\int \frac{(a^x)^2-1}{(a^x)^{\frac{3}{2}}}d(a^x) = \frac{1}{\ln a}\int \sqrt{a^x}d(a^x) - \frac{1}{\ln a}\int (a^x)^{-\frac{3}{2}}d(a^x) = \frac{2}{3\ln a}a^{\frac{3}{2}x} + \frac{2}{\sqrt{a^x}\ln a}$$

1093.

$$\int e^{-(x^2+1)}xdx = -\frac{1}{2}e^{-(x^2+1)}d(-(x^2+1)) = -\frac{1}{2}e^{-(x^2+1)} = -\frac{1}{2e^{x^2+1}}$$

1094.

$$\int x\cdot 7^{x^2}dx = \frac{1}{2}\int 7^{x^2}d(x^2) = \frac{7^{x^2}}{2\ln 7}$$

1095.

$$\int \frac{\sqrt[x]{e}}{x^2}dx = -\int e^{\frac{1}{x}}d\left(\frac{1}{x}\right) = -e^{\frac{1}{x}}$$

1096.

$$\int \frac{5^{\sqrt{x}}}{\sqrt{x}}dx = 2\int 5^{\sqrt{x}}d(\sqrt{x}) = \frac{2\cdot 5^{\sqrt{x}}}{\ln 5}$$

1097.

$$\int \frac{e^x}{e^x-1}dx = \int \frac{1}{e^x-1}d(e^x-1) = \ln|e^x-1|$$

1098.

$$\int e^x\sqrt{a-be^x}dx = -\frac{1}{b}\sqrt{a-be^x}d(a-be^x) = -\frac{2\sqrt{(a-be^x)^3}}{3b}$$

1099.

$$\int (e^{\frac{x}{a}}+1)^{\frac{1}{3}}e^{\frac{x}{a}}dx = a\int \sqrt[3]{e^{\frac{x}{a}}+1}d(e^{\frac{x}{a}}) = \frac{3a\sqrt[3]{(e^{\frac{x}{a}}+1)^4}}{4}$$

1100.

$$\int \frac{dx}{2^x+3} = \frac{1}{3}\int \frac{2^x+3-2^x}{2^x+3}dx = \frac{1}{3}\int dx - \frac{1}{3}\int \frac{2^x}{2^x+3}dx = \frac{x}{3} - \frac{1}{3\ln(a)}\int \frac{d(2^x+3)}{2^x+3} = \frac{x}{3} - \frac{1}{3\ln(a)}\ln|2^x+3|$$

1101.

$$\int \frac{a^xdx}{1+a^{2x}} = \frac{1}{\ln(a)}\int \frac{d(a^x)}{(a^x)^2+1} = \frac{1}{\ln(a)}\arctan a^x$$

1102.

$$\int \frac{e^{-bx}}{1-e^{-2bx}}dx = -\frac{1}{b}\int \frac{d(e^{-bx})}{1-(e^{-bx})^2} = -\frac{1}{2b}\ln\left|\frac{1+e^{-bx}}{1-e^{-bx}}\right|$$

1103.

$$\int \frac{e^tdt}{\sqrt{1-e^{2t}}} = \int \frac{d(e^t)}{\sqrt{1-(e^t)^2}} = \arcsin e^t$$

1104.

$$\int \sin(a+bx)dx = \frac{1}{b}\int \sin(a+bx)d(a+bx) = -\frac{1}{b}\cos(a+bx)$$

1105.

$$\int \cos\frac{x}{\sqrt{2}}dx = \sqrt{2}\int \cos\frac{x}{\sqrt{2}}d(\frac{x}{\sqrt{2}}) = \sqrt{2}\sin\frac{x}{\sqrt{2}}$$

1106.

$$\int(\cos ax{+}\sin ax)^2dx = \int(\cos^2 ax{+}2\cos ax\sin ax{+}\sin^2 ax)dx = \int\left(\frac{1+\cos 2ax}{2}+\sin 2ax+\frac{1-\cos 2ax}{2}\right)dx =$$

$$= \int(1+\sin 2ax)dx = x+\int\sin 2axdx = x+\frac{1}{2a}\int\sin 2axd(2ax) = x-\frac{1}{2a}\cos 2ax$$

1107.

$$\int\frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\int\cos\sqrt{x}d(\sqrt{x}) = 2\sin\sqrt{x}$$

1108.

$$\int\frac{\sin(\lg x)}{x}dx = \ln 10\int\sin(\lg x)d(\lg x) = -\ln(10)\cos(\lg x)$$

1109.

$$\int\sin^2(x)dx = \int\frac{1-\cos 2x}{2}dx = \int\frac{dx}{2}-\frac{1}{2}\int\cos(2x)dx = \frac{x}{2}-\frac{1}{4}\int cos(2x)d(2x) = \frac{x}{2}-\frac{\sin(2x)}{4}$$

1110.

$$\int\cos^2 xdx = \int\frac{1+\cos 2x}{2}dx = \int\frac{dx}{2}+\frac{1}{2}\int\cos 2xdx = \frac{x}{2}+\frac{1}{4}\int\cos(2x)d(2x) = \frac{x}{2}+\frac{\sin(2x)}{4}$$

1111.

$$\int\sec^2(ax+b)dx = \int\frac{dx}{\cos^2(ax+b)} = \frac{1}{a}\int\frac{d(ax+b)}{\cos^2(ax+b)} = \frac{\tan(ax+b)}{a}$$

1112.

$$\int\operatorname{ctg}^2(ax)dx = \frac{1}{a}\int\frac{1-\sin^2(ax)}{\sin^2(ax)}d(ax) = \frac{1}{a}\int\frac{d(ax)}{\sin^2(ax)}-\frac{1}{a}\int d(ax) = -\frac{\operatorname{ctg}(ax)}{a}-x$$

1113.

$$\int\frac{dx}{\sin\frac{x}{a}} = a\int\frac{d(\frac{x}{a})}{\sin\frac{x}{a}} = a\ln|\tan\frac{x}{2a}|$$

1114.

$$\int\frac{dx}{3\cos\left(5x-\frac{\pi}{4}\right)} = \frac{1}{15}\int\frac{d(5x-\frac{\pi}{4})}{\cos(5x-\frac{\pi}{4})} = \frac{1}{15}\ln|\tan(\frac{5x-\frac{\pi}{4}}{2}+\frac{\pi}{4})| = \frac{1}{15}\ln|\tan\left(\frac{5x}{2}+\frac{\pi}{8}\right)|$$

1115.

$$\int\frac{dx}{\sin(ax+b)} = \frac{1}{a}\int\frac{d(ax+b)}{\sin(ax+b)} = \frac{1}{a}\ln|\tan\frac{ax+b}{2}|$$

1116.

$$\int\frac{xdx}{\cos^2 x^2} = \frac{1}{2}\int\frac{d(x^2)}{\cos^x(x^2)} = \frac{1}{2}\tan(x^2)$$

1117.

$$\int x\sin(1-x^2)dx = -\frac{1}{2}\int\sin(1-x^2)d(1-x^2) = \frac{1}{2}\cos(1-x^2)$$

1118.

$$\int\left(\frac{1}{\sin(x\sqrt{2})}-1\right)^2dx=\int\left(\frac{1}{\sin^2(x\sqrt{2})}-\frac{2}{\sin(x\sqrt{2})}+1\right)dx=\int\frac{dx}{\sin^2(x\sqrt{2})}-2\int\frac{dx}{\sin(x\sqrt{2})}+\int dx=$$

$$=\frac{1}{\sqrt{2}}\int\frac{d(x\sqrt{2})}{\sin^2(x\sqrt{2})}-\sqrt{2}\int\frac{d(x\sqrt{2})}{\sin(x\sqrt{2})}+x=-\frac{1}{2}\operatorname{ctg}(x\sqrt{2})-\sqrt{2}\ln|\tan\frac{x\sqrt{2}}{2}|+x$$

1119.

$$\int tg(x)dx=\int\frac{\operatorname{tg}(x)}{1+\operatorname{tg}^2(x)}d(tg(x))=\frac{1}{2}\int\frac{d(tg^2(x)+1)}{tg^2(x)+1}=\frac{1}{2}\ln|tg^2(x)+1|=\frac{1}{2}\ln\frac{1}{\cos^2x}=-\ln|cos(x)|$$

1120.

$$\int ctg(x)dx=-\int\frac{ctg(x)}{1+ctg^2(x)}d(ctg(x))=-\frac{1}{2}\int\frac{d(ctg^2(x)+1)}{ctg^2(x)+1}=-\frac{1}{2}\ln|ctg^2(x)+1|=-\frac{1}{2}\ln\frac{1}{\sin^2x}=\ln|\sin(x)|$$

1121.

$$\int ctg\frac{x}{a-b}dx=(a-b)\int ctg\frac{x}{a-b}d\left(\frac{x}{a-b}\right)=(a-b)\ln|\sin\left(\frac{x}{a-b}\right)|$$

1122.

$$\int\frac{dx}{\operatorname{tg}(\frac{x}{5})}=\int ctg\left(\frac{x}{5}\right)dx=5\int ctg\left(\frac{x}{5}\right)d\left(\frac{x}{5}\right)=5\ln|\sin\left(\frac{x}{5}\right)|$$

1123.

$$\int\frac{tg\sqrt{x}}{\sqrt{x}}dx=2\int tg\sqrt{x}d(\sqrt{x})=-2\ln|\cos\sqrt{x}|$$

1124.

$$\int x\operatorname{ctg}(x^2+1)dx=\frac{1}{2}\int\operatorname{ctg}(x^2+1)d(x^2+1)=\ln|\operatorname{ctg}(x^2+1)|$$

1125.

$$\int\frac{dx}{\sin x\cos x}=2\int\frac{dx}{\sin 2x}=\int\frac{d(2x)}{\sin(2x)}=\ln|tg(x)|$$

1126.

$$\int\cos\frac{x}{a}\sin\frac{x}{a}dx=\frac{1}{2}\int\sin\frac{2x}{a}dx=\frac{a}{4}\int\sin\frac{2x}{a}d\left(\frac{2x}{a}\right)=-\frac{a}{4}\cos\frac{2x}{a}$$

1127.

$$\int\sin^3(6x)\cos(6x)dx=\frac{1}{6}\int\sin^3(6x)d(\sin 6x)=\frac{\sin^4(6x)}{24}$$

1128.

$$\int\frac{\cos(ax)}{\sin^5(ax)}dx=\frac{1}{a}\int\frac{d(\sin(ax)}{\sin^5(ax)}=\frac{\sin^{-4}(ax)}{-4a}=-\frac{1}{4a\sin^4(ax)}$$

1129.

$$\int\frac{\sin(3x)}{3+\cos(3x)}dx=-\frac{1}{3}\int\frac{d(\cos(3x)+3)}{3+\cos(3x)}=-\frac{1}{3}\ln|\cos(3x)+3|$$

1130.

$$\int\frac{\sin(x)\cos(x)}{\sqrt{\cos^2(x)-\sin^2(x)}}dx=\frac{1}{2}\int\frac{\sin(2x)}{\sqrt{\cos(2x)}}dx=-\frac{1}{4}\int\frac{d(\cos(2x))}{\sqrt{\cos(2x)}}=-\frac{1}{2}\sqrt{\cos(2x)}$$

1131.

$$\int \sqrt{1+3\cos^2(x)}\sin(2x)dx = -\frac{1}{2}\int \sqrt{1+3\frac{1+\cos(2x)}{2}}d(\cos(2x)) = -\frac{\sqrt{2}}{4}\int \sqrt{5+3\cos(2x)}d(\cos(2x)) =$$

$$-\frac{\sqrt{2}}{12}\int \sqrt{5+3\cos(2x)}d(5+3\cos(2x)) = \frac{\sqrt{2}}{18}\sqrt{(5+3\cos(2x))^3}$$

1132.

$$\int \operatorname{tg}^3\frac{x}{3}\sec^2\frac{x}{3}dx = \int \frac{\operatorname{tg}^3\frac{x}{3}}{\cos^2\frac{x}{3}}dx = 3\int \operatorname{tg}^3\frac{x}{3}d(\operatorname{tg}\frac{x}{3}) = \frac{3\operatorname{tg}^4\frac{x}{3}}{4}$$

1133.

$$\int \frac{\sqrt{\operatorname{tg}x}}{\cos^2 x}dx = \int \sqrt{\operatorname{tg}(x)}d(\operatorname{tg}(x)) = \frac{2\sqrt{\operatorname{tg}^3(x)}}{3}$$

1134.

$$\int \frac{\operatorname{ctg}^{\frac{2}{3}}x}{\sin^2(x)}dx = \int \operatorname{ctg}^{\frac{2}{3}}x d(\operatorname{ctg}(x)) = \frac{3\operatorname{ctg}^{\frac{5}{3}}x}{5} = \frac{3\sqrt[3]{\operatorname{ctg}^5(x)}}{5}$$

1135.

$$\int \frac{1+\sin(3x)}{\cos^2(3x)}dx = \frac{1}{3}\int \frac{d(3x)}{\cos^2(3x)} - \frac{1}{3}\int \frac{d(cos(3x))}{\cos^2(3x)} = \frac{1}{3}\operatorname{tg}(3x) + \frac{1}{3\cos(3x)}$$

1136.

$$\int \frac{(\cos(ax)+\sin(ax))^2}{\sin(ax)}dx = \int \frac{\cos^2(ax)+2\sin(ax)\cos(ax)+\sin^2(ax)}{\sin(ax)}dx = \int \frac{dx}{\sin(ax)}+2\int \cos(ax)dx =$$

$$+\frac{1}{a}\ln\left|\operatorname{tg}\frac{ax}{2}\right| + \frac{2}{a}\sin(ax)$$

1137.

$$\int \frac{\operatorname{cosec}^2(3x)}{b-a\operatorname{ctg}(3x)}dx = -\frac{1}{3}\int \frac{d(ctg(3x))}{b-a\operatorname{ctg}(3x)} = \frac{1}{3a}\int \frac{d(b-a\operatorname{ctg}(3x))}{b-a\operatorname{ctg}(3x)} = \frac{1}{3a}\ln|b-a\operatorname{ctg}(3x)|$$

1138.

$$\int (2\operatorname{sh}(5x)-3\operatorname{ch}(5x))dx = \frac{2}{5}\int sh(5x)d(5x) - \frac{3}{5}\int ch(5x)d(5x) = \frac{2}{5}ch(5x) - \frac{3}{5}sh(5x)$$

1139.

$$\int sh^2(x)dx = \int \frac{ch(2x)-1}{2}dx = \frac{1}{4}\int ch(2x)d(2x) - \frac{1}{2}x = \frac{sh(2x)-2x}{4}$$

1140.

$$\int \frac{dx}{sh(x)} = \int \frac{sh(x)dx}{sh^2(x)} = \int \frac{d(ch(x))}{ch^2(x)-1} = \frac{1}{2}\ln\left|\frac{ch(x)-1}{ch(x)+1}\right|$$

1141.

$$\int \frac{dx}{ch(x)} = \int \frac{ch(x)dx}{ch^2(x)} = \int \frac{d(sh(x))}{1+sh^2(x)} = \arctan(sh(x))$$

1142.

$$\int \frac{dx}{\operatorname{sh}(x)\operatorname{ch}(x)} = 2\int \frac{dx}{sh(2x)} = \int \frac{d(2x)}{sh(2x)} = \frac{1}{2}\ln\left|\frac{ch(2x)-1}{ch(2x)+1}\right|$$

1143.

$$\int th(x)dx = \int \frac{sh(x)}{ch(x)}dx = \int \frac{d(ch(x))}{ch(x)} = \ln|ch(x)|$$

1144.

$$\int cth(x)dx = \int \frac{ch(x)}{sh(x)}dx = \int \frac{d(sh(x))}{sh(x)} = \ln|sh(x)|$$

Найти неопределенные интегралы:

1145.

$$\int x^5\sqrt{5-x^2}dx = \quad \Longrightarrow (t = 5-x^2) \Longrightarrow \quad = -\frac{1}{2}\int (5-t)^2\sqrt{t}dt = -\frac{1}{2}\int (25-10t+t^2)\sqrt{t}dt = -\frac{25}{2}\int \sqrt{t}dt +$$

$$+5\int t^{\frac{3}{2}}dt - \frac{1}{2}\int t^{\frac{5}{2}}dt = -\frac{25}{3}\sqrt{t^3} + 2\sqrt{t^5} - \frac{1}{7}\sqrt{t^7} = -\frac{25}{3}\sqrt{(5-x^2)^3} + 2\sqrt{(5-x^2)^5} - \frac{1}{7}\sqrt{(5-x^2)^7} =$$

$$\sqrt{(5-x^2)^3}\left(-\frac{25}{3} + 2(5-x^2) - \frac{1}{7}(5-x^2)^2\right) =$$

1146.

$$\int \frac{x^3-1}{x^4-4x+1}dx \quad \Longrightarrow (t = x^4-4x+1) \Longrightarrow \quad \frac{1}{4}\int \frac{d(x^4-4x+1)}{x^4-4x+1} = \frac{1}{4}\ln|x^4-4x+1|$$