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1 <Неопределенный Интеграл>

1.1 <Непосредственное интегрирование>

1°. Основные правила интегрирования.

1) Если $F'(x) = f(x)$, то

$$\int f(x)dx = F(x) + C.$$

где C - произвольная постоянная.

2) $\int Af(x)dx = A \int f(x)dx$, где A - постоянная величина.

3) $\int [f_1(x) \pm f_2(x)]dx = \int f_1(x)dx \pm \int f_2(x)dx$

4) Если $\int f(x)dx = F(x) + C$ и $u = \varphi(x)$, то

$$\int f(u)du = F(u) + C.$$

В частности,

$$\int f(ax + b)dx = \frac{1}{a}F(ax + b) + C \quad (a \neq 0).$$

2° Таблица простейших интегралов.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (I)$$

$$\int \frac{dx}{x} = \ln |x| + C \quad (II)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + C = -\frac{1}{a} \arctan \frac{x}{a} + C_1 \quad (a \neq 0). \quad (III)$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C \quad (a \neq 0) \quad (IV)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c \quad (a \neq 0)$$

$$\int \frac{dx}{\sqrt{x^2 + a}} = \ln |x + \sqrt{x^2 + a}| + C \quad (a \neq 0) \quad (V)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C = -\arccos \frac{x}{a} + C_1 \quad (a > 0) \quad (VI)$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0); \quad \int e^x dx = e^x + C \quad (VII)$$

$$\int \sin x dx = -\cos x + C \quad (VIII)$$

$$\int \cos x dx = \sin x + C \quad (XI)$$

$$\int \frac{1}{\cos^2 x} = \operatorname{tg} x + C \quad (X)$$

$$\int \frac{1}{\sin^2 x} = -\operatorname{ctg} x + C \quad (XI)$$

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + C = \ln |\operatorname{cosec} x - \operatorname{ctg} x| + C \quad (\text{XII})$$

$$\int \frac{dx}{\cos x} = \ln \left| \operatorname{tg} \left(\frac{x}{2} + \frac{\pi}{2} \right) \right| + C = \ln |\operatorname{tg} x + \sec x| + C \quad (\text{XIII})$$

$$\int \operatorname{sh} x dx = \operatorname{ch} x + C \quad (\text{XIV})$$

$$\int \operatorname{ch} x dx = \operatorname{sh} x + C \quad (\text{XV})$$

$$\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C \quad (\text{XVI})$$

$$\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C \quad (\text{XVII})$$

Пример 1.

$$\int (ax^2 + bx + c)dx = \int ax^2 dx + \int bxdx + \int cdx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$$

1031.

$$\int 5a^2 x^6 dx = 5a^2 \int x^6 dx = 5a^2 \cdot \frac{x^7}{7} = \frac{5a^2 x^7}{7}$$

1032.

$$\int (6x^2 + 8x + 3)dx = 6 \int x^2 dx + 8 \int x dx + 3 \int dx = 6 \frac{x^3}{3} + 8 \frac{x^2}{2} + 3x = 2x^3 + 4x^2 + 3x$$

1033.

$$\int x(x+a)(x+b)dx = \int (x^3 + bx^2 + ax^2 + abx)dx = \frac{x^4}{4} + \frac{bx^3}{3} + \frac{ax^3}{3} + \frac{abx^2}{2} = \frac{x^4}{4} + \frac{a+b}{3}x^3 + \frac{ab}{2}x^2$$

1034.

$$\int (a + bx^3)^2 dx = \int b^2 x^6 dx + \int 2abx^3 dx + \int a^2 dx = \frac{b^2 x^7}{7} + \frac{2abx^4}{4} + a^2 x$$

1035.

$$\int \sqrt{2px} dx = \sqrt{2p} \int \sqrt{x} dx = \sqrt{2p} \frac{x\sqrt{x}}{\frac{3}{2}} = \frac{x\sqrt{x}p2\sqrt{2}}{3} = \frac{2\sqrt{2}}{3} \sqrt{x^3}$$

1036.

$$\int \frac{dx}{\sqrt[n]{x}} = \int x^{-\frac{1}{n}} dx = \frac{x^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} = \frac{n}{n-1} \sqrt[n]{x^{n-1}}$$

1037.

$$\int (nx)^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \frac{x^{\frac{1-n}{n}+1}}{\frac{1-n}{n}} = n^{\frac{1-n}{n}+1} \frac{x^{\frac{1-n+n}{n}}}{1-n} = \sqrt[n]{n} \frac{\sqrt[n]{x}}{1-n} =$$

1038.

$$\begin{aligned} \int \left(a^{\frac{2}{3}} - x^{\frac{2}{3}} \right)^3 dx &= \int (a^2 - 3a^{\frac{4}{3}}x^{\frac{2}{3}} + 3a^{\frac{2}{3}}x^{\frac{4}{3}} - x^2) dx = a^2 x - 3\sqrt[3]{a^4} \frac{\sqrt[3]{x^5}}{\frac{5}{3}} + 3\sqrt[3]{a^2} \frac{\sqrt[3]{x^7}}{\frac{7}{3}} - \frac{x^3}{3} = \\ &= a^2 x - \frac{9\sqrt[3]{a^4}}{5} \sqrt[3]{x^5} + \frac{9\sqrt[3]{a^2}}{7} \sqrt[3]{x^7} - \frac{x^3}{3} \end{aligned}$$

1039.

$$\int (\sqrt{x} + 1)(x - \sqrt{x} + 1)dx = \int (x^{\frac{3}{2}} - x + \sqrt{x} + x - \sqrt{x} + 1)dx = \int (x^{\frac{3}{2}} + 1)dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5}\sqrt[5]{x^5}$$

1040.

$$\int \frac{(x^2 + 1)(x^2 - 2)}{\sqrt[3]{x^2}}dx = \int \frac{x^4 - x^2 - 2}{\sqrt[3]{x^2}}dx = \int x^{\frac{10}{3}}dx - \int x^{\frac{4}{3}}dx - 2 \int x^{-\frac{2}{3}}dx = \frac{3\sqrt[3]{x^{13}}}{13} - \frac{3\sqrt[3]{x^7}}{7} - 6\sqrt[3]{x}$$

1041.

$$\begin{aligned} \int \frac{(x^m - x^n)^2}{\sqrt{x}}dx &= \int \frac{x^{2m} - 2x^{m-n} + x^{2n}}{\sqrt{x}}dx = \int (x^{2m-\frac{1}{2}} - 2x^{m-n-\frac{1}{2}} + x^{2n-\frac{1}{2}})dx = \frac{x^{2m+\frac{1}{2}}}{2m+\frac{1}{2}} - \frac{2x^{m-n+\frac{1}{2}}}{m-n+\frac{1}{2}} + \\ &+ \frac{x^{2n+\frac{1}{2}}}{2n+\frac{1}{2}} = \frac{2\sqrt{x^{4m+1}}}{4m+1} - \frac{2\sqrt{x^{2m-2n+1}}}{2m-2n+1} + \frac{2\sqrt{x^{4n+1}}}{4n+1} \end{aligned}$$

1042.

$$\begin{aligned} \int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}}dx &= \int \frac{a^2 - 4\sqrt{a^3}\sqrt{x} + 6ax - 4\sqrt{a}\sqrt{x^3} + x^2}{\sqrt{ax}}dx = a^{\frac{3}{2}} \int \frac{1}{x}dx - 4a \int x^{-\frac{1}{2}}dx + 6\sqrt{a} \int dx - \\ &- 4 \int \sqrt{x}dx + \frac{1}{\sqrt{a}} \int xdx = \sqrt{a^3} \ln|x| - 8a\sqrt{x} + 6\sqrt{ax} - \frac{8}{3}\sqrt{x^3} + \frac{1}{2\sqrt{a}}x^2 \end{aligned}$$

1043.

$$\int \frac{dx}{x^2 + 7} = \int \frac{dx}{x^2 + (\sqrt{7})^2} = \frac{1}{\sqrt{7}} \arctan \frac{x}{\sqrt{7}}$$

1044.

$$\frac{dx}{x^2 - 10} = \frac{dx}{x^2 - (\sqrt{10})^2} = \frac{1}{2a} \ln \left| \frac{x - \sqrt{10}}{x + \sqrt{10}} \right|$$

1045.

$$\int \frac{dx}{\sqrt{x^2 + 4}} = \int \frac{dx}{\sqrt{x^2 + 2^2}} = \ln|x + \sqrt{x^2 + 2}|$$

1046.

$$\int \frac{dx}{\sqrt{8 - x^2}} = \int \frac{dx}{\sqrt{(2\sqrt{2})^2 - x^2}} = \arcsin \frac{x}{2\sqrt{2}}$$

1047.

$$\int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}}dx = \int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{(2-x^2)(2+x^2)}}dx = \int \frac{dx}{\sqrt{2-x^2}} - \int \frac{dx}{\sqrt{2+x^2}} = \arcsin \frac{x}{\sqrt{2}} - \ln|x + \sqrt{2+x^2}|$$

1048. a)

$$\int tg^2 x dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \int \frac{dx}{\cos^2 x} - \int dx = tgx - x$$

1048. б)

$$\int th^2 x dx = \int \frac{sh^2 x}{ch^2 x} dx = \int \frac{ch^2 x - 1}{ch^2 x} dx = \int dx - \int \frac{dx}{ch^2 x} = x - thx$$

1049. а)

$$\int \operatorname{ctg}^2 x dx = \int \frac{\cos^2 x}{\sin^2 x} = \int \frac{1 - \sin^2 x}{\sin^2 x} dx = \int \frac{dx}{\sin^2 x} - \int dx = -\operatorname{ctg} x - x$$

1049. б)

$$\int \operatorname{cth}^2 x = \int \frac{\operatorname{cth}^2 x}{\operatorname{sh}^2 x} dx = \int \frac{1 + \operatorname{sh}^2 x}{\operatorname{sh}^2 x} dx = \int \frac{1}{\operatorname{sh}^2 x} + \int dx = \operatorname{cth}^2 x + x$$

1050.

$$\int 3^x e^x dx = \int (3e)^x dx = \frac{(3e)^x}{\ln(3e)}$$

3°. Интегрирование путем подведения под знак дифференциала.

Пример 2. $\int \frac{dx}{\sqrt{5x-2}} = \frac{1}{5} \int (5x-2)^{-\frac{1}{2}} d(5x-2) = \frac{1}{5} \int u^{-\frac{1}{2}} du = \frac{2}{5} \sqrt{u} = \frac{2}{5} \sqrt{5x-2}$

где было положено $u = x^2$, причем применялись правило (1.1) и табличный интеграл (I).

Пример 3. $\int \frac{x dx}{\sqrt{1+x^4}} = \frac{1}{2} \int \frac{d(x^2)}{\sqrt{1+x^2}} = \frac{1}{2} \ln(x^2 + \sqrt{1+x^4})$

Неявно подразумевалось $u = x^2$, причем применялись правило (1.1) и табличный интеграл (V).

1051.

$$\int \frac{adx}{a-x} \implies (u = a-x) \implies -a \int \frac{du}{u} = -a \ln |u| = -a \ln |a-x|$$

1052.

$$\int \frac{2x+3}{2x+1} dx = \int dx + \int \frac{2dx}{2x+1} \implies (u = 2x+1) \implies = x + \frac{du}{u} = x + \ln |u| = x + \ln |2x+1|$$

1053.

$$\int \frac{1-3x}{3+2x} dx = \implies (u = 3+2x) \implies = \int \frac{1-\frac{3}{2}u+1}{2u} du = \int \frac{du}{u} - \int \frac{3}{4} du = \ln |u| - \frac{3}{4}u = \ln |3+2x| - \frac{9}{4} - \frac{3x}{2}$$

1054.

$$\int \frac{xdx}{a+bx} = \implies (u = bx+a) \implies = \frac{1}{b} \int \frac{\frac{u-a}{b}}{u} du = \frac{1}{b^2} \int \frac{u-a}{u} du = \frac{1}{b^2} u - \frac{a}{b^2} \ln |u| = \frac{a+bx}{b^2} - \frac{a}{b^2} \ln |a+bx|$$

1055.

$$\int \frac{ax+b}{\alpha x+\beta} dx = \implies (u = \alpha x+\beta) \implies = \frac{1}{\alpha^2} \int \frac{u-\beta+\alpha b}{u} du = \frac{u}{\alpha^2} + \frac{\alpha b-\beta}{\alpha^2} \ln |u| = \frac{\alpha x+\beta}{\alpha^2} + \frac{\alpha b-\beta}{\alpha^2} \ln |\alpha x+\beta|$$

1056.

$$\int \frac{x^2+1}{x-1} dx = \implies (u = x-1) \implies = \int \frac{u^2+2u+2}{u} du = \frac{u^2}{2} + 2u + 2 \ln |u| = \frac{x^2-3}{2} + x + 2 \ln |x-1|$$

1057.

$$\int \frac{x^2+5x+7}{x+3} dx = \implies (u = x+3) \implies = \int \frac{u^2-6u+9+5u-15+7}{u} du = \int \frac{u^2-u+1}{u} du =$$

$$= \frac{u^2}{2} - u + \ln|u| = \frac{x^2 + 6x + 9}{2} - x - 3 + \ln|x + 3| = \frac{x^2}{2} + 2x + \frac{3}{2} + \ln|x + 3|$$

1059.

$$\int \left(a + \frac{b}{x-a}\right)^2 dx = \implies (u = x-a) \implies = \int \left(a + \frac{b}{u}\right)^2 du = \int a^2 du + \int 2\frac{ab}{u} du + \int \frac{b^2}{u^2} du = a^2 u +$$

$$+ 2ab \ln|u| + b^2 \frac{u^{-1}}{-1} = a^2(x-a) + 2ab \ln|x-a| - \frac{b^2}{x-a} =$$

1060.

$$\int \frac{x}{(x+1)^2} dx = \implies (u = x+1) \implies = \int \frac{u-1}{u^2} du = \ln|u| - \frac{u^{-1}}{-1} = \ln|x+1| + \frac{1}{x+1}$$

1061.

$$\int \frac{b dy}{\sqrt{1-y}} = \implies (u = 1-y) \implies = -b \int \frac{du}{\sqrt{u}} = -2b\sqrt{u} = -2b\sqrt{1-y}$$

1062.

$$\int \sqrt{a-bx} dx = \implies (u = a-bx) \implies = -\frac{1}{b} \int \sqrt{u} du = -\frac{2}{3b} \sqrt{u^3}$$

1063.

$$\int \frac{x}{\sqrt{x^2+1}} dx = \implies (u = x^2+1) \implies = \frac{1}{2} \int \frac{du}{\sqrt{u}} = \sqrt{x^2+1}$$

1064.

$$\int \frac{\sqrt{x} + \ln x}{x} dx = \int \frac{dx}{\sqrt{x}} + \int \ln x d(\ln x) = 2\sqrt{x} + \frac{\ln^2 x}{2}$$

1065.

$$\int \frac{dx}{3x^2+5} = \frac{1}{3} \int \frac{dx}{x^2+\frac{5}{3}} = \frac{\sqrt{3}}{3\sqrt{5}} \arctan \frac{x\sqrt{3}}{\sqrt{5}} = \frac{1}{\sqrt{15}} \arctan x \sqrt{\frac{3}{5}}$$

1066.

$$\int \frac{dx}{7x^2-8} = \frac{1}{7} \int \frac{dx}{x^2-\frac{8}{7}} = \frac{\sqrt{14}}{28} \ln \left| \frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}} \right| = \frac{\sqrt{14}}{28} \ln \left| \frac{\sqrt{7}x-\sqrt{8}}{\sqrt{7}x+\sqrt{8}} \right|$$

1067.

$$\int \frac{dx}{(a+b)-(a-b)x^2} \quad (0 < b < a) = \frac{1}{(a-b)} \int \frac{dx}{\frac{a+b}{a-b}-x^2} = \frac{1}{2\sqrt{(a-b)(a+b)}} \ln \left| \frac{\sqrt{\frac{a+b}{a-b}}+x}{\sqrt{\frac{a+b}{a-b}}-x} \right| =$$

$$= \frac{1}{2\sqrt{a^2-b^2}} \ln \left| \frac{\sqrt{a+b}+x\sqrt{a-b}}{\sqrt{a+b}-x\sqrt{a-b}} \right|$$

1068.

$$\int \frac{x^2}{x^2+2} dx = \int \frac{x^2+2-2}{x^2+2} dx = \int dx - 2 \int \frac{dx}{x^2+2} = x - \frac{2}{\sqrt{2}} \arctan \frac{x}{\sqrt{2}} = -\sqrt{2} \arctan \frac{x}{\sqrt{2}}$$

1069.

$$\int \frac{x^3}{a^2-x^2} dx = \int \frac{x^3-a^2x+a^2x}{a^2-x^2} dx = a^2 \int \frac{xdx}{a^2-x^2} - \int x dx = -\frac{a^2}{2} \int \frac{d(a^2-x^2)}{a^2-x^2} - \frac{x^2}{2} =$$

$$-\frac{a^2}{2} \ln|a^2-x^2| - \frac{x^2}{2}$$

1070.

$$\int \frac{x^2 - 5x + 6}{x^2 + 4} dx = \int dx - 5 \int \frac{x}{x^2 + 4} dx + 2 \int \frac{dx}{x^2 + 4} = x - \frac{5}{2} \int \frac{d(x^2 + 4)}{x^2 + 4} + \frac{2}{2} \arctan \frac{x}{2} =$$

$$= x - \frac{5}{2} \ln |x^2 + 4| + \arctan \frac{x}{2}$$

1071.

$$\int \frac{dx}{\sqrt{7 + 8x^2}} = \frac{1}{2\sqrt{2}} \int \frac{dx}{\sqrt{x^2 + \frac{7}{8}}} = \frac{1}{2\sqrt{2}} \ln \left| x + \sqrt{x^2 + \frac{7}{8}} \right| = \frac{\sqrt{2}}{4} \ln |2\sqrt{2}x + \sqrt{8x^2 + 7}|$$

1072.

$$\int \frac{dx}{\sqrt{7 - 5x^2}} = \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{\frac{7}{5} - x^2}} = \frac{1}{\sqrt{5}} \arcsin x \sqrt{\frac{5}{7}}$$

1073.

$$\int \frac{2x - 5}{3x^2 - 2} dx = \frac{1}{3} \int \frac{d(x^2)}{x^2 - \frac{2}{3}} - \frac{5}{3} \int \frac{dx}{x^2 - \frac{2}{3}} = \frac{1}{3} \ln \left| x^2 - \frac{2}{3} \right| - \frac{5\sqrt{3}}{6\sqrt{2}} \ln \left| \frac{x - \sqrt{\frac{2}{3}}}{x + \sqrt{\frac{2}{3}}} \right| = \frac{1}{3} \ln \left| x^2 - \frac{2}{3} \right| - \frac{5}{\sqrt{24}} \ln \left| \frac{\sqrt{3}x - \sqrt{2}}{\sqrt{3}x + \sqrt{2}} \right|$$

1074.

$$\int \frac{3 - 2x}{5x^2 + 7} dx = \frac{3}{5} \int \frac{dx}{x^2 + \frac{7}{5}} - \frac{1}{5} \int \frac{2x}{x^2 + \frac{7}{5}} dx = \frac{3}{\sqrt{35}} \arctan x \sqrt{\frac{5}{7}} - \frac{1}{5} \int \frac{d(x^2 + \frac{7}{5})}{x^2 + \frac{7}{5}} = \frac{3}{\sqrt{35}} \arctan x \sqrt{\frac{5}{7}} -$$

$$- \frac{1}{5} \ln \left| x^2 + \frac{7}{5} \right|$$

1075.

$$\int \frac{3x + 1}{\sqrt{5x^2 + 1}} dx = \frac{3}{\sqrt{5}} \int \frac{x}{\sqrt{x^2 + \frac{1}{5}}} dx + \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{x^2 + \frac{1}{5}}} = \frac{3}{2\sqrt{5}} \int \frac{d(x^2 + \frac{1}{5})}{\sqrt{x^2 + \frac{1}{5}}} + \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{x^2 + \frac{1}{5}}} =$$

$$= \frac{1}{\sqrt{5}} \ln |x + \sqrt{x^2 + \frac{1}{5}}| + \frac{3}{\sqrt{5}} \sqrt{x^2 + \frac{1}{5}} = \frac{1}{\sqrt{5}} \ln |x + \sqrt{x^2 + \frac{1}{5}}| + \frac{3}{5} \sqrt{5x^2 + 1}$$

1076.

$$\int \frac{x + 3}{\sqrt{x^2 - 4}} dx = 3 \int \frac{dx}{\sqrt{x^2 - 4}} + \frac{1}{2} \int \frac{d(x^2 - 4)}{\sqrt{x^2 - 4}} = 3 \ln |x + \sqrt{x^2 - 4}| + \sqrt{x^2 - 4}$$

1077.

$$\int \frac{x dx}{x^2 - 5} = \frac{1}{2} \int \frac{d(x^2 - 5)}{x^2 - 5} = \frac{1}{2} \ln |x^2 - 5|$$

1078.

$$\int \frac{x dx}{2x^2 + 3} = \frac{1}{4} \int \frac{d(2x^2 + 3)}{2x^2 + 3} = \frac{1}{4} \ln |2x^2 + 3|$$

1079.

$$\int \frac{ax+b}{a^2x^2+b^2}dx = \frac{1}{2a} \int \frac{d(a^2x^2+b)}{a^2x^2+b^2} + \frac{b}{a^2} \int \frac{dx}{x^2+\frac{b^2}{a^2}} = \frac{1}{2a} \ln|a^2x^2+b| + \frac{1}{a} \arctan \frac{ax}{b}$$

1080.

$$\int \frac{xdx}{\sqrt{a^4-x^4}} = \frac{1}{2} \int \frac{d(x^2)}{\sqrt{a^4-(x^2)^2}} = \frac{1}{2} \arcsin \frac{x^2}{a^2}$$

1081.

$$\int \frac{x^2}{x^6+1}dx = \frac{1}{3} \int \frac{d(x^3)}{(x^3)^2+1} = \frac{1}{3} \arctan x^3$$

1082.

$$\int \frac{x^2}{\sqrt{x^6-1}}dx = \frac{1}{3} \int \frac{d(x^3)}{\sqrt{(x^3)^2-1}} = \frac{1}{3} \ln|x^3+\sqrt{x^6-1}|$$

1083.

$$\int \sqrt{\frac{\arcsin x}{1-x^2}}dx = \sqrt{\arcsin x}d(\arcsin x) = \frac{2}{3} \sqrt{\arcsin^3 x}$$

1084.

$$\int \frac{\arctan \frac{x}{2}}{x^2+4}dx = \frac{1}{2} \int \frac{\arctan \frac{x}{2}}{\frac{x^2}{4}+1}d\left(\frac{x}{2}\right) = \frac{1}{2} \int \arctan \frac{x}{2}d(\arctan \left(\frac{x}{2}\right)) = \frac{\arctan^2 \frac{x}{2}}{4}$$

1085.

$$\begin{aligned} \int \frac{x-\sqrt{\arctan 2x}}{1+4x^2}dx &= \frac{1}{4} \int \frac{x}{x^2+\frac{1}{4}}dx - \frac{1}{2} \int \frac{\sqrt{\arctan 2x}}{1+(2x)^2}d(2x) = \frac{1}{8} \int \frac{d(x^2+\frac{1}{4})}{x^2+\frac{1}{4}} - \frac{1}{2} \int \sqrt{\arctan 2x}d(\arctan 2x) = \\ &= \frac{1}{8} \ln|x^2+\frac{1}{4}| - \frac{1}{3} \sqrt{\arctan^3 2x} \end{aligned}$$

1086.

$$\int \frac{dx}{\sqrt{(1+x^2)\ln(x+\sqrt{1+x^2})}}dx = \int \frac{d(\ln(x+\sqrt{1+x^2}))}{\sqrt{\ln(x+\sqrt{1+x^2})}} = 2\sqrt{\ln(x+\sqrt{1+x^2})}$$

1087.

$$\int ae^{-mx}dx = -\frac{a}{m} \int e^{-mx}d(-mx) = -\frac{a}{m}e^{-mx}$$

1088.

$$\int 4^{2-3x}dx = -\frac{1}{3} \int 4^{4-3x}d(2-3x) = -\frac{4^{2-3x}}{3 \ln 4}$$

1089.

$$\int (e^t - e^{-t})dt = \int e^tdt - \int e^{-t}dt = e^t + \int e^{-t}d(-t) = e^t + e^{-t}$$

1090.

$$\begin{aligned} \int (e^{\frac{x}{a}} + e^{-\frac{x}{a}})^2dx &= \int e^{\frac{2x}{a}}dx + 2 \int e^{\frac{x}{a}-\frac{x}{a}}dx + \int e^{-\frac{2x}{a}}dx = \frac{a}{2} \int e^{\frac{2x}{a}}d\left(\frac{2x}{a}\right) + 2 \int e^0dx - \frac{a}{2} \int e^{-\frac{2x}{a}}d\left(-\frac{2x}{a}\right) = \\ &= \frac{a}{2}e^{\frac{2x}{a}} + 2x - \frac{a}{2}e^{-\frac{2x}{a}} \end{aligned}$$

1091.

$$\int \frac{(a^x - b^x)^2}{a^x b^x} dx = \int \frac{a^{2x} - 2a^x b^x + b^{2x}}{a^x b^x} dx = \int \left(\frac{a}{b}\right)^x dx - 2 \int dx + \int \left(\frac{b}{a}\right)^x dx = \left(\frac{a}{b}\right)^x \frac{1}{\ln a - \ln b} - 2x + \left(\frac{b}{a}\right)^x \frac{1}{\ln b - \ln a} = \frac{1}{\ln a - \ln b} \left(\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x \right) - 2x$$

1092.

$$\int \frac{a^{2x} - 1}{\sqrt{a^x}} dx = \frac{1}{\ln a} \int \frac{(a^x)^2 - 1}{(a^x)^{\frac{3}{2}}} d(a^x) = \frac{1}{\ln a} \int \sqrt{a^x} d(a^x) - \frac{1}{\ln a} \int (a^x)^{-\frac{3}{2}} d(a^x) = \frac{2}{3 \ln a} a^{\frac{3}{2}x} + \frac{2}{\sqrt{a^x} \ln a}$$

1093.

$$\int e^{-(x^2+1)} x dx = -\frac{1}{2} e^{-(x^2+1)} d(-(x^2+1)) = -\frac{1}{2} e^{-(x^2+1)} = -\frac{1}{2e^{x^2+1}}$$

1094.

$$\int x \cdot 7^{x^2} dx = \frac{1}{2} \int 7^{x^2} d(x^2) = \frac{7^{x^2}}{2 \ln 7}$$

1095.

$$\int \frac{\sqrt[x]{e}}{x^2} dx = - \int e^{\frac{1}{x}} d\left(\frac{1}{x}\right) = -e^{\frac{1}{x}}$$

1096.

$$\int \frac{5^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int 5^{\sqrt{x}} d(\sqrt{x}) = \frac{2 \cdot 5^{\sqrt{x}}}{\ln 5}$$

1097.

$$\int \frac{e^x}{e^x - 1} dx = \int \frac{1}{e^x - 1} d(e^x - 1) = \ln |e^x - 1|$$

1098.

$$\int e^x \sqrt{a - be^x} dx = -\frac{1}{b} \sqrt{a - be^x} d(a - be^x) = -\frac{2\sqrt{(a - be^x)^3}}{3b}$$

1099.

$$\int (e^{\frac{x}{a}} + 1)^{\frac{1}{3}} e^{\frac{x}{a}} dx = a \int \sqrt[3]{e^{\frac{x}{a}} + 1} d(e^{\frac{x}{a}}) = \frac{3a \sqrt[3]{(e^{\frac{x}{a}} + 1)^4}}{4}$$

1100.

$$\int \frac{dx}{2^x + 3} = \frac{1}{3} \int \frac{2^x + 3 - 2^x}{2^x + 3} dx = \frac{1}{3} \int dx - \frac{1}{3} \int \frac{2^x}{2^x + 3} dx = \frac{x}{3} - \frac{1}{3 \ln(a)} \int \frac{d(2^x + 3)}{2^x + 3} = \frac{x}{3} - \frac{1}{3 \ln(a)} \ln |2^x + 3|$$

1101.

$$\int \frac{a^x dx}{1 + a^{2x}} = \frac{1}{\ln(a)} \int \frac{d(a^x)}{(a^x)^2 + 1} = \frac{1}{\ln(a)} \arctan a^x$$

1102.

$$\int \frac{e^{-bx}}{1 - e^{-2bx}} dx = -\frac{1}{b} \int \frac{d(e^{-bx})}{1 - (e^{-bx})^2} = -\frac{1}{2b} \ln \left| \frac{1 + e^{-bx}}{1 - e^{-bx}} \right|$$

1103.

$$\int \frac{e^t dt}{\sqrt{1 - e^{2t}}} = \int \frac{d(e^t)}{\sqrt{1 - (e^t)^2}} = \arcsin e^t$$

1104.

$$\int \sin(a + bx) dx = \frac{1}{b} \int \sin(a + bx) d(a + bx) = -\frac{1}{b} \cos(a + bx)$$

1105.

$$\int \cos \frac{x}{\sqrt{2}} dx = \sqrt{2} \int \cos \frac{x}{\sqrt{2}} d\left(\frac{x}{\sqrt{2}}\right) = \sqrt{2} \sin \frac{x}{\sqrt{2}}$$

1106.

$$\begin{aligned} \int (\cos ax + \sin ax)^2 dx &= \int (\cos^2 ax + 2 \cos ax \sin ax + \sin^2 ax) dx = \int \left(\frac{1 + \cos 2ax}{2} + \sin 2ax + \frac{1 - \cos 2ax}{2} \right) dx = \\ &= \int (1 + \sin 2ax) dx = x + \int \sin 2ax dx = x + \frac{1}{2a} \int \sin 2ax d(2ax) = x - \frac{1}{2a} \cos 2ax \end{aligned}$$

1107.

$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = 2 \int \cos \sqrt{x} d(\sqrt{x}) = 2 \sin \sqrt{x}$$

1108.

$$\int \frac{\sin(\lg x)}{x} dx = \ln 10 \int \sin(\lg x) d(\lg x) = -\ln(10) \cos(\lg x)$$

1109.

$$\int \sin^2(x) dx = \int \frac{1 - \cos 2x}{2} dx = \int \frac{dx}{2} - \frac{1}{2} \int \cos(2x) dx = \frac{x}{2} - \frac{1}{4} \int \cos(2x) d(2x) = \frac{x}{2} - \frac{\sin(2x)}{4}$$

1110.

$$\int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx = \int \frac{dx}{2} + \frac{1}{2} \int \cos 2x dx = \frac{x}{2} + \frac{1}{4} \int \cos(2x) d(2x) = \frac{x}{2} + \frac{\sin(2x)}{4}$$

1111.

$$\int \sec^2(ax + b) dx = \int \frac{dx}{\cos^2(ax + b)} = \frac{1}{a} \int \frac{d(ax + b)}{\cos^2(ax + b)} = \frac{\tan(ax + b)}{a}$$

1112.

$$\int \operatorname{ctg}^2(ax) dx = \frac{1}{a} \int \frac{1 - \sin^2(ax)}{\sin^2(ax)} d(ax) = \frac{1}{a} \int \frac{d(ax)}{\sin^2(ax)} - \frac{1}{a} \int d(ax) = -\frac{\operatorname{ctg}(ax)}{a} - x$$

1113.

$$\int \frac{dx}{\sin \frac{x}{a}} = a \int \frac{d(\frac{x}{a})}{\sin \frac{x}{a}} = a \ln \left| \tan \frac{x}{2a} \right|$$

1114.

$$\int \frac{dx}{3 \cos \left(5x - \frac{\pi}{4}\right)} = \frac{1}{15} \int \frac{d\left(5x - \frac{\pi}{4}\right)}{\cos \left(5x - \frac{\pi}{4}\right)} = \frac{1}{15} \ln \left| \tan \left(\frac{5x - \frac{\pi}{4}}{2} + \frac{\pi}{4}\right) \right| = \frac{1}{15} \ln \left| \tan \left(\frac{5x}{2} + \frac{\pi}{8}\right) \right|$$

1115.

$$\int \frac{dx}{\sin(ax + b)} = \frac{1}{a} \int \frac{d(ax + b)}{\sin(ax + b)} = \frac{1}{a} \ln \left| \tan \frac{ax + b}{2} \right|$$

1116.

$$\int \frac{x dx}{\cos^2 x^2} = \frac{1}{2} \int \frac{d(x^2)}{\cos^2(x^2)} = \frac{1}{2} \tan(x^2)$$

1117.

$$\int x \sin(1 - x^2) dx = -\frac{1}{2} \int \sin(1 - x^2) d(1 - x^2) = \frac{1}{2} \cos(1 - x^2)$$

1118.

$$\begin{aligned} \int \left(\frac{1}{\sin(x\sqrt{2})} - 1 \right)^2 dx &= \int \left(\frac{1}{\sin^2(x\sqrt{2})} - \frac{2}{\sin(x\sqrt{2})} + 1 \right) dx = \int \frac{dx}{\sin^2(x\sqrt{2})} - 2 \int \frac{dx}{\sin(x\sqrt{2})} + \int dx = \\ &= \frac{1}{\sqrt{2}} \int \frac{d(x\sqrt{2})}{\sin^2(x\sqrt{2})} - \sqrt{2} \int \frac{d(x\sqrt{2})}{\sin(x\sqrt{2})} + x = -\frac{1}{2} \operatorname{ctg}(x\sqrt{2}) - \sqrt{2} \ln \left| \tan \frac{x\sqrt{2}}{2} \right| + x \end{aligned}$$

1119.

$$\int \operatorname{tg}(x) dx = \int \frac{\operatorname{tg}(x)}{1 + \operatorname{tg}^2(x)} d(\operatorname{tg}(x)) = \frac{1}{2} \int \frac{d(\operatorname{tg}^2(x) + 1)}{\operatorname{tg}^2(x) + 1} = \frac{1}{2} \ln |\operatorname{tg}^2(x) + 1| = \frac{1}{2} \ln \frac{1}{\cos^2 x} = -\ln |\cos(x)|$$

1120.

$$\int \operatorname{ctg}(x) dx = - \int \frac{\operatorname{ctg}(x)}{1 + \operatorname{ctg}^2(x)} d(\operatorname{ctg}(x)) = -\frac{1}{2} \int \frac{d(\operatorname{ctg}^2(x) + 1)}{\operatorname{ctg}^2(x) + 1} = -\frac{1}{2} \ln |\operatorname{ctg}^2(x) + 1| = -\frac{1}{2} \ln \frac{1}{\sin^2 x} = \ln |\sin(x)|$$

1121.

$$\int \operatorname{ctg} \frac{x}{a-b} dx = (a-b) \int \operatorname{ctg} \frac{x}{a-b} d\left(\frac{x}{a-b}\right) = (a-b) \ln \left| \sin \left(\frac{x}{a-b} \right) \right|$$

1122.

$$\int \frac{dx}{\operatorname{tg}\left(\frac{x}{5}\right)} = \int \operatorname{ctg}\left(\frac{x}{5}\right) dx = 5 \int \operatorname{ctg}\left(\frac{x}{5}\right) d\left(\frac{x}{5}\right) = 5 \ln \left| \sin \left(\frac{x}{5} \right) \right|$$

1123.

$$\int \frac{\operatorname{tg}\sqrt{x}}{\sqrt{x}} dx = 2 \int \operatorname{tg}\sqrt{x} d(\sqrt{x}) = -2 \ln |\cos \sqrt{x}|$$

1124.

$$\int x \operatorname{ctg}(x^2 + 1) dx = \frac{1}{2} \int \operatorname{ctg}(x^2 + 1) d(x^2 + 1) = \ln |\operatorname{ctg}(x^2 + 1)|$$

1125.

$$\int \frac{dx}{\sin x \cos x} = 2 \int \frac{dx}{\sin 2x} = \int \frac{d(2x)}{\sin(2x)} = \ln |\operatorname{tg}(x)|$$

1126.

$$\int \cos \frac{x}{a} \sin \frac{x}{a} dx = \frac{1}{2} \int \sin \frac{2x}{a} dx = \frac{a}{4} \int \sin \frac{2x}{a} d\left(\frac{2x}{a}\right) = -\frac{a}{4} \cos \frac{2x}{a}$$

1127.

$$\int \sin^3(6x) \cos(6x) dx = \frac{1}{6} \int \sin^3(6x) d(\sin 6x) = \frac{\sin^4(6x)}{24}$$

1128.

$$\int \frac{\cos(ax)}{\sin^5(ax)} dx = \frac{1}{a} \int \frac{d(\sin(ax))}{\sin^5(ax)} = \frac{\sin^{-4}(ax)}{-4a} = -\frac{1}{4a \sin^4(ax)}$$

1129.

$$\int \frac{\sin(3x)}{3 + \cos(3x)} dx = -\frac{1}{3} \int \frac{d(\cos(3x) + 3)}{3 + \cos(3x)} = -\frac{1}{3} \ln |\cos(3x) + 3|$$

1130.

$$\int \frac{\sin(x) \cos(x)}{\sqrt{\cos^2(x) - \sin^2(x)}} dx = \frac{1}{2} \int \frac{\sin(2x)}{\sqrt{\cos(2x)}} dx = -\frac{1}{4} \int \frac{d(\cos(2x))}{\sqrt{\cos(2x)}} = -\frac{1}{2} \sqrt{\cos(2x)}$$

1131.

$$\int \sqrt{1+3\cos^2(x)} \sin(2x) dx = -\frac{1}{2} \int \sqrt{1+3\frac{1+\cos(2x)}{2}} d(\cos(2x)) = -\frac{\sqrt{2}}{4} \int \sqrt{5+3\cos(2x)} d(\cos(2x)) =$$

$$-\frac{\sqrt{2}}{12} \int \sqrt{5+3\cos(2x)} d(5+3\cos(2x)) = \frac{\sqrt{2}}{18} \sqrt{(5+3\cos(2x))^3}$$

1132.

$$\int \operatorname{tg}^3 \frac{x}{3} \sec^2 \frac{x}{3} dx = \int \frac{\operatorname{tg}^3 \frac{x}{3}}{\cos^2 \frac{x}{3}} dx = 3 \int \operatorname{tg}^3 \frac{x}{3} d(\operatorname{tg} \frac{x}{3}) = \frac{3 \operatorname{tg}^4 \frac{x}{3}}{4}$$

1133.

$$\int \frac{\sqrt{\operatorname{tg} x}}{\cos^2 x} dx = \int \sqrt{\operatorname{tg}(x)} d(\operatorname{tg}(x)) = \frac{2\sqrt{\operatorname{tg}^3(x)}}{3}$$

1134.

$$\int \frac{\operatorname{ctg}^{\frac{2}{3}} x}{\sin^2(x)} dx = \int \operatorname{ctg}^{\frac{2}{3}} x d(\operatorname{ctg}(x)) = \frac{3 \operatorname{ctg}^{\frac{5}{3}} x}{5} = \frac{3 \sqrt[3]{\operatorname{ctg}^5(x)}}{5}$$

1135.

$$\int \frac{1+\sin(3x)}{\cos^2(3x)} dx = \frac{1}{3} \int \frac{d(3x)}{\cos^2(3x)} - \frac{1}{3} \int \frac{d(\cos(3x))}{\cos^2(3x)} = \frac{1}{3} \operatorname{tg}(3x) + \frac{1}{3 \cos(3x)}$$

1136.

$$\int \frac{(\cos(ax) + \sin(ax))^2}{\sin(ax)} dx = \int \frac{\cos^2(ax) + 2 \sin(ax) \cos(ax) + \sin^2(ax)}{\sin(ax)} dx = \int \frac{dx}{\sin(ax)} + 2 \int \cos(ax) dx =$$

$$+\frac{1}{a} \ln \left| \operatorname{tg} \frac{ax}{2} \right| + \frac{2}{a} \sin(ax)$$

1137.

$$\int \frac{\operatorname{cosec}^2(3x)}{b-a \operatorname{ctg}(3x)} dx = -\frac{1}{3} \int \frac{d(\operatorname{ctg}(3x))}{b-a \operatorname{ctg}(3x)} = \frac{1}{3a} \int \frac{d(b-a \operatorname{ctg}(3x))}{b-a \operatorname{ctg}(3x)} = \frac{1}{3a} \ln |b-a \operatorname{ctg}(3x)|$$

1138.

$$\int (2 \operatorname{sh}(5x) - 3 \operatorname{ch}(5x)) dx = \frac{2}{5} \int \operatorname{sh}(5x) d(5x) - \frac{3}{5} \int \operatorname{ch}(5x) d(5x) = \frac{2}{5} \operatorname{ch}(5x) - \frac{3}{5} \operatorname{sh}(5x)$$

1139.

$$\int \operatorname{sh}^2(x) dx = \int \frac{\operatorname{ch}(2x) - 1}{2} dx = \frac{1}{4} \int \operatorname{ch}(2x) d(2x) - \frac{1}{2} x = \frac{\operatorname{sh}(2x) - 2x}{4}$$

1140.

$$\int \frac{dx}{\operatorname{sh}(x)} = \int \frac{\operatorname{sh}(x) dx}{\operatorname{sh}^2(x)} = \int \frac{d(\operatorname{ch}(x))}{\operatorname{ch}^2(x) - 1} = \frac{1}{2} \ln \left| \frac{\operatorname{ch}(x) - 1}{\operatorname{ch}(x) + 1} \right|$$

1141.

$$\int \frac{dx}{\operatorname{ch}(x)} = \int \frac{\operatorname{ch}(x) dx}{\operatorname{ch}^2(x)} = \int \frac{d(\operatorname{sh}(x))}{1 + \operatorname{sh}^2(x)} = \arctan(\operatorname{sh}(x))$$

1142.

$$\int \frac{dx}{\operatorname{sh}(x) \operatorname{ch}(x)} = 2 \int \frac{dx}{\operatorname{sh}(2x)} = \int \frac{d(2x)}{\operatorname{sh}(2x)} = \frac{1}{2} \ln \left| \frac{\operatorname{ch}(2x) - 1}{\operatorname{ch}(2x) + 1} \right|$$

1143.

$$\int th(x)dx = \int \frac{sh(x)}{ch(x)}dx = \int \frac{d(ch(x))}{ch(x)} = \ln |ch(x)|$$

1144.

$$\int cth(x)dx = \int \frac{ch(x)}{sh(x)}dx = \int \frac{d(sh(x))}{sh(x)} = \ln |sh(x)|$$

Найти неопределенные интегралы:

1145.

$$\begin{aligned} \int x^5 \sqrt{5-x^2} dx &= \implies (t = 5-x^2) \implies = -\frac{1}{2} \int (5-t)^2 \sqrt{t} dt = -\frac{1}{2} \int (25-10t+t^2) \sqrt{t} dt = -\frac{25}{2} \int \sqrt{t} dt + \\ + 5 \int t^{\frac{3}{2}} dt - \frac{1}{2} \int t^{\frac{5}{2}} dt &= -\frac{25}{3} \sqrt{t^3} + 2\sqrt{t^5} - \frac{1}{7} \sqrt{t^7} = -\frac{25}{3} \sqrt{(5-x^2)^3} + 2\sqrt{(5-x^2)^5} - \frac{1}{7} \sqrt{(5-x^2)^7} = \\ \sqrt{(5-x^2)^3} \left(-\frac{25}{3} + 2(5-x^2) - \frac{1}{7}(5-x^2)^2 \right) \end{aligned}$$

1146.

$$\int \frac{x^3-1}{x^4-4x+1} dx \implies (t = x^4-4x+1) \implies \frac{1}{4} \int \frac{d(x^4-4x+1)}{x^4-4x+1} = \frac{1}{4} \ln |x^4-4x+1|$$