Report

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Problem Description

Given a set of nodes, each characterized by their (x, y) coordinates in a plane and an associated cost, the challenge is to select exactly 50% of these nodes and form a Hamiltonian cycle.

The goal is to minimize the sum of the total length of the path plus the total cost of the selected nodes.

Distances between nodes are computed as Euclidean distances and rounded to the nearest integer.

Methodology

Hybrid Evolutionary Algorithm

In Hybrid Evolutionary Algorithms evolutionary approaches are combined with other optimization methods. This can include techniques like Local Search, which refines individual solutions, or machine learning methods that guide the search process. In this assignment, we will stick to Local Search solution, particularly Steepest Local Search.

Source code

Link: Source Code

Pseudocode

Hybrid Evolutionary Algorithm

```
FUNCTION HEA(DistanceMatrix, Costs, EndTime, PopSize, Oper, UseLocal):

    StartTime = time()
    Counter = 0

    Population = (generate 20 solutions using SteepestLocalSearch)
    TotalCosts = (compute total costs for each of the solutions from population)

    WHILE (time() - StartTime < EndTime):
        Parent1, Parent2 = (uniformly randomly choose 2 different solutions from
population)

        IF (Oper = 1):
            Child = Operator_1(Parent1, Parent2)
        ELSE IF (Oper = 2):
            Child = Operator_2(Parent1, Parent2)

        IF (UseLocal is True):
            Child = SteepestLocalSearch(Child, DistanceMatrix, Costs)

        ChildTotalCost = GetTotalCost(Child, DistanceMatrix, Costs)
        IF (ChildTotalCost not it TotalCosts):
            MaxTotalCost, Idx = (get worst total cost and its index)
            IF (ChildTotalCost < MaxTotalCost):
                Population[Idx] = Child
                TotalCosts[Idx] = ChildTotalCost
        Counter += 1
    BestSolution, BestTotalCost = (get best solution from population and its total
cost)

    RETURN BestSolution, BestTotalCost, Counter

Operator_1(Parent1, Parent2):
    - get all common edges and nodes
    - place all common edges in empty child in places as they are in one of
parents
    - place all remain common nodes in respective places
    - fill empty places randomly, so that there are no repetitions
Operator_2(Parent1, Parent2):
    - get all common edges and nodes
    - remove from one parent all nodes and edges that are not in common
    - apply Greedy-2-regret-weighted to complete solution
```

Computational Experiments

Results

Table of Cost

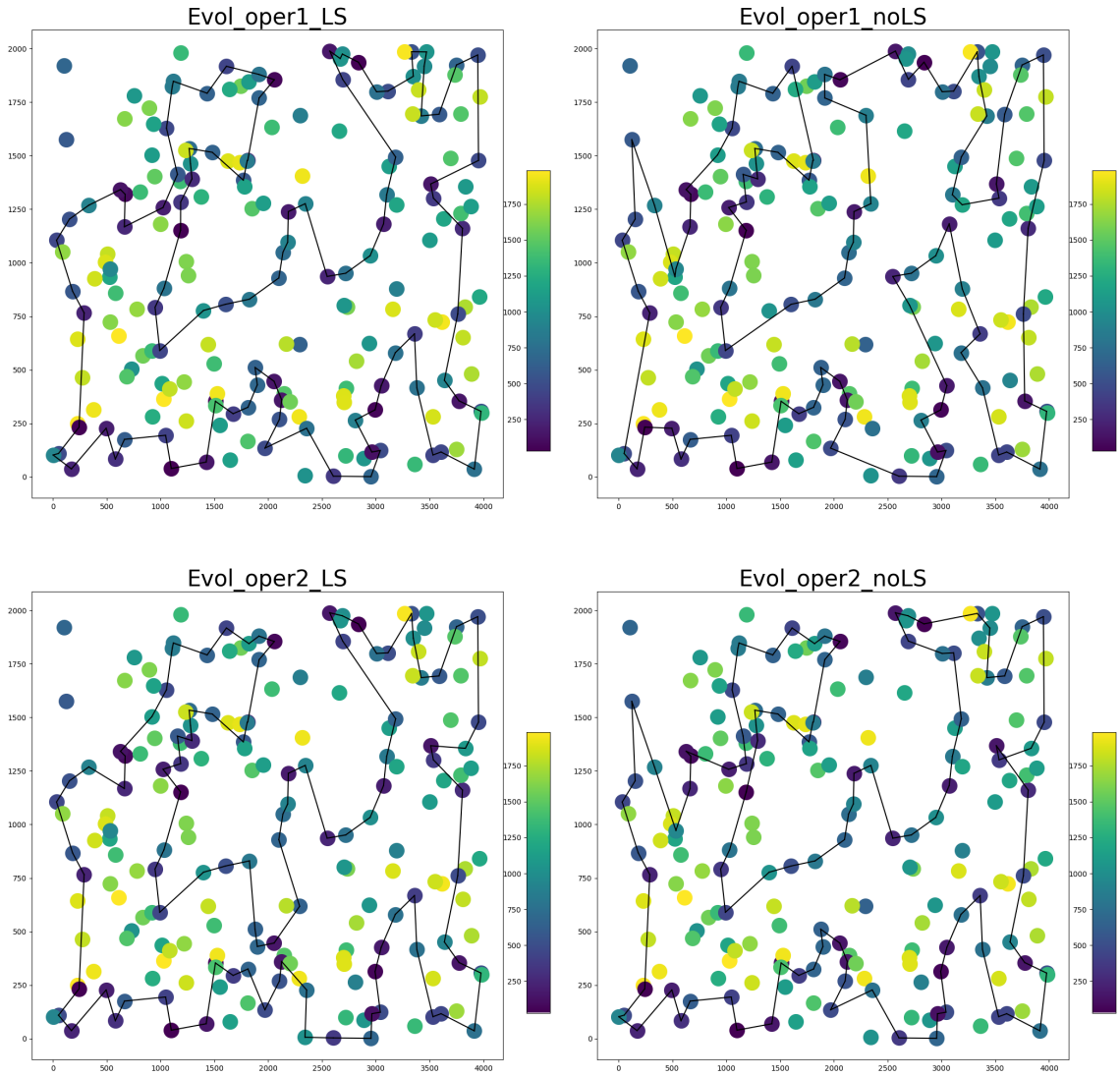
Algorithm	TSPA	TSPB	TSPC	TSPD
RandomSearch	264715.690 (234787.0 – 289096.0)	265095.315 (238995.0 – 287788.0)	214163.21 (192811.0 – 232188.0)	217922.530 (195986.0 – 246928.0)
NearestNeighbor	86319.43 (83561.0 - 93749.0)	77688.285 (75928.0 – 79791.0)	56709.06 (53466.0 – 60369.0)	52472.030 (48240.0 – 57939.0)
GreedyCycle	78049.310 (75990.0 – 81934.0)	71717.795 (68973.0 – 78237.0)	56404.07 (53723.0 – 58868.0)	55334.72 (50717.0 – 61896.0)
Greedy2Regret	117178.570 (110218.0 -124855.0)	121592.715 (111115.0 – 131138.0)	69547.98 (66114.0 – 73603.0)	71900.575 (66024.0 – 76887.0)
Greedy2RegretWeighted	75953.435 (74701.0 – 78510.0)	71428.365 (69147.0 – 77017.0)	54217.24 (51747.0 – 58087.0)	52285.360 (47629.0 – 57787.0)
Greedy-edges-Random	77965.71 (75112.0 - 82142.0)	71018.74 (68315.0 - 74769.0)	51652.62 (49206.0 - 54174.0)	48705.455 (45913.0 - 51873.0)
Greedy-edges-GreedyHeuristic	75438.075 (74521.0 - 77298.0)	70388.815 (67808.0 - 76131.0)	53714.495 (51034.0 - 56971.0)	51525.635 (47281.0 - 57524.0)
Greedy-nodes-Random	90554.405 (83565.0 - 100644.0)	85537.285 (77218.0 - 93542.0)	63718.49 (56594.0 - 71397.0)	61963.345 (54964.0 - 69495.0)
Greedy-nodes-GreedyHeuristic	75512.725 (74521.0 - 77471.0)	70688.61 (68572.0 - 76529.0)	53967.21 (51195.0 - 56739.0)	51145.425 (47368.0 - 57605.0)
Steepest-edges-GreedyHeuristic	75391.6 (74541.0 - 77063.0)	70215.965 (68122.0 - 75907.0)	53529.02 (50972.0 - 57024.0)	51093.845 (46915.0 - 57412.0)
Steepest-nodes-Random	93224.675 (84578.0 - 101323.0)	88096.545 (80827.0 - 96390.0)	65743.78 (58924.0 - 77555.0)	64443.6 (55944.0 - 78053.0)
Steepest-nodes-GreedyHeuristic	75606.625 (74630.0 - 76953.0)	70758.4 (68572.0 - 76545.0)	53807.135 (51203.0 - 57026.0)	51203.305 (46990.0 - 55945.0)
Steepest-CandidateMovesEdges-Random	80856.86 (76709.0 - 85935.0)	73804.335 (69986.0 - 80091.0)	51680.02 (49141.0 - 54538.0)	48294.905 (45672.0 - 52381.0)
Steepest-PreviousDeltas-Random	79913.365 (76798.0 - 87582.0)	73141.7 (68884.0 - 78988.0)	53064.8 (49346.0 - 57601.0)	49794.255 (46581.0 - 55387.0)
Steepest-edges-Random	78307.45 (75390.0 - 83932.0)	71637.29 (68186.0 - 77703.0)	51692.4 (49183.0 - 54617.0)	48647.455 (46150.0 - 51579.0)
MSLS	75178.125 (74562.0 - 75588.0)	68299.225 (67595.0 - 68770.0)	49113.225 (48452.0 - 49647.0)	45531.725 (44622.0 - 46017.0)
ILS	73078.1 (72855.0 - 73279.0)	66442.5 (66117.0 - 66770.0)	47136.4 (46811.0 - 47604.0)	43407.25 (43207.0 - 43949.0)
LSNS_LS	73666.65 (73082.0 - 74700.0)	66879.5 (66505.0 - 67580.0)	47558.15 (46928.0 - 48154.0)	43989.0 (43314.0 - 45305.0)
LSNS_noLS	73671.35 (73153.0 - 74123.0)	67113.7 (66772.0 - 67483.0)	47623.85 (47004.0 - 48236.0)	44279.2 (43587.0 - 45644.0)
HEA_oper1_LS	73169.65 (72885.0 - 73477.0)	66357.2 (66154.0 - 66684.0)	47001.15 (46777.0 - 47269.0)	43359.35 (43181.0 - 43781.0)
HEA_oper1_noLS	75712.5 (74984.0 - 76480.0)	68862.15 (68237.0 - 69963.0)	49805.3 (49015.0 - 50670.0)	46310.8 (45587.0 - 47482.0)
HEA_oper2_LS	73585.9 (73250.0 - 74119.0)	66956.4 (66415.0 - 67311.0)	47651.2 (47084.0 - 48065.0)	43842.45 (43240.0 - 44554.0)
HEA_oper2_noLS	73730.0 (73265.0 - 74184.0)	67069.4 (66722.0 - 67694.0)	47763.4 (47349.0 - 48096.0)	44256.3 (43736.0 - 45018.0)

Table of Time

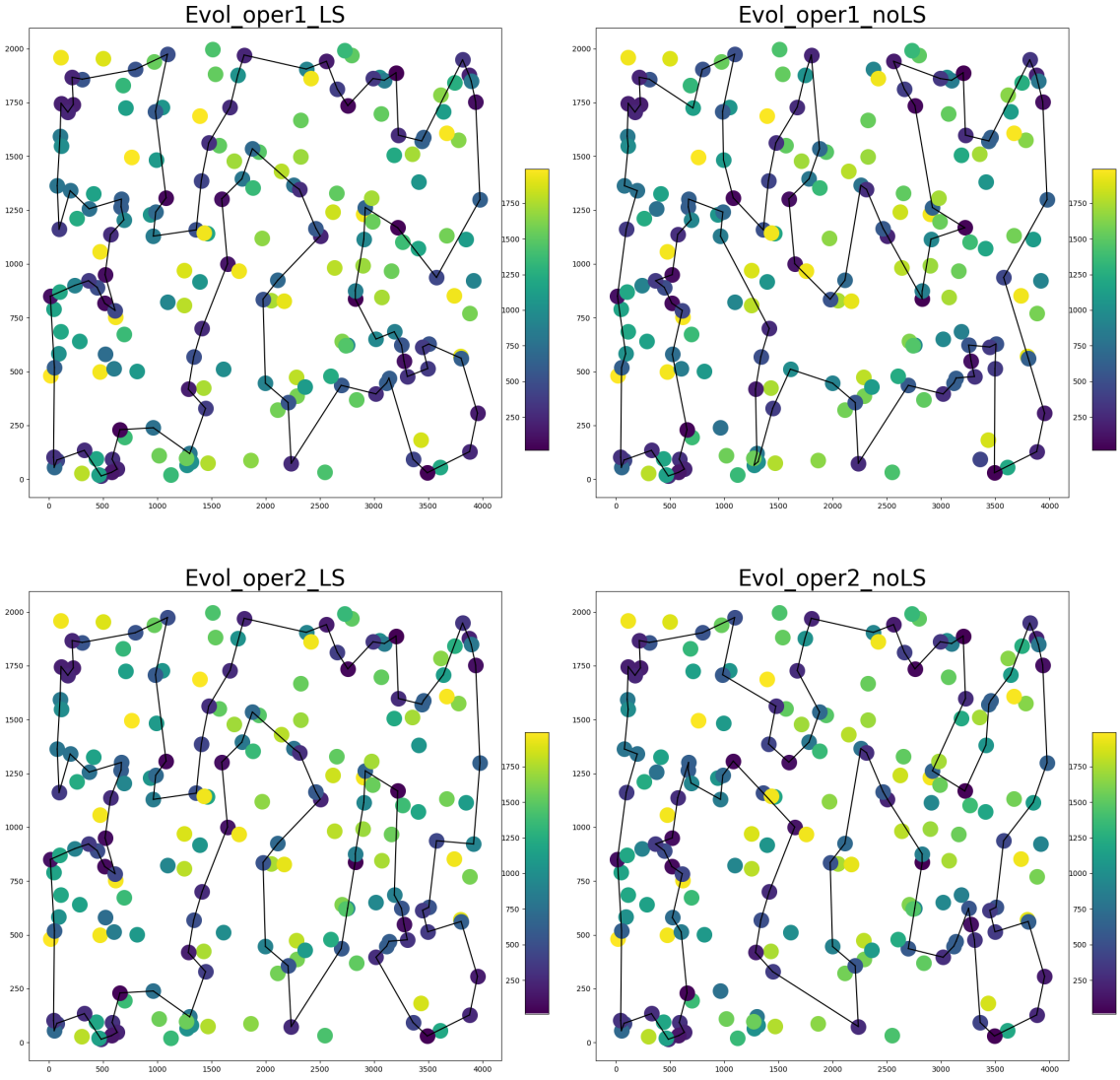
Algorithm	TSPA	TSPB	TSPC	TSPD
Greedy-edges-Random	7.682 (6.255 - 9.005)	7.989 (6.794 - 9.754)	7.739 (6.556 - 9.202)	6.699 (4.754 - 12.083)
Greedy-edges-GreedyHeuristic	0.269 (0.073 - 0.655)	0.418 (0.202 - 0.92)	0.315 (0.069 - 0.831)	0.366 (0.102 - 0.995)
Greedy-nodes-Random	4.005 (3.243 - 5.092)	4.126 (3.348 - 5.451)	3.946 (3.129 - 5.034)	4.469 (3.086 - 9.898)
Greedy-nodes-GreedyHeuristic	0.243 (0.082 - 0.802)	0.291 (0.145 - 0.581)	0.242 (0.073 - 0.663)	0.284 (0.083 - 0.765)
Steepest-edges-GreedyHeuristic	0.538 (0.073 - 1.405)	0.883 (0.337 - 1.591)	0.648 (0.065 - 1.511)	0.73 (0.133 - 2.109)
Steepest-nodes-Random	14.777 (11.815 - 18.468)	14.972 (12.168 - 18.308)	14.376 (11.243 - 18.885)	14.716 (11.481 - 19.12)
Steepest-nodes-GreedyHeuristic	0.335 (0.074 - 1.124)	0.669 (0.345 - 1.219)	0.578 (0.077 - 1.63)	0.628 (0.145 - 1.347)
Steepest-CandidateMovesEdges-Random	1.448 (1.205 - 2.451)	1.476 (1.217 - 1.761)	1.437 (1.203 - 1.788)	1.364 (1.078 - 2.015)
Steepest-PreviousDeltas-Random	6.517 (5.292 - 7.677)	6.523 (5.425 - 8.05)	6.448 (5.265 - 7.948)	6.485 (5.131 - 7.811)
Steepest-edges-Random	12.176 (10.589 - 14.762)	12.516 (11.009 - 14.349)	11.872 (10.033 - 13.814)	12.583 (10.98 - 14.19)
MSLS	1263.926 (1233.821 - 1307.715)	1310.493 (1272.85 - 1428.453)	1266.808 (1237.736 - 1355.937)	1268.967 (1237.212 - 1352.452)
ILS	1264.199 (1264.006 - 1264.605)	1310.183 (1310.003 - 1310.458)	1267.235 (1267.003 - 1267.694)	1269.149 (1269.014 - 1269.642)
LSNS_LS	1264.387 (1264.004 - 1265.019)	1310.405 (1310.04 - 1311.135)	1267.534 (1267.005 - 1268.495)	1269.503 (1269.084 - 1269.988)
LSNS_noLS	1264.393 (1264.003 - 1265.225)	1310.348 (1310.034 - 1310.92)	1267.366 (1267.01 - 1267.841)	1269.263 (1269.032 - 1269.794)
HEA_oper1_LS	1264.138 (1264.006 - 1264.420)	1310.080 (1310.002 - 1310.302)	1267.112 (1267.023 - 1267.417)	1269.086 (1269.021 - 1269.267)
HEA_oper1_noLS	1264.0 (1264.0 - 1264.0)	1310.0(1310.0 - 1310.0)	1267.0 (1267.0- 1267.0)	1269.0 (1269.0 - 1269.0)
HEA_oper2_LS	1264.091 (1264.010 - 1264.237)	1310.105(1310.0 - 1310.314)	1267.102 (1267.0 - 1267.440)	1269.137 (1269.024 - 1269.431)
HEA_oper2_noLS	1264.073 (1264.003 - 1264.203)	1310.054(1310.005- 1310.170)	1267.059 (1267.000 - 1267.161)	1269.074 (1269.0 - 1269.244)

Best Solutions Plots

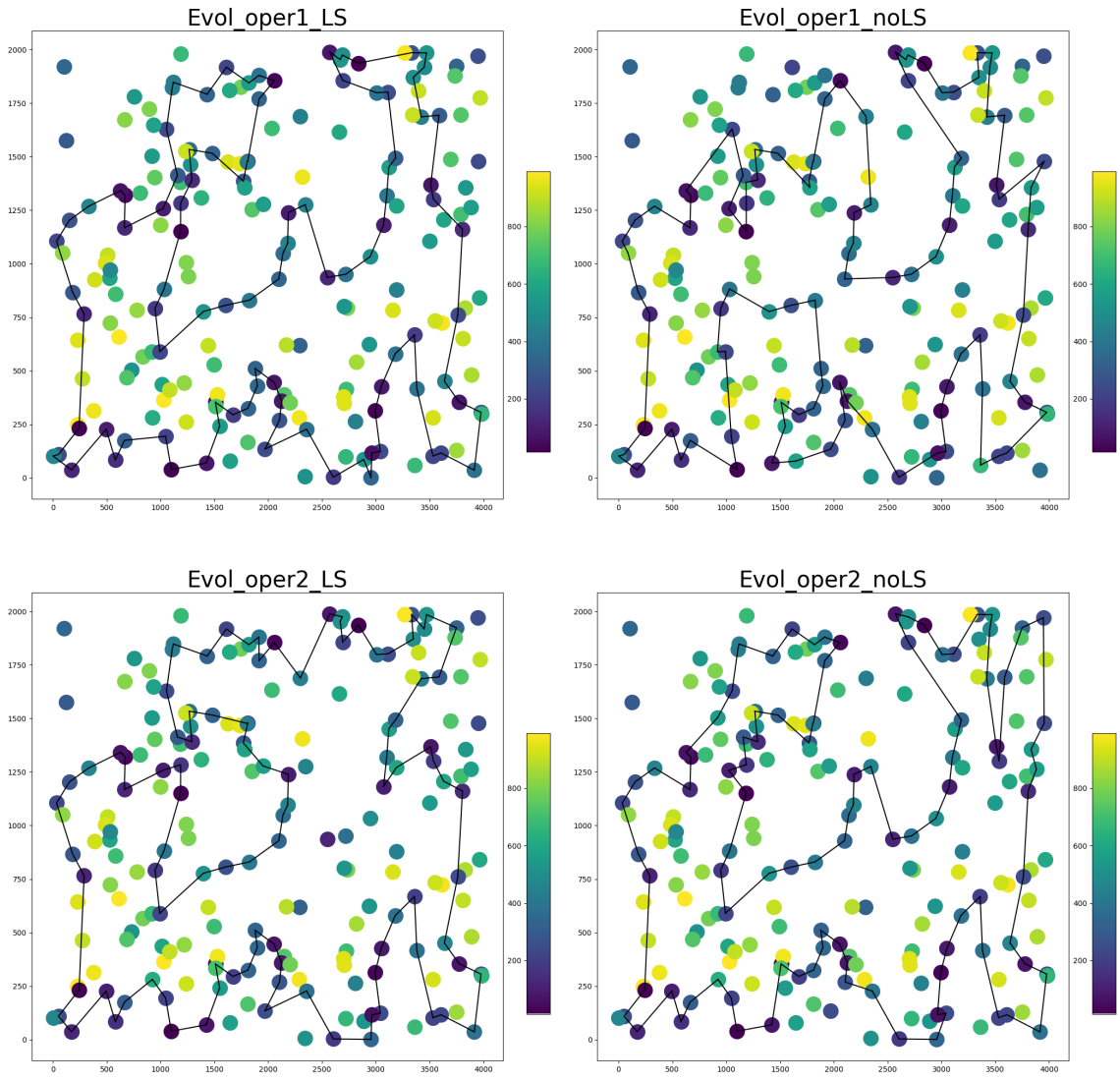
See plots: Plots



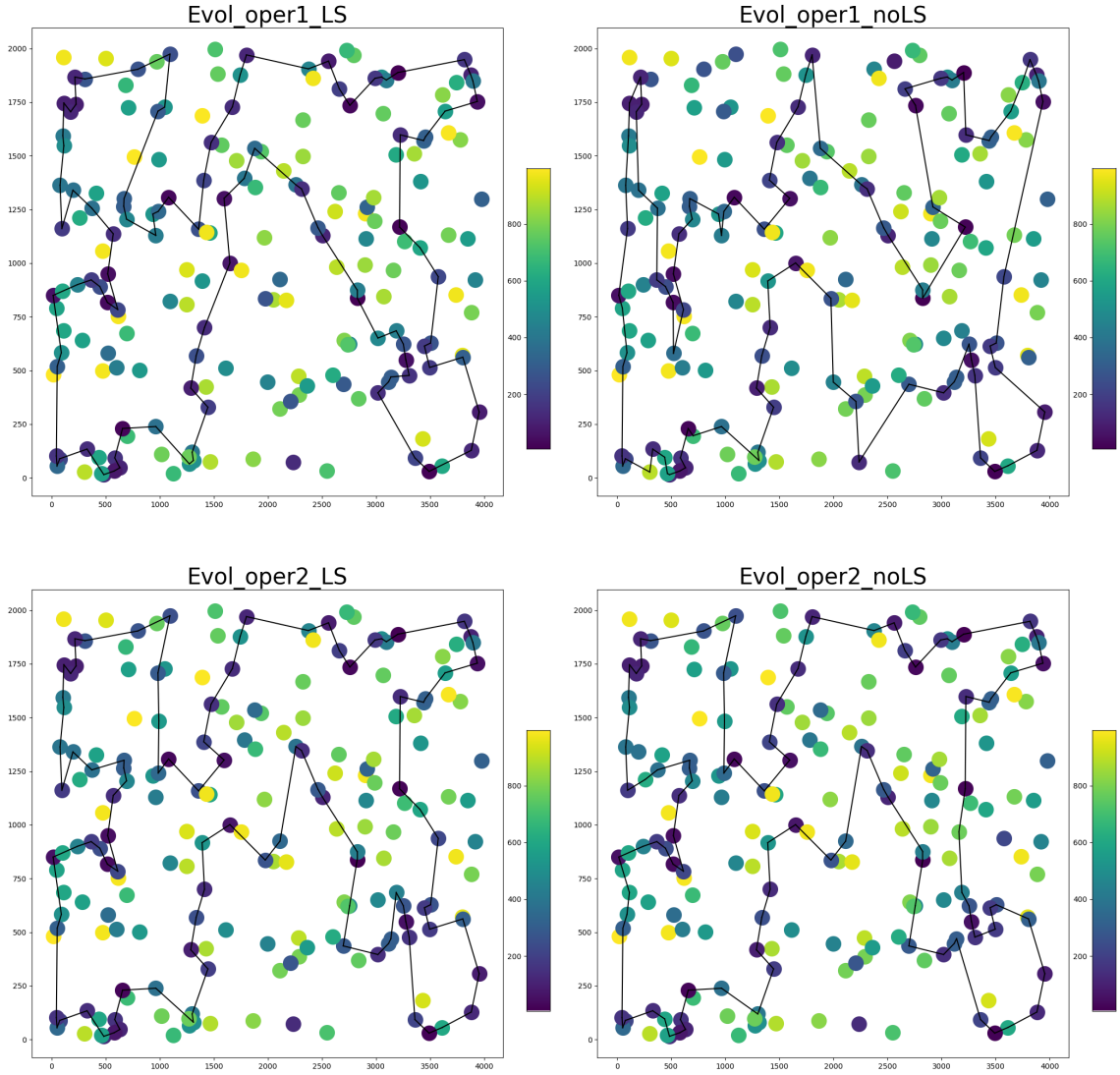
TSPB



TSPC



TSPD



Best solution among all methods so far

TSPA

[48, 106, 160, 11, 152, 130, 119, 109, 189, 75, 1, 177, 41, 137, 199, 192, 175, 114, 4, 77, 43, 121, 91, 50, 149, 0, 19, 178, 164, 159, 143, 59, 147, 116, 27, 96, 185, 64, 20, 71, 61, 163, 74, 113, 195, 53, 62, 32, 180, 81, 154, 144, 141, 87, 79, 194, 21, 171, 108, 15, 117, 22, 55, 36, 132, 128, 145, 76, 161, 153, 88, 127, 186, 45, 167, 101, 99, 135, 51, 112, 66, 6, 172, 156, 98, 190, 72, 12, 94, 89, 73, 31, 111, 14, 80, 95, 169, 8, 26, 92]

Cost: 72855.0

TSPB

[166, 59, 119, 193, 71, 44, 196, 117, 150, 162, 158, 67, 156, 91, 70, 51, 174, 140, 148, 141, 130, 142, 53, 69, 115, 82, 63, 8, 16, 18, 29, 33, 19, 190, 198, 135, 95, 172, 163, 182, 2, 5, 34, 183, 197, 31, 101, 38, 103, 131, 24, 127, 121, 179, 143, 122, 92, 26, 66, 169, 0, 57, 99, 50, 112, 154, 134, 25, 36, 165, 37, 137, 88, 55, 153, 80, 157, 145, 79, 136, 73, 185, 132, 52, 139, 107, 12, 189, 170, 181, 147, 159, 64, 129, 89, 58, 171, 72, 114, 85]

Cost: 66117.0

TSPC

[20, 71, 61, 163, 74, 113, 195, 53, 62, 32, 180, 81, 154, 102, 144, 141, 87, 79, 194, 21, 171, 108, 15, 117, 22, 55, 36, 132, 128, 145, 76, 161, 153, 88, 127, 186, 45, 167, 101, 99, 135, 51, 5, 112, 72, 190, 66, 6, 172, 156, 98, 94, 42, 89, 12, 73, 31, 95, 169, 8, 26, 92, 48, 106, 160, 11, 152, 130, 119, 109, 189, 75, 1, 177, 41, 137, 199, 192, 43, 77, 4, 114, 91, 121, 50, 149, 0, 69, 19, 178, 164, 159, 143, 59, 147, 116, 27, 96, 185, 64]

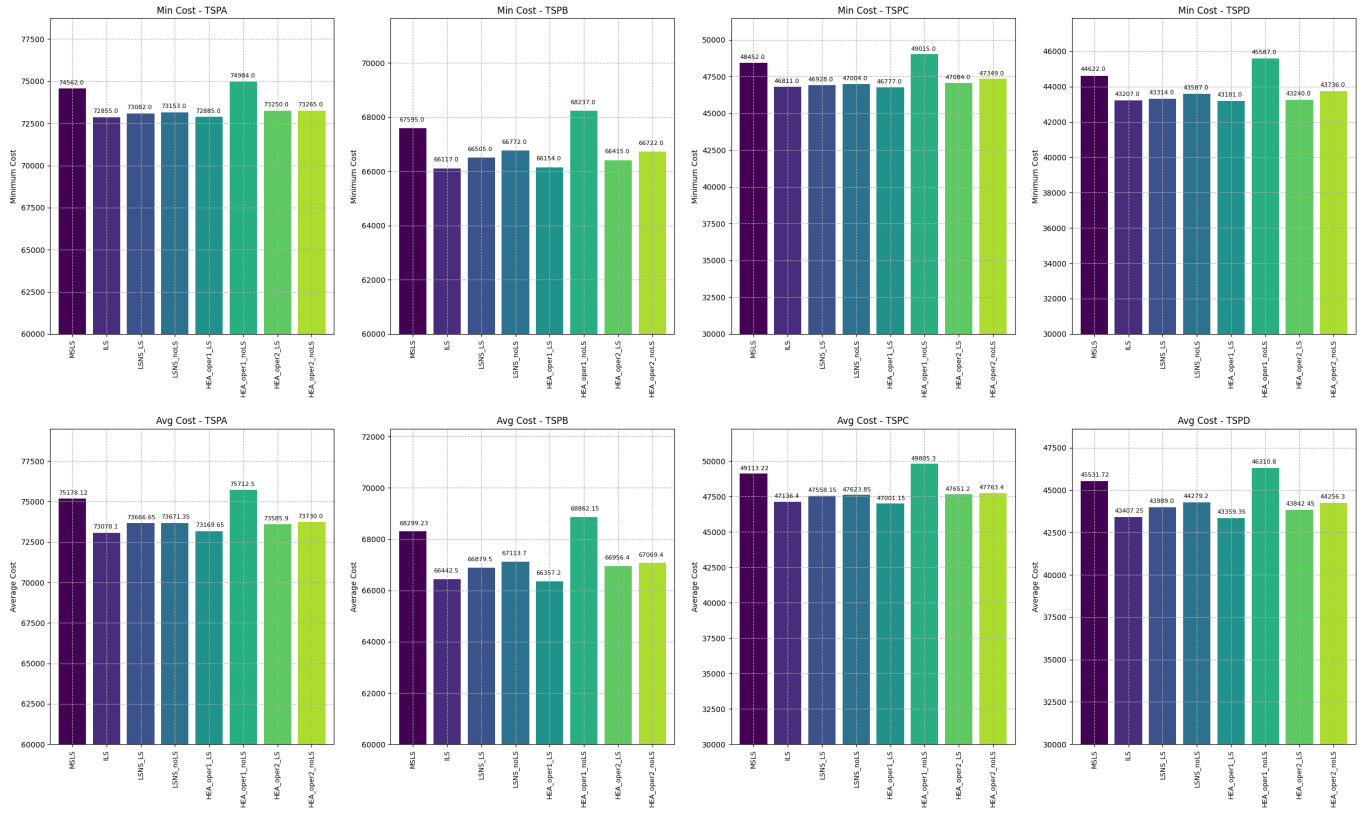
Cost: 46777.0

TSPD

[47, 170, 181, 147, 159, 64, 129, 89, 58, 171, 72, 114, 85, 166, 71, 44, 196, 117, 150, 162, 158, 67, 3, 156, 91, 70, 51, 174, 140, 148, 141, 130, 142, 53, 32, 113, 69, 115, 82, 63, 8, 16, 18, 29, 33, 19, 190, 198, 135, 169, 66, 26, 92, 122, 143, 179, 197, 183, 34, 31, 101, 38, 103, 131, 121, 127, 24, 50, 43, 99, 137, 37, 165, 123, 154, 134, 25, 36, 88, 55, 4, 153, 80, 157, 145, 79, 136, 61, 73, 185, 132, 52, 12, 107, 97, 139, 193, 119, 59, 189]

Cost: 43181.0

Conclusions



When it comes to minimum cost, Hybrid Evolutionary Algorithm with Operator 1 and Local Search applied has shown best performance among all of the rest variants of HEA examined.

Minimum Cost

- In the case of TSPA, both ILS and HEA_oper1_LS reached almost identical minimum cost, with ILS scoring better only by 30 units. Meanwhile, HEA_oper2, whether paired with Local Search (LS) or not, also demonstrated relatively good performance, slightly worse than LSNS.
- For TSPB, ILS narrowly outperformed the other methods. For this instance, the variations of HEA_oper2 surpassed LSNS in terms of effectiveness.
- With TSPC and TSPD, HEA_oper_LS set new best solution among all other methods tested so far. Notably, its version without Local Search provided the poorest result historically.

Overall, the influence of incorporating Local Search is markedly evident in the results, particularly for the TSPC and TSPD instances.

Number of iterations

Algorithm	TSPA	TSPB	TSPC	TSPD
ILS	4197.75 (3991 - 4326)	4160.4 (4054 - 4370)	4206.3(3835 - 4365)	4184.95(4009 - 4349)
LSNS_LS	1827.85 (1328.0 - 2325.0)	1942.75 (1353.0 - 2558.0)	1674.45 (1123.0 - 2196.0)	1823.9 (1249.0 - 2392.0)
LSNS_noLS	2268.0 (1633.0 - 2723.0)	2400.75 (1724.0 - 2931.0)	2326.15 (1654.0 - 2848.0)	2313.25 (1636.0 - 2854.0)
HEA_oper1_LS	5346.1 (3412 - 7191)	7223.35 (4615 - 8928)	3857.9 (3077 - 5475)	4626.95 (3330 - 5847)
HEA_oper1_noLS	3624639.45 (3595274 - 3663433)	3790013.1 (3763971 - 3817952)	3603259.1 (3532825 - 3634250)	3653376.0 (3606872 - 3698766)
HEA_oper2_LS	7670.55 (5536 - 13233)	9007.85 (5469 - 16230)	6512.85 (3794 - 10266)	6944.95 (4534 - 11800)
HEA_oper2_noLS	14150.3 (7998 - 31735)	15206.4 (8896 - 33472)	13260.35 (7623 - 26872)	12414.55 (6152 - 34599)

Regardless of the operator chosen, Hybrid Evolutionary Algorithms consistently perform a significantly higher number of iterations.

The sole exception is HEA_ope1_LS in the TSPC instance, where the iteration count falls below that of ILS.