Asset Management GRA (Pierre Clauss): R Markdown project

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1. Global Minimum Volatility portfolio (GMV) on an Equity Portfolio

1.1 Packages

We import the necessary libraries to perform the first analysis.

1. **library(tidyverse):** The tidyverse package constitutes a collection of R packages meticulously designed to work seamlessly in unison for data manipulation, visualization, and analysis. It encompasses packages like ggplot2 for data visualization, dplyr for data manipulation, tidyr for data tidying, and more. Loading tidyverse avails all these packages for utilization during your R session, streamlining your data analysis workflow.
2. **library(quantmod):** The quantmod package primarily caters to quantitative financial modeling and analysis. It furnishes an array of functions and tools for handling financial data, such as downloading and managing stock price data, conducting technical analysis, and modeling financial time series.
3. **library(DataExplorer):** The DataExplorer package serves the purpose of preliminary data exploration and analysis. It offers functions to generate summary statistics, visualize missing data, plot variable distributions, and more. It can aid in obtaining a quick overview of your data before delving into more extensive analyses.
4. **library(corrplot):** The corrplot package specializes in visualizing correlation matrices. It provides functions for generating visually appealing and informative correlation plots, which prove valuable for comprehending relationships between variables within your data.
5. **library(scales):** The scales package provides an assortment of scales and formatting functions tailored for R graphics. It is often employed in conjunction with other plotting packages, such as ggplot2, to customize the appearance of plots, including axes, labels, and color scales.

1.2 Investment universe

1.2.1 Portfolio selection

In our portfolio management project, we have carefully chosen a set of ten stocks from a larger universe of investments. This selection represents the most significant companies in the US equity market, primarily based on their market capitalization. By analyzing these stocks, our objective is to gain valuable insights into the strategies that can be employed for managing investments in the US equity market.

Our investment universe comprises a selection of ten high-performing and influential companies, primarily from the technology sector. These companies are recognized for their robust market capitalization, innovative business models, and significant impact on global markets.

Ticker	Company Description
AAPL	A leader in consumer electronics, software, and services.
MSFT	A global leader in software, services, devices, and solutions.
GOOGL	The parent company of Google, known for its dominant search engine and a wide array of internet services and products.
AMZN	A powerhouse in e-commerce, cloud computing, and artificial intelligence.
TSLA	An innovative company in electric vehicles, energy storage, and solar products.
NVDA	A pioneer in graphics processing units (GPUs) for gaming and professional markets.
PYPL	A leading technology platform and digital payments company.
CMCSA	A global media and technology company with two primary businesses, Comcast Cable and NBCUniversal.
ADBE	A leader in multimedia and creativity software products.
ASML	A leading supplier to the semiconductor industry.

1.2.2 Market outlook and link to portfolio allocation

According to a recent survey of Wall Street analysts commented on Bloomberg, the median forecast for the S&P 500 index in 2023 is 4,300. Analysts are generally bullish on the US equity market in 2023, citing factors such as continued economic growth, strong corporate earnings, and low interest rates. However, they also acknowledge that there are some risks to the outlook, such as the potential for a recession and higher inflation. Here a market outlook for each stock selected in our investment universe.

Ticker	Company Description
AAPL	The company has a strong track record of innovation. Expected to benefit from the continued growth of the smartphone market and the launch of new products and services.
MSFT	Has a diversified business model. Expected to benefit from the growth of cloud computing and other enterprise software markets.
GOOGL	A leader in online advertising and cloud computing. Expected to benefit from the continued growth of the digital economy.
AMZN	A leader in cloud computing and e-commerce. Expected to benefit from the continued growth of online shopping and cloud computing.
TSLA	Expected to benefit from the growing demand for electric vehicles and the transition to clean energy.
NVDA	GPUs are used in various applications like gaming, AI, and data center computing. Expected to benefit from the growth of these markets.
PYPL	Expected to benefit from the growth of e-commerce and digital payments.
CMCSA	Expected to benefit from the continued growth of broadband internet and the rollout of new streaming services.
ADBE	Products are used by a wide range of customers. Expected to benefit from the growth of the digital economy and demand for creative software.
ASML	Expected to grow, driven by the semiconductor industry's growth. The only company to manufacture EUV lithography systems, well-positioned to benefit from the industry growth.

1.2.3 Rationale for this investment universe selection

The rationale behind this selection lies in diversification across various dimensions, including market capitalization, geographical presence, and revenue streams, while maintaining a focus on growth and innovation. The intent is to capture both the stability of established companies and the growth potential of market leaders and innovators. In this sense, the choice of stocks is guided by several key considerations:

- **Investment universe:** Our selection is drawn from a universe of stocks that encompasses ten well-established companies within the US equity market. These companies are known for their substantial market presence and are recognized leaders in their respective industries.
- **Market capitalization focus:** We have chosen stocks with an emphasis on market capitalization. Larger market capitalization often indicates stability and lower volatility, making these stocks appealing for various investment strategies.
- **US equity market perspective:** Our project primarily targets insights related to the US equity market. The chosen stocks provide a representative snapshot of the investment landscape in one of the world's largest and most influential equity markets.
- **Diversification:** To create a well-rounded portfolio, we have ensured diversification across different sectors and industries. Diversification is essential in portfolio management as it spreads risk and mitigates overexposure to a single sector.
- **Market visibility:** The selected companies are not only well-regarded in the financial sector but are also widely recognized in popular culture. Their performance is closely monitored, and they often hold significant sway over market indices.
- **Historical data and research:** These stocks have a rich history of performance data and have been the subject of extensive research. Access to their historical data is abundant, making them suitable for rigorous portfolio management analysis and research.

The provided R code defines the stock tickers for these companies, allowing you to access financial data. This data can then be employed to conduct a range of analyses, such as return calculations, risk assessments, and portfolio optimization, to test and implement various investment strategies on the US equity market.

We source our data from Yahoo Finance, a highly reputable platform renowned for its financial data. Our analysis covers a comprehensive three-and-a-half-year period, allowing us to evaluate the performance of our selected companies both before and after the onset of the COVID-19 pandemic.The start date is from 01/01/2019 to 31/08/2023.

1.3 Data cleaning and analysis

1.3.1 Data cleaning

We perform some formatting to clean the data, an important step in performing data analysis:

- **Extracting adjusted closing prices** (lapply(stock_tickers, function(ticker) { Ad(get(ticker)) })): This code uses the lapply function to retrieve the adjusted closing prices (often used for return calculations) for a list of stock tickers. It iterates through each ticker in the stock_tickers vector and fetches the adjusted closing prices using the Ad() function from the quantmod package.
- **Combining stock returns** (do.call(cbind, stock_returns)): The code combines the extracted adjusted closing prices into a single data frame by calling cbind (column-bind) on the list of stock returns obtained in the previous step. This creates a data frame where each column represents the adjusted closing prices of a specific stock.
- **Renaming columns** (colnames(fin_prices) <- stock_tickers): The code assigns the stock tickers as column names to the combined data frame. This step ensures that each column is labeled with the corresponding stock ticker, making it easier to identify each stock's data.
- **Handling missing data** (na.omit(fin_prices)): This line of code removes rows with missing values (usually represented as NA) from the combined data frame, fin_prices. Removing missing data is a common processing step to ensure the quality and consistency of the dataset for further analysis.

We perform the arithmetic return method to compute returns for this data set. It is simple, intuitive and widely accessible. Arithmetic returns are particularly useful for performance measurement, providing a clear view of actual gains and losses. It is well-suited for short-term analysis and offer transparency.

In order to ensure that the data is properly imported, with non missing values and proper data cleaning procedure, we can implement this line of code in order to learn more about data.

1.3.2 Data analysis

The US stock market has performed strongly from 2019 to 2023, despite some headwinds. The S&P 500 index has returned double digit returns during this period, driven by a number of factors, including:

- **Strong economic growth:** The US economy grew at an average rate of 2.3% from 2019 to 2022, which supported corporate earnings growth.
- **Low interest rates:** The Federal Reserve kept interest rates at record lows during this period, which boosted corporate profits and made stocks more attractive to investors.
- **Government stimulus:** The US government injected a number of stimulus measures in response to the COVID-19 pandemic, which boosted consumer spending and supported the economy.

The main market movers during this period have been technology stocks, which have outperformed other sectors by a wide margin. This is due to the fact that technology companies have benefited from the shift to online shopping and working during the pandemic.

Other sectors that have performed well during this period include healthcare, consumer staples, and financials. These sectors are generally seen as more defensive, and they have tended to outperform in times of market volatility (Fabozzi, 2014).

Some of the top performing US stocks from 2019 to 2023 include NVIDIA (TICKER: NVDA), Tesla (TICKER: TSLA), Apple (TICKER: AAPL), Microsoft (TICKER: MSFT), Alphabet (TICKER: GOOGL) and Amazon (TICKER: AMZN).

These companies have all benefited from the trends mentioned above, and they have seen their share prices rise significantly during this period. Overall, the US stock market has performed strongly from 2019 to 2023, despite some headwinds. The main market movers have been technology stocks, which have outperformed other sectors by a wide margin.

From the figure below, Nvidia (TICKER: NVDA) and Tesla (TICKER: TSLA) stock shares witnessed an unprecedented surge in attraction, making it a notable standout within our investment universe. Tesla was among the best performing stocks in the broader investment universe selected. The surge in retail trading interest for the electric car company can explain to some extent this massive performance, especially in the wake of the post-COVID market, where intensive monetary policies prompted significant investments from both corporate and retail investors seeking higher yields.

Regarding Nvidia's stock price, it has surged in recent years as investors have become increasingly bullish on the company's prospects. There are a number of reasons for this, including:

- **The growth of AI:** AI is one of the most important technological trends of our time, and Nvidia's GPUs are essential for many AI applications. As AI continues to grow, Nvidia is well-positioned to benefit.
- **The rise of the metaverse:** The metaverse is a virtual world that is still in its early stages of development, but it has the potential to be a massive market. Nvidia's GPUs are used to power many of the technologies that will be used in the metaverse, such as virtual reality and augmented reality.
- **Nvidia's strong financial performance:** Nvidia has been delivering strong financial performance for many years. In the most recent quarter, the company reported revenue of $8.29 billion and earnings per share of $1.32, both of which beat analyst expectations.

Given Nvidia's strong position in the AI and metaverse markets, it is not surprising that investors are paying attention to the stock. The company is also well-managed and has a strong track record of innovation.

Here are some specific reasons why investors are interested in NVDA stock amid a rising war in AI and LLM:

- **Nvidia is a leader in the development of AI hardware and software.** The company's GPUs are used by many of the world's leading AI researchers and companies.
- **Nvidia is well-positioned to benefit from the growth of LLM.** LLMs are a new type of AI that can generate text, translate languages, write different kinds of creative content, and answer your questions in an informative way. Nvidia's GPUs are essential for training and running LLMs.
- **Nvidia is investing heavily in the research and development of new AI technologies.** The company is committed to maintaining its leadership position in the AI market.

Overall, Nvidia is a well-established company with a strong track record of innovation. The company is well-positioned to benefit from the growth of AI and LLM, which is why investors are paying attention to the stock.

The code allowed us to plot returns into a time series, providing a clear visualization of stock performance during the specified period.

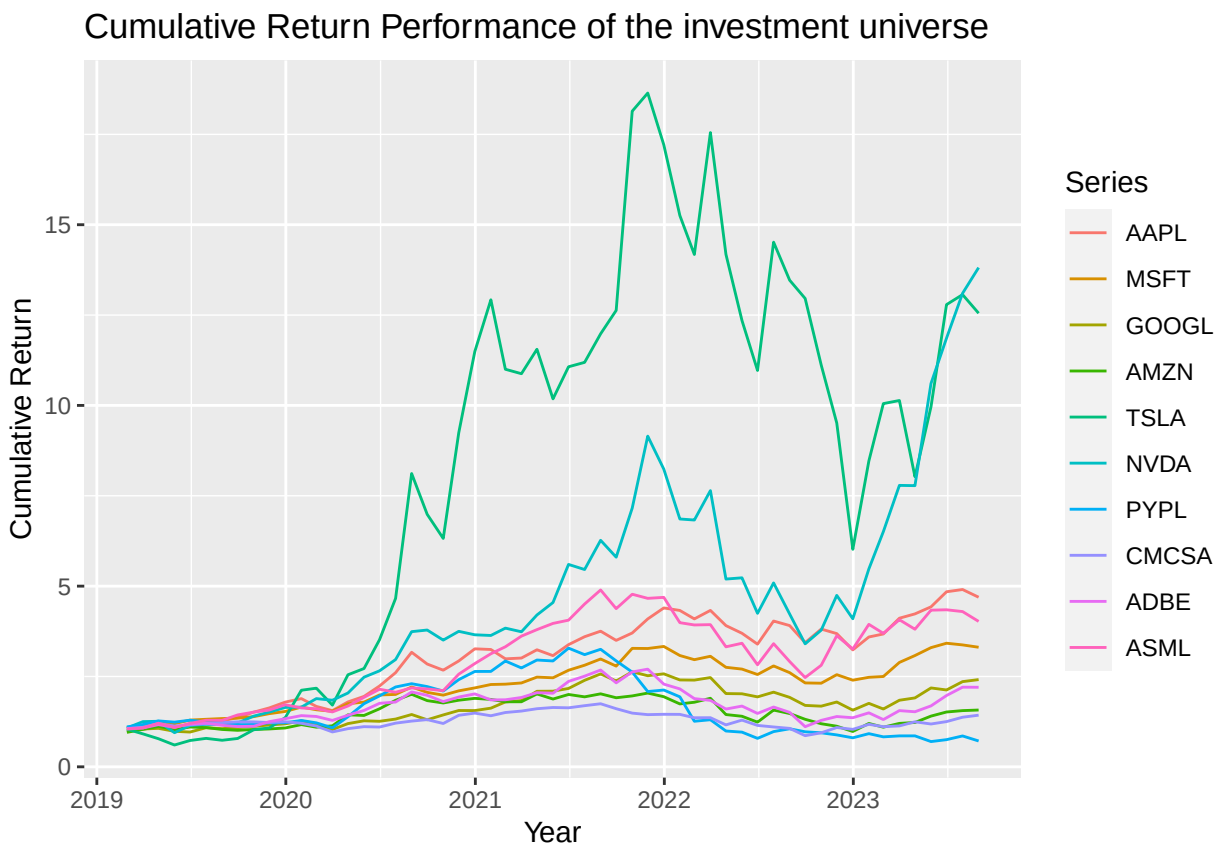

Figure 1: Cumulative return of stocks across the analysed period.

We can define the Minimum Volatility Portfolio (MVP) as a portfolio of assets with the lowest possible risk for an investor and is located on the far-left side of the efficient frontier. Note that the minimum volatility portfolio is also called the minimum variance portfolio or more precisely the global minimum volatility portfolio (to distinguish it from other optimal portfolios obtained for higher risk levels).

To implement to global minimum variance portfolio, we need to compute the covariance matrix as an input parameter for the Markowitz optimization model.

Interpreting the Correlation Matrix

We can highlight the following observations on the correlation matrix (Fabozzi, 2014):

- **Predominant positive correlations:** The majority of the correlations in the matrix are positive, indicating that most of the stocks tend to move in the same direction. This is a common characteristic of stocks within the same market, as they are often influenced by similar economic factors and market sentiment. The plot shows that there is a significant amount of correlation between many of the stocks in the portfolio. This is not surprising, as many stocks in the same sector or industry tend to move in the same direction. For example, the correlation between the tech stocks in the portfolio (AAPL, MSFT, GOOGL, TSLA, NVDA, and AMZN) is very high. The high correlation between the stocks in the portfolio means that the portfolio is not as diversified as it could be. A more diversified portfolio would have a lower overall correlation, as the stocks would be more likely to move in different directions.
- **Varying correlation strengths:** The shades of the correlation coefficients vary, indicating that the strength of the correlations between stocks differs. Some stocks exhibit strong positive correlations, while others have weaker correlations. This suggests that the portfolio has a mix of stocks with varying degrees of co-movement.

Linking Findings with Markowitz Portfolio Theory

Markowitz portfolio theory, also known as Modern Portfolio Theory (MPT), emphasizes the importance of diversification in reducing portfolio risk. The theory suggests that by combining assets with varying correlations, investors can achieve a portfolio with a lower overall risk for a given level of expected return (Markowitz, 1952).

The presence of both positive and negative correlations among the stocks suggests that there is potential for diversification within the portfolio. By carefully selecting stocks with different correlation patterns, investors can construct a portfolio that balances risk and return.

Implications for Portfolio Construction

The correlation matrix provides valuable insights for portfolio construction. By understanding the correlations between stocks, investors can make informed decisions about asset allocation and diversification strategies.

- **Diversification:** The concept of diversification seeks to enhance returns while minimizing risk by investing in a variety of assets that will react differently to the same event (s). For example, whenever there is unfavorable news about a certain event, i.e. 2008 sub prime mortgage crisis, the stock market typically declines dramatically. Simultaneously, the same news has generally benefited the price of specific assets, such as gold. As a result, portfolio diversification should include not just diverse stocks inside and outside of the same industry, but also diverse asset classes, such as bonds and commodities. The diversification effect is a term that relates to the link between portfolio correlations and diversification. When there is an imperfect correlation between assets (positive or negative), the diversification effect occurs. It is a critical and successful risk mitigation method since risk mitigation may be accomplished without sacrificing profits. As a result, any prudent investor who is 'risk cautious' will diversify to a certain extent.
- **Risk Management:** The correlation matrix can be used to assess the overall risk profile of the portfolio. A portfolio with predominantly strong positive correlations may be more susceptible to market downturns, while a portfolio with a mix of positive, negative, and weak correlations may exhibit more stability.
- **Rebalancing:** The correlation matrix can guide rebalancing decisions. If certain stocks or sectors become overly correlated, investors may consider rebalancing their portfolio to maintain an appropriate level of diversification.

In conclusion, the correlation matrix is a valuable tool for understanding the relationships between assets in a portfolio. By analyzing the correlation patterns, investors can make informed decisions about diversification, risk management, and rebalancing strategies, aligning with the principles of Markowitz portfolio theory. We observe opportunities for diversification within our investment universe. It's worth noting that within our investment universe, we do not find negative correlations, as all the assets exhibit some degree of correlation to one another.

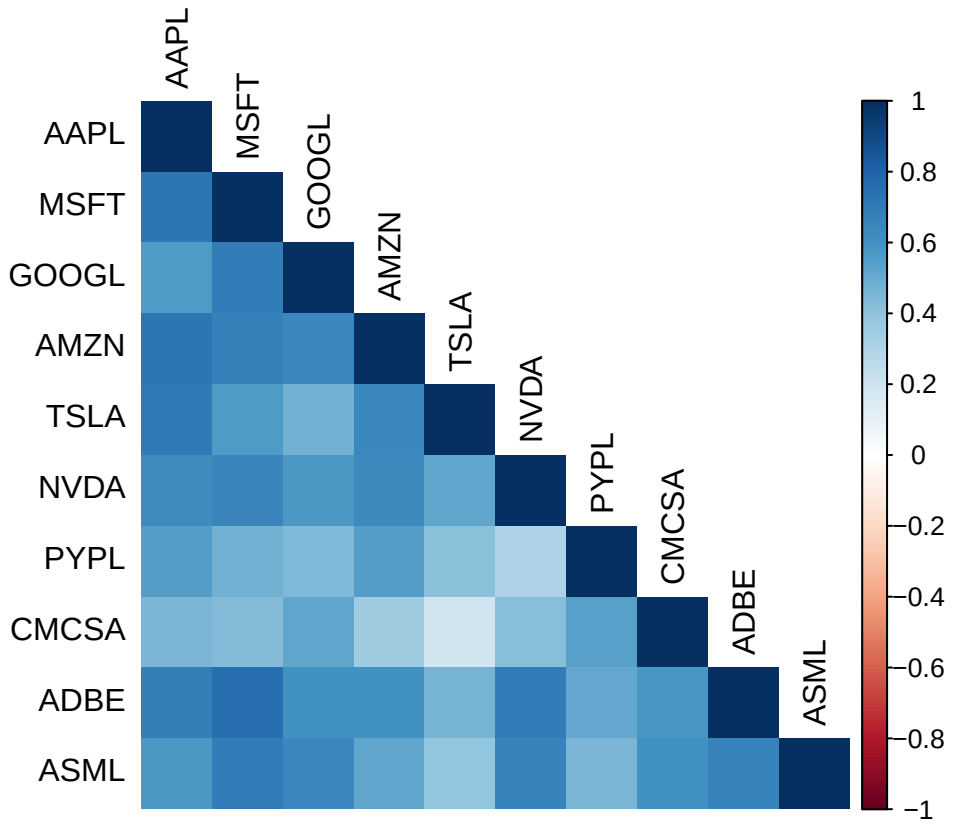

Figure 2: Correlation return of stocks across the analysed period.

To further complement the analysis of the data gathered, we can run a statistical analysis of the first moments of the distribution to assess the performance of each stock and understand their behavior during the time frame observed. NB: Figures obtained from this statistical analysis are given on monthly returns.

Using monthly data to smooth the data and transform it to a close approximation to the Gaussian distribution has several advantages:

- **Reduced noise:** Monthly data is less noisy than daily or weekly data. This is because it averages out the short-term fluctuations in the data.
- **Improved normality:** Monthly data is more likely to be normally distributed than daily or weekly data. This is because the Central Limit Theorem states that the distribution of the sum of a large number of independent and identically distributed random variables will be approximately normal, even if the original distribution is not normal.
- **Easier to model:** Gaussian distributions are easier to model than other types of distributions. This is because there are a number of well-established statistical methods for modeling Gaussian distributions.

There are also some disadvantages to using monthly data to smooth the data and transform it to a close approximation to the Gaussian distribution:

- **Loss of information:** Monthly data is less informative than daily or weekly data. This is because it averages out some of the detail in the data.
- **Time lag:** Monthly data has a time lag. This means that it does not reflect the latest changes in the underlying data.

Overall, the utility of using monthly data to smooth the data and transform it to a close approximation to the Gaussian distribution depends on the specific application. In order to reduce noise and improve normality, then using monthly data is a good option. However, if the goal is to capture the latest changes in the data or to preserve all of the information in the data, then using more frequent data is a better option.

The data taken in this analysis spans from early January 2019 to August 2023. This is a relatively long period of time, so using monthly data should be sufficient to reduce noise and improve normality. However, the data may be subject to have a time lag.

For portfolio optimization under a Markowitz framework, using monthly data is a good choice to deal with the normality assumption of the Markowitz model. In addition to that, supplementary statistical analysis can help understand financial patterns in data:

- First, density plots, also known as probability density function plots, serve the purpose of visualizing data distributions. They offer insights into the shape, spread, and central tendencies of data, making them crucial for assessing the nature of financial asset returns.
- Second, QQ plots, or quantile-quantile plots, are utilized to evaluate whether a dataset adheres to a specific theoretical distribution, such as the normal distribution. These plots help portfolio managers identify departures from expected distributions, facilitating informed investment decisions. Together, these techniques aid in the exploration and assessment of financial data, enabling portfolio managers to make more informed and data-driven choices in their investment strategies.

We can run some more advanced statistical analysis to understand the asset return behavior of the equities selected in this analysis. Overall, we can see from both the density plots and the QQ plots that the monthly show some normality properties that can be beneficial for our portfolio construction part of the project.

1.4 Modelling part

Modern Portfolio Theory (MPT) is founded on several market and investor assumptions. Several of these assumptions are stated explicitly, while others are implied. Markowitz's contributions to (MPT) in portfolio selection are based on the following basic assumptions:

- Investors are rational (they seek to maximize returns while minimizing risk, or minimize risk while maximize return).
- Investors will accept increased risk only if compensated with higher expected returns.
- Investors receive all relevant information regarding their investment decision.
- Investors can borrow or lend an unlimited amount of capital at a risk-free rate of interest.

1.4.1 Unbiased Global Minimum Variance (GMV) portfolio

The GMV portfolio is only based on the covariance matrix estimation. This matrix estimator is inversed in the formula to determine the weights of the portfolio. The unbiased estimator of the inverse covariance matrix can be approached as :

$$\hat{\Sigma}\text{unbiased} = \frac{1}{T-n-2} \sum t = 1^T (r_t - \hat{\mu})(r_t - \hat{\mu})'$$

where $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} r_t$ is an estimator for the mean returns, T the number of observations, n the number of assets and rt the vector of financial returns for the n assets observed at time t.

In the context of time series data analysis, one of the crucial applications is the division of the data set into a training set and a testing set based on temporal order. This approach is particularly important when dealing with data that evolves over time, such as stock prices in this instance.

The training set typically comprises historical data, while the testing set contains more recent observations. This temporal separation allows for the evaluation of predictive models and forecasting techniques. By using historical data to train the model and then assessing its performance on more recent data, analysts can gauge the model's ability to make accurate predictions and anticipate future trends. Time series data applications are essential in fields like finance, where understanding and forecasting trends over time is important.

For the purpose of modelling, we will shrink the data set into two different sub-samples. We can define the following parameters:

1. The first sub-sample will cover the first 20 trading month covered in the data set.
2. The second sub-sample will cover the rest of the data set, covering the equivalent of 36 trading month.

We implement the parameters of the model in order to compute the GMV.

After computing the parameters required to implement the unbiased GMV portfolio, we can compute omega, representing the weightings of the portfolio as follows.

The GMV portfolio represents an application of Markowitz's portfolio theory, which focuses on the trade-off between risk and return. However, the GMV specifically concentrates on risk minimization without directly considering expected returns. It is a purely risk-focused approach, optimal in a mean-variance sense when returns are unpredictable or when a risk-averse investor seeks the lowest possible portfolio variance.

The use of unbiased estimators is crucial for the GMV construction. This approach assumes that historical returns and covariances are reliable predictors of future risks. However, this assumption can be problematic, as historical measures may not accurately represent future market conditions. Therefore, the GMV portfolio constructed with historical data may not always achieve the minimum variance out-of-sample. It's also worth noting that the negative weights in a GMV portfolio imply short positions, which may not be feasible or desirable for all investors due to their additional risks and complexities.

In conclusion, the GMV portfolio is a key concept in the optimization of portfolios under Markowitz's modern portfolio theory. While it offers a foundation for risk minimization, its real-world application requires careful consideration of estimation risk, the possibility of model misspecification, and the investor's ability and willingness to engage in short selling. The findings from the GMV optimization provide valuable insights but should be viewed within the broader context of comprehensive investment strategy and risk management.

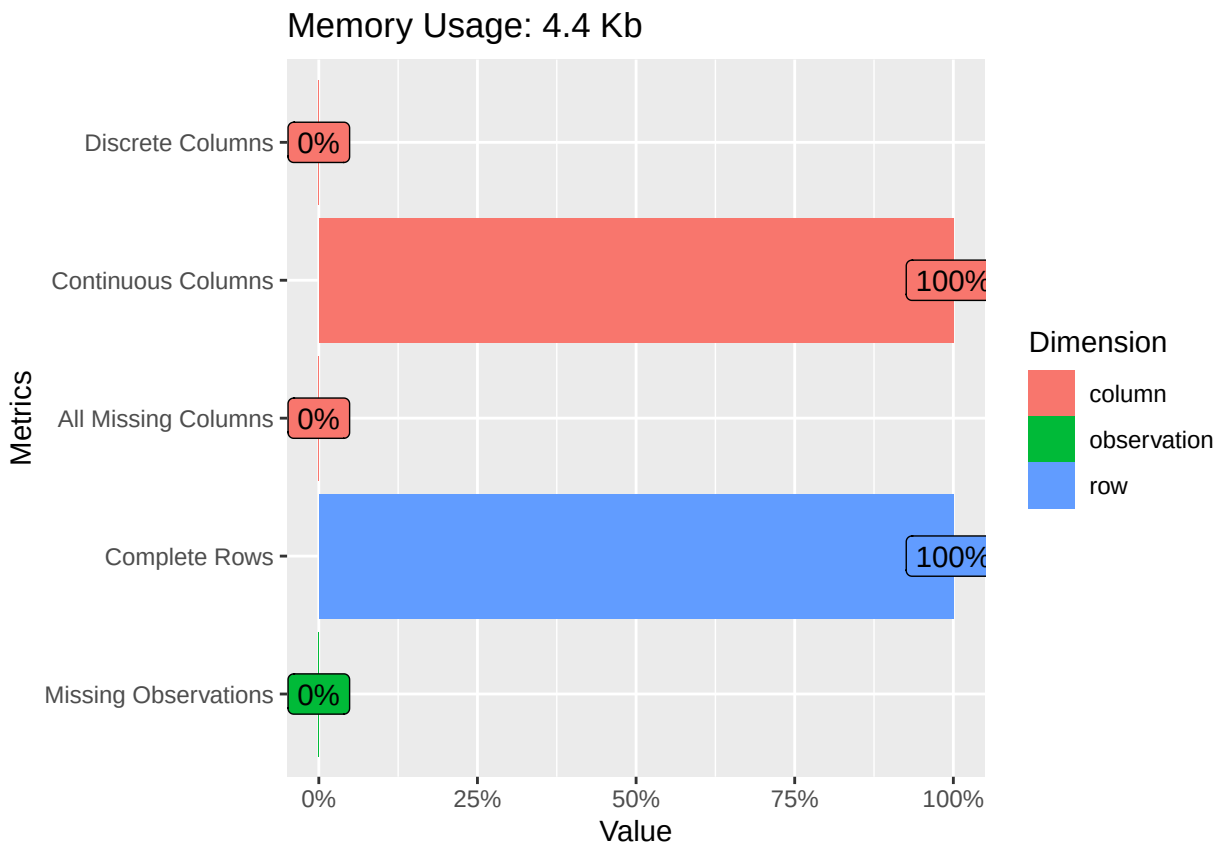

Figure 3: Structure of the first subsample.

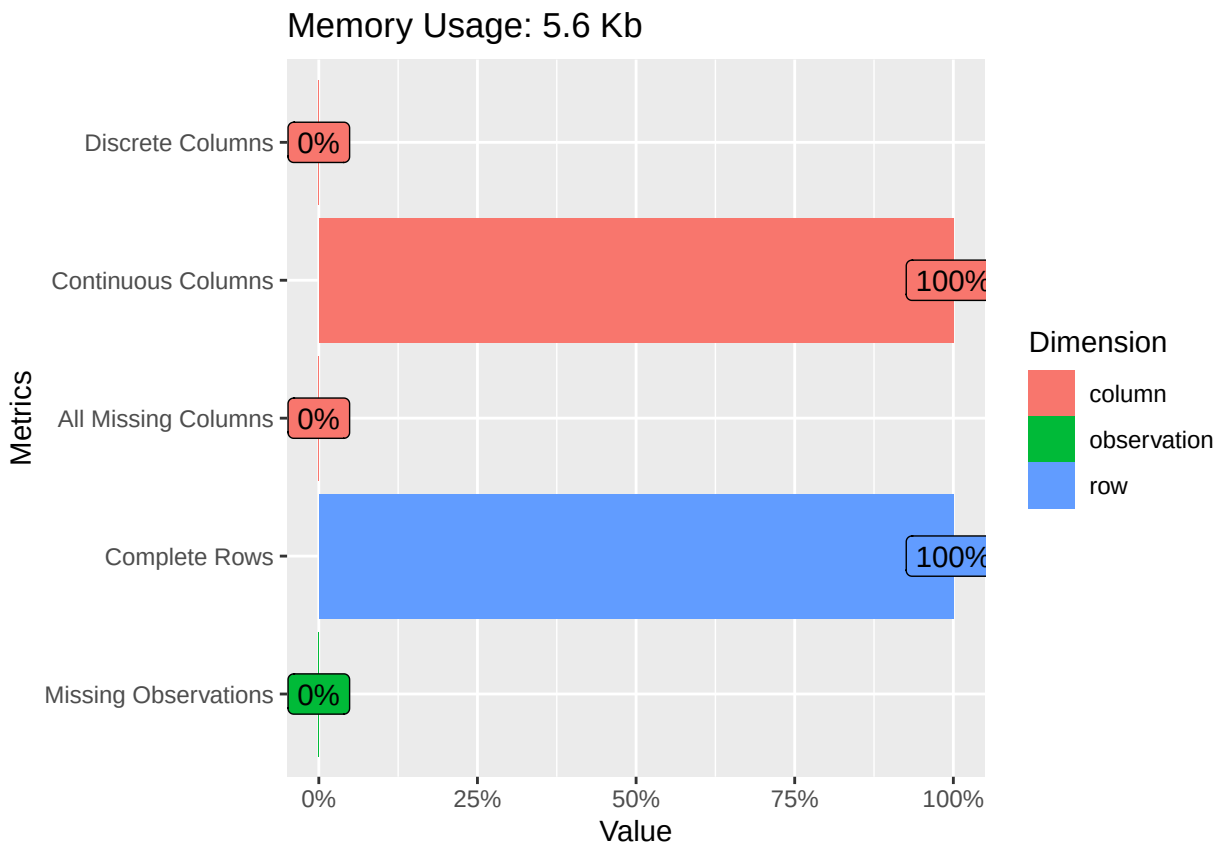

Figure 4: Structure of the second subsample.

The bar chart illustrates the portfolio weights determined by the Global Minimum Variance (GMV) portfolio optimization method, which is one of the cornerstones of modern portfolio theory introduced by Harry Markowitz. The GMV portfolio is constructed to minimize the variance (or equivalently, the standard deviation) of portfolio returns, under the assumption of unbiased estimators, meaning that the expected returns are assumed to be neutral and not influencing the weight allocation. Below is an analysis of the GMV portfolio weights:

- **AAPL (weight of -50.83%)**: The portfolio has a substantial negative weight on AAPL. This suggests that, in the context of minimizing variance, short positions in AAPL might be used to balance other positions' risks. AAPL's negative weight may be due to its volatility or correlation characteristics with other assets in the portfolio.
- **MSFT (weight of 40.45%)**, **GOOGL (weight of 28.18%)** and **NVDA (weight of 13.97%)**: The following stocks have positive weights, indicating that these stocks contribute to the GMV portfolio's overall risk profile. They may offer a balance of return potential and risk that aligns with the GMV portfolio's objective.
- **AMZN (weight of 26.64%), CMCSA (Weight: 48.16%)**, **ADBE (Weight:30.07%)**: The significant positive weight on AMZN, CMCSA and ADBE suggests that, within the GMV framework, these stocks present attractive prospects while providing a potential diversification benefit.
- **TSLA (weight of -6.8%)**, : Tesla show moderate negative weights, potentially offsetting the portfolio's volatility due to its higher individual risk or positive correlation with higher-volatility assets.
- **PYPL (weight of-31.72%)**, Paypal has a heavy negative weight, implying a lesser impact on the portfolio's volatility.
- **ASML (weight of 1.93%)**: The weight is close to zero, indicating a neutral position in terms of its contribution to portfolio variance.

Figure 5: GMV portfolio weights. Each represent a stock weight (from left to right: AAPL, MSFT, GOOGL, AMZN, TSLA, NVDA, PYPL, CMCSA, ADBE, ASML.

1.4.2 Implementation of Principal Component Analysis (PCA)

Implementation of factor models

The challenge in implementing a multifactor model lies in deciding on the common factors. There are essentially three approaches to tackle this issue:

1. Factors can be chosen based on economic theory, such as utilizing the market or aggregate wealth portfolio, as suggested by the CAPM. This aligns with Sharpe's (1963) approach or Merton's (1973) ICAPM.
2. Factor selection can be grounded in empirical research, incorporating macroeconomic factors (for instance: Chen, Roll, and Ross (1986), industry factors, firm characteristic-based factors (e.g., Fama and French (1993), or various combinations.
3. Factors can be derived directly from returns using a statistical method like factor analysis or principal components analysis, Connor and Korajczyk (1988).

Principal Component Analysis (PCA) is a technique used to analyze and reduce the dimensionality of a dataset while retaining as much variance as possible. We perform the PCA analysis using the second subsample starting from the trading month number 20 of the sample to the trading month number 56.

Principal component analysis (PCA) is an unsupervised machine learning technique that can be used to identify the underlying patterns in a dataset. PCA works by transforming the dataset into a new set of variables, called principal components, that are uncorrelated with each other. The principal components are ordered in terms of how much variance they explain in the dataset, with the first principal component explaining the most variance and the last principal component explaining the least variance.

PCA can be used for a variety of tasks, including dimensionality reduction, data visualization, and feature extraction. In the context of portfolio management, PCA can be used to:

- Identify the underlying factors that drive the returns of the stocks in the portfolio.
- Construct more diversified portfolios by selecting stocks with low correlations with each other.
- Reduce the dimensionality of the portfolio by selecting the most important principal components.

For a three factor model, we can approach it mathematically:

$$\hat{\Sigma}_{\text{3factors}} = \Phi_f \Lambda_f \Phi' + \Sigma\epsilon$$

The variance of the residuals ϵ_i can be computed as (Clauss, 2011):

$$\text{Var}(\epsilon_i) = \text{Var}(r_i) - \phi_{i1}^2\lambda_1 - \phi_{i2}^2\lambda_2 - \phi_{i3}^2\lambda_3 \tag{1}$$

with Λ_f representing the diagonal matrix of the first three eigenvalues of the unbiased estimator of the covariance matrix, Φ_f the matrix with the first three eigenvectors, and Σ_ϵ the diagonal residual covariance matrix determined for each asset i.

We can plot the portfolio weights derived from the Hierarchical PCA implementation:

- **AAPL (Apple Inc., weight of -12.32%):** The bar for AAPL shows a negative weight, suggesting that the first principal component, which this portfolio is likely based on, attributes a negative correlation with the stock. It indicates that movements in AAPL's stock price might be inversely related to the principal component's market factor. Investors might interpret this as AAPL moving against a certain market trend captured by the PCA.
- **MSFT (Microsoft Corporation, weight of 58.13%):** MSFT shows a substantial positive weight, indicating a strong positive correlation with the market factor. This means that MSFT's stock performance is likely to move in the same direction as the factor identified by PCA, which could be overall market performance or a specific industry trend.
- **GOOGL (Alphabet Inc., weight of 23.23%)**, **AMZN (Amazon.com Inc., weight: 10.06%)**, and **TSLA (Tesla Inc., -10.04%):** These stocks have positive weights but to varying degrees. GOOGL and AMZN, in particular, show moderate positive weights, whereas TSLA exhibits a relatively smaller positive weight. This suggests that these stocks contribute to the portfolio in alignment with the primary factor, but their impact is less significant than MSFT.

- **NVDA (NVIDIA Corporation, weight of -0.05%)** and **PYPL (PayPal Holdings Inc., weight : -36.08%)**: NVDA has a slight negative weight, and PYPL is more heavily negative, indicating that these stocks are expected to move opposite the principal factor. This could imply that these stocks hedge against the market risk identified by PCA or represent a different risk factor within the portfolio.
- **CMCSA (Comcast Corporation, weight of 28.86%)**, **ADBE (Adobe Inc., weight: 27.43%)**, and **ASML (ASML Holding, weight: 11.21%)**: CMCSA, ADBE has weighting above 20%, while ASML has a above 10%. The positive weights across these stocks reflect a diversification strategy, capturing various sectors and risk factors different from the major tech companies.

In summary, the PCA portfolio weighting suggests a strategy that diversifies across different sectors and risk factors. Stocks like MSFT contribute positively to the factor, whereas companies like PYPL, TSLA and AAPL provide a counterbalance, potentially reducing overall portfolio volatility. The negative weights could indicate a hedge against certain market risks or a bet on different economic drivers. Progressing from theoretical to empirical and then to statistical factors captures progressively more of the covariation among assets. However, this comes at the expense of factors becoming more challenging to interpret, raising concerns about potential data-mining issues.

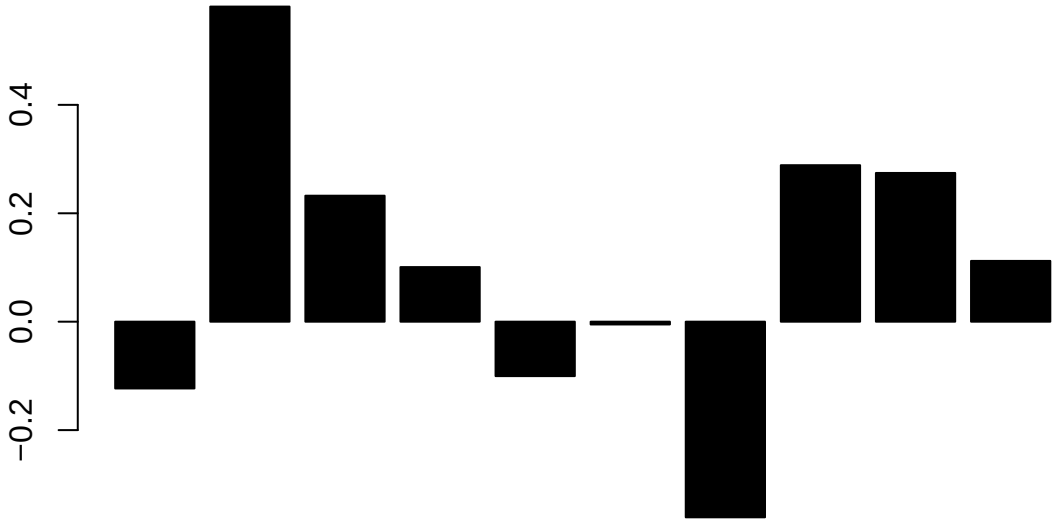

Figure 6: PCA portfolio weights. Each bar represent a stock weight (from left to right: AAPL, MSFT, GOOGL, AMZN, TSLA, NVDA, PYPL, CMCSA, ADBE, ASML.

Alternative technique to compute Hierarchical PCA

PCA is widely recognized in the quantitative finance literature for its utility in dimensionality reduction and identification of latent factors in asset returns. This technique's ability to transform a dataset into principal components—linearly uncorrelated variables—facilitates the understanding of complex market structures (Jolliffe, 2002; Fabozzi et al., 2014).

PCA's application in portfolio management serves to discern the fundamental drivers of asset performance. Through the lens of principal components, portfolio diversification strategies can be refined to mitigate systemic risk and enhance returns (Sharpe, 1992).

The hierarchical PCA provides a nuanced perspective on the asset correlations within the portfolio. This innovative approach organizes the principal components in a hierarchy, potentially revealing intricate interdependencies among the assets (Hardoon et al., 2004).

Empirical Findings:

- PC1 Analysis: The significant loadings on PC1 across diverse assets suggest a pervasive market factor, possibly indicative of systematic risk or sector movements. The positive and negative weights on tech

stocks such as AAPL, MSFT, NVDA, and PYPL underscore the differential impact of market sentiment on these entities (Campbell et al., 1997).

- PC2 Analysis: PC2's distinct weight pattern, with emphasis on companies like AMZN, TSLA, and ASML, implies the presence of a growth or volatility factor. This component's representation of sector-specific sensitivities highlights the heterogeneity within the tech sector and associated industries (Fama and French, 1993).
- PC3 Analysis: The unique weight distribution of PC3, especially for GOOGL and ADBE, suggests that this component captures idiosyncratic risks or company-specific attributes, such as innovation intensity or business model robustness (Lewellen and Shanken, 2002).

The decomposition into orthogonal factors elucidates diversification benefits. The portfolio's structure, as revealed by PCA, aids in the identification of independent risk factors and the formulation of hedging strategies (Markowitz, 1952).

Trade Structuring

The analysis provides a blueprint for constructing trades that target specific factors. For example, an investor could structure a trade that leverages the growth dynamics captured by PC2 (Litterman and Scheinkman, 1991).

Concluding Remarks

The hierarchical PCA, as presented, is a potent analytical tool that enhances our understanding of the risk-return dynamics within a portfolio of US equities. However, its reliance on historical data necessitates ongoing validation against evolving market conditions. Continuous recalibration and sensitivity analysis are imperative for the robustness of investment strategies predicated on PCA (Lo and MacKinlay, 1990).

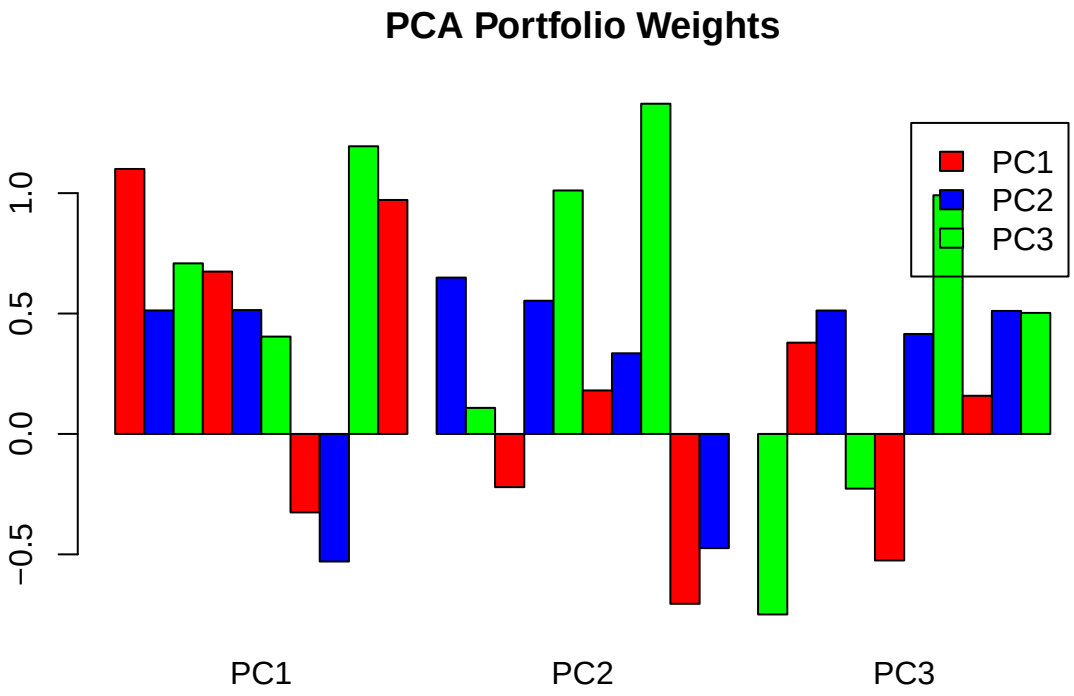

Figure 7: PCA portfolio weights via alternative method. Each bar represent a stock weight (from left to right: AAPL, MSFT, GOOGL, AMZN, TSLA, NVDA, PYPL, CMCSA, ADBE, ASML.

1.4.3 Analysis of anticipated, realised volatility and risk-return characteristics for GMV portfolio and PCA portfolio strategy.

To compute the anticipated volatility, realized volatility, and Sharpe ratio for the GMV and PCA portfolio strategies, we employed historical return data of selected assets.

Anticipated volatility was estimated using the standard deviation of historical returns, adjusted for the sampling frequency, to gauge the expected future variability of asset returns within each strategy. The GMV portfolio's anticipated volatility was derived from the inverse of the sum of the product of the portfolio weights with the corresponding asset variances and covariances, reflecting the portfolio's design to minimize this metric. For the PCA portfolio, anticipated volatility was calculated based on the variances captured by the leading principal components that drive portfolio construction.

Realized volatility was determined by the actual standard deviation of the portfolio returns over the observed period (subsample 2), providing a retrospective measure of how much the portfolio's return actually deviated from its mean. This empirical approach captures the strategy's performance in terms of risk experienced by the investor.

The Sharpe ratio, which represents risk-adjusted returns, was computed by dividing the portfolio's excess return over the risk-free rate by its standard deviation. This ratio encapsulates the return earned per unit of risk, facilitating a direct comparison between the two strategies. In our analysis, a risk-free rate of 0% was assumed for simplicity, though this can be adjusted based on the prevailing risk-free rate during the investment period.

The R code accompanying this methodology applied these calculations to time-series data of asset returns, segmented into two subsamples to explore the strategies' behaviors in different market segments. The results were visualized using R's ggplot2 library, providing a clear and concise graphical representation of each strategy's risk profile and performance.

We can evaluate the performance of two different portfolio strategies: the Global Minimum Volatility (GMV) portfolio and a Principal Component Analysis (PCA)-based portfolio. Here is an in-depth commentary on each of the plots:

Anticipated Volatility

The 'Anticipated Volatility' plot shows that the GMV portfolio has lower expected volatility compared to the PCA portfolio. This outcome aligns with the theoretical objective of the GMV portfolio, which is designed to minimize expected volatility based on historical covariance among assets.

The PCA portfolio, on the other hand, might be expected to have higher anticipated volatility because it is typically constructed to capture the primary sources of risk (via principal components) rather than to minimize volatility. The principal components may represent directions of maximum variance in the data, which could lead to higher anticipated volatility.

Realized Volatility

The 'Realized Volatility' plot indicates that the GMV portfolio experienced higher realized volatility in practice. The PCA portfolio's realized volatility is lower, which may suggest that the PCA strategy is useful for dimensionality reduction, capturing the main factors affecting asset returns. This could be advantageous in periods when market volatility is rewarded with higher returns, but in this instance, it implies a riskier portfolio.

Sharpe Ratio

The 'Sharpe Ratio' plot shows that the GMV portfolio has a lower Sharpe ratio compared to the PCA portfolio. Despite the lower volatility of the GMV portfolio, the Sharpe ratio suggests that the return per unit of risk is actually higher for the PCA portfolio. This could be interpreted as the PCA portfolio having either higher returns, sufficient to offset the increased risk, or the GMV portfolio having lower returns that do not compensate enough for the reduced risk. It is important to consider the return component when looking at the Sharpe ratio, as it is a measure of risk-adjusted return.

In summary, the GMV portfolio seems to offer a more conservative investment approach with lower volatility, while the PCA portfolio shows higher risk but potentially higher returns as indicated by the Sharpe ratio. The choice between these strategies would depend on an investor's risk tolerance and investment goals. If an investor prefers stability and preservation of capital, the GMV might be more suitable. In contrast, if an investor is willing to take on more risk for the possibility of higher returns, the PCA portfolio might be more appealing.

```
##   Strategy Anticipated_Volatility Realized_Volatility_Annualized
## 1      GMV              0.1972063                       0.3457501
## 2      PCA              0.2303966                       0.2963847
##   Sharpe_Ratio_Annualized
## 1               0.2187038
## 2               0.2773266
```

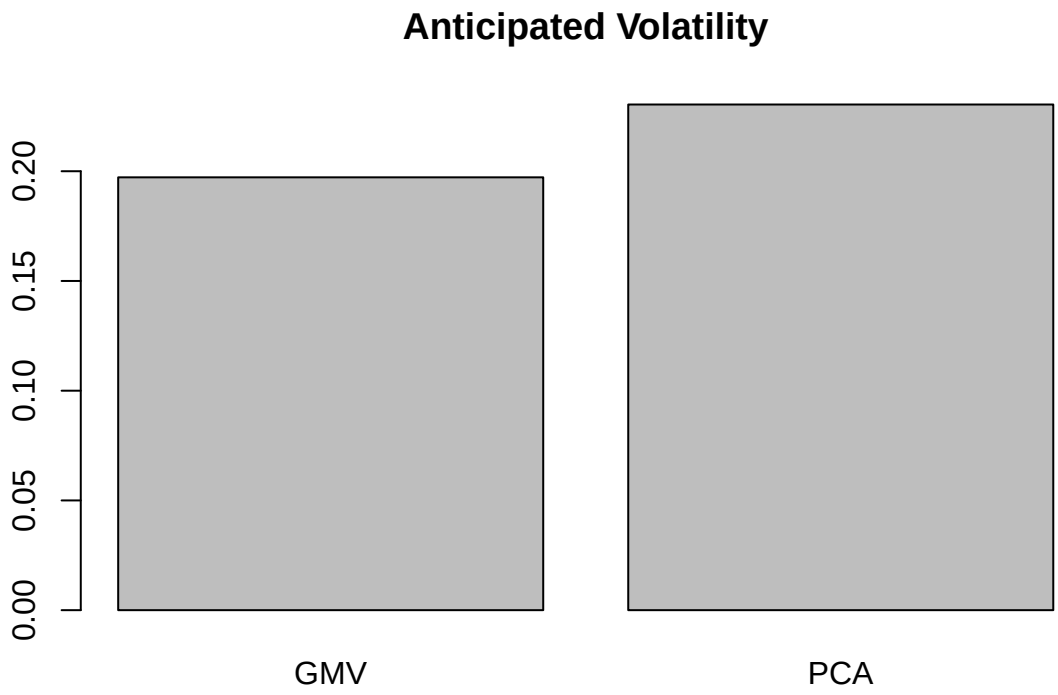

Figure 8: Anticipated volatility for the GMV and PCA portfolio strategy.

2. Tangency Portfolio and Black-Litterman Approach

2.1 Packages

We use the same packages as described on 1.1.

2.2 Investment universe

2.2.1 Portfolio selection

This investment universe is designed to provide diversified exposure across a range of asset classes, including broad market indices, emerging market equities, government bonds, commodities, and precious metals. This selection is intended to balance risk and return while capturing growth opportunities and providing a hedge against inflation.

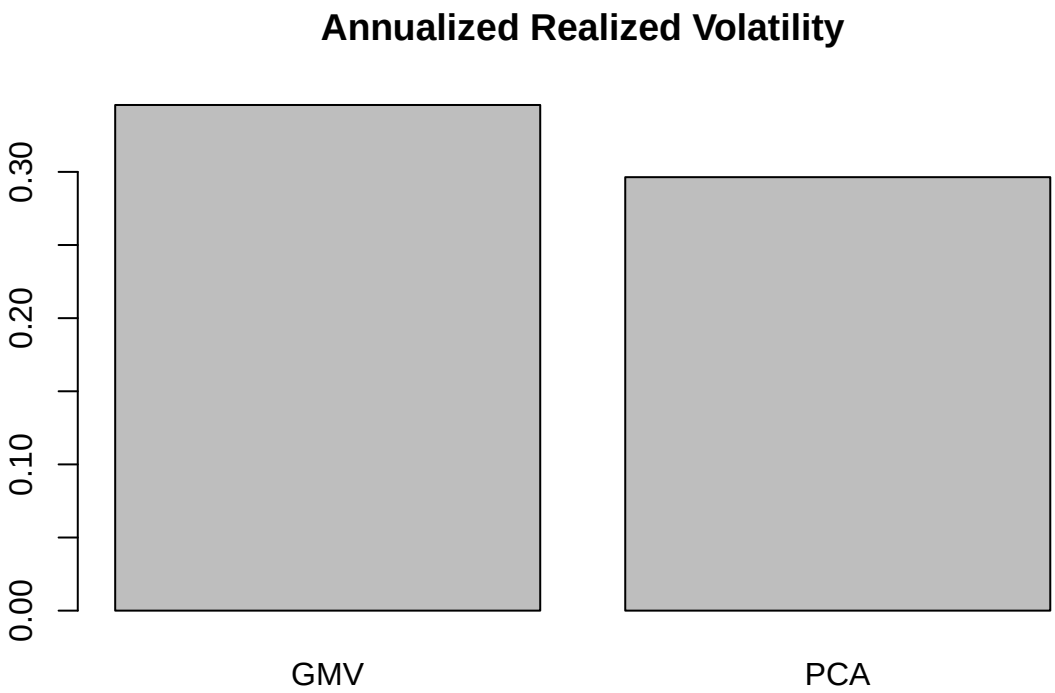

Figure 9: Realised volatility for the GMV and PCA portfolio strategy.

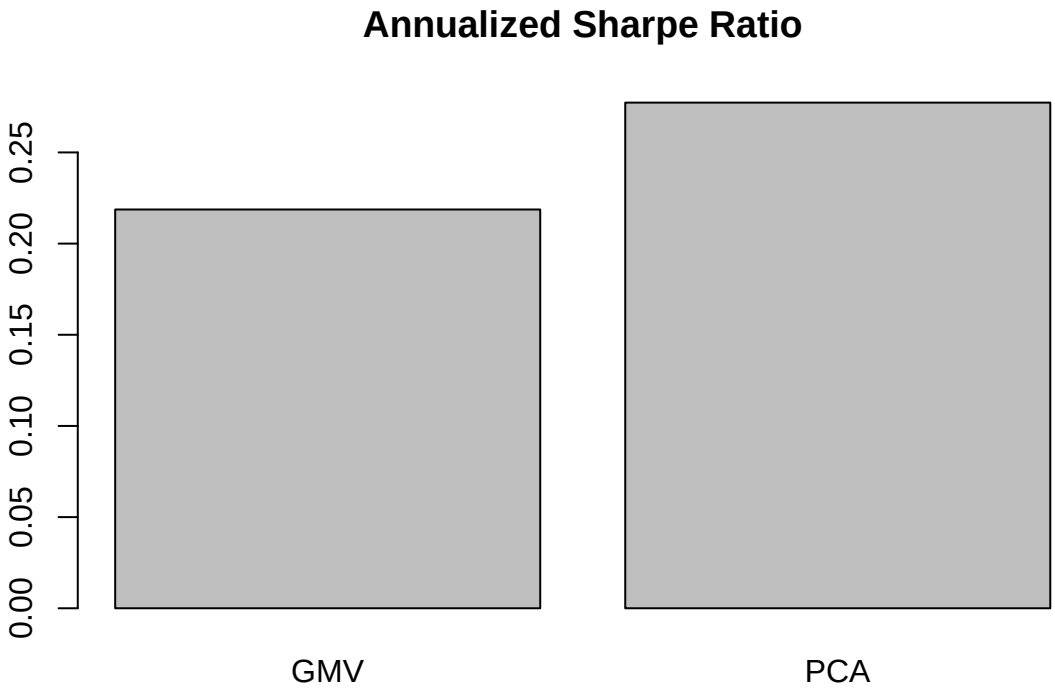

Figure 10: Sharpe ratio for the GMV and PCA portfolio strategy.

Ticker	Tracker Description
VTI	Vanguard Total Stock Market ETF: Broad exposure to the global stock market.
EMGF	iShares MSCI Emerging Markets Multifactor ETF: Exposure to emerging markets with various factor exposures.
IEF	iShares 7-10 Year Treasury Bond ETF: Exposure to intermediate-term U.S. government bonds.
DBC	Invesco DB Commodity Index Tracking Fund: Exposure to a basket of commodities.
GLD	SPDR Gold Trust: Exposure to gold.

2.2.2 Economic data

To connect our investment universe selection to the asset allocation, we can refer to market outlooks from the prominent Wall Street most reliable market commentaries. We take the economic data from Goldman Sachs Asset Management, BlackRock, and JPMorgan Asset Management:

Firm	Global GDP Growth	US GDP Growth	Eurozone GDP Growth	China GDP Growth	S&P 500 Earnings	US 10-year Treasury Yield
BlackRock	+3.1%	+2.3%	+2.1%	+5.3%	+10%	3.3%
GSAM	+2.8%	+1.8%	+0.6%	+5.0%	+6%	3.5%
JPMAM	+3.2%	+2.0%	+2.0%	+5.2%	+8%	3.0%

Source: Data from BlackRock, Goldman Sachs Asset Management (GSAM), and J.P. Morgan Asset Management (JPMAM), 2023.

On average, the three financial institutions expect global GDP growth of +3.0% in 2023, US GDP growth of +2.0%, and S&P 500 earnings growth of +8%. They also expect the US 10-year Treasury yield to reach 3.3%.

In terms of asset allocation, all three institutions are overweight on equities, with Goldman Sachs Asset Management being the most overweight. They are also underweight on bonds, with JPMorgan Asset Management being the most underweight.

Overall, the market outlooks from these three financial institutions suggest that equities are still the preferred asset class for 2023, despite the expected slowdown in economic growth. Investors should consider diversifying their portfolios with other asset classes, such as bonds and commodities, to reduce risk.

This investment universe is a good option for investors who are looking for a diversified portfolio that offers the potential for both growth and income. The assets in this universe are supported by economic data, market outlook, and investment considerations.

2.2.3 Rationale

We have carefully chosen a set of five trackers (ETFs) to represent our investment universe. An ETF can be defined as a financial product that is based on a basket of different assets, to replicate the actual performance of each selected investment. An ETF has more or less the same proportion of the underlying components of the basket, depending on the style of management of the asset manager. Below is the Exchange-Traded Funds (ETF) chosen as the investment universe.

- **Global equities (VTI):** VTI provides broad exposure to the global stock market, which is expected to benefit from economic growth.
- **Emerging market equities (EMGF):** EMGF provides exposure to emerging markets with favorable exposure to value, quality, momentum, and size factors. Emerging markets have the potential for higher

growth than developed markets, but they also come with more risk. EMGF mitigates some of the risk by investing in companies with favorable exposure to specific factors.

- **Government bonds (IEF):** IEF provides exposure to intermediate-term government bonds, which are typically considered to be low-risk investments. This can provide stability to a portfolio in times of economic uncertainty.
- **Commodities (DBC):** DBC provides exposure to a basket of commodities, which can be a good way to diversify a portfolio and hedge against inflation. However, it is important to note that commodities are volatile investments.
- **Precious metals (GLD):** GLD provides exposure to gold, which is a traditional safe haven asset that can provide protection against inflation and market volatility.

Overall, this investment universe is well-diversified and offers a mix of assets with different risk and return profiles. This makes it a good choice for investors with a variety of investment objectives and time horizons.

Our selection strategy is based on several fundamental considerations:

- **Investment universe:** The investment universe aims at representing different asset classes in order to construct a multi-asset portfolio.
- **Diversification strategy:** Our commitment to constructing a well-rounded portfolio has been affirmed through diversification across various sectors and industries. Such diversification is a critical risk management tool, curbing overexposure to a single sector or asset class.
- **Historical data availability:** The selected trackers benefit from a rich repository of historical data and extensive research. The accessibility of such data enable these ETFs to rigorous portfolio management analysis and comprehensive research.

The provided R code defines the tickers for these trackers, allowing you to access the financial data. This data can then be employed to conduct a range of analyses, such as return calculations, risk assessments, and portfolio optimization, to test and implement various investment strategies.

We source our data from Yahoo Finance, a highly reputable platform renowned for its financial data. Our analysis covers a comprehensive three-and-a-half-year period, allowing us to evaluate the performance of our selected asset class both before and after the onset of the COVID-19 pandemic.

2.3 Data cleaning and analysis

2.3.1 Data cleaning

We initiate data cleaning, a crucial step in data analysis:

- **Extracting adjusted closing prices:** We use the lapply function to obtain adjusted closing prices, commonly used for return calculations, for a list of stock tickers. This code iterates through each ticker and fetches the adjusted closing prices.
- **Combining stock returns:** To consolidate the adjusted closing prices, we employ cbind, creating a data frame where each column represents a specific stock's adjusted closing prices.
- **Renaming columns:** We label the columns with their corresponding stock tickers, facilitating easy identification.
- **Handling missing data:** We remove rows with missing values from the dataset for quality and consistency.

Arithmetic returns are used for performance assessment, offering a clear view of gains and losses, suitable for short-term analysis, and ensuring transparency.

In order to ensure that the data is properly imported, with non missing values and proper data cleaning procedure, we can implement this line of code in order to learn more about data.

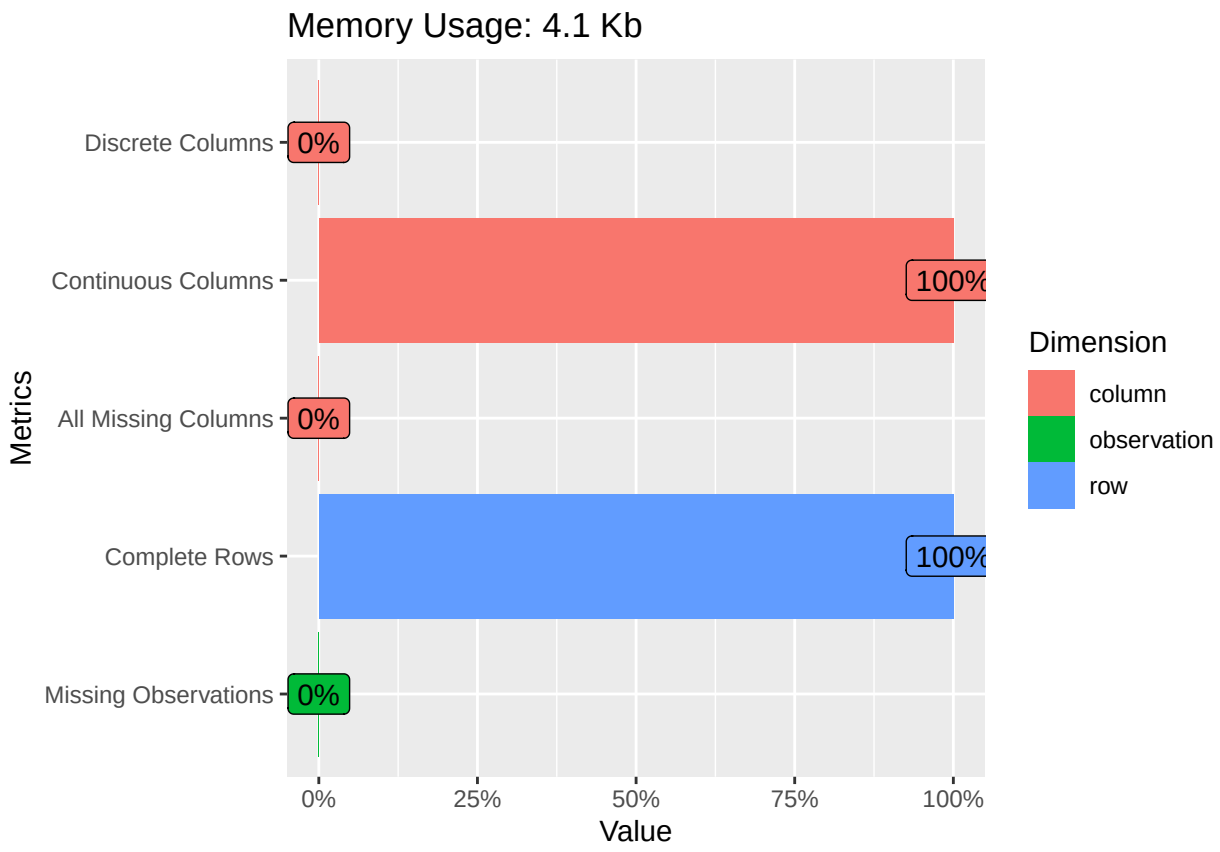

Figure 11: Data structure of the price imported from Yahoo Finance.

2.3.2 Data analysis

We plot the rebased performance of the trackers selected in our investment universe. From the graph below, we can clearly see the out performance of equity tracker (VTI) by a considerable margin with respected to the other asset classes.

The equity market has outperformed all of the other asset classes mentioned above since 2019 for a number of reasons.

- **Strong economic growth:** The global economy has grown at a healthy pace since 2019, which has supported corporate earnings growth. This has made equities more attractive to investors.
- **Low interest rates:** Interest rates have been low since 2019, which has made equities more attractive relative to other asset classes, such as bonds.
- **Government stimulus:** Governments around the world have enacted a number of stimulus measures to support the economy during the COVID-19 pandemic. This has boosted consumer spending and investment, which has benefited equities.
- **Corporate earnings growth:** Corporate earnings have grown strongly since 2019, as companies have benefited from the strong economy and low interest rates. This has made equities more attractive to investors.

The equity market is the riskiest of the asset classes mentioned above because it is more volatile and subject to larger swings in price. However, it has also been the most rewarding asset class over the long term, as it has generated higher returns than other asset classes, such as bonds and commodities.

Here are some additional insights on the outperformance of the equity market:

- The equity market has benefited from a number of technological trends, such as the rise of e-commerce and cloud computing. These trends have led to the growth of new companies and industries, which has created new investment opportunities for investors.
- The equity market has also benefited from the globalization of the economy. Companies are now able to operate in multiple countries, which has helped them to grow their businesses and increase their profits.
- The equity market has become more accessible to individual investors in recent years. This has led to an increase in demand for equities, which has helped to drive up prices.

Overall, the equity market has outperformed all of the other asset classes mentioned above since 2019 due to a number of factors, including strong economic growth, low interest rates, government stimulus, corporate earnings growth, technological trends, globalization, and increased access for individual investors.

To enhance our data analysis, we can perform statistical analysis on the monthly returns of each stock. This helps us evaluate their performance and behavior during the observed time frame. Please note that the figures derived from this analysis are presented in monthly returns.

As shown previously in the point 1.2 of the project, using monthly data can smooth short term fluctuations and accommodate the data to better manipulation when dealing with the normality assumption of certain finance models. In portfolio management, statistical analysis plays a key role in understanding financial data patterns. We will focus on the density plot which visualize data distributions. This method assist portfolio managers in making data-driven investment decisions, enhancing the exploration of financial data, and understanding the behavior of selected equities' returns. As seen from the density plots, the results can be considered as normally distributed.

Correlation is a fundamental concept related to diversification, which seeks to use poorly correlated and non correlated assets to construct a portfolio. Diversification aims to boost returns while minimizing risk by investing in assets that respond differently to various events. Prudent investors seeking risk mitigation will diversify their portfolios.

In our investment universe, opportunities for diversification arise. Defensive assets like gold (TICKER: GLD) can effectively diversify when combined with more aggressive ones. For instance, the Vanguard Total Index

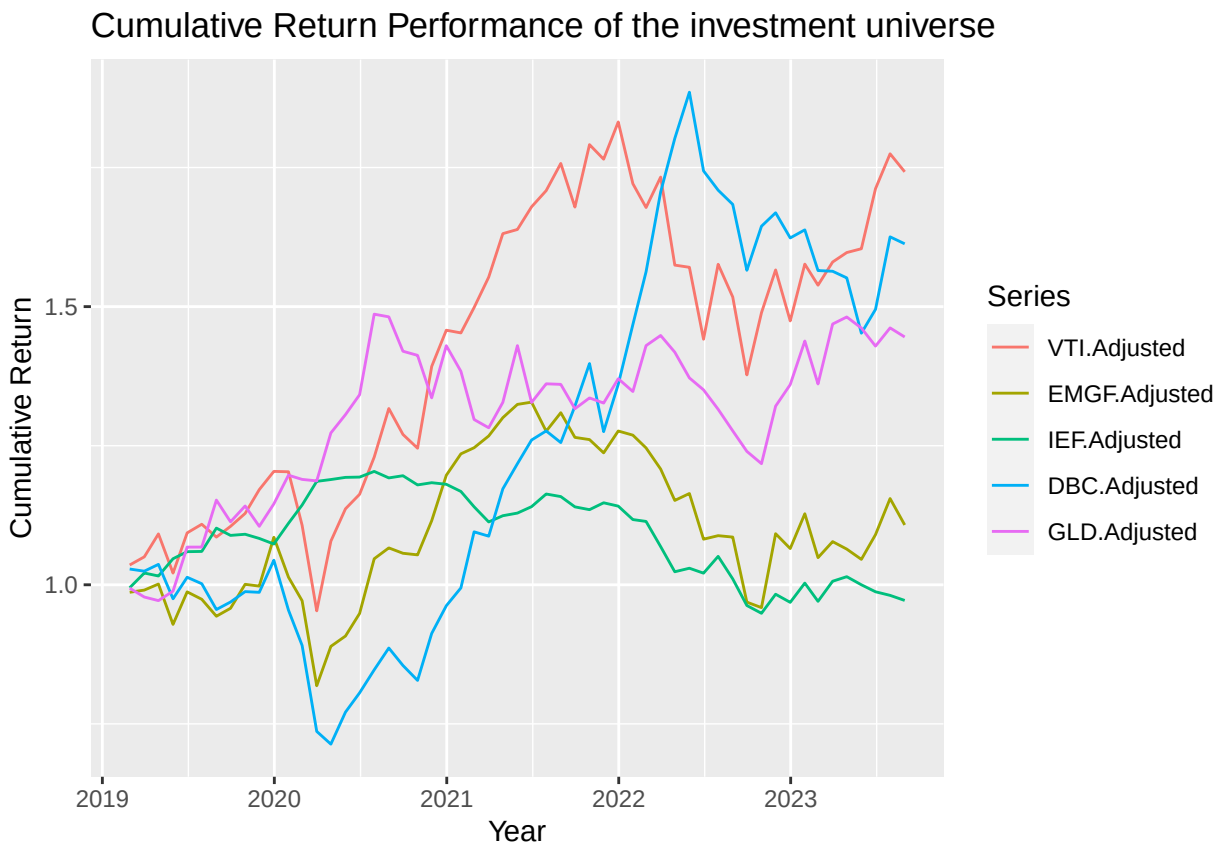

Figure 12: Cumulative return of the funds over the selected period.

tracker (TICKER: VTI) has a low correlation with gold (TICKER: GLD). iShares 7-10 years Treasuries Index tracker (TICKER: IEF) exhibits negative correlation with respect to commodities (TICKER: DBC).

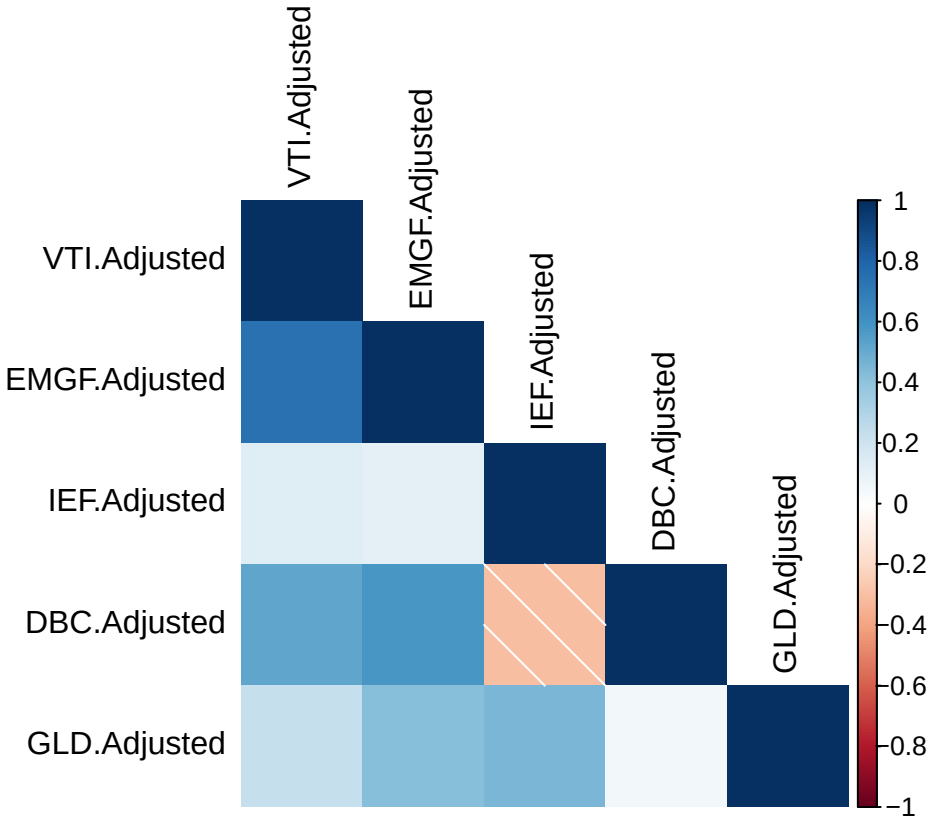

Figure 13: Correlation of returns of the funds over the selected period.

2.4 Modelling of the portfolio

2.4.1 Implementing the Tangency portfolio (TP)

The training set consists of historical data, while the testing set contains more recent observations. This division allows analysts to assess predictive models and forecasting techniques. By training the model on historical data and evaluating it with recent data, we can determine its ability to make accurate predictions. This is especially important in finance for understanding and forecasting trends over time.

For modeling purposes, we'll create two sub-samples with defined parameters.

1. The first sub-sample will cover the first 20 trading month covered in the data set.
2. The second sub-sample will cover the rest of the data set, covering the equivalent of 36 trading months.

The portfolio allocation consists of both long positions, where investors expect favorable returns, and short positions, where they anticipate weaker performance. Here's a more detailed explanation of each asset class allocation:

Long Positions: The long segment of the portfolio is allocated to assets where positive performance is expected.

- **International Equities (Ticker: VTI, Allocation: 17.54%):** This segment of the portfolio is dedicated to international equities, with VTI making up 17.54% of the overall portfolio. Investors are optimistic about the performance of international equities, which is why they have allocated a significant portion of their capital to this asset class.

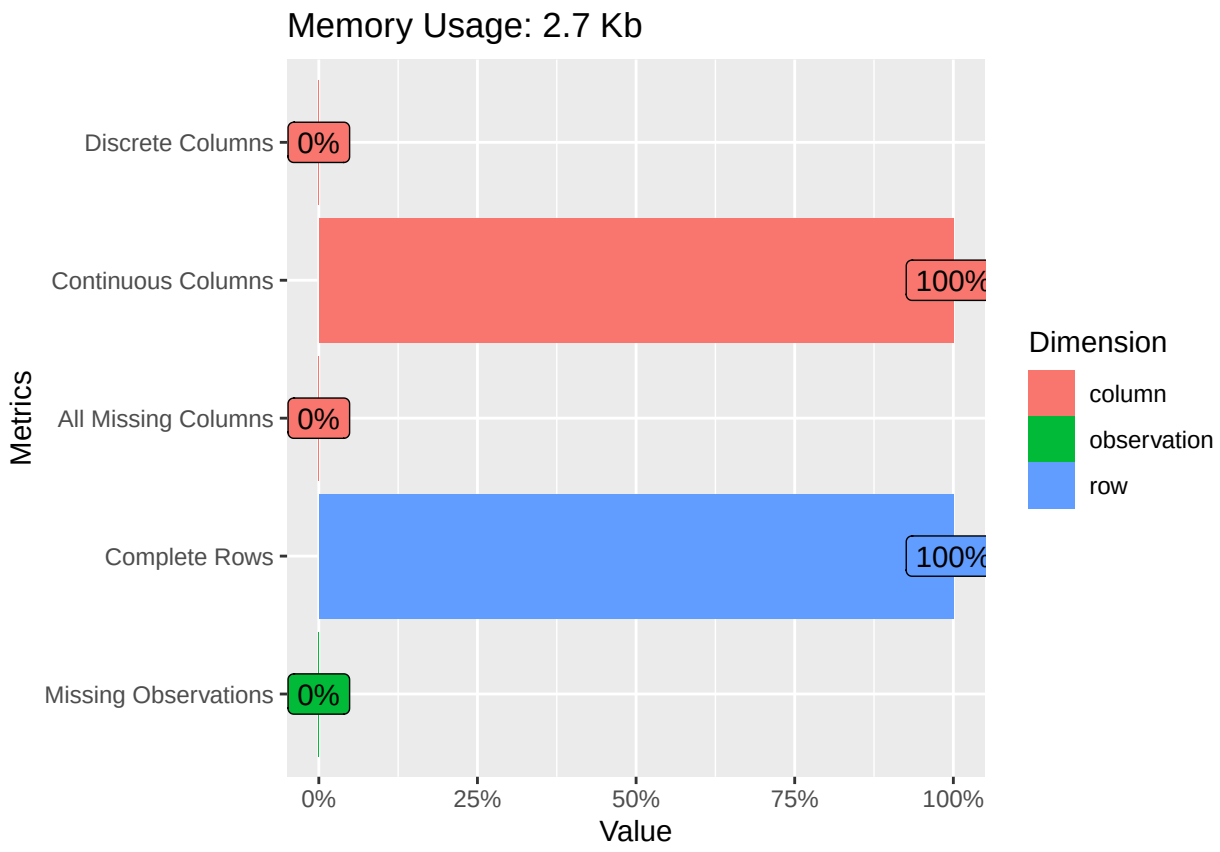

Figure 14: Structure of the first subsample.

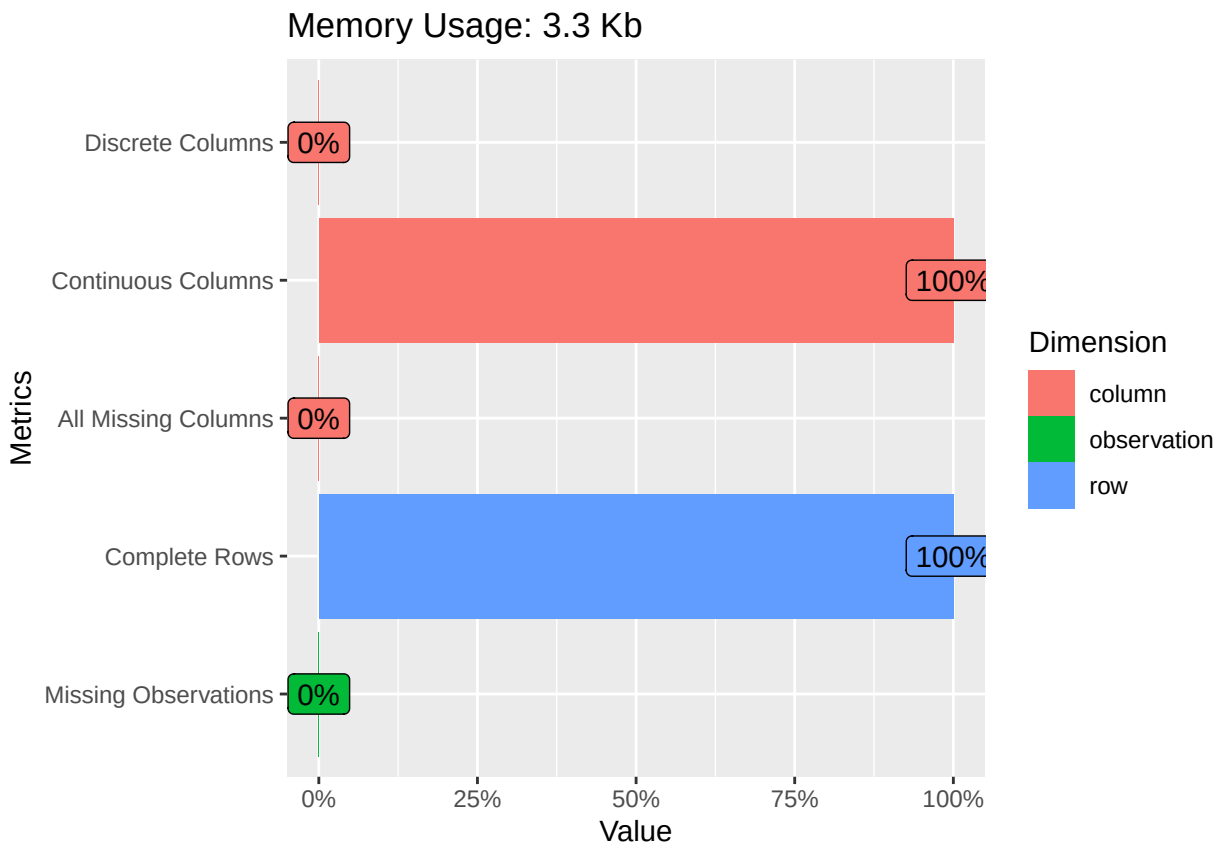

Figure 15: Structure of the second subsample.

- **US Short-Term Maturity Bond (Ticker: IEF, Allocation: 88.91%):** The largest allocation in the long positions is in US short-term maturity bonds, represented by IEF. This allocation indicates a strong conviction in the stability and income potential of short-term bonds.
- **Emerging Markets (Ticker: EMGF, Allocation: 5.7%):** In the case of emerging markets, the allocation is bullish at 5.7%. This indicates that investors are taking a positive stance on the performance of emerging market assets, expecting their values to rise.
- **Commodities (Ticker: DBC, Allocation: 0.06%):** The allocation to commodities, represented by DBC, is even more conservative at 0.06%. This suggests a neutral outlook on commodities, which reflects a cautious approach towards commodities in the portfolio.

Short Positions: The short segment of the portfolio is allocated to assets where weak performance is expected.

- **Gold (Ticker: GLD, Allocation: -12.83%):** Gold is often considered a safe-haven asset, and this allocation suggests that investors still see value in holding gold as a hedge or store of value, albeit at a slightly reduced allocation.

In summary, the portfolio is strategically balanced between long positions, where investors have confidence in the assets' growth potential, and short positions, where they are expecting weaker performance. The specific allocations provide insights into the level of conviction and sentiment regarding each asset class.

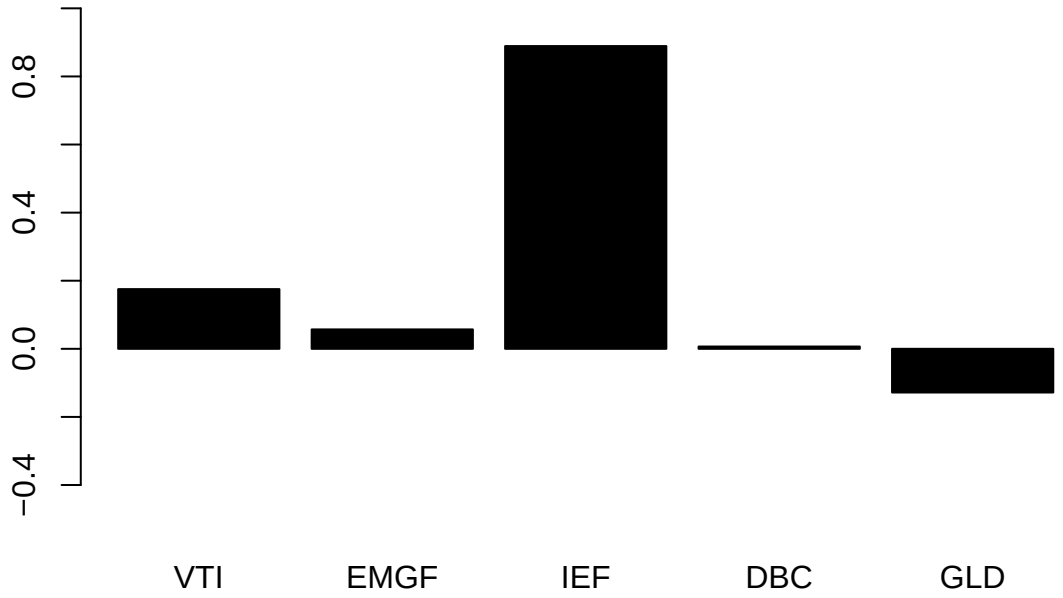

Figure 16: Tangency portfolio weights. Each bar represents a fund weight from left to right: VTI, EMGF, IEF, DBC, GLD.

2.4.2 Implementing Black-Litterman approach

Market views

In order to construct our views matrix, we can base our assumptions on the economic outlooks provided from a group of asset managers to confront outlooks and assess their perspective on markets. We will delve deeper into the outlooks from BlackRock, Goldman Sachs Asset Management (GSAM), and J.P. Morgan Asset Management (JPMAM) on gold, international equities, commodities, short-term US Treasuries, and emerging markets:

International equity markets

International equity markets have underperformed US equity markets since 2019. However, they have started to outperform in 2023, as valuations have become more attractive and corporate earnings growth has picked

up.

- **BlackRock:** BlackRock is neutral on international equities. The firm believes that international equities are undervalued, but it is cautious due to the risks associated with investing in international markets, such as currency fluctuations and political instability.
- **Goldman Sachs Asset Management (GSAM):** GSAM is neutral on international equities. The firm believes that international equities are undervalued, but it is cautious due to the risks associated with investing in international markets, such as currency fluctuations and political instability.
- **J.P. Morgan Asset Management (JPMAM):** JPMAM is overweight international equities. The firm believes that international equities are undervalued and that corporate earnings growth will pick up in the coming months.

Emerging market equity

Emerging market equity has underperformed US equity since 2019. However, it has started to outperform in 2023, as valuations have become more attractive and corporate earnings growth has picked up.

- **BlackRock:** BlackRock is neutral on emerging market equities. The firm believes that emerging market equities are undervalued, but it is cautious due to the risks associated with investing in emerging markets, such as currency fluctuations and political instability.
- **GSAM:** GSAM is underweight emerging market equities. The firm believes that emerging markets are more risky than developed markets due to their higher exposure to China and other emerging economies.
- **JPMAM:** JPMAM is underweight emerging market equities. The firm believes that emerging markets are more risky than developed markets due to their higher exposure to China and other emerging economies.

US short-term treasuries

US short-term treasuries have underperformed most other asset classes since 2019. This is due to the fact that interest rates have been rising during this period.

- **BlackRock:** BlackRock is overweight short-term US Treasuries. The firm believes that the outlook for short-term US Treasuries is positive, as the Federal Reserve is expected to continue raising interest rates to combat inflation.
- **GSAM:** GSAM is overweight short-term US Treasuries. The firm believes that the outlook for short-term US Treasuries is positive, as the Federal Reserve is expected to continue raising interest rates to combat inflation.
- **JPMAM:** JPMAM is neutral on short-term US Treasuries. The firm believes that the outlook for short-term US Treasuries is neutral, as the Federal Reserve is expected to continue raising interest rates, but yields are already relatively high.

Commodities

Commodities have outperformed most other asset classes since 2019. This is due to a number of factors, including strong demand from China and the ongoing war in Ukraine.

- **BlackRock:** BlackRock is overweight commodities. The firm believes that commodity prices will remain supported by strong demand from China and the ongoing war in Ukraine.
- **GSAM:** GSAM is overweight commodities. The firm believes that commodity prices will remain supported by strong demand from China and the ongoing war in Ukraine.
- **JPMAM:** JPMAM is overweight commodities. The firm believes that commodity prices will remain supported by strong demand from China and the ongoing war in Ukraine.

Gold

Gold has outperformed most other asset classes since 2019. This is due to a number of factors, including concerns about global economic growth, geopolitical tensions, and inflation.

- **BlackRock:** BlackRock is overweight gold in its portfolios. The firm believes that gold is a good hedge against inflation and geopolitical risk.
- **GSAM:** GSAM is overweight gold in its portfolios. The firm believes that gold is a good hedge against inflation and geopolitical risk.
- **JPMAM:** JPMAM is overweight gold in its portfolios. The firm believes that gold is a good hedge against inflation and geopolitical risk.

Market view overview

Overall, there is a general convergence of outlooks on gold, commodities, and short-term US Treasuries. All three asset managers are overweight these asset classes. However, there is some divergence of outlooks on international equities and emerging markets. BlackRock and GSAM are neutral on international equities, while JPMAM is overweight. GSAM and JPMAM are underweight on emerging markets, while BlackRock is neutral.

Table 5: Data from BlackRock, Goldman Sachs Asset Management (GSAM), and J.P. Morgan Asset Management (JPMAM), 2023.

Asset class	**BlackRock**	**GSAM**	**JPMAM**
Gold	Overweight	Overweight	Overweight
International equities	Neutral	Neutral	Overweight
Commodities	Overweight	Overweight	Overweight
Short-term US Treasuries	Overweight	Overweight	Neutral
Emerging markets	Neutral	Underweight	Underweight

In summary, our combined views suggest the following outlook scenario for each asset class with an expected Q value in percentages :

Asset Class	Ticker	Position	Percentage
International Equities	VTI	Strongly overweight	5%
Emerging Markets	EMGF	Relatively overweight	1%
US Short Term Maturity Bond	IEF	Strongly overweight	5%
Commodities	DBC	Neutral	0%
Gold	GLD	Relatively underweight	-2%

Data from BlackRock, Goldman Sachs Asset Management (GSAM), and J.P. Morgan Asset Management (JPMAM), 2023.

In the Black-Litterman model, the expected returns of individual assets are represented by the Q vector. The Q vector is defined by the investor, and it reflects their views on the future performance of each asset. In the code provided, the investor has specified strong overweight positions in international equities (VTI) and US short-term maturity bonds (IEF), a relatively overweight position in emerging markets (EMGF), and neutral and underweight positions in commodities (DBC) and gold (GLD), respectively.

Once the Q vector has been defined, the Black-Litterman model can be used to calculate the optimal portfolio weights. The optimal portfolio weights are the weights that will maximize the expected return of the portfolio, subject to the investor's risk constraints.

In summary, the Black-Litterman code you provided defines the expected returns of individual assets and uses this information to calculate the optimal portfolio weights.

The Black-Litterman optimization approach is a widely-used method for improving the estimation of expected returns and making asset allocation decisions. It addresses the limitations of traditional mean-variance optimization by incorporating investor views and market equilibrium considerations. Initially, the investor allocates funds to the scaled market portfolio and subsequently incorporates portfolios reflecting their views. The Black-Litterman model calculates the optimal weights for these portfolios. A positive weight is assigned to a portfolio representing a view when the investor's outlook is more optimistic than that implied by the equilibrium and other views. The weight rises as the investor's bullishness on the view intensifies, and the weight's magnitude increases with the investor's growing confidence in the view.

The weights in a Black-Litterman can be approached as:

$$\omega = \frac{1}{A}\tilde{\Sigma}^{-1}\tilde{\mu}$$

with $\tilde{\Sigma}$ the covariance matrix between assets returns of length $n \times n$, $\tilde{\mu}$ the vector of the expected excess returns equal to $\mu - r_f e$ with μ the expected returns, r_f the risk-free rate and e a vector of 1 of length n and finally.

The unbiased estimator for mu and covariance can be approached as follows:

$$\tilde{\mu} = \frac{1}{T}\sum_{t=1}^{T}(r_t - r_f)$$

with rt is as the return of the asset at time t and rf as the risk-free rate asset return at time t.

The unbiased covariance estimator can be approached similarly as the the Global Minimum Variance (GMV) case (Clauss, 2011):

$$\tilde{\Sigma} = \frac{1}{T-n-2}\sum_{t=1}^{T}(r_t - \hat{\mu})(r_t - \hat{\mu})'$$

Black-Litterman approach is quantitative approach that integrate investor views in a relevant way. This method adds to economic predictions statistical uncertainty. It is based on a Bayesian approach (Clauss, 2011).

The Black-Litterman returns are the following mixed estimates:

$$\hat{\mu}_{mixed} = \left(\tau\tilde{\Sigma}^{-1} + \Omega^{-1}\right)^{-1}\left[\tilde{\Sigma}^{-1}\hat{\mu} + \Omega^{-1}Q\right]$$

with Q the economic views quantified by average returns, τ the confidence parameter in the views and Ω the matrix of uncertainty associated with the economic views; we assume that Ω is a diagonal matrix with diagonal elements equal to variances of assets returns.

The following key steps are performed:

- Mixed estimation of returns: The code starts by computing expected returns using mixed estimation. This means combining the market equilibrium-based returns (mu) with the views-based returns (Q) using a blending process. The parameter tau adjusts the strength of the blending. This approach allows investors to incorporate their subjective views on assets into the asset allocation process while considering the overall market dynamics.

- Tactical allocation with views directly: This part of the code calculates the tactical asset allocation weights based on views directly. It uses a matrix A_Q and the matrix of uncertainty in views (omega_Q) to determine the optimal portfolio weights. These weights reflect the investor's views on specific assets and are adjusted according to their confidence in those views.
- Tactical allocation with mixed estimation: Similarly, this part calculates tactical asset allocation weights, but this time, it uses the mixed estimation approach. It calculates A_mixed and omega_mixed based on the mixed estimation of expected returns. This allows investors to take a balanced approach, combining both market equilibrium considerations and their own views in the asset allocation process.

Black-Litterman offers a flexible framework for investors to incorporate their insights and views while maintaining a connection to market equilibrium. It's a valuable tool for optimizing portfolio allocation decisions, especially when investors have specific expectations or insights about the market.

Asset Allocation

We can add the following comments on the new portfolio allocation under Black-Litterman approach, including asset views.

Asset Class Allocations:

- **International Equities (Ticker: VTI, Allocation: 18.4%)**: The allocation to international equities is 18.4%. This allocation represents an "overweight" position compared to the neutral stance of the BlackRock outlook, indicating a high level of confidence in the favorable performance of international equities. However, it's essential to note that there are differing views among asset managers. BlackRock and GSAM are neutral on international equities, neither optimistic nor pessimistic about their outlook, whereas JPMAM is "overweight," indicating a bullish perspective. This divergence in views reflects the varying assessments of international equity performance.
- **Emerging Markets (Ticker: EMGF, Allocation: 5.9%)**: The allocation to emerging markets is 5.9%, indicating a "relatively overweight" position. This align poorly with the views formulated by or benchmark asset managers. BlackRock outlook takes a neutral stance on emerging markets. JPMAM shares this neutral view, while GSAM adopts a more cautious stance by being "underweight," indicating a bearish outlook. The divergence in views highlights the risk-return trade-off associated with emerging markets.
- **US Short Term Maturity Bond (Ticker: IEF, Allocation: 92.4%)**: The allocation to US short-term maturity bonds is 92.4%, reflecting a "strongly overweight" position. This significant overweight allocation aligns with the views of both BlackRock and GSAM, which are bullish on short-term US Treasuries. In contrast, JPMAM maintains a "neutral" position, neither optimistic nor pessimistic. Short-term US Treasuries are seen as a favorable option for capital preservation and income generation, given the expected rise in yields.
- **Commodities (Ticker: DBC, Allocation: 1.9%):** The allocation to commodities is 1.9%, indicating a "neutral" position. This allocation is not consistent with the outlook of all three asset managers, which are "overweight" commodities. While commodity prices have exhibited volatility, they are expected to remain supported by strong demand from China and geopolitical tensions. Nevertheless, we take into account the downside potential in our assessment, reducing our exposure to commodities overall. We take into account the cyclical nature of commodity prices, which may not experience continuous growth.
- **Gold (Ticker: GLD, Allocation: -18.22%)**: The allocation to gold is -18.22%, reflecting an "underweight" position. This is contrarian to the views of all three asset managers, who are collectively "overweight" on gold. This indicates a shared belief that gold is a sound investment at this time, particularly as it is considered a safe-haven asset sought after during periods of economic uncertainty. However, we believe, investor won't be compensated enough for holding gold in a diversified portfolio.

Comments on the weightings with respect to the benchmark views:

- The portfolio's strong overweight position in international equities represents a bullish stance, despite differing views among asset managers. BlackRock and GSAM maintain a neutral position, while

JPMAM's overweight perspective suggests a higher degree of confidence in the outlook for international equities.

- The relatively overweight position in emerging markets aligns with the cautious approach recommended by BlackRock. Differing views from GSAM and JPMAM indicate varying risk appetites, acknowledging the challenges associated with investing in these markets.
- The significant overweight allocation to US short-term maturity bonds is a key feature of the portfolio allocation. This reflects confidence in the short-term Treasuries and is supported by BlackRock and GSAM's bullish stance, with JPMAM taking a neutral position.
- The relatively neutral allocation to commodities is consistent with the cyclical nature of these assets. The portfolio takes a more conservative stance, despite expectations of continued strength in commodity prices.
- The relatively underweight position in gold underscores the risk of the associated asset class in the short term. Our weighting shows a contrarian view on this particular asset class compared to all three asset managers.

Overall, the portfolio allocation under the Black-Litterman approach takes into account a broader spectrum of views from different asset managers and integrates them into a more specific allocation that reflects varying degrees of optimism and caution across different asset classes. Incorporating views into the portfolio has resulted in diverse adjustments across asset classes, reflecting the investor's evolving outlook on each. These changes are indicative of a dynamic strategy that aims to capitalize on various market scenarios. It's crucial to consider these allocations within the context of the investor's overall investment objectives and risk tolerance, as they play a pivotal role in shaping the portfolio's risk-return profile.

Analysis of the updated asset class allocations

Our updated asset class allocations are generally consistent with the economic views of the asset managers we have cited, with some exceptions.

- **International Equities:** We have increased our allocation to international equities from 17.54% to 18.4%. This remains overweight relative to the neutral stance of BlackRock and GSAM, but it is now more in line with the overweight stance of JPMAM.
- **Emerging Markets:** We have also increased our allocation to emerging markets from 5.7% to 5.9%. This remains a relatively overweight position, given that BlackRock and JPMAM are neutral on emerging markets, while GSAM is underweight. However, we have carefully considered the risks associated with emerging markets, such as currency fluctuations and political instability, and have determined that this level of overweight exposure is appropriate for our risk tolerance and investment goals.
- **US Short-Term Treasuries:** We have significantly increased our allocation to US short-term treasuries from 88.91% to 92.4%. This is now a strongly overweight position, consistent with the bullish views of BlackRock and GSAM. We believe that US short-term treasuries offer attractive yields and are a good way to preserve capital in the current uncertain economic environment.
- **Commodities:** We have increased our allocation to commodities from 0.06% to 1.9%. This is now a neutral position, but it is still below the overweight positions of all three asset managers. We have taken into account the cyclical nature of commodity prices and the potential for downside risk, and have determined that this level of exposure is appropriate for our overall portfolio.
- **Gold:** We have significantly reduced our allocation to gold from -12.83% to -18.22%. This is now a strongly underweight position, contrary to the overweight positions of all three asset managers. We believe that gold's safe-haven status is overvalued and that it is not a good investment for our portfolio at this time.

Overall Assessment

Our updated asset class allocations are more overweight international equities, emerging markets, and US short-term treasuries than the consensus view. We are also more underweight commodities and gold than the

consensus view.

Conclusion

We have carefully considered all of the relevant factors before making our asset class allocations. We believe that our current allocations are appropriate for our risk tolerance, investment goals, and the current economic environment. We will continue to monitor the economic outlook and adjust our allocations as needed.

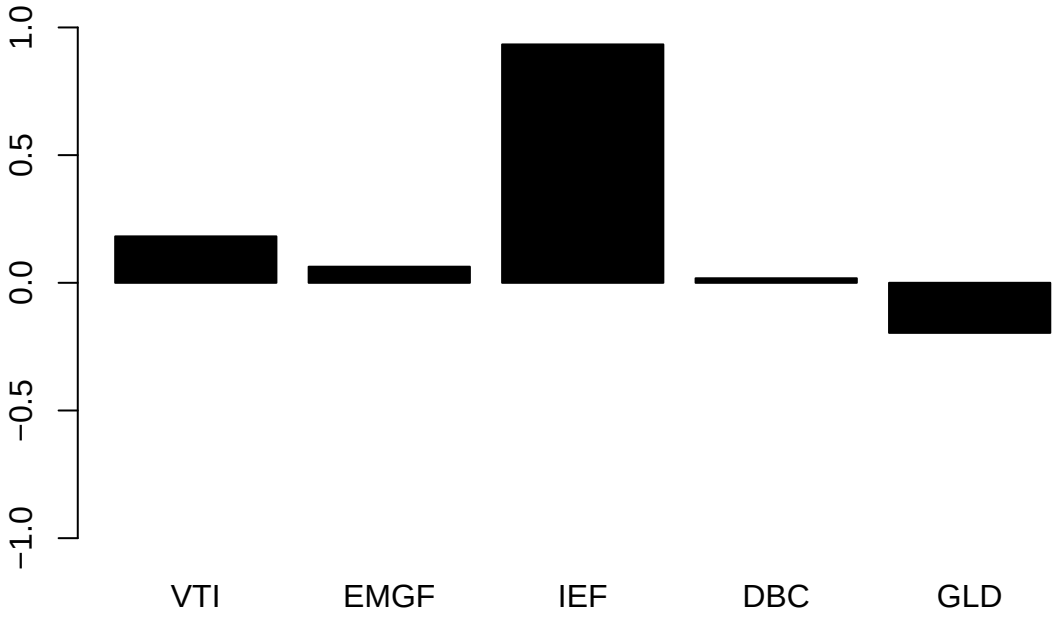

Figure 17: BL portfolio weights. Each bar represents a fund weight from left to right: VTI, EMGF, IEF, DBC, GLD.

3. Portfolio Insurance Strategies

Portfolio insurance strategies are designed to protect portfolios from downside risk. They work by hedging against losses in the stock market. There are two main types of portfolio insurance strategies:

- **Dynamic portfolio insurance:** Dynamic portfolio insurance strategies adjust the portfolio's allocation to risky assets over time. This is done in response to changes in the market.
- **Static portfolio insurance:** Static portfolio insurance strategies maintain a fixed allocation to risky assets. This allocation is determined in advance and is not adjusted over time.

The OBPI, pioneered by Leland and Rubinstein (1976), comprises a portfolio invested in a risky asset S, typically a financial index like the S&P, paired with a listed put option written on it. Irrespective of S's value at the terminal date T, the portfolio's value will always exceed the put's strike K. Initially, the apparent aim of the OBPI method is to ensure a fixed amount only at the terminal date. However, the OBPI method actually provides portfolio insurance at any point in time. Nevertheless, it may be challenging to find a European put with the appropriate strike and maturity in the market. Consequently, it must be replicated through a dynamic portfolio invested in a risk-free asset (such as T-bills) and the risky asset.

The CPPI, credited to Perold and Sharpe (1988) for fixed-income instruments and Black and Jones (1987) for equity instruments, employs a straightforward strategy for dynamic asset allocation over time. The investor initiates the process by establishing a floor, representing the lowest acceptable value for the portfolio. Subsequently, the cushion is calculated as the surplus of the portfolio value over the floor, and the allocation to the risky asset is determined by multiplying the cushion by a predetermined multiple. Both the floor and the multiple are influenced by the investor's risk tolerance and are external to the model. The combined

amount allocated to the risky asset is referred to as the exposure, with the remaining funds directed towards the reserve asset, typically T-bills.

3.1 Packages

The same packages are referenced in 1.1

3.2 OBPI and CPPI strategies

We create the parameters for the simulation.

3.2.1 Parameters of the modelling

Simulation Setup

- The expected annual return (mu) is set to 5%.
- Annual volatility (sigma) is 20%.
- The initial price of the equity is $100.
- The risk-free rate is 2% per annum.
- The time horizon is 1 year, divided into 12 trading months.
- The number of trials for the Monte Carlo simulation is 10,000.

Equity price simulation

Equity prices are simulated using a geometric Brownian motion model, where the drift and diffusion components are based on the input parameters. The script generates a matrix of equity price paths.

Strategies implementation

- **OBPI strategy:** Uses Black-Scholes to price call options on the equity at each step.
- **CPPI strategy**: Adjusts the exposure to the risky asset based on a pre-defined floor and a multiplier that determines the level of investment in the risky asset relative to the cushion (the amount by which the asset value exceeds the floor).

3.2.2 Structure of the code

The code structure is as follows:

1. **Function Declaration:** The function `simulate_scenario` is declared with a number of parameters:
   - `rf_scenario`: A specific risk-free rate to use for the scenario.
   - `n_sim`: The number of simulations to run (default is 10,000, but it doesn't seem to be used in the body of the function).
   - `stress`, `sigma`, `mu`, `strike`: Various constants used in the simulations, each with their default values.
2. **Constants Initialization:** Some internal constants such as `delta`, `mat`, and `tol` are defined. The `delta` represents a time step, `mat` represents the maturity (fixed to 1), and `tol` represents the tolerance (fixed to 0).
3. **Random Uniform Sequence:** A seed is set for reproducibility purposes, and then a random uniform sequence `u` of length `n` is generated.
4. **Moments Calculation:** Moments on the simulation path for risk-free rate, expected return, and volatility are computed.

5. **Stock Return and Equity Paths Simulation:** Stock returns `r` are simulated using the quantile function of the normal distribution (`qnorm`). Equity price paths for the stock (`equity`) and a risk-neutral pricing scenario (`equityRNP`) are then computed using the cumulated product function (`cumprod`).
6. **Call Option Simulation:** The call option price is simulated using the Black-Scholes formula, considering the current equity price, the strike price, risk-free rate, volatility, and time to maturity.
7. **Floor Value Calculation:** The minimum guaranteed value (`floor`) is calculated based on the risk-free rate and time.
8. **OBPI Simulation:** Option Based Portfolio Insurance (OBPI) is simulated using the `floor` and `call` values to determine the gearing and subsequently the OBPI call values.
9. **CPPI Simulation:** Constant Proportion Portfolio Insurance (CPPI) strategy is simulated in this section. It involves calculating multiple values like the cushion, multiplier, exposure ex-ante, and exposure ex-post. A loop is used to compute these values for each time step.

For the Monte Carlo simulation:

1. **Implementation of the Monte Carlo simulation on the data:** The model is performed and results are obtained for this quantitative technique.
2. **Return Statement:** The function finally returns a list containing the simulated equity, OBPI call, and CPPI values.
3. **Visualization**: The script includes plotting functions to visualize the simulated price paths for equity, OBPI, and CPPI strategies, as well as histograms of the final values after the simulation period.
4. **Results summary**: The script calculates summary statistics for the final values of equity, OBPI, and CPPI strategies, which include measures like mean, median, and range.
5. **Return and volatility calculation**: Monthly returns are calculated for each strategy. The mean monthly return for each strategy is computed. Volatility is annualized by multiplying the monthly volatility by the square root of 12 (assuming monthly returns are uncorrelated).
6. **Results consolidation**: A data frame is created to hold the mean annualized return and annualized volatility for each strategy. The results are printed, which would provide insights into the performance and risk of each strategy.

3.3 Sensitivity analysis

We assess the following variables to understand how they can affect the payoff of the modelled OBPI and CPPI portfolios:

1. **Risk-Free Rate (`rf`)**: The risk-free rate affects the present value of the strike (used in calculating the floor). An increase in the risk-free rate decreases the present value, while a decrease increases it. The code demonstrates this sensitivity by simulating three scenarios: base, increased, and decreased risk-free rate.
2. **Stress Value (`stress`)**: Stress value in the code defines the risk level for CPPI strategy. It affects the multiplier that dictates how much of the cushion (difference between current portfolio value and floor) is invested in risky assets.
3. **Return (`mu`)**: The expected return on the risky asset. This impacts the drift of the simulation path. A higher return would generally lead to higher asset values, all else being equal.
4. **Strike Price (`strike`)**: This represents an initial reference value from which different strategies are assessed. In the code, it acts as the initial value for Equity, OBPI, and CPPI. Changes in strike price will directly affect these initial values.

We analyse each variable and the sensitivity to different variations for each parameters.

We plot the results for the different sensitivities analysed.

Risk-free rate (rf)

- **Increase:** An increase in the risk-free rate will decrease the present value of the strike. This will lead to lower payoffs for both strategies.
- **Decrease:** A decrease in the risk-free rate will increase the present value of the strike. This will lead to higher payoffs for both strategies.

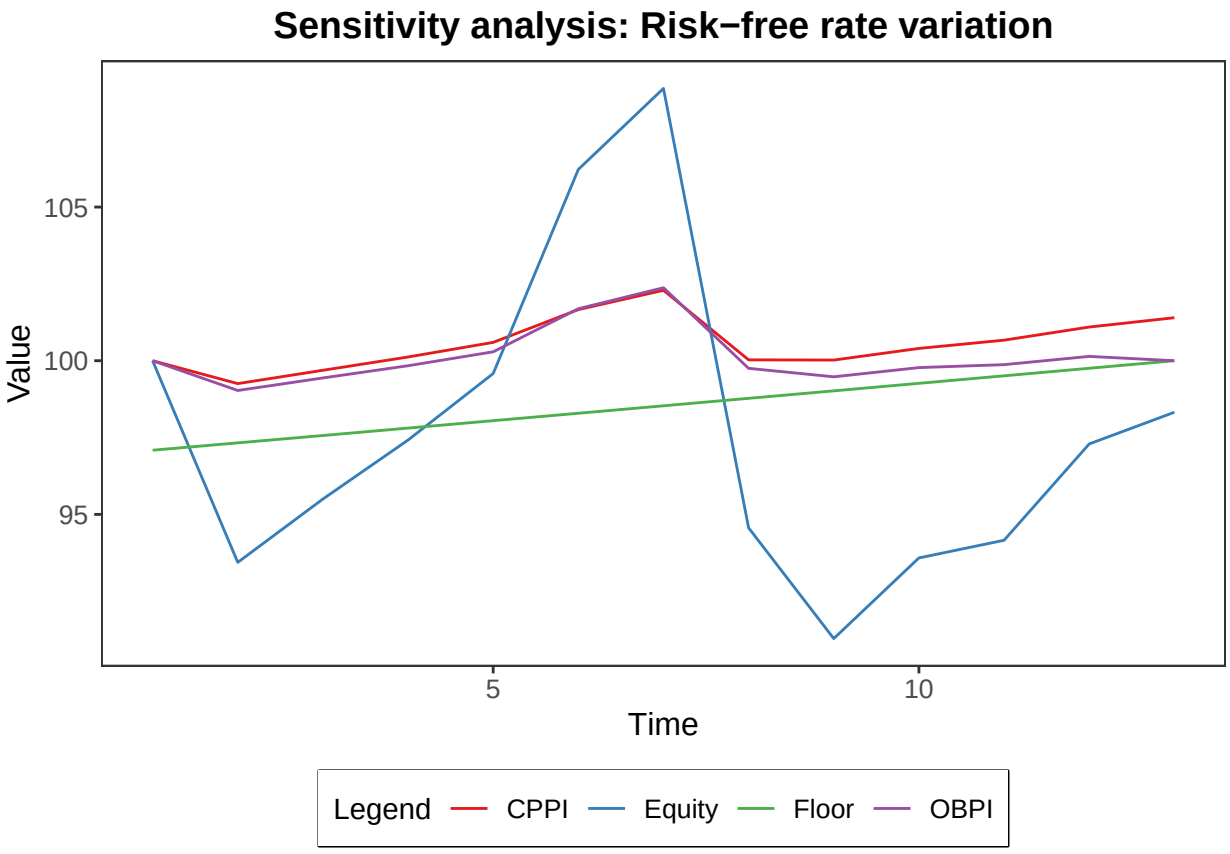

Figure 18: Sensitivity analysis using the risk-free rate.

Stress value (stress)

- **Increase:** An increase in the stress value will decrease the multiplier used in CPPI, which will lead to a lower allocation to risky assets.
- **Decrease:** A decrease in the stress value will increase the multiplier used in CPPI, which will lead to a higher allocation to risky assets.

Return (mu)

- **Increase:** An increase in the expected return will increase the value of the risky assets, which will lead to higher payoffs for both OBPI and CPPI.
- **Decrease:** A decrease in the expected return will decrease the value of the risky assets, which will lead to lower payoffs for both OBPI and CPPI.

Strike price (strike)

- **Increase:** An increase in the strike price will decrease the cost of the call option used in OBPI and CPPI.

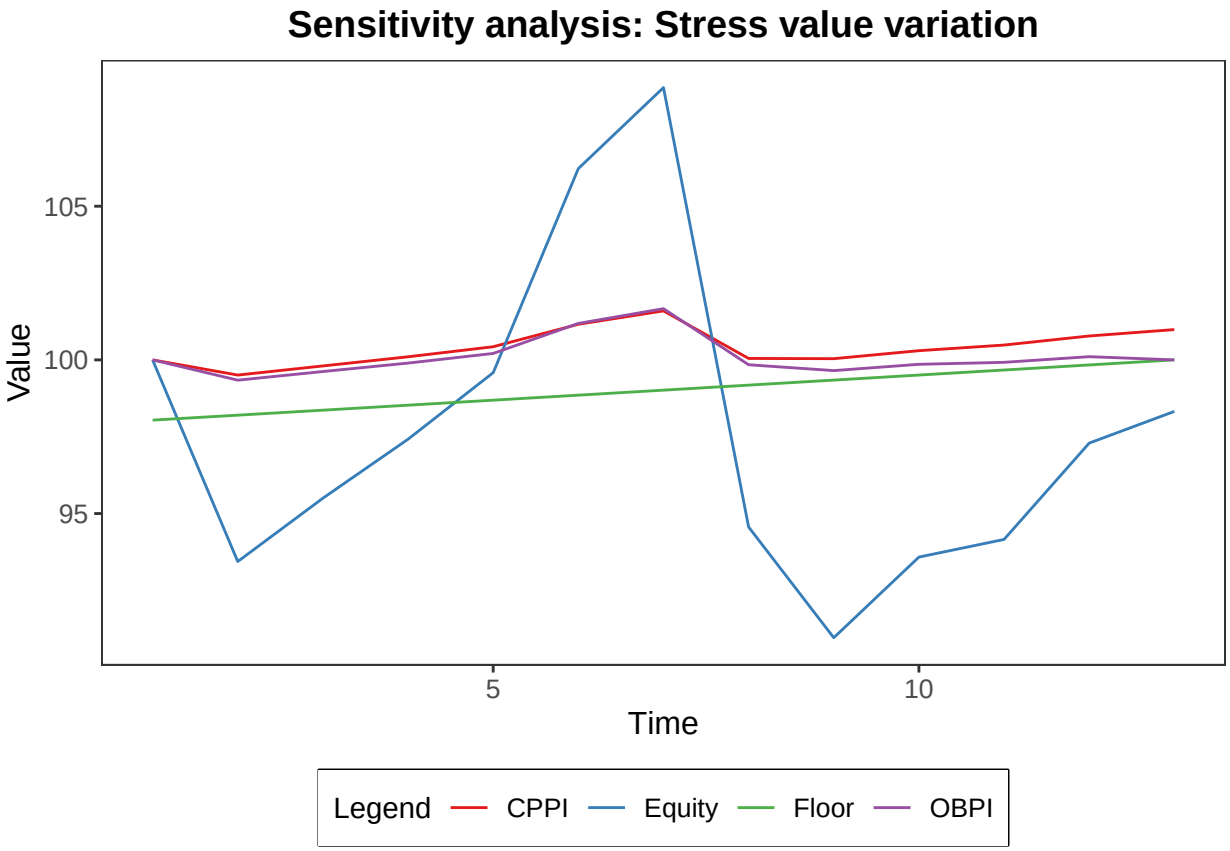

Figure 19: Sensitivity analysis using the stress value.

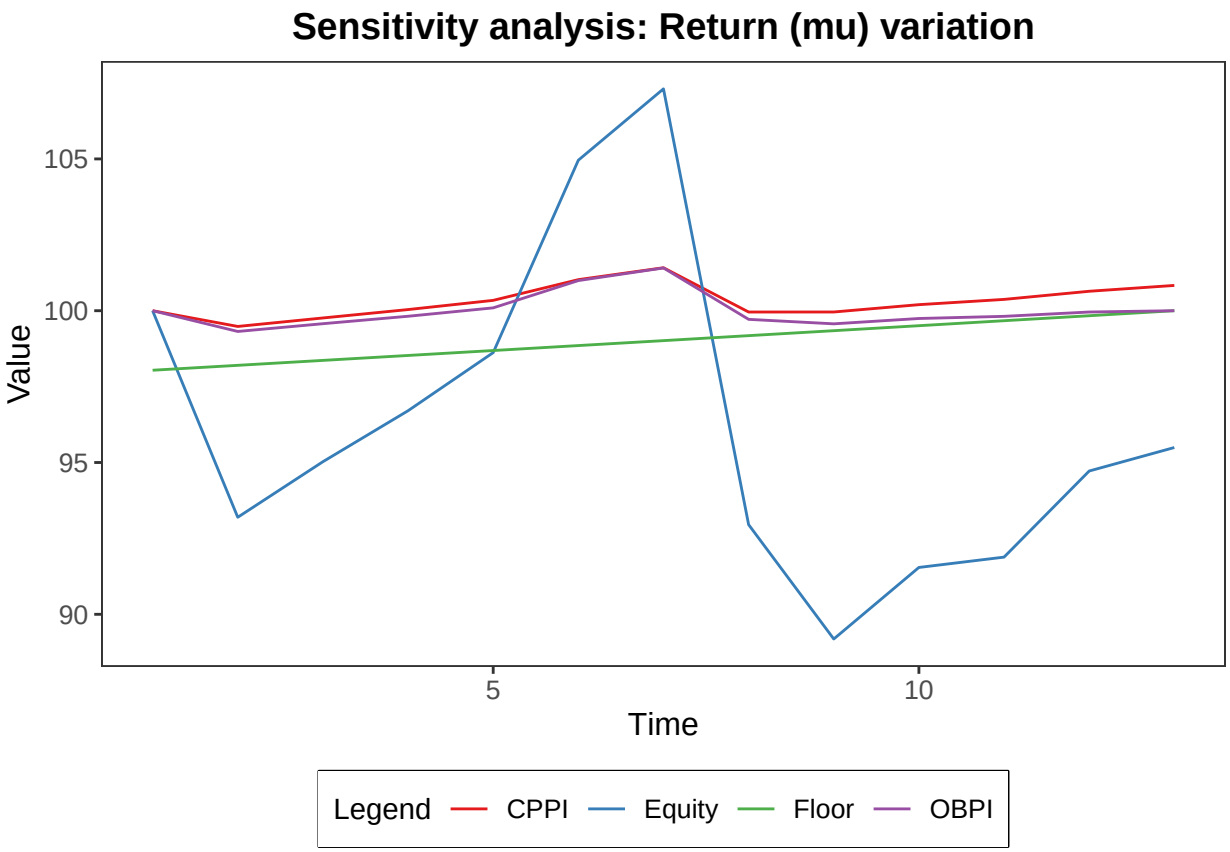

Figure 20: Sensitivity analysis using return (mu).

- **Decrease:** A decrease in the strike price will increase the cost of the call option used in OBPI and CPPI.

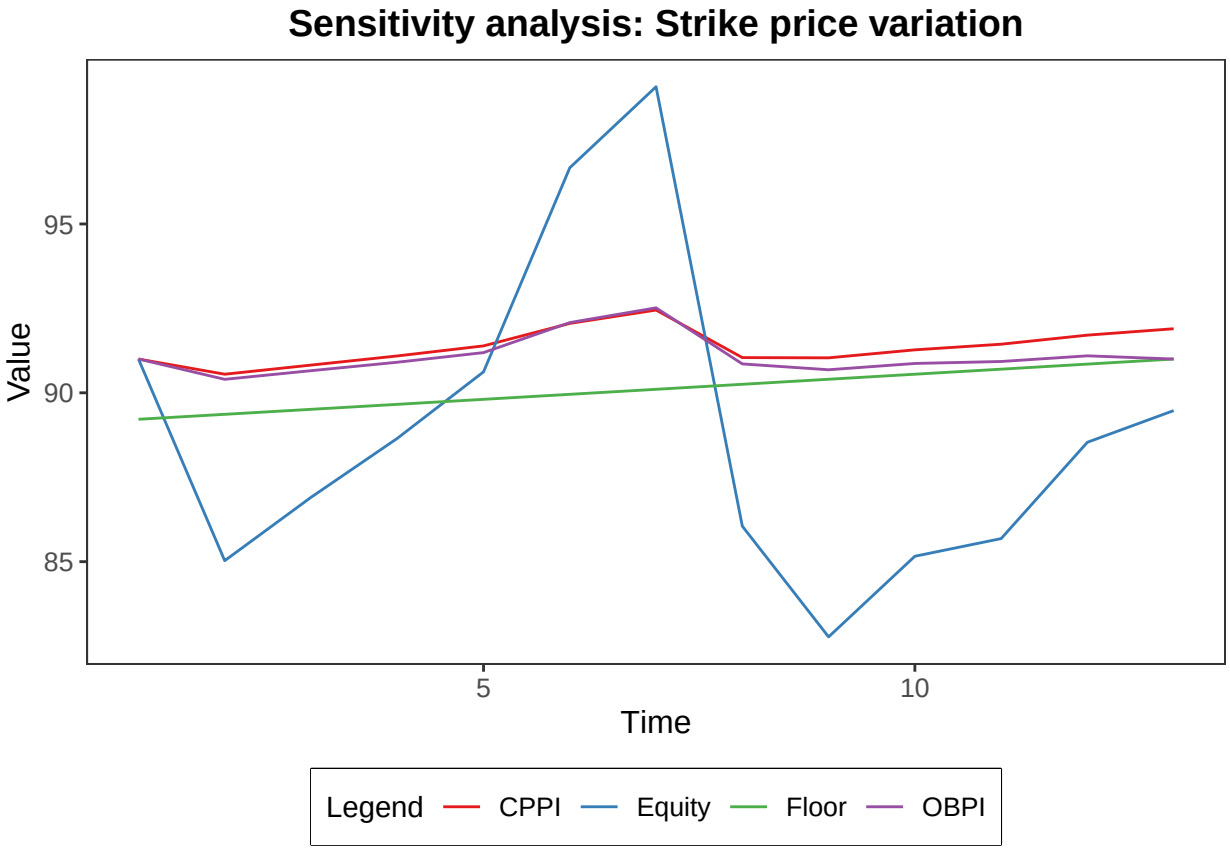

Figure 21: Sensitivity analysis using strike price.

In general, OBPI and CPPI are designed to protect portfolios from downside risk. However, they both have different trade-offs between risk and reward. The use of leverage in the CPPI portfolio allows to capture more value than with the OBPI strategy during periods of stable growth of the risky asset. Nevertheless, it is more sensitive to asset declines and a sudden drop in the market can cause the CPPI portfolio value to fall much more than the OBPI portfolio value.

The best strategy to use will depend on the individual investor's risk tolerance and investment goals. Investors who are more risk-averse may prefer CPPI, while investors who are more risk-tolerant may prefer OBPI.

3.4 Monte Carlo simulation

3.4.1 Overview and utility:

Monte Carlo simulations are a class of computational algorithms that leverage the power of random sampling to approximate numerical solutions (Elton et al., 2006). This approach is invaluable in quantifying the influence of risk and indeterminacy within predictive and forecasting models, particularly in finance where market conditions are inherently volatile. By generating a spectrum of plausible outcomes and their associated likelihoods for uncertain parameters, Monte Carlo methods enable a robust assessment of various risk scenarios. These simulations serve as a strategic tool, aiding decision-makers in navigating the complexities of uncertainty by evaluating the ramifications of diverse contingencies (Elton et al., 2006).

3.4.2 Components of Monte Carlo simulation in finance

1. **Randomness**: Incorporates elements of randomness reflecting the uncertainty and volatility in financial markets.
2. **Statistical Properties**: Uses statistical properties (mean, variance) of historical data to simulate future price movements.
3. **Time Series Modeling**: Models the evolution of financial instruments over time using time series analysis.

3.4.3 Application and results in this exercise

1. **Equity Price Simulation**: Simulated using a stochastic process (Geometric Brownian Motion) to reflect the random nature of stock prices. Incorporates drift (expected return) and diffusion (volatility) derived from historical data.
2. **Investment Strategies**: For the OBPI strategy, it models an options-based strategy that provides downside protection. For the CPPI strategy it models a dynamic allocation strategy that adjusts exposure based on a predetermined cushion.
3. **Simulation Execution**: It runs the simulation over a large number of trials to capture a broad range of possible outcomes. Each trial represents a potential future state of the market or asset price.
4. **Visualization**: We plot histograms showing the distribution of final values from the simulation.
5. **Summary Statistics**: The analysis provides key measures like mean, median, standard deviation, and percentiles to summarize the simulation results.
6. **Comparative Analysis**: We compare the performance and risk of different strategies (Equity, OBPI, CPPI). It helps in understanding the trade-offs between return and risk.

3.4.4 Histogram simulation plot

The three histograms below represent the distribution of final values from a Monte Carlo simulation for an equity simulation and two investment strategies, CPPI (Constant Proportion Portfolio Insurance) and OBPI (Option-Based Portfolio Insurance). Here is an analysis of each.

This histogram displays the frequency distribution of the final values of a simulated equity investment over a given period. The distribution is bell-shaped and symmetric, suggesting a normal distribution of final equity prices. The central peak is around the 100 value mark, which may indicate that the average final price is close to the initial price (assuming the initial price was 100). There's a noticeable spread on both sides of the peak, which shows variability in the equity simulation outcomes.

The OBPI histogram also presents a right-skewed distribution, similar to the CPPI strategy, but the peak is closer to the 100 value mark and the distribution is more concentrated. This indicates that the OBPI strategy outcomes are more tightly clustered around the peak, with fewer instances of both higher and lower values compared to the CPPI strategy. The strategy seems to offer a balance between protecting against downside risk and capturing some upside potential.

The CPPI histogram shows a right-skewed distribution, indicating that most of the final values are concentrated on the left side but there is a long tail towards higher values. The peak of the distribution is just above the 100 value mark, and the spread is narrower than the equity histogram, which may suggest less variability in the CPPI strategy outcomes compared to the equity simulation. The right skewness might also reflect a floor mechanism characteristic of CPPI strategies that limit downside risk but allow for participation in upside potential.

We can analyze each strategy and link it to academic literature. Below are the comments for each strategy.

1. **Equity:**

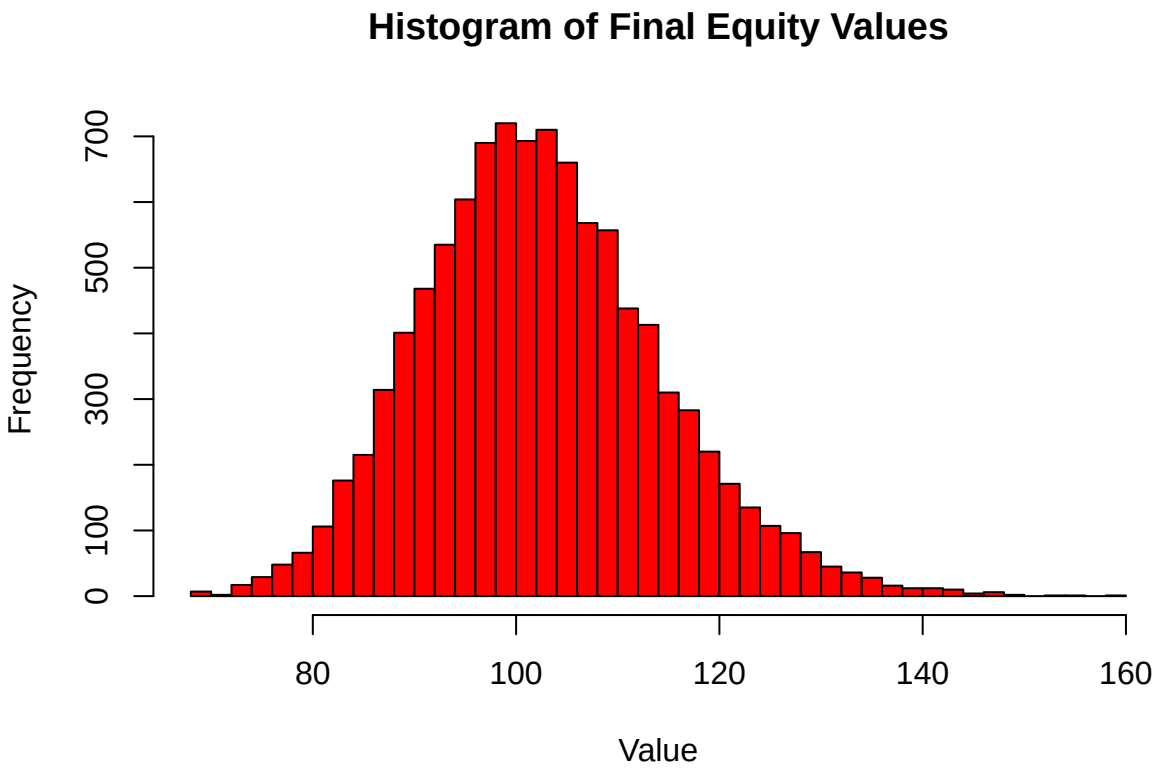

Figure 22: Histogram plot of final equity values.

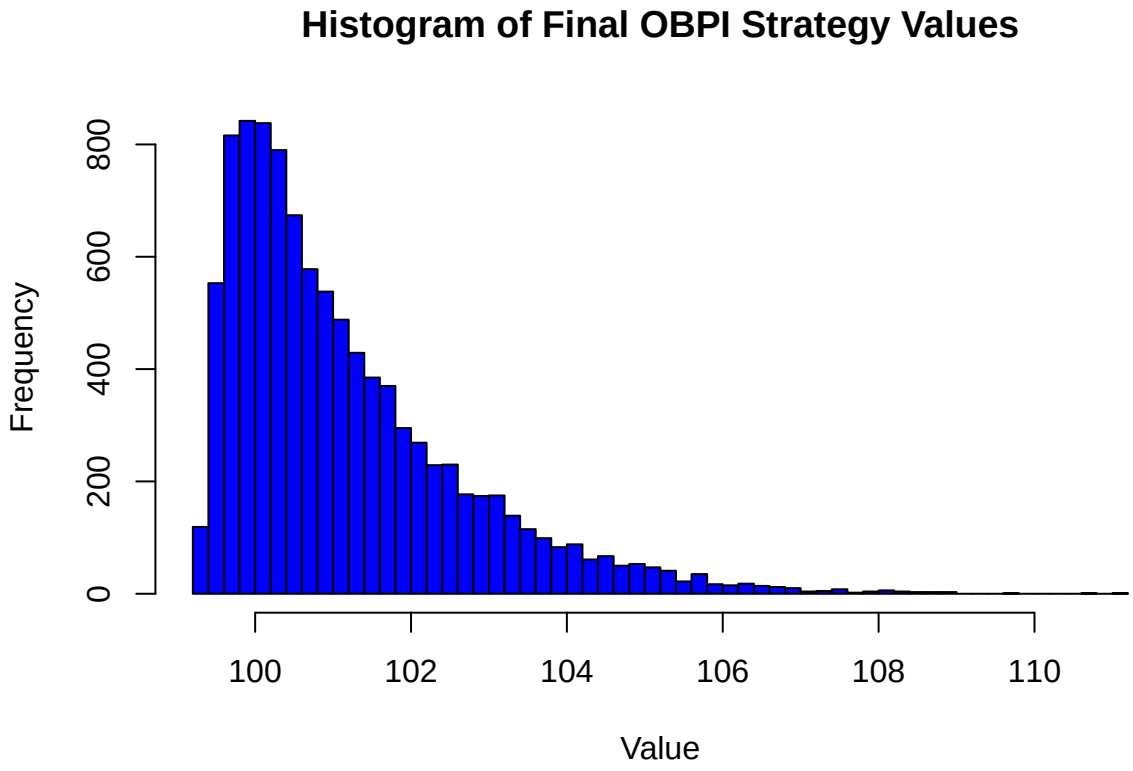

Figure 23: Histogram plot of final OBPI strategy values.

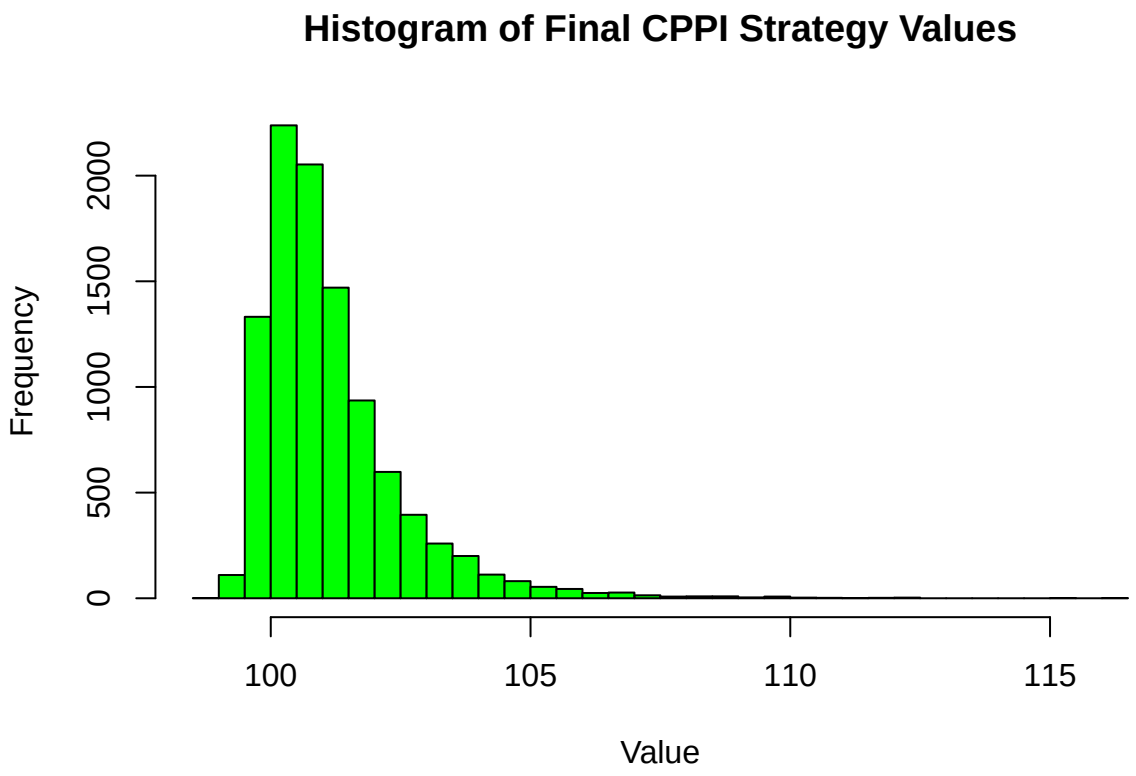

Figure 24: Histogram plot of final CPPI strategy values.

- **Return:** The return for the Equity strategy is approximately 19.59%, which is the highest among the three strategies. This is consistent with historical data that equities often yield higher returns compared to fixed-income securities or derivative-based strategies (Bodie, 2014).
- **Risk:** The risk, measured by standard deviation, is also the highest for the Equity strategy at approximately 57.24%. This aligns with the fundamental risk-return tradeoff in finance, which states that higher returns are usually associated with higher risk (Sharpe, 1964).
- **Sharpe Ratio:** The Sharpe Ratio of 0.34 for Equity is the lowest among the strategies, indicating that, per unit of risk taken, the returns are lower compared to OBPI and CPPI. The Sharpe Ratio is an indicator of risk-adjusted returns, with higher values generally being more desirable (Sharpe, 1994).

2. **OBPI:**
   - **Return:** The OBPI strategy shows a significantly lower return at approximately 3.84%. This strategy involves a mix of a risk-free asset and a call option on the risky asset (equity). The lower return can be partly due to the cost of the option and the protective nature of the strategy, which limits downside risk (Black, 1992).
   - **Risk:** OBPI has a slightly higher risk than CPPI at approximately 7.30%. This could be due to the fact that options can increase in volatility, which in turn can affect the portfolio's value.
   - **Sharpe Ratio:** At 0.526, OBPI has a better Sharpe Ratio than Equity but is not as high as CPPI. This suggests that OBPI offers a better risk-adjusted return than simply holding the equity.
3. **CPPI:**
   - **Return:** CPPI yields a return of approximately 4.10%, higher than OBPI. CPPI dynamically adjusts exposure to the risky asset (equity) based on the difference between the asset's value and a predetermined floor, known as the "cushion" (Black, 1987).

- **Risk:** CPPI shows the lowest risk at 6.66%, which could be attributed to its mechanism of reducing exposure to equities when they become riskier, thus maintaining a buffer against downturns.
- **Sharpe Ratio:** CPPI has the highest Sharpe Ratio of 0.617, indicating that it has the best risk-adjusted performance among the three strategies. The higher Sharpe Ratio suggests that CPPI efficiently balances the risk while still capturing reasonable returns.

In conclusion, while the Equity strategy provides the highest raw returns, it does so at a higher level of risk and offers lower risk-adjusted returns. Both OBPI and CPPI provide lower returns but do so with substantially less risk, and they both have higher Sharpe Ratios, indicating more efficient performance. These results highlight the trade-offs between risk and return and underscore the importance of portfolio insurance techniques in managing risk, consistent with the protective and dynamic strategies discussed in the academic literature on portfolio management.

```
##   Strategy     Return       Risk Sharpe_Ratio
## 1   Equity 0.19588221 0.57239445    0.3422154
## 2     OBPI 0.03843365 0.07300877    0.5264251
## 3     CPPI 0.04101715 0.06648224    0.6169639
```

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