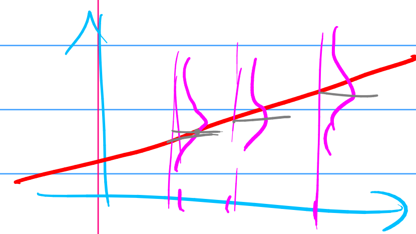
$$p(x_1, \dots, x_n) = \dots\dots\dots$$

Lin. regr.: $\bar{w}, y_1, y_2, \dots, y_N$

$$p(\bar{w}, y_1, \dots, y_N | X) = p(\bar{w}) \cdot \prod_{n=1}^{N} p(y_n | \bar{w}, \bar{x}_n) =$$



$$= \mathcal{N}(\bar{w} | \dots) \cdot \prod_{n=1}^{N} \mathcal{N}(y_n | \bar{w}^T \bar{x}_n, \sigma^2)$$

Markov chain

$$X_1 \to X_2 \to \dots \to X_{T-1} \to X_T$$

$$p(x_1, \dots, x_T) = p(x_1) p(x_2 | x_1) p(x_3 | x_2) \dots p(x_T | x_{T-1})$$

$$p(x_1, \dots, x_T) = p(x_1) p(x_2 | x_1) p(x_3 | \cancel{x_1}, x_2) \dots p(x_T | x_1, \cancel{x_2, \dots}, x_{T-1})$$

HMM

$$\begin{tikzcd} X_1 \arrow[r] \arrow[d] & X_2 \arrow[r] \arrow[d] & \dots \arrow[r] & X_T \arrow[d] \\ y_1 & y_2 & & y_T \end{tikzcd}$$

$$p(x_1, \dots, x_T, y_1, \dots, y_T) =$$
$$= p(x_1) p(y_1 | x_1) p(x_2 | x_1) p(y_2 | x_2) p(x_3 | x_2) p(y_3 | x_3) \dots p(y_T | x_T)$$

$$\boxed{\begin{aligned} p(\bar{x}, \bar{y}) = {} & p(x_1) p(x_2 | x_1) \dots p(x_T | x_1 \dots x_{T-1}) \\ & p(y_1 | x_1 \dots x_{T-1}) p(y_2 | y_1, x_1, \dots x_{T-1}) \\ & \dots p(y_T | x_1 \dots x_T, y_1, \dots, y_{T-1}) \end{aligned}}$$

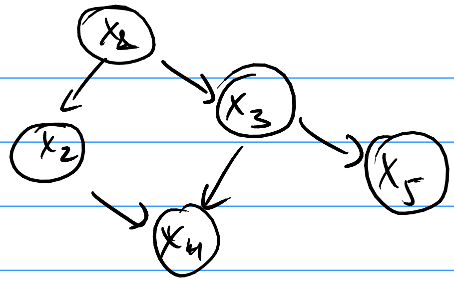
Clustering $\overbrace{\bar{x}_1, \dots, \bar{x}_N}^{D}, \overbrace{\bar{z}_1, \dots, \bar{z}_N}^{\text{latent}}, \overbrace{\bar{\mu}_1, \dots, \bar{\mu}_K, \Sigma_1, \dots, \Sigma_K, \bar{\pi}}^{\text{params}}$

$$p(\bar{x}_1, \dots, \bar{x}_N, \bar{z}_1, \dots, \bar{z}_N, \bar{\mu}_1 \dots \bar{\mu}_K, \Sigma_1, \dots, \Sigma_K, \bar{\pi}) = p(\bar{\mu}_1) \dots p(\bar{\mu}_K) p(\Sigma_1) \dots p(\Sigma_K) p(\bar{\pi}) \times$$

$$\times \prod_{n=1}^{N} p(\bar{z}_n | \bar{\pi}) \cdot p(\bar{x}_n | \bar{z}_n, \mu_{..}, \Sigma_{..})$$

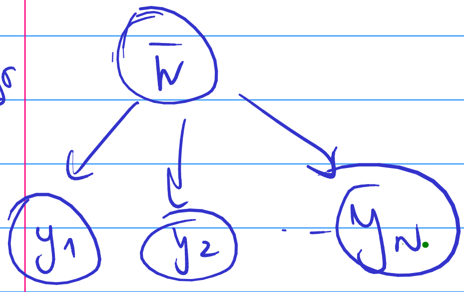
Directed graphical model

$$p(x_1,\dots x_n) = \prod_{i=1}^{n} p(x_i \mid par(x_i))$$

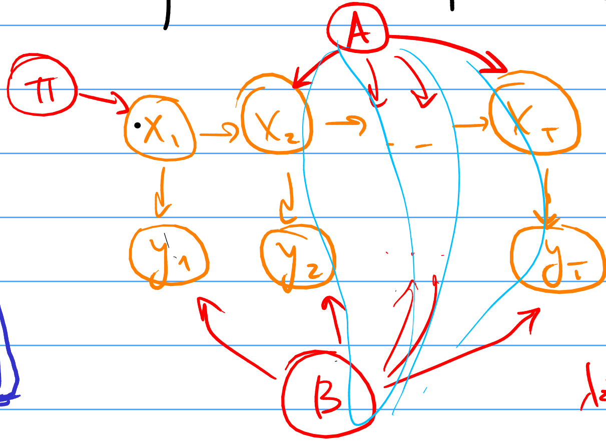


$$p(x_1 \dots x_5) =$$
$$= p(x_1)\, p(x_2 \mid x_1)\, p(x_3 \mid x_1)$$
$$p(x_4 \mid x_2, x_3)\;\; p(x_5 \mid x_3)$$

lin. regr

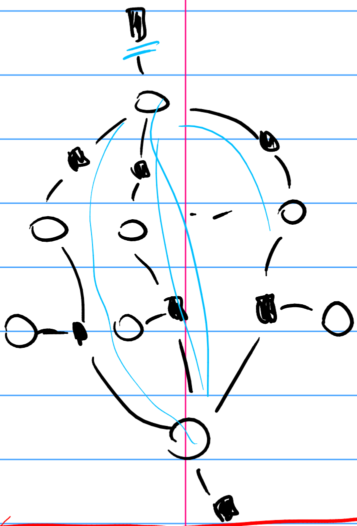


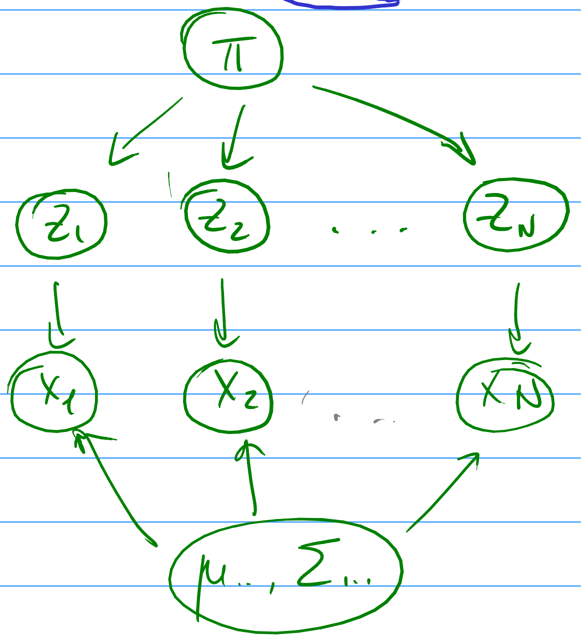
HMM

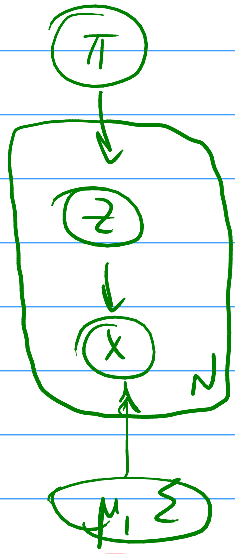


$\lambda = (\pi, A, B)$

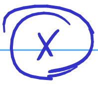
clustering



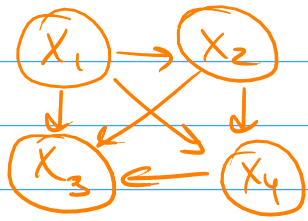




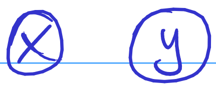
n=1

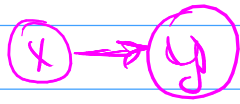


$p(x) = p(x)$



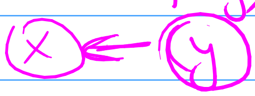
n=2





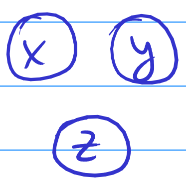
$p(x,y) = p(x)\,p(y)$

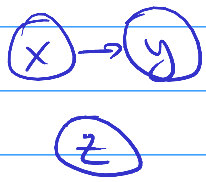
$p(x,y) = p(x)\,p(y \mid x)$
$= p(y)\,p(x \mid y)$

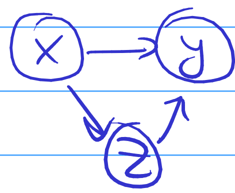


$p(x_1)\,p(x_2 \mid x_1)$
$p(x_4 \mid x_1, x_2)$
$p(x_3 \mid x_1, x_2, x_4)$

n=3



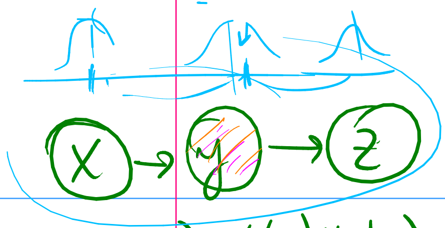




$p(x,y,z) = p(x)\,p(y)\,p(z)$

$p(x,y,z) = p(z)\,p(x)\,p(y \mid x)$

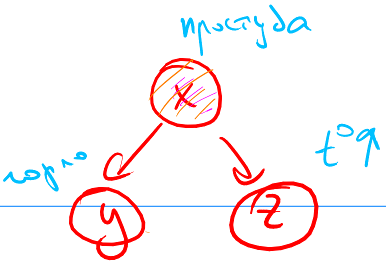
$p(x)\,p(z \mid x)$
$p(y \mid x, z)$



$$p(x,y,z)=p(x)p(y|x)p(z|y)$$

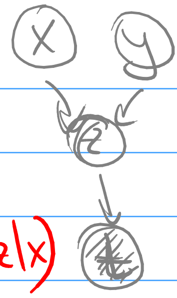
$$p(x,z|y) \overset{?}{=} p(x|y)p(z|y)$$

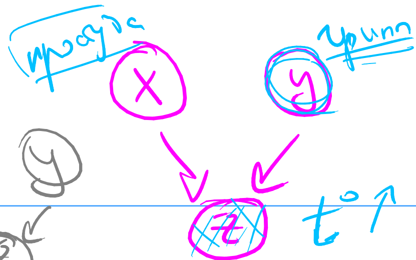
$$\frac{p(x,y,z)}{p(y)} = \frac{p(x)p(y|x)p(z|y)}{p(y)}$$



$$p(x,y,z) = p(x)p(y|x)p(z|x)$$

$$p(y,z|x) \overset{?}{=} p(y|x)\cdot p(z|x)$$





$$p(x,y,z) = p(x)p(y)p(z|x,y)$$

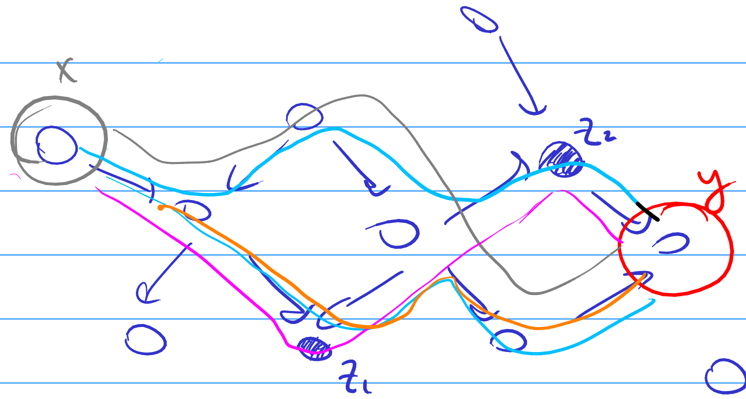
$$p(x,y) \overset{?}{=} p(x)p(y)$$

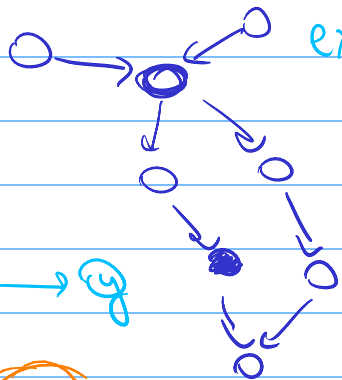
$$\int p(x,y,z)\,dz = p(x)p(y)\int p(z|x,y)\,dz$$

$$p(x,y|z) \neq p(x|z)\,p(y|z)$$

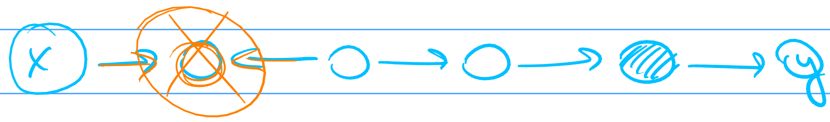
explaining away

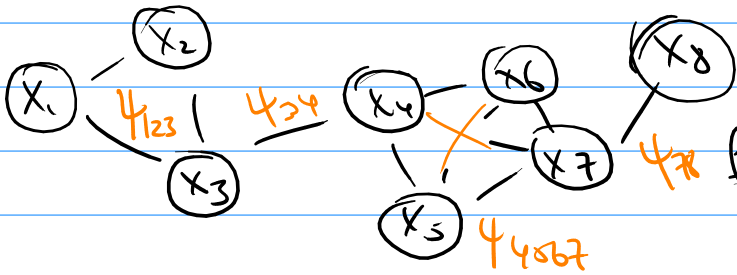
Thm.

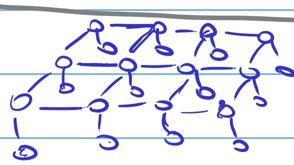




$$p(x,y|z) = ?$$





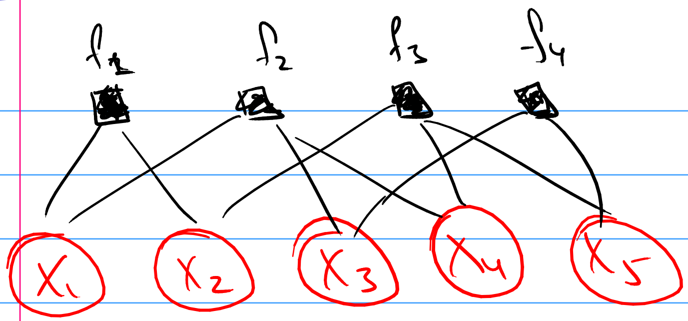


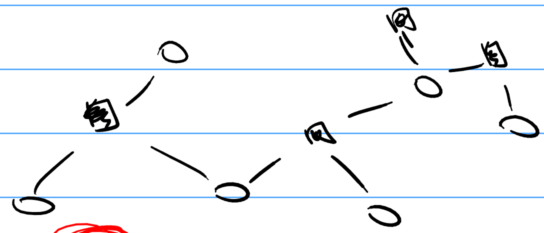
$$f(x_1,x_2,\dots,x_8) = \psi_{123}(x_1,x_2,x_3)\cdot \psi_{34}(x_3,x_4)\cdot \psi_{4567}(\dots)\cdot\psi_{78}(\dots)$$

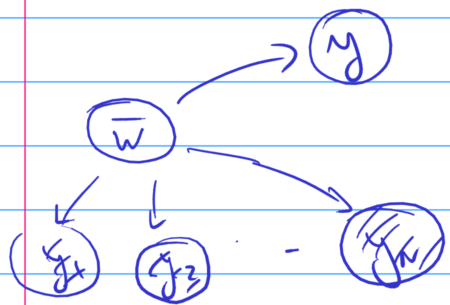
$$p(x_1 \dots x_8) = \frac{1}{Z}\psi_{123}\dots\psi_{78}$$

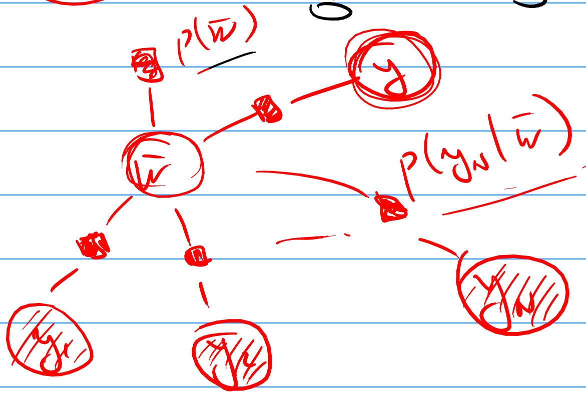
Фактор-графы

$$F(x_1 .. x_5) = f_1(x_1, x_2) f_2(x_1, x_3, x_4) f_3(x_2, x_4, x_5) f_4(x_3, x_5)$$



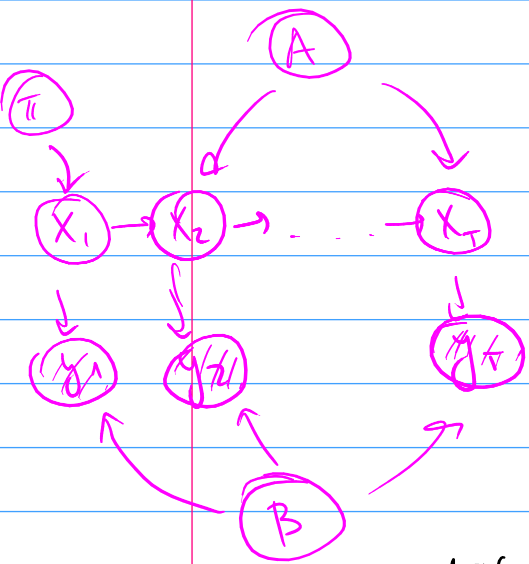






$$p(\bar{w}|D) \propto p(\bar{w}) \prod p(y_n|\bar{w})$$

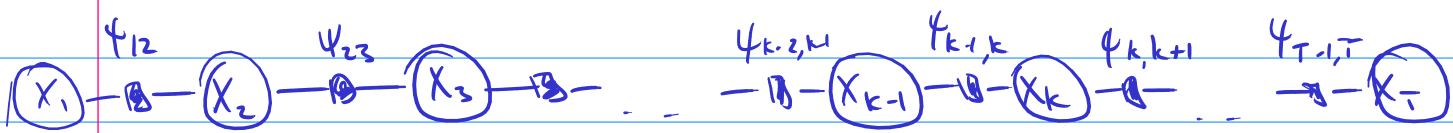
$$\int p(y, \bar{w}|D)\, d\bar{w}$$



$$\int \underline{p(\pi, A, B, X|y)}\, dx_1 dx_2 .. dx_T$$

$$F(x_1, .. x_N) = \underline{\prod f_i(X_i)}$$

$$f(x_k) = \sum_{x_1 .. x_{k-1}, x_{k+1} .. x_N} F(x_1 .. x_N)$$



$$F(x_1 .. x_T) = \psi_{12}(x_1, x_2)\, \psi_{23}(x_2, x_3) \, ..\, \psi_{T-1,T}(x_{T-1}, x_T)$$

$$f(x_k) = \underbrace{\sum_{x_1 .. x_{k-1}, x_{k+1} .. x_T}}_{2^{T-1}} \psi_{12} \cdot \psi_{23} \cdot \,..\, \cdot \psi_{T-1,T} =$$

$$= \sum_{\underbrace{x_1 \dots x_{k-1}}} \sum_{\underbrace{x_{k+1} \dots x_T}} \left(\underbrace{\psi_{12} \cdots \psi_{k-2,k-1} \cdot \psi_{k-1,k}} \cdot \underbrace{\psi_{k,k+1} \cdots \psi_{T-1,T}} \right) =$$

$$= \left(\sum_{x_1 \dots x_{k-1}} \psi_{12} \cdots \psi_{k-1,k} \right) \times \left(\sum_{x_{k+1} \dots x_T} \psi_{k,k+1} \cdots \psi_{T-1,T} \right)$$

$$\underline{\quad}(x_k) \qquad\qquad \underline{\quad}(x_k)$$

$$\sum_{x_1 - x_{k-1}} \psi_{12} \cdots \psi_{k-1,k} = \sum_{x_1} \sum_{x_2} \cdots \sum_{x_{k-1}} \left(\psi_{12} - \psi_{k-1,k} \right) =$$

$$2^{k-1}$$

$$= \sum_{x_2 \, x_{k-1}} \left(\psi_{23} \psi_{34} - \psi_{k-1,k} \cdot \underbrace{\left(\sum_{x_1} \psi_{12} \right)}_{(x_1, x_2)} \right) =$$

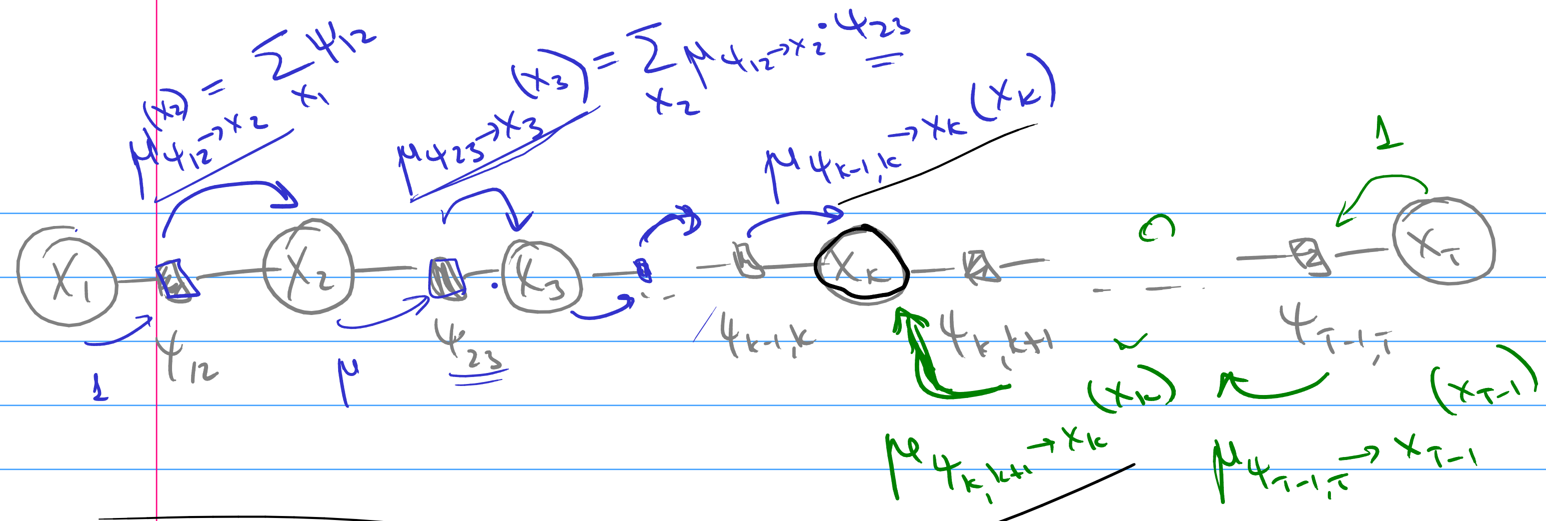
$$= \sum_{x_3 - x_{k-1}} \left(\psi_{34} - \psi_{k-1,k} \cdot \sum_{x_2} \left(\psi_{23} \cdot \left(\sum_{x_1} \psi_{12} \right) \right) \right) =$$

$$= \sum_{x_{k-1}} \psi_{k-1,k} \left[\sum_{x_{k-2}} \psi_{k-2,k-1} \cdots \sum_{x_3} \psi_{34} \left(\sum_{x_2} \psi_{23} \left[\sum_{x_1} \psi_{12} \right] \right) \cdots \right]$$

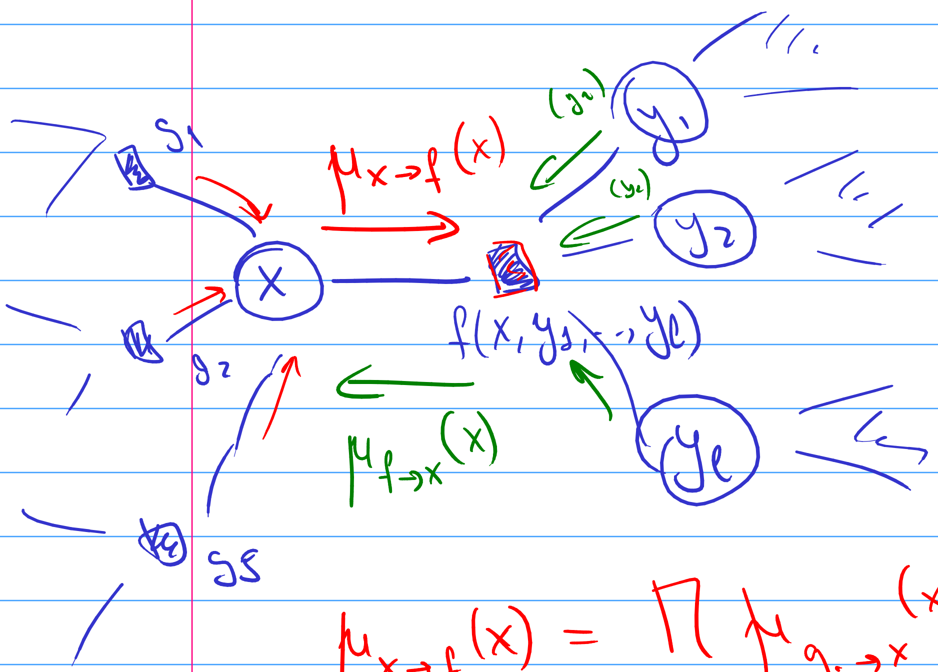
ψ_k $\quad 2 \quad 2 \quad 2$

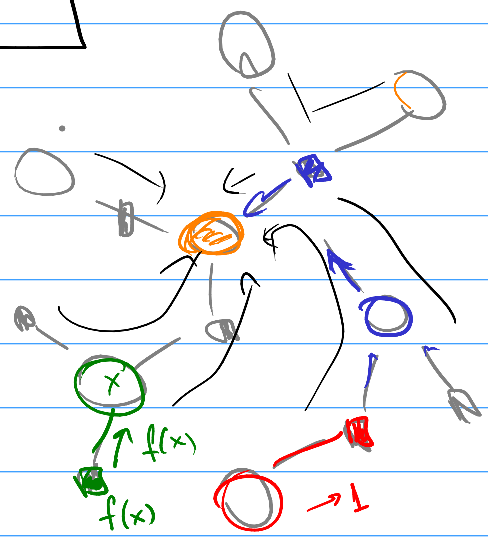
$$\sum_{x_{k+1} \dots x_T} \psi_{k,k+1} \psi_{k+1,k+2} - \psi_{T-1,T} =$$

$$= \sum_{x_{k+1}} \psi_{k,k+1} \left[\sum_{x_{k+2}} \psi_{k+1,k+2} \cdots \left[\sum_{x_{T-1}} \psi_{T-2,T-1} \left[\sum_{x_T} \psi_{T-1,T} \right] \right] \right]$$



$$\boxed{f(x_k) = \mu_{\psi_{k-1,k}\to x_k}(x_k) \cdot \mu_{\psi_{k,k+1}\to x_k}(x_k)}$$



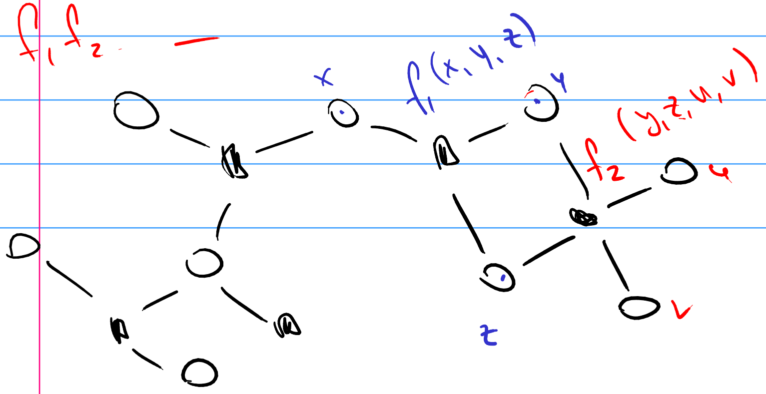


Message passing

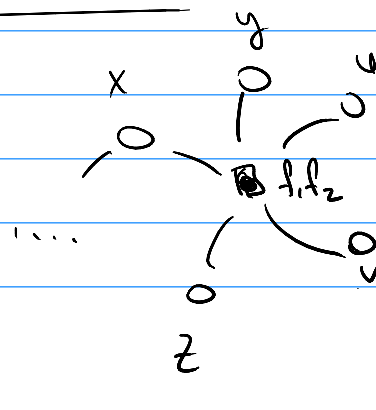
$$\mu_{x\to f}(x) = \prod_i \mu_{g_i\to x}(x)$$

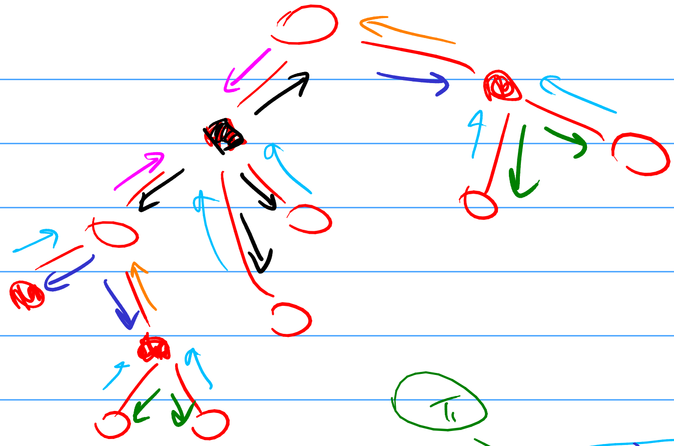
$$\mu_{f\to x}(x) = \sum_{y_1 \ldots y_l} f(x, y_1, \ldots, y_l) \cdot \prod_{j=1}^{l} \mu_{y_j\to f}(y_j)$$

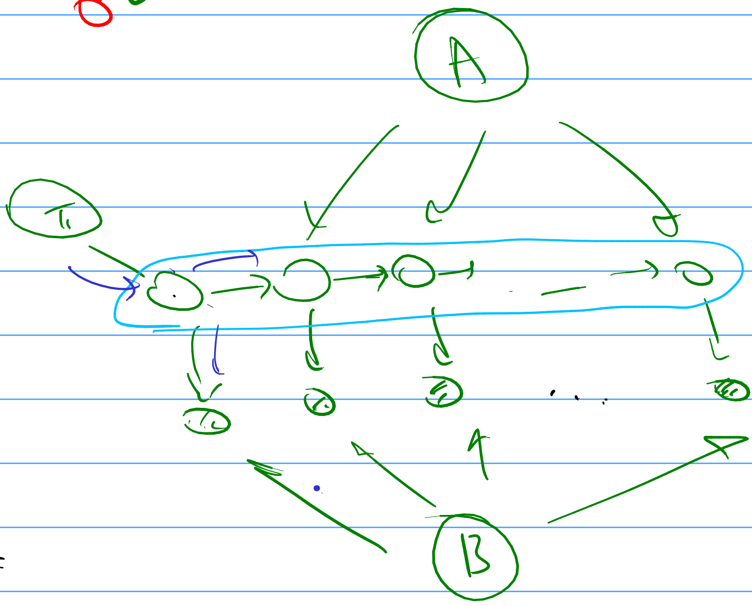
$F = \ldots f_1 f_2 \ldots$



....





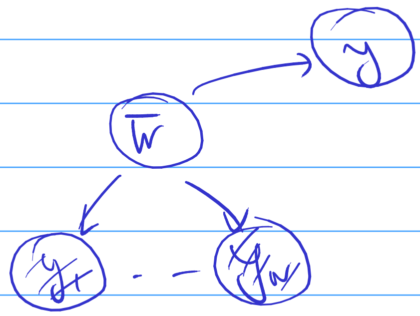


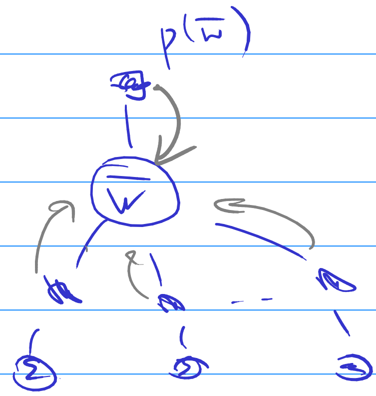
$$\underbrace{p(\pi, A, B \mid D)}\ \propto$$

$$\propto p(\pi, A, B, D) =$$

$$= \underbrace{\sum_{Q} p(\pi, A, B, D, Q)}$$

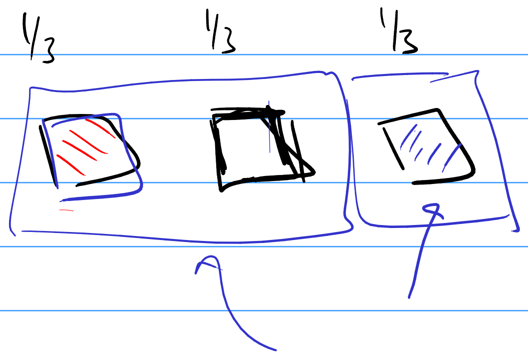
$$p(y \mid D) = \int p(\bar{w}, y \mid D)\, d\bar{w}$$





$$p(\bar{w} \mid D) =$$

$$= p(\bar{w}) \cdot \prod_{n} p(y_n \mid \bar{w})$$



GNN