Introduction to Mobile Robotics course, Lecture 5

Mapping

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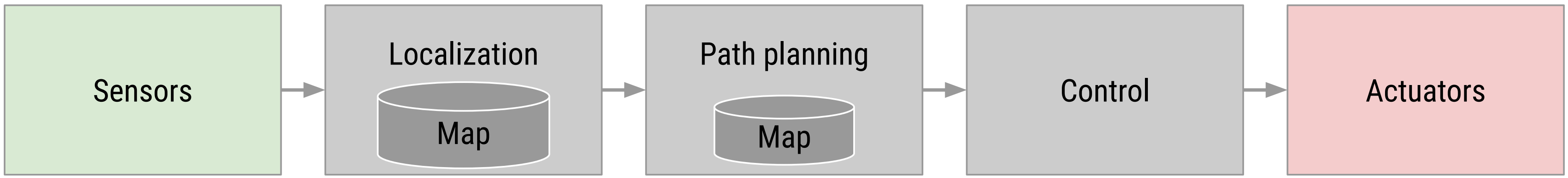
Materials by Oleg Shipitko

Outline

1. Mapping problem definition
2. Maps types
3. Topological maps
4. Features/landmarks maps
5. Occupancy grid maps

(SIMPLIFIED) CONTROL SCHEME OF MODERN MOBILE ROBOT



Mapping problem definition

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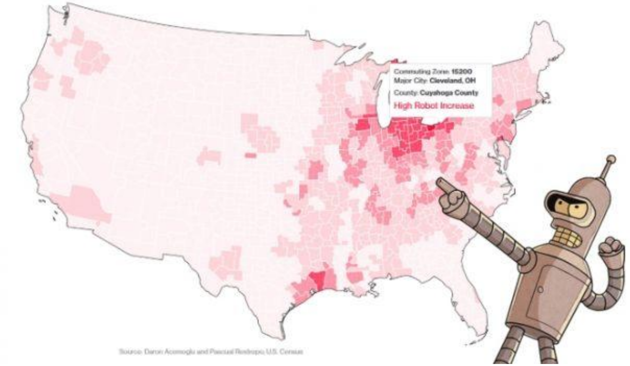
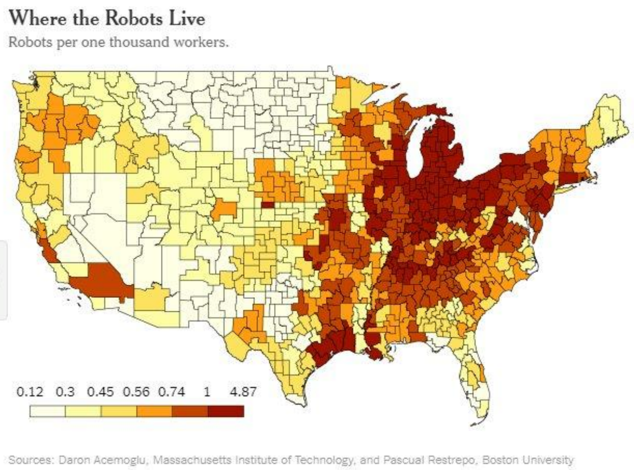
01

WHAT IS MAPPING?

Mapping (in robotics) refers to the process of modeling the environment and presenting it in a form that is convenient for further use in navigation (localization, planning and motion implementation).





WHY MAPPING IS TOUGH?

- ❑ Sensor measurement errors generate incomplete and/or inconsistent data
- ❑ Errors in determining ego-position (localization) also lead to similar contradictions
- ❑ How to integrate data over time?
- ❑ How to understand that we have already visited a place?

FORMAL MAPPING PROBLEM DEFINITION

Given:

$\mathbf{x}_{1:t}$ – all previous robot states (poses)

$\mathbf{z}_{1:t}$ – sensors measurements

Find:

map – map of the environment

Maps types

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WHICH TYPES OF MAPS ARE THERE?

Metric

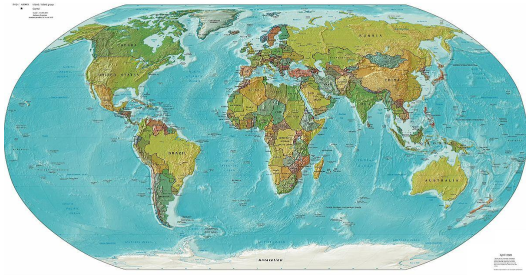
- ❑ Reflect the world in the form of 2D or 3D space
- ❑ Objects are set by their coordinates
- ❑ Distance between objects is measured in meters

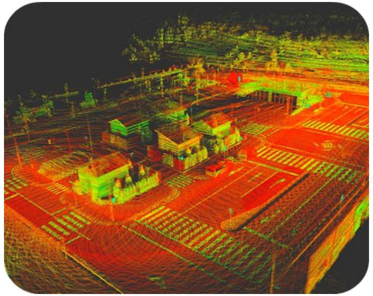
Topological

- ❑ Reflect the world in the form of places (locations) and connections (transitions) between them
- ❑ Distances between objects can be stored in links

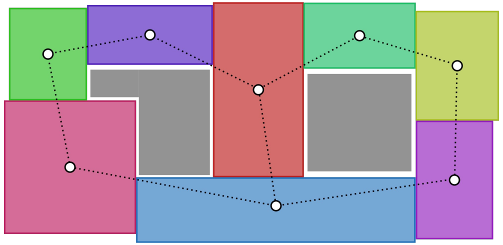
WHICH TYPES OF MAPS ARE THERE?

Metric





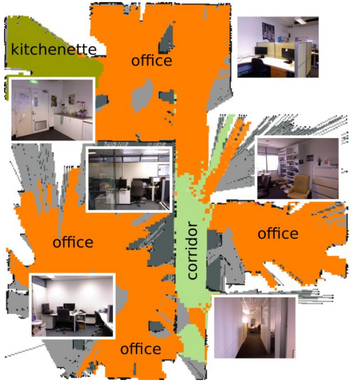
Topological



WHICH TYPES OF MAPS ARE THERE?

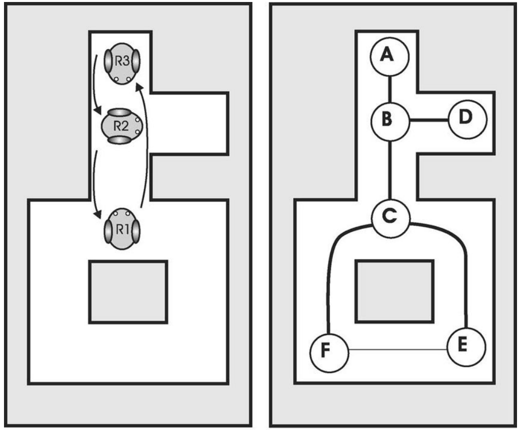
Metric

- Occupancy grid maps
- Feature-based maps, landmark-based
- (Sometimes) Semantic maps



Topological

- Graphs



(a) (b)

Source: Peasgood, M., Clark, C. M., & McPhee, J. (2008). A complete and scalable strategy for coordinating multiple robots within roadmaps. IEEE Transactions on Robotics, 24(2), 283-292.
Sünderhauf N. et al. Place categorization and semantic mapping on a mobile robot //2016 IEEE international conference on robotics and automation (ICRA). – IEEE, 2016. – C. 5729-5736.`

Topological maps

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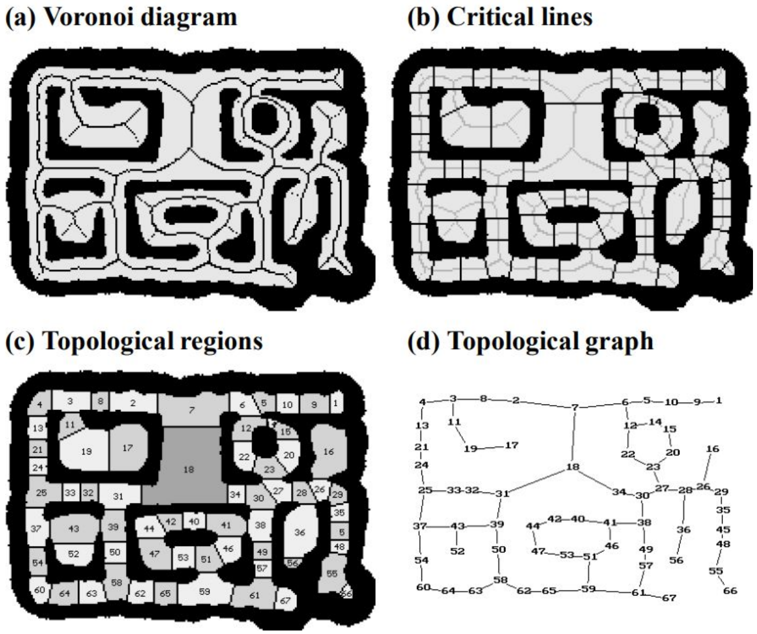
TOPOLOGICAL MAPS

- It is a set of locations (**nodes**) and transitions between them (**edges**).
- **Location** is usually the space in which the robot can be reliably positioned (localized) and / or the point of making a decision about the direction of further motion. For example, a room in the case of a building.
- The locations are connected with each other by **transitions** containing a certain law of robot control, according to which the transition can be carried out.



Thrun, Sebastian, and Arno Bücken. "Integrating grid-based and topological maps for mobile robot navigation." *Proceedings of the National Conference on Artificial Intelligence*. 1996.

MAIN DRAWBACK

- Topological representation does not exist (or is difficult to achieve) for large open spaces/

Features/landmarks maps

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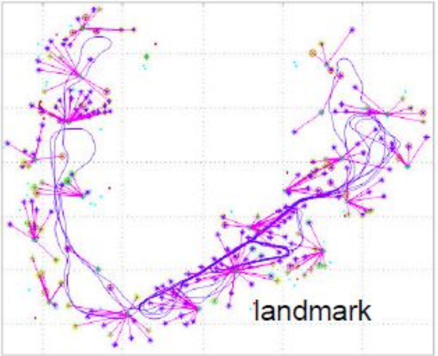
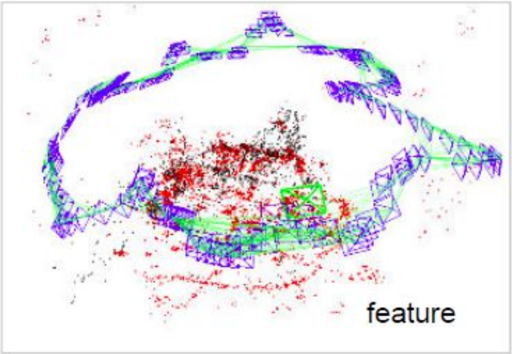
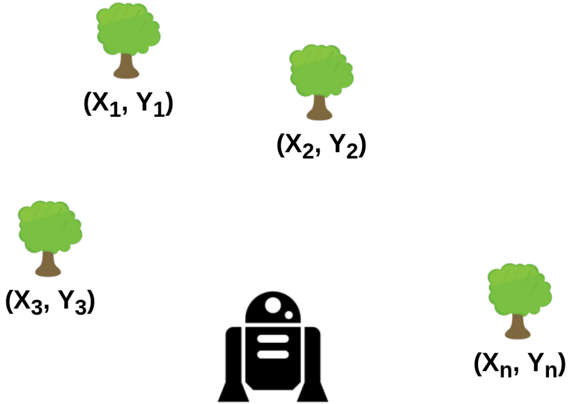
04

FEATURE / LANDMARKS MAPS

- Store features specified by their coordinates in space
- Anything can be used as feature / landmark: trees, road signs, doors, image keypoints …
- Very compact space representation



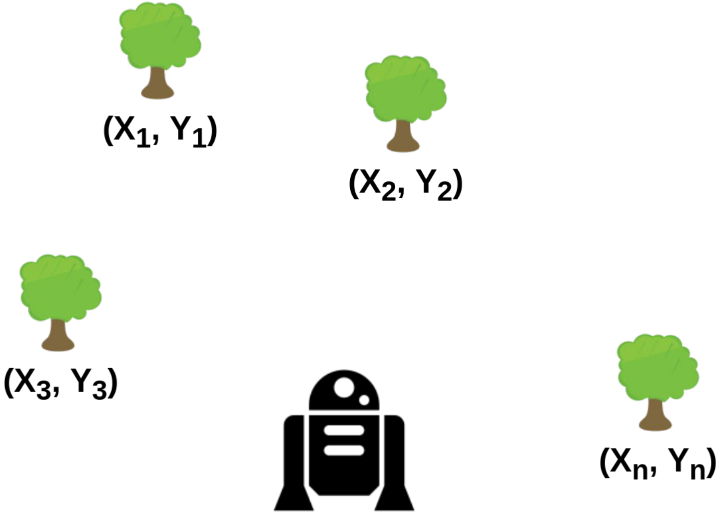




FEATURES VS. LANDMARKS

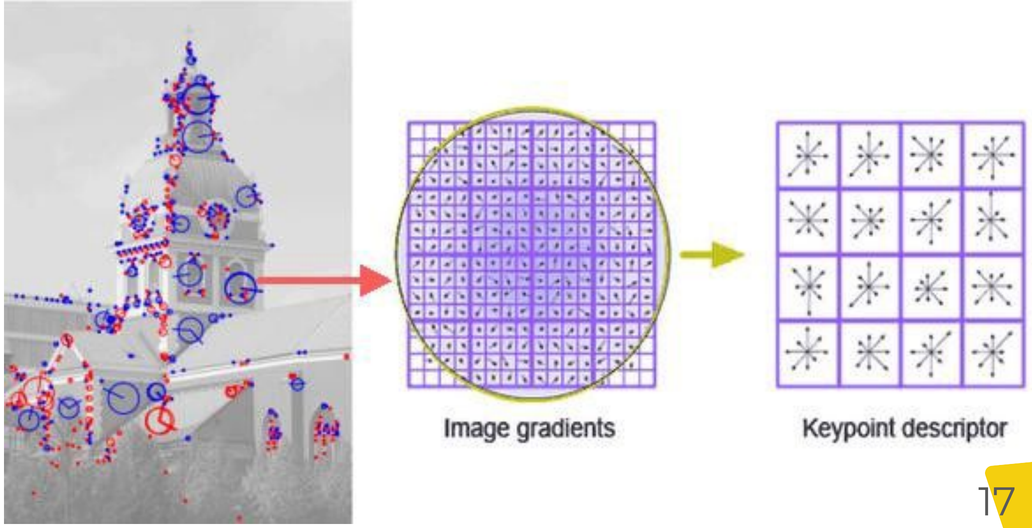
Landmarks

- Natural or man-made objects.



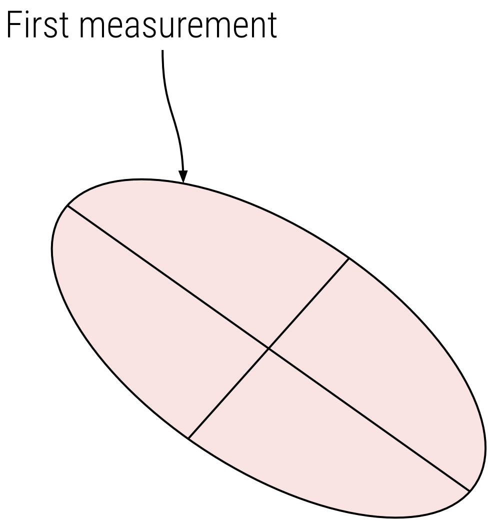
Features

- Artificially built-up structures. Usually more abstract than landmarks.



FEATURE / LANDMARKS MAPS GENERATION

- Most often, the Kalman filter and its modifications are used to build feature maps
- Each feature is encoded with its own spatial coordinates
- The estimate of the landmark position is iteratively refined with each new measurement (detection)



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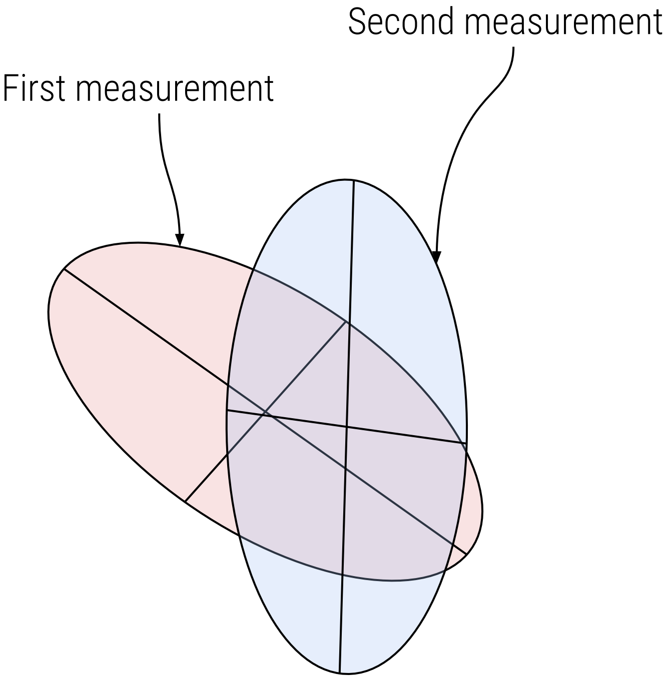

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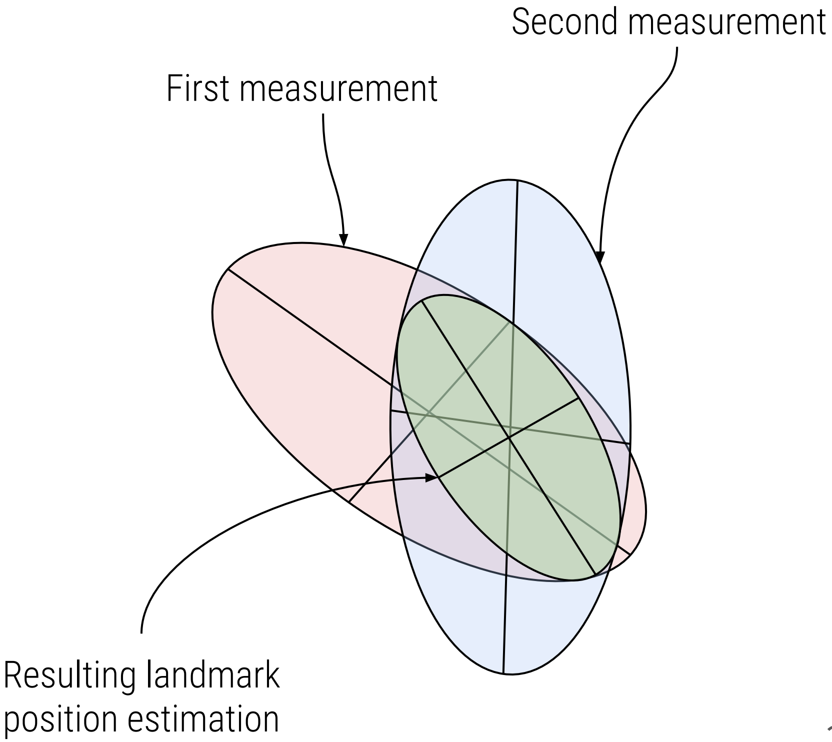

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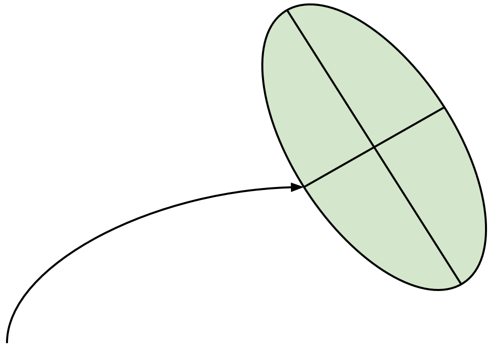

Resulting landmark position estimation

Occupancy grid maps

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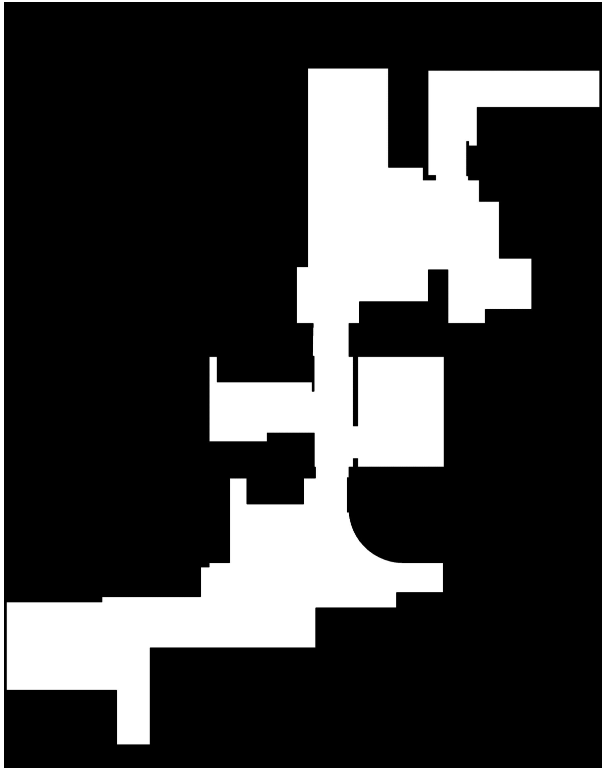
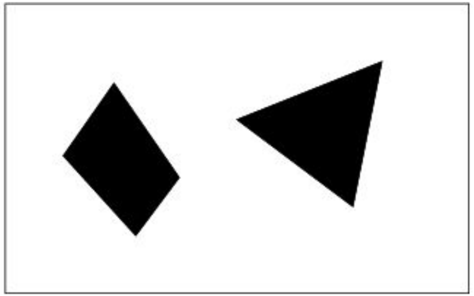
OCCUPANCY GRID MAPS

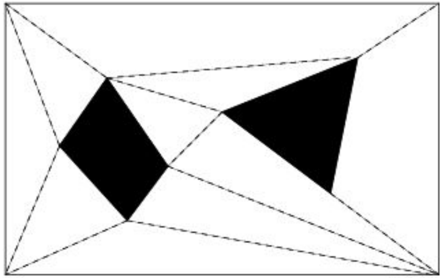
- The most popular maps format
- Space is discretized into cells
  - Usually regular grid is used
- The probability that the cell is free (passable) or occupied (impassable) is estimated



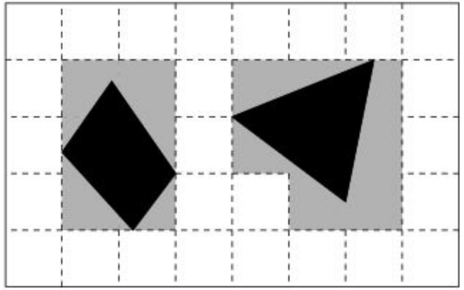
SPACE DISCRETIZATION APPROACHES



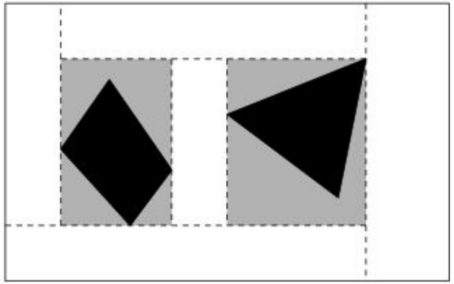
Metric map of the environment



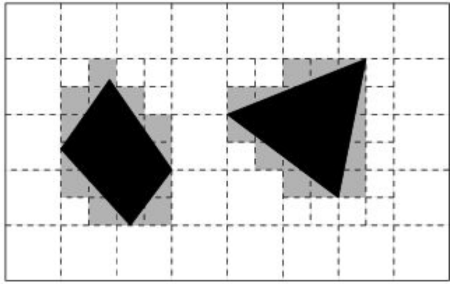
Exact cell decomposition



Regular cell decomposition



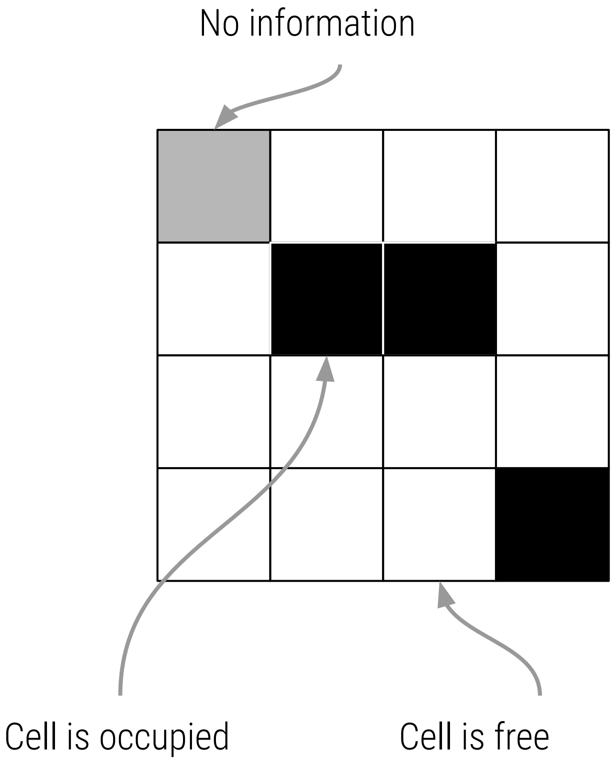
Rectangular cell decomposition



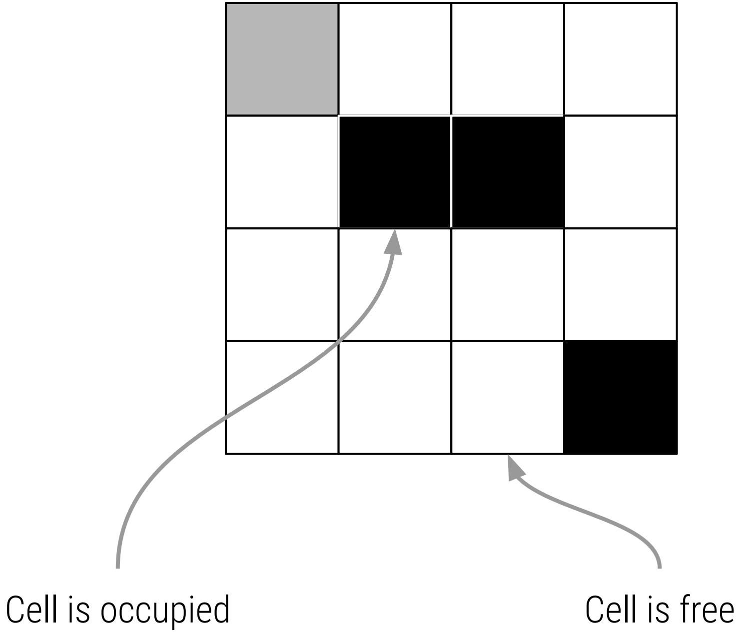
Quadtree decomposition

OCCUPANCY GRID MAPS

- Each cell is a binary random variable
  - $p(m_{x,y}) = 1$ – cell is free
  - $p(m_{x,y}) = 0$ – cell is occupied
  - $p(m_{x,y}) = 0.5$ – we know nothing on the cell state

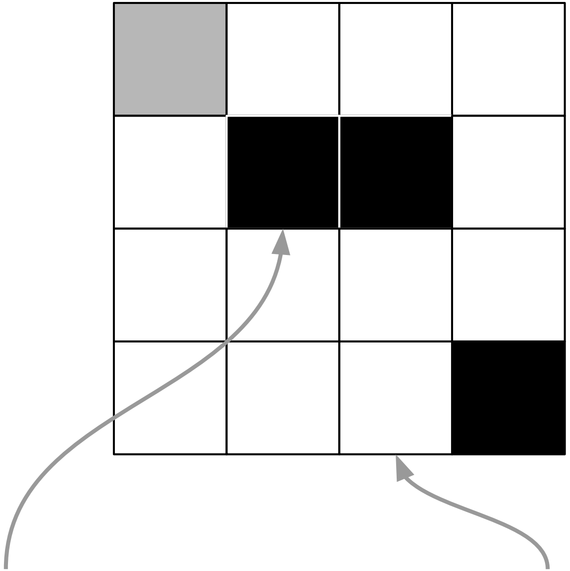


ASSUMPTIONS



ASSUMPTIONS

1. The area described by the cell is entirely occupied or free

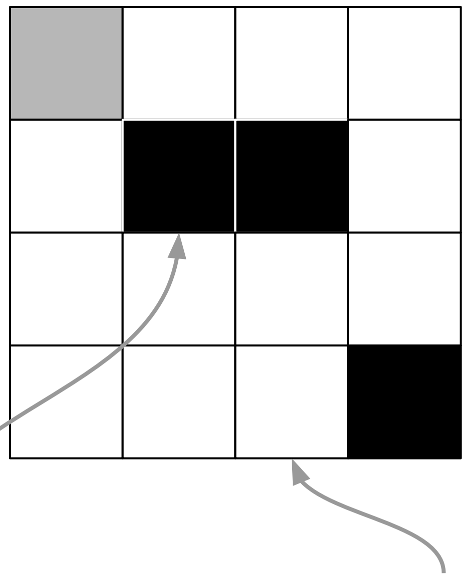


The whole cell is occupied

The whole cell is free

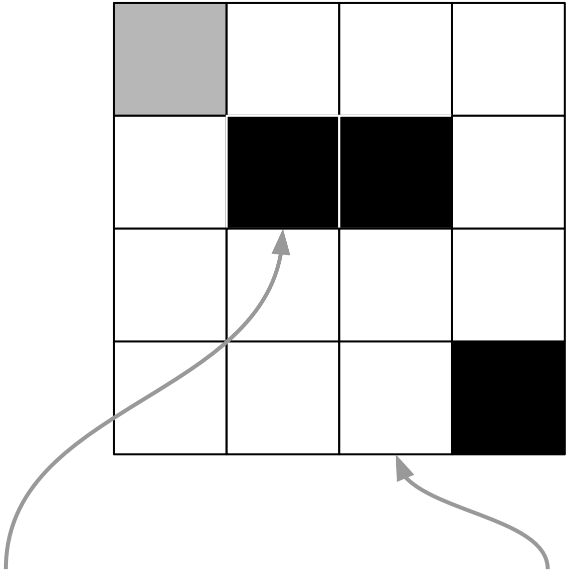
ASSUMPTIONS

1. The area described by the cell is entirely occupied or free
2. The world is static



ASSUMPTIONS

1. The area described by the cell is entirely occupied or free
2. The world is static
3. Cells are independent



The whole cell is **always** occupied (**regardless of neighboring**)

The whole cell is **always** free (**regardless of neighboring**)

MAP REPRESENTATION

The probability of a map is given by the product of the (independent) probabilities of all its cells.

$$p(\mathrm{map}) = \prod_{x,y} p(m_{x,y})$$

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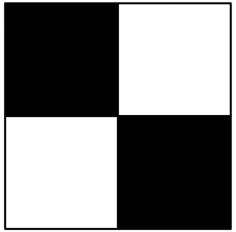

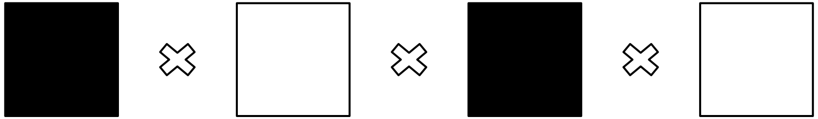

PROBABILISTIC MAPPING PROBLEM DEFINITION

Given the vector of all consecutive sensors measurements $\mathbf{z}_{1:t} = \mathbf{z}_0 \ldots \mathbf{z}_t$, and robot (sensor) poses $\boldsymbol{x}_{1:t} = \boldsymbol{x}_0 \ldots \boldsymbol{x}_t$, it is needed to build / recover the most probable map.

$$p(\mathrm{map} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t}) = \prod_{x,y} p(m_{x,y} | \mathbf{z}_{1:t}, \mathbf{x}_{1:t})$$

PROBABILISTIC MAPPING PROBLEM DEFINITION

1. Let's count two numbers:

   a. How many times have we observed a cell – $C_{x,y}$

   b. We will increase or decrease $O_{x,y}$ by 1 each time we observe an obstacle or a free zone in the cell, respectively

2. Let's calculate the probability of a cell being occupied as:

$$p(m_{x,y}) = \frac{O_{x,y} + C_{x,y}}{2C_{x,y}}$$

BAYESIAN ESTIMATION

$$p(m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t}) =$$

BAYESIAN ESTIMATION

$$p(m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t}) =$$

Bayes' rule

$$= \frac{p(\mathbf{z}_t|m_{x,y},\mathbf{z}_{1:t-1},\mathbf{x}_{1:t})p(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t})} =$$

BAYESIAN ESTIMATION

$$p(m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t}) =$$

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Markov property

$$= \frac{p(\mathbf{z}_t|m_{x,y},\mathbf{x}_t)p(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1})}{p(\mathbf{z}_t|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t})} =$$

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Independence property

$$= \frac{p(m_{x,y}|\mathbf{z}_t,\mathbf{x}_t)p(\mathbf{z}_t|\mathbf{x}_t)p(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1})}{p(m_{x,y})p(\mathbf{z}_t|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t})}$$

BAYESIAN ESTIMATION

$$p(\neg m_{x,y}|\mathbf{z}_{1:t}, \mathbf{x}_{1:t}) =$$

Similarly, for the opposite event

$$= \frac{p(\neg m_{x,y}|\mathbf{z}_t, \mathbf{x}_t)p(\mathbf{z}_t|\mathbf{x}_t)p(\neg m_{x,y}|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t-1})}{p(\neg m_{x,y})p(\mathbf{z}_t|\mathbf{z}_{1:t-1}, \mathbf{x}_{1:t})}$$

BAYESIAN ESTIMATION

Let's calculate the probability ratio:

$$\frac{p(m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t})}{p(\neg m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t})} = \frac{\dfrac{p(m_{x,y}|\mathbf{z}_t,\mathbf{x}_t)p(\mathbf{z}_t|\mathbf{x}_t)p(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1})}{p(m_{x,y})p(\mathbf{z}_t|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t})}}{\dfrac{p(\neg m_{x,y}|\mathbf{z}_t,\mathbf{x}_t)p(\mathbf{z}_t|\mathbf{x}_t)p(\neg m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1})}{p(\neg m_{x,y})p(\mathbf{z}_t|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t})}}$$

BAYESIAN ESTIMATION

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The ratio of the probabilities of cell occupancy and freedom under the condition of new measurements

Recursive member (the same ration for previous measurement / moment of time)

Ratio of prior probabilities (e.g. $p(m_{x,y})=0.5$ if we didn't know anything about the map at the beginning)

LOGARITHMIC ODDS

Let's call an odds:

$$\text{odds}(X) = \frac{p(X)}{1 - p(X)}$$

Let's call a logarithmic odds:

$$\text{logodds}(X) = log\frac{p(X)}{1 - p(X)}$$

MAPPING WITH AN INVERSE MODEL IN LOGARITHMIC FORM

Let's calculate the logarithmic odds from:

$$\frac{p(m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t})}{p(\neg m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t})} = \frac{p(m_{x,y}|\mathbf{z}_t,\mathbf{x}_t)}{1-p(m_{x,y}|\mathbf{z}_t,\mathbf{x}_t)} \frac{p(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1})}{1-p(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1})} \frac{1-p(m_{x,y})}{p(m_{x,y})}$$

And we get:

$$\text{logodds}(m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t}) = \text{logodds}(m_{x,y}|\mathbf{z}_t,\mathbf{x}_t) + \text{logodds}(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1}) - \text{logodds}(m_{x,y})$$

MAPPING WITH AN INVERSE MODEL IN LOGARITHMIC FORM

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And we get:

$$\mathrm{logodds}(m_{x,y}|\mathbf{z}_{1:t},\mathbf{x}_{1:t}) = \mathrm{logodds}(m_{x,y}|\mathbf{z}_t,\mathbf{x}_t) + \mathrm{logodds}(m_{x,y}|\mathbf{z}_{1:t-1},\mathbf{x}_{1:t-1}) - \mathrm{logodds}(m_{x,y})$$

Inverse measurement / observation model

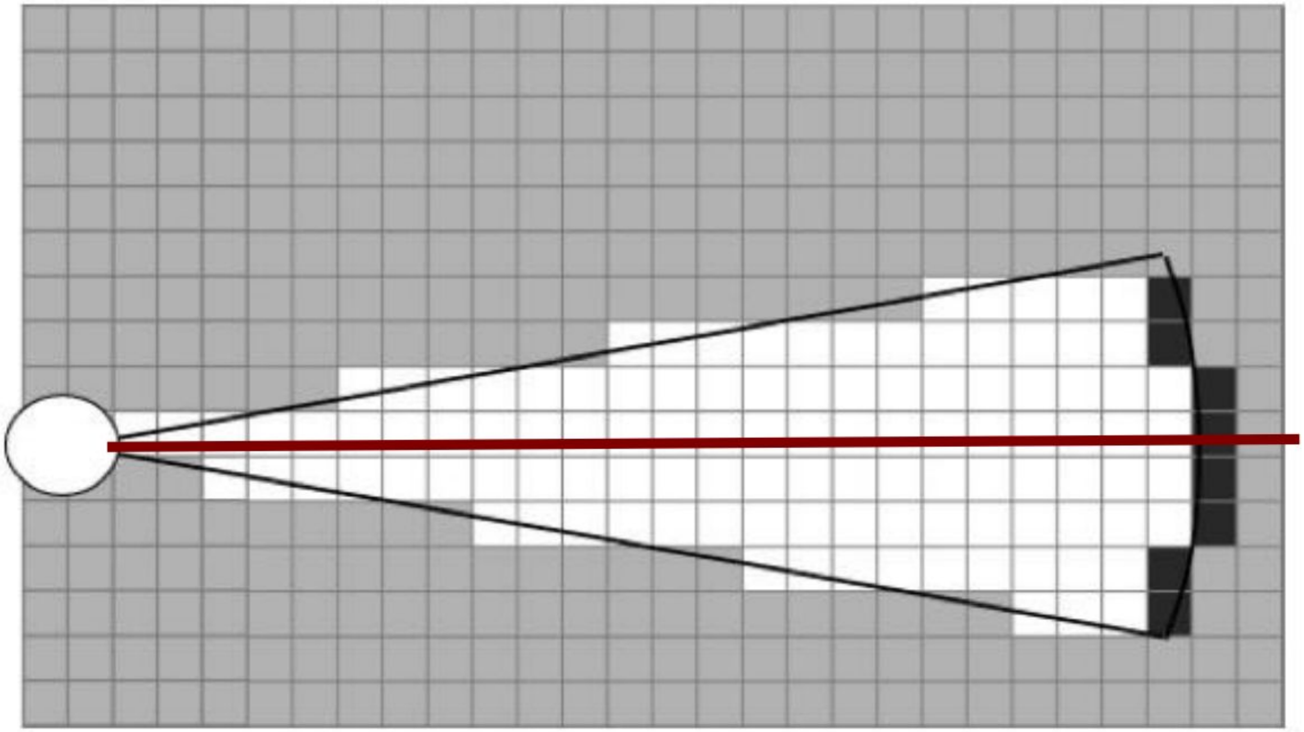
$$\textbf{occupancy\_grid\_mapping}(\{l_{t-1,i}\}, x_t, z_t)\textbf{:}$$

```
for all cells m_i do
    if m_i in perceptual field of z_t then
        l_{t,i} = l_{t-1,i} + inv_sensor_model(m_i, x_t, z_t) - l_0
    else
        l_{t,i} = l_{t-1,i}
    endif
endfor
return {l_{t,i}}
```

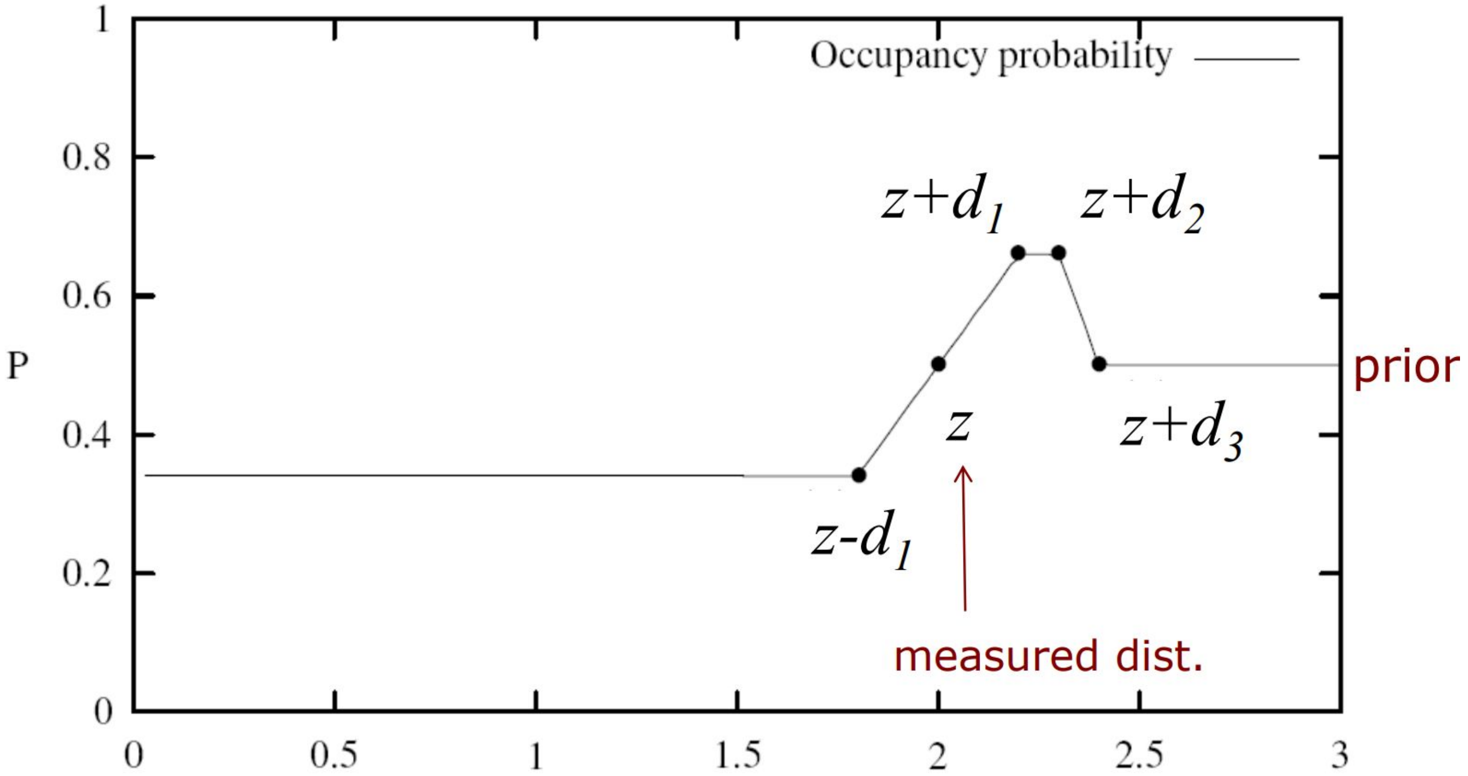
highly efficient, we only have to compute sums

Source: Gian Diego Tipaldi, Wolfram Burgard

EXAMPLE OF A SIMPLE SONAR MEASUREMENT MODEL

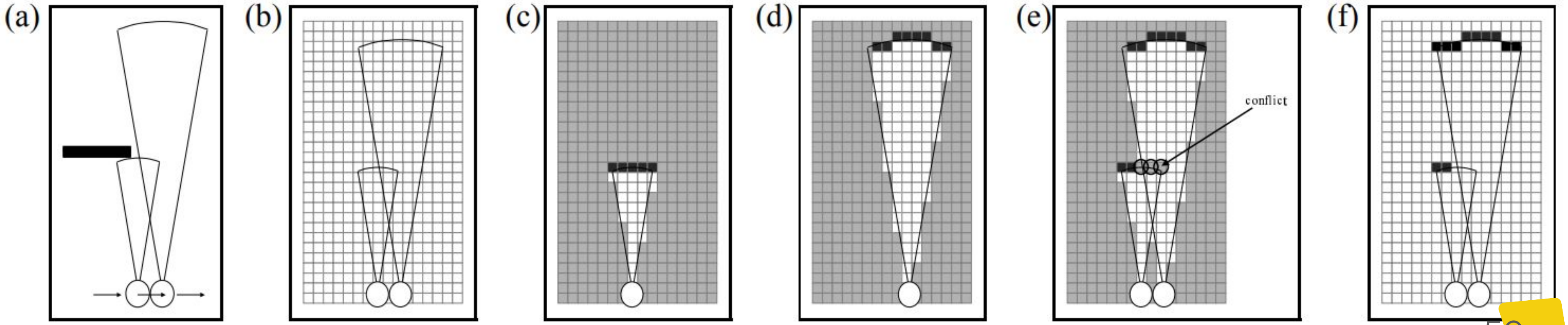


EXAMPLE OF A SIMPLE SONAR MEASUREMENT MODEL



WHAT IS THE DISADVANTAGE OF MAPPING WITH A INVERSE MODEL

- The inverse model considers the cells of the map as **independent** random variables
- This approach fails to explain contradicting data



MAPPING WITH FORWARD MODEL

- In contrast to mapping with an inverse model, we will consider the search for the **entire map** as an optimization problem in the space of all possible maps
- We will try to find such a *map* that maximizes the probability of all obtained measurements:

$$\hat{map} = \arg\max_{map} p(z|x, map)$$

ADDITIONAL RESOURCES

1. Probabilistic Robotics, Chapter 9
2. Topological Mapping. Benjamin Kuipers
3. Robot Mapping. Gian Diego Tipaldi, Wolfram Burgard
4. Learning Occupancy Grids with Forward Models. Sebastian Thrun

Thanks for attention!

Questions? Additions? Welcome!

girafe
ai