80. Categories, Functors, and Natruel Transformations.

(OI) Categories.

A contegory & consists of

(a). (objects) $Ob(\mathcal{C})$. the class of dijects in \mathcal{C} . (denote as $x \to Y$)

(6) (morphisms) \(\forall \times\) \(\forall \times

S.t. $Hom_{\ell}(x, \hat{Y}) = Hom_{\ell}(x', \hat{Y}) \iff x = x', \hat{Y} = \hat{Y}$

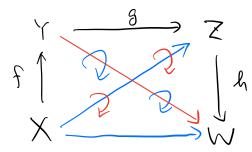
(c) (Composition law) & X-Y. Z & Ob(e), we have a map.

Home
$$(X,Y) \times Home (Y,Z) \xrightarrow{\circ} Home(X,Y)$$

 $(f,g) \longrightarrow g \circ f$

which satisfy the following two axioms

(1). (Associativity)



(2) (identity)
$$\forall X \in Ob(e) \exists X \xrightarrow{I_X} X$$

S.t.
$$h \circ I_X = h$$
. $I_x \circ k = k$

Example

$$(1) C = (Set)$$
 (Top)

$$Hom_{\ell}\circ p(X,Y) := Hom_{\ell}(Y,X).$$

lerminology

$$X, X' \in Ob(\mathcal{C}). \quad X \xrightarrow{f} X'$$

$$f \text{ is an isomorphism.} \iff \exists x' \xrightarrow{f} X.$$

$$f \circ f = I_{x'}$$

$$f \circ f = I_{x'}$$

(O. 2) (Functors.)

D. C. C. Categories

for write as the A covariant/contravariant functor F: e -> c' (not a map

just write like this)

Consists of (a) a rule of associating to each X EDb(E) an object F(x) E Ob(e)

> (b). a map Home (x, Y) F Home, (F(x), F(Y)) for each pair X, Y. E Ob(E).

St.
$$F(I_x) = I_{F(x)}$$
 and $f(I_x) = I_{F(x)}$ and

ie F(gof)=F(g)oF(f) F(f) = F(g)

Examples

$$\begin{cases} X^{ep} = X \\ Hom_{e}(X,Y) \rightarrow X \xrightarrow{f} Y \xrightarrow{ep} Y \in Hom_{e^{e}}(Y,X) \end{cases}$$

(2).
$$\forall X \in Ob(e)$$
. $h_{X}: e \rightarrow (Set)$

$$h_X(Y) := Hom_{\mathcal{C}}(Y,X), \forall Y \in Ob(\mathcal{C}).$$

$$h_{x}(f) := h_{x}(Y) \xrightarrow{\circ f} h_{x}(Y') \qquad \forall Y \xrightarrow{f} Y$$

$$Hom_{e}(Y,X) \qquad Hom_{e}(Y',X)$$

is a rule of associating to each $X \in Ob(e)$ a morphism.

$$F_{r}(x) \xrightarrow{T(x)} F_{z}(x)$$
 St. for each $X \xrightarrow{f} Y$ we have $F_{r}(x) \xrightarrow{T(x)} F_{z}(x)$

$$F_{r}(f) \downarrow Q \qquad \downarrow F_{r}(f)$$

$$F_{r}(f) \downarrow T(f) \xrightarrow{T(f)} F_{z}(f)$$

•
$$C \xrightarrow{F_1} C'$$
 $F_1 \xrightarrow{T} F_2$
 $F_3 \xrightarrow{F_3} F_3$

natural equivalence of

$$F_{i}(x) \xrightarrow{T(x)} F_{z}(x)$$
 is an isom, for each $X \in Ob(e)$