全纯函数

holomorphic

Ω 平面区域 → 连通开集

Ex: $f(z)=z^2$ $\quad g(z)=\bar{z}$ $\quad h(z)=|z|^2$

$$z_0\in\Omega,\ \frac{f(z_0+h)-f(z_0)}{h}=\frac{(z_0+h)^2-z_0^2}{h}=\frac{2hz_0+h^2}{h}=2z_0+h\to 2z_0$$

$$\frac{g(z_0+h)-g(z_0)}{h}=\frac{\bar{h}}{h}$$ 不存在 limit

$h(z)=z\bar{z}$ 只有 $z=0$ 有导数.

C-R d.e. 用了 看 $\mathbb{R}^2\to\mathbb{R}^2$ 的引导 → 这个一下子就完事了!

也写了两个方向趋近

$g\circ f(z)$.

$$\frac{g\circ f(z+h)-g\circ f(z)}{h}=\frac{g\circ f(z+h)-g\circ f(z)}{f(z+h)-f(z)}\,\frac{f(z+h)-f(z)}{h}$$

$$=g'(f(z))f'(z)$$

formal derivative

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$$

$$\frac{\partial}{\partial \bar z} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial f}{\partial y}\right)$$

$$f(z) = u(x,y) + i\,v(x,y)$$

$$f(\bar z) = u(x,-y) + i\,v(x,-y)$$

$$\overline{f(\bar z)} = u(x,-y) - i\,v(x,-y)$$

$$\overline{f(z)} = u(x,y) - i\,v(x,y).$$

X

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\overline{\frac{\partial f}{\partial z}} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - i\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\frac{\partial \bar f}{\partial \bar z} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$= \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - i\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

we have $\overline{\dfrac{\partial f}{\partial z}} = \dfrac{\partial \bar f}{\partial \bar z}$ check

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就是算子

复偏导. 定义式. 1 形式定义 (直接给公式)

2. 增量定义. (增量方程).

记忆式 (不严格). $f(z) \to f(x,y)$

$$\begin{cases} x = \frac{1}{2}(z+\bar z) \\ y = \frac{-i}{2}(z-\bar z) \end{cases}$$

$\Rightarrow$ use 链式法则.

且. z, $\bar{z}$ 形式上独立

满足"偏导"的各种性质