L Outer measure. on $\mathbb{R}$

$m^*(E) = \inf\{ \sum_n \ell(I_n) : \{I_n\}_{n\geq 1}$ 是一列开区间并且 $E \subset \bigcup_n I_n \}$ 定义在 $2^{\mathbb{R}}$ 上

O.M
- m^*(至多可数集) $=0$. $m^*(\phi)=0$ 例 2.2.1
- 单调性 Thm 2.2.1
- 次可加性. Thm 2.2.3

and. in particular
- I 为区间. $m^*(I) = \ell(I)$ Thm 2.2.2
- 平移
- 外正则性 T2. $m^*(E) = \inf\{m^*(Q) : E \subset Q,\ Q \text{ is open}\}$

$\Downarrow$ m^*-measurable $m^*(A) \geq m^*(A\cap E) + m^*(A \cap E^c)$ Thm 2.3.5 $\Rightarrow$ 极限性. Thm2.3.6

可数可加性

L measurable sets . and L measure. on $\mathbb{R}$.

and.
- 平移性 2.4
- 正则性 2.5.

(G$_\delta$集 . F$_\sigma$集)

Ω

$\downarrow$

σ-algebra
- ϕ. (零测集) $\in \Omega$ Thm 2.3.2
- 补 Thm 2.3.4
- 可数交并.

and
- 区间 $\in \Omega$ Thm 2.3.3
- 开,闭集 $\in \Omega$ Thm 2.3.4. (Borel sets $\subset \Omega$)

不可测例子. $E(x) = \{y \in [0,1] : y - x \in \mathbb{Q}\}$

存在不是Borel集的可测集. P41

L measure on $\mathbb{R}^n$ almost the same.

Thm. 2.7.1. $P \overset{\text{measurable}}{\subset} \mathbb{R}^p$ "直积的测度"

$Q \overset{\text{measurable}}{\subset} \mathbb{R}^q$

$\Rightarrow P \times Q \overset{\text{measurable}}{\subset} \mathbb{R}^{p+q}$ $m(P\times Q) = m(P)\cdot m(Q)$

Thm 2.7.2. difference set is measurable.