以《统计学习方法》例11.1为例

条件随机场的三种形式：

1、参数化形式

[1]原始形式

只保留特征取值为1的条件后：

[2]总矩阵形式

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | y1=1 | y1=2 | y2=1 | y2=2 | y3=1 | y3=2 |
| y1=1 |  |  | 0 | 1 |  |  |
| y1=2 |  |  | 0 | 0 |  |  |
| y2=1 | 0 | 0 |  |  | 0 | 1 |
| y2=2 | 1 | 0 |  |  | 0 | 0 |
| y3=1 |  |  | 0 | 0 |  |  |
| y3=2 |  |  | 1 | 0 |  |  |

[3]简化矩阵形式

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | y1=1 | y1=2 |  | y2=1 | y2=2 |  | y3=1 | y3=2 |
| start | 0 | 0 | y1=1 | a | b | y2=1 | e | f |
|  |  |  | y1=2 | c | d | y2=2 | g | h |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | y1=1 | y1=2 |  | y2=1 | y2=2 |  | y3=1 | y3=2 |
| start | A | B | y1=1 | C | D | y2=1 | E | F |
|  |  |  | y2=2 | C | D | y2=2 | E | F |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | y1=1 | y1=2 | y2=1 | y2=2 | y3=1 | y3=2 |
|  | A | B | C | D | E | F |

2、向量内积形式

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| y1, y2, y3 | 1,1,1 | 1,1,2 | 1,2,1 | 1,2,2 | 2,1,1 | 2,1,2 | 2,2,1 | 2,2,2 |
| t | a+e | a+f | b+g | b+h | c+e | c+f | d+g | d+h |
| s | A+C+E | A+C+F | A+D+E | A+D+F | B+C+E | B+C+F | B+D+E | B+D+F |

计算非规范化概率

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| y1, y2, y3 | 1,1,1 | 1,1,2 | 1,2,1 | 1,2,2 | 2,1,1 | 2,1,2 | 2,2,1 | 2,2,2 |
| p | exp(3.2) | exp(3.9) | exp(4.3) | exp(3.2) | exp(3.1) | exp(3.8) | exp(2.8) | exp(1.7) |

3、矩阵形式

由简化矩阵

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | y1=1 | y1=2 |  | y2=1 | y2=2 |  | y3=1 | y3=2 |
| start | a01 | a02 | y1=1 | b11 | b12 | y2=1 | c11 | c12 |
|  |  |  | y2=2 | b21 | b22 | y2=2 | c21 | c22 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| y1, y2, y3 | 1,1,1 | 1,1,2 | 1,2,1 | 1,2,2 |
| P | a01+b11+c11 | a01+b11+c12 | a01+b12+c21 | a01+b12+c22 |
| y1, y2, y3 | 2,1,1 | 2,1,2 | 2,2,1 | 2,2,2 |
| P | a02+b21+c11 | a02+b21+c12 | a02+b22+c21 | a02+b22+c22 |

二、CRF概率计算，前向概率与后向概率

以11.2为例,3个结点，n=3

1、前向概率与后向概率

[1]

[2]

[3]注：

注意到，其实是对应。那么为什么，可以从Part1 3矩阵形式中看出，所以

2、非规范化条件概率计算

[1]

[2]

3、规范化因子(11.2证明)

首先个人认为《统计学习方法》（第一版第20次印刷）Page200关于Z(x)的公式有误

以11.2为例简单证明一下，不做普遍性证明。n=3

证明如下：