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Novel fake news spreading model with similarity on PSO-based networks

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ABSTRACT

This paper proposes a fake news spreading model with similarity taken into account, assuming that the similarity between individuals can affect the transmission rate. Simulations show that the similarity of two connected nodes and the product of their degrees are positively correlated when the network temperature is small, and the similarity of two connected nodes decreases as the product of their degrees increases. Thus the transmission rate can be expressed as the function of their degrees in the proposed model. The theoretic analysis demonstrates the critical threshold is related to both the influence coefficient and the similarity function. Simulation results show a smaller influence coefficient leads to a larger critical threshold and smaller final density of stiflers.

1. Introduction

In recent years, online social networks have become one of the main access to news and facilitated the production and dissemination of fake news and rumors [1]. A survey found that 62 percent of US adults obtain news through social media [2]. Users post and repost information, news, photos, videos on their social network accounts which are seldom verified. In the 2016 United States presidential election, the fake news about pro-Trump was shared for 30 million times, and the fake news about pro-Clinton was shared for 7.6 million times [3]. People's trust in social media is much lower than traditional print and TV outlets. Moreover, it is proposed in [4] that fake news is diffused far more rapidly and widely than the truth.

Complex network is a fundamental tool used by researchers for understanding and modeling social, biological, and physical systems [5–7], in which the nodes represent individuals or organizations, and the edges represent the interactions among them [8]. For social networks, the nodes represent the individuals and the edges represent the relationship between them. Real-world networks are mostly heterogeneous networks, where a few nodes have very large degrees, and the vast majority of nodes have very small degrees. The distribution of nodes follows a power-law distribution (scale-free property) $P(k) \sim k^{-\gamma}$, where k denotes the degree of nodes, and γ is a parameter that ranges between 2 and 3 for most real-world networks [9].

Epidemic spreading is a common phenomenon in the complex networks such as virus spreading in computer networks, and fake news spreading in social networks. The modeling of epidemic spreading can be used to investigating how diseases

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spread among individuals [10]. Mean-field approximation has been very successful in modeling epidemic spreading processes, and the most common mean-field models are the susceptible–infected–susceptible (SIS) model [11,12] and the Susceptible–Infected–Removed (SIR) model [13,14]. It is worth mentioning that the epidemic threshold in an infinite-size scale-free network tends to vanish for both the SIS and the SIR models.

The spreading of the fake news and rumors can be seen as 'infection of the mind'. Daley and Kendall [15] introduced the DK model to modeling the rumor spreading, which is the original model of rumor spreading. In this model, the individuals are classified into three different categories, ignorants (denoted by I, those who have not got the rumor), spreaders (denoted by S, those who are spreading the rumor), and stiflers (denoted by R, those who know the rumor and do not spread it). The rumor spreads when a spreader is involved in a pairwise contact. When a spreader meets an ignorant, the ignorant becomes a spreader. When a spreader meets another spreader or stifler, the spreader becomes a stifler. Maki and Thompson [16] developed the DK model to the MK model, in which they proposed that the rumor is spread by directed contacts of the spreaders with other individuals. Moreover, when a spreader meets another spreader, only the initiative spreader turns into a stifler.

Both the DK and the MK models assume that the individuals contact with each other randomly, which is a homogenous mixing approximation. Moreno et al. [17] investigated the dynamics of the rumor spreading model on the heterogeneous networks, the results show that network heterogeneity has a considerable effect on the dynamics of rumor spreading. Nekovee et al. [18] investigated the threshold behavior of the rumor spreading on both the homogenous and the heterogeneous networks, the results show that the heterogeneous social networks are prone to spread rumors. Roshani et al. [19] proposed a generalized rumor spreading model on top of heterogeneous networks with consideration of degree-biased transmission rate and nonlinear infectivity, showing that the threshold of their model is larger than that of the standard rumor spreading model.

In the above-mentioned rumor spreading models on heterogenous networks, it is assumed that the degree of nodes is a key factor affecting the spreading. It was proposed in [20,21] that the degree, known as popularity, is just one dimension of attractiveness; another dimension is similarity. That is, new connections are connected to both the popular nodes and the similar nodes in growing networks, which is called the Popularity and Similarity Optimization(PSO) algorithm. For instance, a new Twitter user will not only follow the influential individuals but also follow similar individuals such as his/her friends or families or those who have similar interests, even if the similar individuals are not popular. In the social networks, when an ignorant individual receives fake news from a spreader, the more similar they are, the ignorant is more likely to believe the fake news and spread it.

In this paper, we propose a fake news spreading model on the PSO-based networks, in which the similarity is taken into account. For simplicity, a commonly-used SIR model is adopted due to its effectiveness in describing the dynamics of much real-world rumor spreading phenomena. In this model, the individuals are classified into three different categories, ignorants (denoted by I, those who have not got the fake news), spreaders(denoted by S, those are spreading the fake news), and stiflers (denoted by R, those who know the fake news and do not spread it). When an ignorant interacts with a spreader, the ignorant turns to a spreader with probability λ. When a spreader interacts with another spreader or stifler, the spreader turns to be a stifler with probability σ, while a spreader may also become a stifler with probability δ spontaneously without any contact. It is worth noting that λ depends on the similarity of the two individuals in our model.

The contributions of this paper include:

1. A fake news spreading model with similarity on the PSO-based networks is proposed. To the best of our knowledge, this is the first time when the fake news spreading model is investigated on the PSO-based networks.
2. Also for the first time, the similarity between individuals is introduced into modeling fake news spreading on the PSO-based networks.
3. The critical threshold and maximum spreading are obtained for the proposed model.

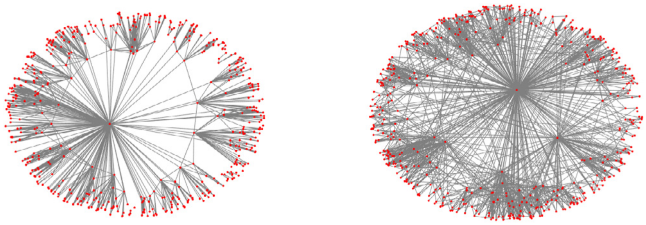
The remainder of this paper is organized as follows. Section 2 provides preliminaries on the PSO-based networks, the hyperbolic network model, and the standard rumor spreading model on the heterogeneous networks. In Section 3, the fake news spreading model with similarity on the PSO-based networks is proposed. Section 4 presents simulation results on the proposed model. Section 5 concludes the paper and outlines some future work.

2. Preliminaries

2.1. PSO-based networks

In this section, a simple way to generate a PSO-based network is reviewed and more details can be found in [20].

It was proposed in [20] that popularity is just one facet of node attractiveness in growing networks, and the similarity is another important aspect that cannot be neglected. In real-world networks, new nodes seek to link to both the popular nodes and the similar nodes. To describe this phenomenon, the PSO algorithm model is proposed in which the links are generated with the balance of both the popularity and the similarity, as follows. First, the network is empty at the initial time $t = 0$. Second, at each time t ($t = 1, 2, 3 \ldots$), new node t is generated, and t is also called the age of the node. It is assumed that the older nodes are more popular due to they have more opportunities to attract connections. The nodes can be placed on a disk, and each of them corresponds to a pair of polar coordinates, the radius coordinate t, and the



(a) (b)

Fig. 1. Network topology of two PSO-based networks, the number of nodes $N = 500$, the radius of disk $R = 11$, average degree $\langle k \rangle = 6$, power-law degree distribution exponent $\gamma = 3$. (a) Temperature $T = 0$, average clustering coefficient is 0.75. (b) Temperature $T = 0.5$, average clustering coefficient is 0.41.

angular coordinate θ_t, where θ_t is a random number between 0 and 2π. Finally, each new node t connects to the m nodes which have the m smallest of the values $s\theta_{ts}$, where the parameter m is related to the average node degree $m = \frac{\bar{k}}{2}$, and θ_{ts} is the angular distance between nodes t and s ($s < t$). It is worth stressing that $s\theta_{ts}$ includes both the popularity and the similarity.

The power-law exponent of the generated network can be adjusted by replacing $s\theta_{ts}$ with $s^{\beta}\theta_{ts}$,$(0 \leq \beta \leq 1)$ in the PSO-based networks model, the clustering coefficient of the generated network can be controlled by the network temperature T. The difference between the BA network model and the PSO-based networks model is that new nodes connect to nodes of degree k with the same probability in the BA model, while new nodes connect to the most similar nodes in the k-degree nodes in the PSO-based networks model.

2.2. Hyperbolic network model

It was proposed in [22,23] that PSO-based networks can be embedded in the hyperbolic space.

A simple way to form a PSO-based network in the hyperbolic space contains three steps. First, place N nodes on a disk with radius R, the radius of nodes follows $\rho(r) = \alpha e^{\alpha(r-R)}$ and the angular of nodes follows $\rho(\theta) = \frac{1}{2\pi}$, where the parameter $\alpha > 0$. Hence, each node corresponds to a (r, θ) coordinate pair, where r represents polar radius and θ represents the polar angular. Second, for each pair of nodes, the hyperbolic distance between them is calculated by $d = \frac{1}{\zeta}\text{arc}\dot{c}osh(\cosh\zeta r\cosh\zeta r' - \sinh\zeta r\sinh\zeta r'\cos\Delta\theta)$, where the coordinates of the two nodes are (r, θ) and (r', θ'), and $\Delta\theta = \pi - \left|\pi - \left|\theta - \theta'\right|\right|$ denotes the angular distance between the two nodes. $\zeta = \sqrt{|K|}$ and is constant, where K ($K < 0$) denotes the constant curvature of hyperbolic space. Finally, whether two nodes are connected depends on the connection probability $p(d) = \frac{1}{e^{(\zeta/2T)(d-R)}+1}$, where d is the hyperbolic distance between two nodes, and T is the network temperature. when the hyperbolic distance between every two nodes is less than R, the two nodes are connected.

Two generated PSO-based networks with 500 nodes are shown in Fig. 1, the power-law degree distribution $P(k) \sim k^{-3}$. When a node is located near the center of the disk, that means it is more popular. When the angular distance between two nodes is smaller, the two nodes are more similar. The average clustering coefficient is 0.75 for the network with temperature $T = 0$, and the average clustering coefficient is 0.41 for the network with temperature $T = 0.5$.

2.3. Standard SIR rumor spreading model on heterogeneous networks

In this subsection, a commonly-used SIR rumor spreading model on heterogeneous networks is reviewed, and more details can be found in [18]. The individuals are classified into three different categories, the ignorants, the spreaders, and the stiflers. The densities of the ignorants, the spreaders and the stiflers with degree k at time t are denoted as $I_k(t)$, $S_k(t)$, and $R_k(t)$ respectively. Moreover, $I_k(t)$, $S_k(t)$, and $R_k(t)$ satisfy the condition $I_k(t) + S_k(t) + R_k(t) = 1$. The mean-field equations for the dynamics of rumor spreading on complex networks with arbitrary degree correlations are as follows,

$$\frac{dI_k(t)}{dt} = -k\lambda I_k(t)\sum_l S_l(t)P(l|k), \tag{1a}$$

$$\frac{dS_k(t)}{dt} = k\lambda I_k(t)\sum_l S_l(t)P(l|k)$$
$$- k\sigma S_k(t) \times \sum_l [S_l(t) + R_l(t)]P(l|k) - \delta S_k(t), \tag{1b}$$

$$\frac{dR_k(t)}{dt} = k\sigma S_k(t)\sum_l [S_l(t) + R_l(t)]P(l|k) + \delta S_k(t). \tag{1c}$$

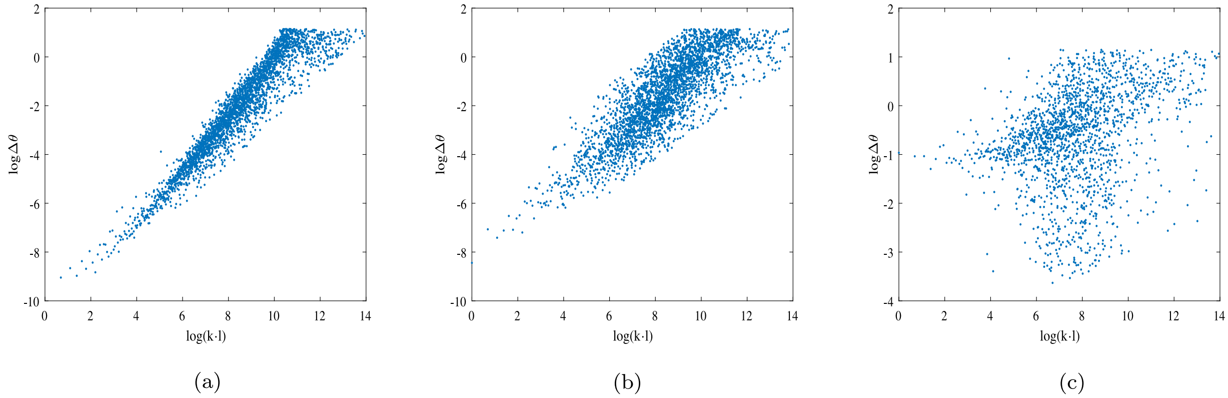


Fig. 2. Simulations on the correlation of angular distance $\Delta\theta$ between each pair of nodes and the product of their degrees k and l on PSO-based networks, the size of networks $N = 5000$, average degree $\langle k \rangle = 6$, power-law degree distribution exponent $\gamma = 2.2$. (a) Temperature $T = 0$, (b) temperature $T = 0.5$, (c) temperature $T = 1$.

Where λ denotes the probability that an ignorant turns to a spreader, when an ignorant interacts with a spreader. σ denotes the probability that an ignorant turns to a spreader, when a spreader interacts with another spreader or stifler. δ denotes the probability that a spreader becomes a stifler without any contact. $P(l|k)$ is the conditional probability of a node with degree k connecting to a node with degree l. For uncorrelated networks, $P(l|k) = lP(l)/\langle k \rangle$, where $P(k)$ is the degree distribution and $\langle k \rangle$ is the average degree of the network. $\sum_l S_l(t)P(l|k)$ is the probability that a link originating from an individual with degree k points to a spreader at time t. $\sum_l [S_l(t) + R_l(t)]P(l|k)$ is the probability that a link originating from an individual with degree k points to a spreader or a stifler at time t.

3. Fake news spreading model with similarity

3.1. The relationship between the similarity of two connected nodes and the product of their degrees

In this subsection, the relationship between the similarity of two connected nodes and the product of their degrees is investigated by simulations. The PSO-based networks with 5000 nodes are generated by the method mentioned in Section 2.2 for simulation, the average degrees of these networks are set as $\langle k \rangle = 6$, T is varying from 0 to 1. Fig. 2 shows the relationship between the angular distance $\Delta\theta$ of each pair of connected nodes and the product of their degrees on PSO-based networks. For $T = 0$, Fig. 2(a) shows that the angular distance $\Delta\theta$ between a pair of connected nodes increases as the product of their degrees $k \cdot l$ increases, which is positively correlated. For $T = 0.5$, Fig. 2(b) shows the correlation is smaller than that of $T = 0$. For $T = 1$, Fig. 2(c) shows the correlation disappears.

To investigate the effect of the temperature T on the correlations, the Pearson correlation coefficient of the angular distance $\Delta\theta$ and the product of the degrees $k \cdot l$ is adopted for analysis. Fig. 3(a) shows that the Pearson correlation coefficient decreases as the temperature T increases. For smaller T, if a pair of nodes with lower degrees can be connected, the similarity between them must be larger, while a pair of nodes with larger degrees can be connected, the similarity between them inclines to be smaller. Fig. 3(b) shows that the average clustering coefficient decreases as the temperature T increases.

In this paper, our model is proposed under the assumption that the angular distance of a pair of connected nodes and their product of the degrees is highly correlated, that is, the temperature is smaller or the average clustering coefficient of the network is larger.

3.2. Model

The fake news spreading model with similarity will be proposed in this subsection. In the diffusion process of fake news, when an ignorant individual receives fake news from a spreader, the more similar they are, the ignorant is more likely to believe the fake news and spread it. Based on this assumption, our model differs from the standard model in terms of spreading probability λ. λ is constant in the standard model, while the spreading rates between two connected nodes are related to their similarity in our model, which are denoted as λ_S.

Each individual of the network can be in one of three states, ignorants, spreaders, and stiflers, and the proposed model is composed of two possible transitions. The first one is denoted by I-S, when a spreader i publishes fake news on the online social network, an ignorant j may spread this fake news with the probability $\lambda_S = \frac{\lambda C}{\Delta\theta(i,j)}$, where λ denotes transmission probability and is a constant, C denotes influence coefficient and is a positive constant, which describes the influence of the similarity on the spreading. $\Delta\theta(i, j)$ denotes the angular distance between individuals i and j, and the angular distance

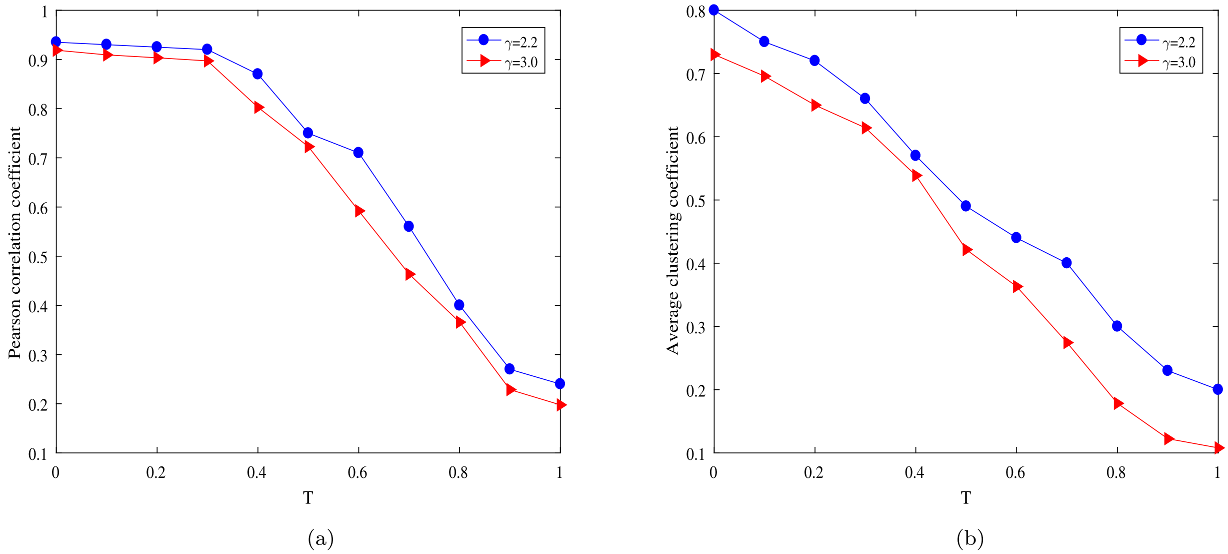


Fig. 3. (a) Simulations on the Pearson correlation coefficient of the angular distance $\Delta\theta$ and the product of the degrees $k \cdot l$ for different temperature T. (b) Simulations on the relationship between average clustering coefficient and the temperature T.

between two individuals is smaller means they are more similar. When an ignorant individual contacts a spreader, the more similar they are, the ignorant is more likely to become a spreader. The second transmission is denoted by S-R, When a spreader interacts with another spreader or stifler, the spreader turns to be a stifler with probability σ, where σ denotes the transmission probability and is a constant. Meanwhile, a spreader may also stop spreading the fake news spontaneously with probability δ due to forgetting or other reasons.

Based on the standard rumor spreading model, the fake news spreading model with similarity is described by the following equations,

$$\frac{dI_k(t)}{dt} = -k\lambda I_k(t)\sum_l \frac{C}{\Delta\theta(l,k)} S_l(t)P(l|k), \tag{2a}$$

$$\begin{aligned}\frac{dS_k(t)}{dt} &= k\lambda I_k(t)\sum_l \frac{C}{\Delta\theta(l,k)} S_l(t)P(l|k) \\ &- k\sigma S_k(t) \times \sum_l [S_l(t)+R_l(t)]P(l|k) - \delta S_k(t),\end{aligned} \tag{2b}$$

$$\frac{dR_k(t)}{dt} = k\sigma S_k(t) \times \sum_l [S_l(t)+R_l(t)]P(l|k) + \delta S_k(t). \tag{2c}$$

As presented in Section 3.1, the similarity between a pair of connected nodes is related to the product of their degrees. Based on this, the angular distance $\Delta\theta(l,k)$ in Eqs. (2a) and (2b) can be expressed as a function of their degrees, which is $\Delta\theta(l,k) = \phi(k)\phi(l)$, where ϕ is the similarity function and increases as the degree of nodes increases. In this paper, we focus on uncorrelated networks where the conditional probability satisfies $P(l|k) = \frac{lP(l)}{\langle k\rangle}$. Therefore, by replacing $\Delta\theta(l,k)$ and $P(l|k)$ with $\phi(k)\phi(l)$ and $\frac{lP(l)}{\langle k\rangle}$ in Eq. (2), the following equations are obtained,

$$\frac{dI_k(t)}{dt} = -\frac{\lambda kC}{\langle k\rangle\phi(k)} I_k(t)\sum_l \frac{l}{\phi(l)} S_l(t)P(l), \tag{3a}$$

$$\begin{aligned}\frac{dS_k(t)}{dt} &= \frac{\lambda kC}{\langle k\rangle\phi(k)} I_k(t)\sum_l \frac{l}{\phi(l)} S_l(t)P(l) \\ &- \frac{\sigma k}{\langle k\rangle} S_k(t) \times \sum_l l[S_l(t)+R_l(t)]P(l) - \delta S_k(t)\end{aligned} \tag{3b}$$

$$\frac{dR_k(t)}{dt} = \frac{\sigma k}{\langle k\rangle} S_k(t) \times \sum_l l[S_l(t) + R_l(t)]P(l) + \delta S_k(t). \tag{3c}$$

3.3. The threshold and maximum spreading

In this subsection, the threshold and maximum spreading of the proposed model in Eq. (3) are analyzed.

Eq. (3a) can be integrated directly, yielding

$$I_k(t) = I_k(0) exp[-\frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi(t)], \tag{4}$$

where $I_k(0)$ is the density of ignorant individuals with degree k at the initial time $t = 0$, and it is set to 1 for simplicity. $\Phi(t)$ is an auxiliary function which is defined as

$$\Phi(t) = \sum_k p(k) \frac{k}{\phi(k)} \int_0^t S_k(t')dt' = \int_0^t \langle \frac{k}{\phi(k)} S_k(t')\rangle dt'. \tag{5}$$

Multiplying Eq. (3b) with $\frac{kp(k)}{\phi(k)}$ and summing over all k gives

$$\begin{aligned} \frac{d\Phi}{dt} &= \langle \frac{k}{\phi(k)}\rangle - \left\langle \frac{k}{\phi(k)} exp[-\frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi(t)]\right\rangle - \delta\Phi \\ &- \frac{\sigma}{\langle k\rangle} \int_0^t \left\{ \langle k\rangle - \left\langle k exp[-\frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi(t')]\right\rangle \right\} \\ &\times \langle \frac{k^2}{\phi(k)} S_k(t')\rangle dt'. \end{aligned} \tag{6}$$

As $t \to \infty$, $S(k) \to 0$, thus $\frac{d\Phi}{dt} = 0$, and Eq. (6) becomes

$$\begin{aligned} 0 &= \langle \frac{k}{\phi(k)}\rangle - \left\langle \frac{k}{\phi(k)} exp[-\frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi_\infty]\right\rangle - \delta\Phi_\infty \\ &- \frac{\sigma}{\langle k\rangle} \int_0^\infty \left\{ \langle k\rangle - \left\langle k \cdot exp[-\frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi(t')]\right\rangle \right\} \\ &\times \langle \frac{k^2}{\phi(k)} S_k(t')\rangle dt'. \end{aligned} \tag{7}$$

When $\sigma = 0$, the solution can be obtained by solving Eq. (7). When $\sigma \neq 0$, Eq. (7) should be solved for leading order in σ. Therefore, to obtain $S_k(t)$ for zeroth order in σ, Eq. (3b) is integrated and gives

$$\begin{aligned} S_k(t) &= 1 - exp[-\frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi(t)] \\ &- \delta \int_0^t e^{\delta(t-t')} \times [1 - exp[-\frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi(t')]]dt' + O(\sigma). \end{aligned} \tag{8}$$

When the transmission rate is close to the critical threshold, both $\Phi(t)$ and Φ_∞ are small. Let $\Phi(t) = \Phi_\infty f(t)$, where $f(t)$ is a finite function. Working to the leading order in Φ_∞ gives

$$S_k(t) \approx -\delta \frac{\lambda kC}{\langle k\rangle \phi(k)} \Phi_\infty \int_0^t e^{\delta(t-t')} f(t')dt' + O(\Phi_\infty^2) + O(\sigma). \tag{9}$$

Inserting Eq. (9) in Eq. (7) and expanding the exponential to the relevant order in Φ_∞ gives

$$\begin{aligned} 0 &= \Phi_\infty[\lambda C \frac{\langle k^2/\phi^2(k)\rangle}{\langle k\rangle} - \delta] \\ &- \lambda^2 C^2 \frac{\left\langle k^3/\phi^3(k)\right\rangle}{\langle k\rangle^2} (\frac{1}{2} + \sigma\delta \frac{\langle k^2\rangle}{\langle k\rangle}) \Phi_\infty^2 + O(\sigma^2) + O(\Phi_\infty^3). \end{aligned} \tag{10}$$

The non-trivial solution of Eq. (10) is given by:

$$\Phi_\infty = \frac{\lambda C \frac{\langle k^2/\phi^2(k)\rangle}{\langle k\rangle} - \delta}{\lambda^2 C^2 \frac{\langle k^3/\phi^3(k)\rangle}{\langle k\rangle^2} (\frac{1}{2} + \sigma\delta \frac{\langle k^2\rangle}{\langle k\rangle})}. \tag{11}$$

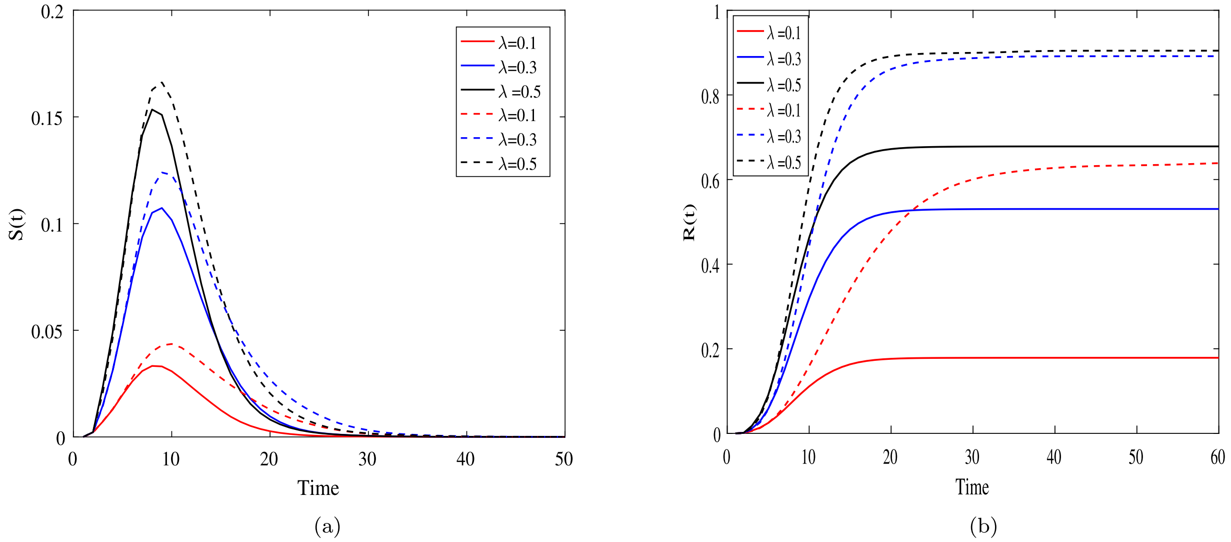


Fig. 4. Monte–Carlo simulations on the fake news spreading model with similarity on PSO-based network for temperatures $T = 0$ (solid curves) and $T = 0.3$ (dashed curves), transmission rate $\sigma = 0.2$, $\delta = 0.1$, influence coefficient $C = 0.3$, average degree $\langle k\rangle = 6$, (a) Time evolution of the density of spreaders for different λ, (b) Time evolution of the density of stiflers for different λ.

To obtain a positive value for Φ_∞, the following inequality should be satisfied,

$$\frac{\lambda}{\delta} \geq \frac{\langle k\rangle}{C\langle k^2/\phi^2(k)\rangle}. \tag{12}$$

Thus, the critical threshold of the proposed model in Eq. (3) is obtained, which is independent of σ. In particular, for $\delta = 1$ the critical threshold is given by

$$\lambda_c \geq \frac{\langle k\rangle}{C\langle k^2/\phi^2(k)\rangle}. \tag{13}$$

Moreover, the maximum spreading R is given by

$$R = \sum_k P(k)(1 - exp[-\frac{\lambda k C}{\langle k\rangle\phi(k)}\Phi_\infty]), \tag{14}$$

which depends on the form of $P(k)$.

4. Simulations

In this section, Monte-Carlo simulations are carried out on the PSO-based networks that are generated by the method in Section 2.2. All generated networks are heterogeneous networks with 5000 nodes, and the power-law degree distribution exponent is set as 3, the fake news source is a single spreader that is randomly selected. The results in this section are based on Monte-Carlo simulations for 5000 times.

Fig. 4 shows the time evolution of the density of spreaders and stiflers for different values of the spreading process rate λ. The transmission rates σ and δ are set as 0.2 and 0.1, respectively, and the influence coefficient C is set as 0.3, the average degree is set as 6.0, the temperatures T are set as 0 and 0.3, respectively. Fig. 4(a) shows that the number of individuals who spread the fake news increases as the spreading rate λ increases, moreover, for the larger T, a larger population spreads the fake news than the smaller T. Fig. 4(b) shows that the final densities of the stiflers on the network with $T = 0$ (solid curves) are smaller than that of the network with $T = 0.3$.

Fig. 5 shows the time evolution of the density of spreaders and stiflers for different influence coefficient C. The transmission rates λ, σ and δ are set as 0.3, 0.2 and 0.1, respectively, the average degree is set as 6.0, the temperatures T are set as 0. The results indicate that both the number of spreaders and the number of stiflers increases as the influence coefficient C increases.

Fig. 6 displays the effect of influence coefficient C on the critical threshold. Fig. 6(a) shows critical thresholds $\lambda = 0.04, 0.02$, and 0.01 (as the black arrows denoted in the figure) for $C = 0.05$, 0.1 and 0.2. Fig. 6(b) shows the fake news is prone to outbreak as C increases, which is consistent with the theoretical analysis. Fig. 7 shows the effect of the average degree on the critical threshold and final densities of the stiflers in the proposed model. Fig. 7(a) shows critical thresholds $\lambda = 0.07, 0.02$, and 0.01 (as the black arrows denoted in the figure) for different average degrees $\langle k\rangle = 3.5, 5.5$ and 7,

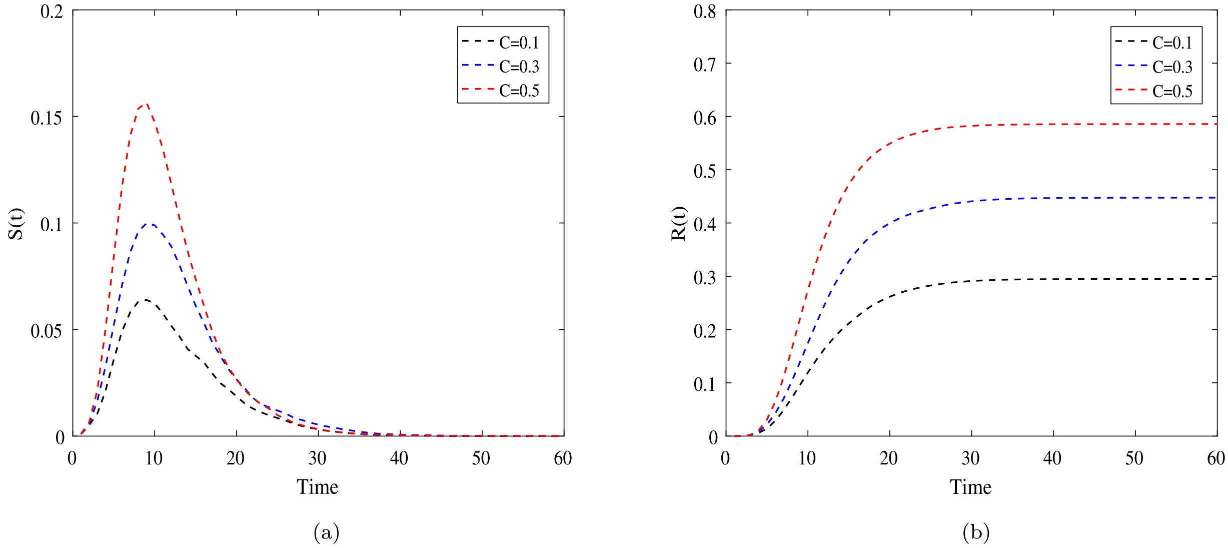


Fig. 5. Monte–Carlo simulations on the fake news spreading model with similarity on PSO-based network, transmission rate $\lambda = 0.3$, $\sigma = 0.2$, $\delta = 0.1$, temperatures $T = 0$, average degree $\langle k \rangle = 6$, (a) Time evolution of the density of spreaders for different influence coefficient C (b) Time evolution of the density of stiflers for different influence coefficient C.

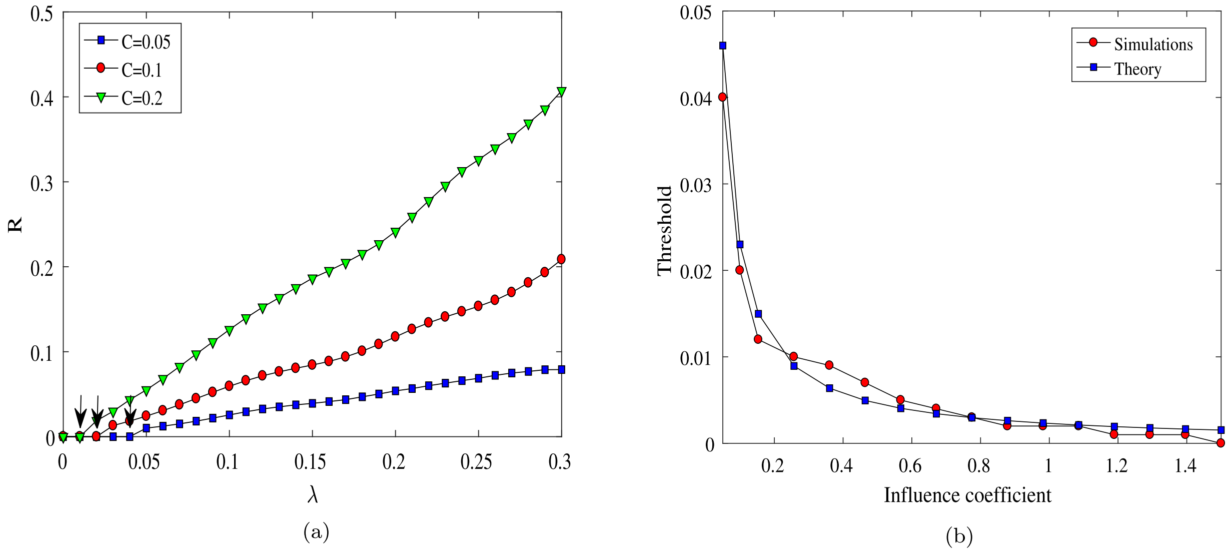


Fig. 6. Effect of the influence coefficient C on the critical threshold in the fake news spreading model with similarity. (a) The final density of stiflers for different influence coefficient $C = 0.05, 0.1$, and 0.2, transmission rate $\sigma = 0.5$, $\delta = 1$, average degree $\langle k \rangle = 6$, temperatures $T = 0$. (b) Comparison of the theoretical critical threshold and the simulation results, average degree $\langle k \rangle = 6$, temperatures $T = 0$.

which indicates the fake news is prone to outbreak as the average degree increases, and Fig. 7(b) shows the final density of the stiflers increases as the average degree of nodes increases.

5. Conclusion

In this paper, the fake news spreading model with the similarity has been proposed and investigated on the PSO-based networks. The relationship between the similarity of nodes and the product of their degrees is analyzed by simulations, and the results show that the similarity of two connected nodes decreases as the product of their degrees increases. Moreover, the Pearson correlation coefficient of angular distance between two connected nodes and the product of their degrees decreases as the temperature T increases. Based on this, the fake news spreading model with similarity is proposed, in which the similarity between two connected nodes is expressed as the product of the function of their

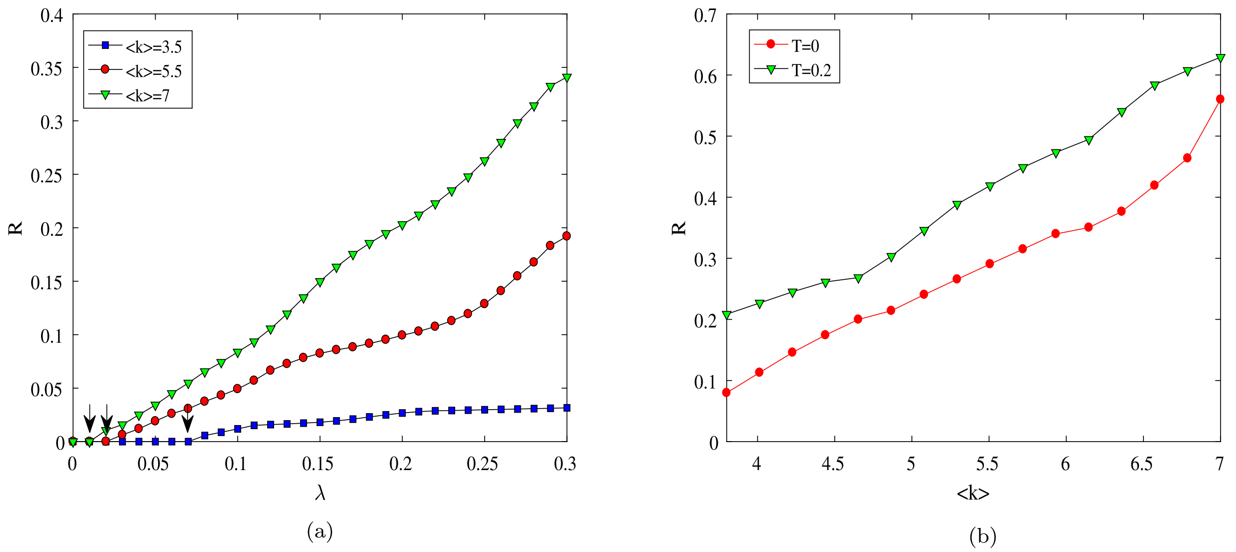


Fig. 7. Monte-Carlo simulations on the effect of average degree on threshold and final densities of the stiflers in the fake news spreading model with similarity. (a) The final density of stiflers for different average degrees $\langle k\rangle = 3.5$, 5.5, and 7, transmission rate $\sigma = 0.5$, $\delta = 1$, influence coefficient $C = 0.1$, temperature $T = 0$, (b) Effect of average degree ⟨k⟩ on the final densities of the stiflers, transmission rate $\lambda = 0.2$, $\sigma = 0.2$, $\delta = 0.1$, influence coefficient $C = 0.2$.

degrees. Then a steady-state analysis is conducted to investigate the critical threshold, and the results show the critical threshold is related to both the influence coefficient and the similarity function. Simulation results show a larger T leads to a larger final density of the stiflers, and a smaller influence coefficient leads to a larger critical threshold and smaller final density of stiflers.

The proposed model can also be applied to describe the rumor spreading on online social networks. Moreover, it can be extended to other rumor or fake news spreading models, such as the SI model and SEIR model, which will be further studied in the near future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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