Fast Star Pattern Recognition Using Spherical Triangles

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Ideal Star Tracker Algorithm

- Fast - particularly for tumbling situations
- Reliable
  - Maximize correct determinations
  - Minimize incorrect determinations
- Consume little in computer resources such as RAM, storage and processor capability

Star Pattern Recognition

- Method by which star tracker determines the stars to which it is pointed
- Many methods, many proprietary
  - Angle, Magnitude, “a priori,” lost-in-space
- Angle Method & Spherical Triangle Method
  - Both methods tested and results compared

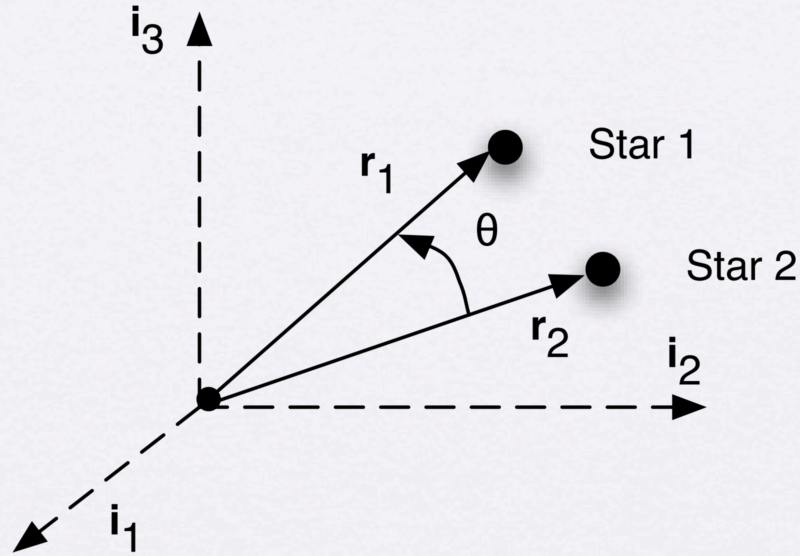
Methods

Angle Method

- Angles between stars stored in catalog

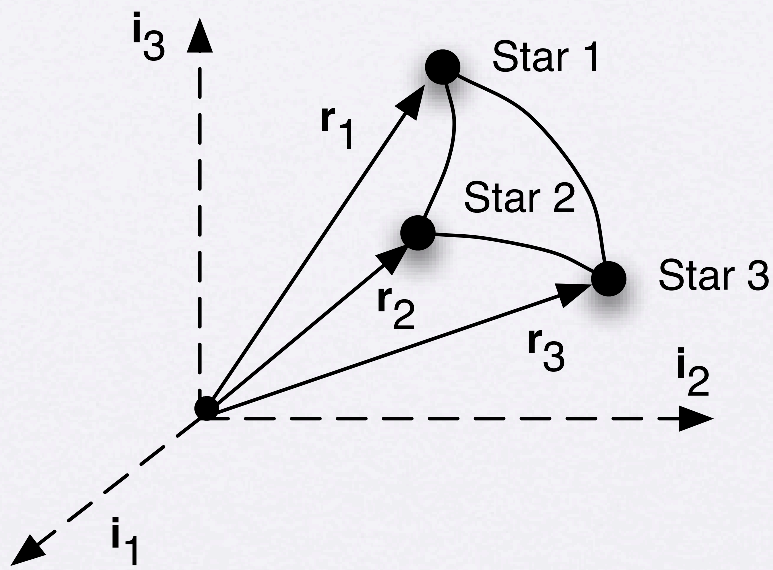
- Cosine of angles between stars in field of view matched to cosine of angles in catalog

- Must have at least two stars in field of view to be useful



Spherical Triangle Method

- Catalog of spherical triangles made
- Area and polar moment of spherical triangle made from stars in FOV matched to those in catalog
- Must have at least three stars in field of view to work.



Measurement Error

- Matching angles and spherical triangles made difficult because of star tracker position measurement error
- Typical star tracker error has normal distribution and a standard deviation (σ) of 87 microradians

Measurement Error

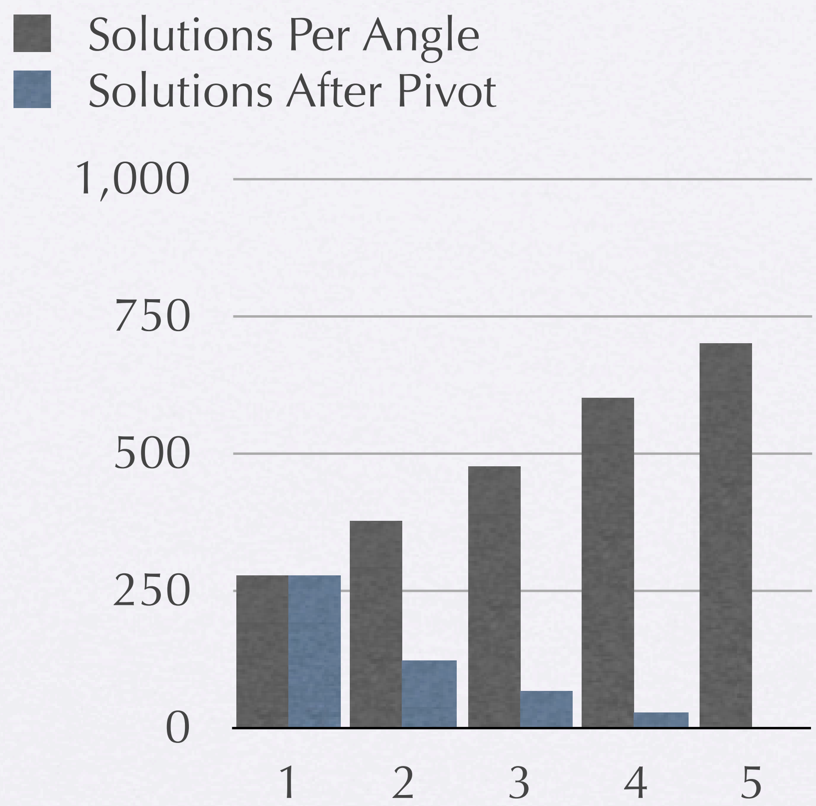
- Choose a σ bound for measurement
- Larger σ means greater probability of correct angle or spherical triangle existing within bounds, but also makes it more likely more than one angle or spherical triangle can be the correct solution.
- Probability of true position lying within 3σ bound of measurement is 99.7%.

Measurement Error

- Typical angle measurement will result in a hundred or more possible solutions.
- Typical spherical triangle area measurement will result in hundreds or thousands of possible solutions.
- Method used to approach a single solution is called “pivoting.”

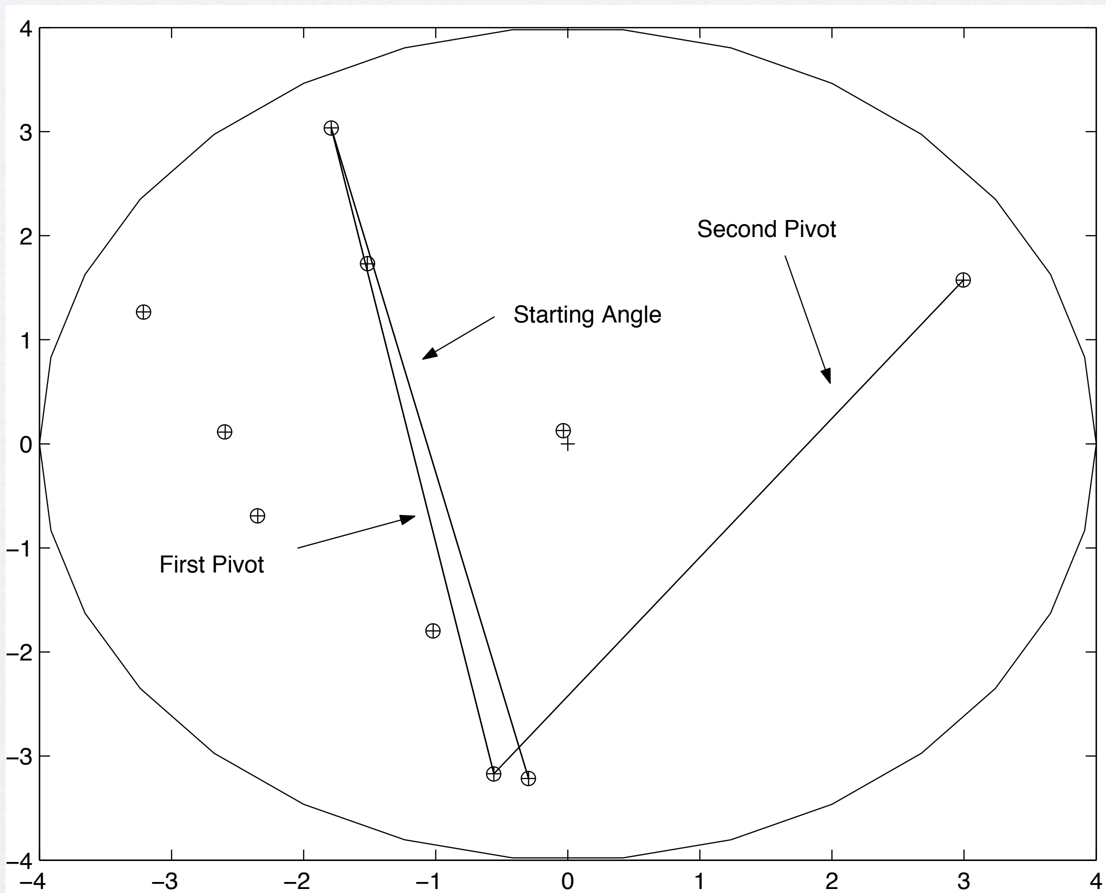
Pivoting With Angles

- If two angles are known to share a star in common, the solution to both must also have a star in common
- Multiple pivots can ultimately lead to a single solution
- Pivot ordering is computationally expensive



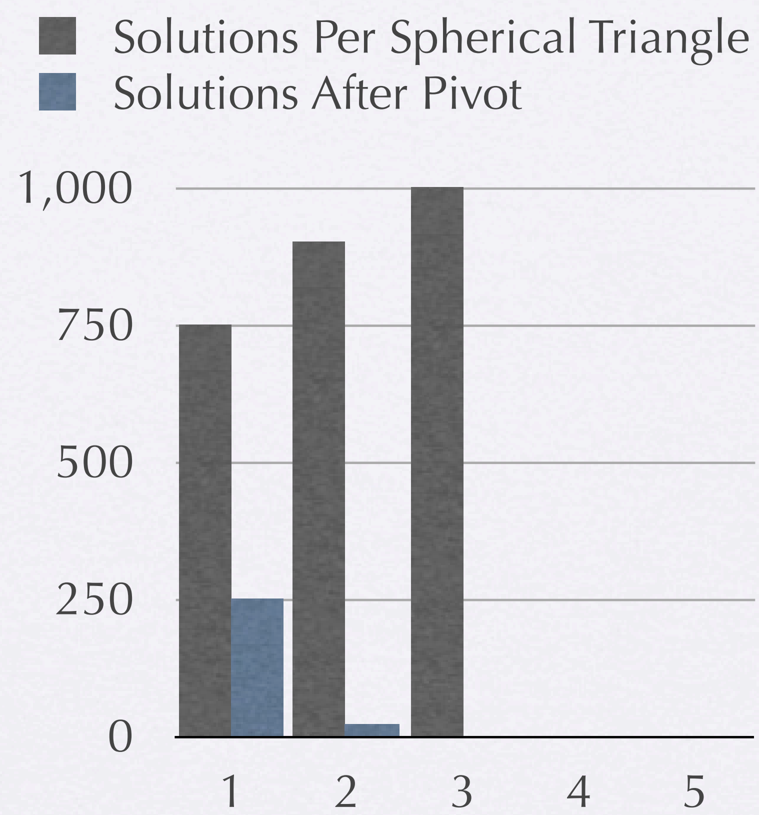
Pivoting to Solution

- Matlab output shown here



Pivoting With Spherical Triangles

- Similar to angle pivots
- If two spherical triangles are known to share two stars in common, the solution to both must also have two stars in common
- Will use polar moment also reduce possible solutions



Pivoting to Solution

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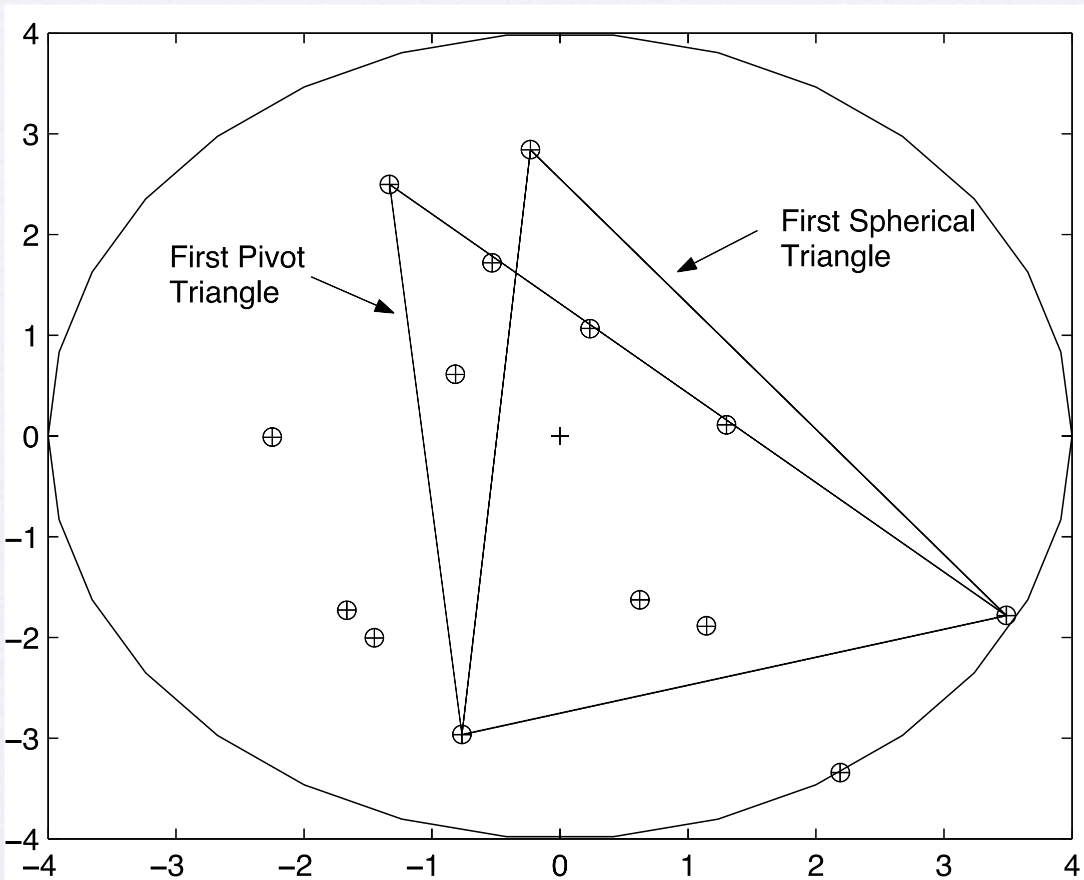
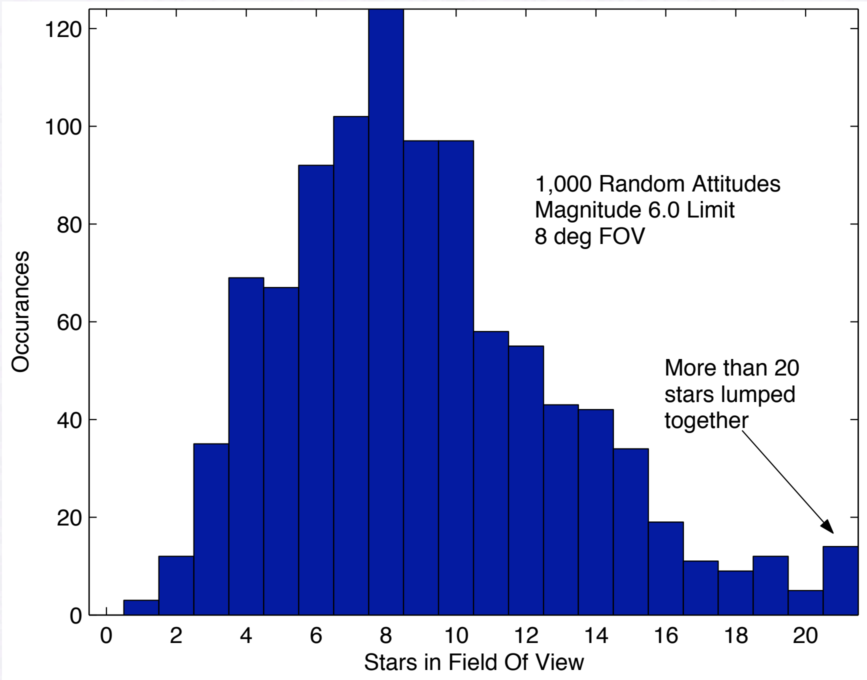


Catalog Creation

Star Catalog

- Mag 6, 8 deg FOV star tracker assumed
- Star catalog used has 8,118 stars magnitude 6.0 or brighter
- Entire celestial sphere for lost-in-space testing



Angle Catalog Requirements

- Angles between pairs of stars, sorted by angle
- Stars that make up the angle
- K-Vector to speed finding a particular pair of stars by it's angle

Angle Catalog Created

- 106,308 angles found
- 12 MB file

Spherical Triangle Catalog Requirements

- Area of triangles formed by triplets of stars, sorted by area.
- Polar Moment of triangles
- Stars that make up the angle
- K-Vector to speed finding a particular pair of stars by it's area

Spherical Triangle Area

- Area calculated directly from vectors pointing to stars that make up the triangle

$$\mathcal{A} = 4\tan^{-1}\sqrt{\tan\frac{s}{2}\tan\frac{s-a}{2}\tan\frac{s-b}{2}\tan\frac{s-c}{2}}$$

$$s = \frac{1}{2}(a+b+c)$$

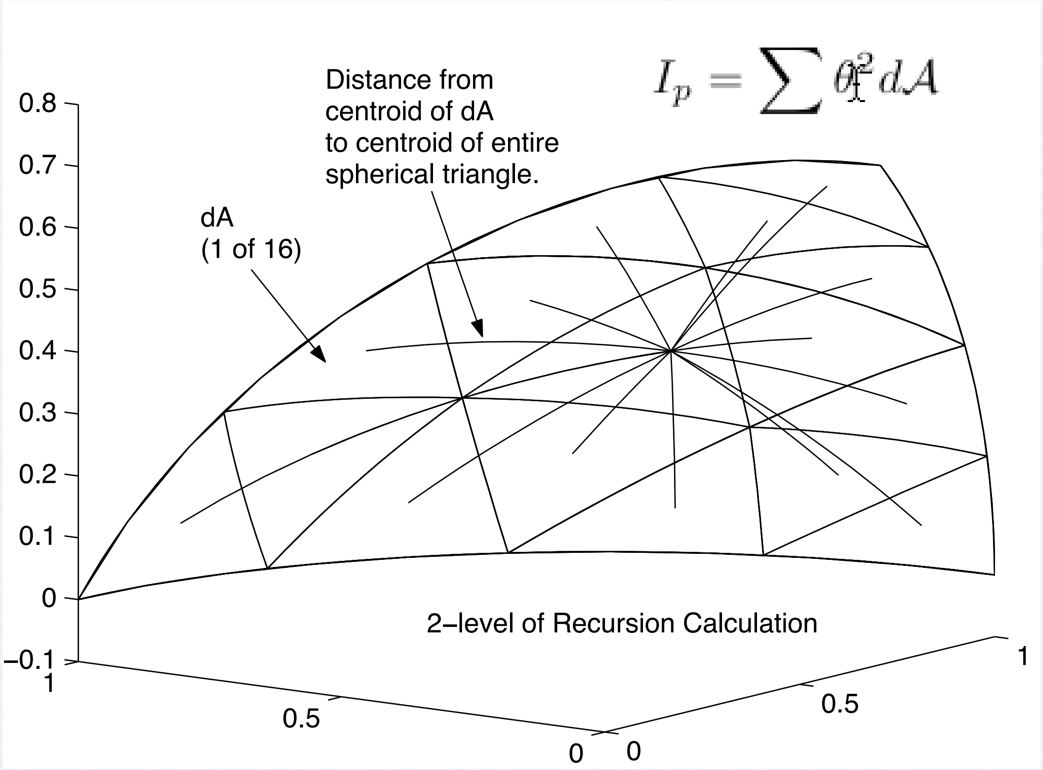
$$a = \cos^{-1}\left(\frac{\mathbf{b}_1 \cdot \mathbf{b}_2}{|\mathbf{b}_1||\mathbf{b}_2|}\right)$$

$$b = \cos^{-1}\left(\frac{\mathbf{b}_2 \cdot \mathbf{b}_3}{|\mathbf{b}_2||\mathbf{b}_3|}\right)$$

$$c = \cos^{-1}\left(\frac{\mathbf{b}_3 \cdot \mathbf{b}_1}{|\mathbf{b}_3||\mathbf{b}_1|}\right)$$

Spherical Triangle Polar Moment

- Equal to the sum of infinitesimal areas multiplied by the square of its distance from the axis of interest, in radians



Spherical Triangle Catalog Created

- 662,799 spherical triangles found
- 162 MB file

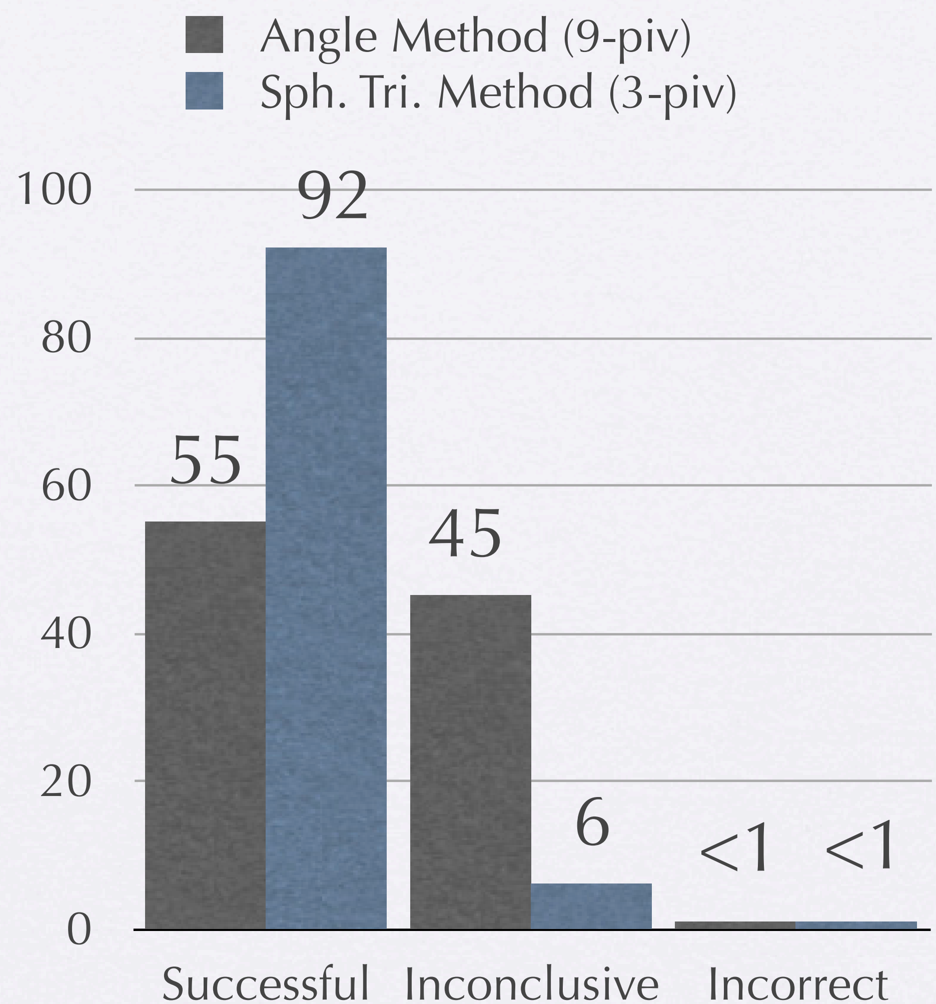
Testing Both Methods

Method of Testing

- 1,000 random attitudes for angle and spherical triangle algorithms tested
- Results of compiled
  - Correct, Cannot, Inconclusive & Incorrect
- False Star Inclusion

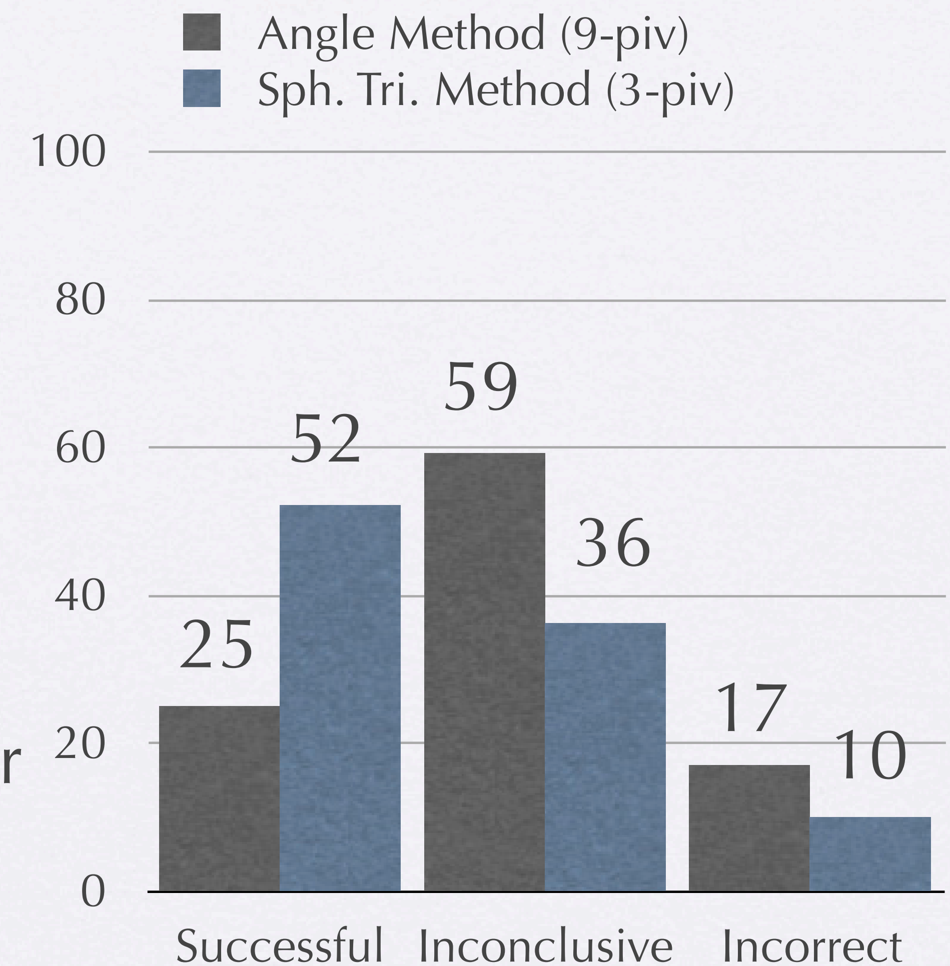
Overall Results, No False Star

- Spherical Triangle method almost twice as likely to get correct result.
- Without false star, incorrect results reflect 3σ bounds.



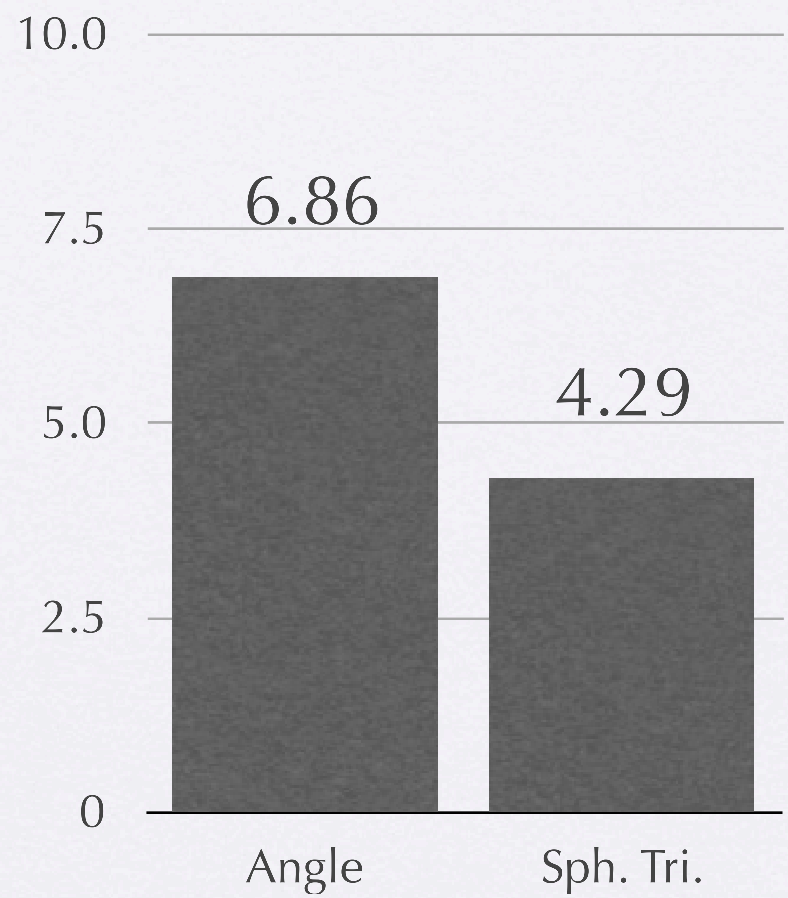
Overall Results, With False Star

- When a false star is included, the spherical triangle method is still twice as likely to reach correct result.
- Fewer incorrect results because fewer stars are needed.



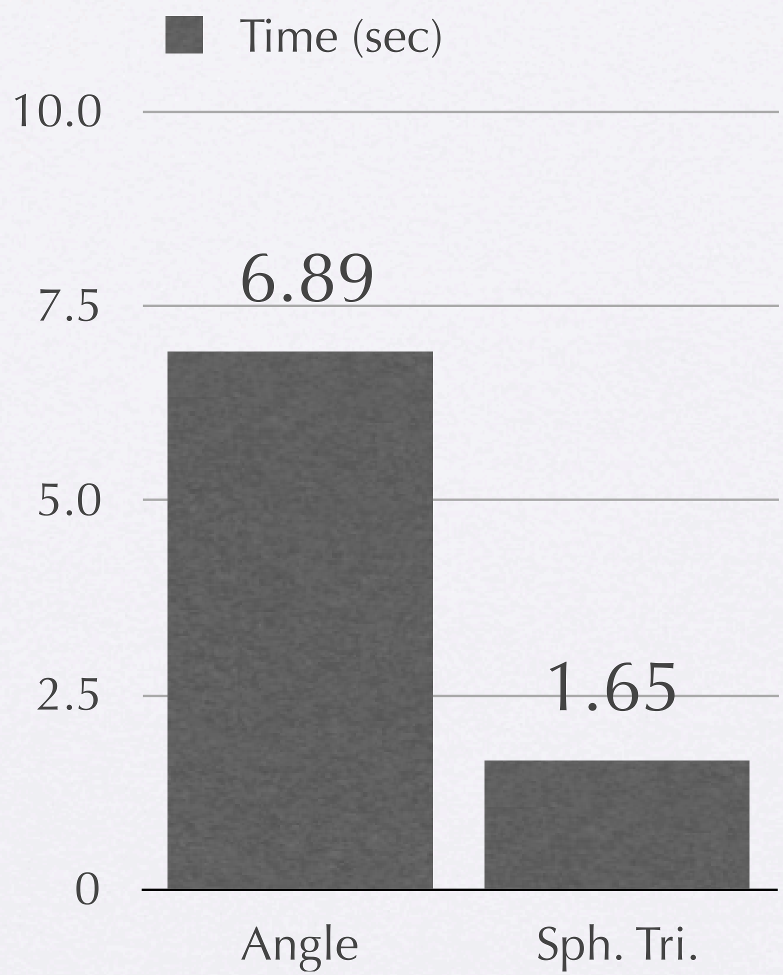
Average Stars Needed

- Fewer pivots means fewer stars.
- Fewer stars improves odds of successful result with false star.



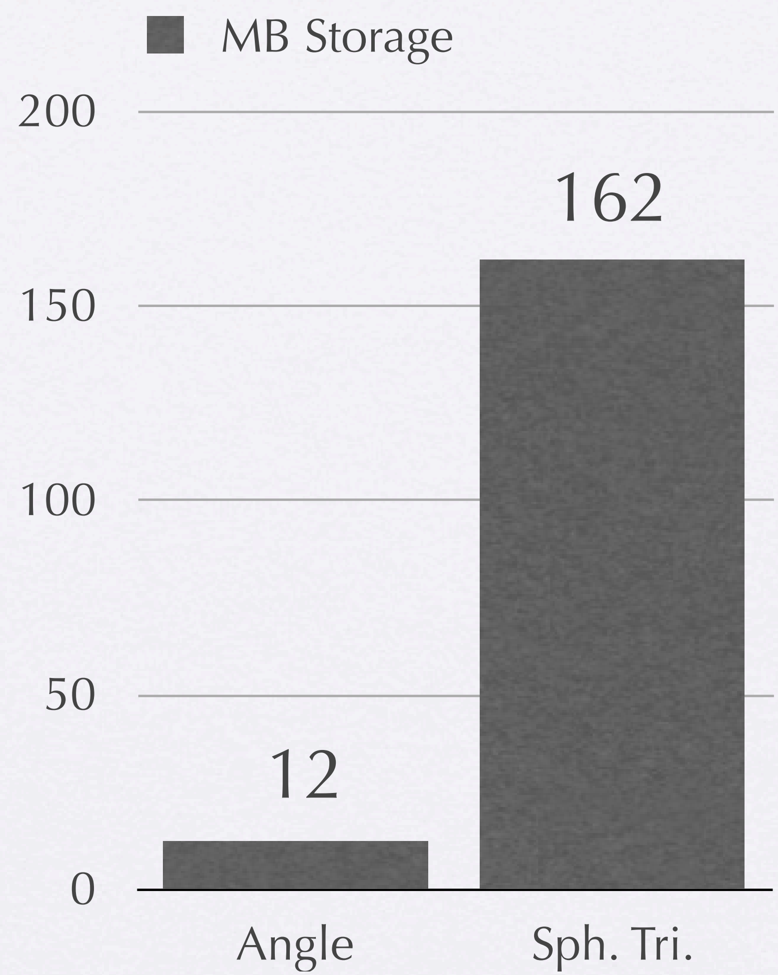
CPU Time

- Greatest amount of CPU time spent determining order of pivots
- Spherical Triangle Method, despite more involved math, is faster overall, since fewer pivots are required.



Catalog Storage Requirements

- All this success comes at the expense of storage space.
- Spherical triangle catalog is more than ten times larger than the angle catalog.
- Storage always getting cheaper.



Questions?