Fast Star Pattern Recognition Using Spherical Triangles

Thesis Defense

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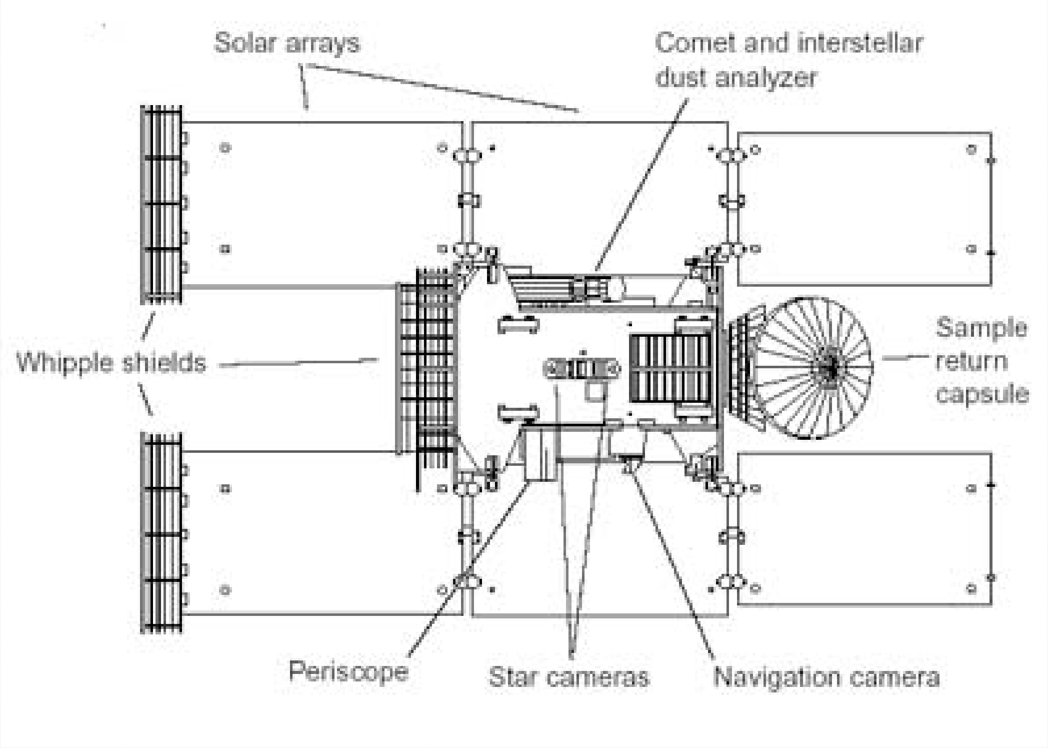
Star Trackers

- Imaging device which tries to determine attitude by determining what stars lie within its field of view
- Used by many satellites and spacecraft for attitude control
- Typical star tracker has 8 deg field of view and is sensitive to magnitude six stars

Star Trackers In The News

Congratulations Stardust!



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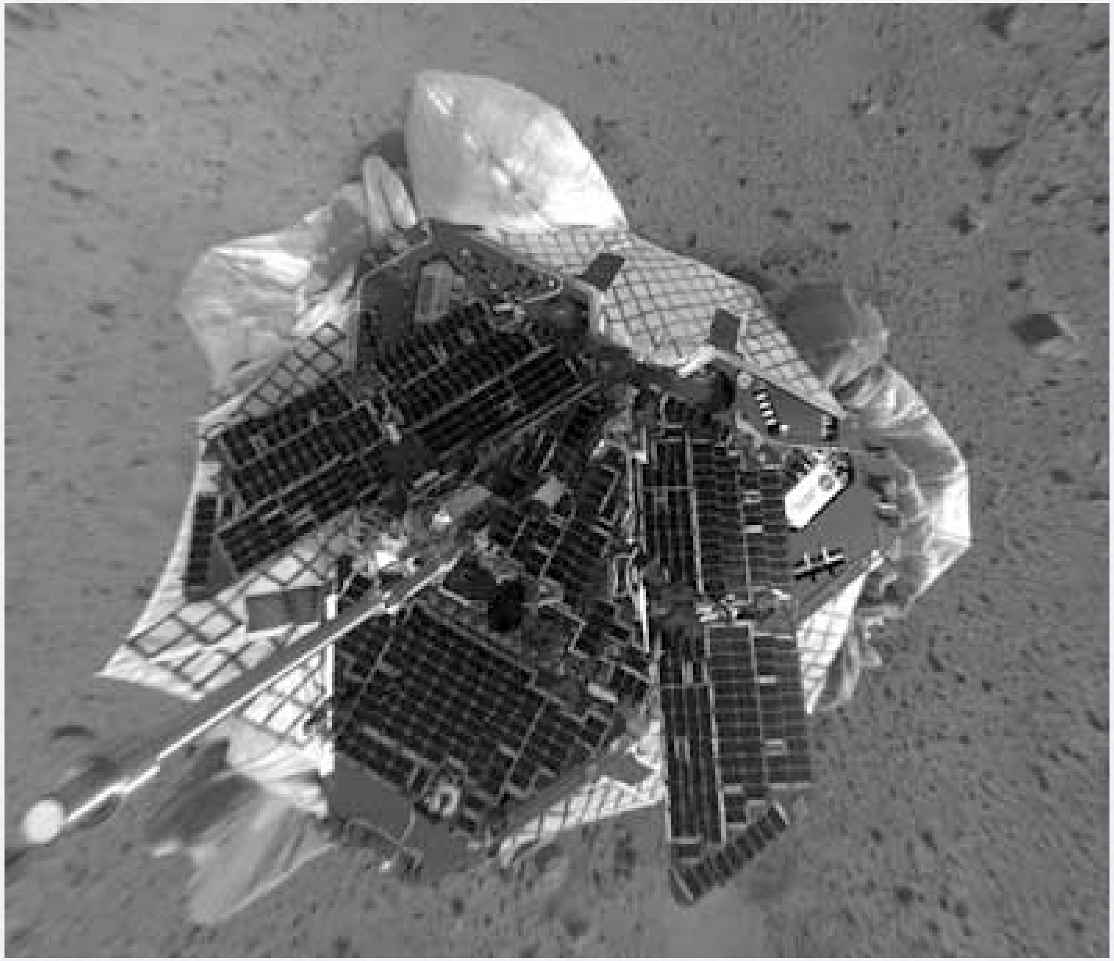
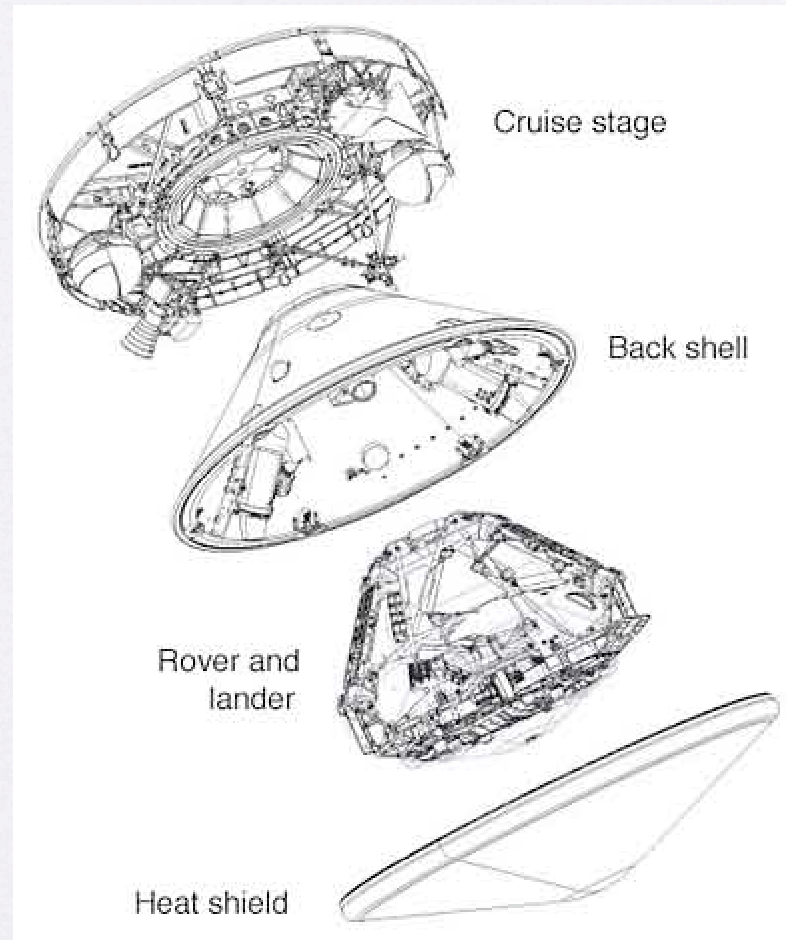
Congratulations Spirit!





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Ideal Star Tracker

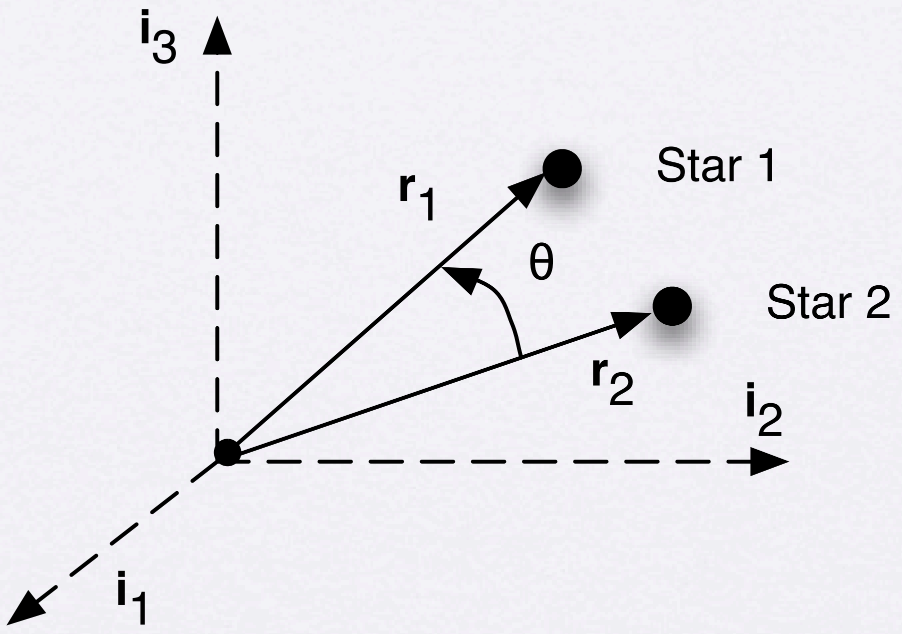
- Fast - particularly for tumbling situations
- Reliable
    - Maximize correct determinations
    - Minimize incorrect determinations
- Consume little in computer resources such as RAM, storage and processor capability

Star Pattern Recognition

- Method by which star tracker determines the stars to which it is pointed
- Many methods, many proprietary
- Angle Method
- Spherical Triangle Method
- Both methods tested and results compared

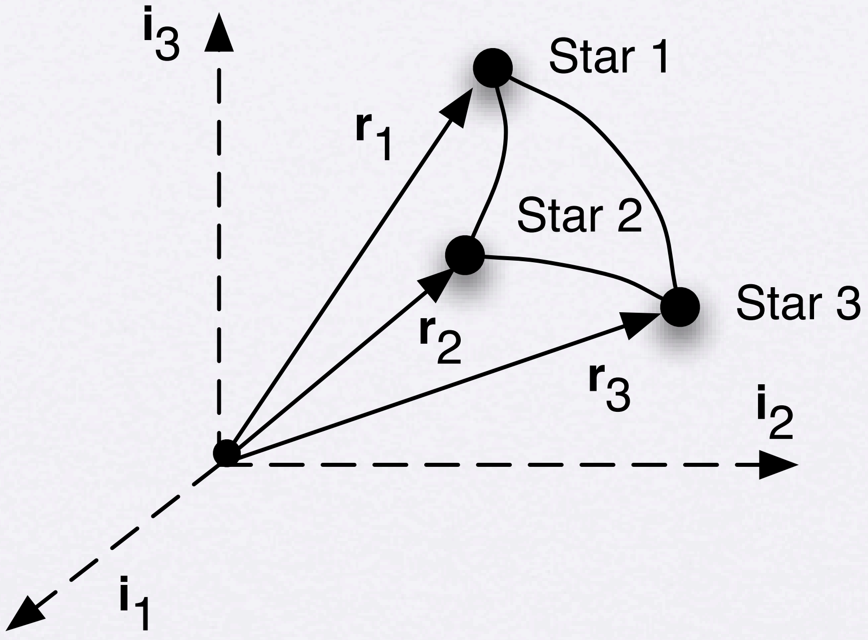
Angle Method

- Angles between stars stored in catalog
- Cosine of angles between stars in field of view matched to cosine of angles in catalog
- Must have at least two stars in field of view to be useful



Spherical Triangle Method

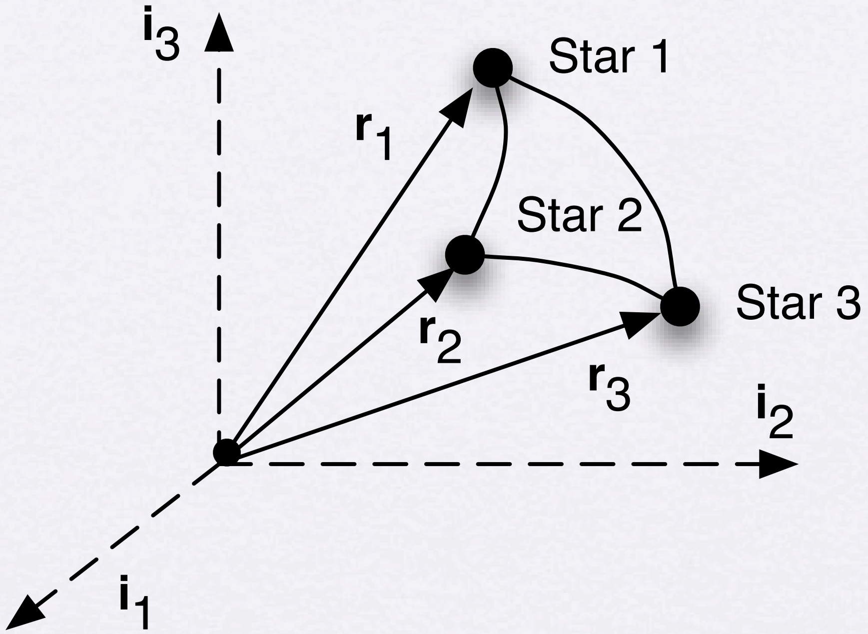
- Catalog of spherical triangles made
- Area and polar moment of spherical triangles matched to those in catalog
- Must have at least three stars in field of view to work.



Spherical Triangle Method

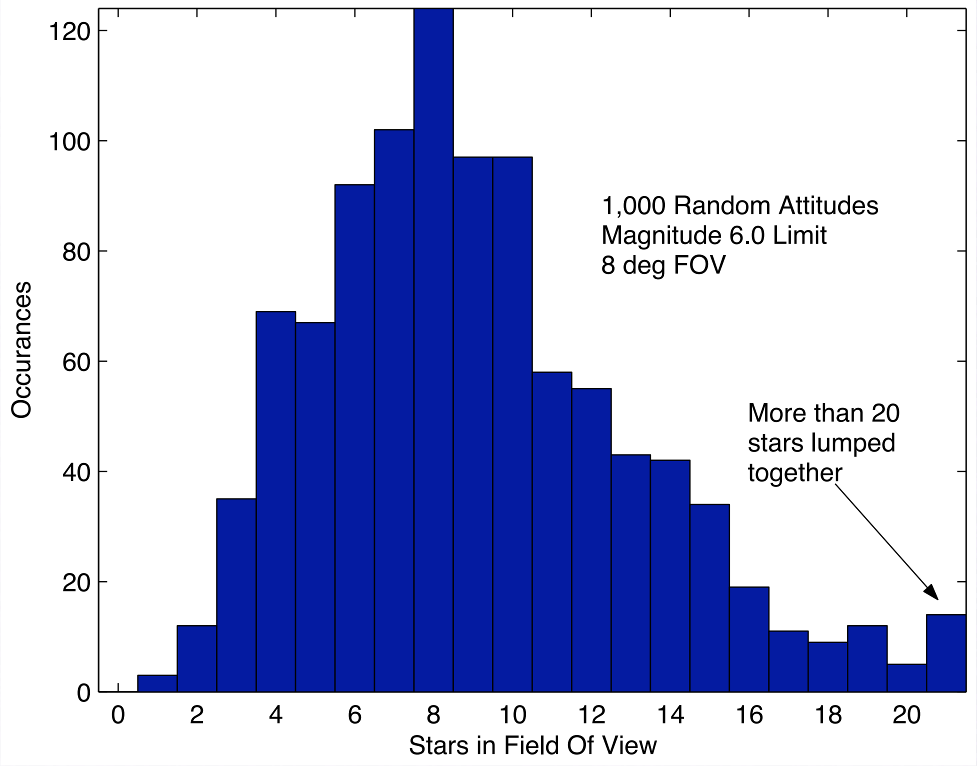
- Area determined from vectors pointing to each star

- Polar moment determined by summation of elements



Star Catalog

- Star catalog used has 8,118 stars magnitude 6.0 or brighter
- Earth-Centric Inertial (ECI) Coordinate System
- Random attitude testing reveals probability of seeing certain number of stars in 8 deg FOV



Effects of Measurement Error

- Matching made difficult because of star tracker position measurement error
- Error typically has normal distribution and a standard deviation (σ) of 87 microradians
- Upper and lower bounds placed on measurement based on chosen σ
- Probability of true position lying within 3σ bound of measurement is 99.7%

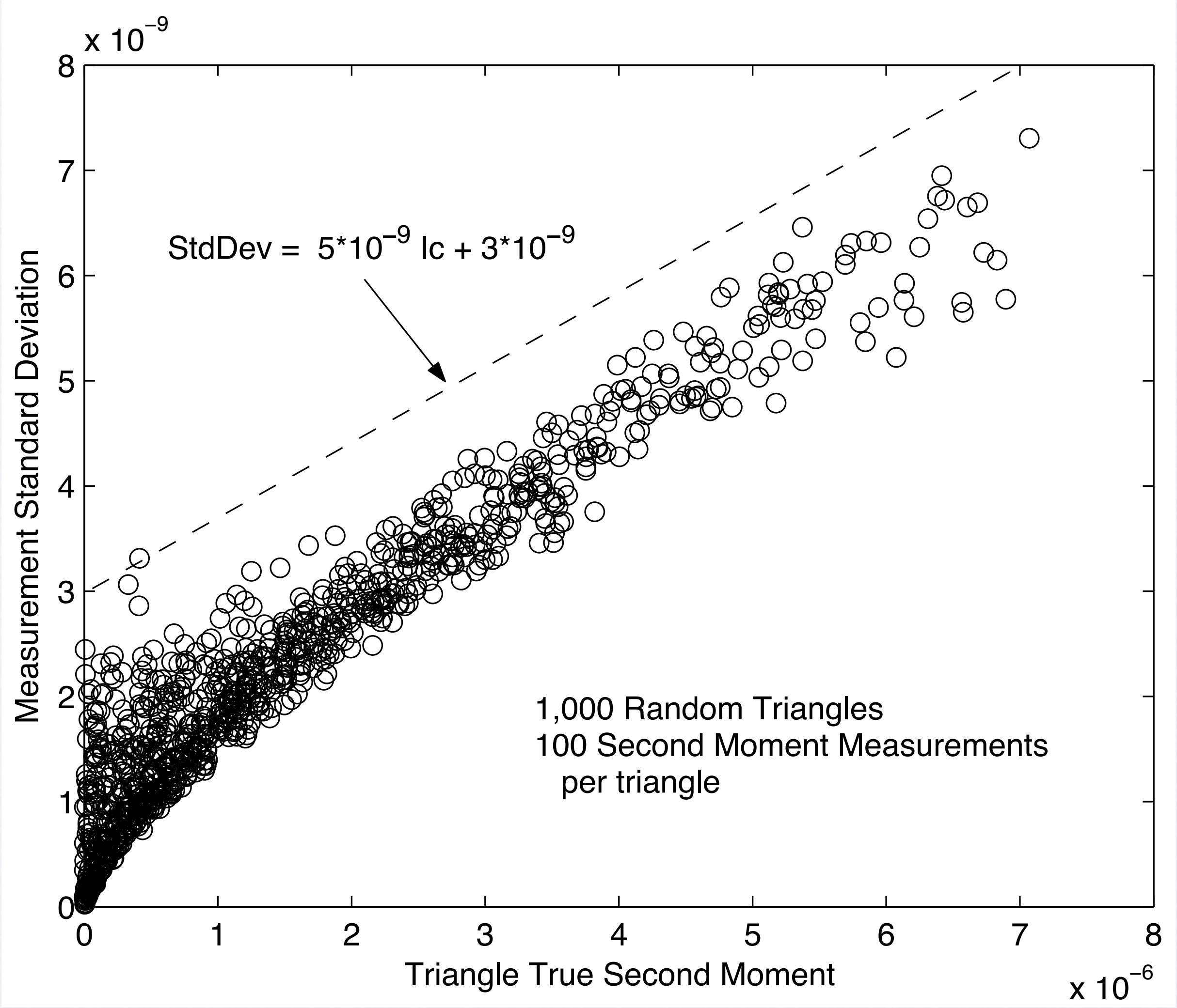
Effects of Measurement Error

- Standard Deviation of Area
  - Covariance analysis made
  - Standard Deviation can be determined analytically from vectors pointing to vertices of spherical triangle

Effects of Measurement Error

- Standard Deviation of Polar Moment determined graphically.

- Method overestimates actual standard deviation, but ensures meeting standard deviation requested

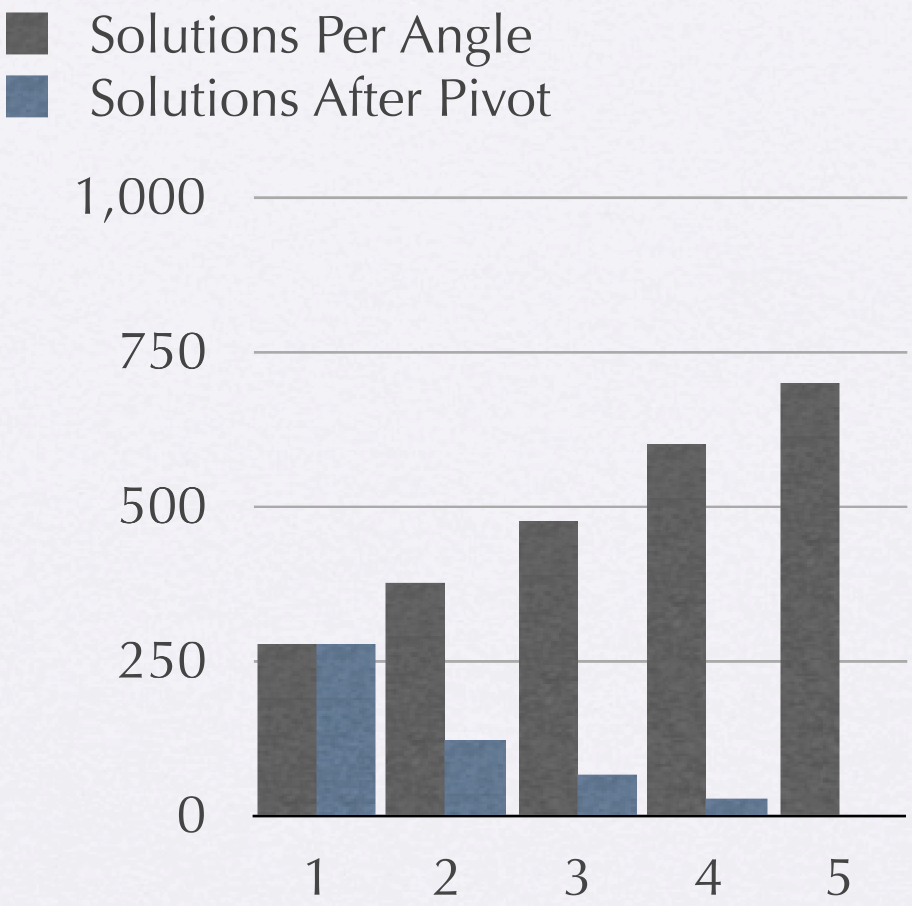


Effects Of Measurement Error

- Within 3σ bounds can lie hundreds or thousands of possible solutions to an angle or spherical triangle in FOV
- Solution is “pivoting” - a method by which many angles or spherical triangles in the FOV can be used together to approach a single solution

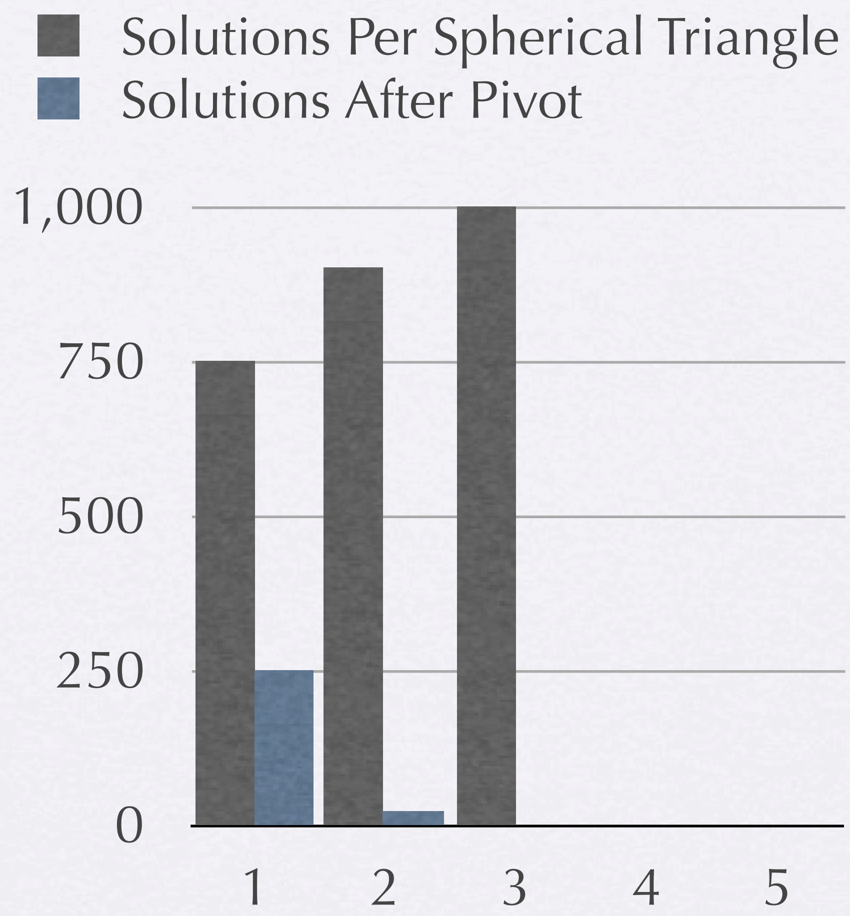
Pivoting With Angles

- If two angles are known to share a star in common, the solution to both must also have a star in common
- Multiple pivots can ultimately lead to a single solution
- Computationally expensive



Pivoting With Spherical Triangles

- Similar to angle pivots
- If two spherical triangles are known to share two stars in common, the solution to both must also have two stars in common
- Will use polar moment also reduce possible solutions



Triangle & Spherical Triangle Catalog Creation

- Brute-force testing of every combination of stars is difficult at best
    - Angle Method - 33 million combinations
    - Spherical Triangle Method - 89 billion combinations!
- Spherical Triangle testing with Matlab would take common hardware weeks

Bettter Data Structure Required

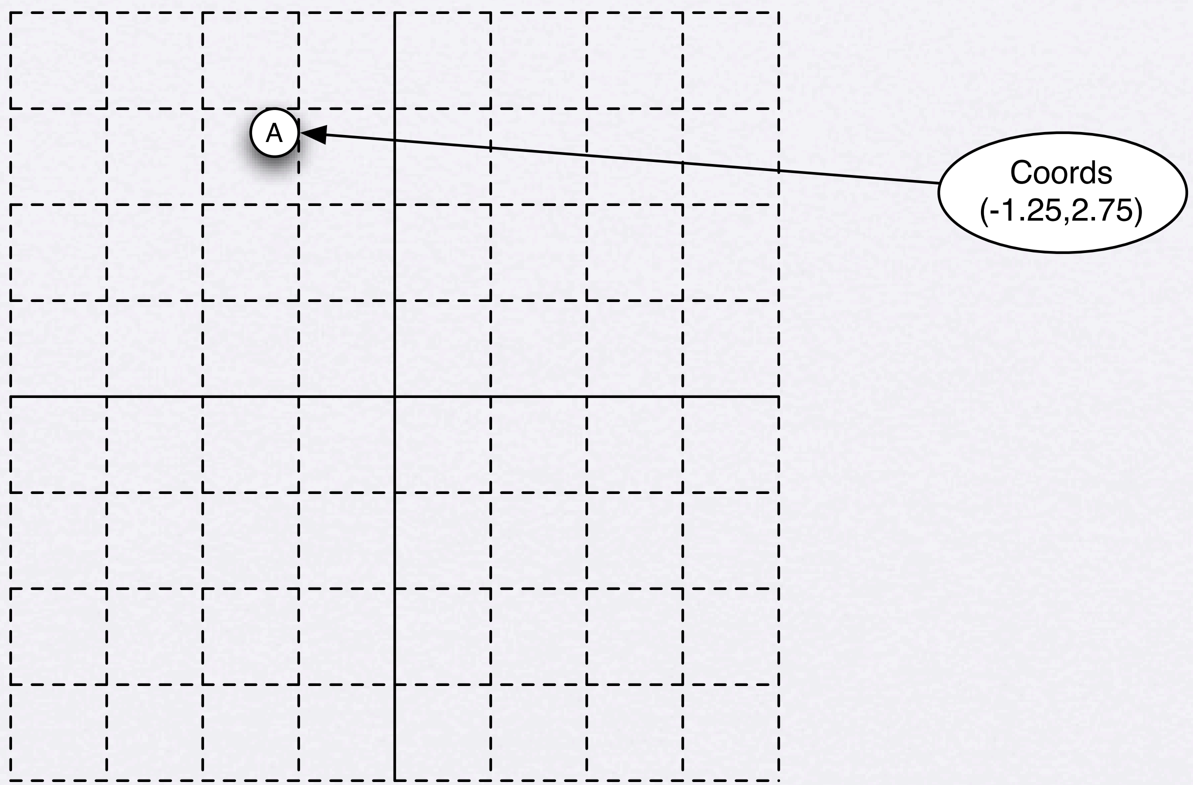
- Need way to reduce combinations to test
- Data structure needed that will store stars in such a way that stars obviously far apart will not be tested.
- Quad-Tree Structure used

Quad-Tree Data Structure

- Commonly used to store objects in 2-D space
- Object's position in space can be inferred from its position in the quad-tree
- Objects residing in same area of space can be quickly found
- Will be modified so that searches for objects near another can be made

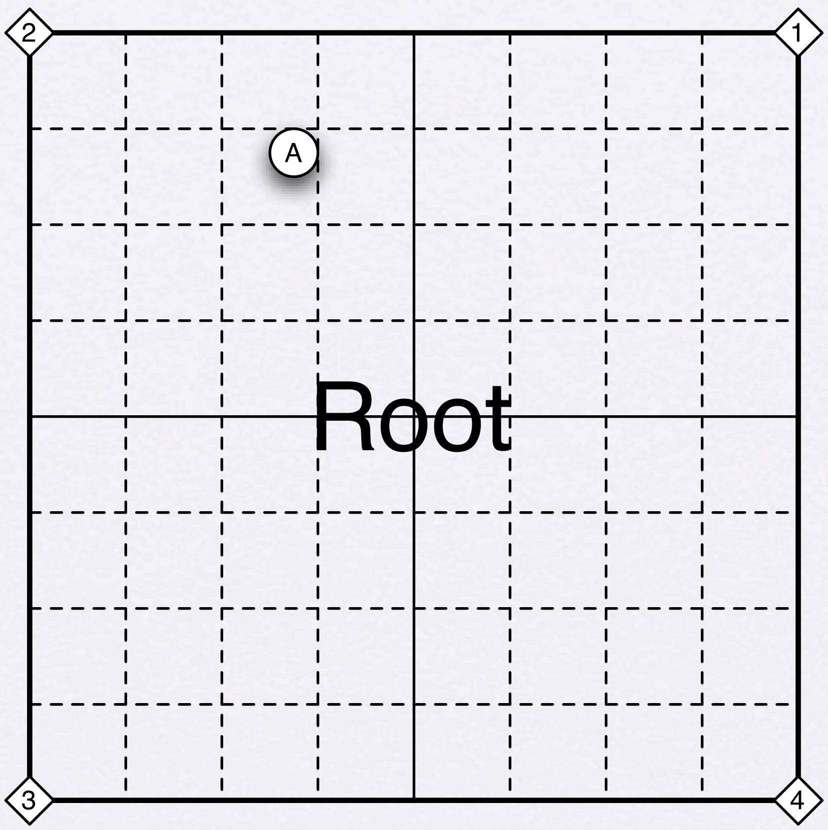
2-D Quad-Tree Example

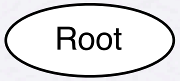
- Start with an 8x8 unit space, with object A



2-D Quad-Tree Example

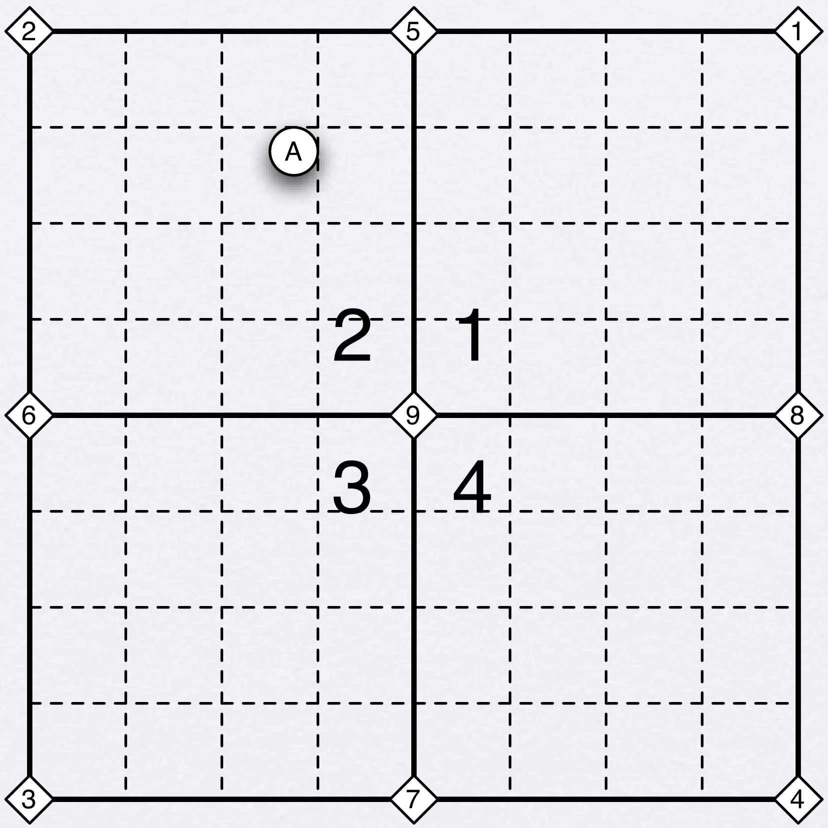
- Start with Root Quadrant & four Vertices

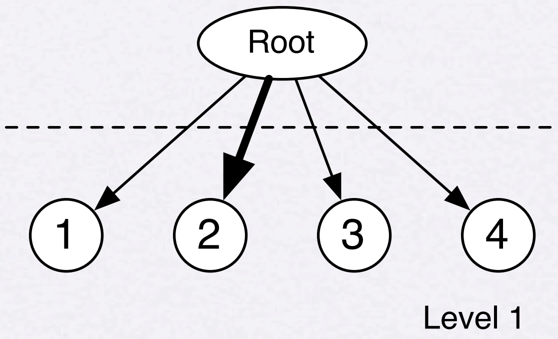




2-D Quad-Tree Example

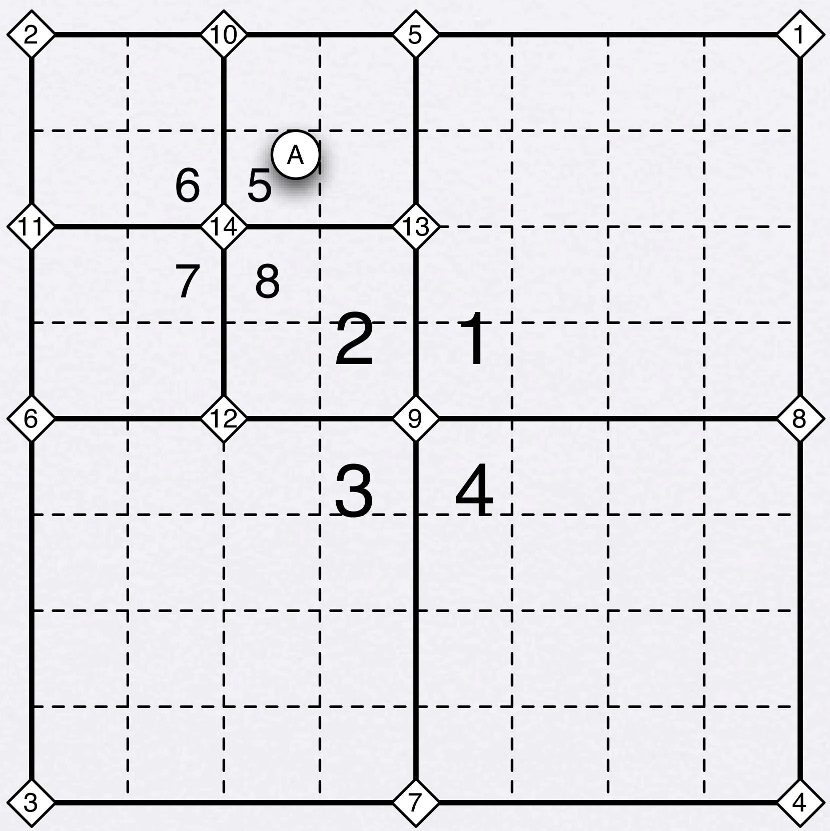
- Divide into four first-level quadrants. Add vertices as required.

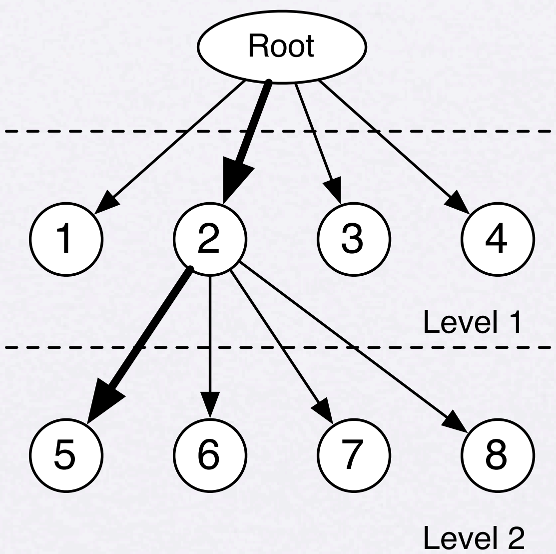




2-D Quad-Tree Example

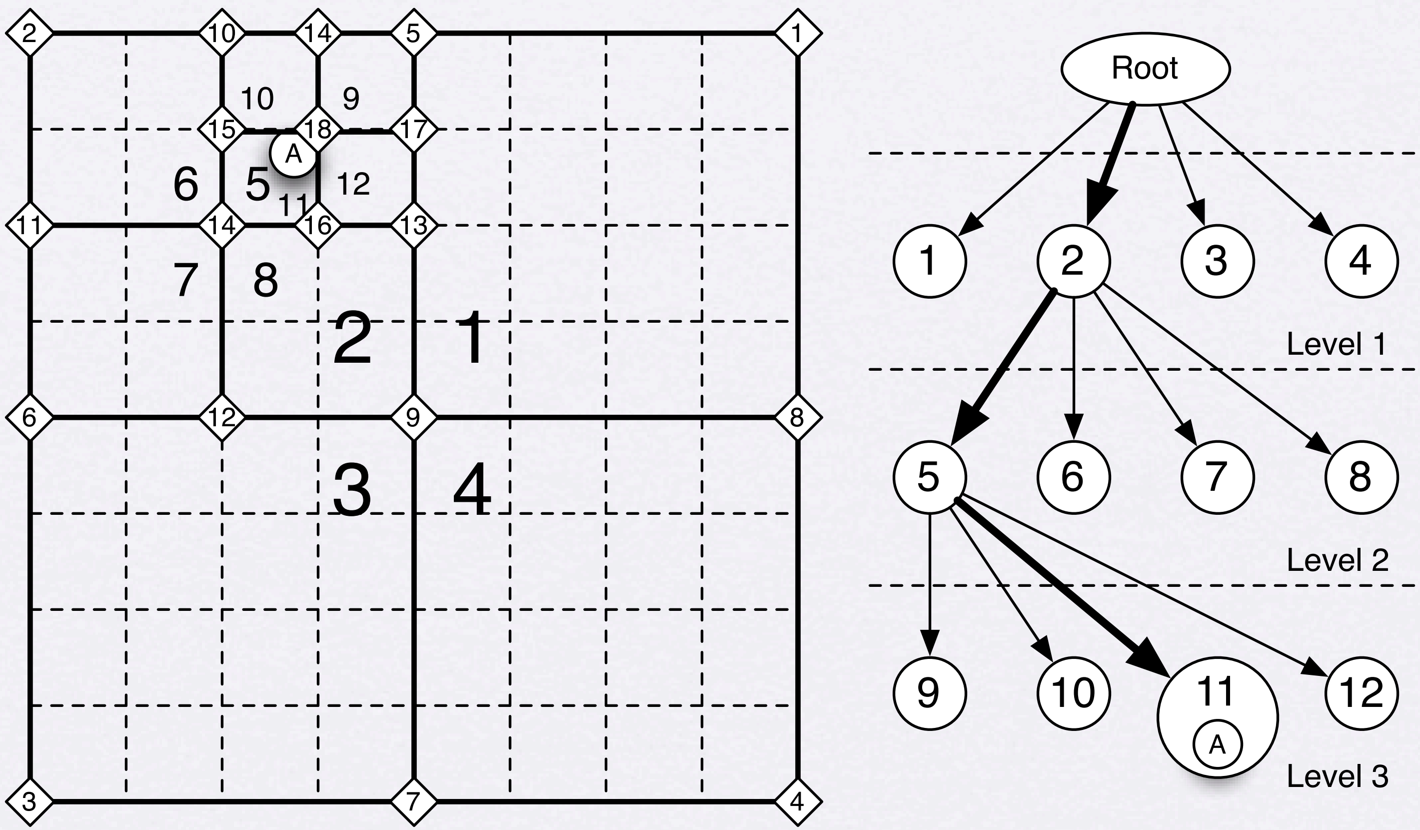
- Divide again to improve precision of location





2-D Quad-Tree Example

- Divide again, target level reached. Vertices store quads. Object A stored in quadrant 11.

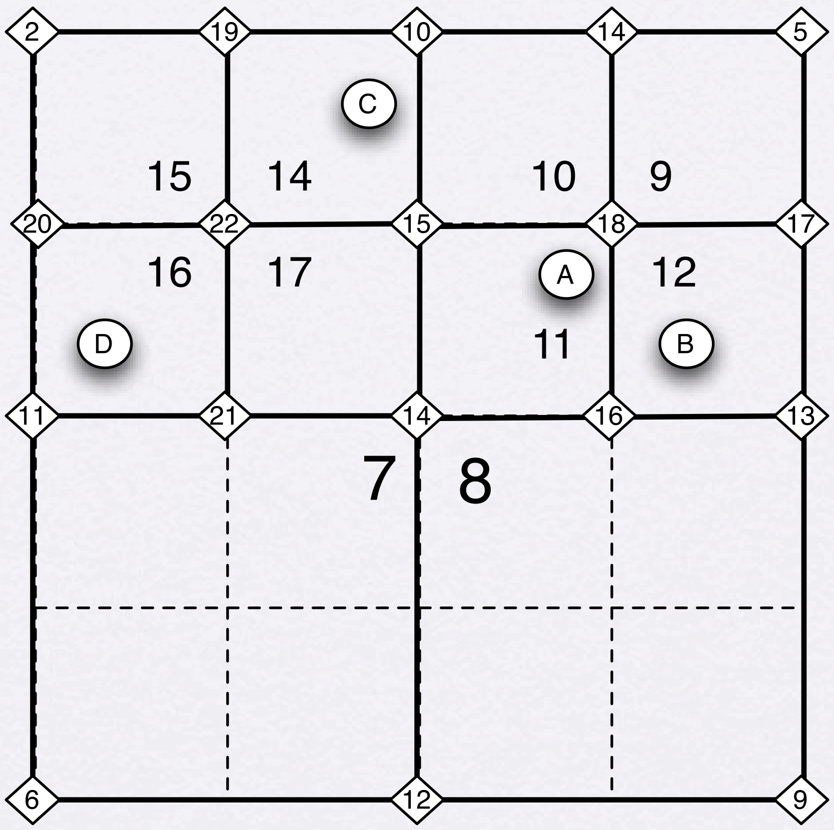


2-D Quad Tree Searches

- Easy to find what objects lie within a particular area of space
- Harder to determine what objects lie within a certain distance of another object
- Vertex structures are now used

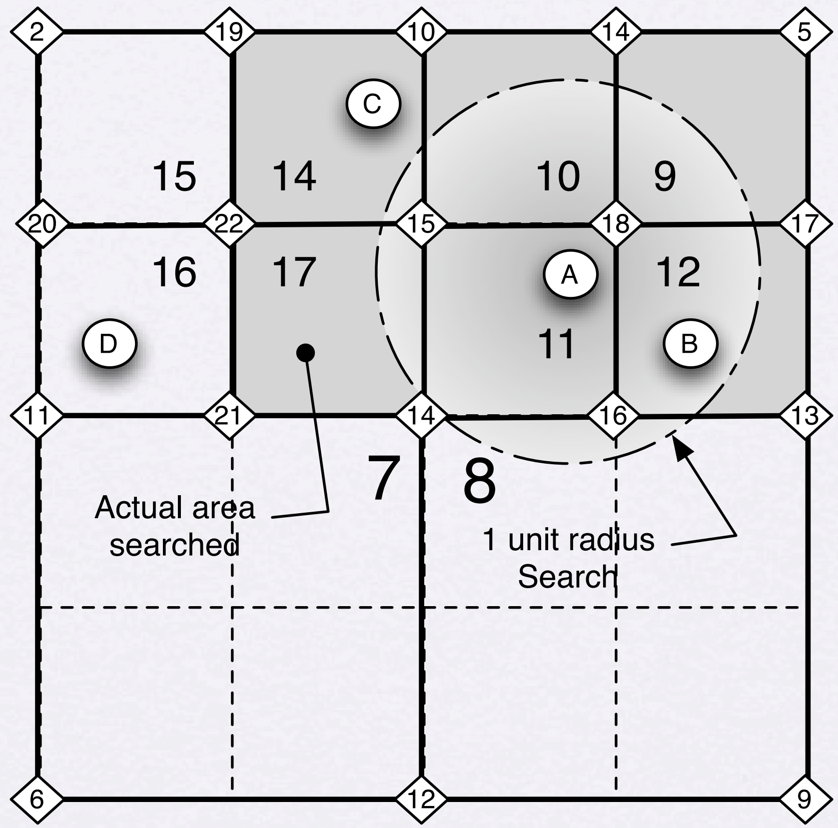
Proximity Searches

- Additional objects placed into quad tree.
- Nodes and vertices added as necessary
- Vertices store level-three nodes only
- Want to know what's within one unit of object A



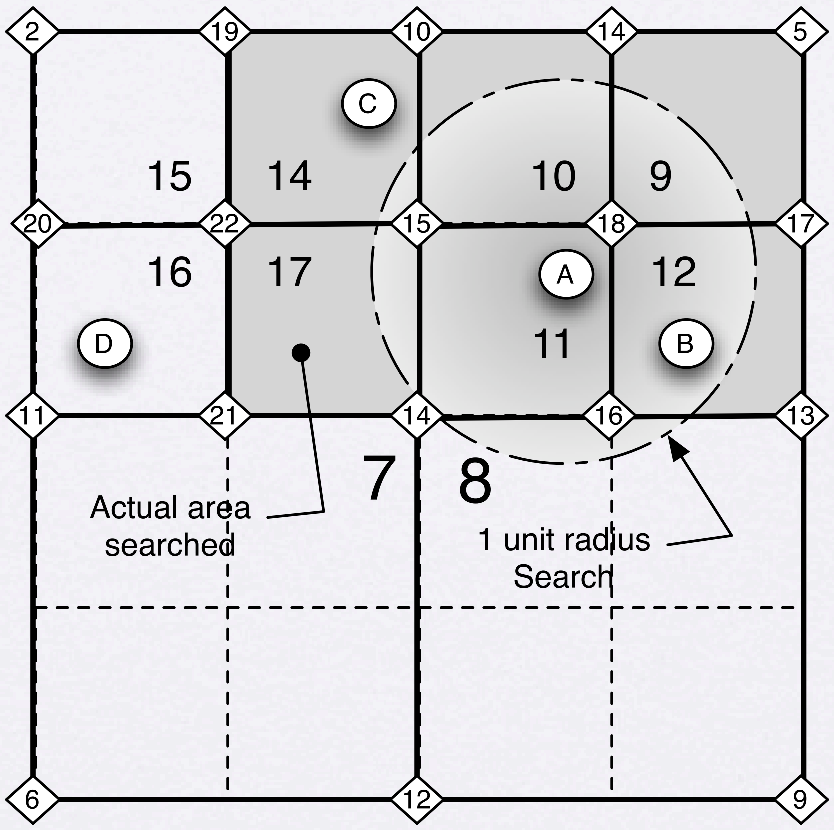
Proximity Searches

- Object A in node 11, node touches vertices 14, 15, 16 and 18
- Vertices combined touch quadrants 9, 10, 11, 12, 14 and 17
- Need to test objects in those nodes for proximity to object A.



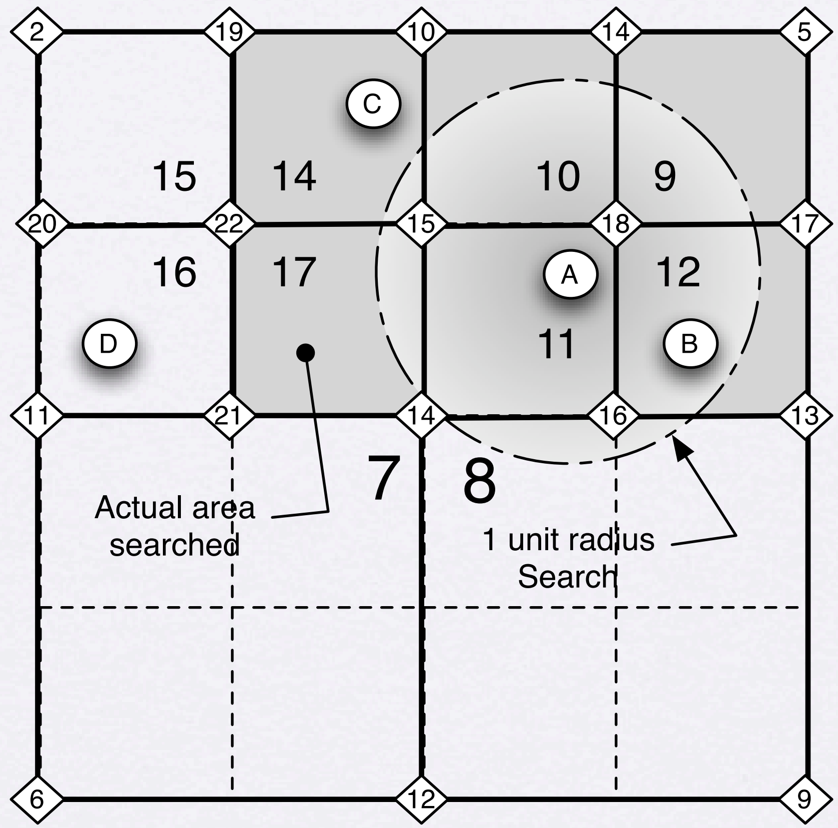
Proximity Searches

- Object B within one unit, object C is not.
- Object D is not tested
- Not necessary to scan node 8
- Reduces amount of work necessary when lots of objects exist



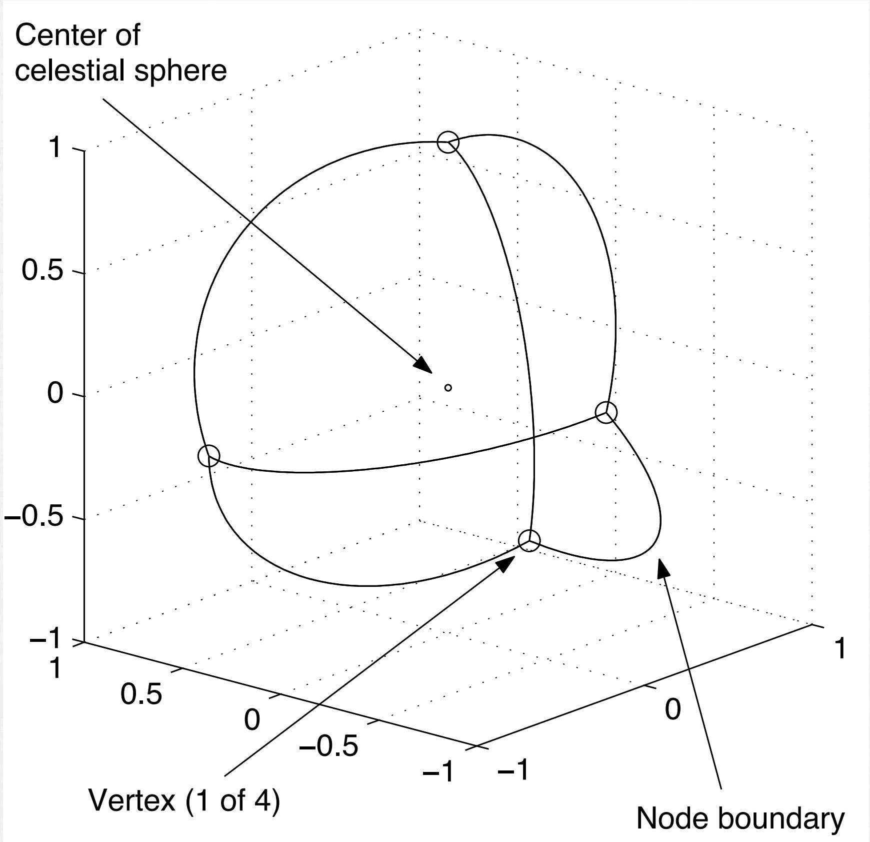
Proximity Searches

- Depth of quad-tree determines distance that can be reliably searched.
- Limit is width and height of a target level quadrant.
- Search distance too big for quad-tree depth - objects may be missed
- Search distance to small for quad-tree depth - inefficient searches



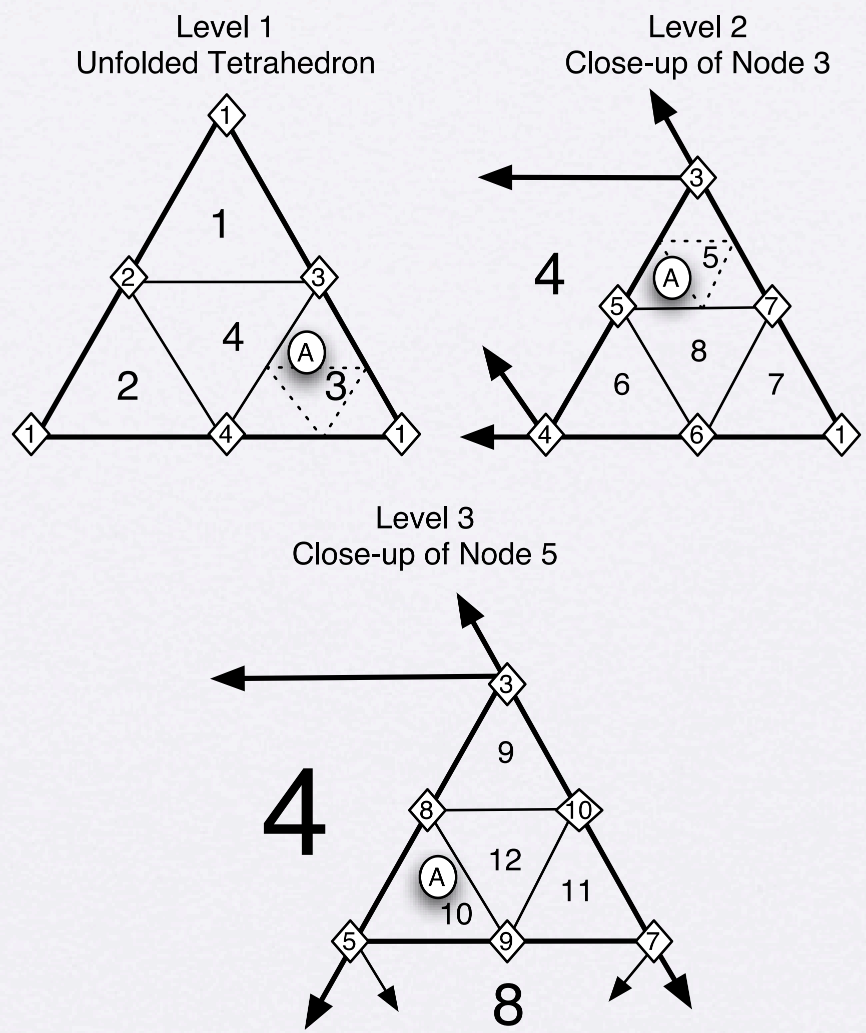
Spherical Quad-Tree

- Modify quad-tree structure for dividing the surface of a sphere.
- Divide surface into four spherical triangles.
- First-level of quad-tree is a spherical tetrahedron



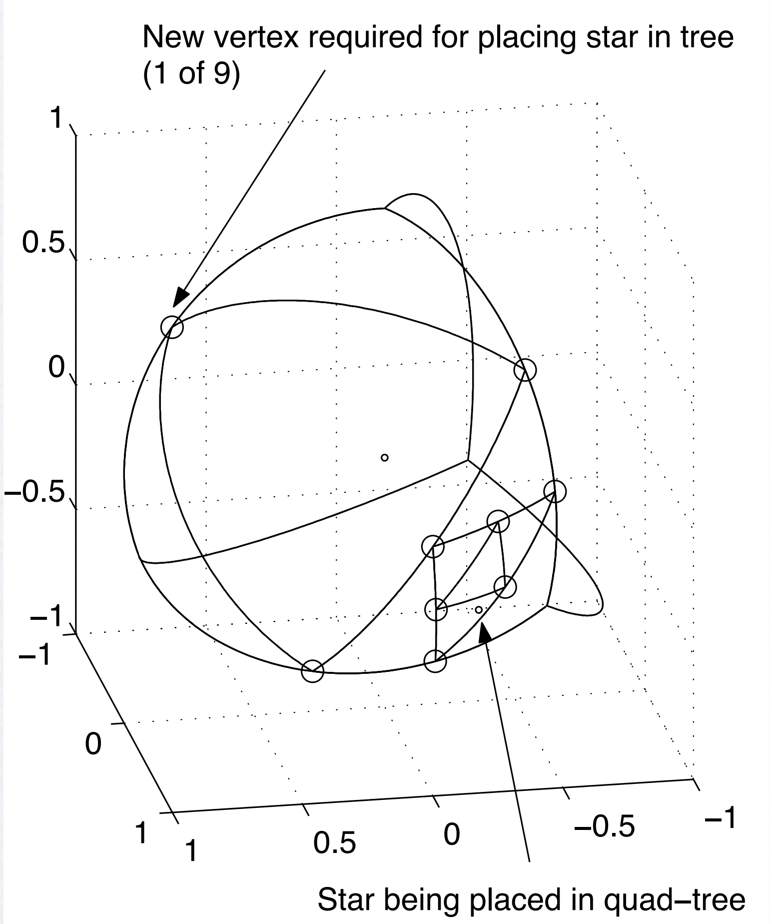
Triangular Quad-Tree Structure

- Unfolded, 2-D version of spherical tetrahedron
- Each triangle can be divided into four more triangles
- A inserted in same manner as 2-D quad-tree - quadrants now call nodes.
- Vertices added as required



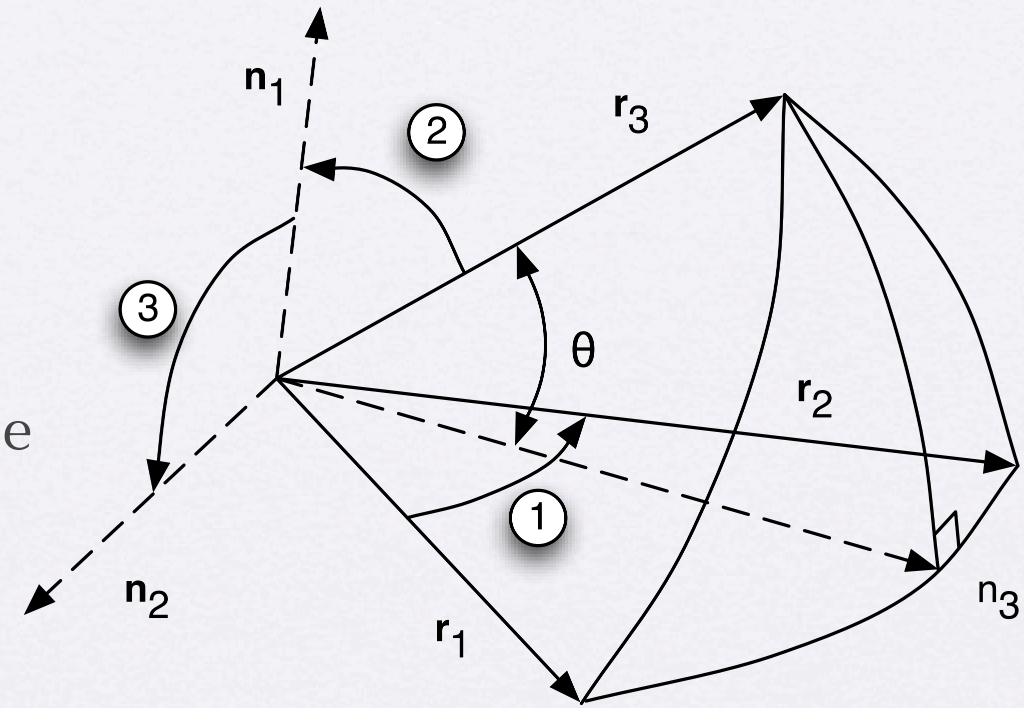
Spherical Quad-Tree of Stars

- Matlab output showing a star being inserted into spherical quad-tree.
- Each spherical triangle is broken down into four smaller spherical triangles forming quad-tree. Vertices added as necessary.
- Notice uneven sizes and shapes of quad-tree.

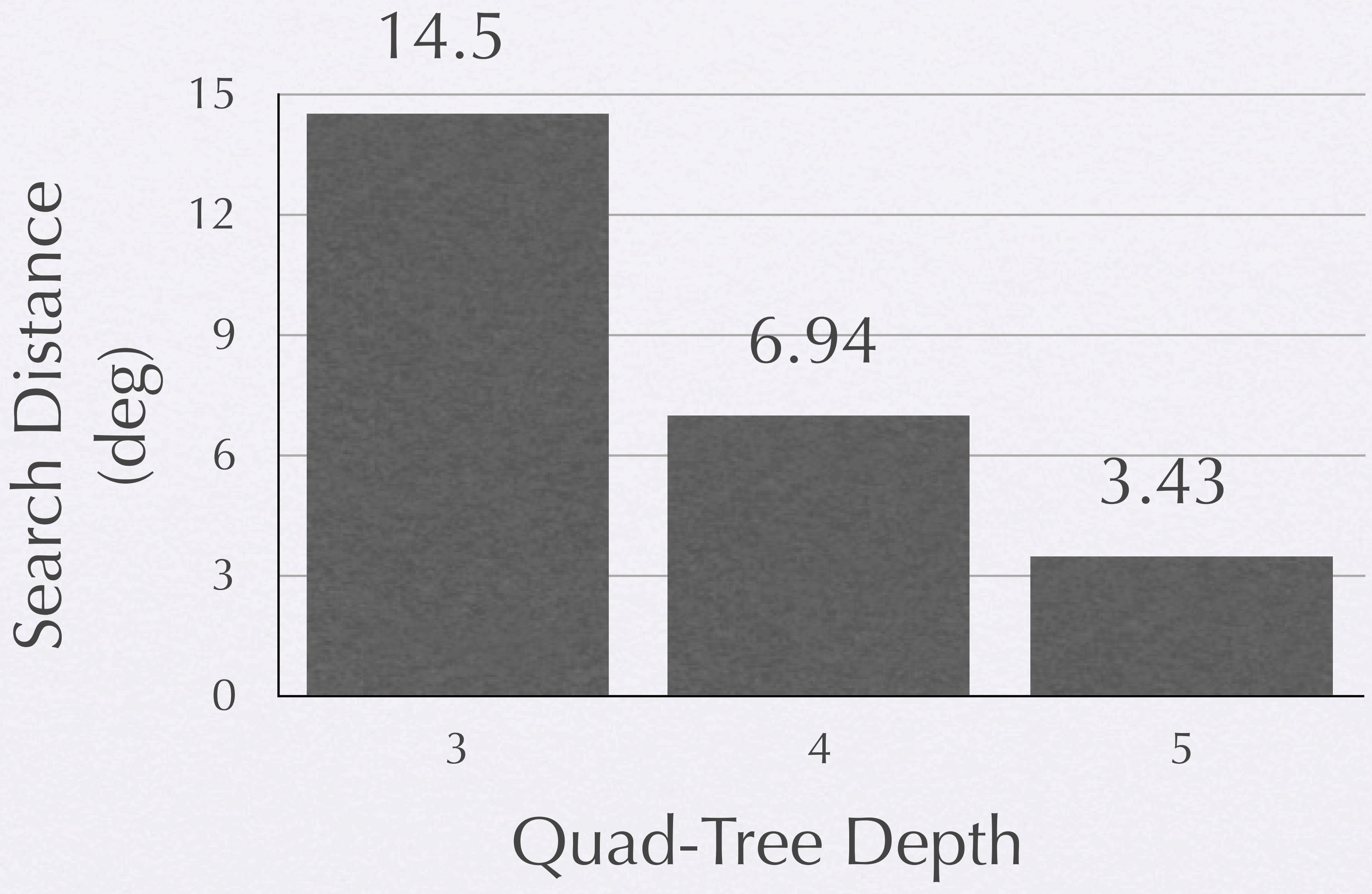


Search Limit

- Complicated because spherical triangles at the same level have different shapes and sizes.
- Maximum search distance is the smallest height (in deg) of any spherical triangle at target level
- Three steps to determining height



Maximum Search Distance

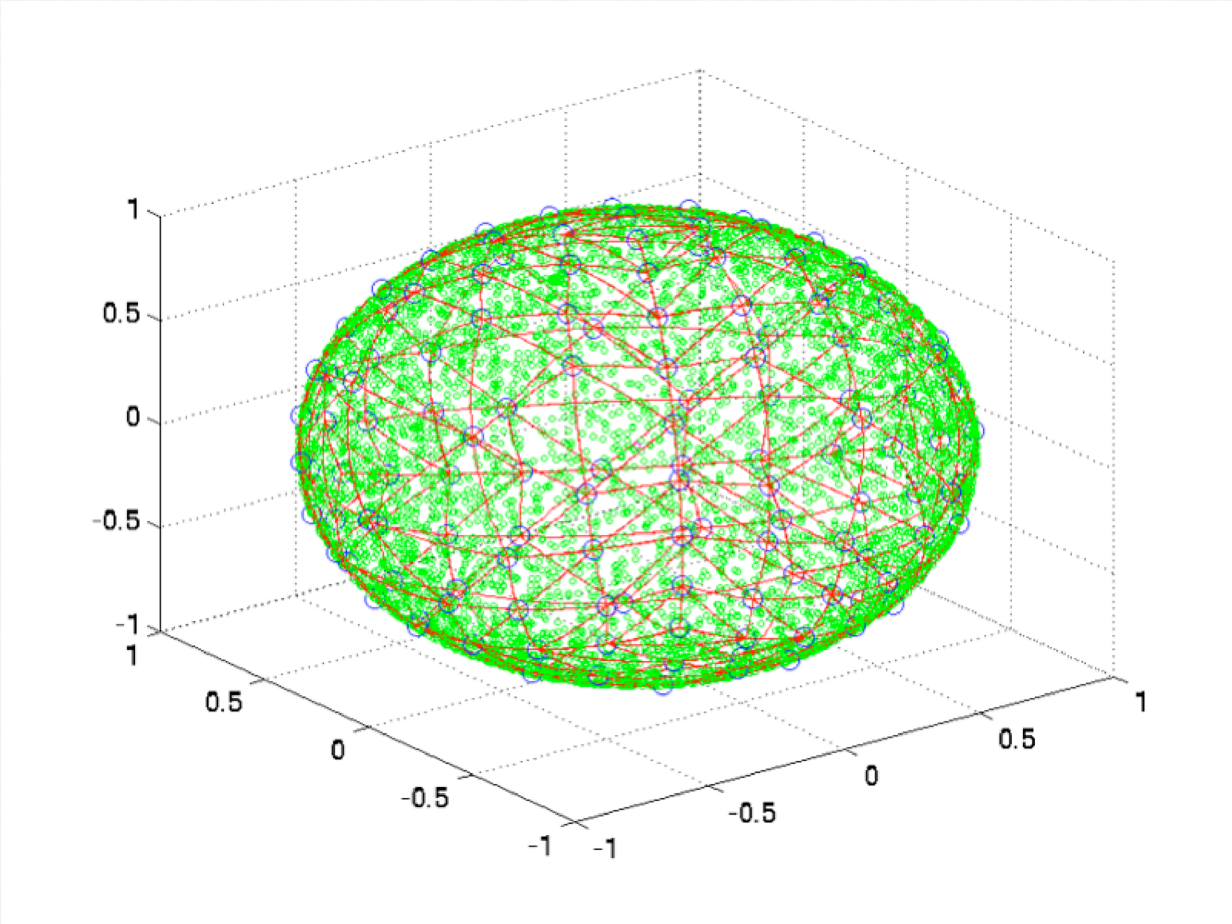


Search Problem

- Star Tacker has an 8 deg field of view
- Two choices:
  - Use a three level quad-tree to get 8 deg FOV angle and spherical triangles.
  - Use a four level quad-tree and settle for 6.94 deg FOV angles and spherical triangles. This was chosen.

Finished Quad-Tree

- Four-level quad-tree
- 8,188 Magnitude 6 Stars and brighter
- 340 Nodes
- 130 Vertices
- Constructed in minutes



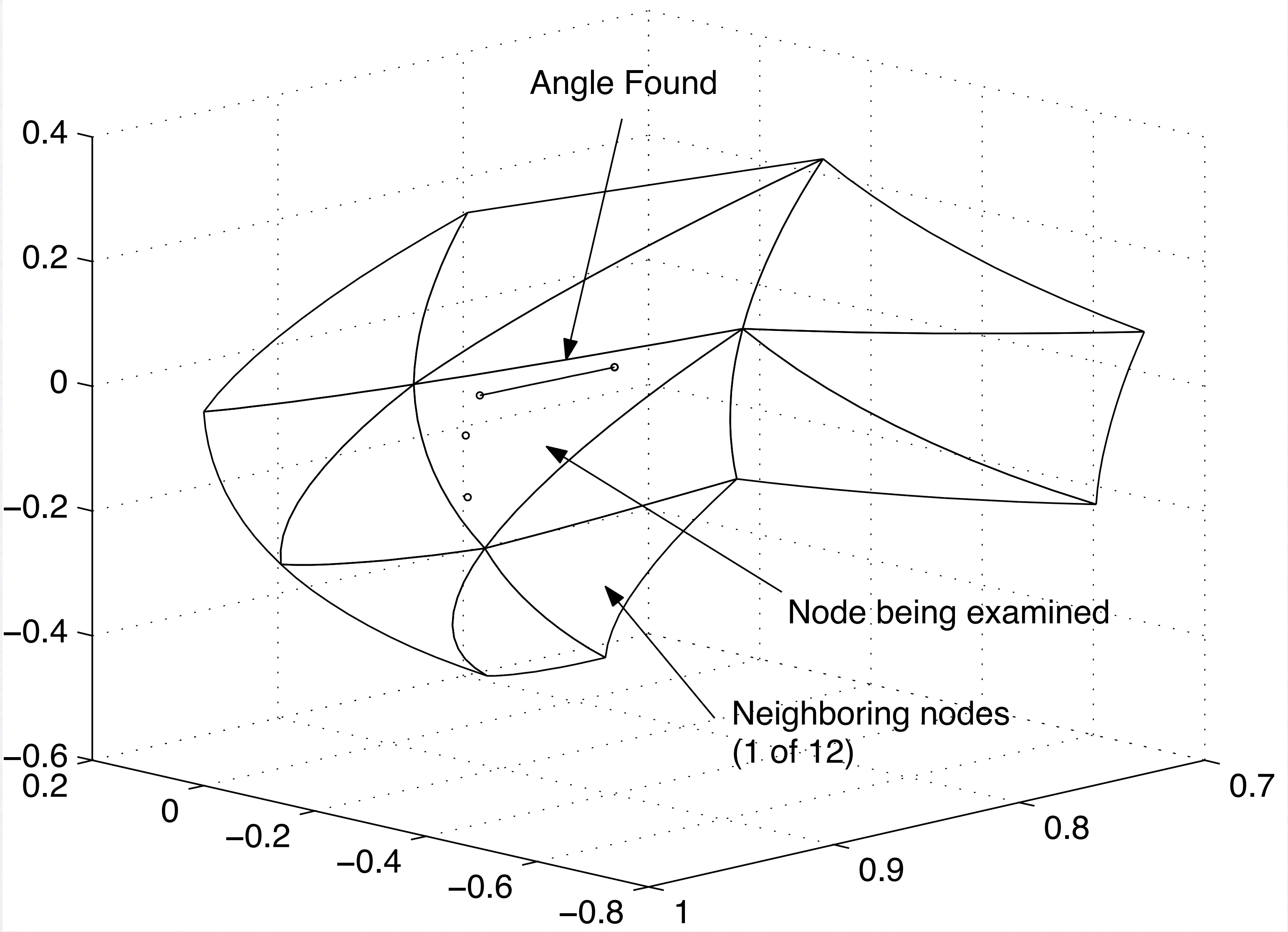
Angle Catalog Construction

- Catalog created by going node to node through spherical quad-tree
- Two kinds of angles to look for:
  - Single-node angles
  - Double-node angles

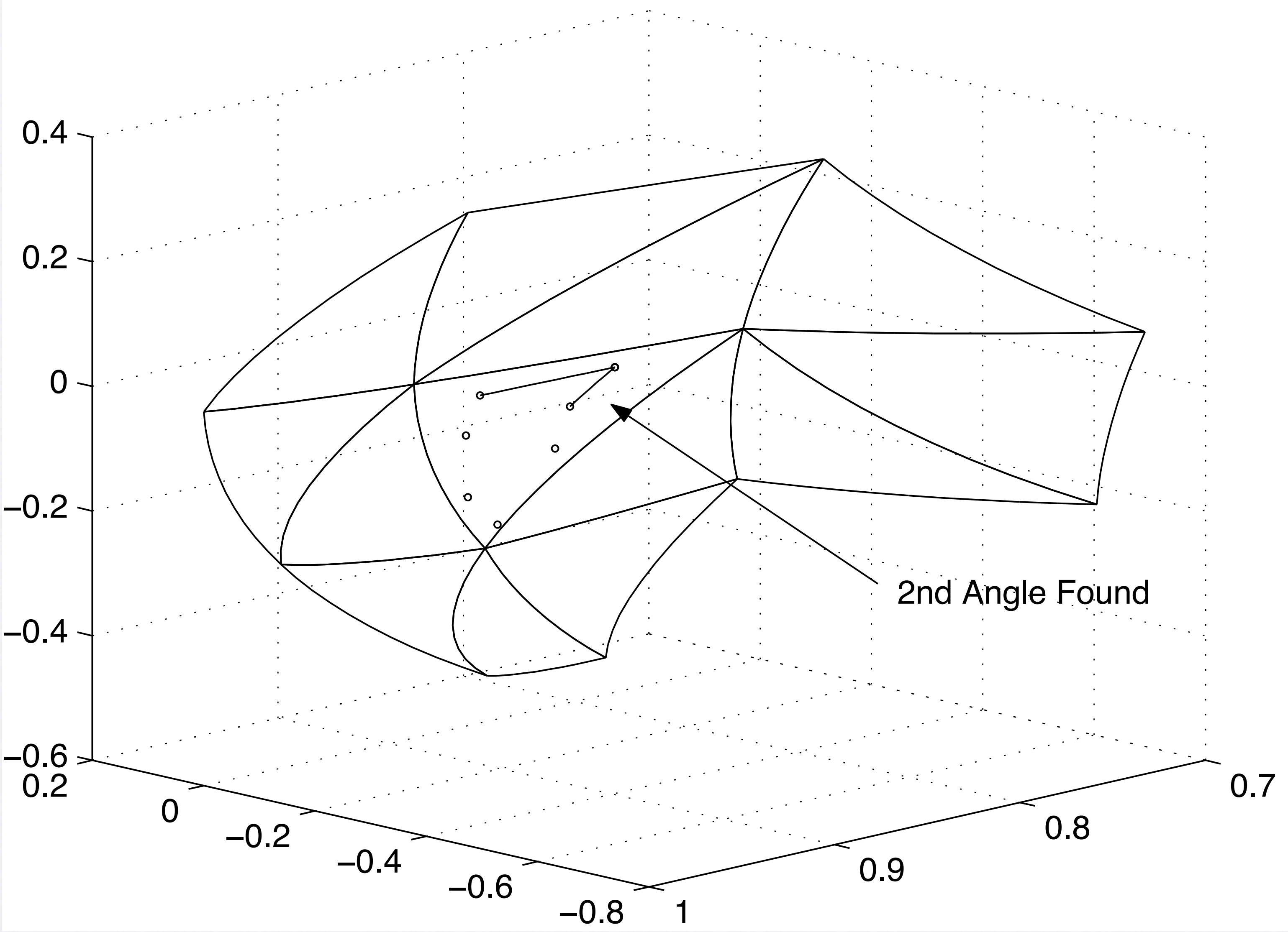
Single-Node Angles

- Both stars that make up angle are in the node being examined

Single-Node Angles



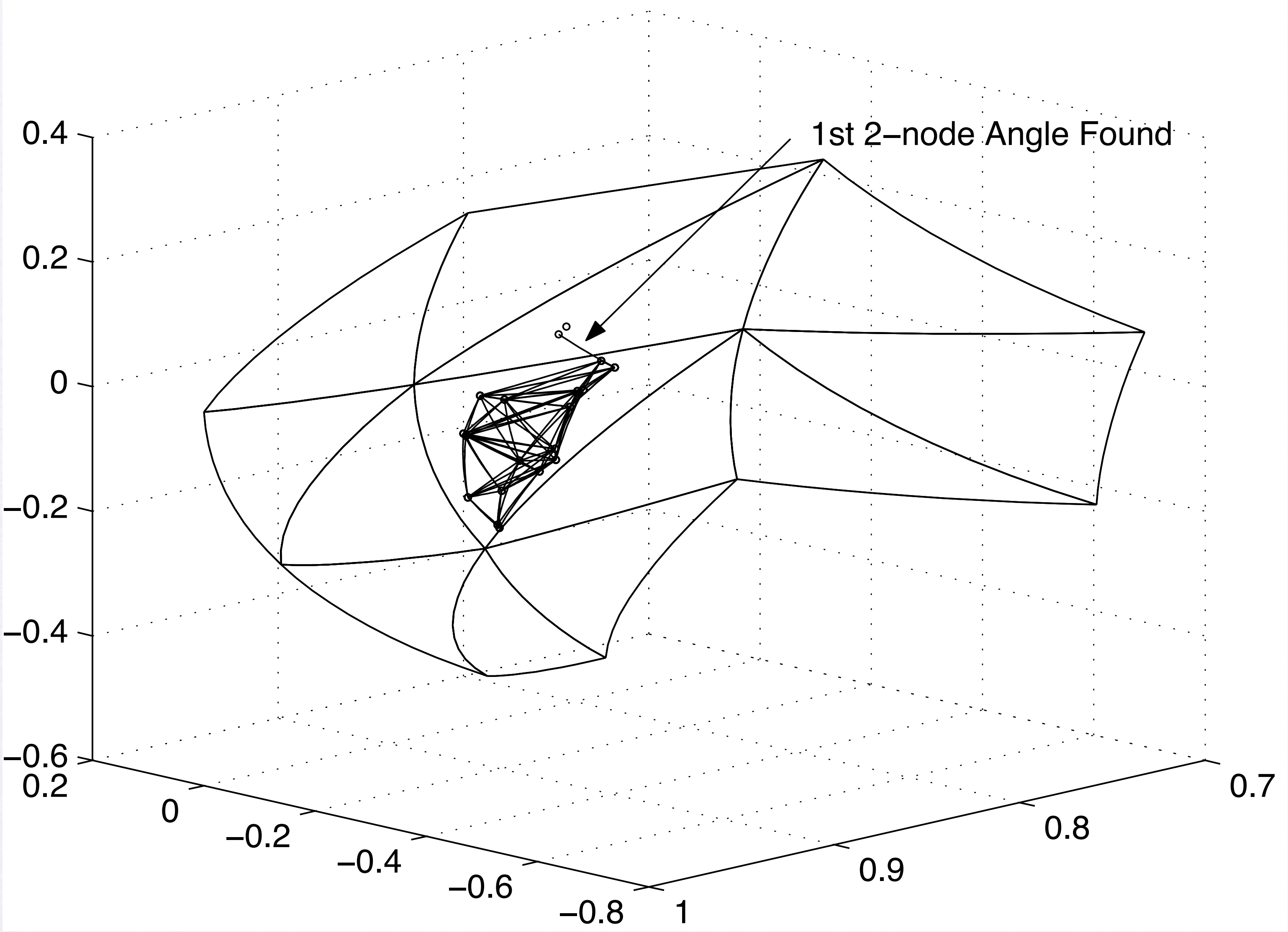
Single-Node Angles



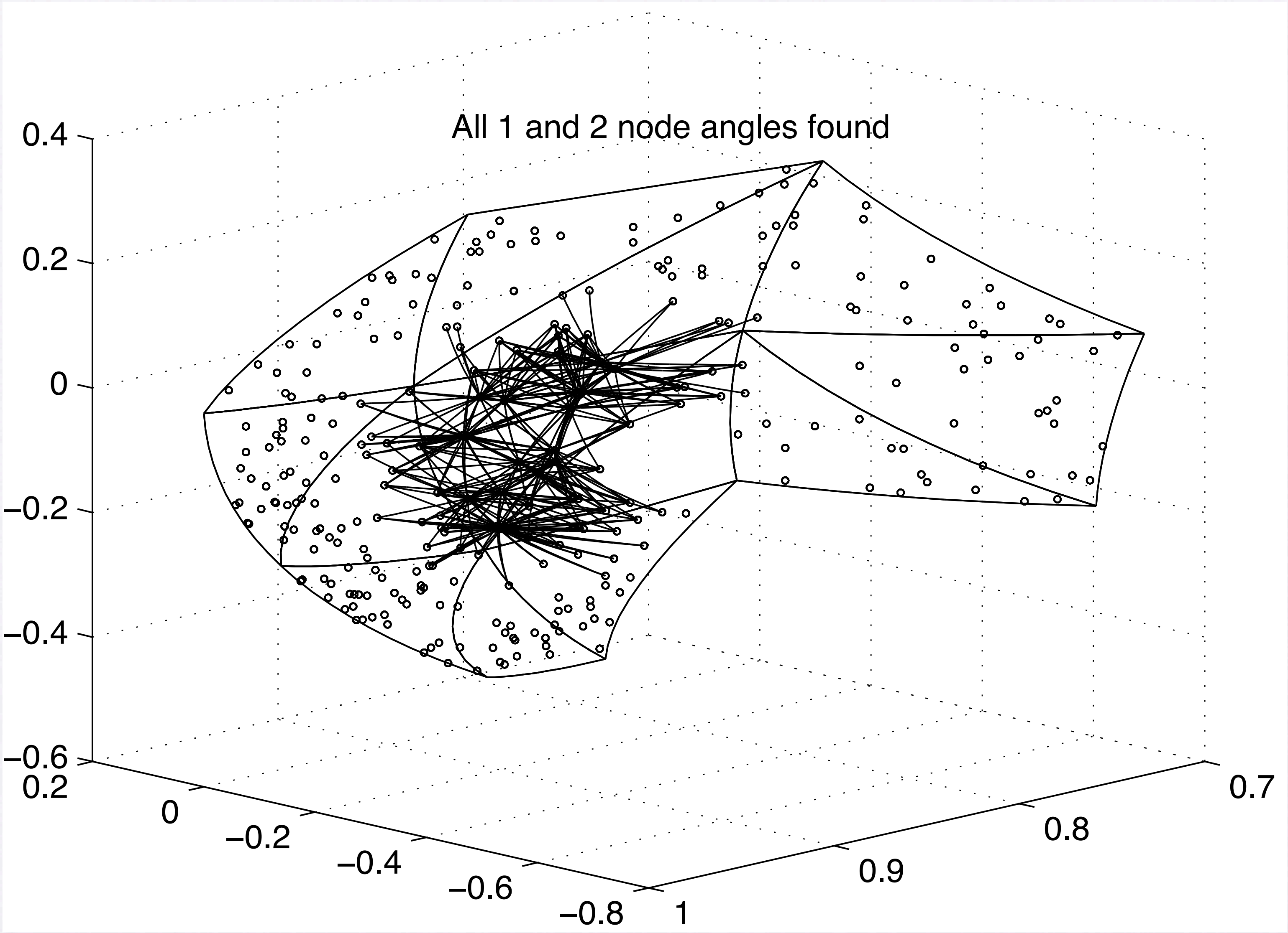
Two-Node Angles

- Angles made when one star is in the node being examined, and the other in a neighboring node.
- Neighboring nodes take note that they've been searched for two-node angles by the node being examined.
- Keeps same angle from being counted twice.

Two-Node Angles



First Node Finished



Angle Catalog Results

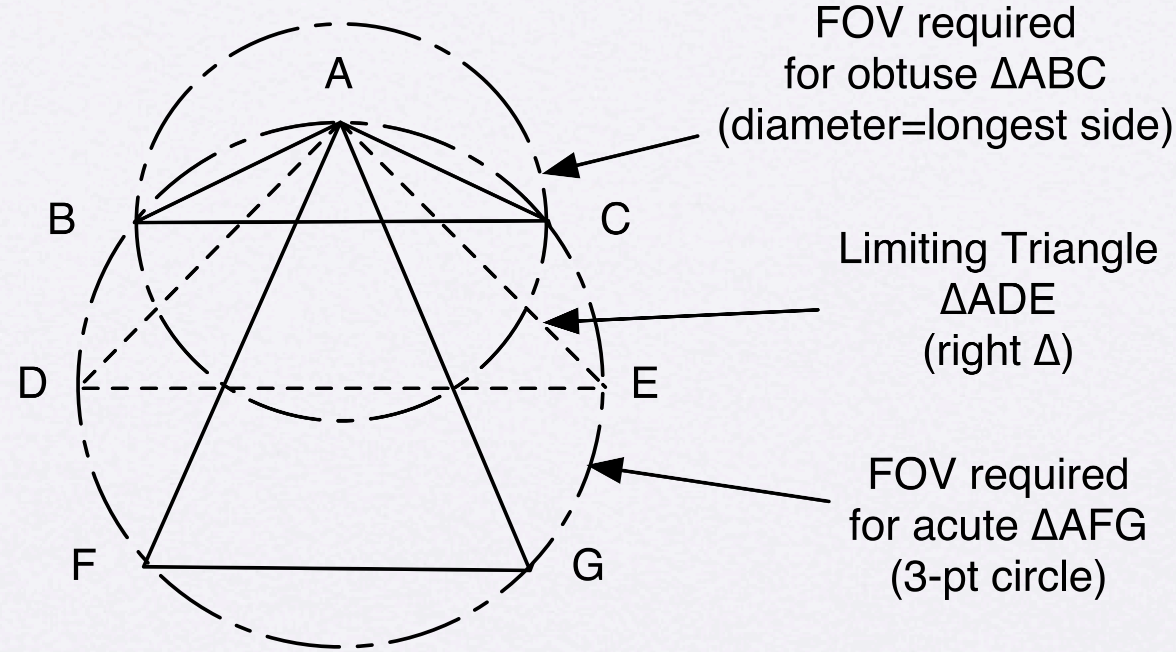
- Specifications:
  - Magnitude 6.0 and brighter
  - 6.94 degrees or less
- 106,308 angles found
- Less than an hour to compute
- 12 MB file

Spherical Triangle Catalog

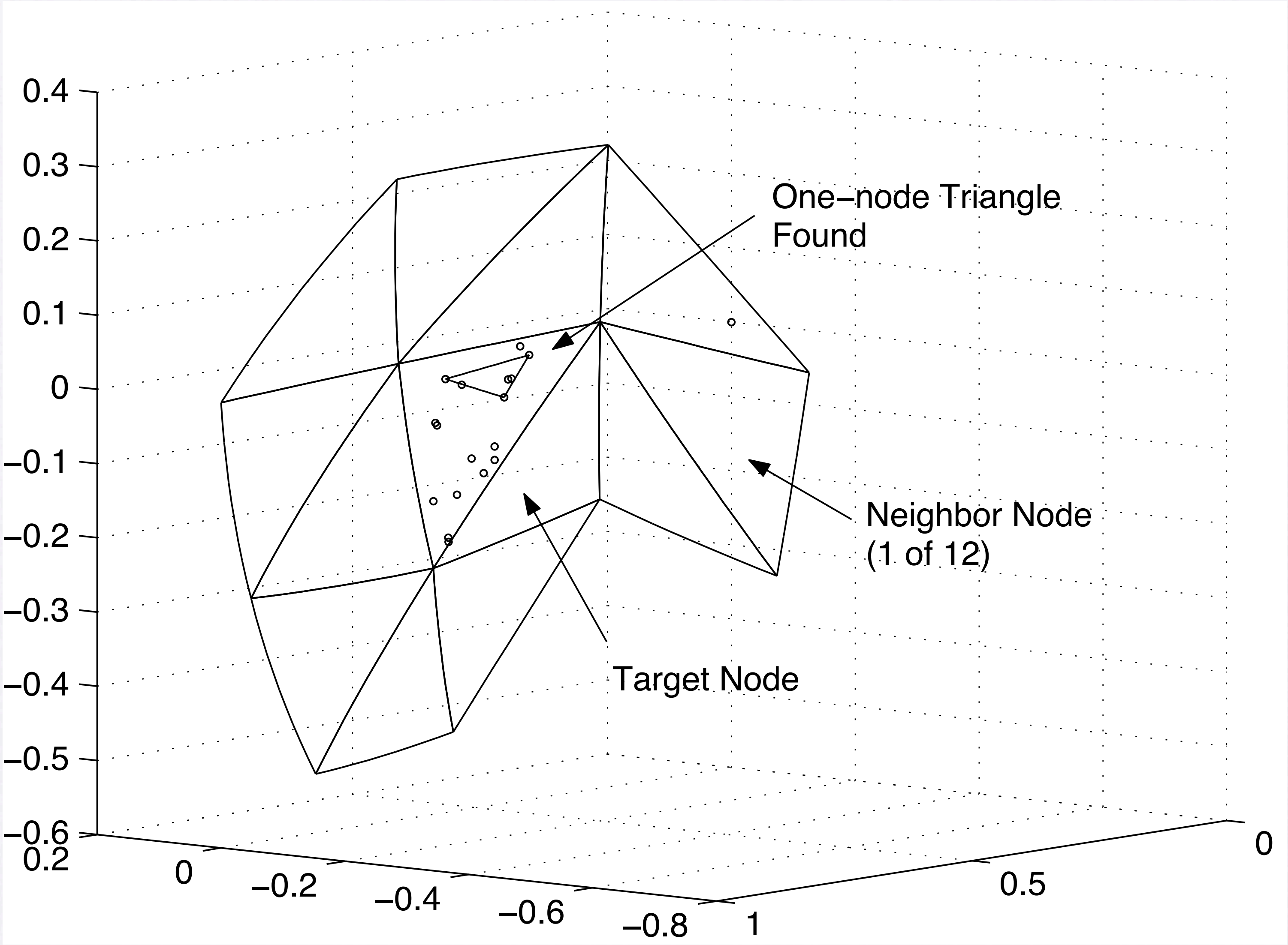
- Similar in construction to angle catalog
- Three kinds of spherical triangles
  - One-Node spherical triangles
  - Two-Node Spherical Triangles
  - Three Node Spherical Triangles

FOV Calculation

- Two situations
- Obtuse Triangle is only the length of the longest side
- Acute Triangle requires equation for 3-pt circle
- Planar triangle is fine for this calculation



One-Node Sph. Triangles



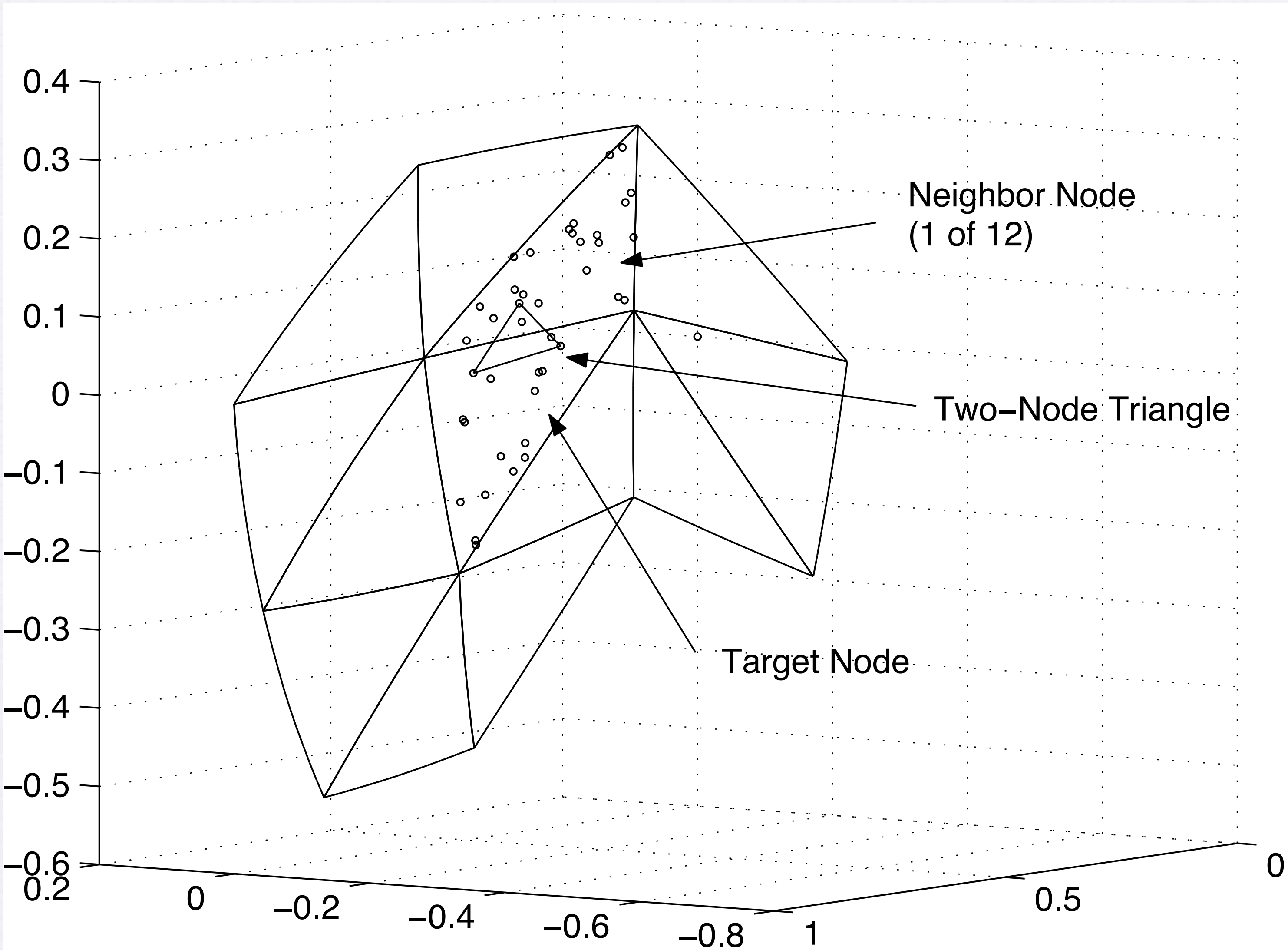
Two & Three Node Spherical Triangles

- Two-node spherical triangles have one star in one node, two stars in a neighboring node, or vice-versa.

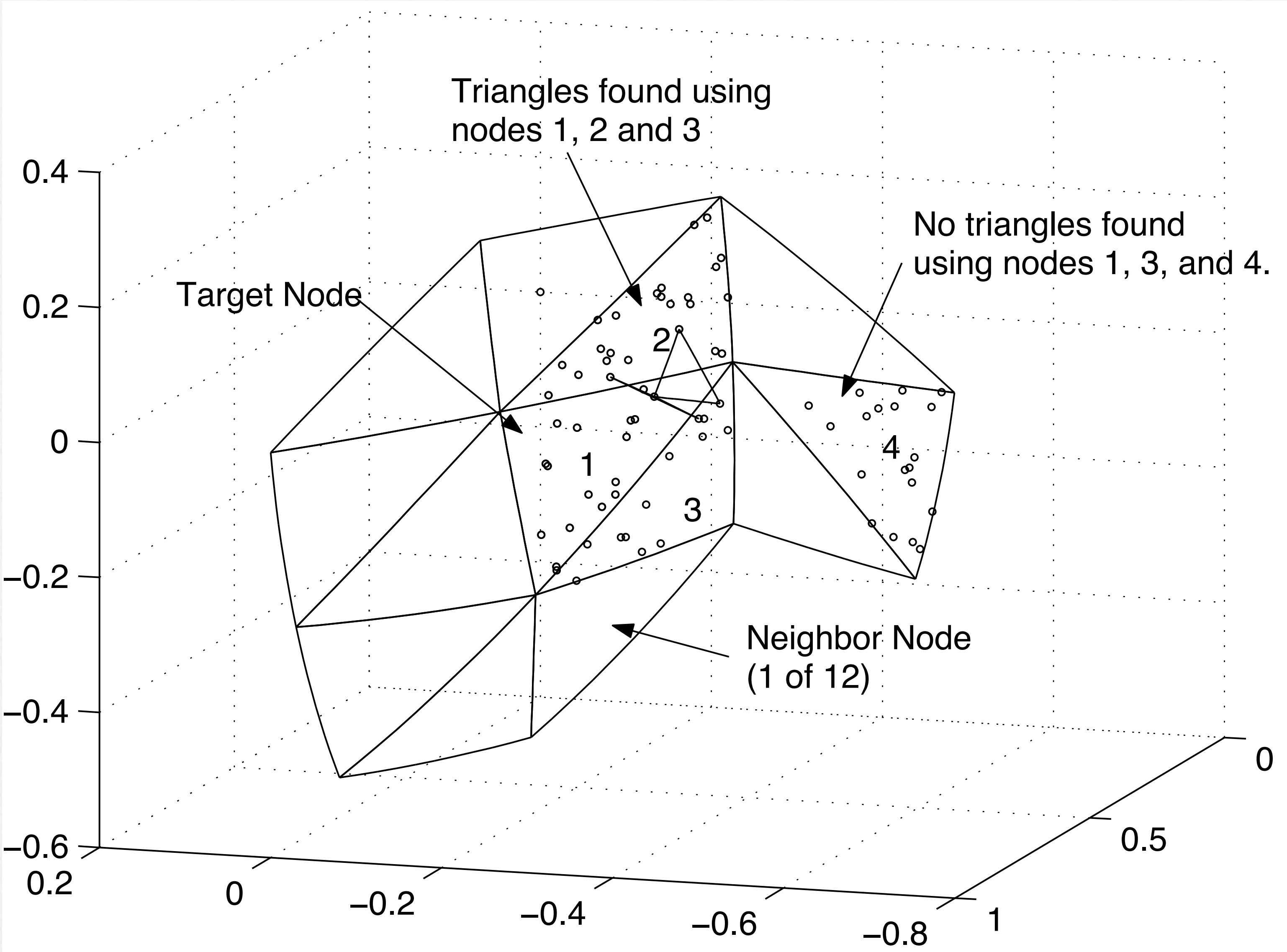
- Three-node spherical triangles have one star in node being examined, and one in two neighboring nodes.

- Neighboring nodes keep track of examinations by other nodes so spherical triangles aren't counted more than once.

Two-Node Sph. Triangles



Three-Node Sph. Triangles



Spherical Triangle Catalog Early Results

- Specifications:
  - Magnitude 6.0 and brighter
  - 6.94 degrees or less
- 662,799 spherical triangles found
- Several hours to compute
- 81 MB file

Area & Polar Moment

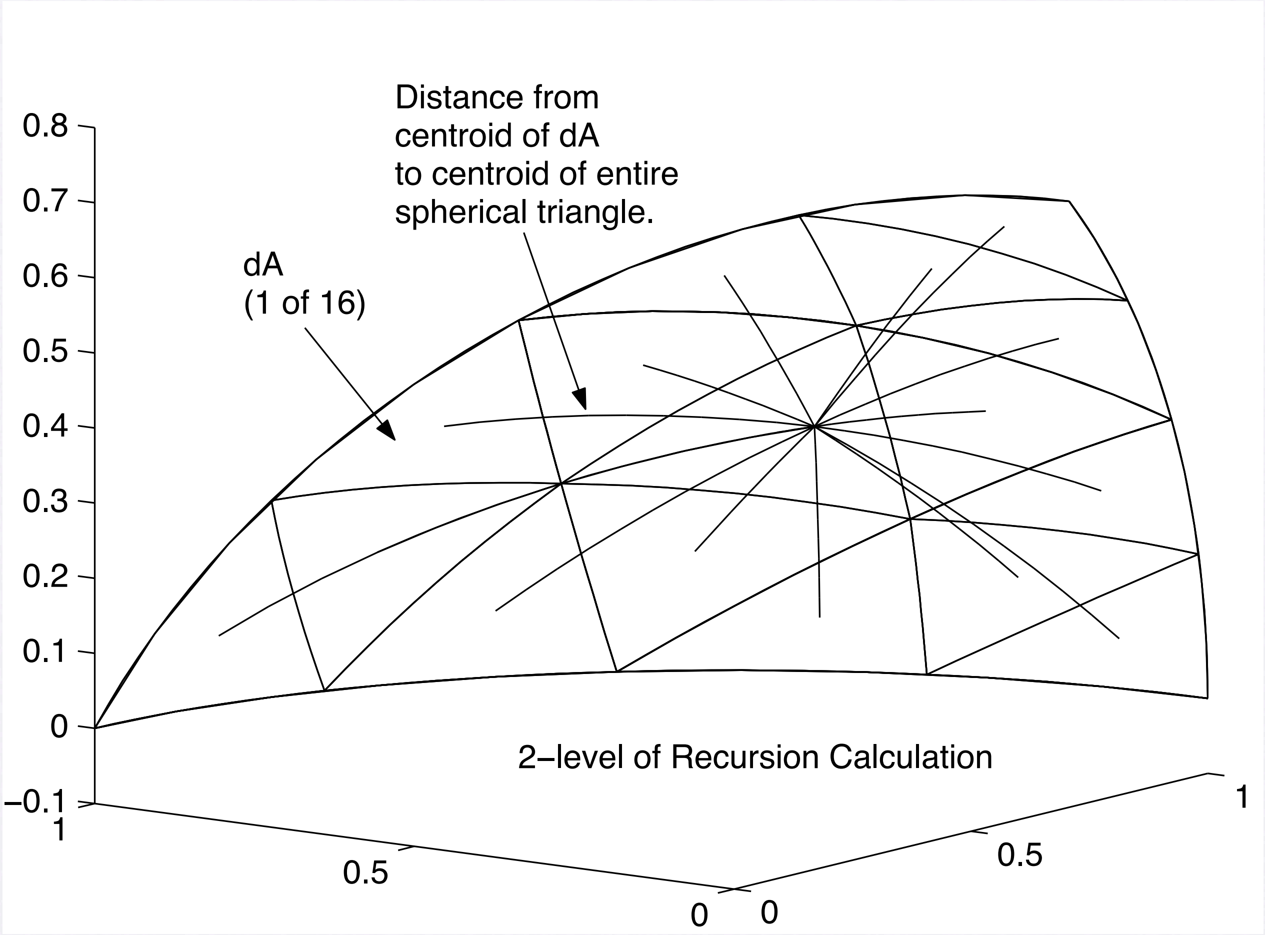
- Spherical Triangle Catalog still needs area and polar moments to be calculated
- Area calculated directly from vectors pointing to stars that make up the triangle
- Polar Moment calculated by summation of terms

Polar Moment

- Similar to second moment, exept it is calculated about an axis perpendicular to the plane being examined
- Equal to the sum of infinitesimal areas multiplied by the square of its distance from the axis of interest.
- Axis of interest is the centroid
- Distance measured in radians of arc

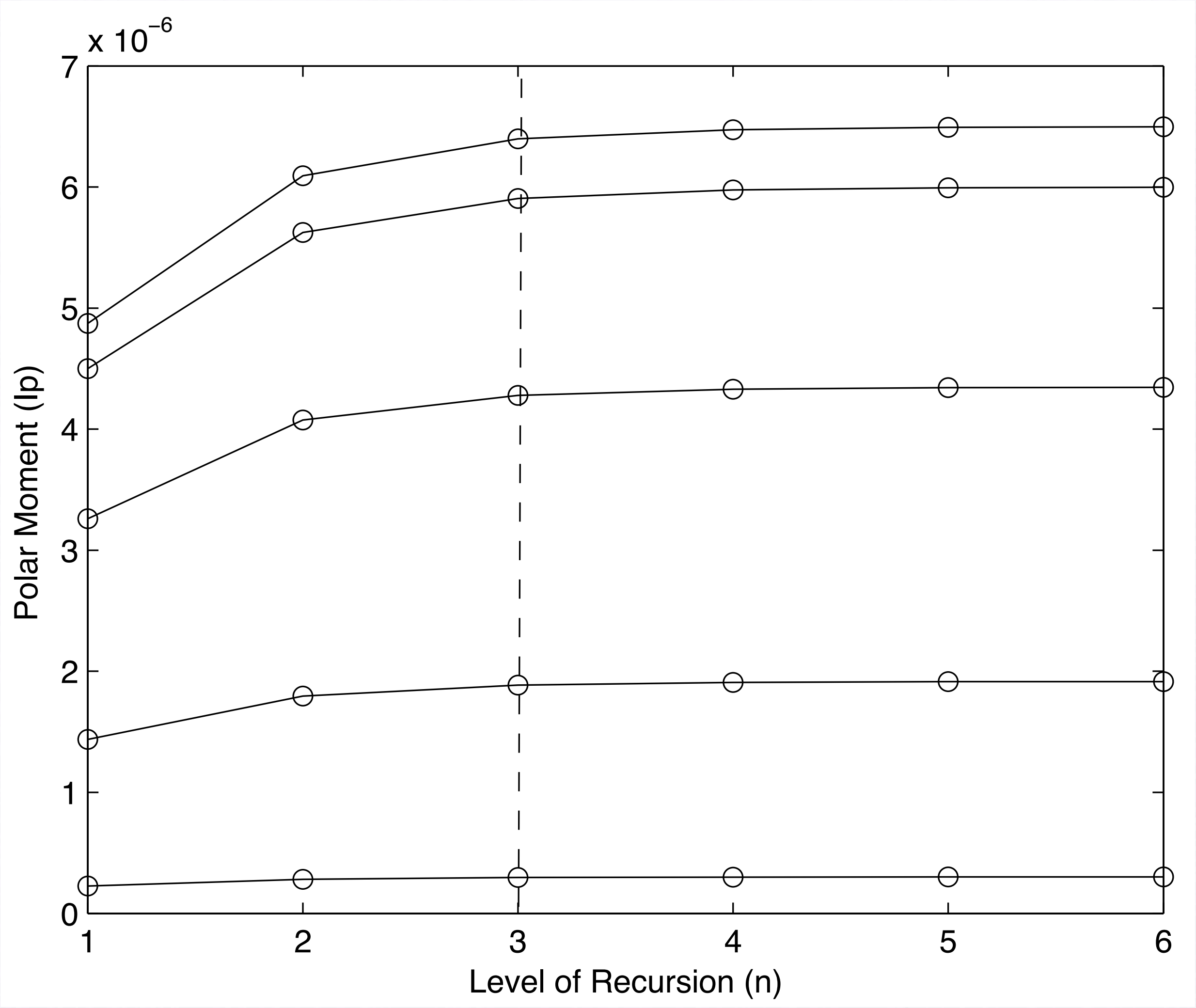
Polar Moment Algorithm

- Recursive algorithm which breaks spherical triangles into smaller triangles to whatever level needed.



Level of Recursion Needed

- Tested convergence to find what level of recursion was necessary



Level of Recursion Needed

- Three Levels (64 triangles) is “close” to convergence, and was chosen.
  - Takes eight hours of computing time. Longest single operation.
- Four levels (256 triangles) would be better but would require 4 times as long
  - Little effect, if any, on results

Spherical Triangle Catalog Results

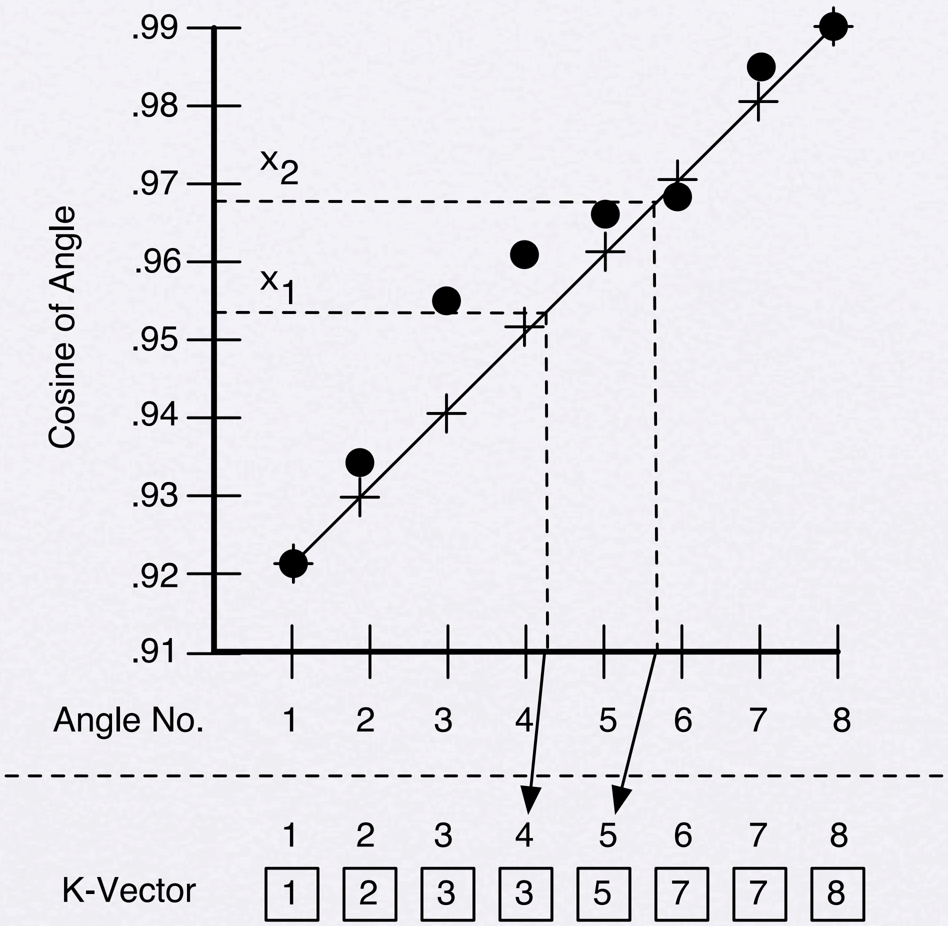
- Catalog is now finished.
- Including area and polar moment raises file size to 162 MB.

K-Vector Generation

- K-Vector is a data structure used for locating items that have a property within a specific range from a list of items sorted by that property
- Angles - must sort by cosine of angle
- Spherical Triangles - must sort by area
- Must be able to fit an equation to the sorted data as a function of its position in the list

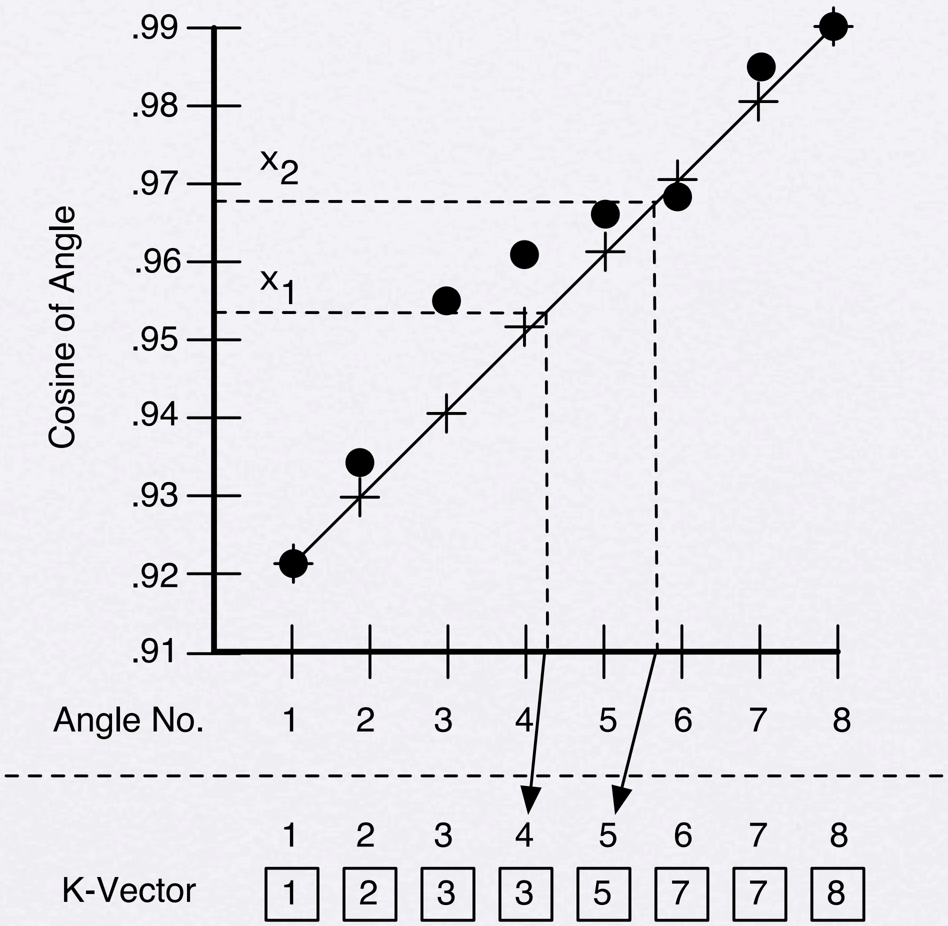
K-Vector Demonstration

- Example for angle method
- Eight angles sorted by cosine of angle
- Equation to connect first and last points determined. In this case, a line is a good choice.



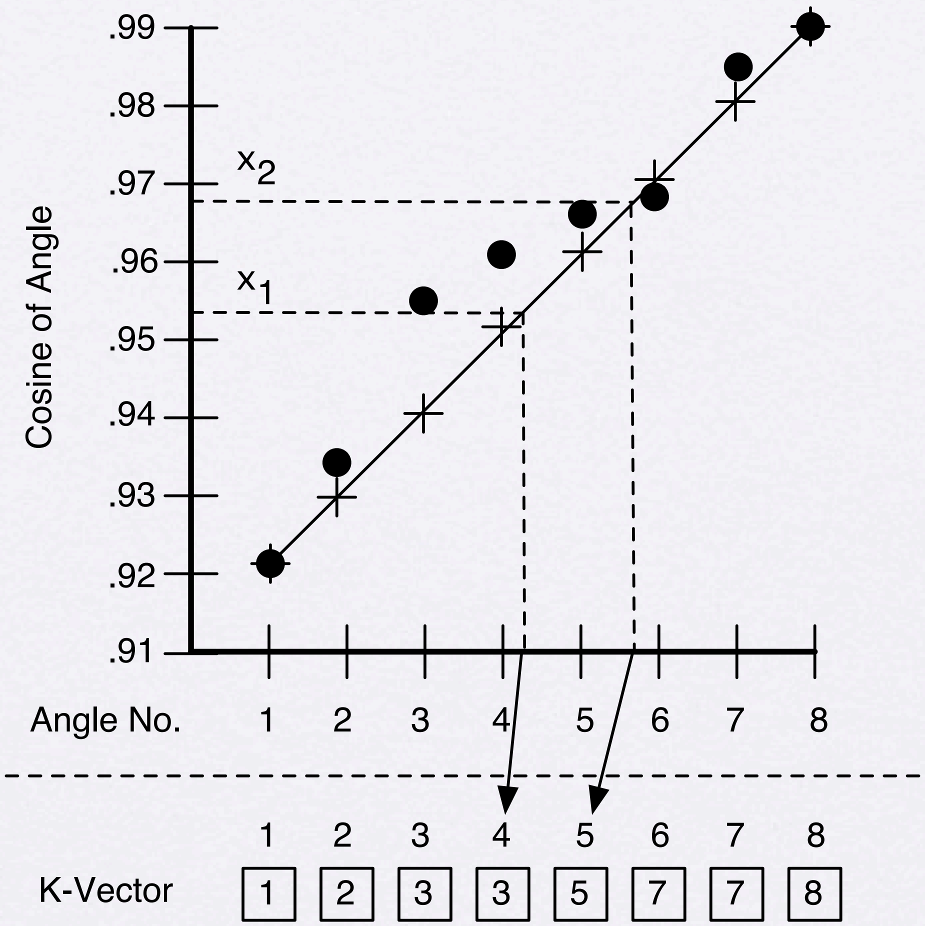
K-Vector Demonstration

- Need to construct the K-Vector
- At each point along the line, the position of the first data point above the line is stored in the K-Vector
- First and last elements of K-Vector are always first and last points of data



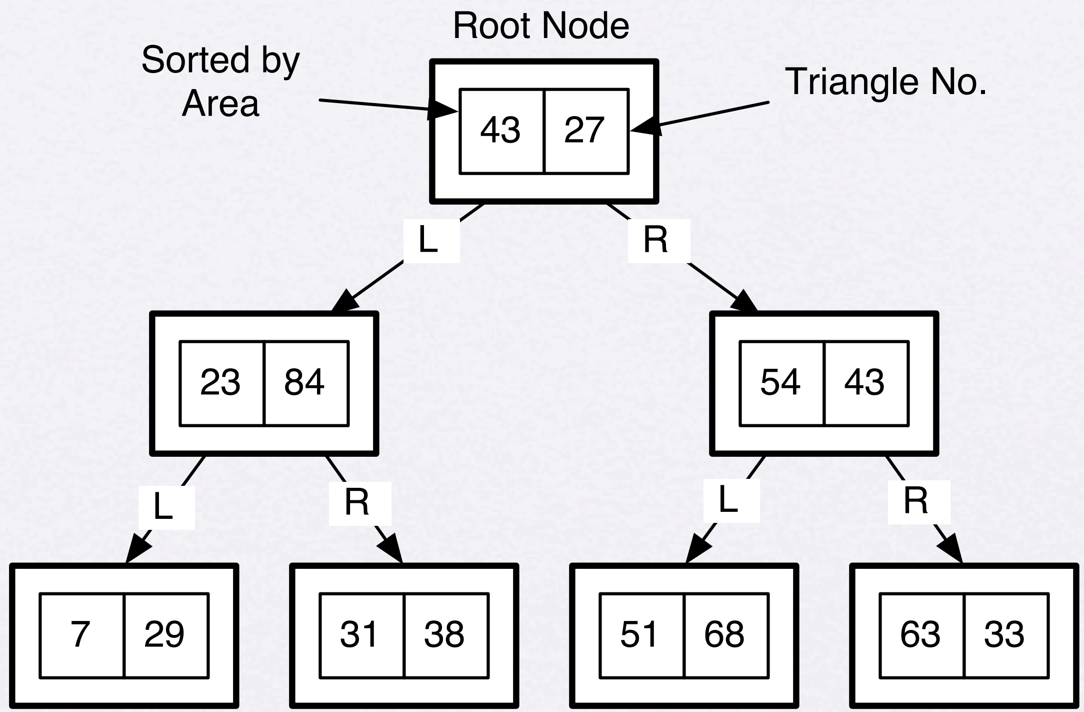
K-Vector Demonstration

- Angles with cosines between x_1 and x_2 wanted
- Use line connecting first and last point to find K-Vector elements 4 and 5
- K-Vector says angles 3 through 5 are have cosines between x_1 and x_2.



Sorting Angle and Spherical Triangle Catalogs

- Sorted using binary tree algorithm
- Recursive algorithm used to retrieve a sorted pointer list



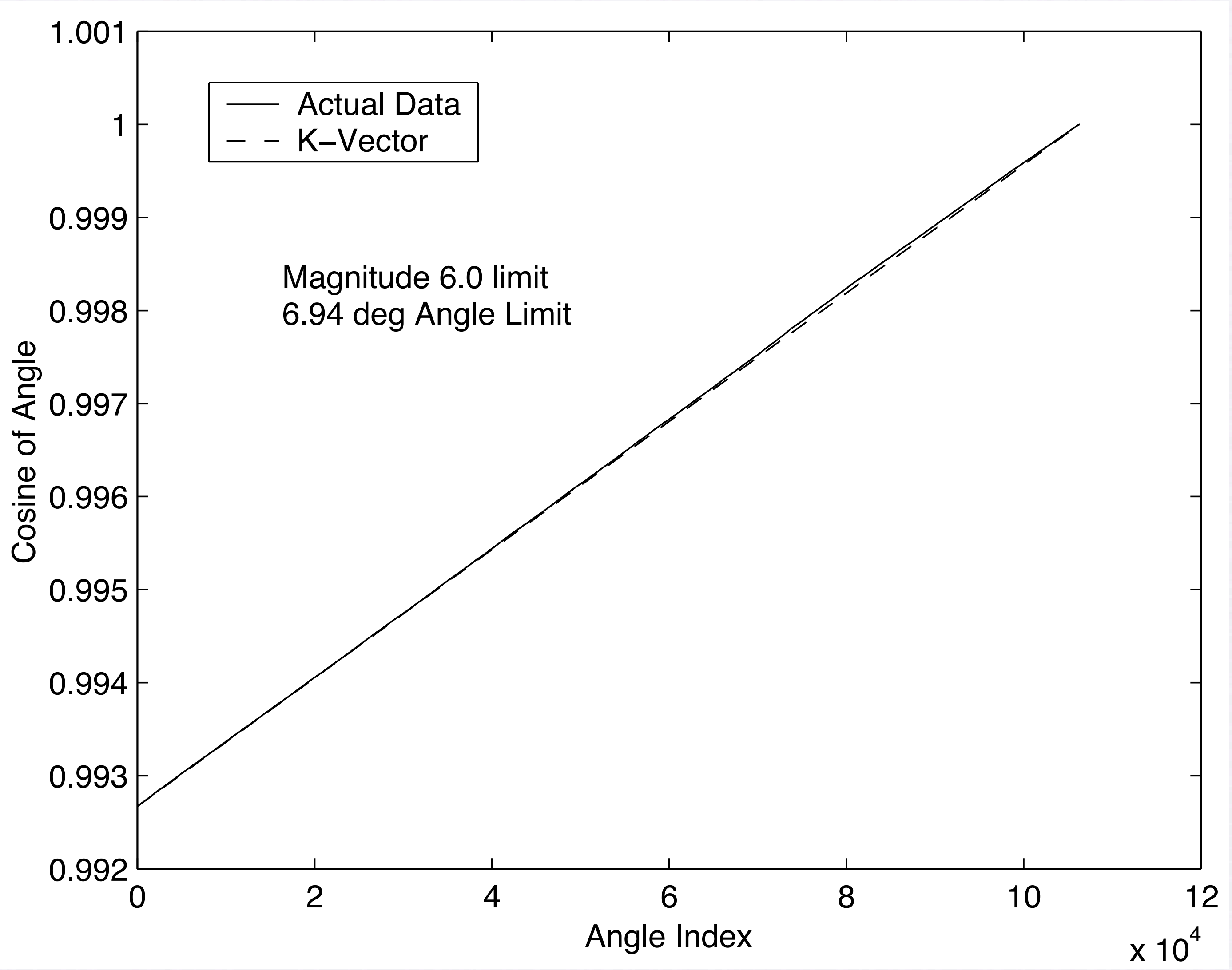
Sorted Pointers

- Instead of moving actual data around, pointer array used

Triangle	1	2	3	4	5
Area	12	34	9	22	16
Polar Moment	78	23	26	53	84

Element	1	2	3	4	5
Pointer Array after sort by area	3	1	5	4	2

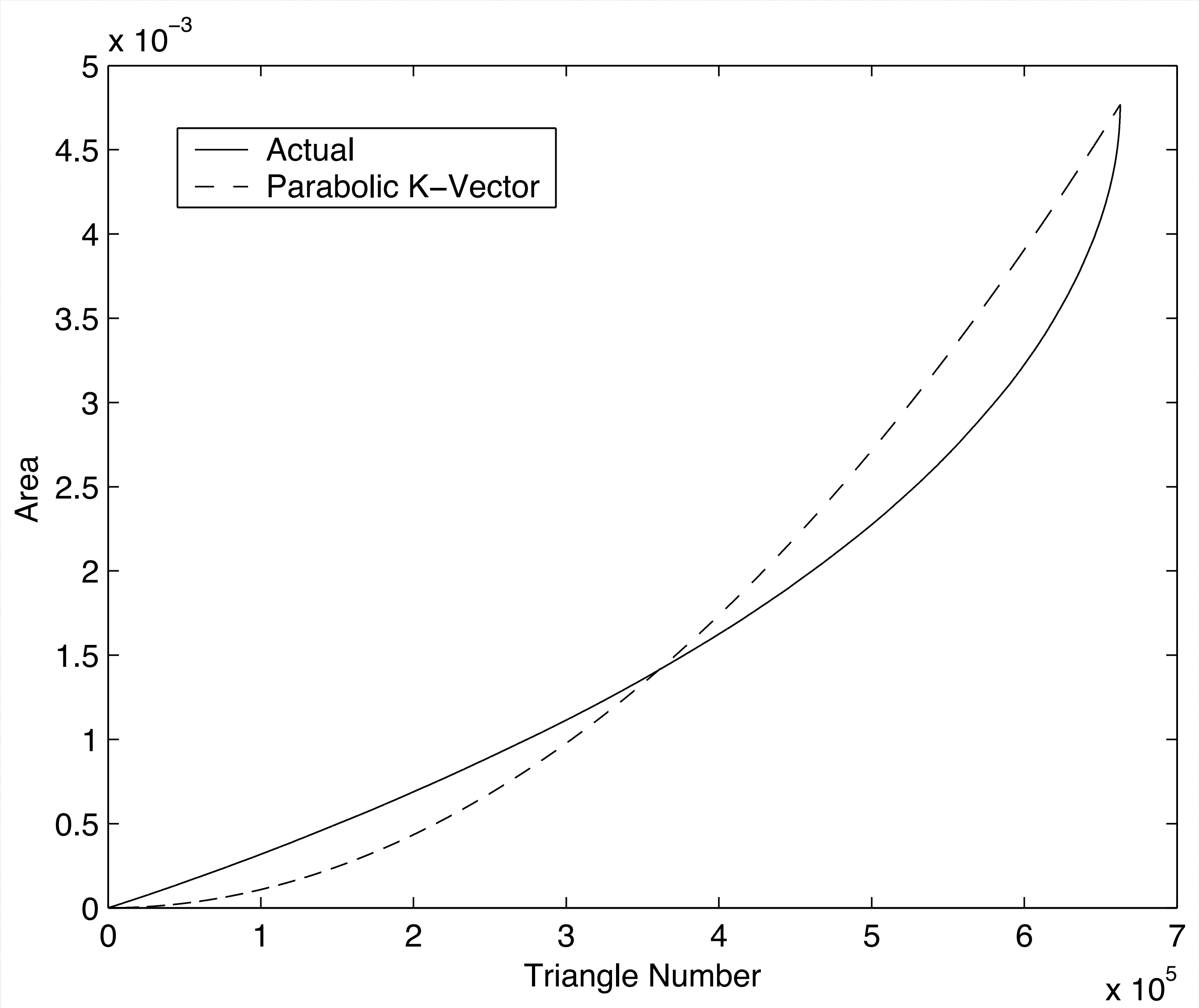
Angle K-Vector Result



Spherical Triangle K-Vector

- K-Vector works best when line connecting first and last points closely matches data
- Plotting spherical triangle area vs. spherical triangle number looks “sort of” parabolic
- Parabolic equation used for K-Vector

Spherical Triangle K-Vector



Ready for Star Pattern Recognition

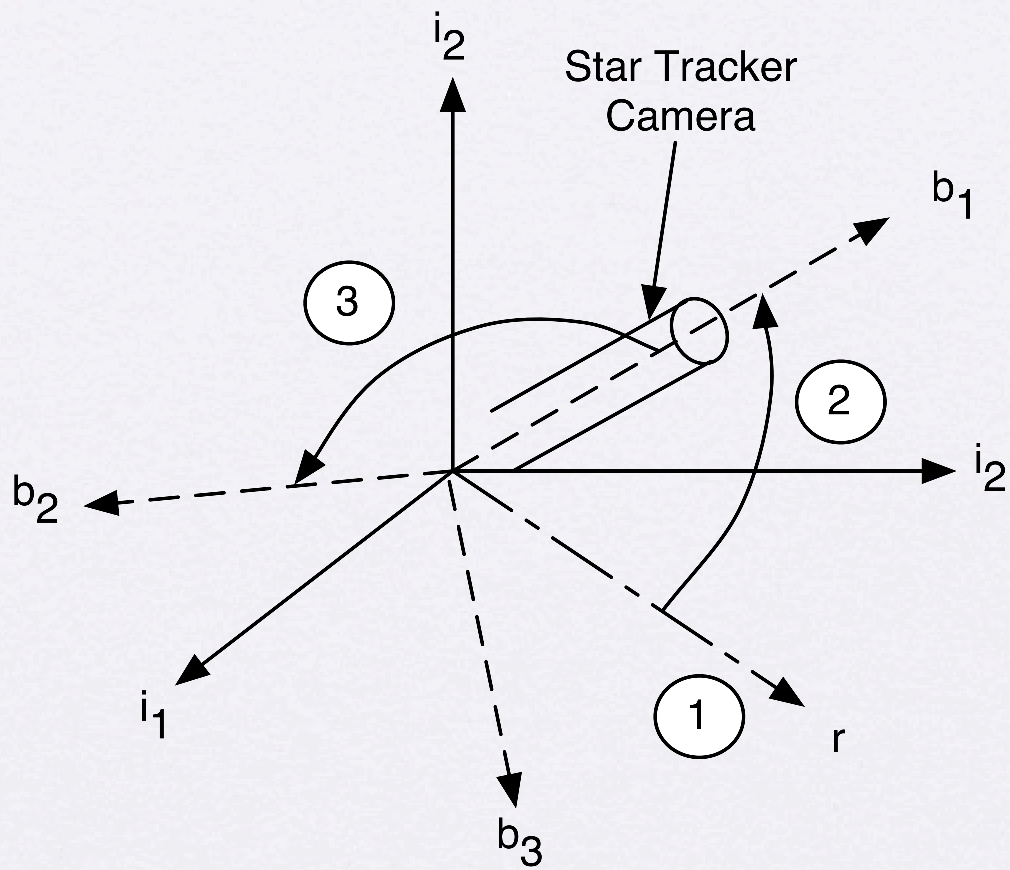
- Angle Method
    - Angle catalog, with cosine of angles
    - Pointer array sorted by cosine of angle
- Spherical Triangle Method
    - Spherical triangle catalog with area and polar moment
    - Pointer Array sorted by area

Method of Testing

- 1,000 random attitudes for angle and spherical triangle algorithms tested
- Results of compiled
    - Correct, Cannot, Inconclusive & Incorrect
- Methods compared

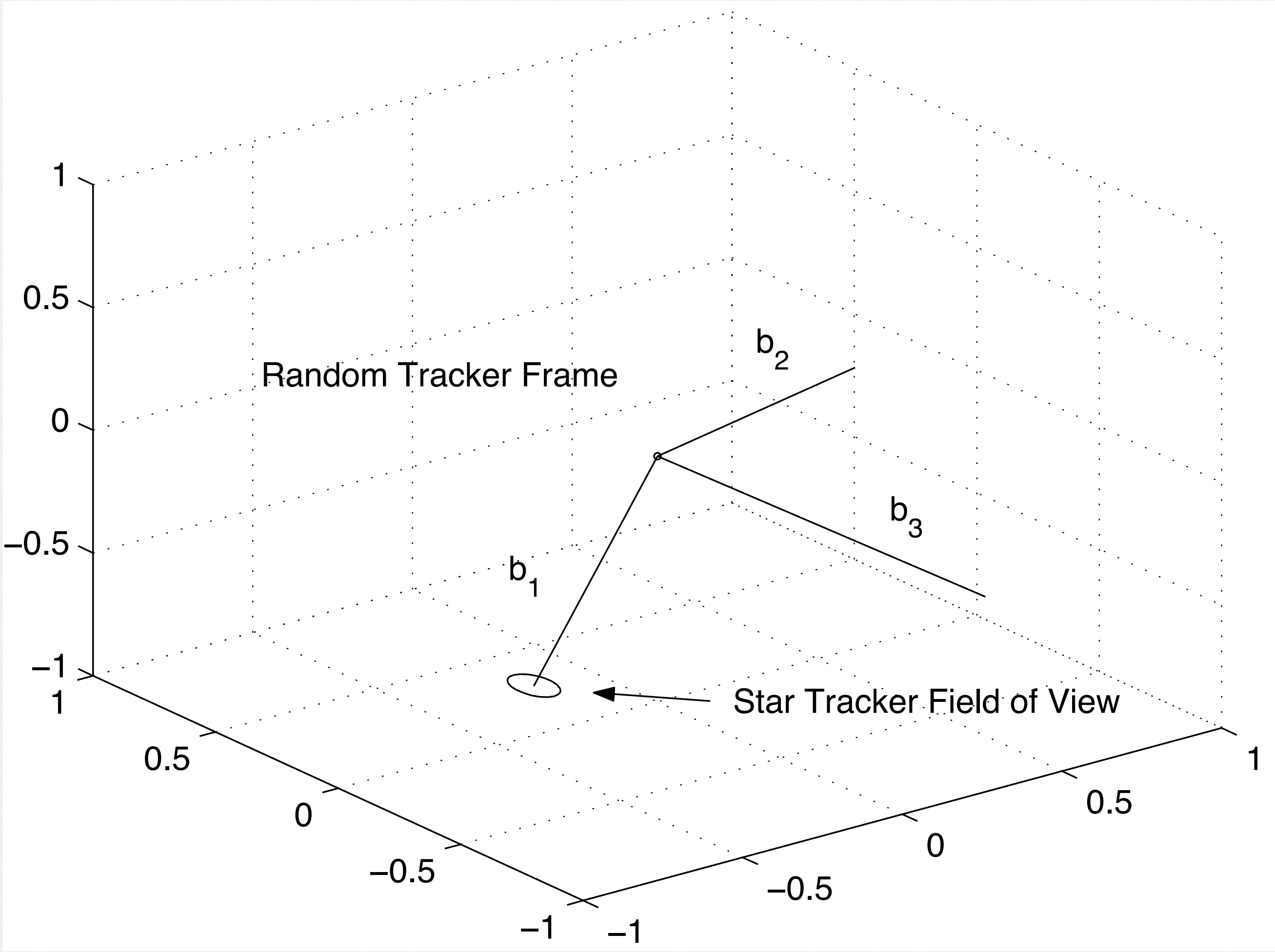
Random Attitude

- Random direction for star tracker camera created
- Three steps to creating random attitude



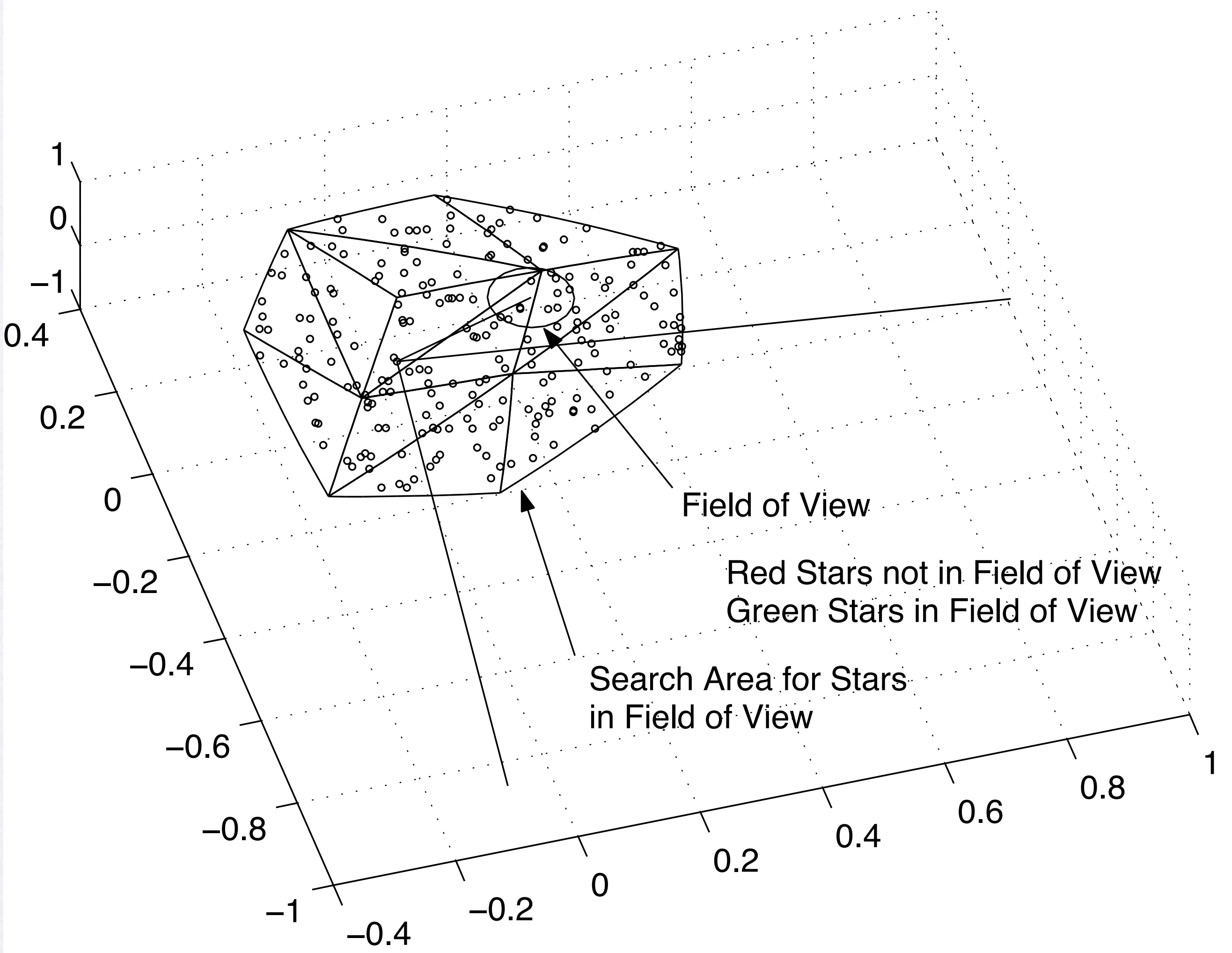
Random Attitude

- Matlab output showing random star tracker attitude and field of view



Random Attitude

- Determination of stars within field of view for testing
- Numbered for patten recognition algorithm
- Spherical quad-tree used, since it was available.



Coordinate Transform

- Stars given in ECI coordinate systems
- Need to convert to frame of tracker so that it can be seen that methods employed are attitude independent

Addition of Error

- Star positions are so far true
- Random error with normal distribution added to star positions in frame of star tracker to create measurements for star pattern recognition algorithms
- Star tracker measurements have standard deviation of 87 microradians
- Feed star positions measurements into algorithms.

Angle Method

- Three Distinct Sections of Code
  - Angle Measurements
  - Pivot Ordering
  - Pivoting to Solutions

Angle Measurements

- All combinations of two stars in FOV are examined.
- Cosine of each angle determined
- 3σ range for angle for which to search for solution determined
- Range of possible solution angles to each angle determined using K-Vector

Pivot Sort

- One angle will never be enough to find solution. Pivoting will be required.
- Start with largest angle - it has the least number of possible solutions
- Angle to which to pivot must have one star in common with previous angle.
- If more than one angle qualifies, the one with the largest angle is chosen

Pivot Sort

- Linked List Structure Used
- Order of pivoting is: 4, 3, 1 and 2

Pivot Order:	2		3		1		Start	
Angle No. In FOV	**1**		**2**		**3**		**4**	
Cosine of Angle	.95		.96		.93		.92	
Star Nos. in FOV	7	2	5	7	9	2	3	9
Pointer Min	35		30		40		40	
Pointer Max	47		37		50		50	
Next Link	2		EOL		1		3	

Pivot Sort

- Magnitude 6.0 star sensitivity - common to have 10 or more stars in FOV.
- Many combinations of two stars
- Computationally intensive - best to limit pivots if possible

Pivoting to Solution

- One pivot demonstrated here

Finalists

Angle No.	Star 1	Star 2
27	15	44
32	35	38
54	4	76
19	76	13
33	54	42

Pivot Angle Possible Solutions

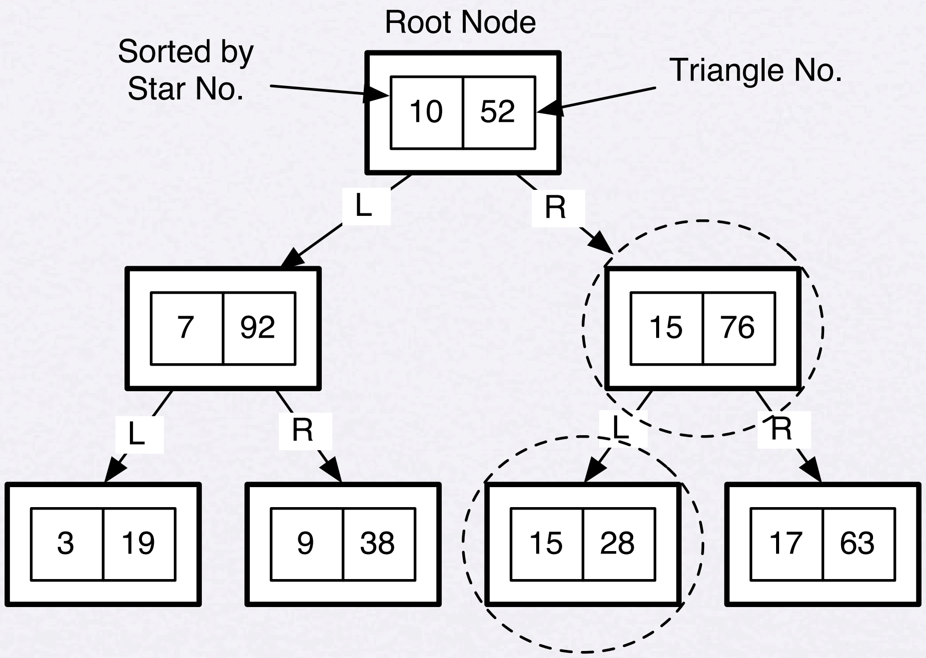
Angle No.	Star 1	Star 2
23	56	3
10	23	15
35	52	97
46	12	76
5	33	73
34	15	30
66	4	43

New Finalists

Angle Nos.
10 (27)
46 (54)
66 (54)

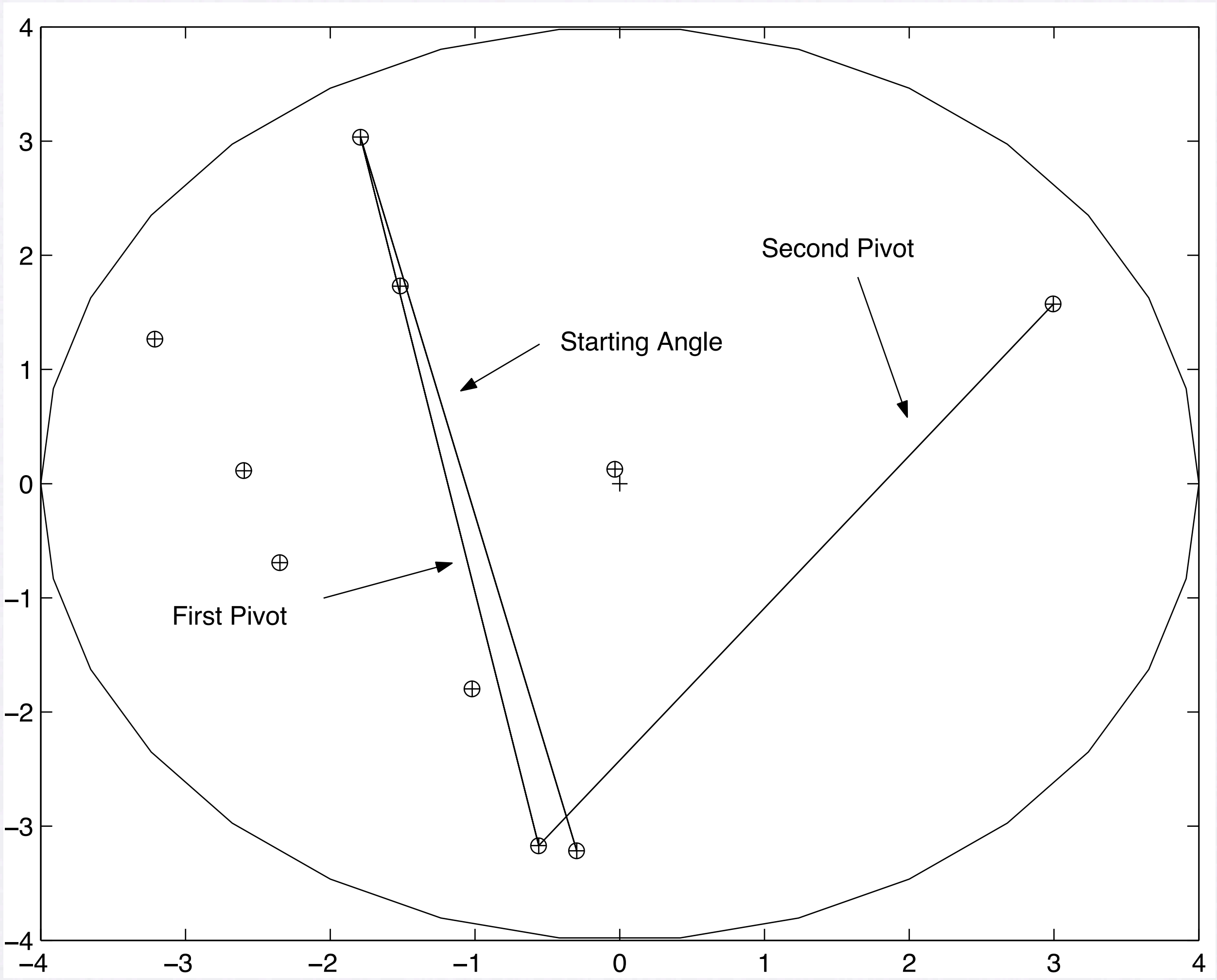
Pivoting to Solution

- Two binary trees used to speed finding angles with one matching star
- If one angle has 100 possible s solutions, and the pivot angle 150, tree will be only 7 elements deep, requireing 700 tests instead of 15,000.



Pivoting to Solution

- Matlab output shown here



Pivoting to Solution

- Pivoting continues until single solution is reached - all stars that make up angles used for pivoting will be known
- Possible to not reach a solution
  - Run out of angles to which to pivot or pivot limit is reached
  - So many possible solutions that number of finalists after pivoting begins to rise

Spherical Triangle Method

- Similar to Angle Method
- Three Distinct Sections of Code
  - Spherical Triangle Measurements
  - Pivot Ordering
  - Pivoting to Solutions

Spherical Triangle Measurements

- All combinations of three stars in FOV are examined.
- Area of each spherical triangle determined
- 3σ range of area for which to search for solution determined
- Range of possible solution spherical triangles to each spherical triangle in FOV determined using K-Vector

Pivot Sort

- One spherical triangle will never be enough to find solution. Pivoting will be required.
- Start with spherical triangle with largest area - it has the least number of possible solutions
- Pivot triangle must have two stars in common with previous triangle
- If more than one triangle qualifies, the one with the largest area is chosen

Pivot Sort

- Linked List Structure Used

- Order of pivoting is: 2, 1, 4 and 3.

Pivot Order:	1	Start	3	2
Sph Tri No. In FOV	**1**	**2**	**3**	**4**
Area	25	35	27	20
Star Nos. in FOV	4 7 6	4 1 7	1 6 8	6 1 7
Pointer Min	65	87	54	35
Pointer Max	80	95	64	61
Next Link	4	1	EOL	3

Pivot Sort

- Magnitude 6.0 star sensitivity - common to have 10 or more stars in FOV.
- Combinations of 3 stars - Even more combinations than the angle method
- Very computationally intensive - best to limit pivots if possible

Pivoting to Solution

- One pivot demonstrated here

Finalists

Sph.Tri No.	Star 1	Star 2	Star 3
27	15	44	27
32	35	38	64
54	29	15	26
19	4	13	14
33	54	42	6

Pivot Triangle
Possible Solutions

Sph.Tri No.	Star 1	Star 2	Star 3
63	15	44	97
23	15	27	31
3	4	76	78
25	23	3	11
53	54	15	26
68	4	14	55
52	53	42	46

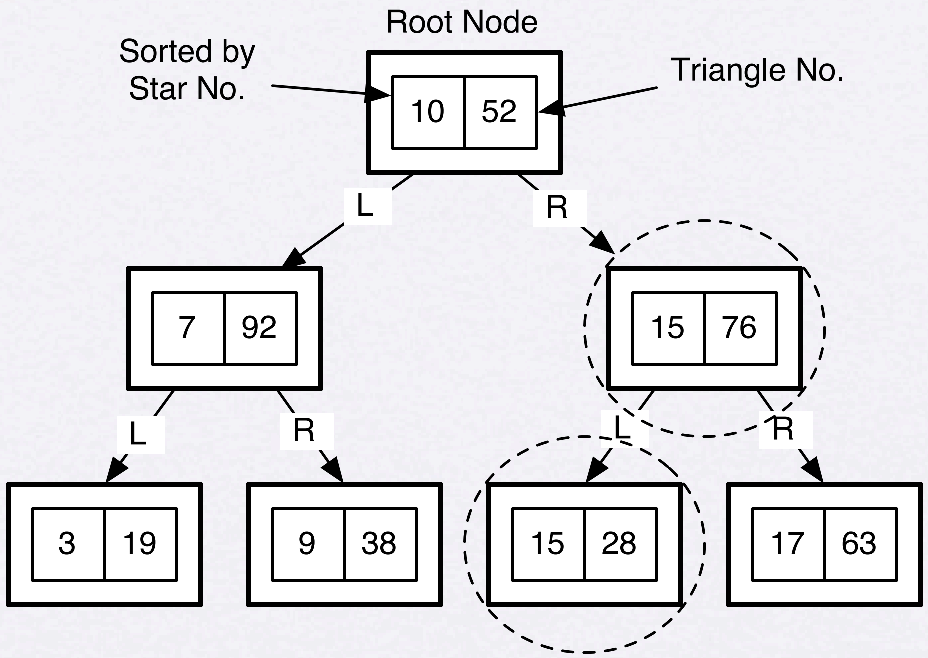
New
Finalists

Sph.Tri. Nos.
23 (27)
53 (54)
68 (19)

Pivoting to Solution

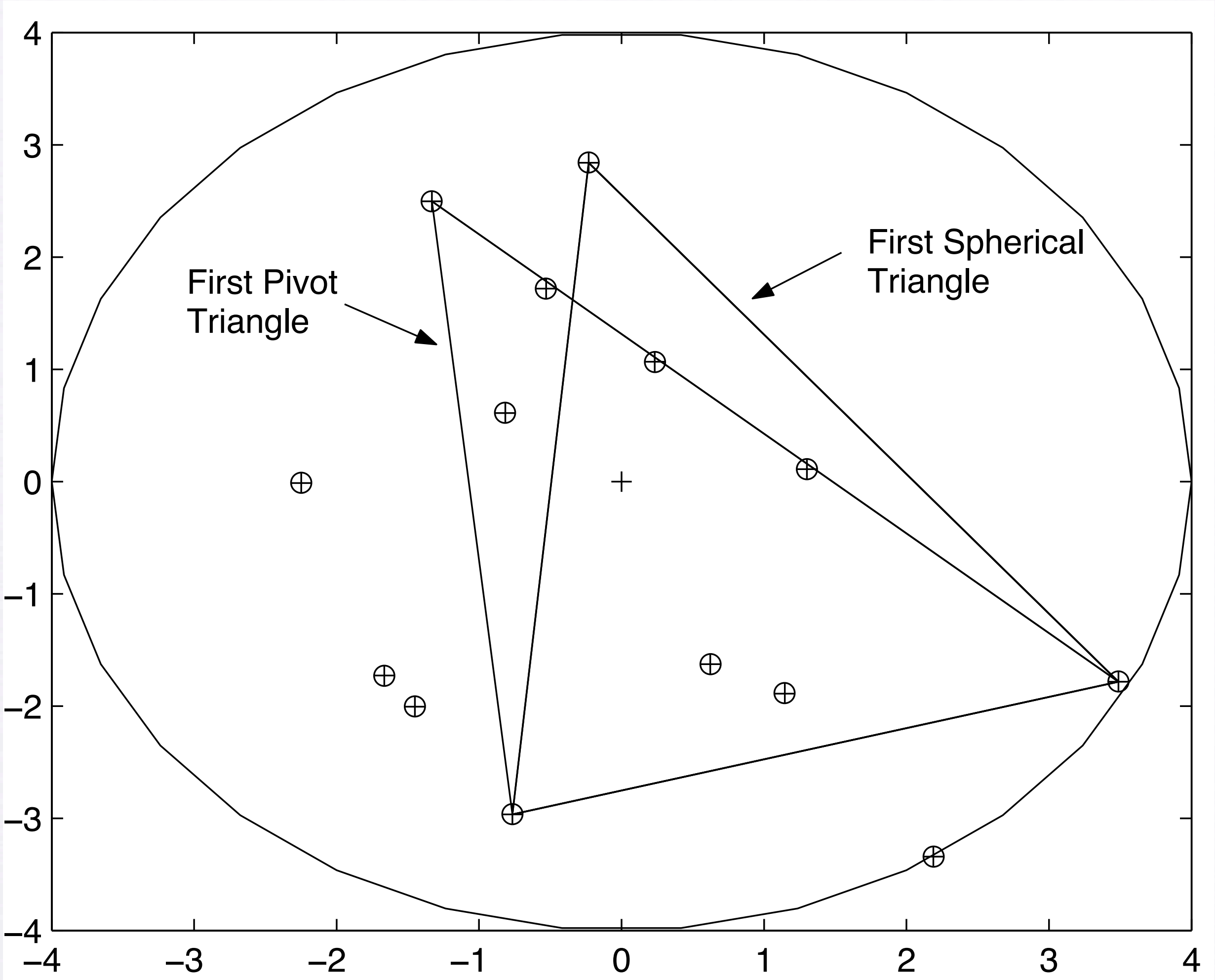
- Same as used by angle method, except three binary trees are used to speed finding angles with one matching star

- Generally fewer possible solutions for pivoting than the angle method, but many more combinations to test.



Pivoting to Solution

- Matlab output shown here



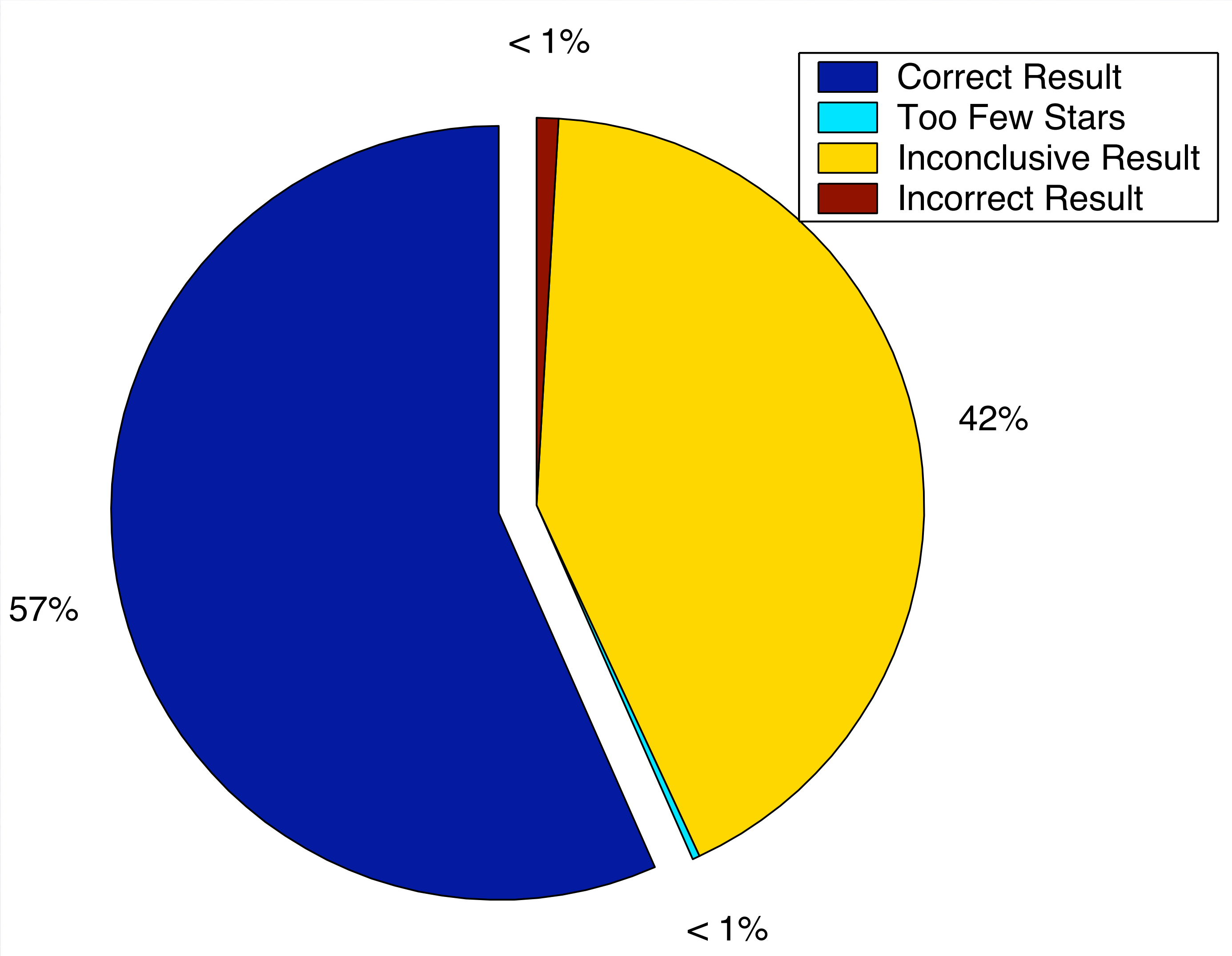
Pivoting to Solution

- Pivoting continues until single solution is reached - all stars that make up spherical triangles used for pivoting will be known
- Possible to not reach a solution
  - Run out of spherical angles to which to pivot or pivot limit is reached
  - So many possible solutions that number of finalists after pivoting begins to rise

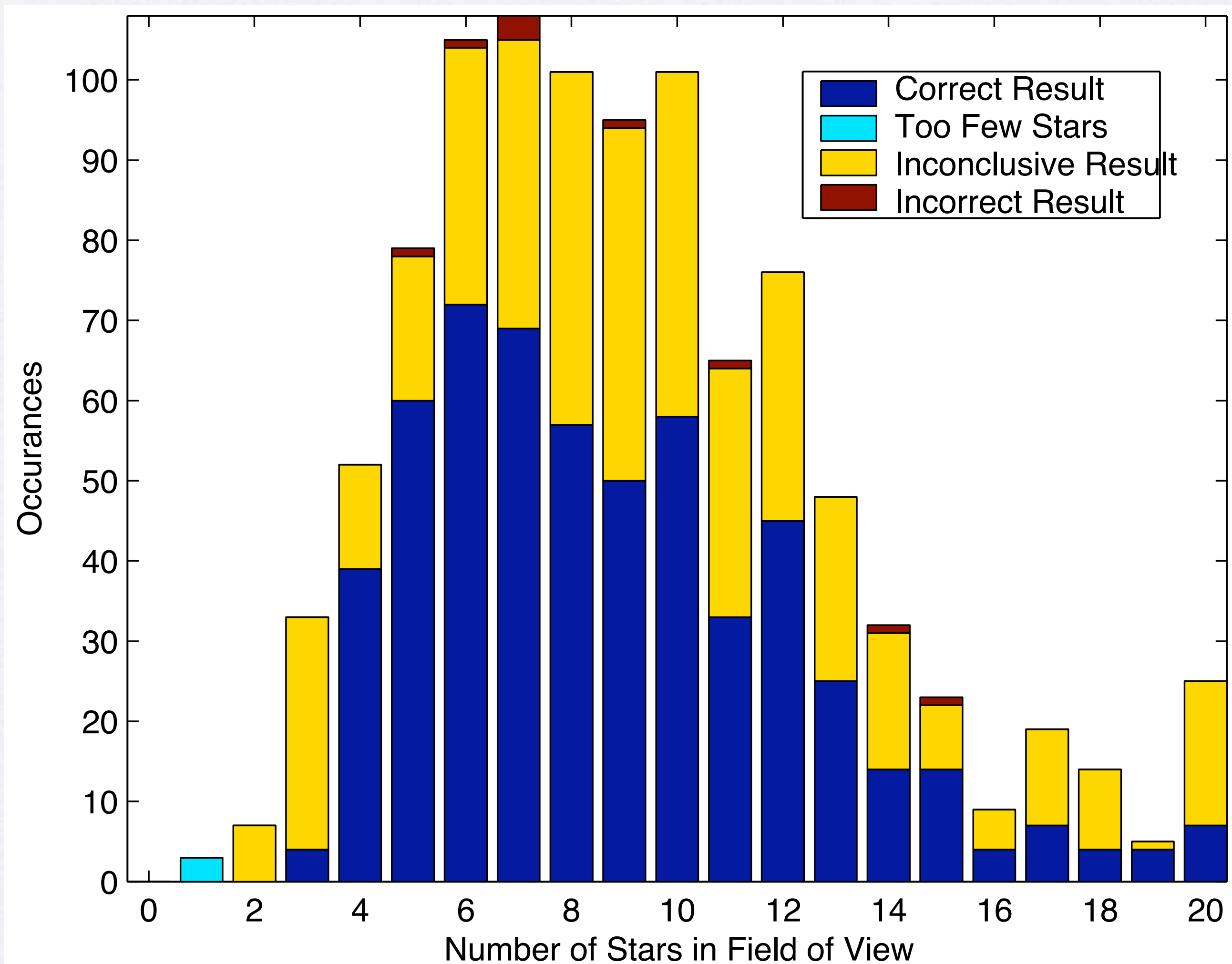
Results for Angle Method

- 1,000 Random Attitudes
- Magnitude 6 Stars
- 8 deg FOV, 6.94 deg Angle Limit
- 10,000 possible solution limit
- 1,000 pivot limit (no limit)

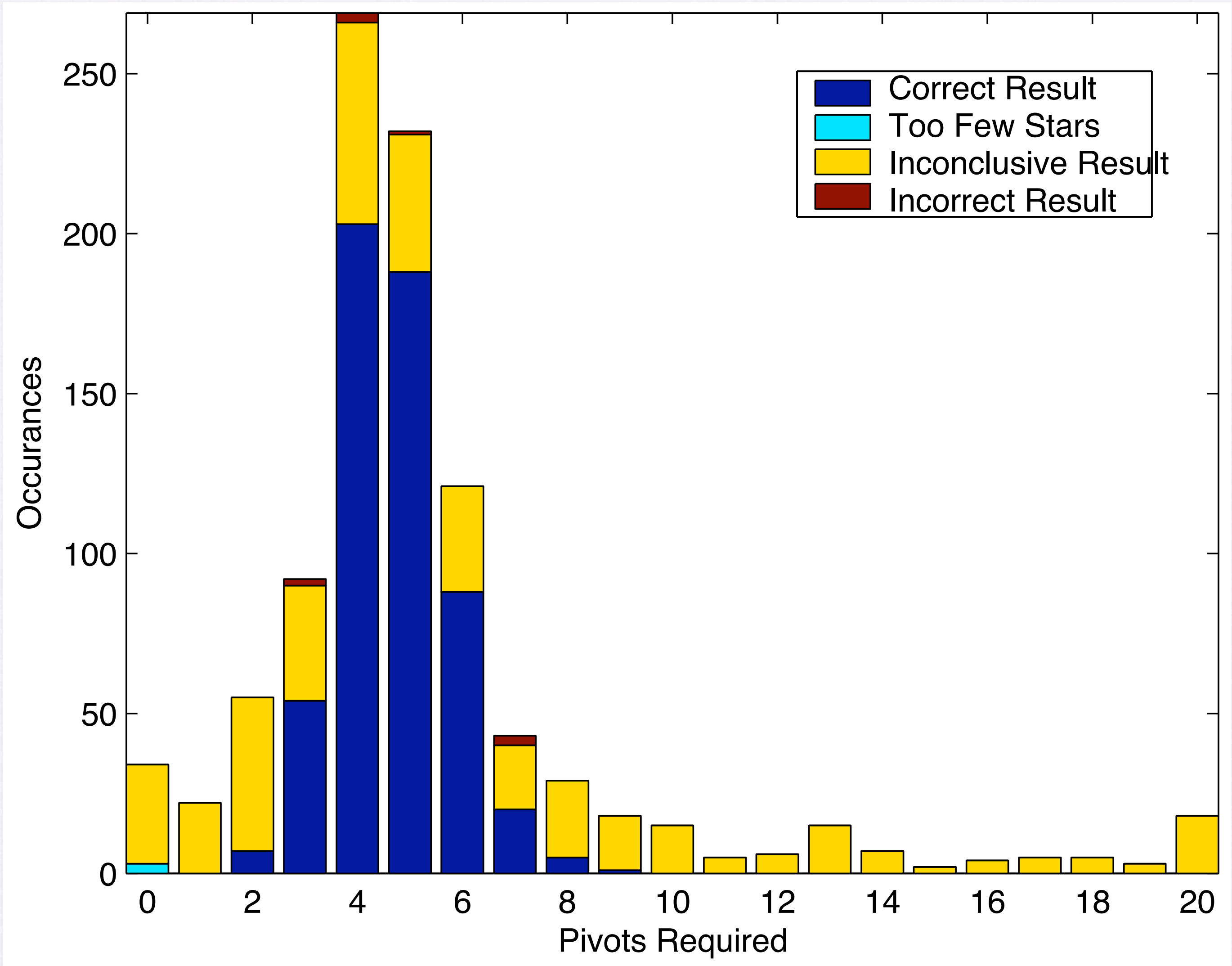
Overall Results for Angle Method



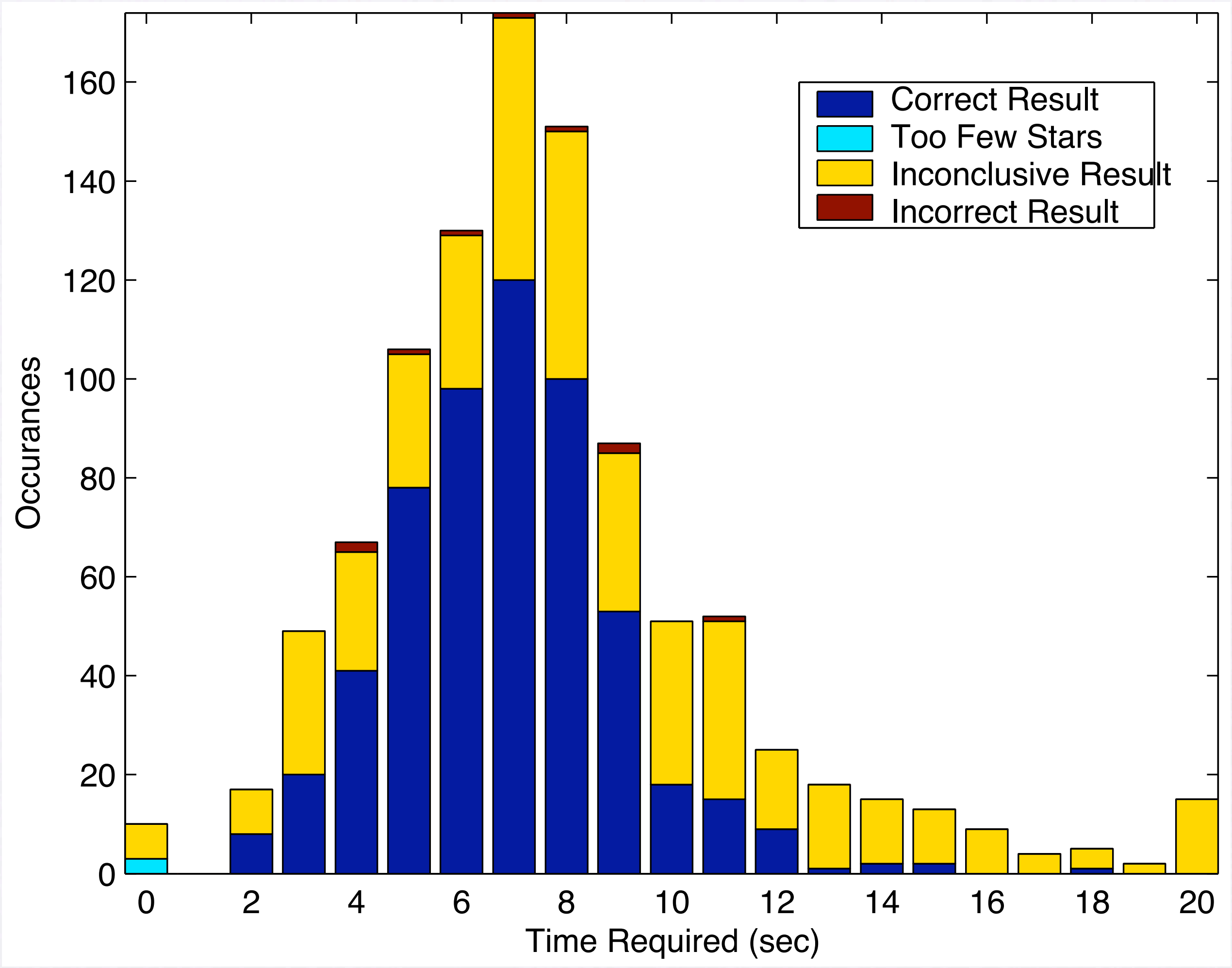
Distribution of Results for Angle Method



Pivots Required for Angle Method



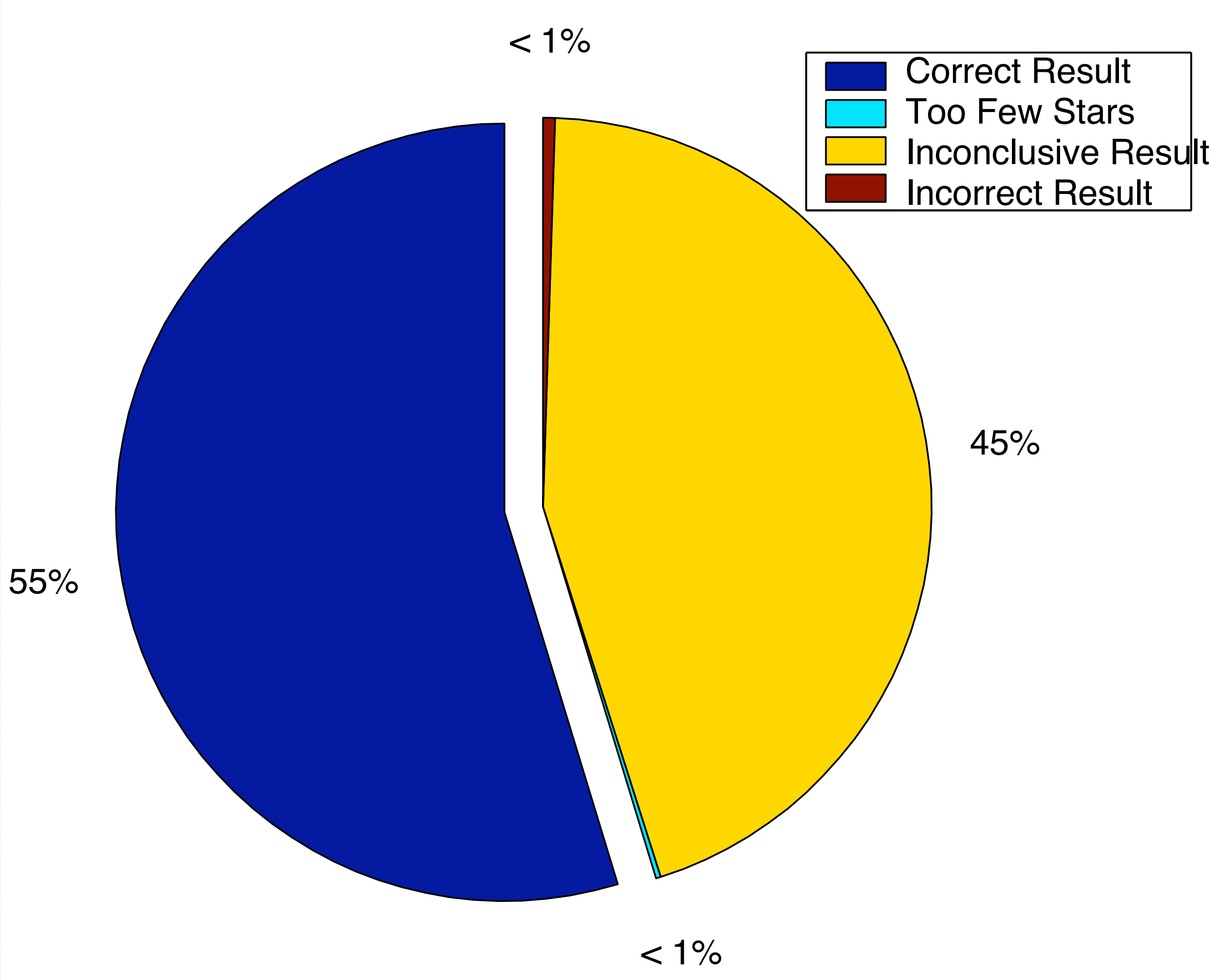
CPU Time Required for Angle Method



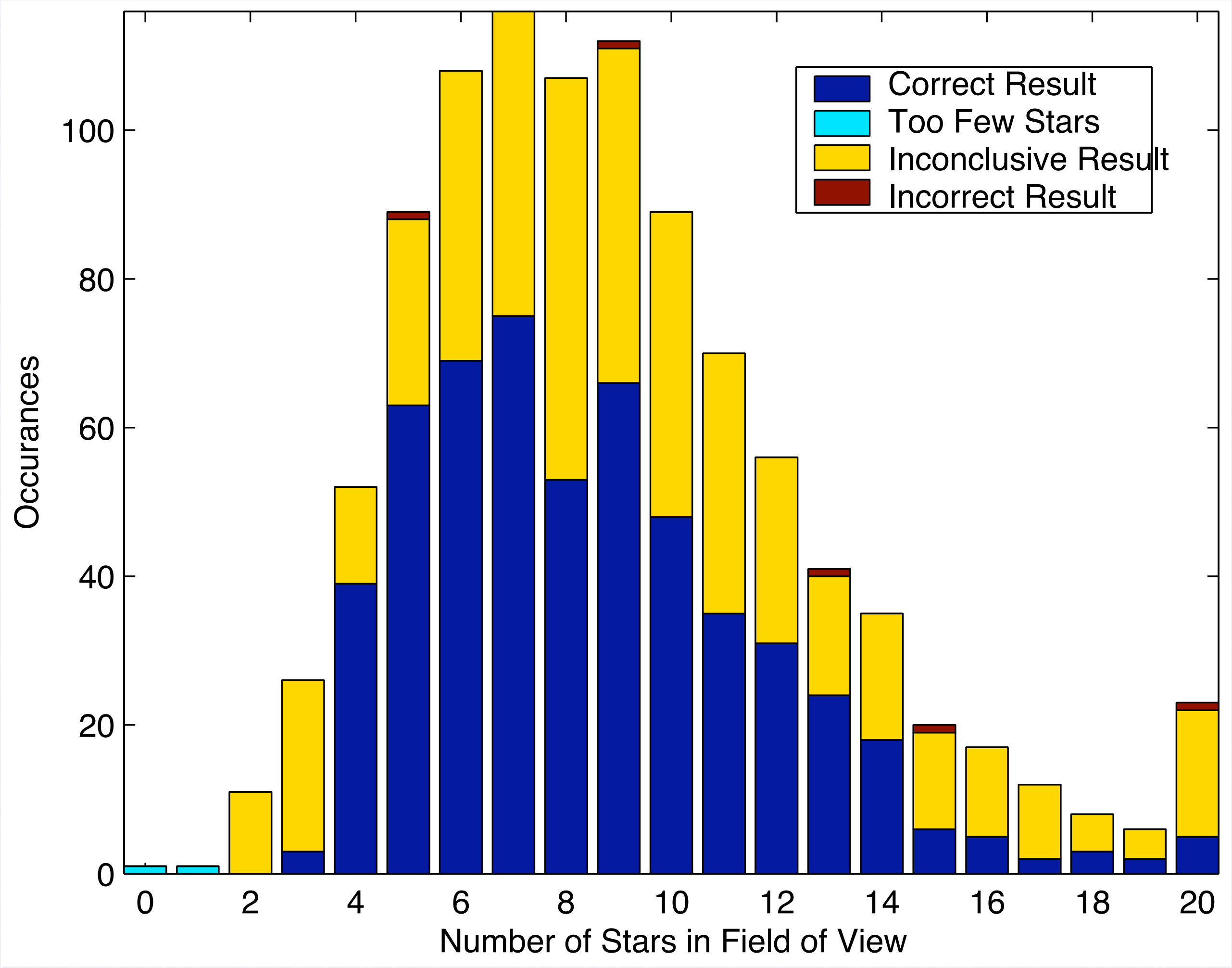
Results for Angle Method With 9-Pivot Limit

- 1,000 Random Attitudes
- Magnitude 6 Stars
- 8 deg FOV, 6.94 deg Angle Limit
- 10,000 possible solution limit
- 9-pivot limit (no limit)

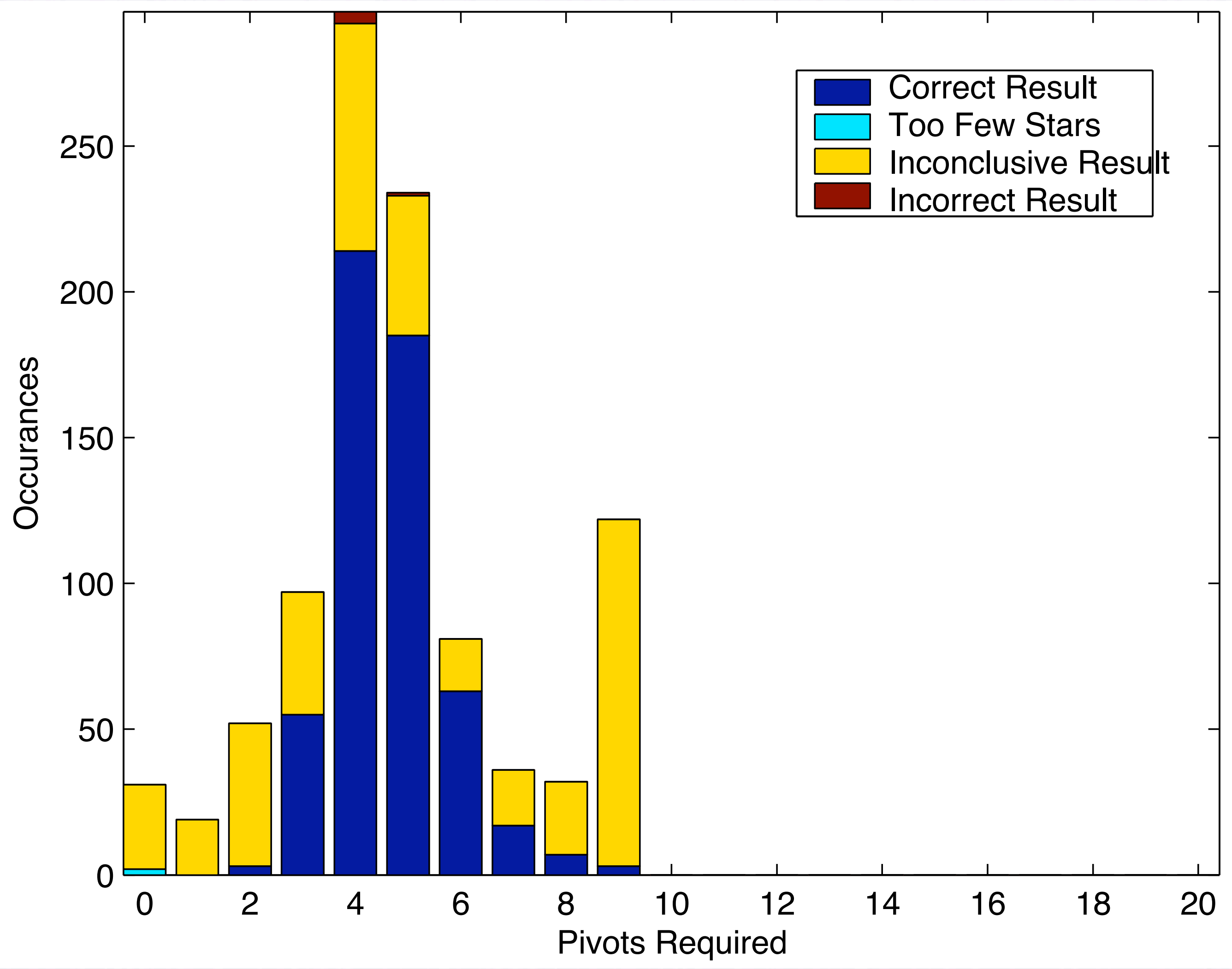
Overall Result for Angle Method with 9-Pivot Limit



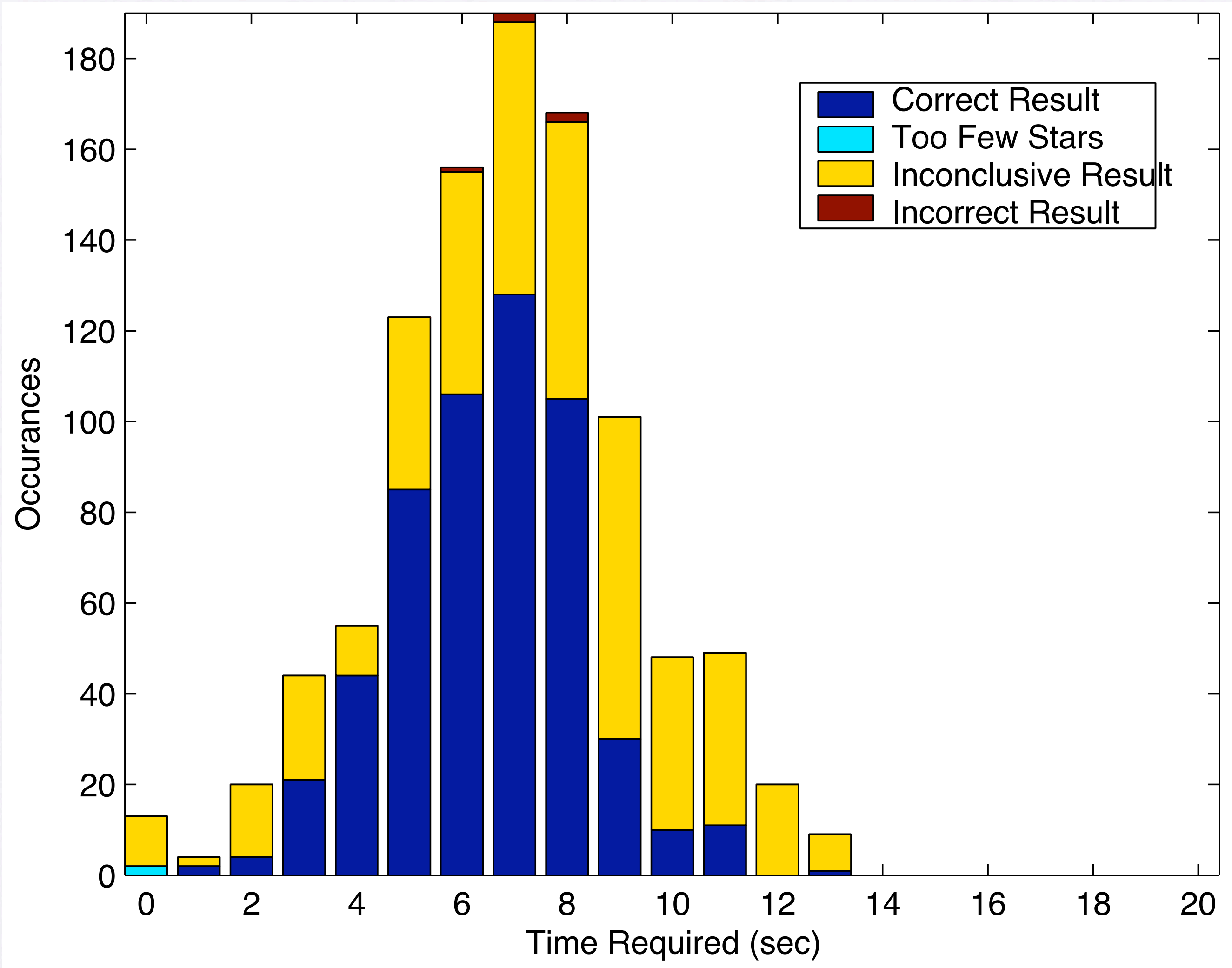
Distribution of Results for Angle Method with 9-Pivot Limit



Pivots Required for Angle Method with 9-Pivot Limit



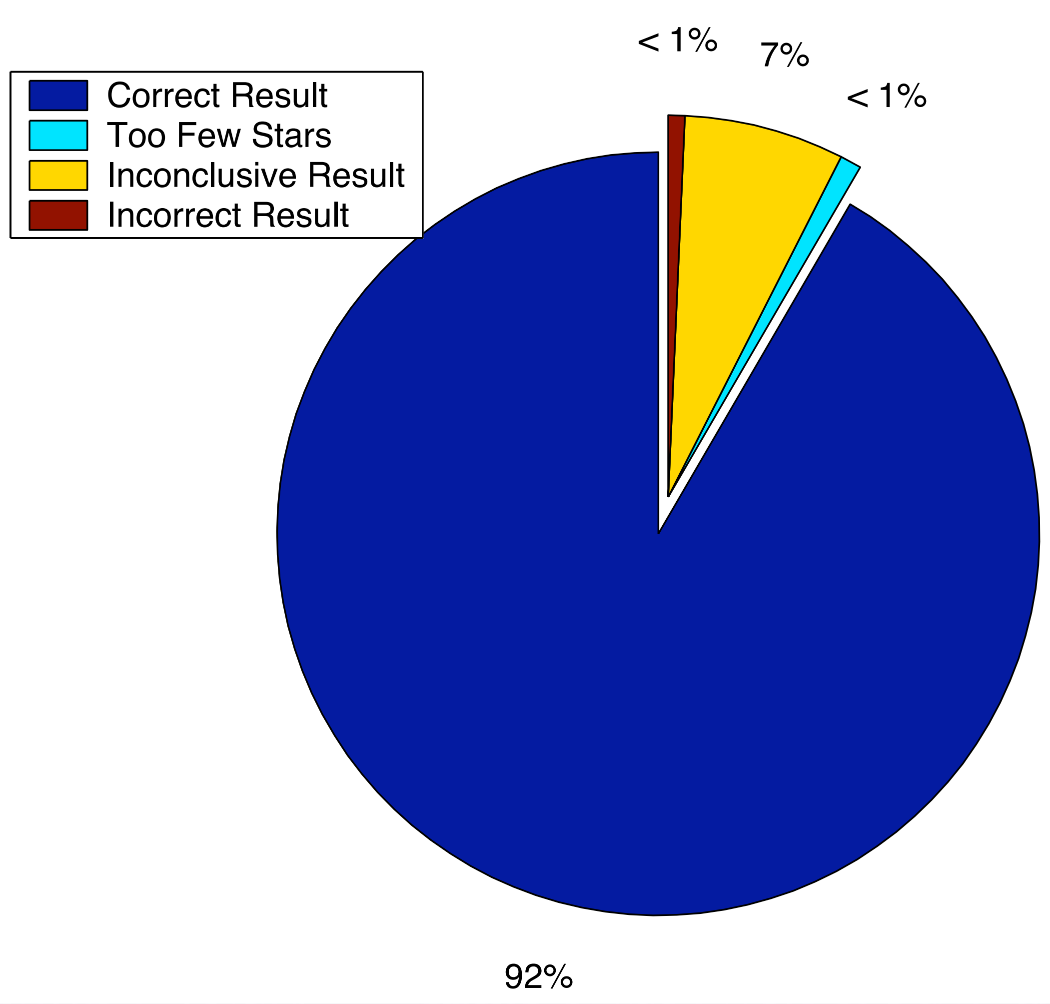
CPU Time Required for Angle Method with 9-Pivot Limit



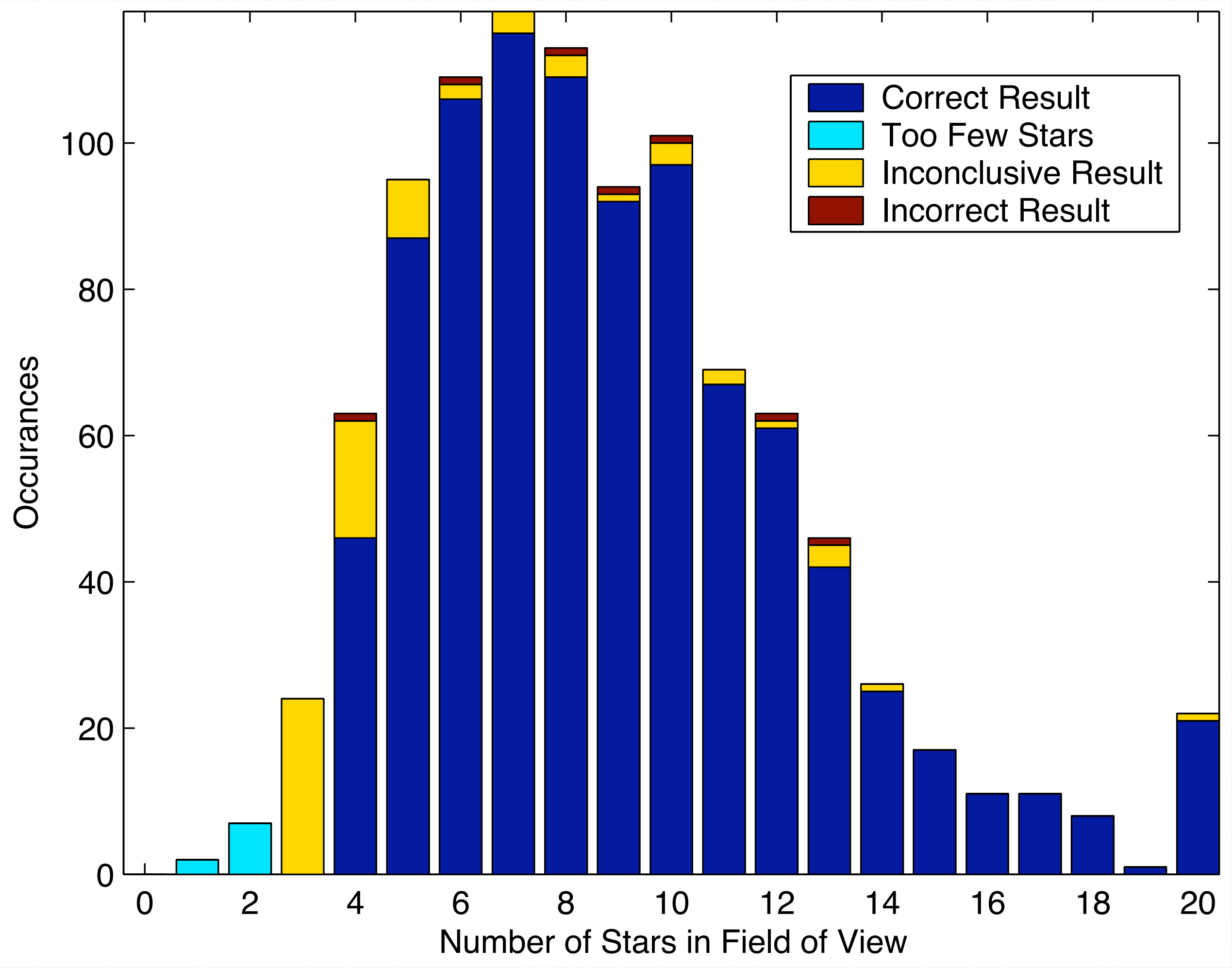
Results for Spherical Triangle Method

- 1,000 Random Attitudes
- Magnitude 6 Stars
- 8 deg FOV, 6.94 deg FOV Limit
- 10,000 possible solution limit
- 1,000-pivot limit (no limit)

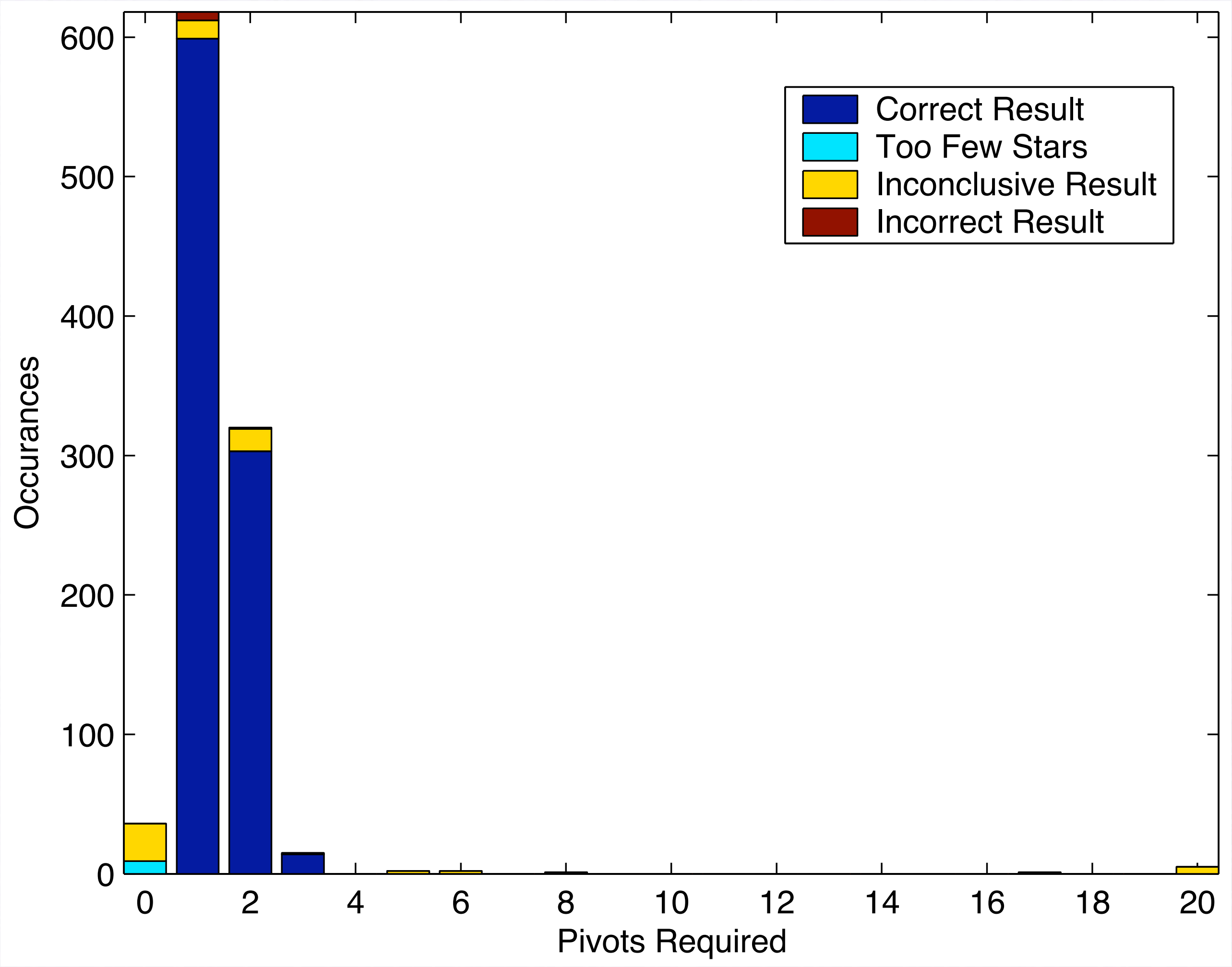
Overall Result for Spherical Triangle Method



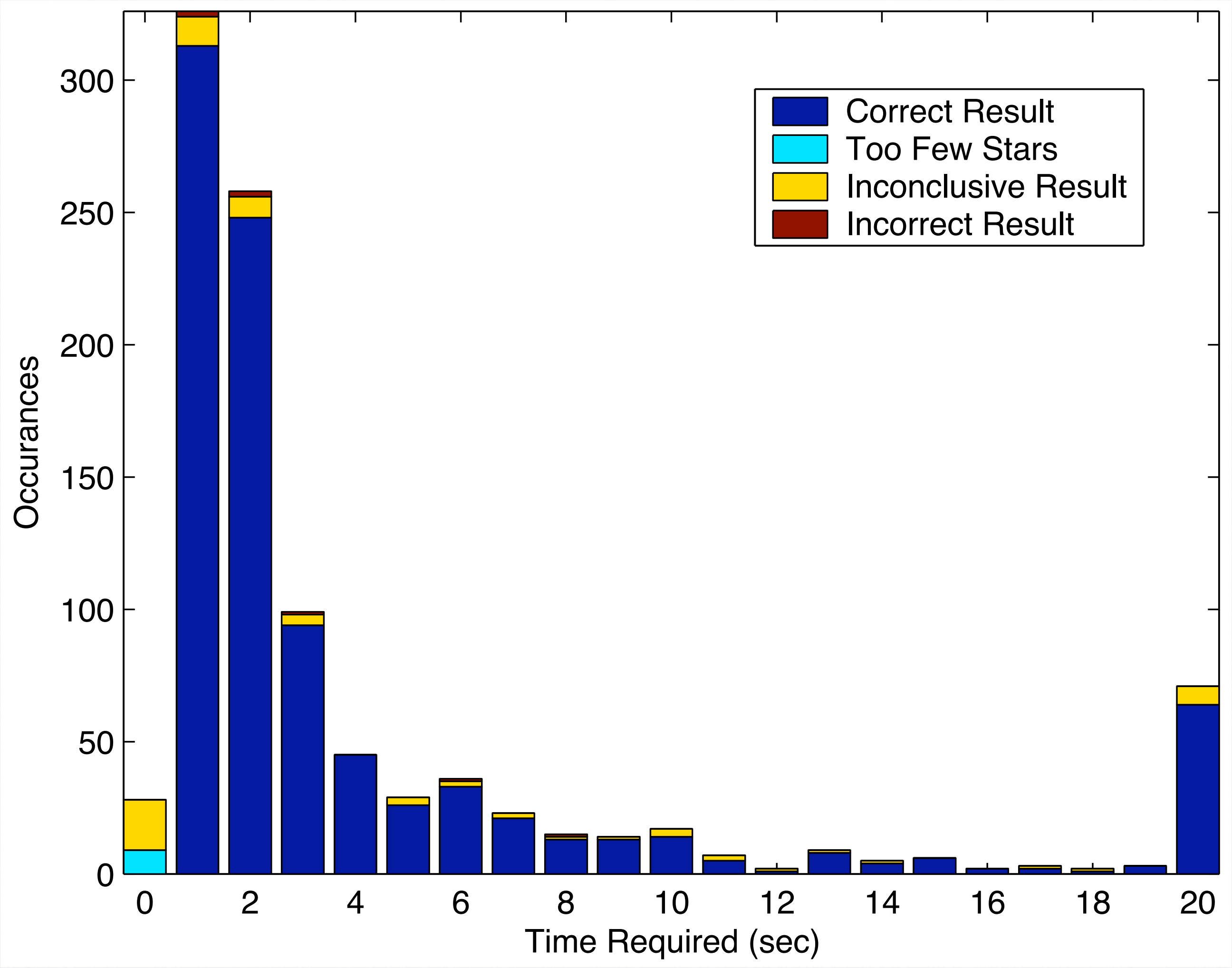
Distribution of Results for Spherical Triangle Method



Pivots Required for Spherical Triangle Method



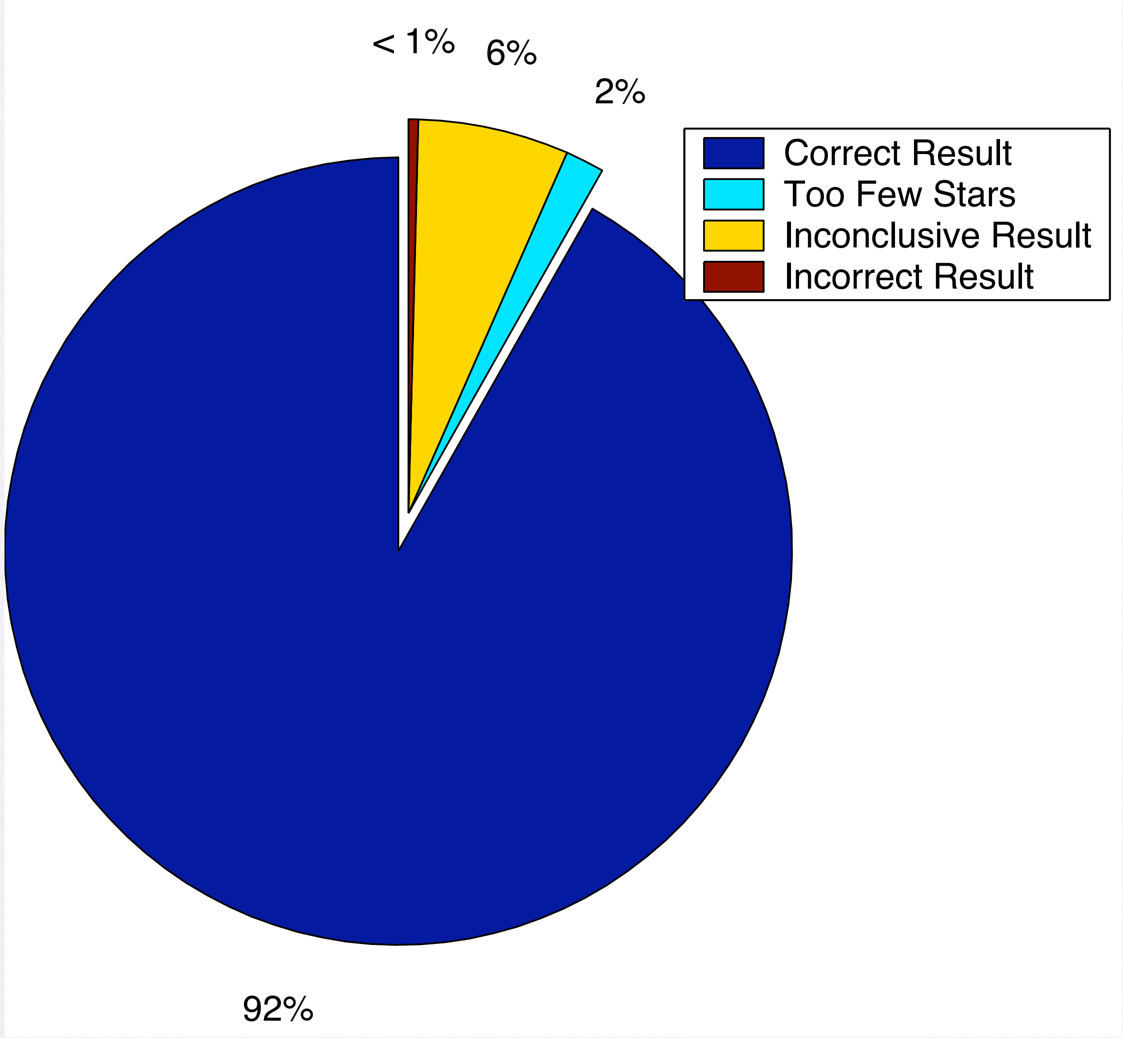
CPU Time Required for Spherical Triangle Method



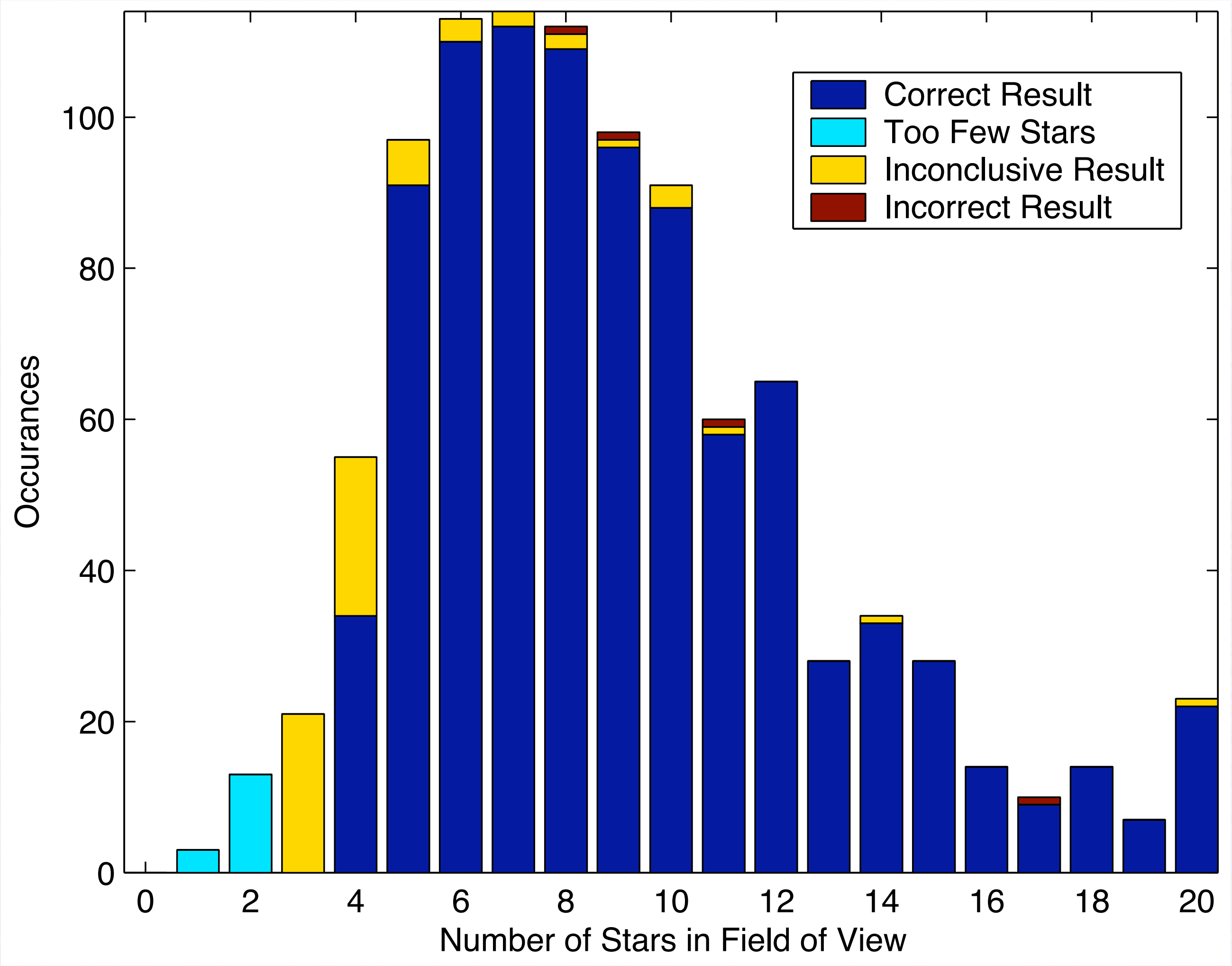
Results for Spherical Triangle Method With 3-Pivot Limit

- 1,000 Random Attitudes
- Magnitude 6 Stars
- 8 deg FOV, 6.94 deg Angle Limit
- 10,000 possible solution limit
- 3-pivot limit

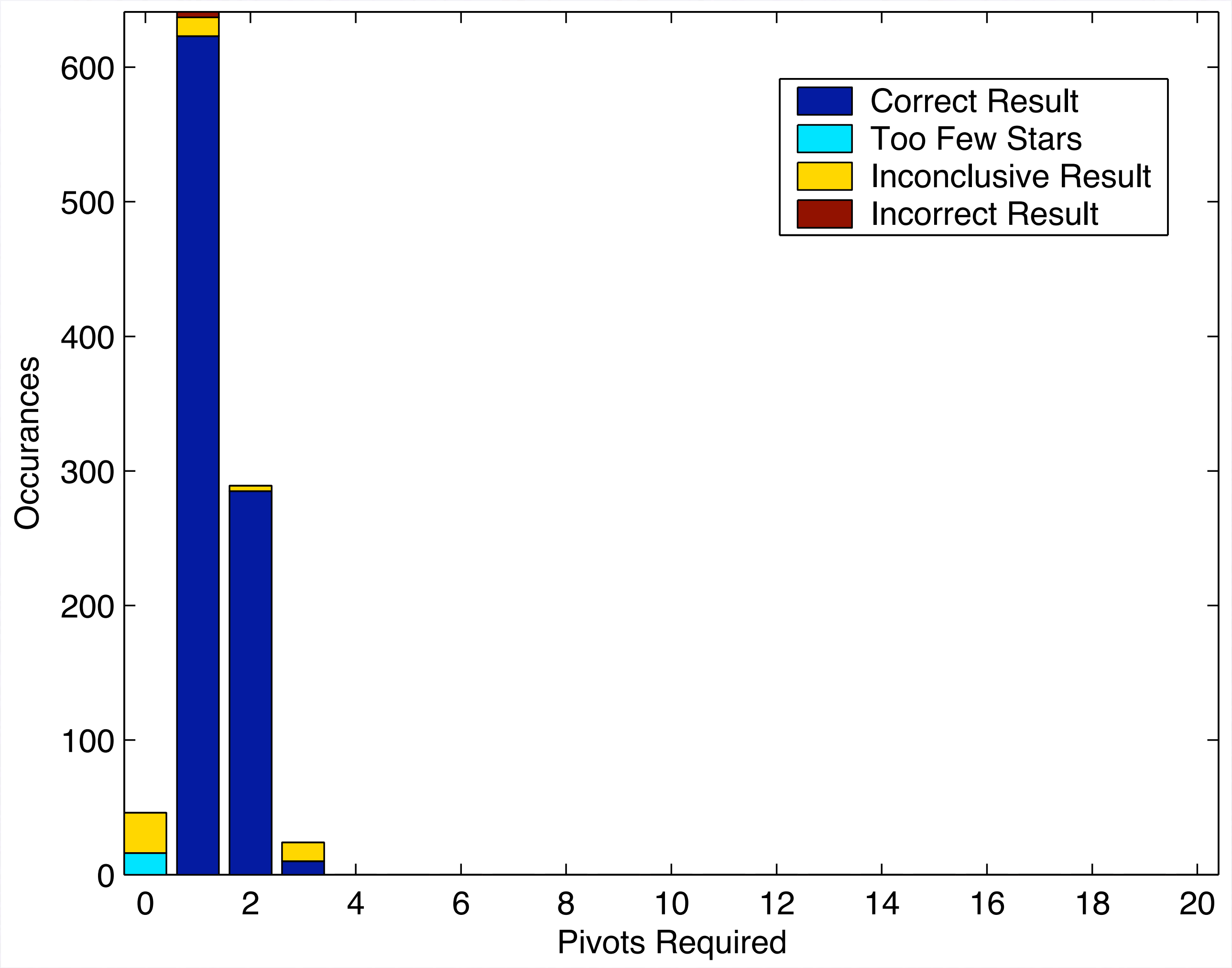
Overall Result for Sph. Tri. Method with 3-Pivot Limit



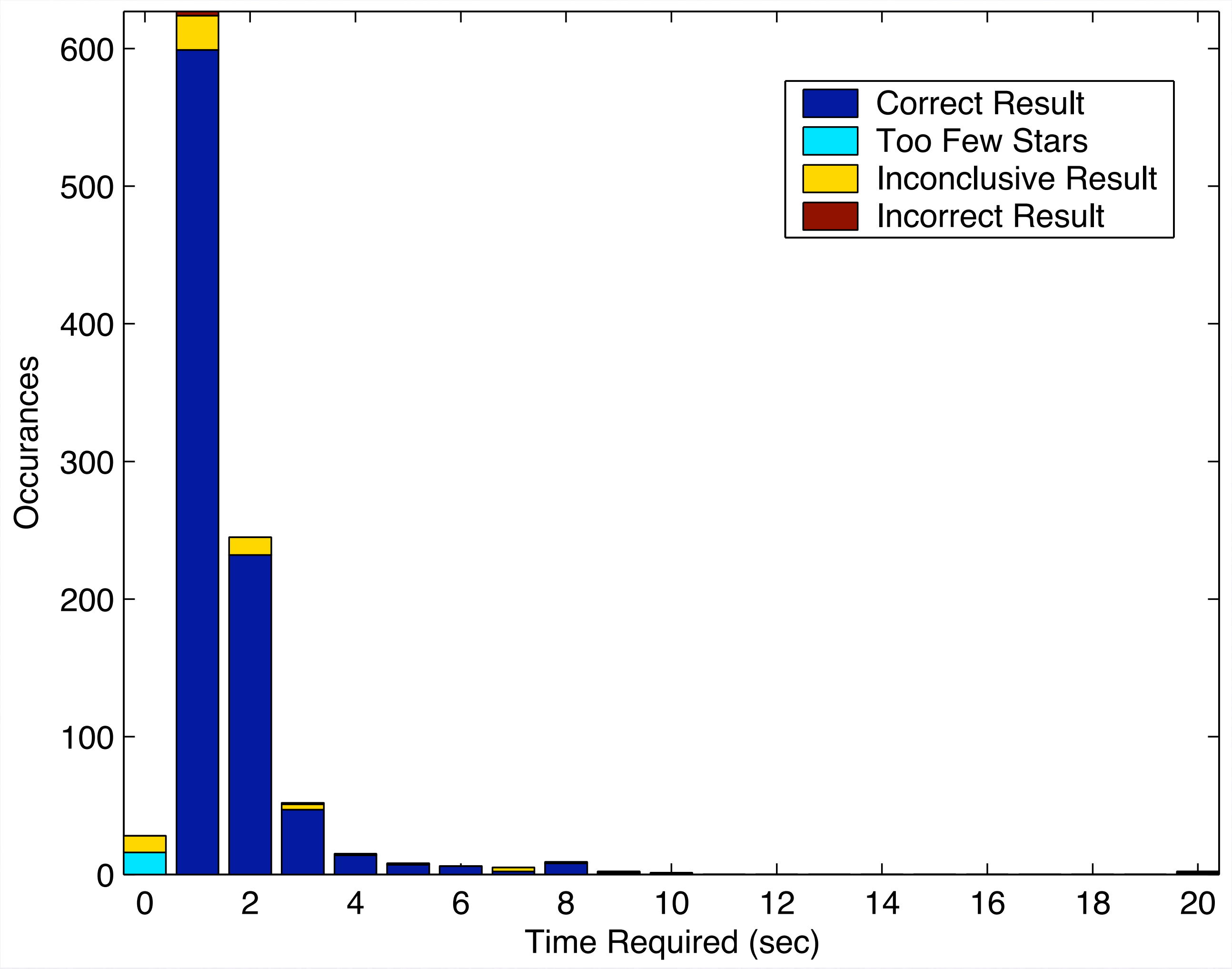
Distribution of Results for Sph. Tri. Method with 3-Pivot Limit



Pivots Required for Sph. Tri. Method with 3-Pivot Limit



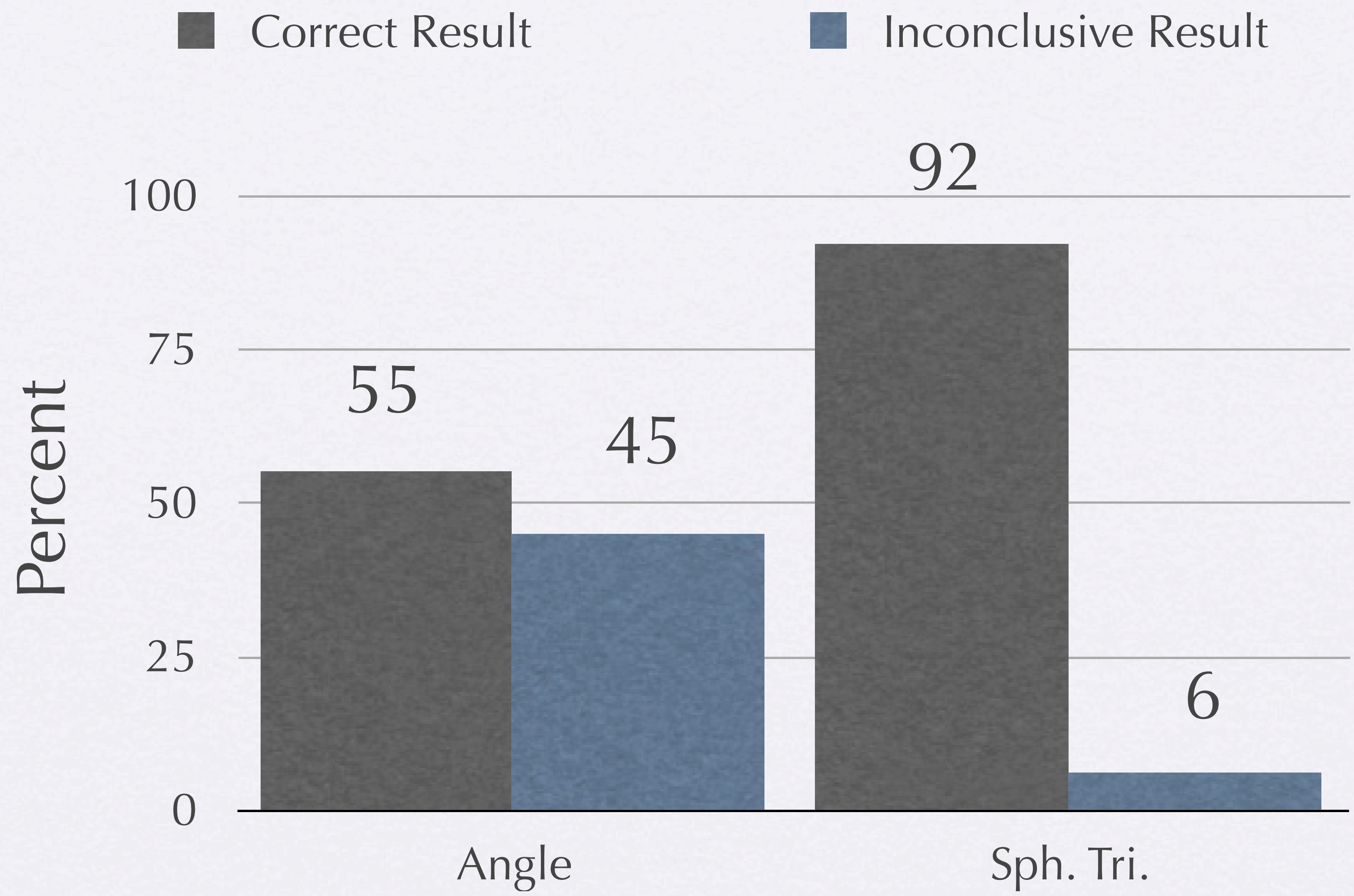
CPU Time Required for Sph. Tri. Method with 3-Pivot Limit



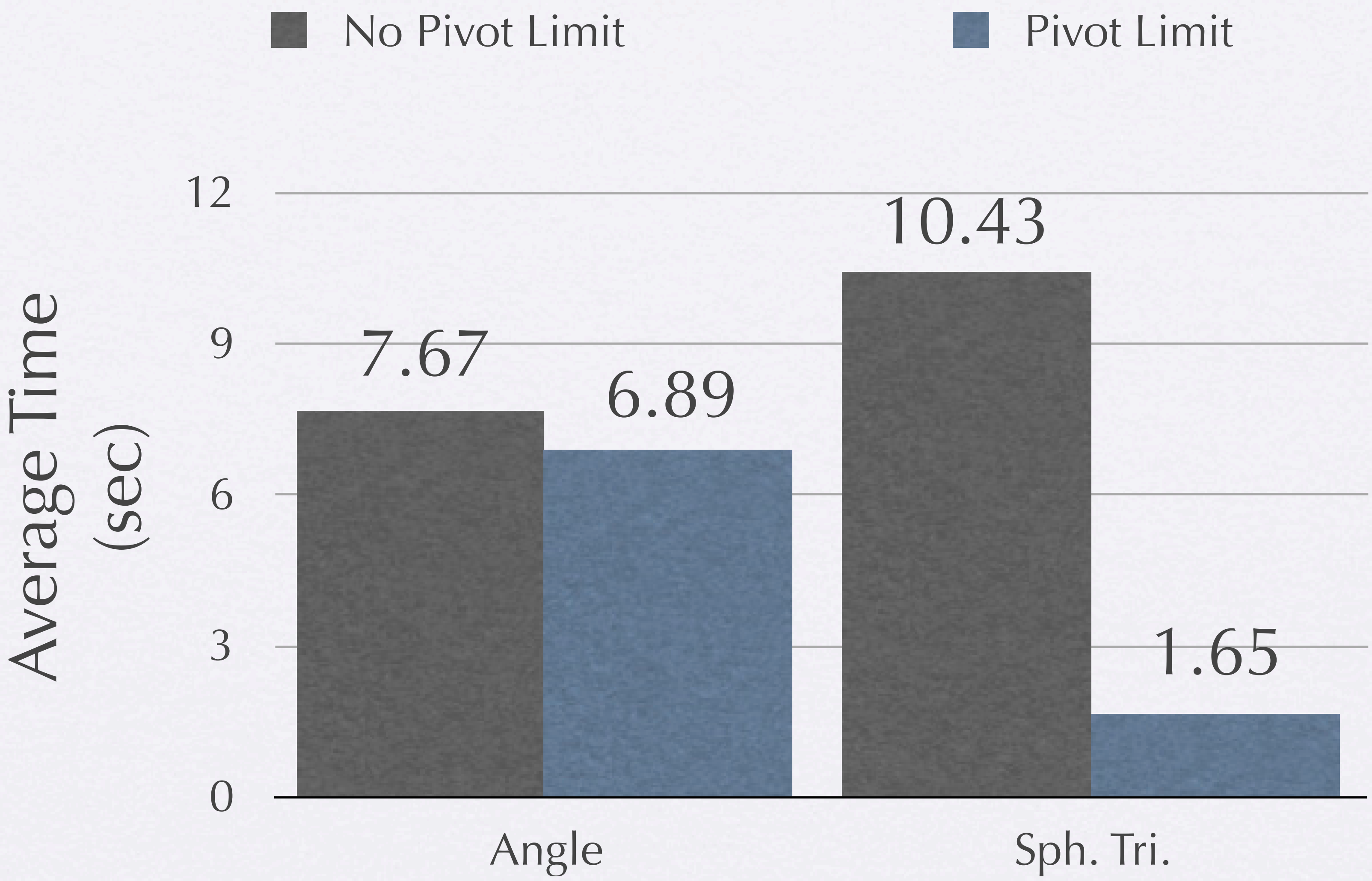
Comparison Of Results

- Success Rate
- CPU Time Required

Overall Success Comparison



CPU Time Comparison



Conclusions

- Spherical Triangle Method faster and more likely to obtain a correct result
- Similar rates of incorrect attitudes
  - Result of the 3σ bound on solution
- Data storage requirements of spherical triangle method may be troublesome

Directions to Continue

- Explore different fields of view and magnitude sensitivity
- Determine polar moment and standard deviation of polar moment analytically
- Incorporate quad-tree into star pattern recognition algorithm
- Spherical pent-tree with dodecahedron base to make more efficient

Thank you!

- To everyone for coming
- To Dr. Mook and Dr. Singh for for accepting to chair my thesis defense
- To Dr. Crassidis, for his help, for his guidance, and for giving me the opportunity to explore this very exciting field of aerospace engineering