

Fast Star Pattern Recognition Using Spherical Triangles

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Problem of Star Pattern Recognition

Star Trackers

- Imaging device which tries to determine attitude by determining what stars lie within its field of view
- Used by many satellites and spacecraft for attitude control
- Typical star tracker has 8 deg field of view and is sensitive to magnitude six stars

Ideal Star Tracker

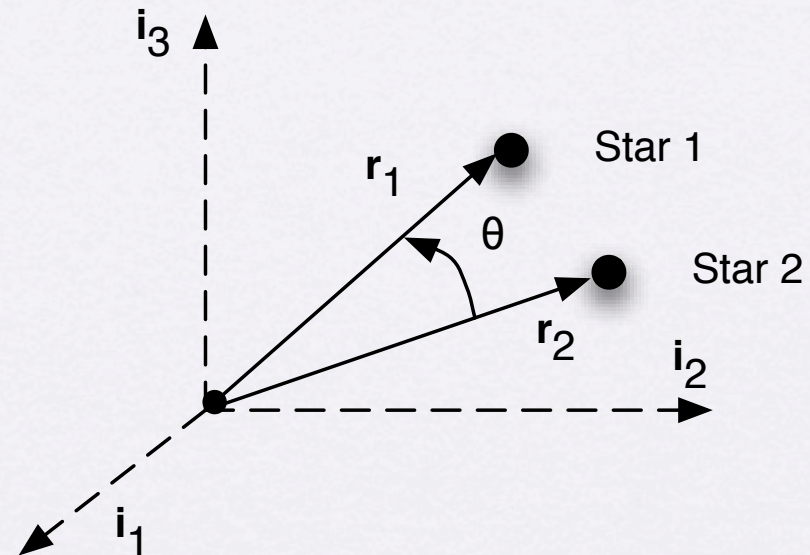
- Fast - particularly for tumbling situations
- Reliable
 - Maximize correct determinations
 - Minimize incorrect determinations
- Consume little in computer resources such as RAM, storage and processor capability

Star Pattern Recognition

- Method by which star tracker determines the stars to which it is pointed
- Many methods, many proprietary
 - Angle, Magnitude, “a priori,” lost-in-space
- Angle Method & Spherical Triangle Method
 - Both methods tested and results compared

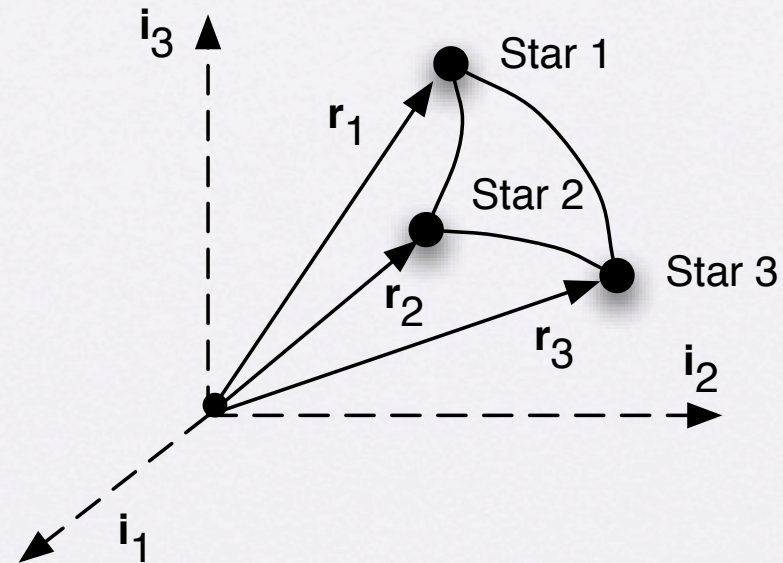
Angle Method

- Angles between stars stored in catalog
- Cosine of angles between stars in field of view matched to cosine of angles in catalog
- Must have at least two stars in field of view to be useful



Spherical Triangle Method

- Catalog of spherical triangles made
- Area and polar moment of spherical triangle made from stars in FOV matched to those in catalog
- Must have at least three stars in field of view to work.



Effects of Measurement Error

- Matching angles and spherical triangles made difficult because of star tracker position measurement error
- Typical star tracker error has normal distribution and a standard deviation (σ) of 87 microradians

Effects of Measurement Error

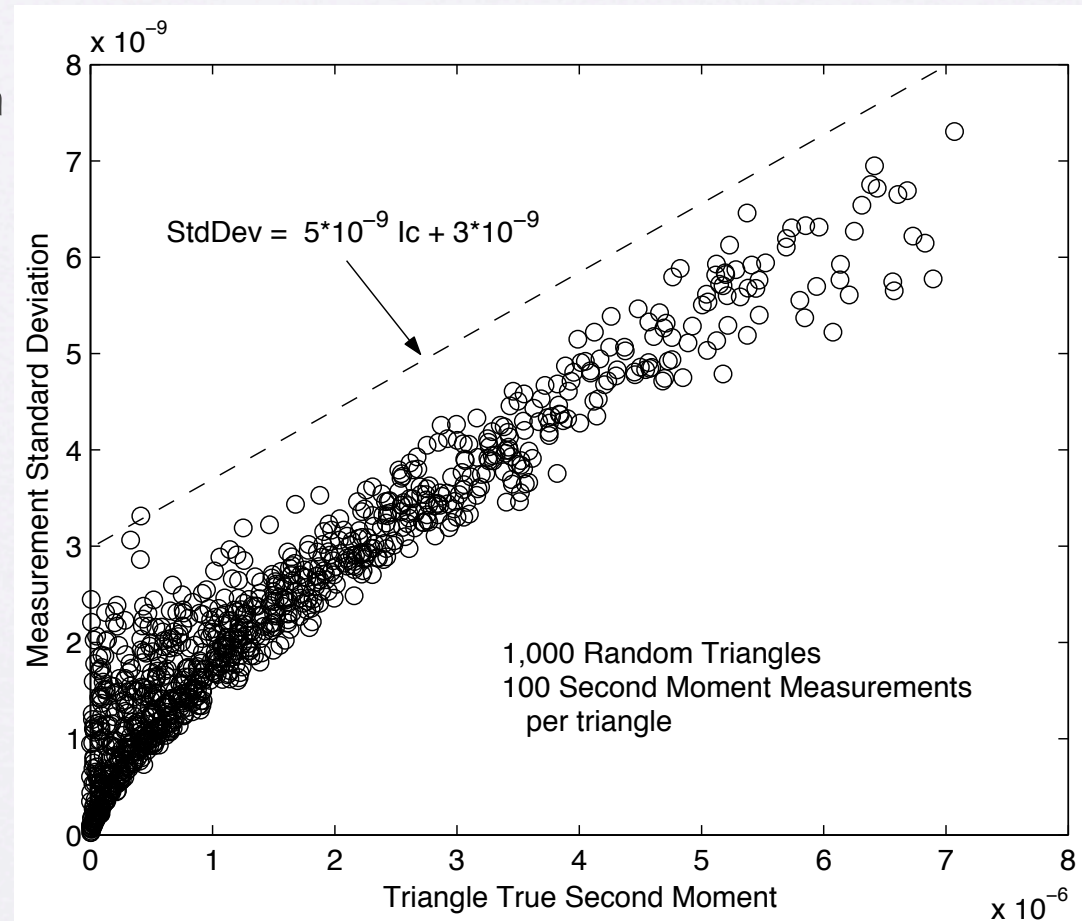
- Choose a σ bound for measurement
- Larger σ means greater probability of correct angle or spherical triangle existing within bounds, but also makes it more likely more than one angle or spherical triangle can be the correct solution.
- Probability of true position lying within 3σ bound of measurement is 99.7%.

Effects of Measurement Error

- Standard Deviation of Area
 - Covariance analysis made
 - Standard Deviation can be determined analytically from vectors pointing to vertices of spherical triangle

Effects of Measurement Error

- Standard Deviation of Polar Moment determined graphically.
- Method overestimates actual standard deviation, but ensures meeting standard deviation requested



Effects of Measurement Error

- Typical angle measurement will result in a hundred or more possible solutions.
- Typical spherical triangle area measurement will result in hundreds or thousands of possible solutions.
- Method used to approach a single solution is called “pivoting.”

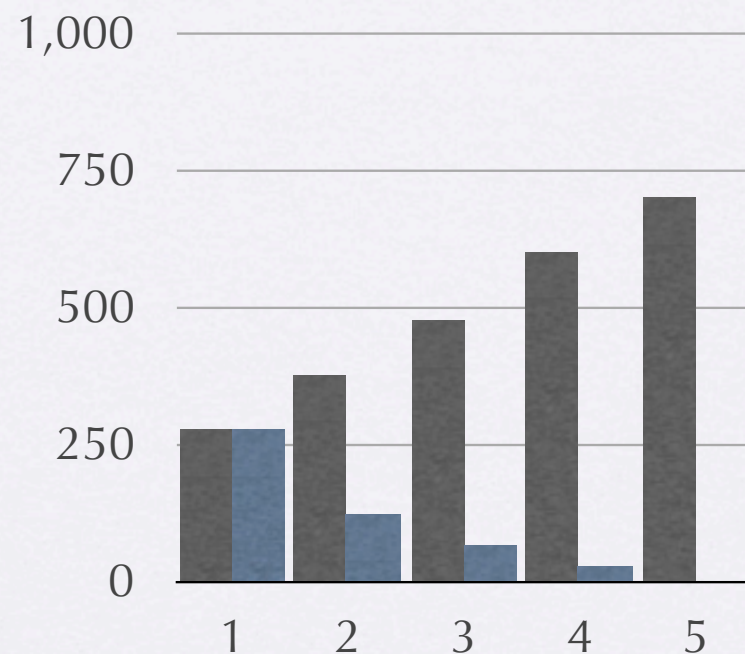
Pivoting With Angles

- If two angles are known to share a star in common, the solution to both must also have a star in common

- Multiple pivots can ultimately lead to a single solution

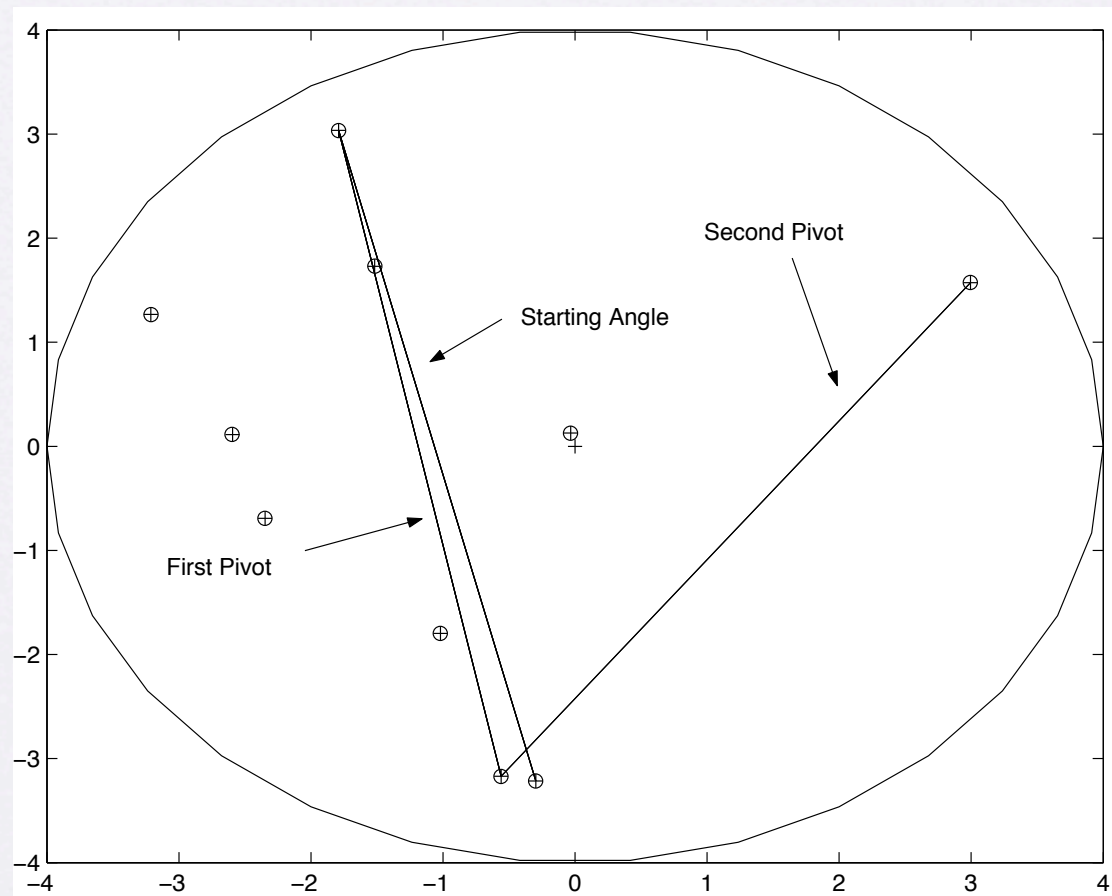
- Computationally expensive

■ Solutions Per Angle
■ Solutions After Pivot



Pivoting to Solution

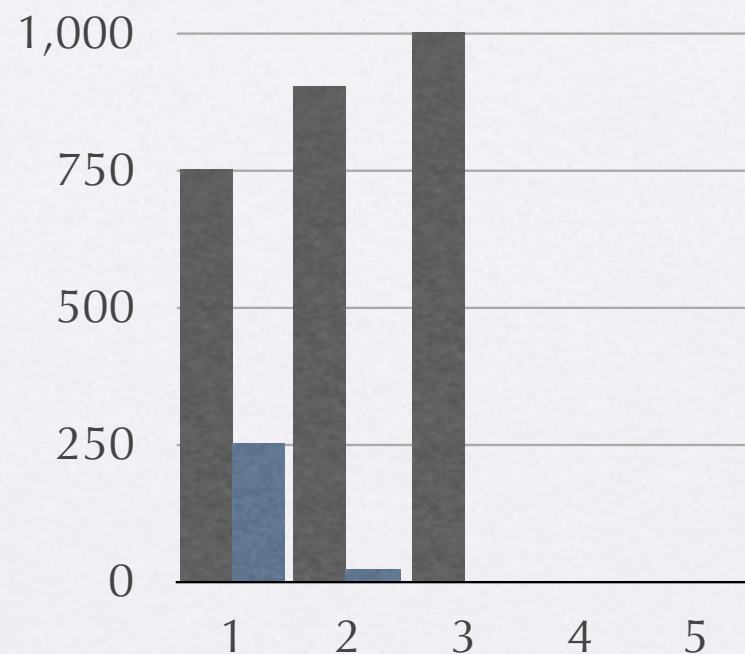
- Matlab output shown here



Pivoting With Spherical Triangles

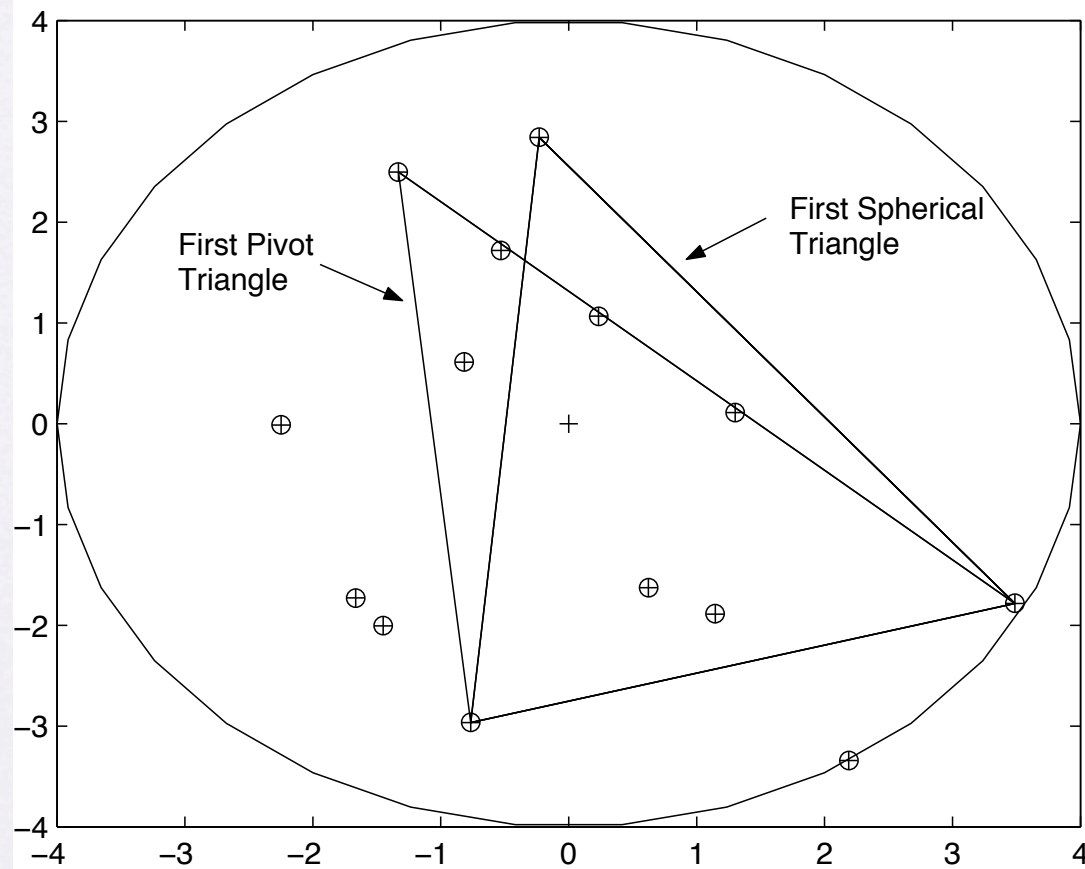
- Similar to angle pivots
- If two spherical triangles are known to share two stars in common, the solution to both must also have two stars in common
- Will use polar moment also reduce possible solutions

■ Solutions Per Spherical Triangle
■ Solutions After Pivot



Pivoting to Solution

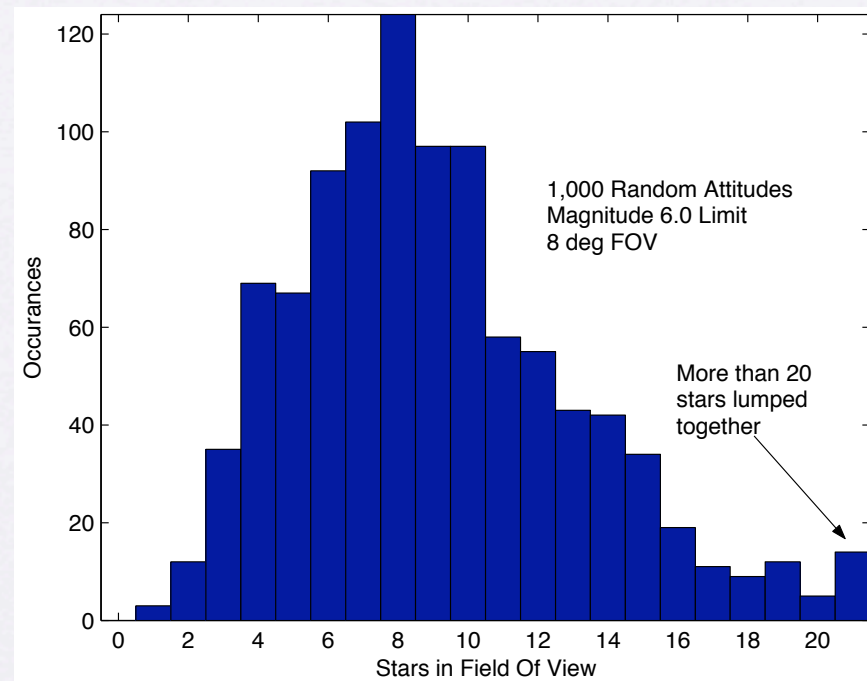
- Matlab output shown here



Catalog Creation

Star Catalog

- Star catalog used has 8,118 stars magnitude 6.0 or brighter
- Entire Celestial Sphere for lost-in-space testing.
- Random attitude testing reveals probability of seeing certain number of stars in 8 deg FOV



Angle Catalog Requirements

- Angles between pairs of stars, sorted by angle
- Stars that make up the angle
- K-Vector to speed finding a particular pair of stars by it's angle

Angle Catalog Created

- 106,308 angles found
- Less than an hour to compute
- 12 MB file

Spherical Triangle Catalog Requirements

- Area of triangles formed by triplets of stars, sorted by area.
- Polar Moment of triangles
- Stars that make up the angle
- K-Vector to speed finding a particular pair of stars by it's area

Spherical Triangle Area

- Area calculated directly from vectors pointing to stars that make up the triangle

$$\mathcal{A} = 4 \tan^{-1} \sqrt{\tan \frac{s}{2} \tan \frac{s-a}{2} \tan \frac{s-b}{2} \tan \frac{s-c}{2}}$$

$$s = \frac{1}{2}(a + b + c)$$

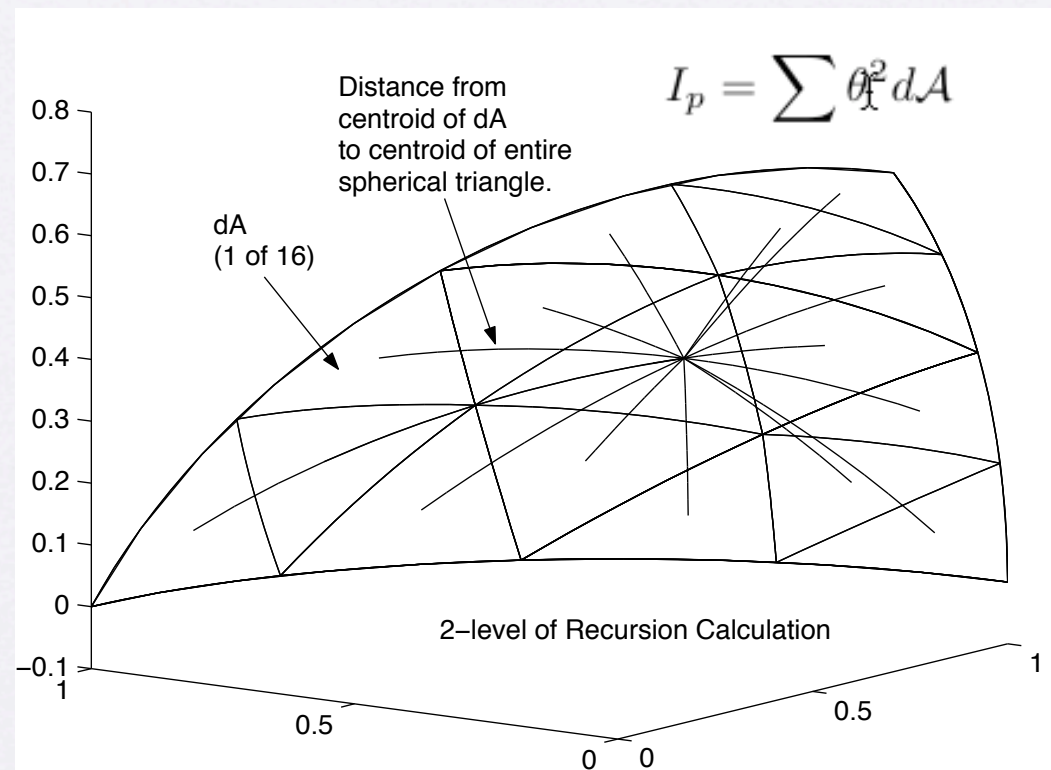
$$a = \cos^{-1} \left(\frac{\mathbf{b}_1 \cdot \mathbf{b}_2}{|\mathbf{b}_1||\mathbf{b}_2|} \right)$$

$$b = \cos^{-1} \left(\frac{\mathbf{b}_2 \cdot \mathbf{b}_3}{|\mathbf{b}_2||\mathbf{b}_3|} \right)$$

$$c = \cos^{-1} \left(\frac{\mathbf{b}_3 \cdot \mathbf{b}_1}{|\mathbf{b}_3||\mathbf{b}_1|} \right)$$

Spherical Triangle Polar Moment

- Equal to the sum of infinitesimal areas multiplied by the square of its distance from the axis of interest, in radians



Spherical Triangle Catalog Created

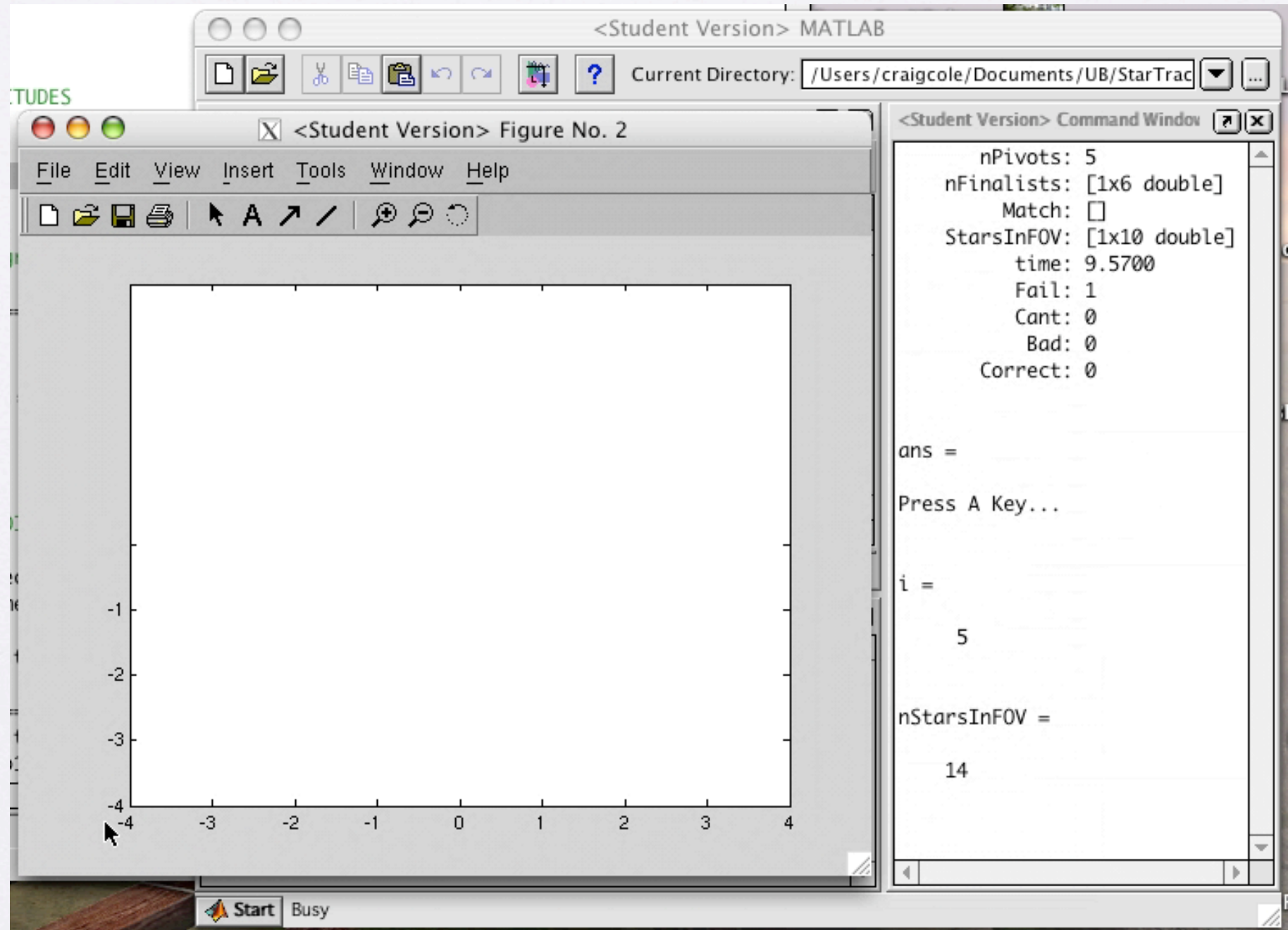
- 662,799 spherical triangles found
- 9 hours to compute
 - Most time spent calculating polar moment
- 162 MB file

Testing Both Methods

Method of Testing

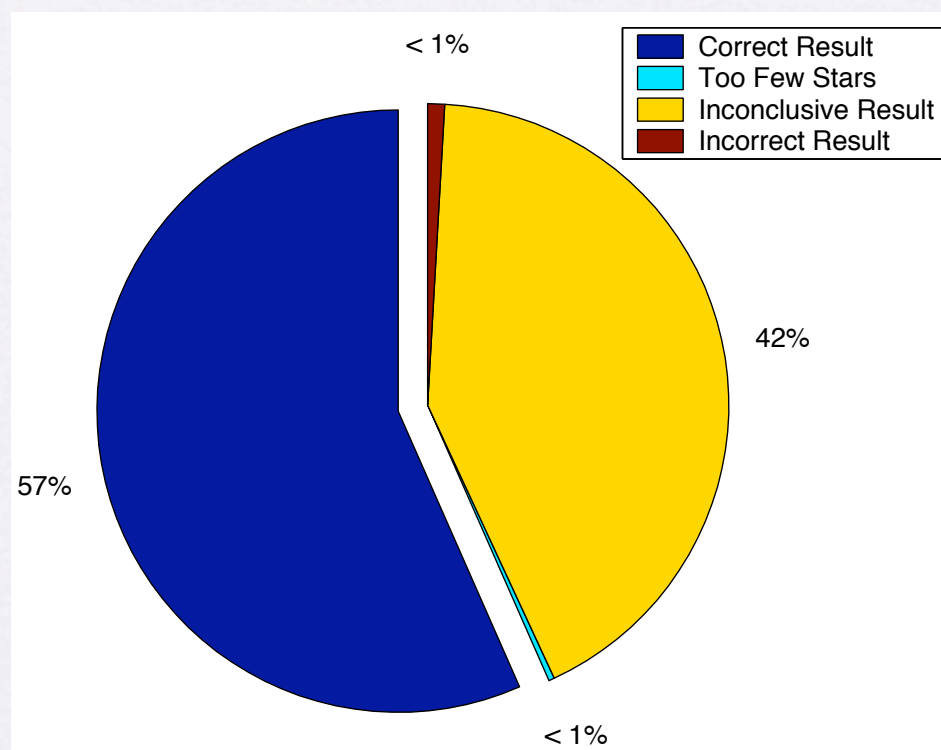
- 1,000 random attitudes for angle and spherical triangle algorithms tested
 - Stars in FOV converted to tracker frame, noise added.
- Results of compiled
 - Correct, Cannot, Inconclusive & Incorrect
- False Star Inclusion

Angle Method



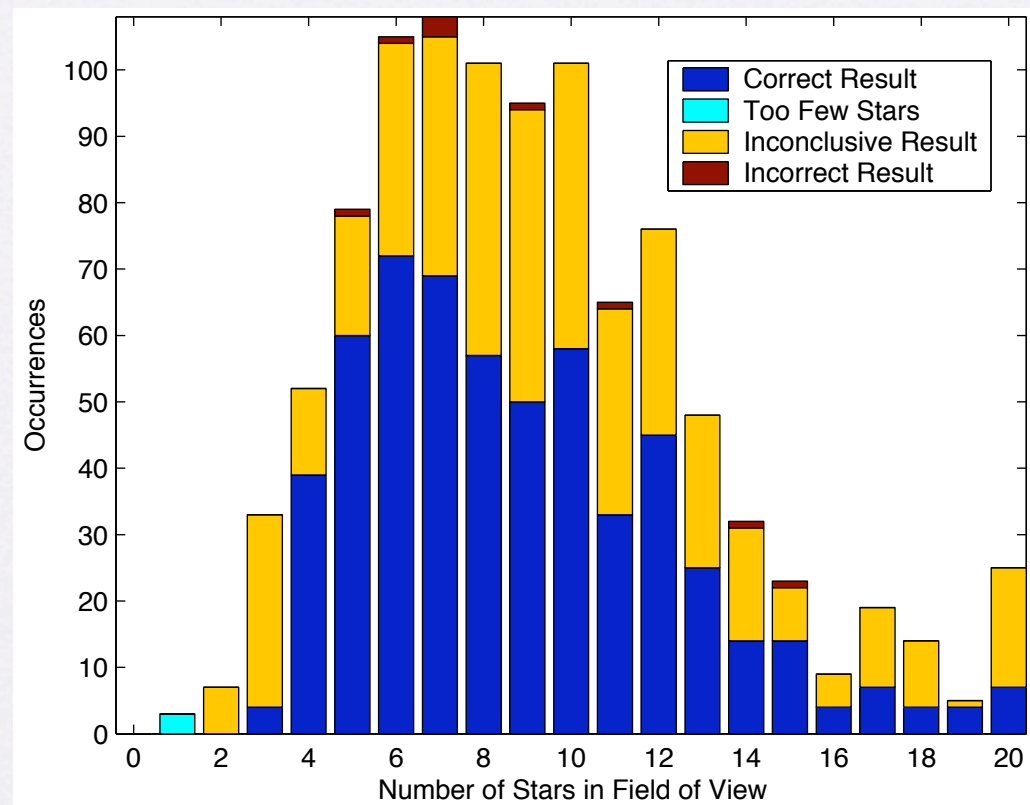
Overall Results for Angle Method

- Successful just over half of tests.
- $<1\%$ Incorrect matches 3σ expectations
- Rarely too few stars



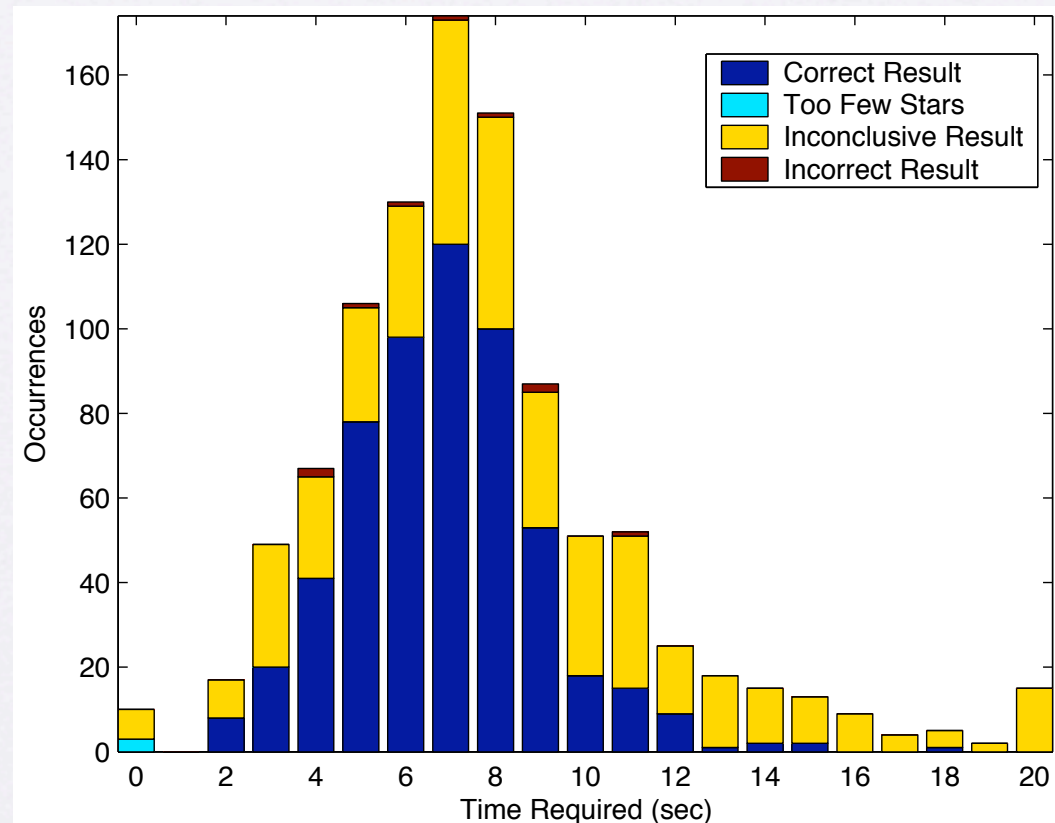
Distribution of Results for Angle Method

- Rarely works with three stars
- Best results with 4 or 5 stars
- Results don't improve with number of stars in FOV.



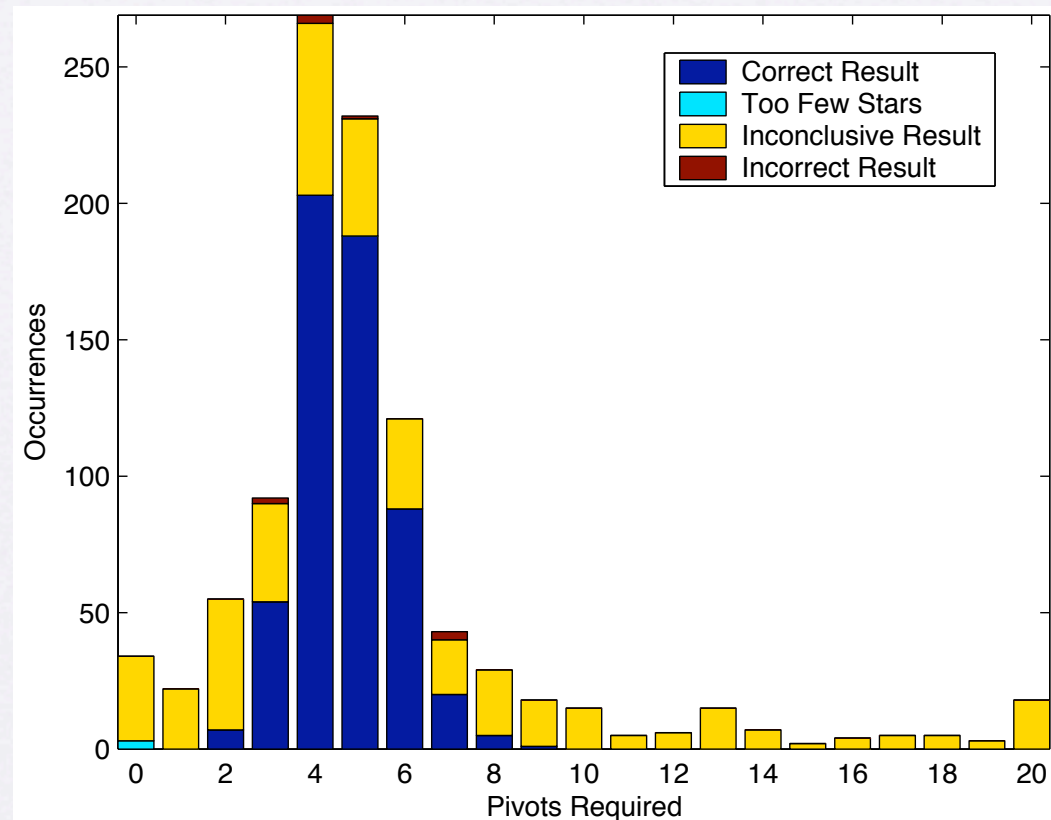
Time Distribution for Angle Method

- Average of 7.67 seconds required
- Standard deviation is 3.60 seconds.



Pivot Distribution for Angle Method

- Average of 5.48 pivots (7.48 stars required)
- No correct solution if more than 9 pivots were necessary.
- Limit to 9 pivots.



Angle Method With Pivot Limits

- 9-pivot Limit reduces success only slightly.
- Number of pivots and time required drops significantly.

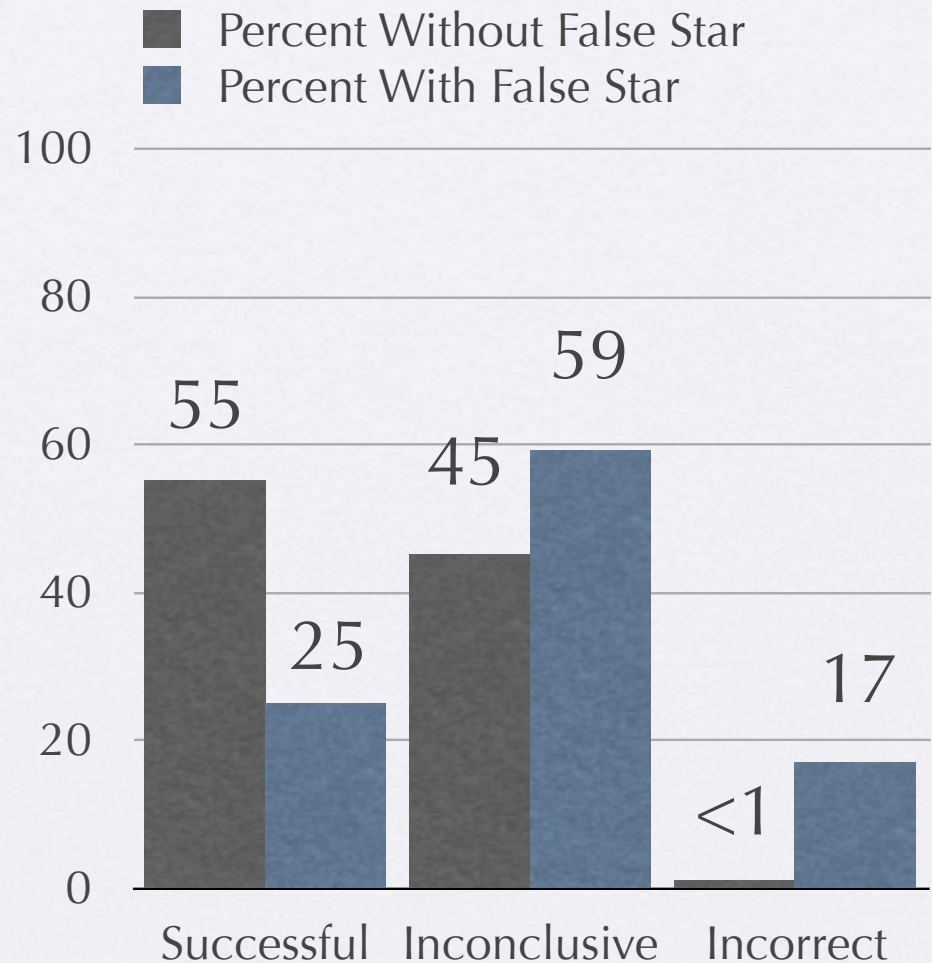
	No Pivot Limit	9-Pivot Limit
Successful Result	57%	55%
Avg. Pivots Required	5.48	4.86
Time (sec)	7.67	6.89

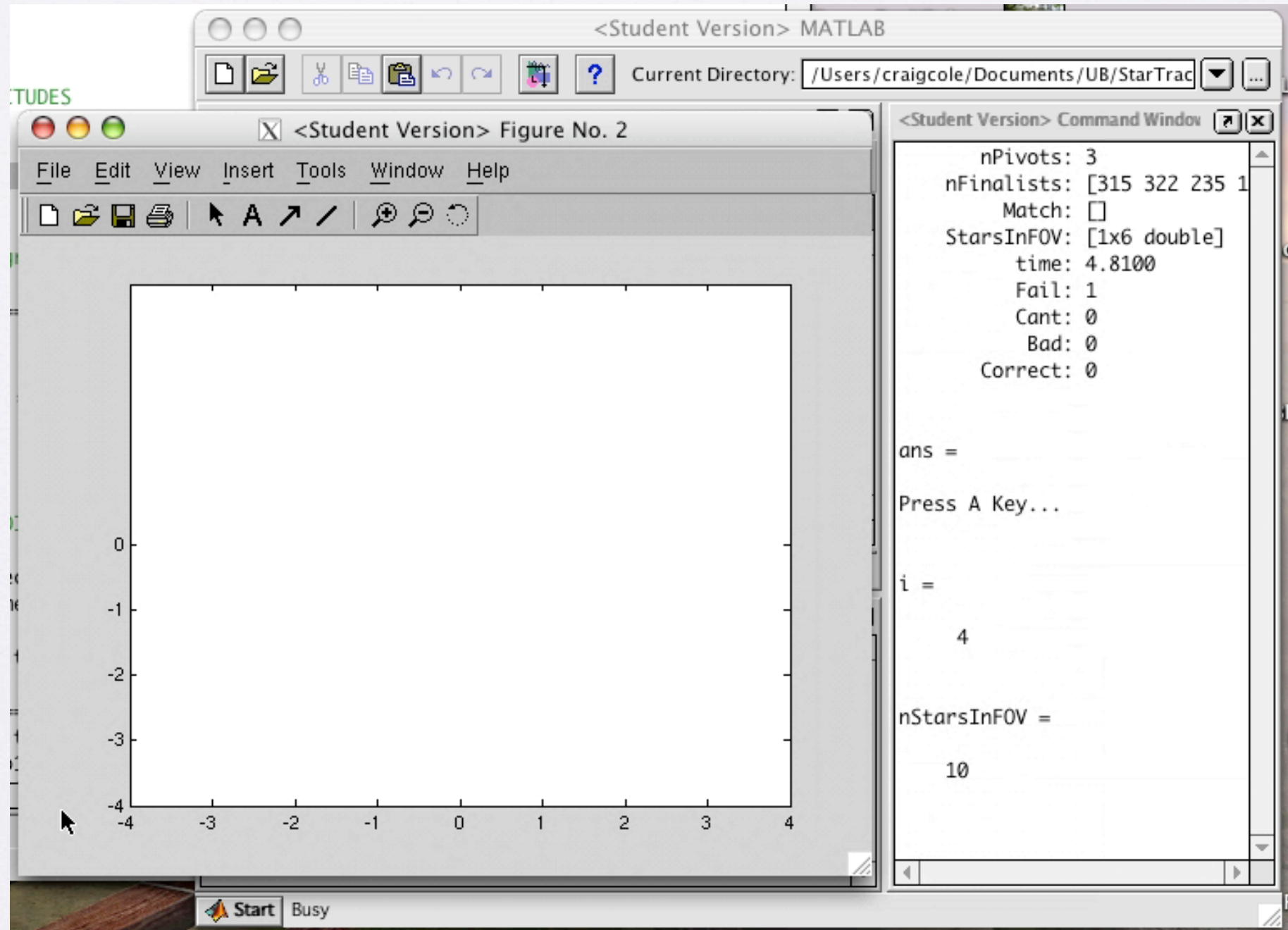
False Star Inclusion

- Debris in star field can look like a stars to the star tracker.
- Angle method with 9-pivot limit tested with a “false star” included in the field of view.

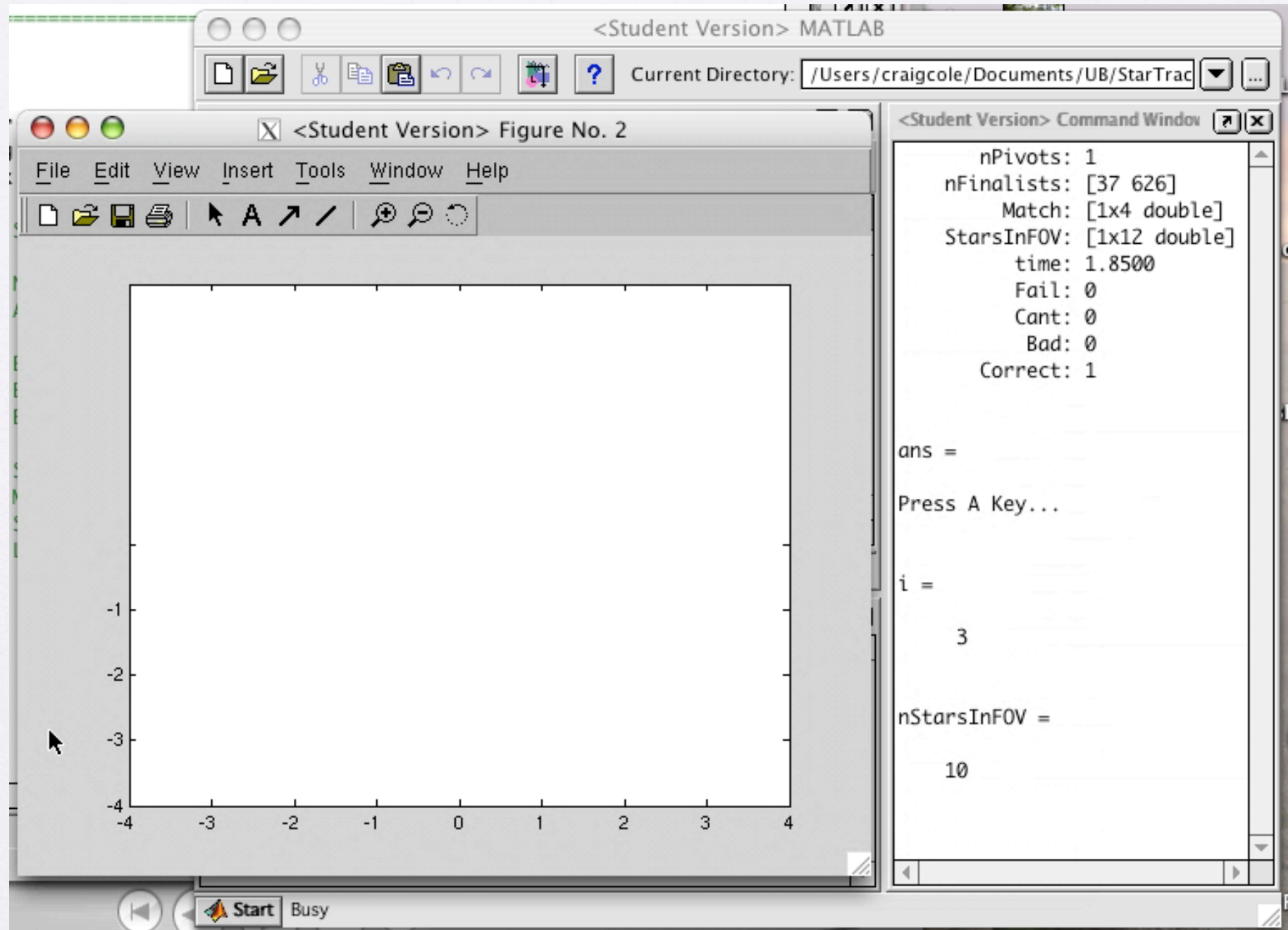
Angle Method With False Star Inclusion

- Success rate drops in half
- Angle method will cannot be correct if false star is used.
- Fewer stars needed, less likely false star is used.



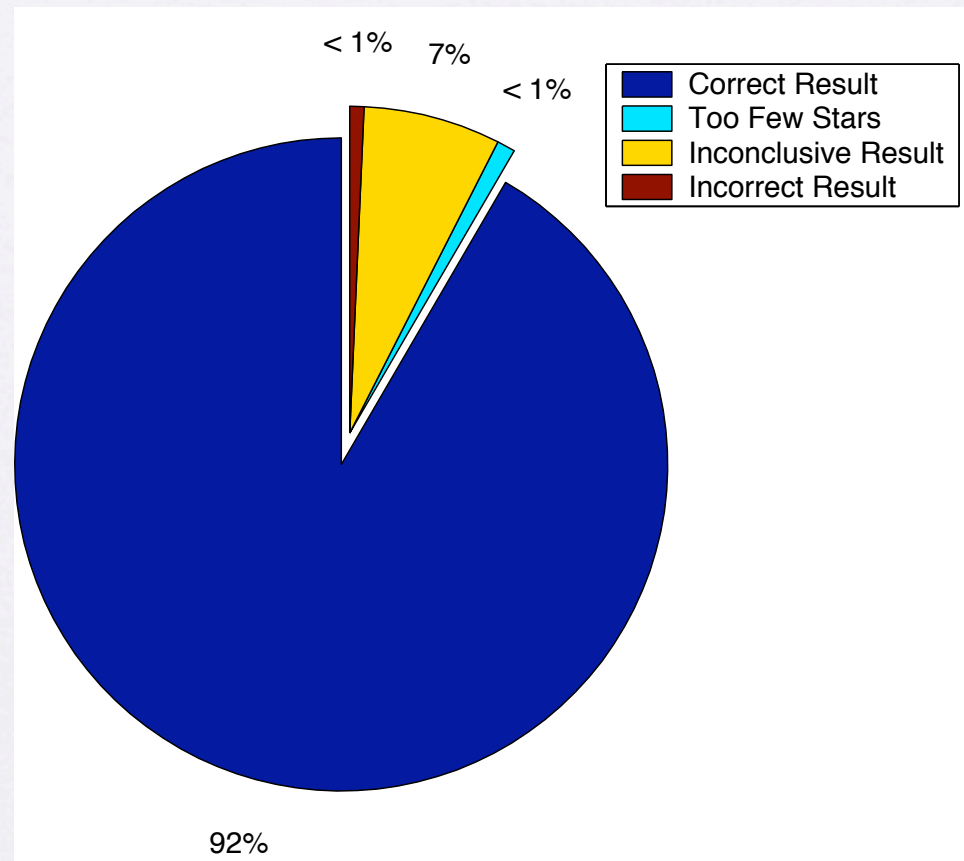


Spherical Triangle Method



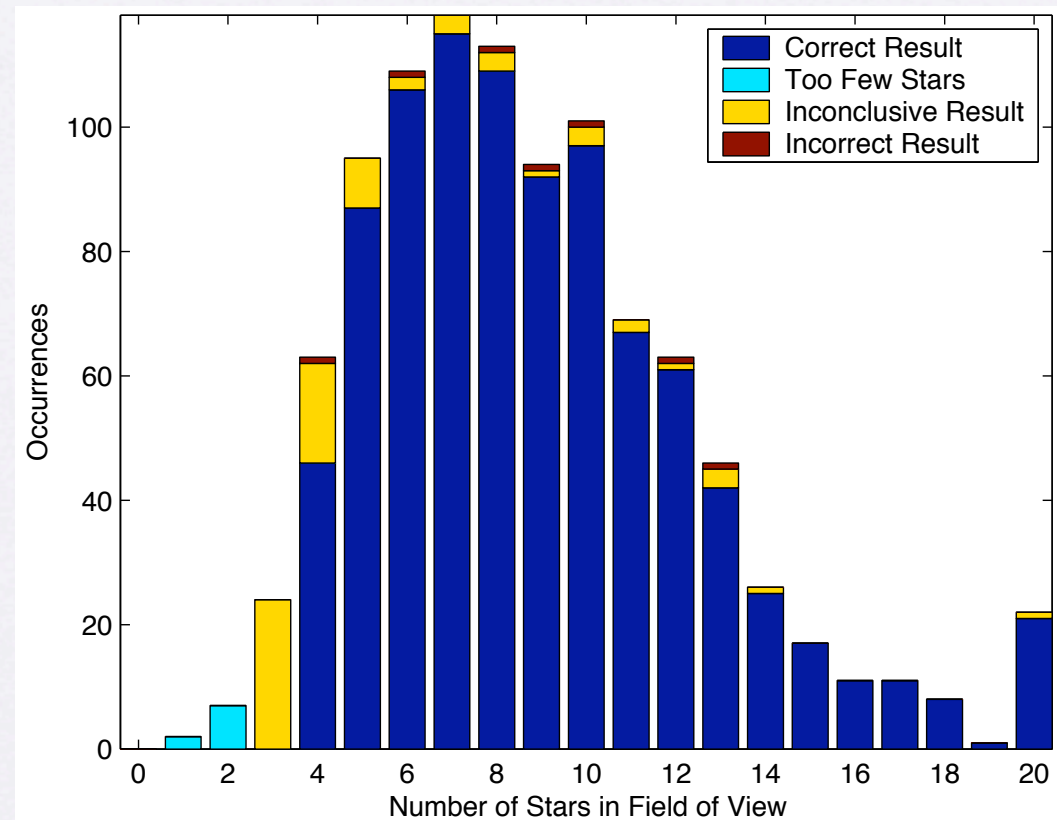
Overall Results for Spherical Triangle Method

- Successful nine times out of ten.
- $<1\%$ Incorrect matches 3σ expectations
- Still less than 1% too few stars



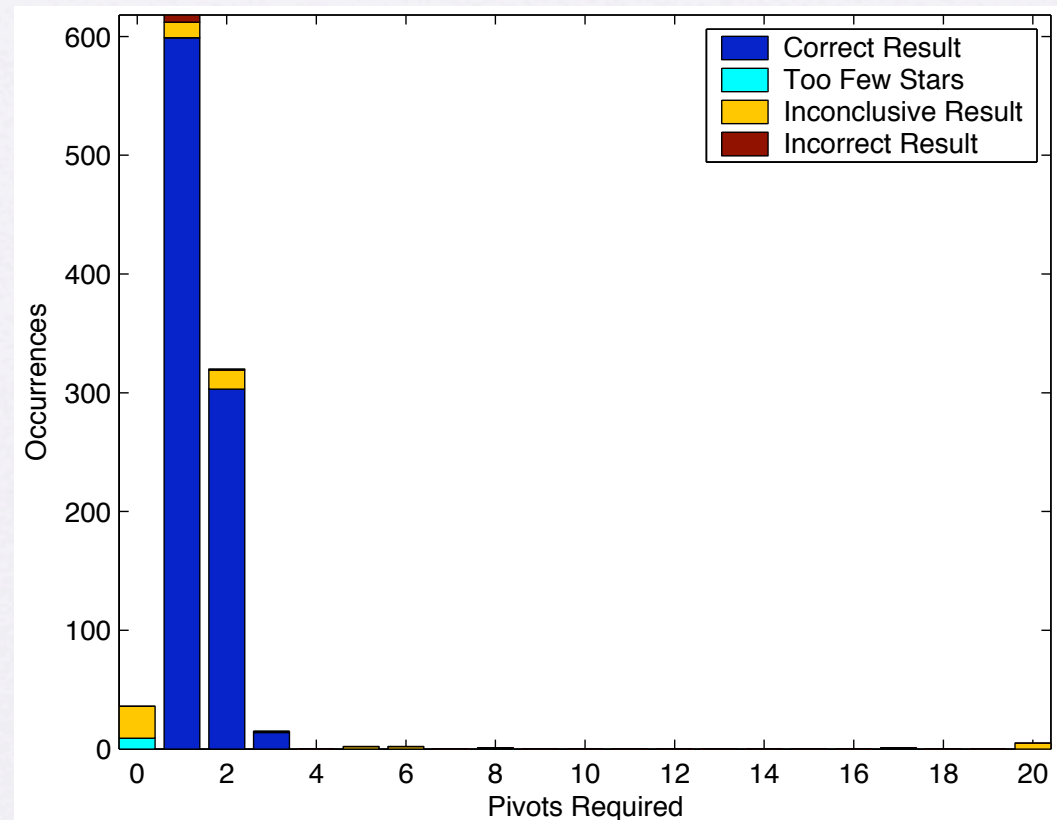
Distribution of Results for Spherical Triangle Method

- Doesn't work until 4 stars in FOV.
- Success rate does not drop off as number of stars in FOV increase.



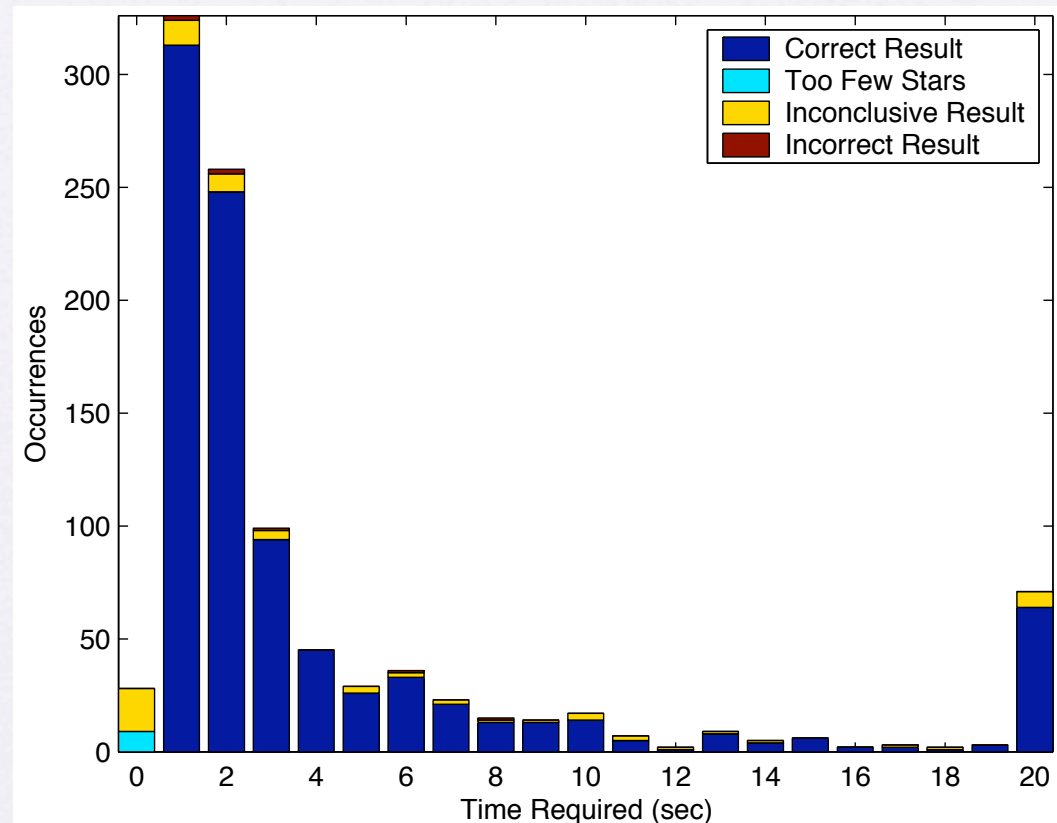
Time Distribution for Spherical Triangle Method

- Average of 7.67 seconds required
- Standard deviation is 3.60 seconds.



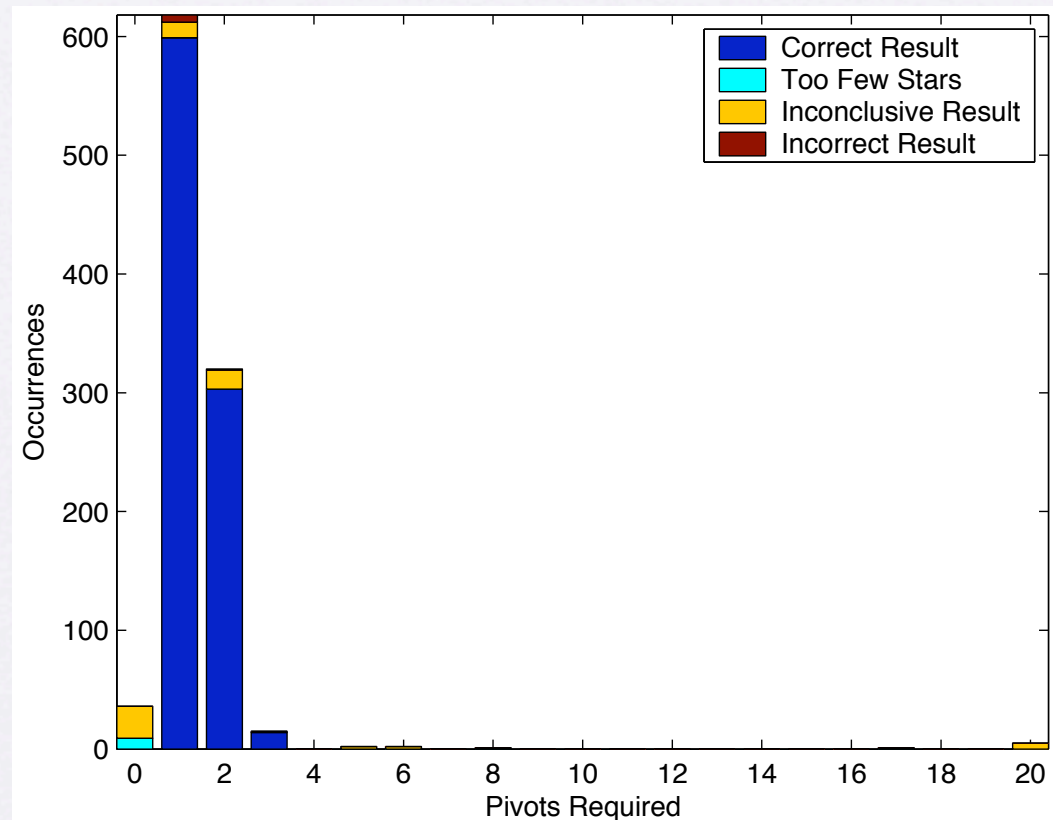
Time Distribution for Spherical Triangle Method

- Average of 10.43 seconds required.
- Times greater than 20 seconds grouped together.
- Many long, yet positive solutions.



Pivot Distribution for Spherical Triangle Method

- Average of 1.73 pivots required. (4.73 stars)
- Some “high-pivots” successful.
- Rarely need more than 3 pivots.



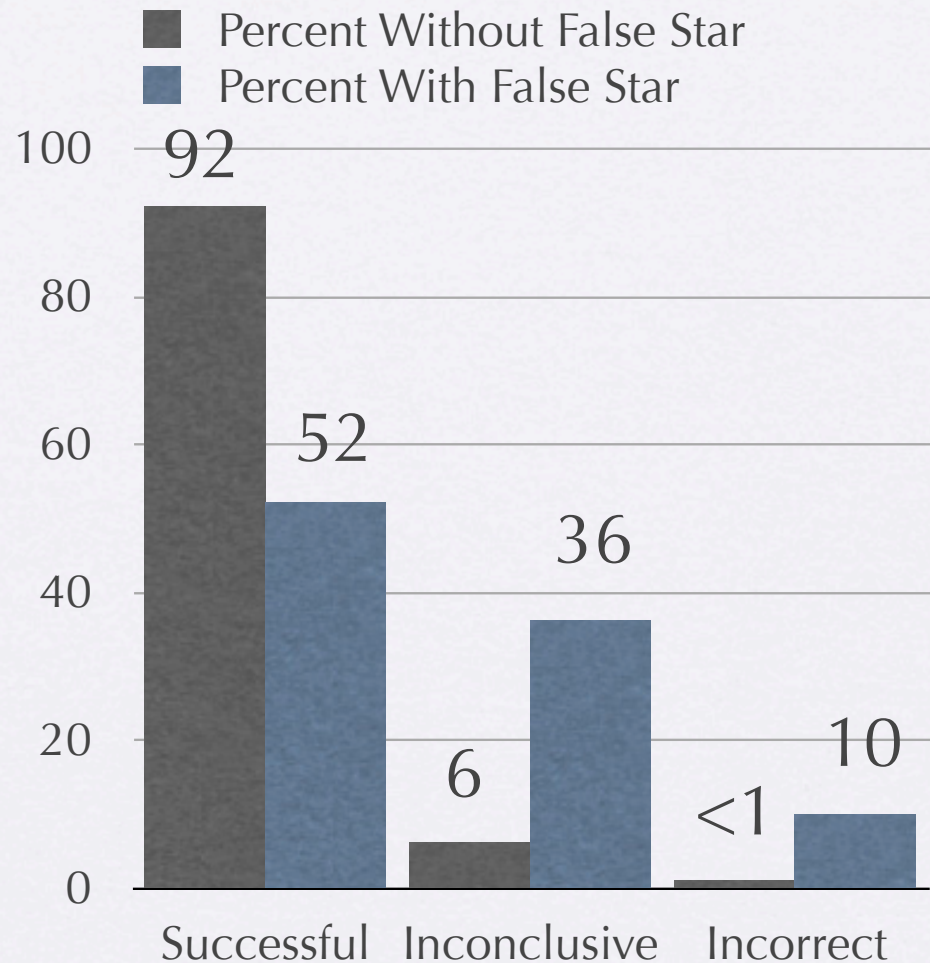
Spherical Triangle Method With Pivot Limits

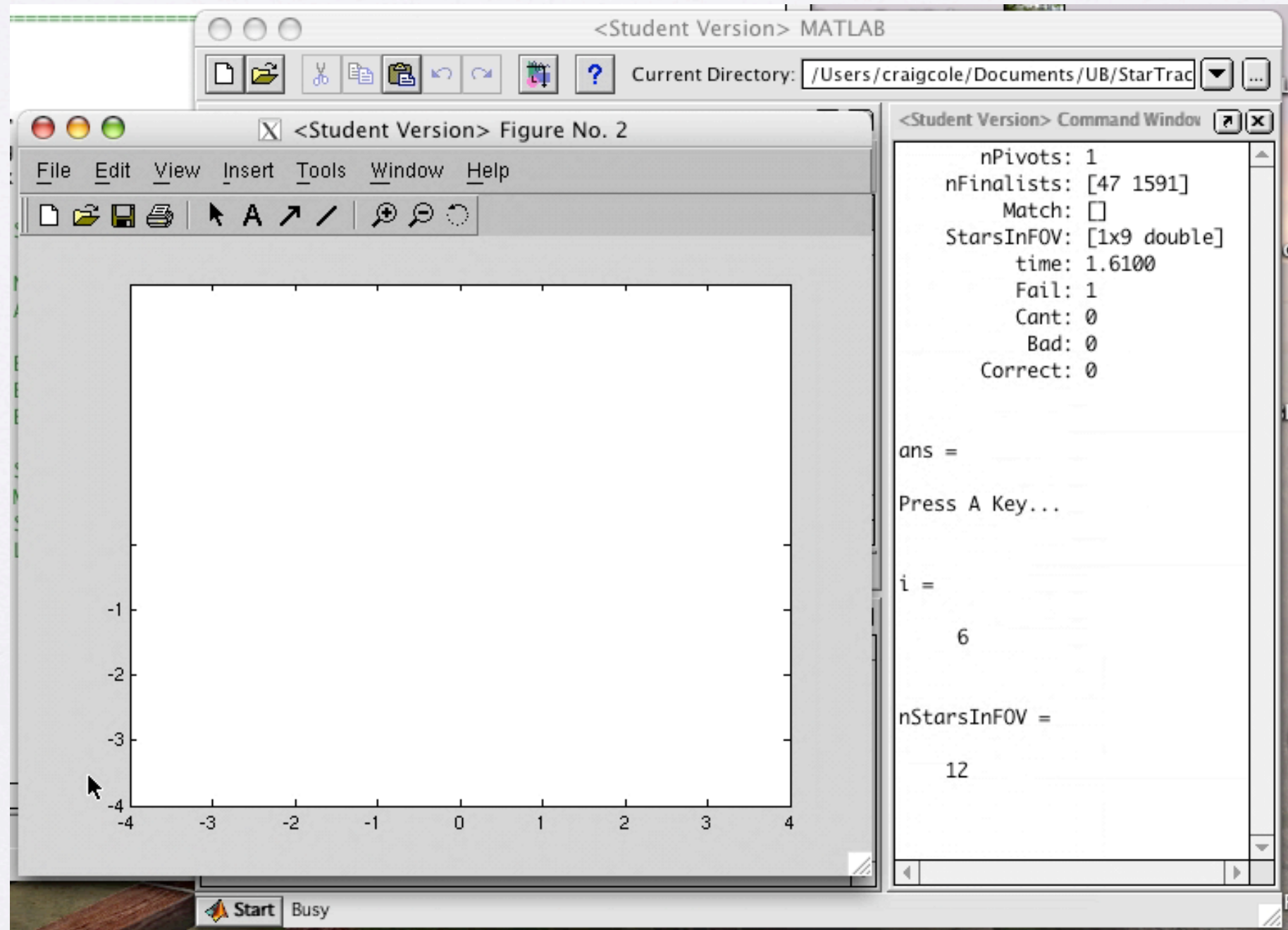
- No significant change in success.
- Pivot ordering takes large percentage of CPU time
- Big drop in total time.

	No Pivot Limit	3-Pivot Limit
Successful Result	92%	92%
Avg. Pivots Required	1.73	1.29
Time (sec)	10.43	1.65

Spherical Triangle Method With False Star Inclusion

- Success rate drops in half
- Angle method will cannot be correct if false star is used.
- Fewer stars needed, less likely false star is used.

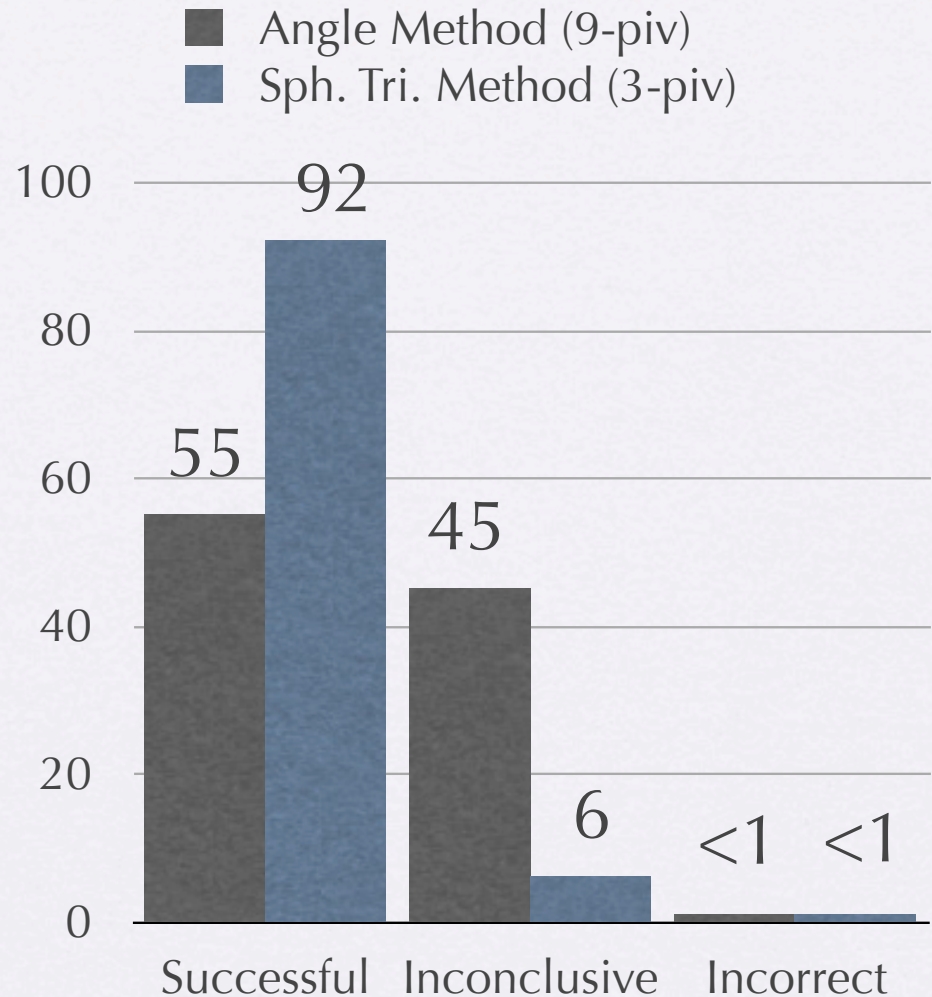




Comparison of Angle and Spherical Triangle Method and Conclusions

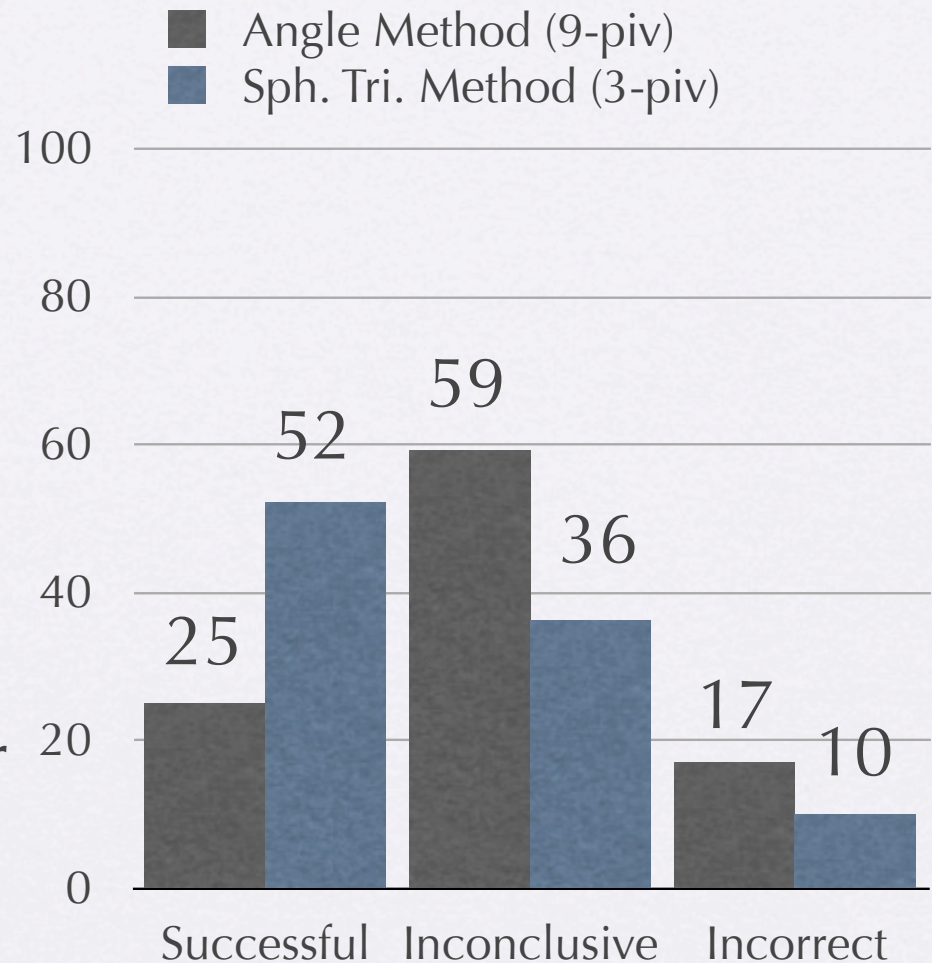
Overall Results

- Spherical Triangle method almost twice as likely to get correct result.
- Without false star, incorrect results reflect 3σ bounds.



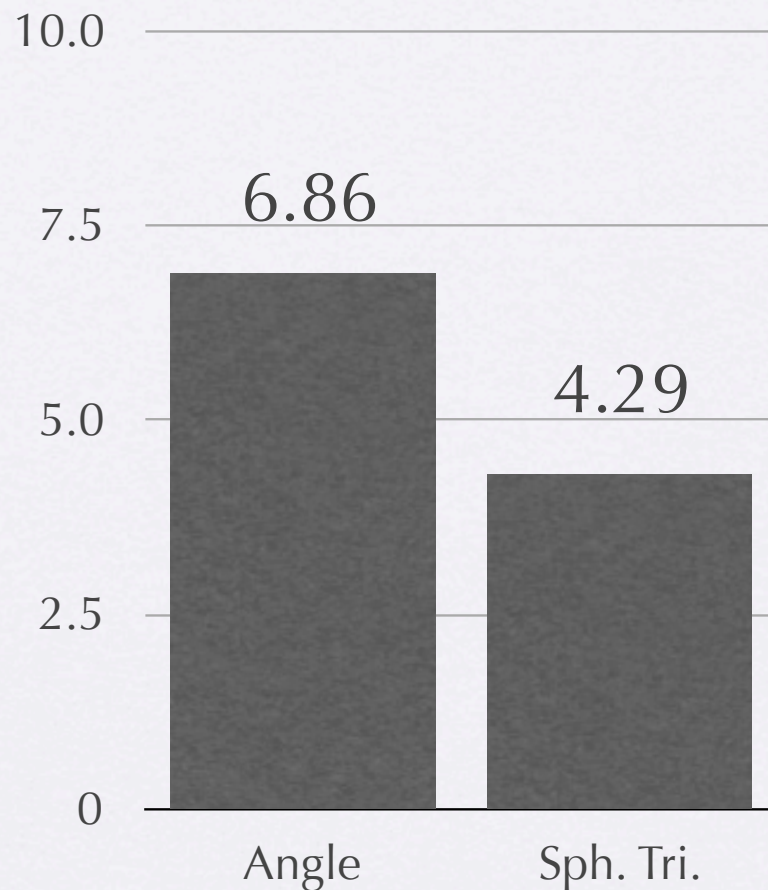
Overall Results

- When a false star is included, the spherical triangle method is still twice as likely to reach correct result.
- Fewer incorrect results because fewer stars are needed.



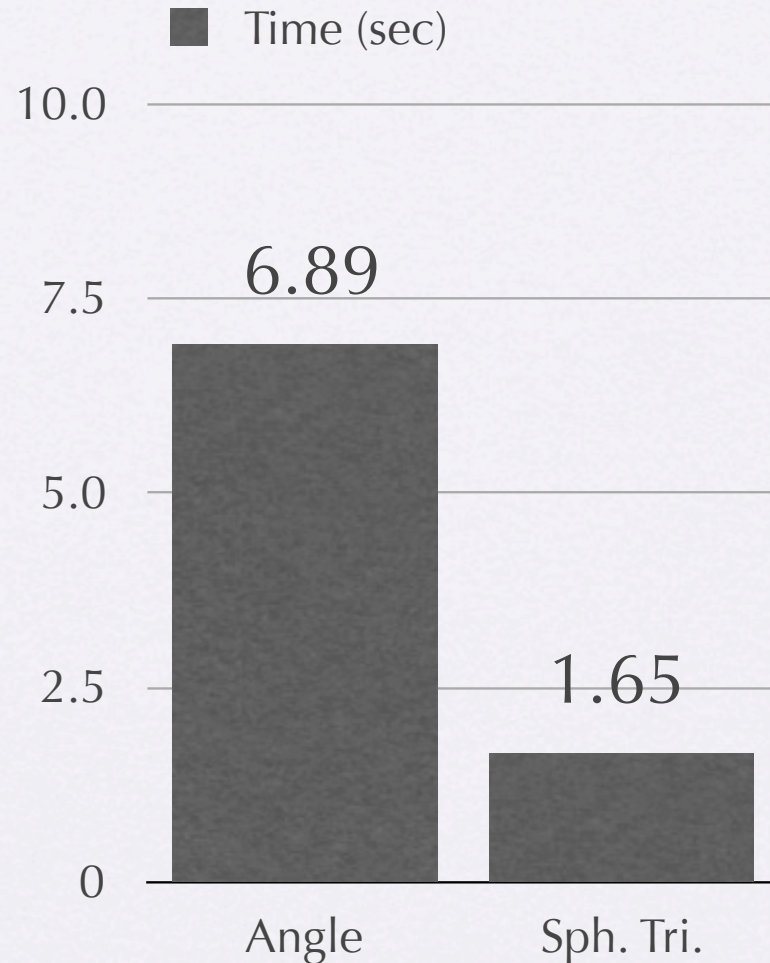
Average Stars Needed

- Fewer pivots means fewer stars.
- Fewer stars improves odds of successful result with false star.



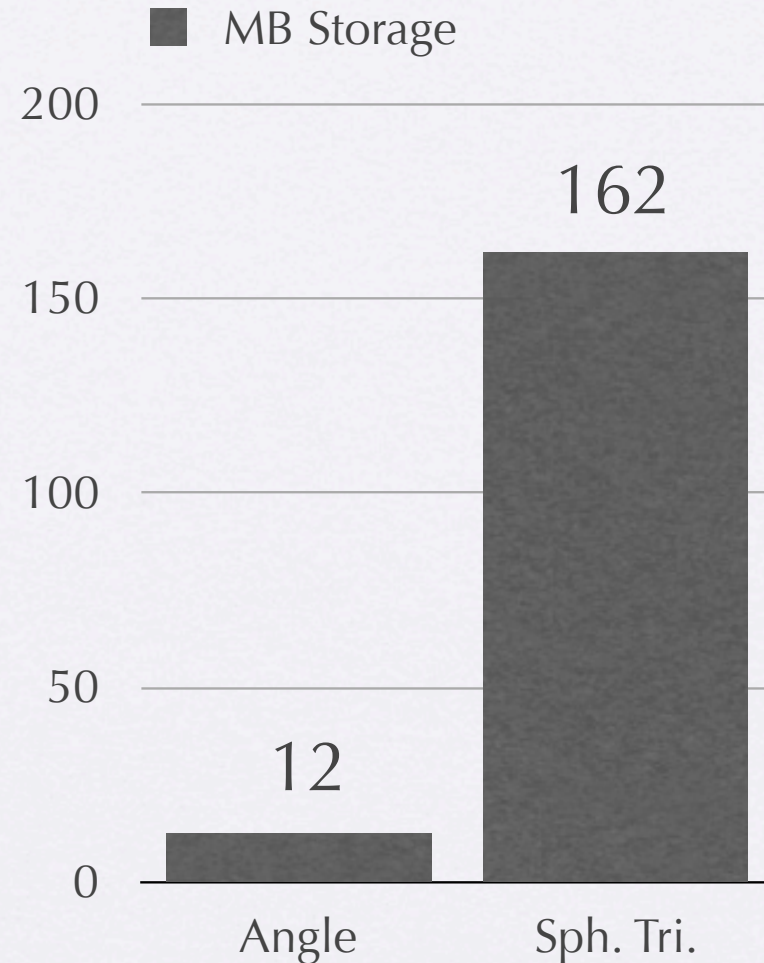
CPU Time

- Even if a false star is included, the spherical triangle method is twice as likely to reach correct result.
- Fewer incorrect results because fewer stars are needed.



Catalog Storage Requirements

- All this success comes at the expense of storage space.
- Spherical triangle catalog is more than ten times larger than the angle catalog.
- Storage always getting cheaper.



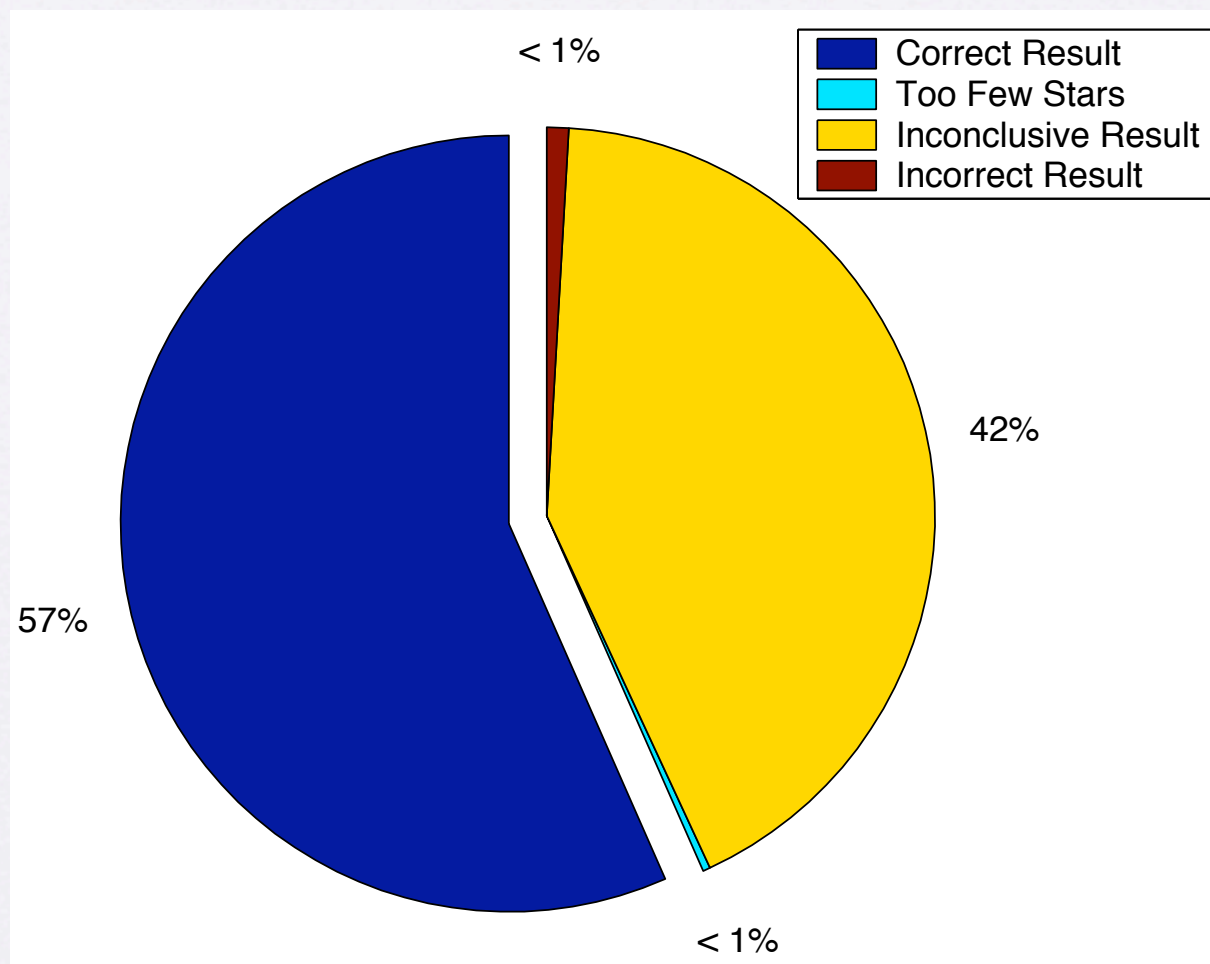
Directions to Continue

- Explore different fields of view and magnitude sensitivity
- Explore changes in sigma bounds on success and failure.
- Explore methods for retesting FOV using different pivot orders when inconclusive results are reached.

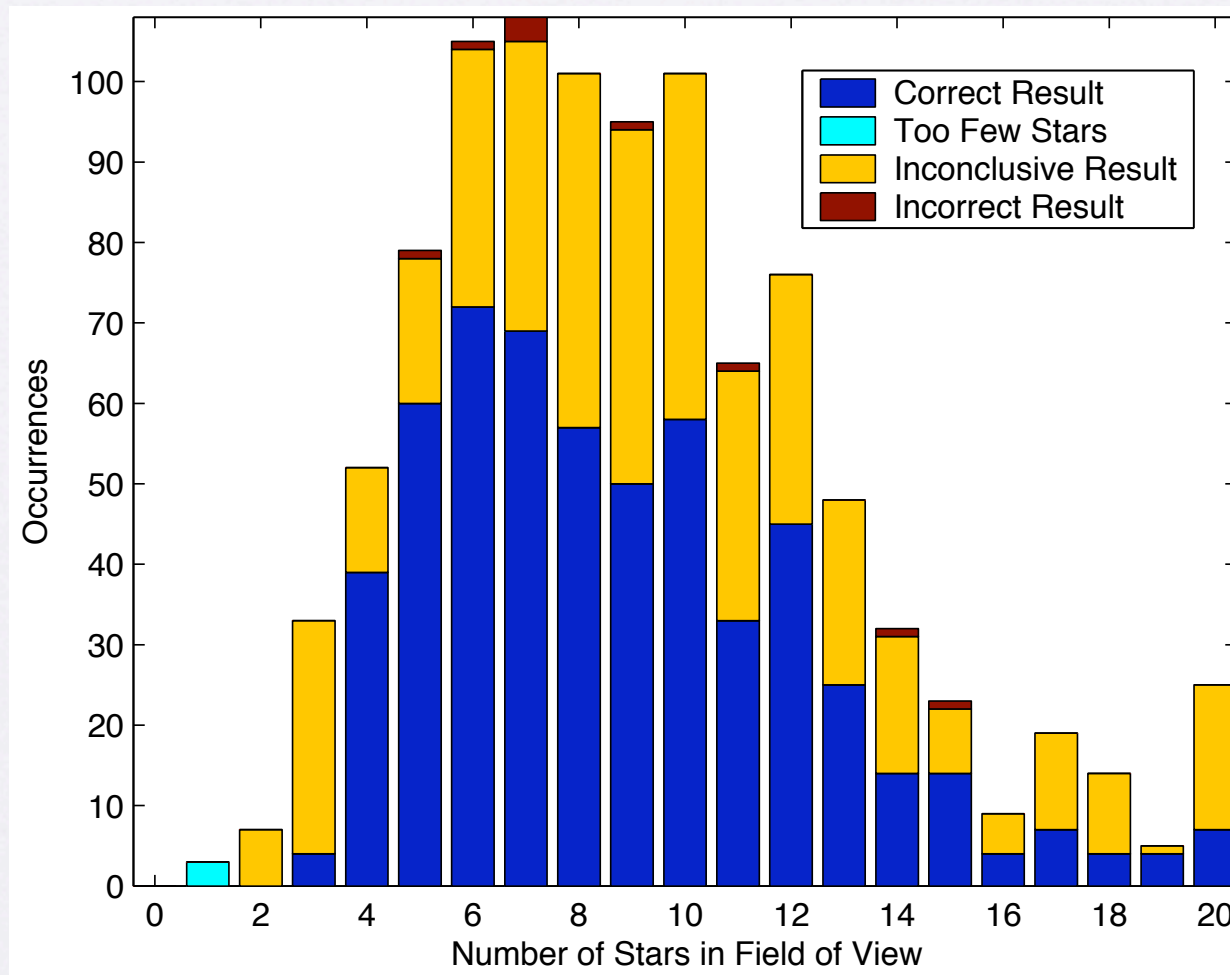
Appendix A.

Angle Method with No Pivot Limit

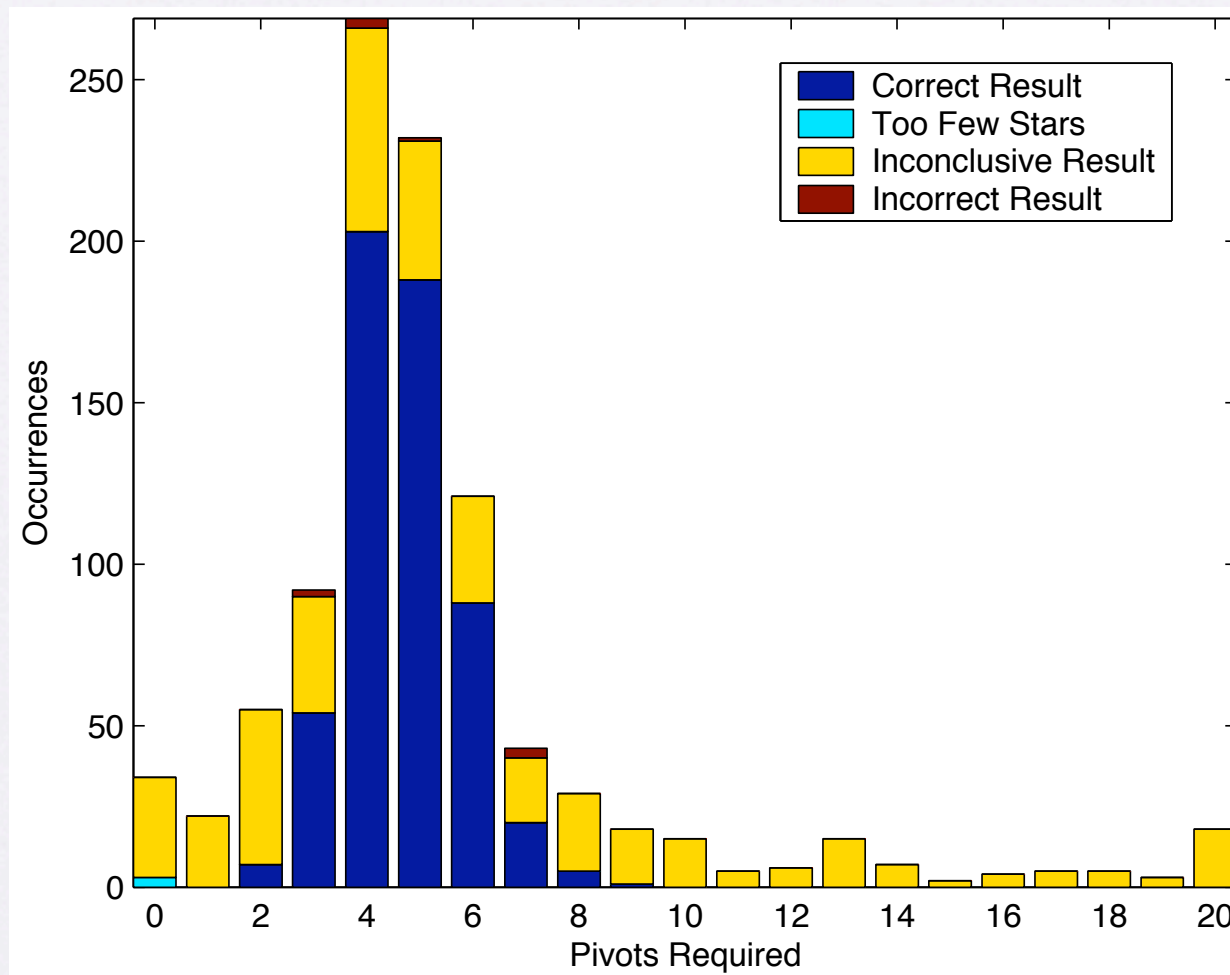
Overall Results for Angle Method



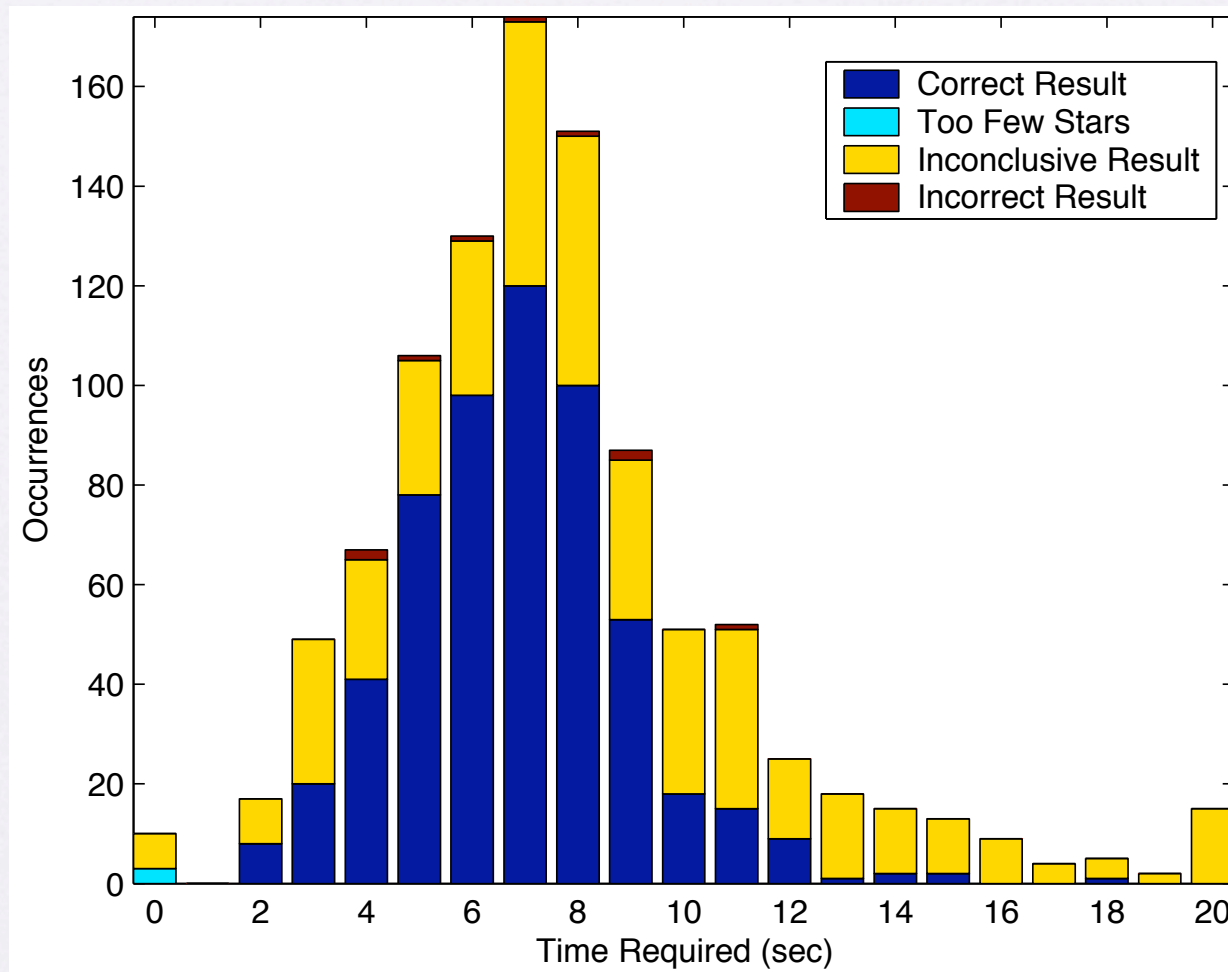
Distribution of Results for Angle Method



Pivots Required for Angle Method

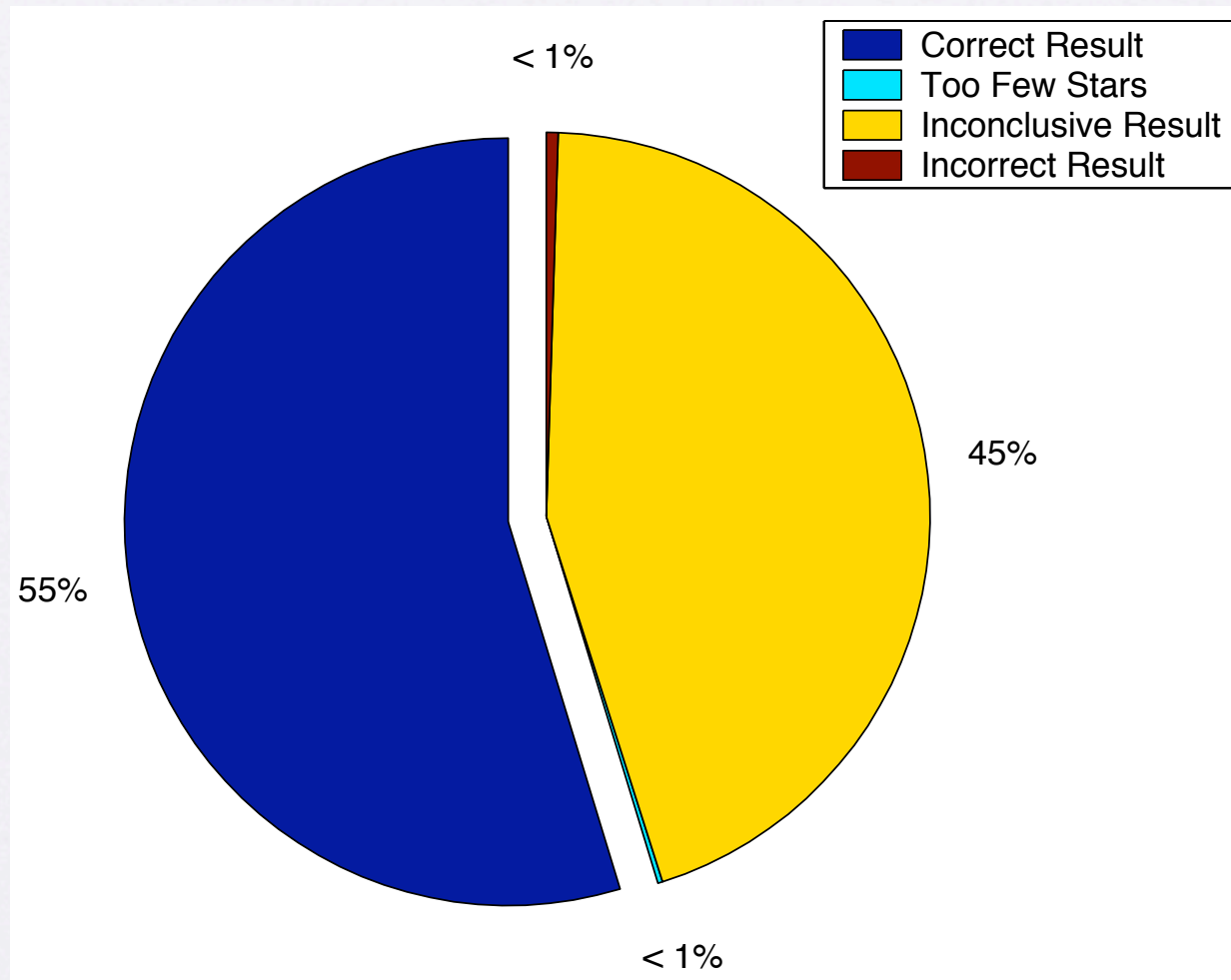


CPU Time Required for Angle Method

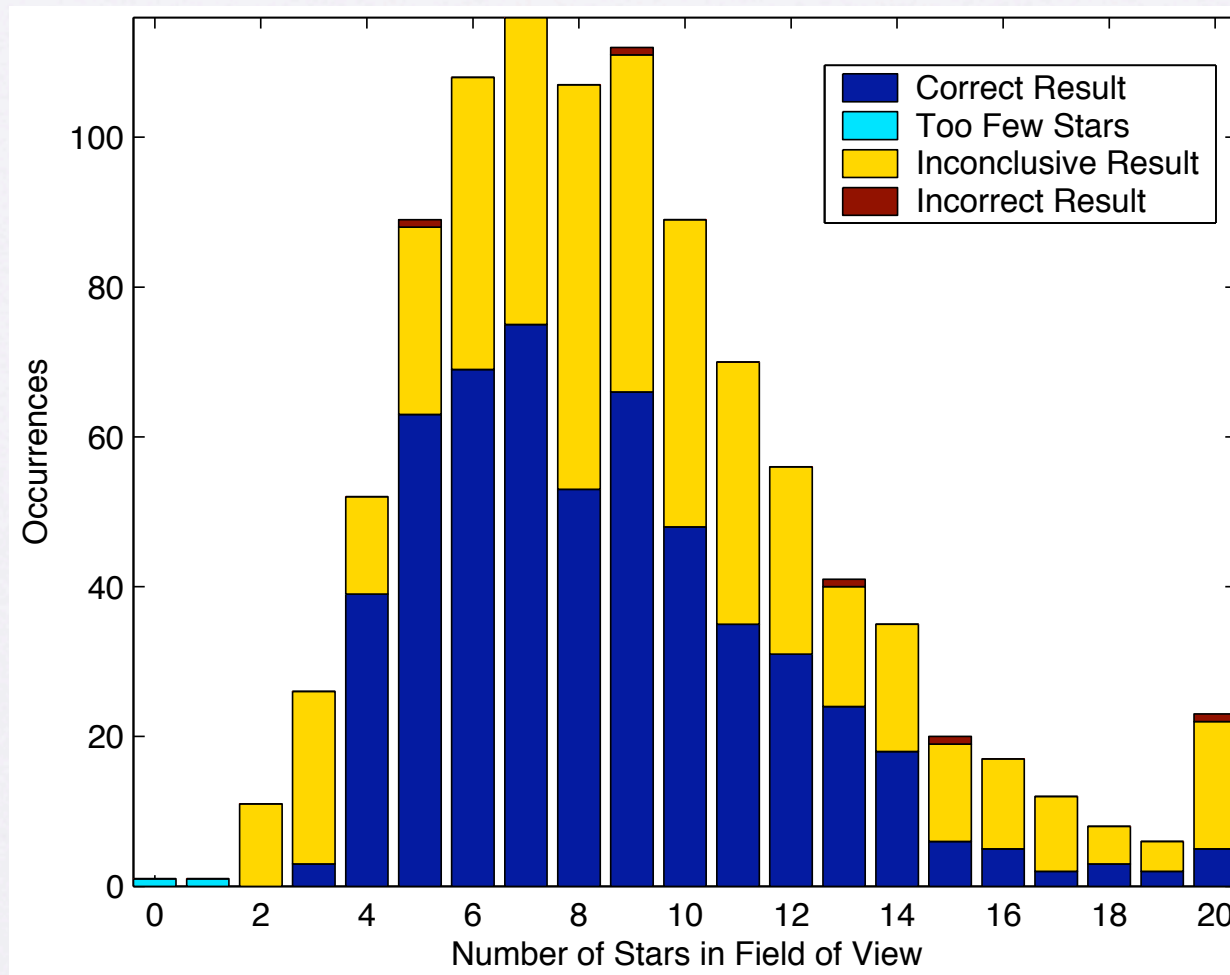


Appendix B. Angle Method, 9-Pivot Limit

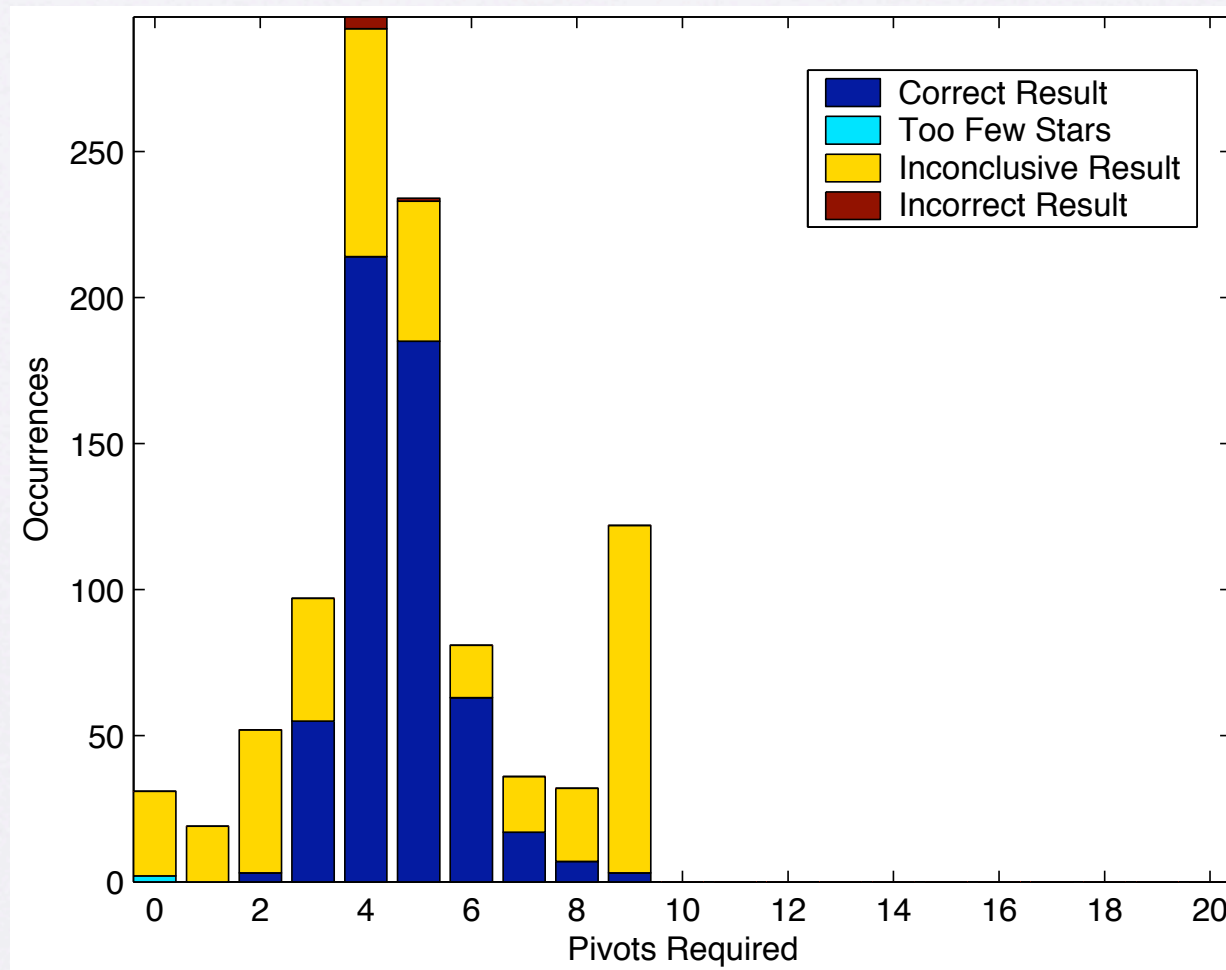
Overall Result for Angle Method with 9-Pivot Limit



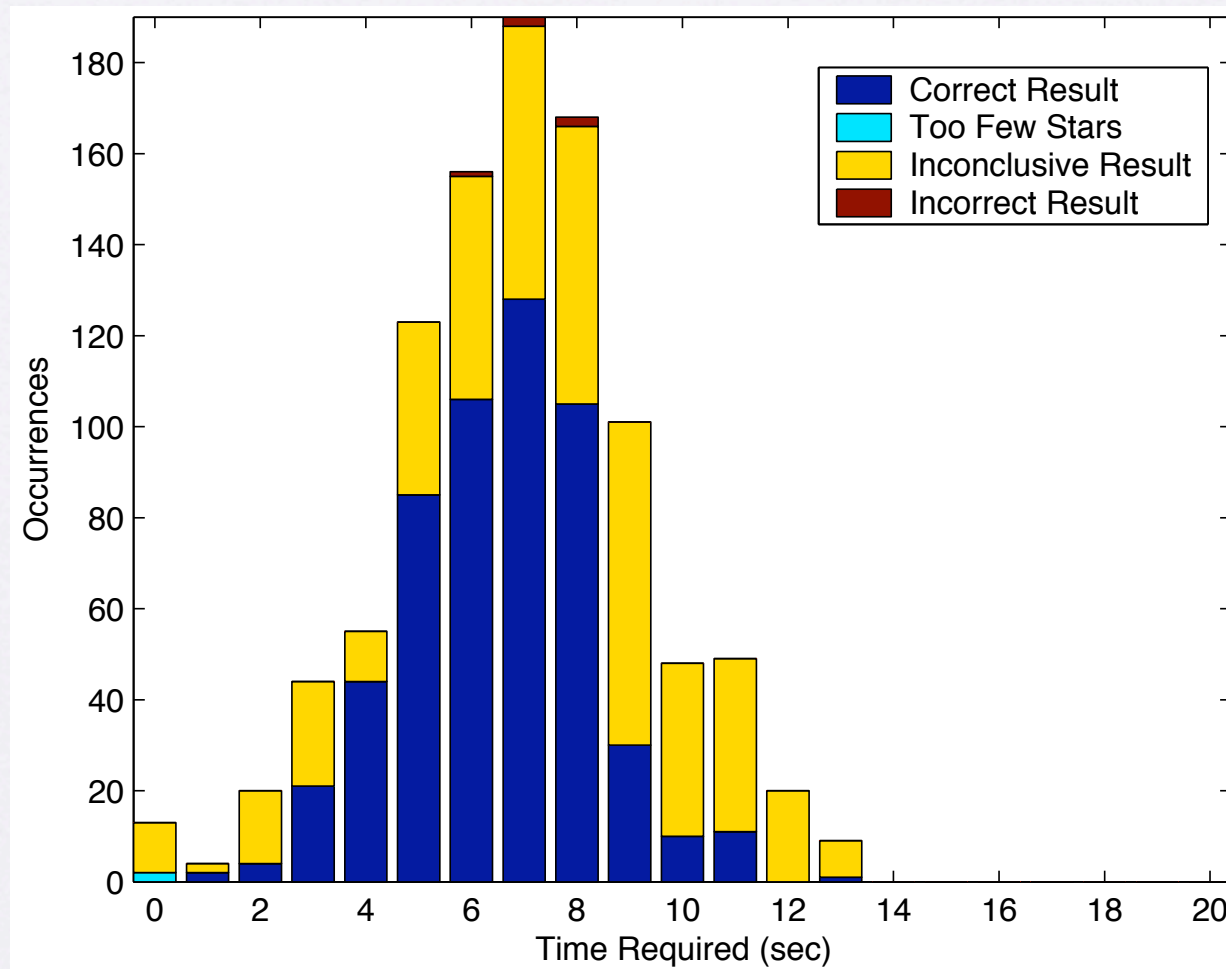
Distribution of Results for Angle Method with 9-Pivot Limit



Pivots Required for Angle Method with 9-Pivot Limit

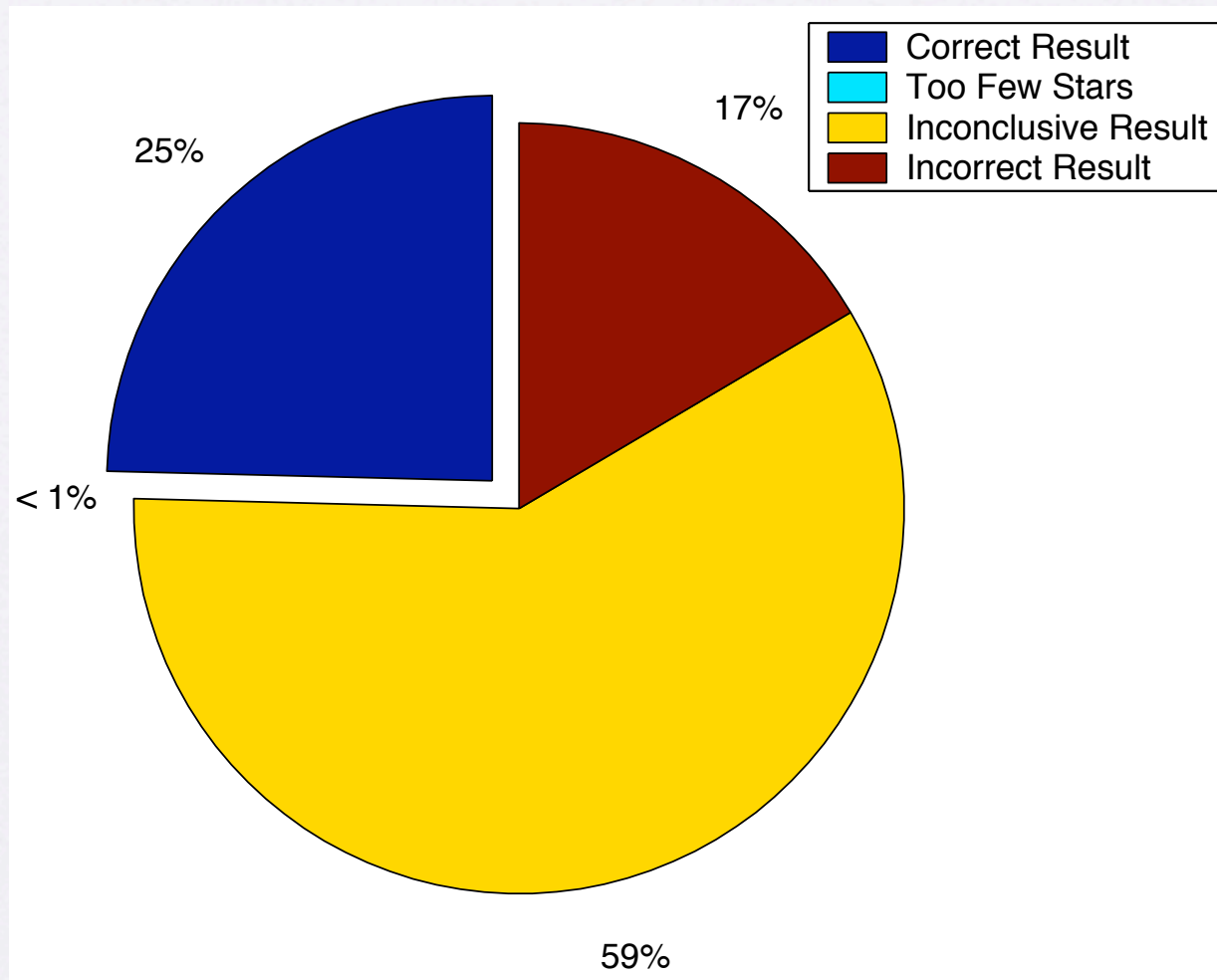


CPU Time Required for Angle Method with 9-Pivot Limit

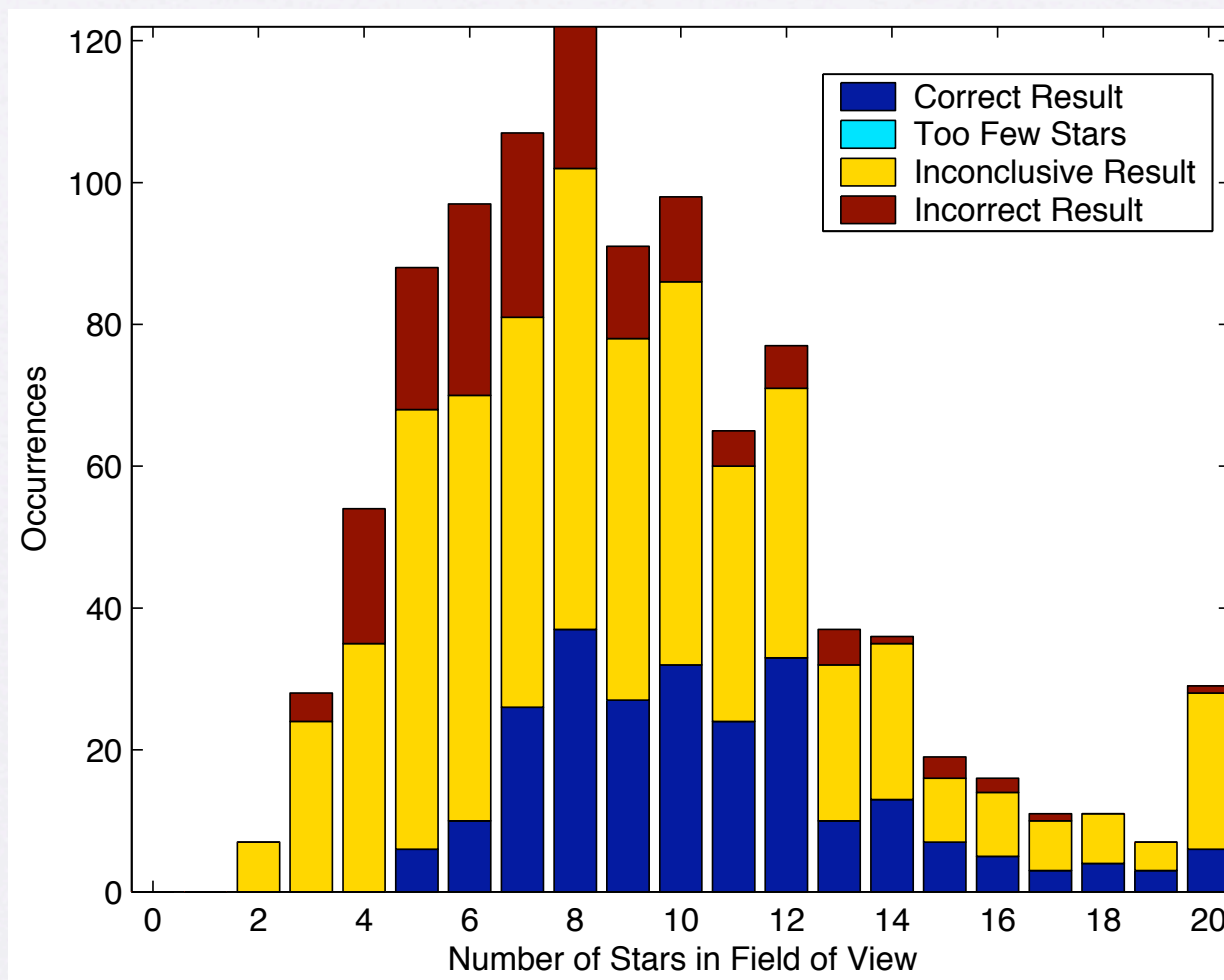


Appendix C.
Angle Method,
9-Pivot Limit,
False Star Inclusion

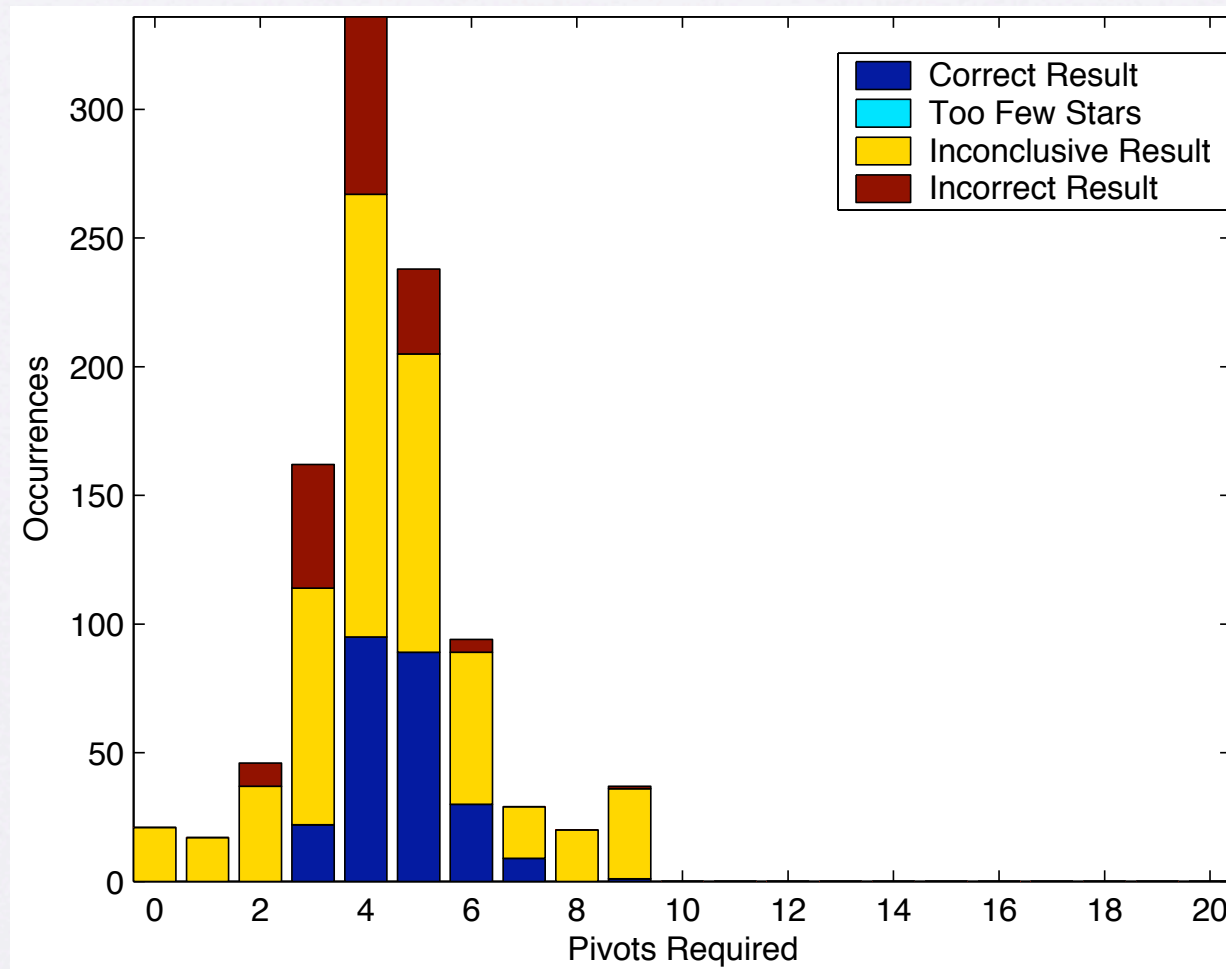
Overall Result for Angle Method (9-Pivot) w/False Star



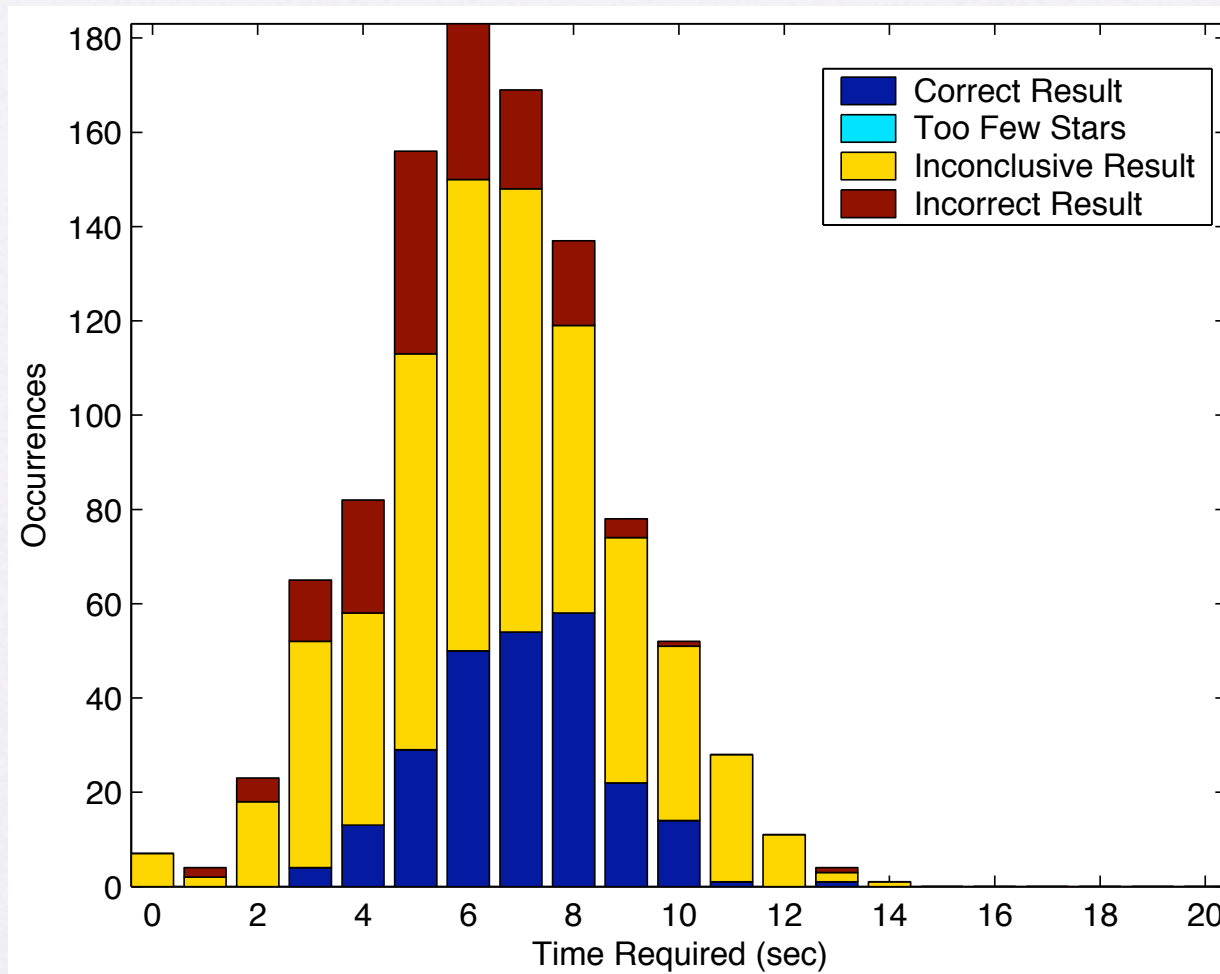
Distribution of Results for Angle Method (9-Pivot) w/False Star



Pivots Required for Angle Method (9-Pivot) w/False Star

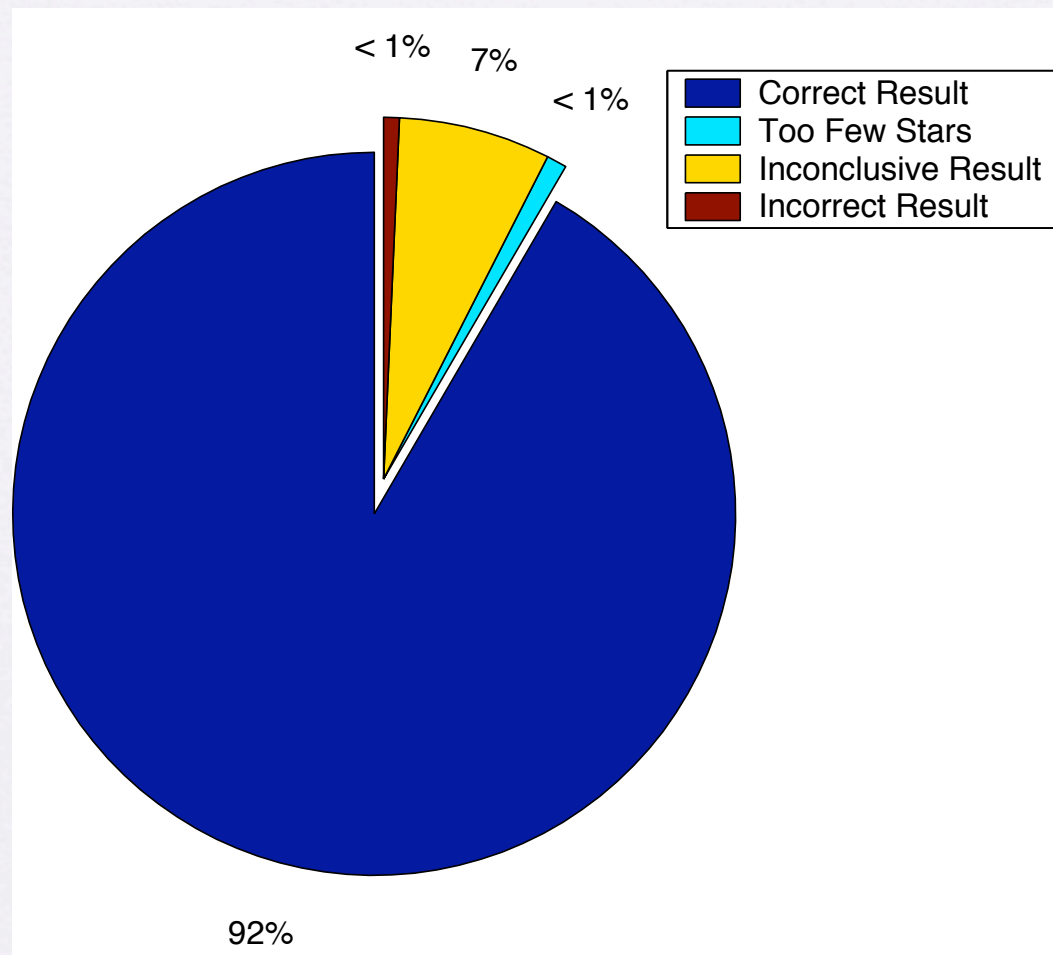


CPU Time Required for Angle Method (9-Pivot) w/False Star

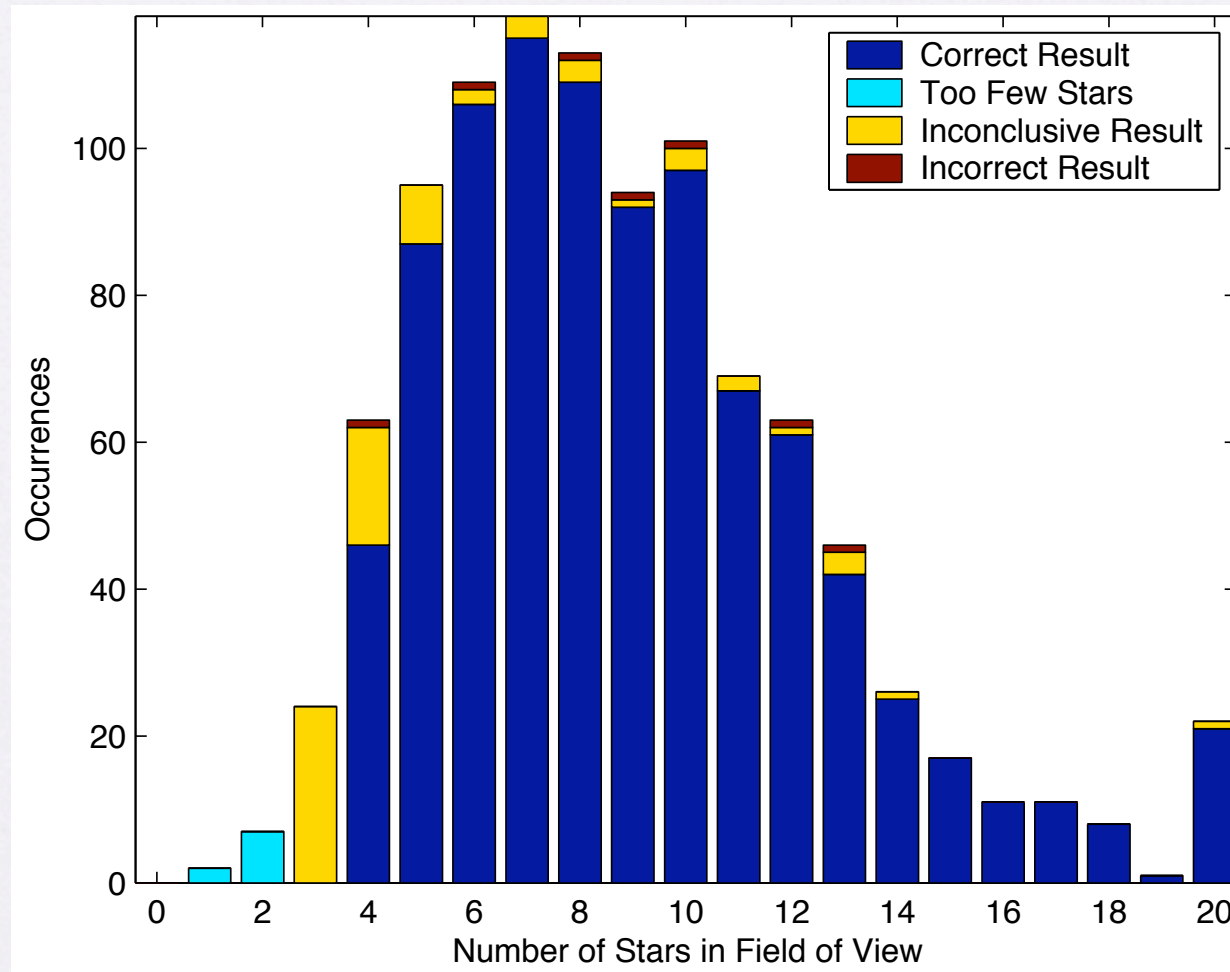


Appendix D.
Sph. Tri. Method,
No Pivot Limit

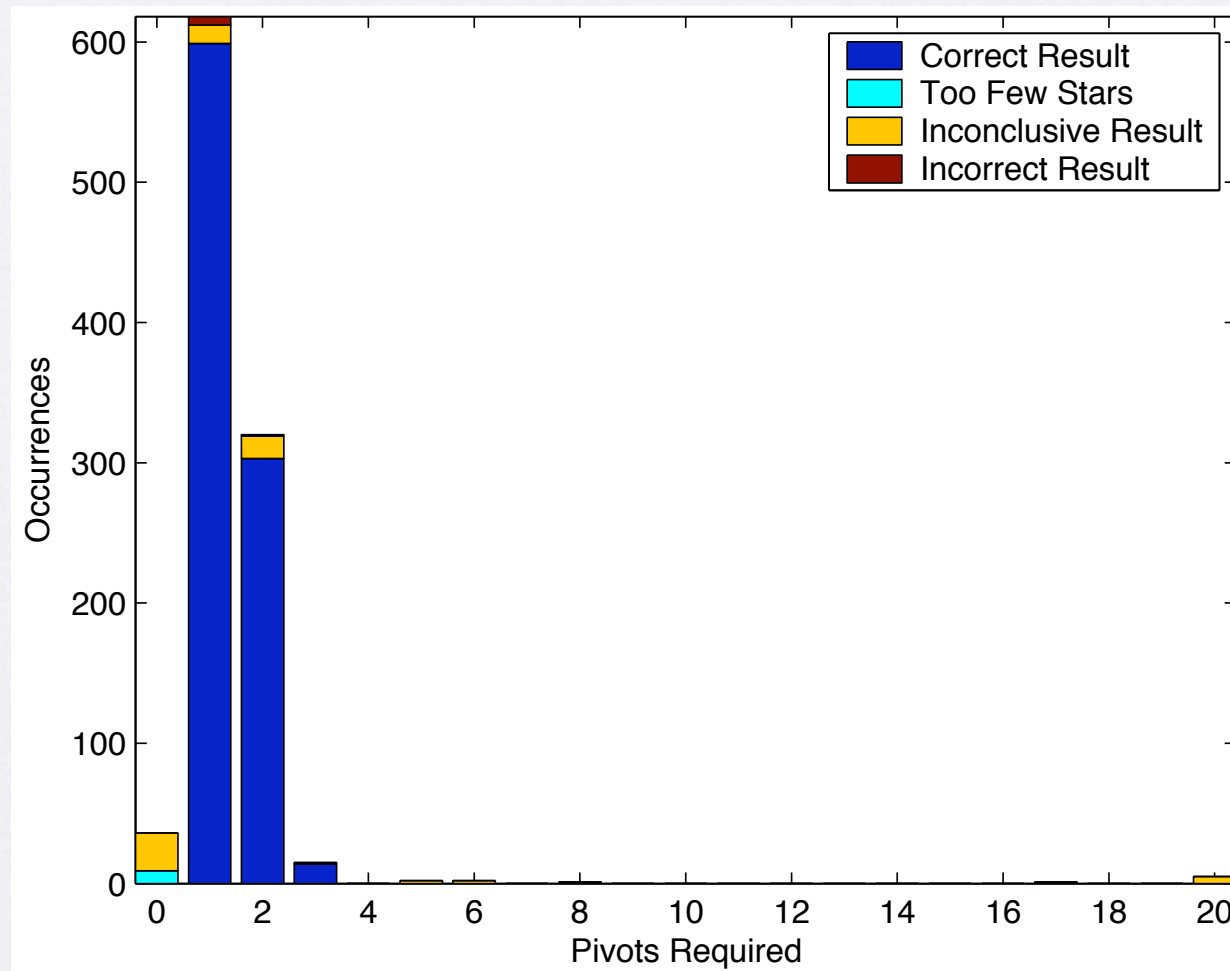
Overall Result for Spherical Triangle Method



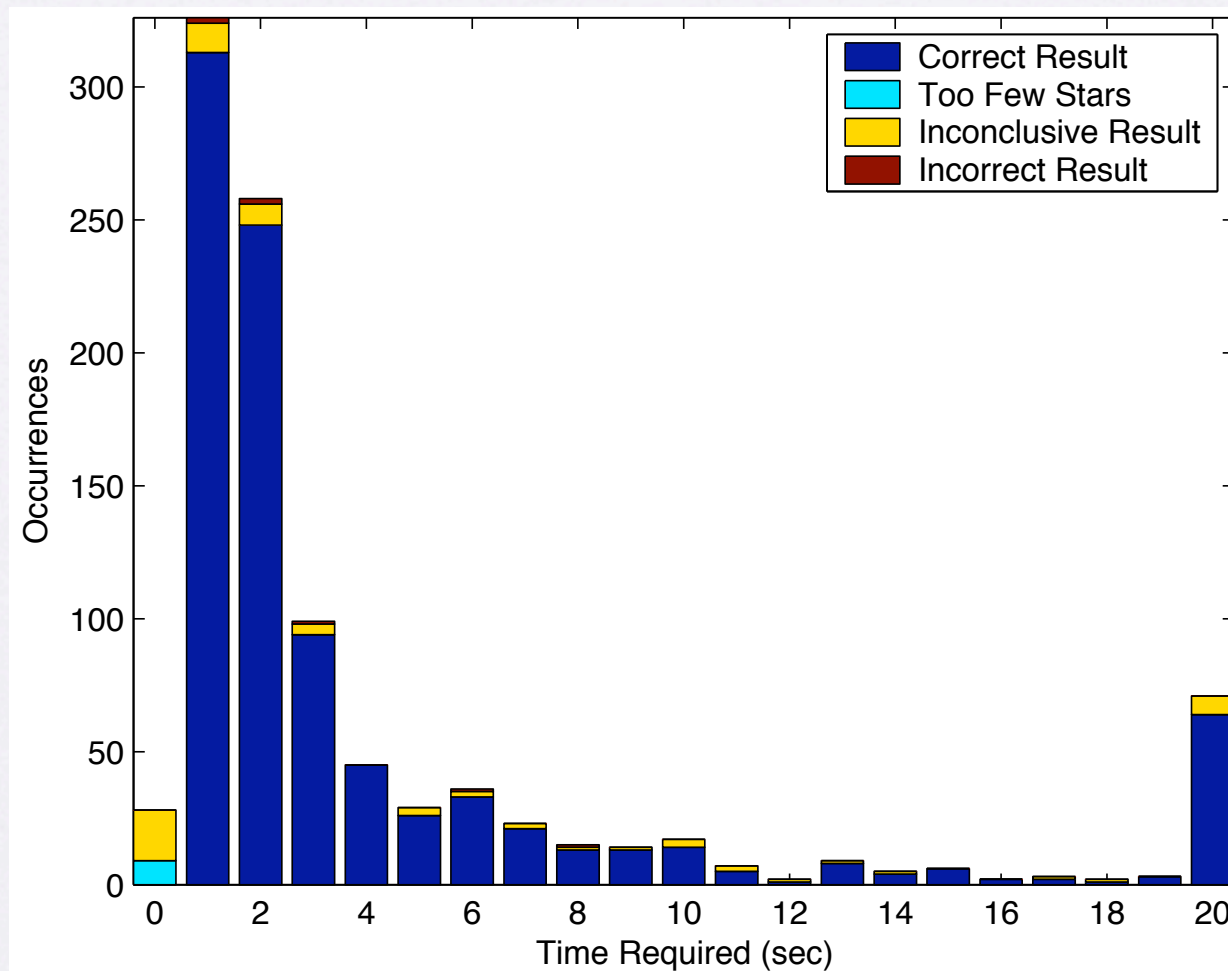
Distribution of Results for Spherical Triangle Method



Pivots Required for Spherical Triangle Method

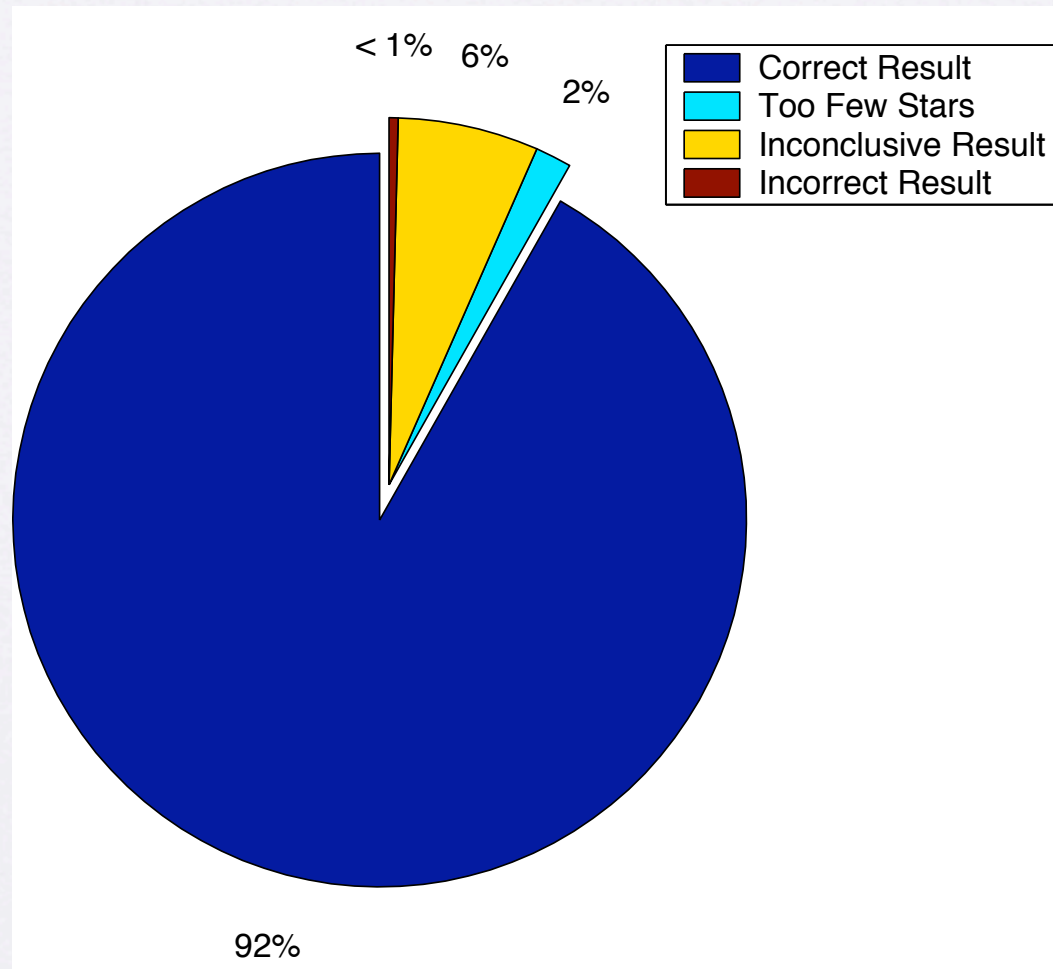


CPU Time Required for Spherical Triangle Method

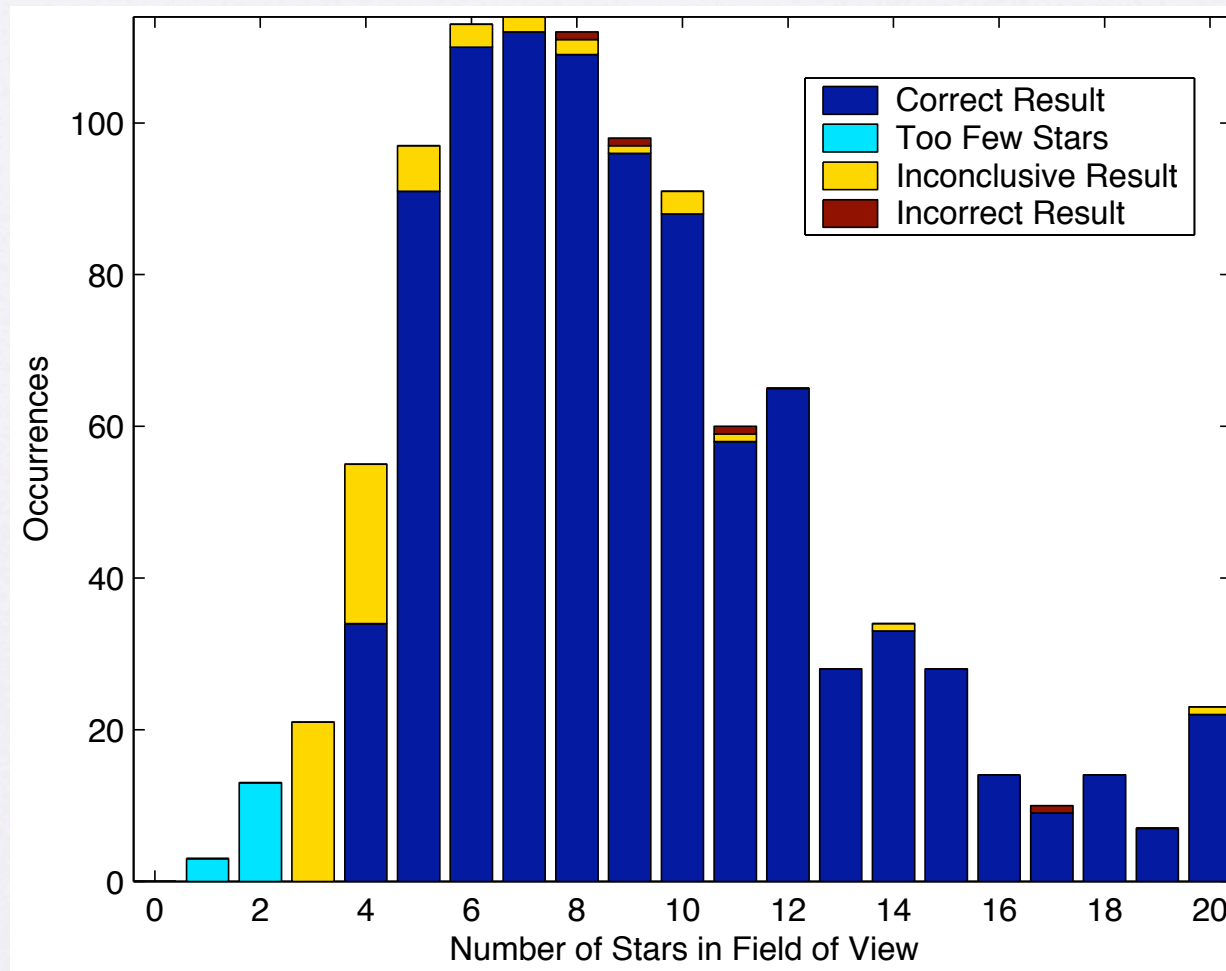


Appendix E.
Sph. Tri. Method,
3-Pivot Limit

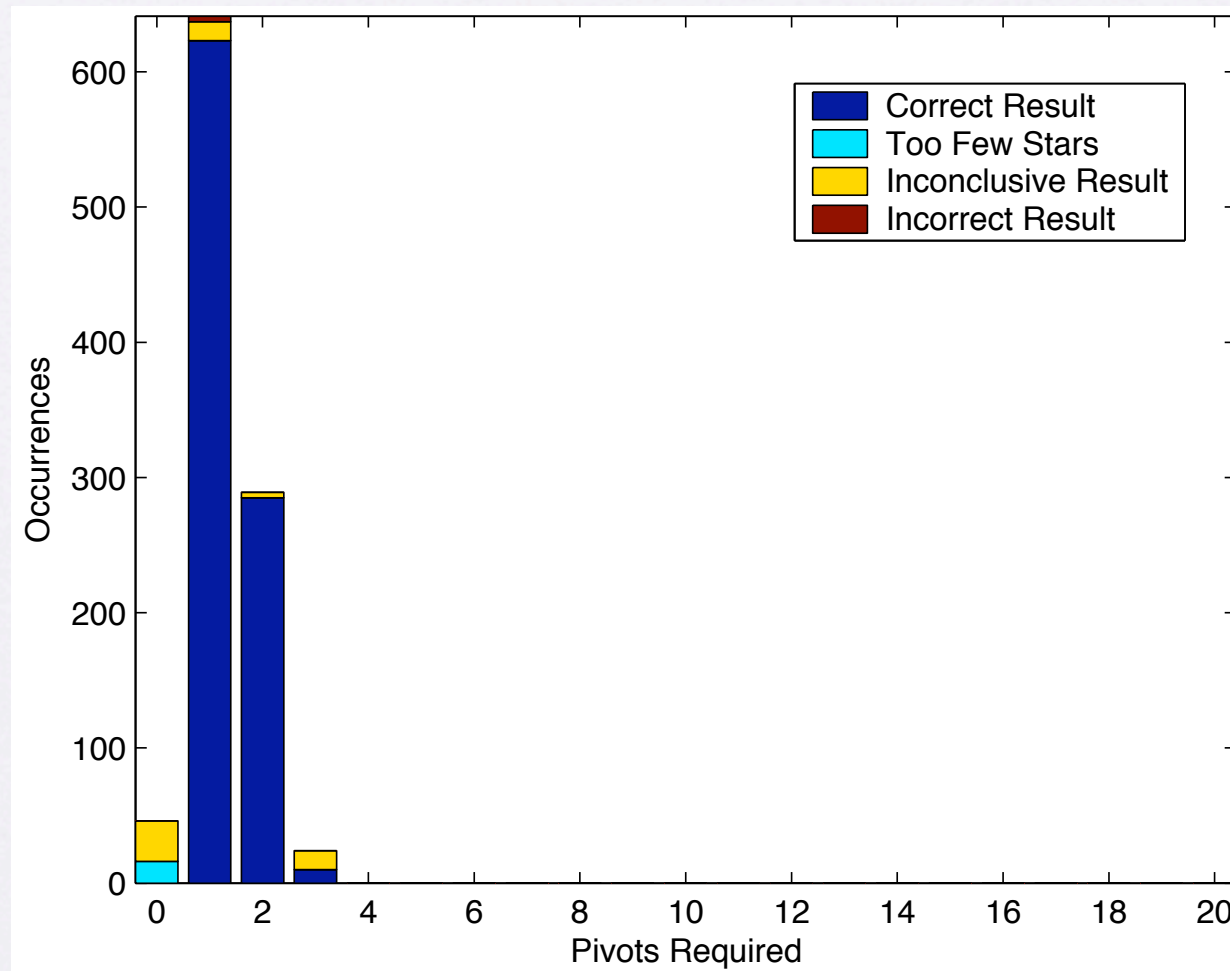
Overall Result for Sph. Tri. Method with 3-Pivot Limit



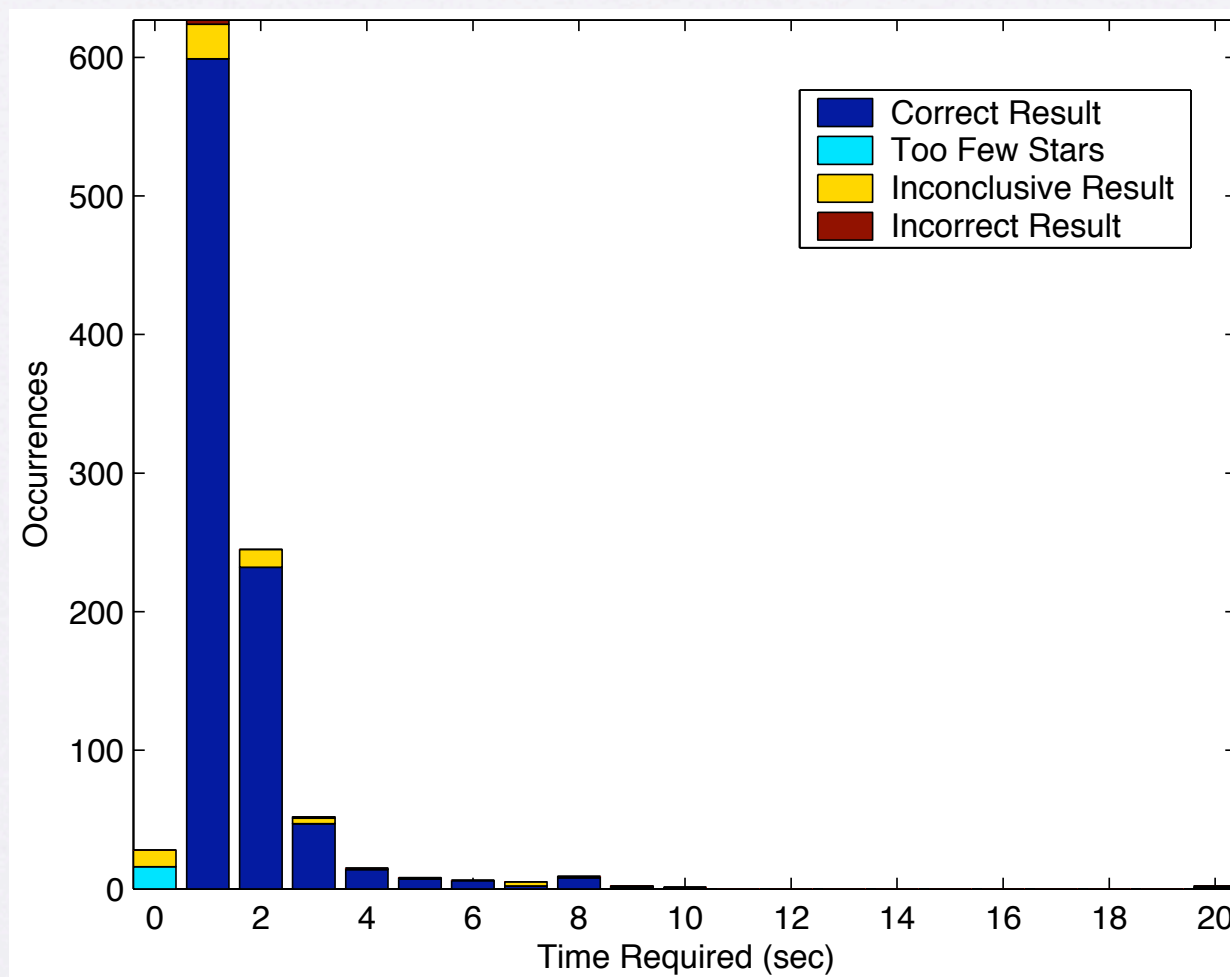
Distribution of Results for Sph. Tri. Method with 3-Pivot Limit



Pivots Required for Sph. Tri. Method with 3-Pivot Limit

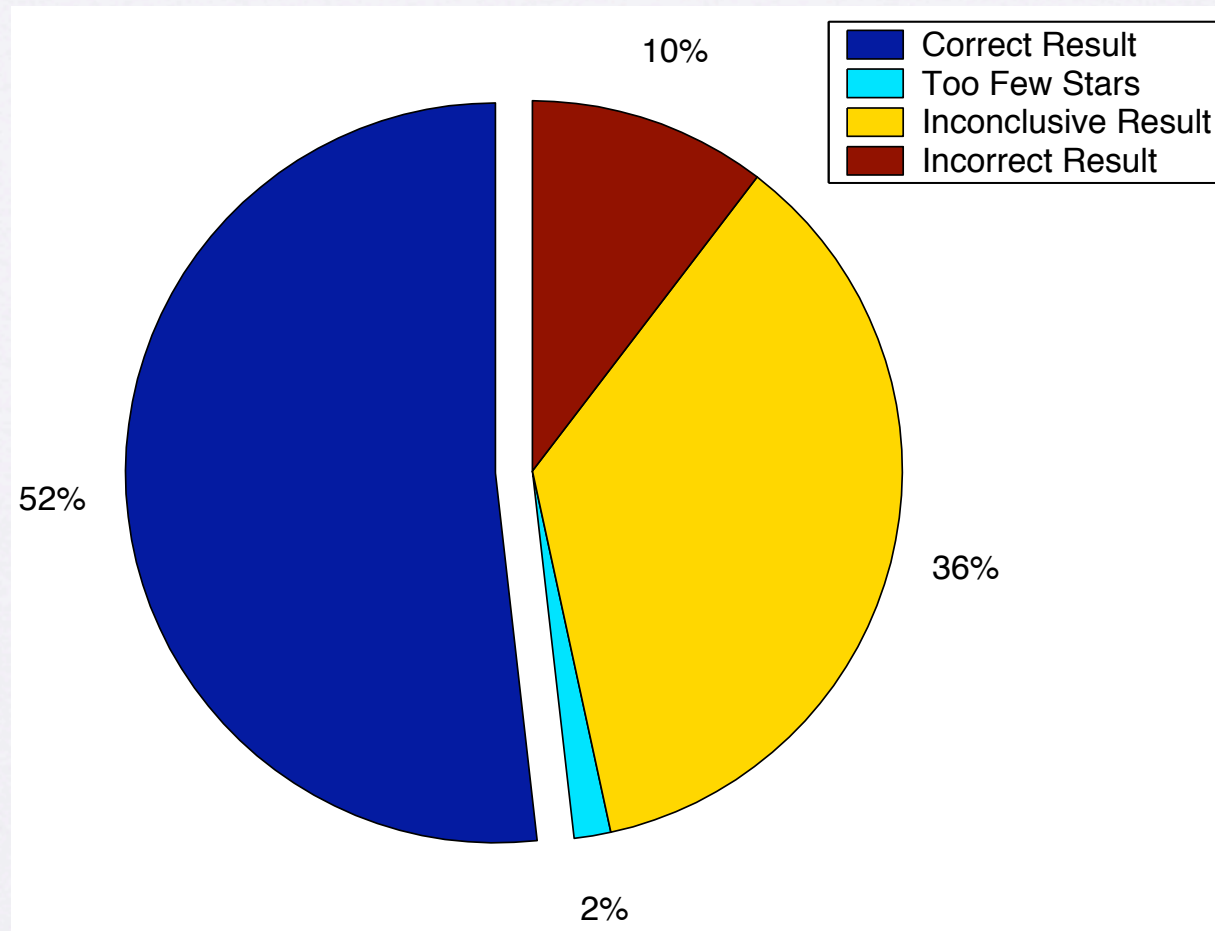


CPU Time Required for Sph. Tri. Method with 3-Pivot Limit

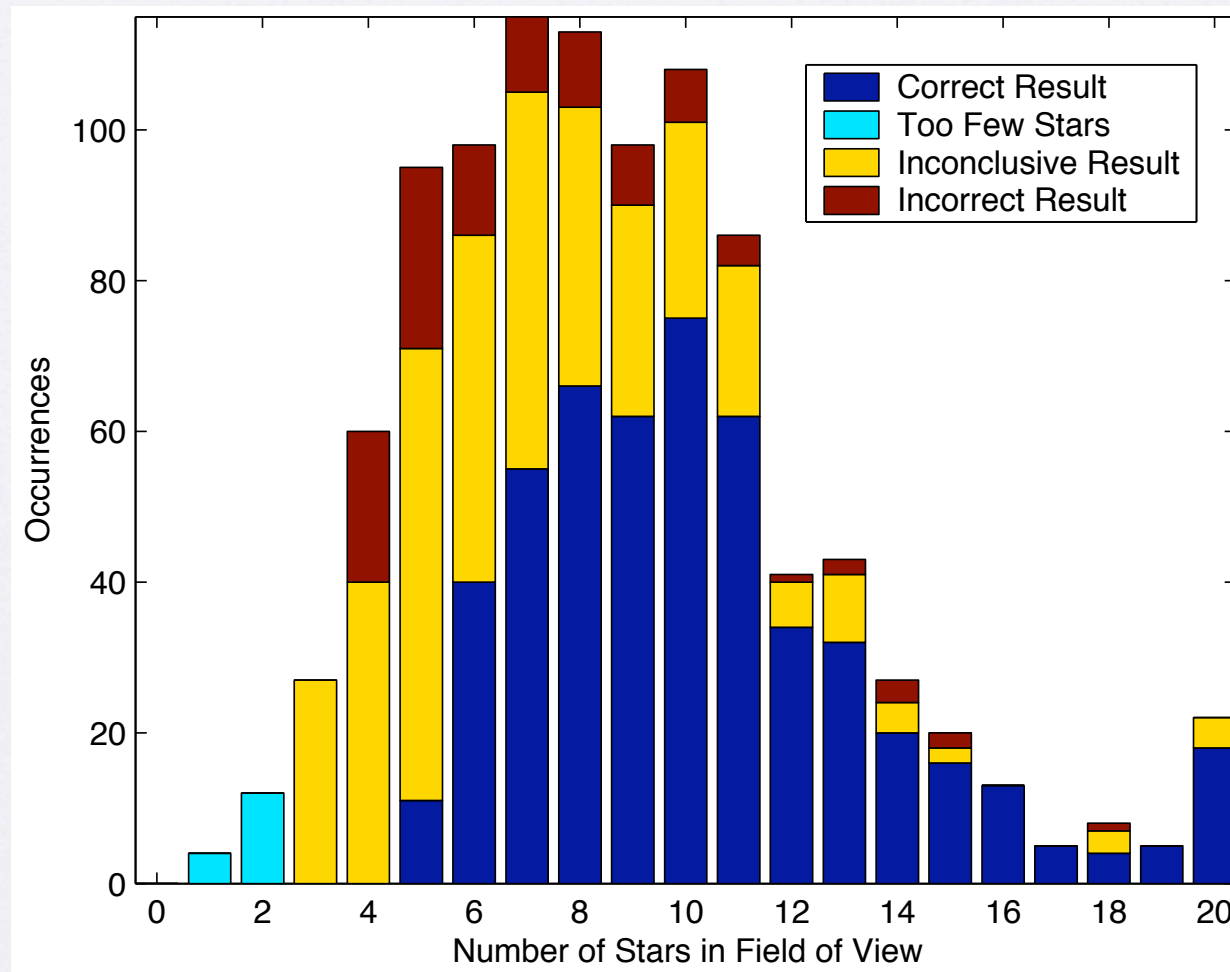


Appendix F.
Sph. Tri. Method,
3-Pivot Limit,
False Star Inclusion

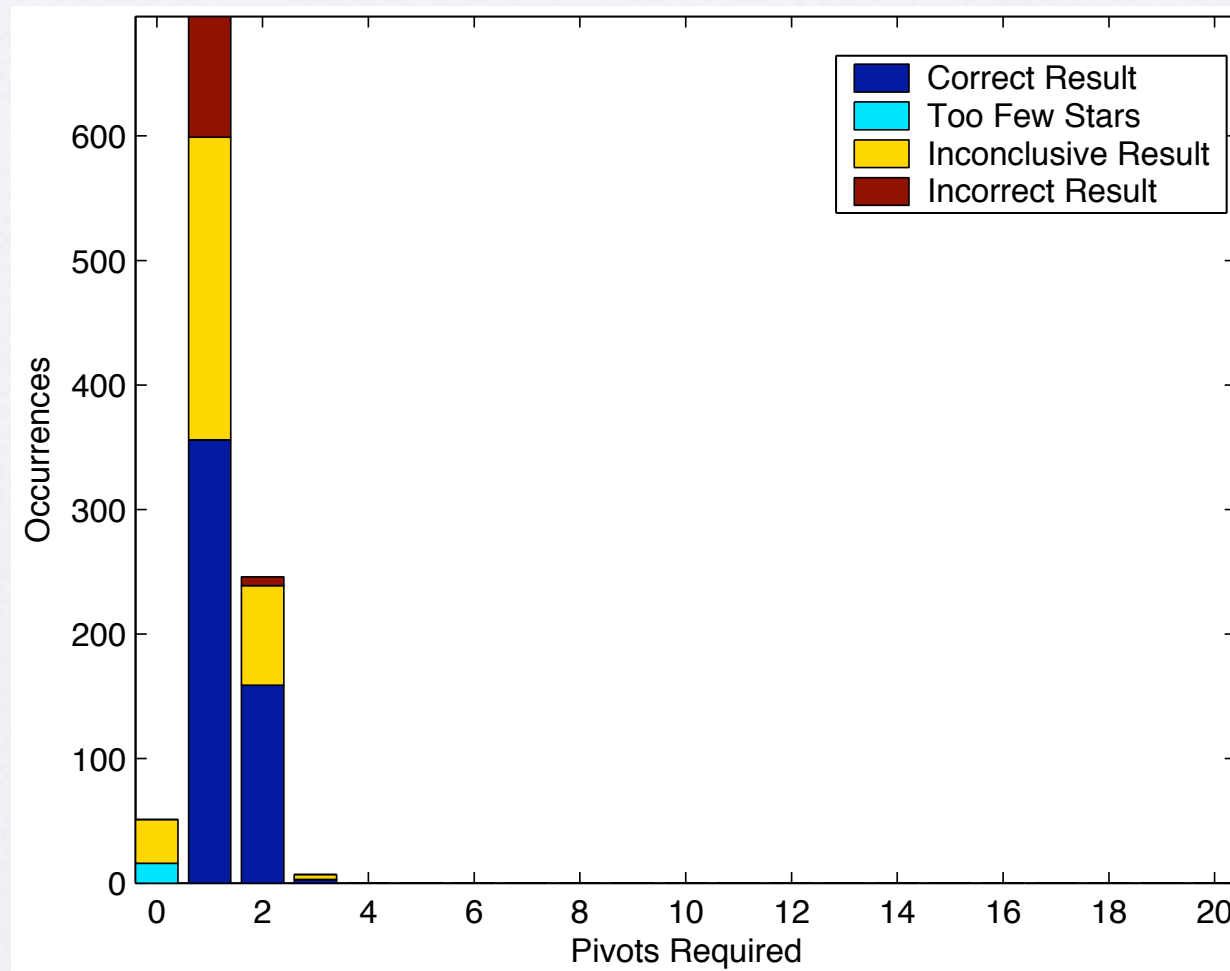
Overall Result for Sph. Tri. Method (3-Pivot) w/False Star



Distribution of Results for Sph. Tri. Method (3-Pivot) w/False Star



Pivots Required for Sph. Tri. Method (3-Pivot) w/False Star



CPU Time Required for Sph. Tri. Method (3-Pivot) w/False Star

