

ASP Final

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Due: December 29th, 2022

1 Details of Beamformers

- assume uniform linear array(ULA) with N isotropic antennas with inter-element spacing $d = \frac{\lambda}{2}$
- the steering vector for ULAs

$$\mathbf{a}(\theta_i) = [1 \ \exp(j\pi \sin \theta) \ \exp(j2\pi \sin \theta) \ \dots \ \exp(j(N-1)\pi \sin \theta)]^T$$

- array data model

$$\mathbf{x}(t) = \mathbf{a}(\theta)s_1(t) + \mathbf{n}(t) = \mathbf{1}(Ae^{j2\pi ft}) + \mathbf{n}(t)$$

$$\begin{aligned} \text{where } \mathbb{E}[A] &= \mathbf{0} \quad \text{and} \quad \mathbb{E}[|A|^2] = \sigma_1^2 \\ \text{where } \mathbb{E}[\mathbf{n}(t)] &= \mathbf{0} \quad \text{and} \quad \mathbb{E}[\mathbf{n}(t)\mathbf{n}^H(t)] = \sigma_n^2 \mathbf{I} \end{aligned}$$

1.1 The beamformer with uniform weights

1.1.1 weight vector

$$\mathbf{w} = \mathbf{1}/N$$

1.1.2 beamformer output

$$\begin{aligned} y(t) &= \mathbf{w}^H \mathbf{x}(t) \\ &= (Ae^{j2\pi ft}) + (\frac{1}{N} \mathbf{1}^H \mathbf{n}(t)) \end{aligned} \tag{1}$$

1.1.3 beam pattern

$$\begin{aligned} B_\theta(\theta) &= (\frac{1}{N} \mathbf{1}^H) \mathbf{a}(\theta) \\ &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j\pi \sin \theta \cdot n} \\ &= e^{j\frac{N-1}{2}\pi \sin(\theta)} \times \frac{1}{N} \times \frac{\sin(\frac{N}{2}\pi \sin \theta)}{\sin(\frac{1}{2}\pi \sin \theta)} \end{aligned} \tag{2}$$

- DOA $\theta_1 = 0$:N-times SNR enhancement
- DOA $\theta_1 = \theta$:SNR depending on $B_\theta(\theta)$
- The maximum of $B_\theta(\theta)$ occurs at $\theta=0$

1.2 The beamformer with array steering

1.2.1 weight vector

$$\mathbf{w} = \frac{1}{N} \mathbf{a}(\theta_s) = \frac{1}{N} [1 \quad e^{j(\pi \sin \theta_s)} \quad e^{j(\pi \sin \theta_s) \times 2} \quad \dots \quad e^{j(\pi \sin \theta_s) \times (N-1)}]^T$$

1.2.2 beam pattern

$$\begin{aligned} B_\theta(\theta) &= \mathbf{w}^H \mathbf{a}(\theta) \\ &= e^{j \frac{N-1}{2} (\pi \sin \theta - \pi \sin \theta_s)} \times \frac{1}{N} \times \frac{\sin[\frac{N}{2} (\pi \sin \theta - \pi \sin \theta_s)]}{\sin[\frac{1}{2} (\pi \sin \theta - \pi \sin \theta_s)]} \end{aligned} \quad (3)$$

- The maximum of $|B_\theta(\theta)|$ occurs at $\theta = \theta_s$

1.3 The MVDR beamformer

1.3.1 Interference rejection

- $\mathbf{x}(t) = \underbrace{\mathbf{a}(\theta_s)\mathbf{s}(t)}_{\text{Signal}} + \underbrace{\mathbf{a}(\theta_i)\mathbf{i}(t)}_{\text{Interference}} + \underbrace{\mathbf{n}(t)}_{\text{Noise}}$
- $\mathbf{R} = \underbrace{\sigma_s^2 \mathbf{a}(\theta_s)\mathbf{a}^H(\theta_s)}_{\text{Signal}} + \underbrace{\sigma_i^2 \mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)}_{\text{Interference}} + \underbrace{\sigma_n^2 \mathbf{I}}_{\text{Noise}}$ where $\mathbf{R} = \mathbb{E}[\mathbf{x}(t)\mathbf{x}^H(t)]$
- $\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t)$
 $= \underbrace{\mathbf{w}^H \mathbf{a}(\theta_s)\mathbf{s}(t)}_{\text{Signal}} + \underbrace{\mathbf{w}^H \mathbf{a}(\theta_i)\mathbf{i}(t)}_{\text{Interference}} + \underbrace{\mathbf{w}^H \mathbf{n}(t)}_{\text{Noise}}$

1.3.2 Optimization problem(with the distortionless constraint)

$$\begin{aligned} &\arg \min_{\mathbf{w}} \quad \mathbb{E}[|y(t)|^2] \\ &\text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_s) = 1, \\ &\quad \quad \quad y(t) = \mathbf{w}^H \mathbf{x}(t) \end{aligned} \quad (4)$$

1.3.3 weight vector

$$\mathbf{w}_{MVDR} = \frac{\mathbf{R}^{-1} \mathbf{a}(\theta_s)}{\mathbf{a}^H(\theta_s) \mathbf{R}^{-1} \mathbf{a}(\theta_s)}$$

1.4 The LCMV beamformer

1.4.1 Interference rejection

- $\mathbf{x}(t) = \underbrace{\mathbf{a}(\theta_s)\mathbf{s}(t)}_{\text{Signal}} + \underbrace{\mathbf{a}(\theta_i)\mathbf{i}(t)}_{\text{Interference}} + \underbrace{\mathbf{n}(t)}_{\text{Noise}}$
- $\mathbf{R} = \underbrace{\sigma_s^2 \mathbf{a}(\theta_s)\mathbf{a}^H(\theta_s)}_{\text{Signal}} + \underbrace{\sigma_i^2 \mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)}_{\text{Interference}} + \underbrace{\sigma_n^2 \mathbf{I}}_{\text{Noise}}$ where $\mathbf{R} = \mathbb{E}[\mathbf{x}(t)\mathbf{x}^H(t)]$

- $\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t)$

$$= \underbrace{\mathbf{w}^H \mathbf{a}(\theta_s) \mathbf{s}(t)}_{\text{Signal}} + \underbrace{\mathbf{w}^H \mathbf{a}(\theta_i) \mathbf{i}(t)}_{\text{Interference}} + \underbrace{\mathbf{w}^H \mathbf{n}(t)}_{\text{Noise}}$$

1.4.2 Optimization problem(with the more constraints)

$$\begin{aligned} & \arg \min_{\mathbf{w}} \quad \mathbb{E}[|y(t)|^2] \\ & \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{g}, \\ & \quad \quad \quad y(t) = \mathbf{w}^H \mathbf{x}(t) \end{aligned} \tag{5}$$

1.4.3 weight vector

$$\mathbf{w}_{LCMV} = \mathbf{R}^{-1} \mathbf{C} (\mathbf{C}^H \mathbf{R}^{-1} \mathbf{C})^{-1} \mathbf{g}$$

2 Denoise $\tilde{\theta}_s(t)$ and $\tilde{\theta}_i(t)$

- I use EMD(Empirical Mode decomposition) to denoise $\tilde{\theta}_s(t)$ and $\tilde{\theta}_i(t)$.

2.1 Advantages of EMD algorithm

- When we use EMD, we could easily denoise $\tilde{\theta}_s(t)$ and $\tilde{\theta}_i(t)$ in time domain, rather than transforming the signals into frequency domain then progressing.
- When we use EMD, we could denoise $\tilde{\theta}_s(t)$ and $\tilde{\theta}_i(t)$ well.

2.2 EMD(take denoising $\tilde{\theta}_s(t)$ for example)

2.2.1 EMD details

STEP 1:let $y(n)=x(n)$ and $n=1$ and $k=1$, where $x(n)$ is the sample

STEP 2:Find the local peaks

STEP 3:Connect local peaks and get $e_{max}(n)$

STEP 4:Find the local dips

STEP 5:Connect local dips and get $e_{min}(n)$

STEP 6:Compute the mean $m(n)=\frac{e_{max}(n)+e_{min}(n)}{2}$

STEP 7:Compute the residue $d(n)=y(n)-m(n)$

STEP 8:Find $d_{max}(n)$:the local peaks of $d(n)$

STEP 9:Find $d_{min}(n)$:the local dips of $d(n)$

STEP 10:Check whether the local peaks of $d(n)$ are all greater than 0

STEP 11:Check whether the local dips of $d(n)$ are all smaller than 0

STEP 12:Check whether $|\frac{u_1(n)+u_0(n)}{2}|$ is smaller than thr , where $u_1(n)$ and $u_0(n)$ are the upper and lower envelope of $d(n)$ respectively

STEP 13:If step 10 to step 12 are satisfied, set $c_n(n)=d(n)$ and continue to step 14, otherwise, set $y(n)=d(n)$ and $k=k+1$ and repeat step 2 to step 6(if k is greater than 1000, stop the iteration)

STEP 14:Calculate $x_o(n)=x(n)-\sum_{s=1}^n c_s(n)$

STEP 15:Check whether $x_o(n)$ is a function with no more than one extreme point(let n_{max} :the total number of the local maxima of $x_o(n)$ and n_{min} :the total number of the local minima of $x_o(n)$)

STEP 16:If step 15 is satisfied, the empirical mode decomposition is completed, otherwise, set $y(n)=x_o(n)$ and $n=n+1$ and repeat step 2 to step 7

2.2.2 EMD algorithm

Algorithm 1 An algorithm with EMD

Require: the samples $\tilde{\theta}_s(n)$ and the threshold thr

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 $y(n) \leftarrow \tilde{\theta}_s(n)$   
 $x_o(n) \leftarrow \tilde{\theta}_s(n)$   
 $thr \leftarrow 0.2$   
 $k \leftarrow 1$   
while 1 do  
  while 1 do  
     $m(n) = \frac{e_{max}(n) + e_{min}(n)}{2}$   
     $k \leftarrow k + 1$   
     $d(n) = y(n) - m(n)$   
    if  $d_{max}(n) > 0 \wedge d_{min}(n) < 0 \wedge |\frac{u_1(n) + u_0(n)}{2}| < thr \vee k > 1000$  then  
       $c_n(n) \leftarrow d(n)$   
      break  
    else  
       $y(n) \leftarrow d(n)$   
    end if  
  end while  
   $x_o(n) = \tilde{\theta}_s(n) - \sum_{s=1}^n c_S(n)$   
  if  $n_{max} + n_{max} \leq 3$  then  
    break  
  else  
     $y(n) \leftarrow x_o(n)$   
     $n \leftarrow n + 1$   
  end if  
end while  
 $\hat{\theta}_s(n) \leftarrow x_o(n)$ 
```

3 Plot the estimated DOAs $\hat{\theta}_s(n)$ and $\hat{\theta}_i(n)$

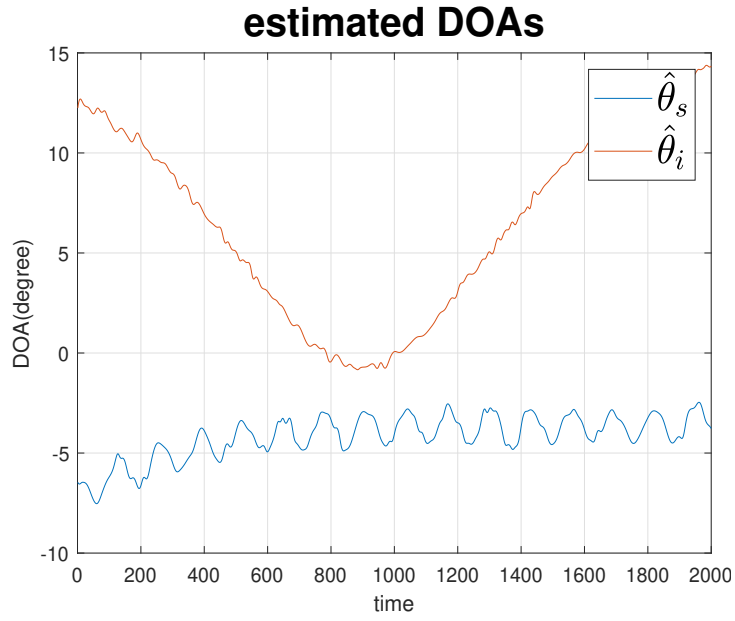


Figure 1: estimated DOAs

3.1 The noisy DOAs $\tilde{\theta}_s(n)$ and $\tilde{\theta}_i(n)$

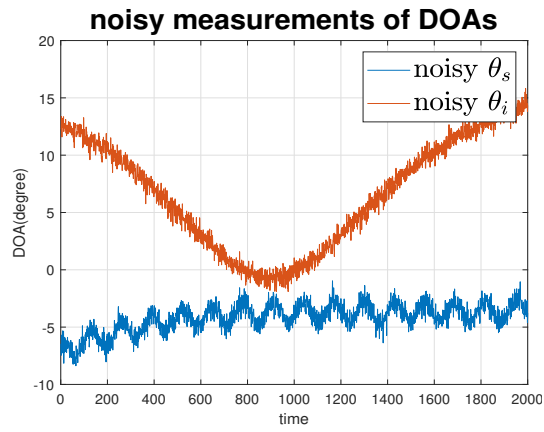


Figure 2: noisy DOAs

4 Design my beamformer

4.1 Advantages of my beamformer

Compared to the LCMV beamformer, my beamformer could beamform the signal in the time interval, in which DOAs of the source signal and the interference signal are similar. Thus, the estimated signal will be more similar to the source signal.

4.2 Details

STEP 1: Beamforming the signal with array steering beamformer (the most similar to the real signal)

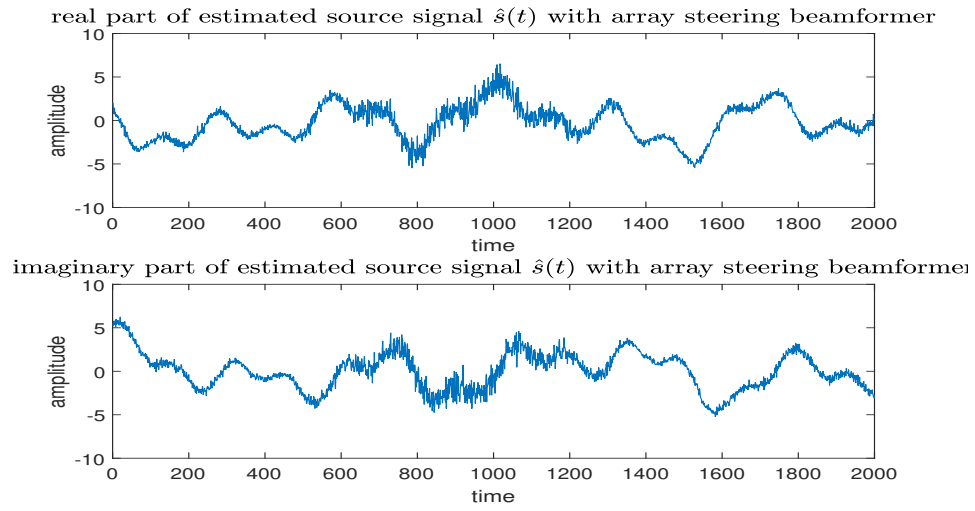


Figure 3: signal with array steering beamformer

STEP 2: Beamforming the signal with uniform weighting beamformer

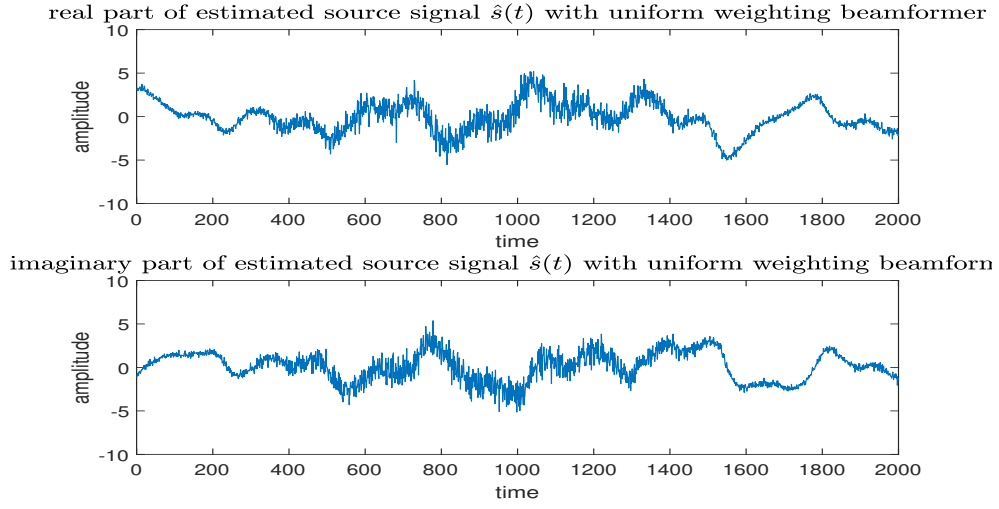


Figure 4: signal with uniform weighting beamformer

STEP 3: Calculate $\hat{\mathbf{R}}$ with SMI(Sample Matrix Inversion)

STEP 4: Using $\hat{\mathbf{R}}$ to beamform the signal with MVDR beamformer

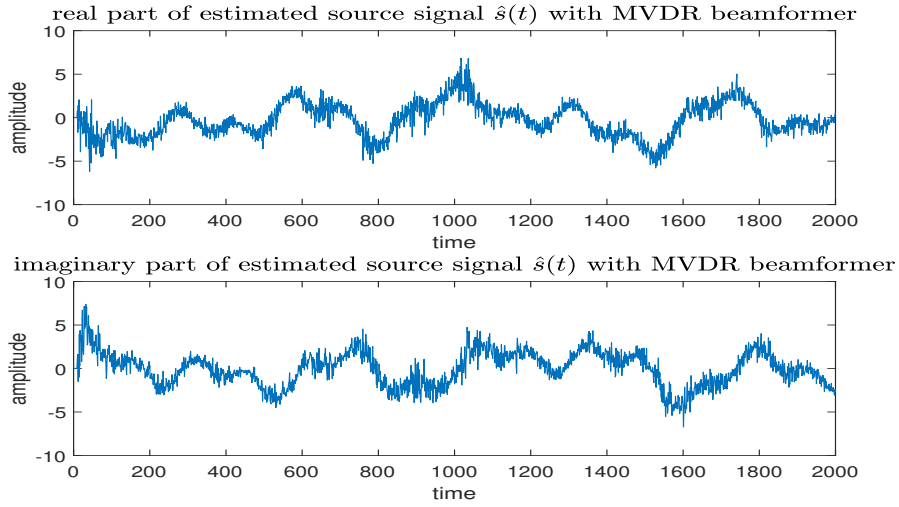


Figure 5: signal with MVDR beamformer

STEP 5: Using $\hat{\mathbf{R}}$ to beamform the signal with LCMV beamformer

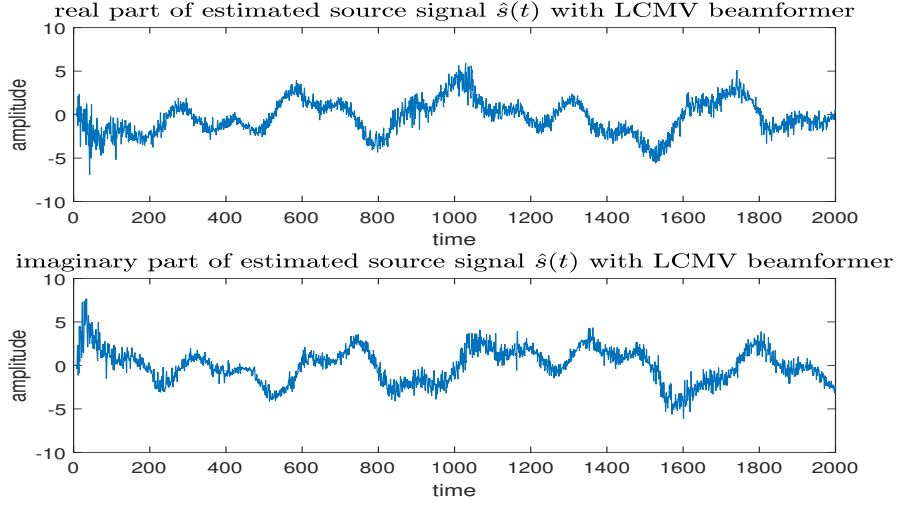


Figure 6: signal with LCMV beamformer

STEP 6:Based on LCMV beamformer,add more constraints.First, let $\mathbf{C} = [\mathbf{a}(\theta_s) \mathbf{a}(\theta_i - 1) \mathbf{a}(\theta_i) \mathbf{a}(\theta_i + 1)]$
STEP 7:By observing the estimated DOAs,we could find that, the interference is very serious from time 670 to 1172.So I change the vector $\mathbf{g} = [1 \ 10^{-4} \ 10^{-4} \ 10^{-4}]^T$ from time 670 to 1172.

4.3 Algorithm

Algorithm 2 An algorithm with SMI

Require: σ_o^2 and μ and steering vector $\mathbf{a}(\theta_s)$ and the sample $x(n)$

- 1: $\mathbf{P}(0) = \frac{1}{\sigma_o^2} \mathbf{I}$
 - 2: $\hat{\mathbf{w}}(0) = \frac{\mathbf{a}(\theta_s)}{N}$
 - 3: **for** $K=1,2,\dots,2000$ **do**
 - 4: $\mathbf{g}(K) = \frac{\mu^{-1} \mathbf{P}(K-1) \mathbf{x}(K)}{1 + \mu^{-1} \mathbf{x}^H(K) \mathbf{P}(K-1) \mathbf{x}(K)}$
 - 5: $\mathbf{P}(K) = \mu^{-1} \mathbf{P}(K-1) - \mu^{-1} \mathbf{g}(K) \mathbf{x}^H(K) \mathbf{P}(K-1)$
 - 6: **end for**
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Algorithm 3 An algorithm with my beamformer

Require: σ_o^2 and μ and steering vector $\mathbf{a}(\theta_s)$ and interference steering vector $\mathbf{a}(\theta_i)$ and the sample $x(n)$

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1:  $\mathbf{P}(0) = \frac{1}{\sigma_o^2} \mathbf{I}$ 
2:  $\hat{\mathbf{w}}(0) = \frac{\mathbf{a}(\theta_s)}{N}$ 
3: for  $K=1,2,\dots,2000$  do
4:    $\mathbf{C}(K) = [\mathbf{a}(\theta_s)(K) \ \mathbf{a}(\theta_i - 1)(K) \ \mathbf{a}(\theta_i)(K) \ \mathbf{a}(\theta_i + 1)(K)]$ 
5:    $\mathbf{g}(K) = \frac{\mu^{-1} \mathbf{P}(K-1) \mathbf{x}(K)}{1 + \mu^{-1} \mathbf{x}^H(K) \mathbf{P}(K-1) \mathbf{x}(K)}$ 
6:    $\mathbf{P}(K) = \mu^{-1} \mathbf{P}(K-1) - \mu^{-1} \mathbf{g}(K) \mathbf{x}^H(K) \mathbf{P}(K-1)$ 
7:   if  $K \geq 672 \wedge K \leq 1172$  then
8:      $\mathbf{g} \leftarrow [1 \ 10^{-4} \ 10^{-4} \ 10^{-4}]^T$ 
9:   else
10:     $\mathbf{g} \leftarrow [1 \ 0 \ 0 \ 0]^T$ 
11:   end if
12:    $\mathbf{w}_{LCMV}(K) = \mathbf{P} \mathbf{C}^H \mathbf{P} \mathbf{C}^{-1} \mathbf{g}$ 
13:    $\mathbf{y}(K) = \mathbf{w}_{LCMV}(K) \mathbf{x}(K)$ 
14: end for
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5 Plot the real and imaginary parts of estimated source signal $\hat{s}(t)$

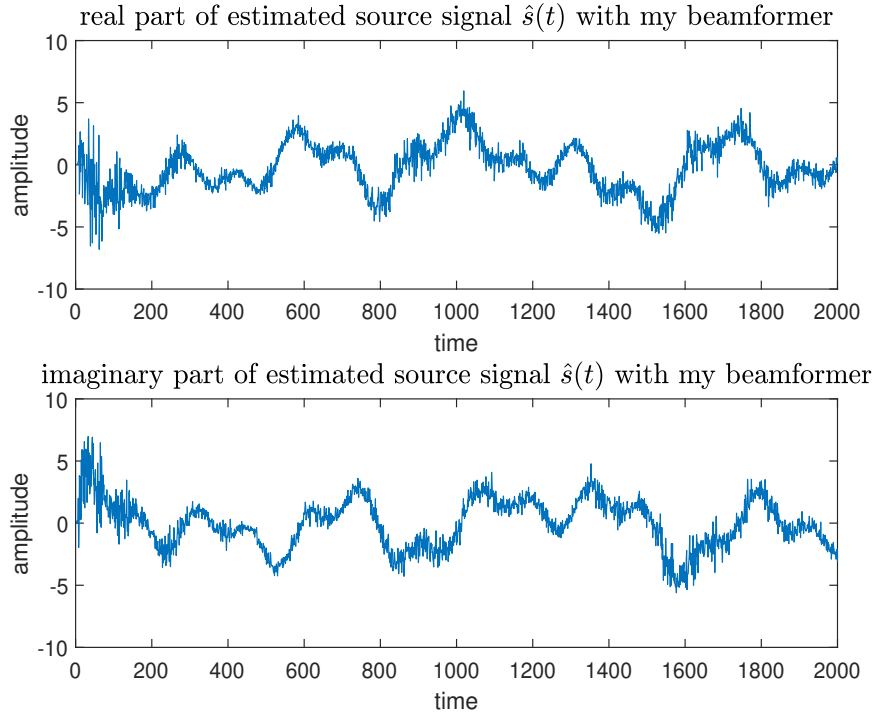


Figure 7: signal with my beamformer