

Multiscale Symbolic Phase Transfer Entropy in Financial Time Series Classification

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Received 2 January 2017

Accepted 2 April 2017

Published 2 May 2017

Communicated by Wei-Xing Zhou

We address the challenge of classifying financial time series via a newly proposed multiscale symbolic phase transfer entropy (MSPTE). Using MSPTE method, we succeed to quantify the strength and direction of information flow between financial systems and classify financial time series, which are the stock indices from Europe, America and China during the period from 2006 to 2016 and the stocks of banking, aviation industry and pharmacy during the period from 2007 to 2016, simultaneously. The MSPTE analysis shows that the value of symbolic phase transfer entropy (SPTE) among stocks decreases with the increasing scale factor. It is demonstrated that MSPTE method can well divide stocks into groups by areas and industries. In addition, it can be concluded that the MSPTE analysis quantify the similarity among the stock markets. The symbolic phase transfer entropy (SPTE) between the two stocks from the same area is far less than the SPTE between stocks from different areas. The results also indicate that four stocks from America and Europe have relatively high degree of similarity and the stocks of banking and pharmaceutical industry have higher similarity for CA. It is worth mentioning that the pharmaceutical industry has weaker particular market mechanism than banking and aviation industry.

Keywords: Multiscale symbolic phase transfer entropy; symbolic phase transfer entropy; stock market; similarity.

1. Introduction

Recently, economics has become an active research field for physicists. A number of studies have successfully applied statistical mechanics to the economic systems. Physicists have attempted to apply the concepts and methods of statistical physics, such as the correlation function, multifractal, spin models, complex

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networks, and information theory to study stock markets [1–11]. As a consequence, all those companies in the stock market are correlated and interconnected, so the interaction therein is highly nonlinear, unstable and long-ranged.

Information is an important keyword in analyzing the market or in estimating the stock price of a given company. More specifically, it includes two important indicators: the direction and strength. A key measure of information is known as entropy. The transfer entropy is a commonly used tool of nonlinear systems. The transfer entropy which measures the information flow between nonlinear systems has been recently introduced by Schreiber [12]. Kaiser and Schreiber [13], Verdes [14] and Kuniyoshi [15] point out that all kinds of technologies have been proposed to estimate the transfer entropy from the observed values [16–21]. However, most of the technologies highly require data, need to adjust parameters, and are sensitive to noise. The shortcomings hinder the wide application of transfer entropy in various fields. Staniek and Lehnertz numerically illustrated that symbolic transfer entropy is a powerful and fast calculation method in quantifying dominating direction of information flow between time series from coupling system of the same and different structures [22]. Lobier *et al.* proposed phase transfer entropy (Phase TE) as a measure to infer the direct relationship between neural oscillations [23]. Therefore, considering faults and the bottleneck problem in the study of transfer entropy method, we propose the improved method of transfer entropy to overcome the limitations of existing methods, which is novel in detection of the information flow among nonlinear systems.

Traditional measures of information flow of time series only quantify the direction and strength between nonlinear systems on a single time scale. However, many physiologic variables, such as heart rate, fluctuate in a very complex manner and present information flow over multiple time scales [24–34]. An innovative measure based on a weighted sum of scale-dependent entropies, which is coarse-grained entropies over multiple scales, was proposed by Zhang [35, 36]. Accordingly, multiscale symbolic phase transfer entropy (MSPTE) is introduced to quantify the direction and strength of information flow among systems and distinguish different area of the markets simultaneously in this paper.

The remainder of this paper is organized as follows. In Sec. 2, we present the methodologies of MSPTE. We show data description in Sec. 3 and perform the detail analysis and present our results in Sec. 4. The conclusions are presented at the last section.

2. Methodology

2.1. Symbolic phase transfer entropy

The symbolic phase transfer entropy is symbolic transfer entropy based on phase. That is to say, before we deal with the ordinary time series, we extract phase time series from the original time series. Then, we deal with the phase time series.

Therefore, we define symbolic phase transfer entropy in the following steps:

Step 1. For the given time series $X(i)$ and $Y(i)$, its instantaneous phase time-series $\theta^x(t)$ and $\theta^y(t)$ is separately obtained by $X(i)$ and $Y(i)$ with Hilbert transform [37]. For the given time series x , its analytic series ζ is defined as

$$\zeta(t) = x(t) + j\hat{x}(t) = A(t)e^{j\theta(t)}, \quad (1)$$

$x(t)$ with Hilbert transform gets $\hat{x}(t)$

$$\hat{x}(t) = \frac{1}{\pi} P.V. \cdot \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau. \quad (2)$$

P.V. denotes the integral of the Cauchy principal value. In Eq. (1), $A(t)$ is the instantaneous amplitude and $\theta(t)$ is the instantaneous phase of $x(t)$.

Step 2. We calculate the symbolic transfer entropy of instantaneous phase time-series. Then, we can get two instantaneous phase time-series $\theta^x(t)$ and $\theta^y(t)$. First, we reconstruct the phase space of $\theta^x(t)$ and $\theta^y(t)$, and then the symbolic processing is carried out. The method of symbolic processing is based on the phase space reconstruction and associated with the definition of permutation entropy [38]. For an arbitrary i , $\theta^x(i) = \{\theta^x(i), \theta^x(i+1), \dots, \theta^x(i+(m-1)l)\}$ got by phase space reconstruction are arranged in an ascending order $\{\theta^x(i+(k_{i1}-1)l) \leq \theta^x(i+(k_{i2}-1)l) \leq \dots \leq \theta^x(i+(k_{im}-1)l)\}$, where l is the time delay, and m denotes the embedding dimension. In case of equal values, for example, while $\theta^x(i+(k_{i1}-1)l) = \theta^x(i+(k_{i2}-1)l)$ we write $\theta^x(i+(k_{i1}-1)l) \leq \theta^x(i+(k_{i2}-1)l)$ if $k_{i1} \leq k_{i2}$, therefore we insure that every θ_i^x is uniquely mapped onto one of the $m!$ possible permutations. A symbol is thus defined as $\hat{x}_i = (k_{i1}, k_{i2}, \dots, k_{im})$, and with the relative frequency of symbols we estimate joint and conditional probabilities of the sequence of permutation indices. With the relative frequencies of the symbols, we estimate the joint and conditional probabilities of the sequence of permutation index. Finally, we compute the corresponding probability. The symbolic phase transfer entropy from Y to X is

$$\begin{aligned} \text{SPTE}_{Y,X} &= - \sum p(\hat{\theta}_{i+\delta}^x, \hat{\theta}_i^x, \hat{\theta}_i^y) \log p(\hat{\theta}_{i+\delta}^x | \hat{\theta}_i^x) \\ &\quad + \sum p(\hat{\theta}_{i+\delta}^x, \hat{\theta}_i^x, \hat{\theta}_i^y) \log p(\hat{\theta}_{i+\delta}^x | \hat{\theta}_i^x, \hat{\theta}_i^y) \\ &= \sum p(\hat{\theta}_{i+\delta}^x, \hat{\theta}_i^x, \hat{\theta}_i^y) \log \frac{p(\hat{\theta}_{i+\delta}^x | \hat{\theta}_i^x, \hat{\theta}_i^y)}{p(\hat{\theta}_{i+\delta}^x | \hat{\theta}_i^x)}, \end{aligned} \quad (3)$$

where

$$p(\hat{\theta}_{i+\delta}^x | \hat{\theta}_i^x, \hat{\theta}_i^y) = \frac{p(\hat{\theta}_{i+\delta}^x, \hat{\theta}_i^x, \hat{\theta}_i^y)}{p(\hat{\theta}_{i+\delta}^x, \hat{\theta}_i^y)}, \quad (4)$$

$$p(\hat{\theta}_{i+\delta}^x | \hat{\theta}_i^x) = \frac{p(\hat{\theta}_{i+\delta}^x, \hat{\theta}_i^x)}{p(\hat{\theta}_{i+\delta}^x)}. \quad (5)$$

2.2. Multiscale symbolic phase transfer entropy

$x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ are two time series of length- N . We first construct consecutive coarse-grained time series from the original series x and y with the scale factor τ . The coarse-graining process is as follows: we divide the original time series into non-overlapping segments of length τ and then calculate the average of data points in each segment. Generally, each element of the coarse-grained time series $x_j^{(\tau)}$ and $y_j^{(\tau)}$ are calculated referring to the equation

$$\begin{aligned} x_j^{(\tau)} &= \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} x_i, 1 \leq j \leq \frac{N}{\tau}, \\ y_j^{(\tau)} &= \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} y_i, 1 \leq j \leq \frac{N}{\tau}. \end{aligned} \quad (6)$$

For scale one ($\tau = 1$), the time series $x^{(1)}$ and $y^{(1)}$ are the original time series. The length of each coarse-grained time series is equal to $\frac{N}{\tau}$.

Next, we calculate symbolic phase transfer entropy for coarse-grained time series plotted as a function of the scale factor τ .

3. Data

In an effort to illustrate the corresponding capability of the MSPTE method for real-world dynamics, we also analyze six stock indices of the America, Europe and China during the period from 3 January, 2006 to 11 March, 2016 which are listed in Table 1 and six stocks of banking, aviation industry and pharmacy during the period from 5 November, 2007 to 15 July, 2016 which are listed in Table 2. We obtain data sets from the website of <http://finance.yahoo.com> and <http://www.10jqka.com.cn>, respectively. Due to stock markets have the different trading dates, we complement the asynchronous data with cubic spline interpolation and then reconnect the remaining parts of the original series to obtain the same length time series. The overall run of indices after the preprocessing is displayed in Figs. 1 and 2.

Denoting the stock market index as $x(t)$, the logarithmic daily return is defined by $g(t) = \ln(x(t)) - \ln(x(t-1))$. The normalized daily return is defined as $R(t) = (g(t) - \langle g(t) \rangle) / \sigma$, where σ is the standard deviation of the series $g(t)$.

Table 1. The list of six stock markets.

| Num | Area | Index |
|-----|---------|------------|
| 1 | Europe | DAX |
| 2 | Europe | FTSE |
| 3 | America | NAS |
| 4 | America | S&P500 |
| 5 | China | ShangZheng |
| 6 | China | ShenCheng |

Table 2. The list of six stock markets.

| Num | Index | Abbreviation | Industry |
|-----|---|--------------|----------|
| 1 | China Construction Bank | CCB | Banking |
| 2 | China Merchants Bank | CMB | Banking |
| 3 | AIR CHINA | CA | Aviation |
| 4 | China Southern Airlines | CZ | Aviation |
| 5 | TRT Health International | TRT | Pharmacy |
| 6 | Zhejiang Huahai Pharmaceutical Co., Ltd | Huahai | Pharmacy |

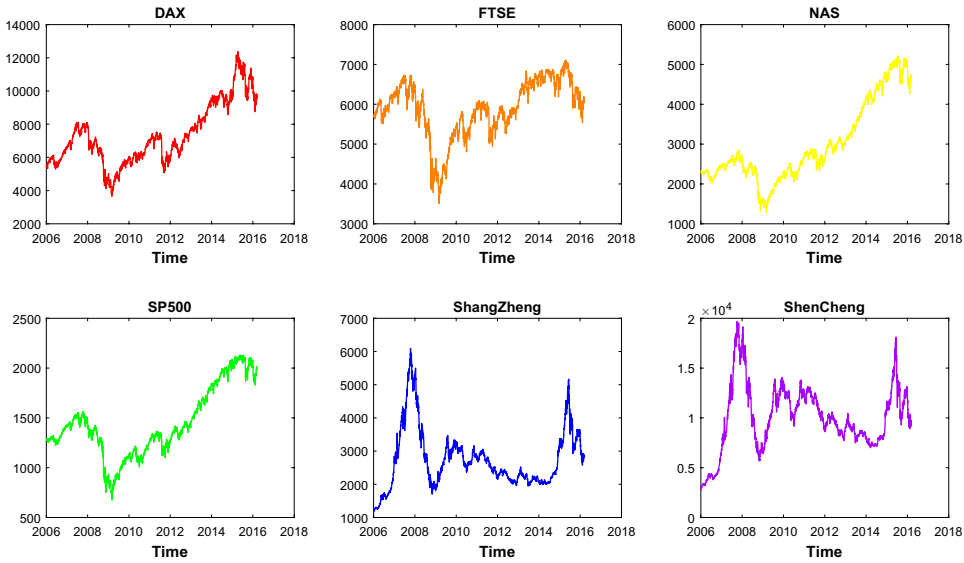


Fig. 1. Stocking closing prices of DAX, FTSE, NAS, S&P500, ShangZheng and ShenCheng.

4. Analysis and Results

In this section, we first discuss MSPTE to analyze the daily records of six stock exchange indices plotted in Fig. 1. We present the results of MSPTE analysis on the six stock indices in Fig. 3. First, the value of MSPTE decreases with the increasing scale factor τ in general. In fact, the MSPTE analysis on the logarithmic return series can be viewed as the entropy measure under different resolutions. The smaller the resolution is, the higher the degree of coarse-grained time series is. However, the degree of coarse-grained time series is determined by scale factor τ . That means that the large τ corresponds to the small resolution. From Fig. 3, we find that the entropy measure of each time series decreases with the increasing scale factor τ , indicating a decreasing complexity of each stock market over the timescale. The log return series for MSPTE analysis will offset some positive and negative data in calculation, which is why the value of MSPTE decreases with the increasing scale factor τ .

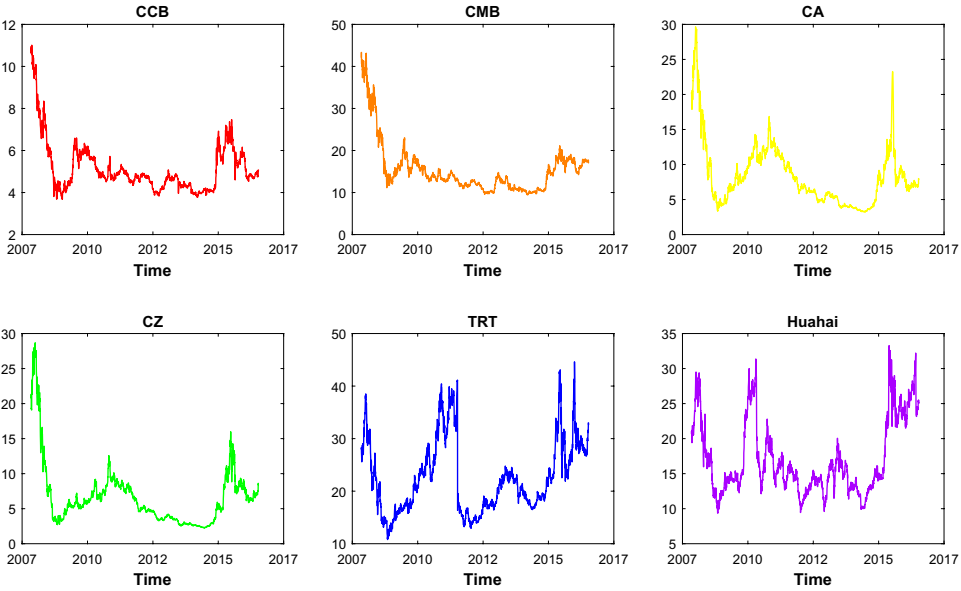


Fig. 2. Stock closing prices of CCB, CMB, CA, CZ, TRT and Huahai.

Figure 3(a) depicts results of MSPTE between DAX and all the other stock indices. The MSPTE values of the markets in same country are much closer, which is because the business behaviors of the stock markets in same country are influenced by similar rules and the mutual influences between markets. The results show that the information transfers can be divided into three groups. It is generally known that the economic factors of stock indices in the same country should be similar, but for the different stock markets, they belong to different countries, which have different economic background and political factors, so the similarity between two stock indices from two different countries should be weaker than from the same country. That is to say that in the same country, the similarity can be higher and have stronger auto-correlations. The first group belongs to ShangZheng with ShenCheng. The second group belongs to NAS with S&P500. The last group consists of FTSE. Obviously, the classification results are consistent with region division. Those behaviors due to the stock markets in same area are influenced by similar rules and the mutual influence between markets. Furthermore, the value of MSPTE from DAX to FTSE is clearly lower than the values of MSPTE from DAX to other stock indices. It indicates that information transfer between the two stocks from the same country is far less than that from different areas. Besides those, the values of SPTE from DAX to the stock indices of China are larger than America. In fact, the symbolic phase transfer entropy (SPTE), in some respects, presents the similarity between stock indices. The larger the value of MSPTE, the lower similarity among stock indices.

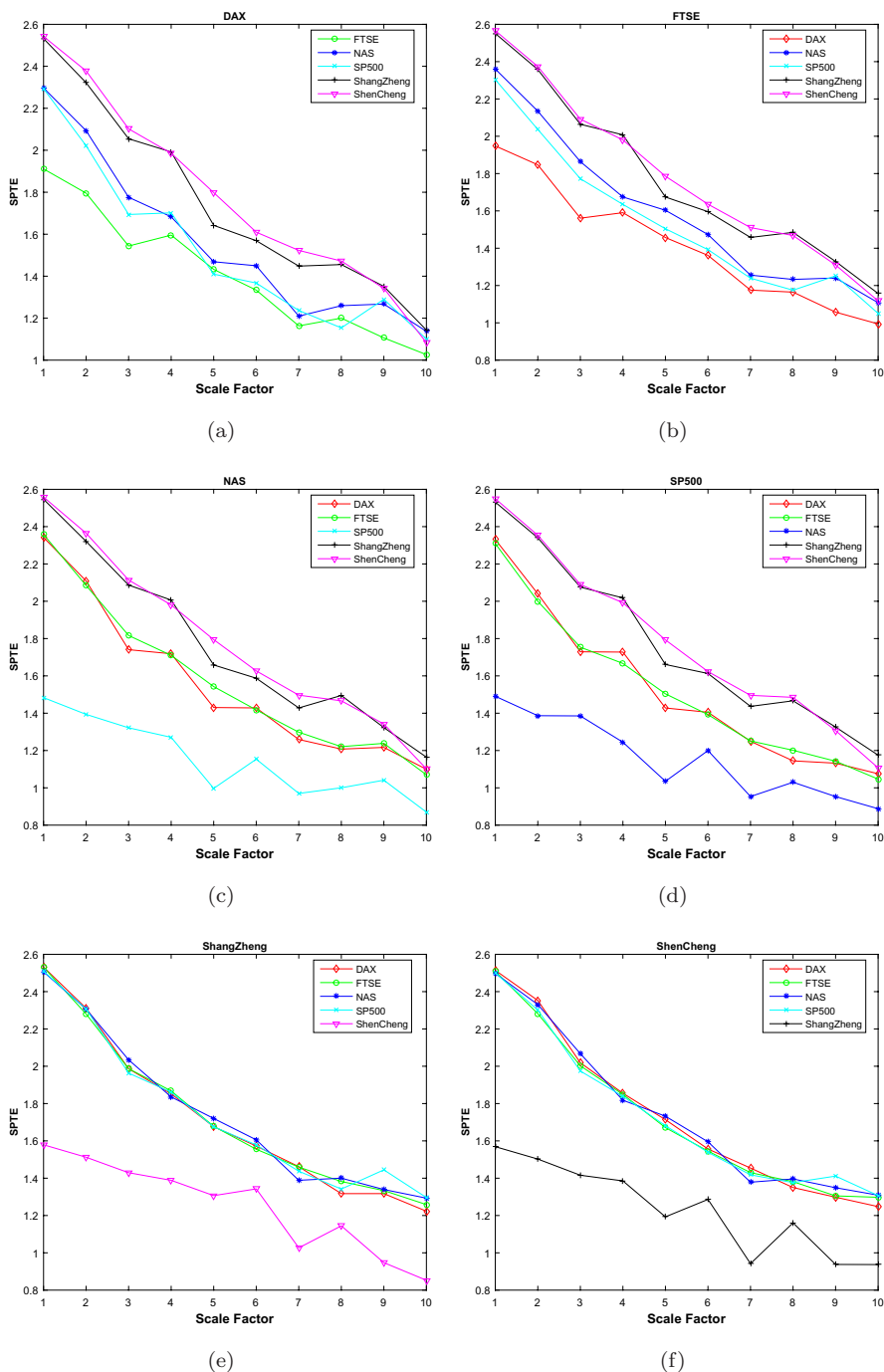


Fig. 3. The results of multiscale symbolic phase transfer entropy between (a) DAX with the others, (b) FTSE with the others, (c) NAS with the others, (d) S&P500 with the others, (e) ShangZheng with the others and (f) ShenCheng with the others, respectively.

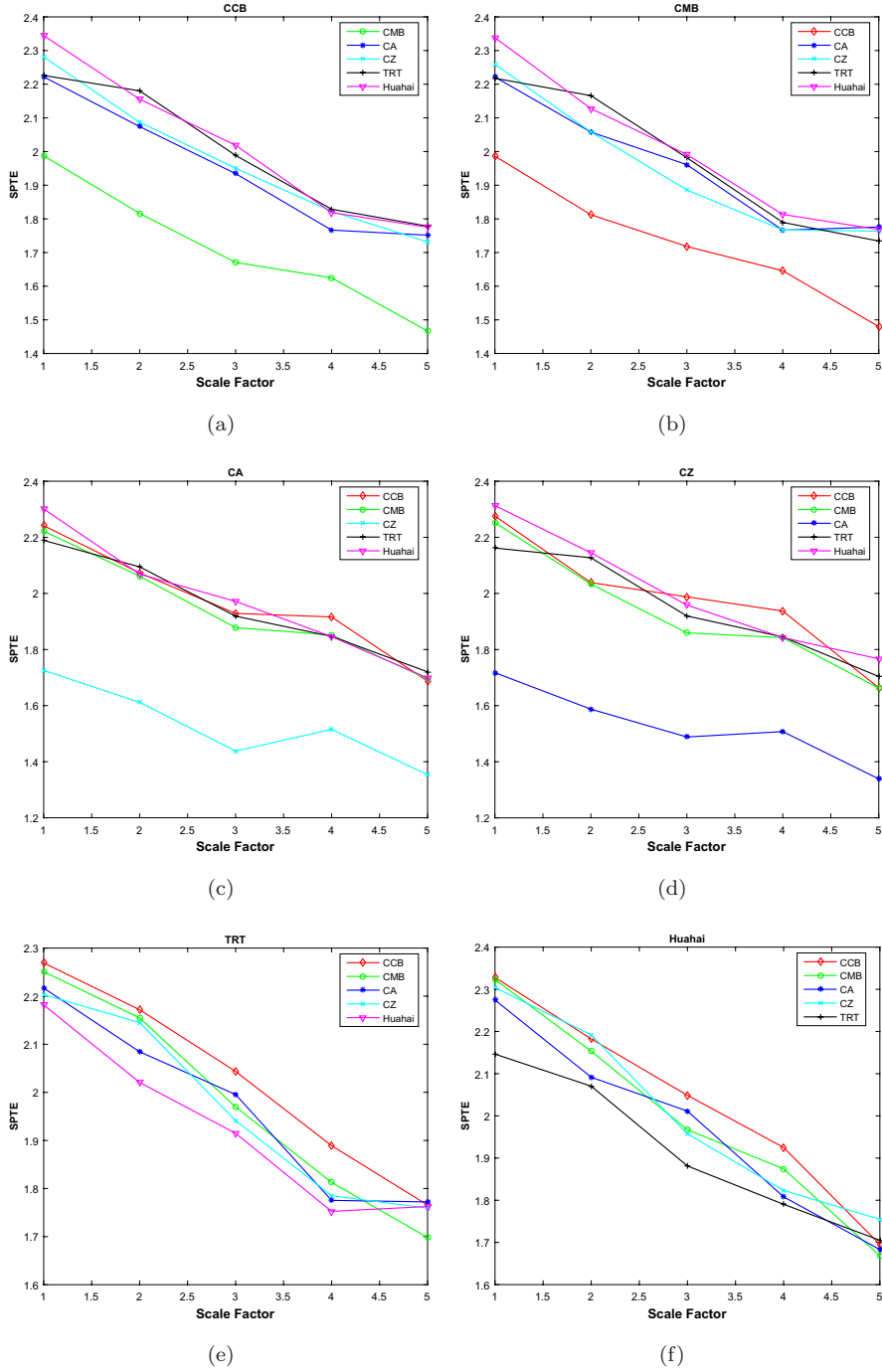


Fig. 4. The results of multiscale symbolic phase transfer entropy between (a) CCB with the others, (b) CMB with the others, (c) CA with the others, (d) CZ with the others, (e) TRT with the others and (f) Huahai with the others, respectively.

Similarly, the results of Figs. 3(a)–3(d) reveal that the MSPTE in the same region (USA or China) are very similar to each other. Moreover, it can be seen that the first four maps in Fig. 3 have strong similarities. In Fig. 3(e) and 3(f) it cannot divide the stock markets by their areas except the stock indices from China. This is because four stocks from America and Europe have relatively high degree of similarity for the stocks of China.

Then, the MSPTE method is applied to six stocks of banking, aviation industry and pharmacy to investigate information transfer among the daily price returns plotted in Fig. 2. The MSPTE results are shown in Fig. 4. Because it is difficult to distinguish the stock indices of different industries when the scale changes in a wide range, the max-scale is set to 5. Compared to the obvious differences between the two stock indices from different countries, those from different industries are little significant due to the similar economic factors from the same country. Similarly, with the increasing scale factor τ , the value of the MSPTE decreases in general. The MSPTE between the two stocks from the same industry is far less than the MSPTE between stocks from different industries. In Figs. 4(e) and 4(f), there is subtle difference in results between the MSPTE of internal pharmaceutical industry and the MSPTE from pharmaceutical industry to other industries. That suggests pharmaceutical industry has weaker particular market mechanism than banking and aviation industry.

Figures 4(a) and 4(b) depict the results of CCB and CMB with all the other indices separately. It is obvious that the MSPTE values of the markets in same industry are much closer. This result is consistent with the division of the industries. Additionally, according to Fig. 4(a), we can see that the MSPTEs: CCB versus TRT, CCB versus Huahai are closer when τ ranges from 2 to 5 than $\tau = 1$, and CCB versus CA, CCB versus CZ are more similar when τ ranges from 2 to 3 than other scales. And in Fig. 4(b), the MSPTEs for CMB versus TRT and CMB versus Huahai are more similar when τ ranges from 2 to 5 than $\tau = 1$ as well.

From Fig. 4(c), we find the values of MSPTE are close to each other. It cannot divide CCB, CMB, TRT and Huahai by their industries, which indicates CCB, CMB, TRT and Huahai have higher similarity for CA. In Fig. 4(d), it can divide the six indices into three groups: (1) CCB and CMB; (2) CA and CZ; (3) TRT and Huahai, which is also classified by their industries. Besides, in Fig. 4(d) we can see that the MSPTEs: CZ versus TRT, CZ versus Huahai are closer when τ ranges from 2 to 5 than $\tau = 1$. It is found that the MSPTEs: CZ versus CCB, CZ versus CMB are more similar when τ ranges from 1 to 2 than other scales in Figs. 4(d)–4(f). And in Fig. 4(e) we can find that the MSPTEs: TRT versus CA, TRT versus CZ are more like when τ ranges from 4 to 5 than other scales.

5. Conclusions

In this paper, we investigated the information transfer of European stock markets (DAX and FTSE), American stock markets (NAS and S&P500) and Chinese

stock markets (ShangZheng and ShenCheng) for the period from 2006 to 2016 and banking (CCB and CMB), aviation industry (CA and CZ) and pharmacy (TRT and Huahai) during the period from 2007 to 2016 by applying MSPTE methods. We find the MSPTE can well divide stock indices into groups by areas and industries. And in some way, the MSPTE reflects the similarity among stock indices. The larger the value of MSPTE, the lower similarity among stock indices. Therefore, the MSPTE between the two stocks from the same area is far less than the MSPTE between stocks from different areas, which indicates the stock markets from the same region have the particular market mechanism of every region. In addition, four stocks from America and Europe have relatively higher degree of similarity than China. And the stocks of banking and pharmaceutical industry have higher similarity with CA. It is worth noting that the pharmaceutical industry has weaker particular market mechanism than banking and aviation industry. To sum up, MSPTE is proved to be effective in quantifying the information flow among systems and distinguishing different stock markets simultaneously.

In summary, we introduce a novel classification method — MSPTE for financial time series, which can be also applied to a wide range of data sets. This classification of MSPTE may provide very useful information about the underlying dynamical processes. When we use the method, we should make full analysis of the background of time series. Using this method, we make a preliminary research now, expecting that the MSPTE will have a good performance in the analysis of classification and clustering around the subject of big data.

Acknowledgments

We acknowledge support from the National Natural Science Foundation of China (Nos. 61673005, 61304145 and 61371130).

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