

# GP\_pr90

November 28, 2024

```
[1]: # imports
import numpy as np
import xarray as xr
import pandas as pd
import matplotlib.pyplot as plt
import cartopy.crs as ccrs
from esem import gp_model
from eofs.xarray import Eof
from utils import *
import gpflow
```

```
2024-11-22 05:29:07.339814: E
external/local_xla/xla/stream_executor/cuda/cuda_fft.cc:477] Unable to register
cuFFT factory: Attempting to register factory for plugin cuFFT when one has
already been registered
WARNING: All log messages before absl::InitializeLog() is called are written to
STDERR
E0000 00:00:1732278547.351846 251285 cuda_dnn.cc:8310] Unable to register cuDNN
factory: Attempting to register factory for plugin cuDNN when one has already
been registered
E0000 00:00:1732278547.355785 251285 cuda_blas.cc:1418] Unable to register
cuBLAS factory: Attempting to register factory for plugin cuBLAS when one has
already been registered
2024-11-22 05:29:07.370371: I tensorflow/core/platform/cpu_feature_guard.cc:210]
This TensorFlow binary is optimized to use available CPU instructions in
performance-critical operations.
To enable the following instructions: AVX2 FMA, in other operations, rebuild
TensorFlow with the appropriate compiler flags.
```

```
[2]: # list of experiment data used for training

train_files= ['ssp126', 'ssp370', 'ssp585', 'historical', 'hist-GHG',
↵ 'hist-aer']
```

### 0.0.1 prepare data

```
[3]: # get data
Xtrain, eof_solvers = get_Xtrain(train_files)
Ytrain_pr90 = get_Ytrain(train_files)['pr90'].values.reshape(-1, 96*144)

Xtest = get_Xtest('ssp245', eof_solvers)
Ytest = xr.open_dataset('../test/outputs_ssp245.nc').compute()
pr90_truth = 86400 * Ytest['pr90'].mean('member') # convert pr to mm/day,
↳ default unit is kg/m²/s
```

```
[4]: # drop rows including nans
train_nan_mask = Xtrain.isna().any(axis=1).values
Xtrain = Xtrain.dropna(axis=0, how='any')
Ytrain_pr90 = Ytrain_pr90[~train_nan_mask]
assert Xtrain.shape[0]==Ytrain_pr90.shape[0]

test_nan_mask = Xtest.isna().any(axis=1).values
Xtest = Xtest.dropna(axis=0, how='any')
pr90_truth = pr90_truth[~test_nan_mask]
```

```
[5]: # Standardize predictor fields requiring standardization (non-EOFs)
train_CO2_mean, train_CO2_std = Xtrain['CO2'].mean(), Xtrain['CO2'].std()
train_CH4_mean, train_CH4_std = Xtrain['CH4'].mean(), Xtrain['CH4'].std()

Xtrain['CO2'] = (Xtrain['CO2'] - train_CO2_mean) / train_CO2_std
Xtrain['CH4'] = (Xtrain['CH4'] - train_CH4_mean) / train_CH4_std

Xtest['CO2'] = (Xtest['CO2'] - train_CO2_mean) / train_CO2_std
Xtest['CH4'] = (Xtest['CH4'] - train_CH4_mean) / train_CH4_std
```

```
[6]: # Standardize predictand fields
train_pr90_mean, train_pr90_std = Ytrain_pr90.mean(), Ytrain_pr90.std()
Ytrain_pr90 = (Ytrain_pr90 - train_pr90_mean) / train_pr90_std
```

### 0.0.2 Model

```
[7]: kernel_CO2 = gpflow.kernels.Matern32(active_dims=[0]) # active_dims specifies
↳ which dimension the kernel is applied to
kernel_CH4 = gpflow.kernels.Matern32(active_dims=[1])

kernel_BC = gpflow.kernels.Matern32(lengthscales=5 * [1.], active_dims=[2, 3,
↳ 4, 5, 6])
kernel_S02 = gpflow.kernels.Matern32(lengthscales=5 * [1.], active_dims=[7, 8,
↳ 9, 10, 11])

kernel = kernel_CO2 + kernel_CH4 + kernel_BC + kernel_S02
```

```
I0000 00:00:1732278556.287521 251285 gpu_device.cc:2022] Created device
/job:localhost/replica:0/task:0/device:GPU:0 with 79379 MB memory: -> device:
0, name: NVIDIA A100-SXM4-80GB, pci bus id: 0000:01:00.0, compute capability:
8.0
```

```
[8]: np.random.seed(5)

"""
In Gaussian Processes, a mean function represents the "prior mean" or the
↳ expected value
of the function at any input point before observing any data.
"""

mean = gpflow.mean_functions.Constant()

model = gpflow.models.GPR(data=(Xtrain.astype(np.float64), # cast to float64
↳ because gpflow requires numerical stability
                                Ytrain_pr90.astype(np.float64)),
                           kernel = kernel,
                           mean_function = mean)
```

```
[9]: # define optimizer
optimizer = gpflow.optimizers.Scipy()

# train
optimizer.minimize(model.training_loss,
                   variables=model.trainable_variables,
                   options=dict(dis=True, maxiter=1000))
```

```
WARNING: All log messages before absl::InitializeLog() is called are written to
STDERR
```

```
I0000 00:00:1732278560.829441 251345 cuda_solvers.cc:178] Creating GpuSolver
handles for stream 0x5649e4255d30
```

```
This problem is unconstrained.
```

```
RUNNING THE L-BFGS-B CODE
```

```
* * *
```

```
Machine precision = 2.220D-16
```

```
N =          18      M =          10
```

```
At X0          0 variables are exactly at the bounds
```

```
At iterate    0      f=  1.46798D+07      |proj g|=  6.21221D+05
```

```
At iterate    1      f=  1.45440D+07      |proj g|=  1.01968D+05
```

```
At iterate    2      f=  1.45285D+07      |proj g|=  9.99489D+04
```

At iterate	3	f=	1.44452D+07	proj g =	3.58827D+05
At iterate	4	f=	1.42922D+07	proj g =	6.25518D+05
At iterate	5	f=	1.41021D+07	proj g =	4.52557D+05
At iterate	6	f=	1.40326D+07	proj g =	1.06766D+05
At iterate	7	f=	1.40282D+07	proj g =	1.22619D+04
At iterate	8	f=	1.40276D+07	proj g =	1.43959D+04
At iterate	9	f=	1.40264D+07	proj g =	4.71371D+04
At iterate	10	f=	1.40240D+07	proj g =	9.25431D+04
At iterate	11	f=	1.40186D+07	proj g =	1.48191D+05
At iterate	12	f=	1.40100D+07	proj g =	1.84034D+05
At iterate	13	f=	1.40008D+07	proj g =	1.37278D+05
At iterate	14	f=	1.39966D+07	proj g =	4.85063D+04
At iterate	15	f=	1.39958D+07	proj g =	8.26892D+03
At iterate	16	f=	1.39957D+07	proj g =	5.80296D+03
At iterate	17	f=	1.39953D+07	proj g =	4.77829D+03
At iterate	18	f=	1.39924D+07	proj g =	1.87259D+04
At iterate	19	f=	1.39909D+07	proj g =	1.52986D+04
At iterate	20	f=	1.39888D+07	proj g =	1.51650D+04
At iterate	21	f=	1.39874D+07	proj g =	1.25454D+04
At iterate	22	f=	1.39864D+07	proj g =	7.01579D+03
At iterate	23	f=	1.39861D+07	proj g =	2.28178D+03
At iterate	24	f=	1.39860D+07	proj g =	3.08149D+03
At iterate	25	f=	1.39856D+07	proj g =	9.74069D+03
At iterate	26	f=	1.39849D+07	proj g =	4.02553D+03

At iterate	27	f=	1.39843D+07	proj g =	1.28718D+04
At iterate	28	f=	1.39835D+07	proj g =	5.55241D+03
At iterate	29	f=	1.39826D+07	proj g =	1.22881D+04
At iterate	30	f=	1.39822D+07	proj g =	8.24564D+03
At iterate	31	f=	1.39821D+07	proj g =	1.36367D+03
At iterate	32	f=	1.39821D+07	proj g =	1.36846D+03
At iterate	33	f=	1.39821D+07	proj g =	9.38599D+02
At iterate	34	f=	1.39819D+07	proj g =	1.66793D+04
At iterate	35	f=	1.39817D+07	proj g =	1.15253D+04
At iterate	36	f=	1.39805D+07	proj g =	1.13503D+04
At iterate	37	f=	1.39798D+07	proj g =	2.00358D+04
At iterate	38	f=	1.39789D+07	proj g =	6.29111D+03
At iterate	39	f=	1.39783D+07	proj g =	9.61583D+03
At iterate	40	f=	1.39781D+07	proj g =	3.83890D+03
At iterate	41	f=	1.39780D+07	proj g =	1.09278D+03
At iterate	42	f=	1.39779D+07	proj g =	3.11077D+03
At iterate	43	f=	1.39778D+07	proj g =	9.51519D+02
At iterate	44	f=	1.39777D+07	proj g =	9.80848D+02
At iterate	45	f=	1.39777D+07	proj g =	6.27246D+03
At iterate	46	f=	1.39776D+07	proj g =	4.49350D+03
At iterate	47	f=	1.39776D+07	proj g =	1.48108D+03
At iterate	48	f=	1.39775D+07	proj g =	2.09269D+03
At iterate	49	f=	1.39773D+07	proj g =	1.21856D+03
At iterate	50	f=	1.39771D+07	proj g =	8.25550D+02

At iterate	51	f=	1.39771D+07	proj g =	8.74418D+03
At iterate	52	f=	1.39769D+07	proj g =	6.25965D+03
At iterate	53	f=	1.39767D+07	proj g =	1.57274D+03
At iterate	54	f=	1.39764D+07	proj g =	2.34693D+03
At iterate	55	f=	1.39762D+07	proj g =	3.52594D+03
At iterate	56	f=	1.39762D+07	proj g =	2.32412D+03
At iterate	57	f=	1.39760D+07	proj g =	1.86973D+03
At iterate	58	f=	1.39760D+07	proj g =	9.61779D+02
At iterate	59	f=	1.39759D+07	proj g =	1.43788D+03
At iterate	60	f=	1.39758D+07	proj g =	9.12383D+02
At iterate	61	f=	1.39757D+07	proj g =	8.22320D+02
At iterate	62	f=	1.39757D+07	proj g =	5.93475D+03
At iterate	63	f=	1.39756D+07	proj g =	5.07342D+02
At iterate	64	f=	1.39756D+07	proj g =	1.02594D+03
At iterate	65	f=	1.39756D+07	proj g =	1.68745D+03
At iterate	66	f=	1.39756D+07	proj g =	1.99615D+03
At iterate	67	f=	1.39756D+07	proj g =	4.29550D+03
At iterate	68	f=	1.39755D+07	proj g =	3.12065D+03
At iterate	69	f=	1.39755D+07	proj g =	9.87579D+02
At iterate	70	f=	1.39755D+07	proj g =	9.04254D+02
At iterate	71	f=	1.39755D+07	proj g =	1.21805D+03
At iterate	72	f=	1.39754D+07	proj g =	5.55287D+02
At iterate	73	f=	1.39754D+07	proj g =	2.34076D+03
At iterate	74	f=	1.39754D+07	proj g =	4.65380D+02

At iterate	75	f=	1.39754D+07	proj g =	5.32640D+02
At iterate	76	f=	1.39754D+07	proj g =	2.92541D+03
At iterate	77	f=	1.39754D+07	proj g =	4.58982D+03
At iterate	78	f=	1.39754D+07	proj g =	1.64129D+03
At iterate	79	f=	1.39754D+07	proj g =	4.04985D+03
At iterate	80	f=	1.39754D+07	proj g =	8.74768D+02
At iterate	81	f=	1.39754D+07	proj g =	3.00055D+03
At iterate	82	f=	1.39753D+07	proj g =	3.93557D+03
At iterate	83	f=	1.39753D+07	proj g =	4.29627D+03
At iterate	84	f=	1.39753D+07	proj g =	6.03157D+03
At iterate	85	f=	1.39753D+07	proj g =	8.43935D+02
At iterate	86	f=	1.39753D+07	proj g =	1.68740D+03
At iterate	87	f=	1.39753D+07	proj g =	3.64708D+03
At iterate	88	f=	1.39752D+07	proj g =	5.85611D+03
At iterate	89	f=	1.39752D+07	proj g =	7.43095D+03
At iterate	90	f=	1.39751D+07	proj g =	1.09647D+04
At iterate	91	f=	1.39750D+07	proj g =	4.49521D+03
At iterate	92	f=	1.39749D+07	proj g =	7.60894D+03
At iterate	93	f=	1.39749D+07	proj g =	1.36631D+03
At iterate	94	f=	1.39748D+07	proj g =	1.23062D+03
At iterate	95	f=	1.39748D+07	proj g =	1.52596D+03
At iterate	96	f=	1.39748D+07	proj g =	9.27421D+02
At iterate	97	f=	1.39748D+07	proj g =	4.71367D+02
At iterate	98	f=	1.39748D+07	proj g =	8.18500D+02

At iterate	99	f=	1.39748D+07	proj g =	1.29868D+03
At iterate	100	f=	1.39748D+07	proj g =	1.35876D+02
At iterate	101	f=	1.39748D+07	proj g =	1.11274D+02
At iterate	102	f=	1.39748D+07	proj g =	8.31806D+01
At iterate	103	f=	1.39748D+07	proj g =	8.30047D+01
At iterate	104	f=	1.39748D+07	proj g =	5.11195D+02
At iterate	105	f=	1.39748D+07	proj g =	3.11663D+02
At iterate	106	f=	1.39748D+07	proj g =	2.34212D+02
At iterate	107	f=	1.39748D+07	proj g =	4.19002D+02
At iterate	108	f=	1.39748D+07	proj g =	1.26851D+03
At iterate	109	f=	1.39748D+07	proj g =	1.30270D+03
At iterate	110	f=	1.39748D+07	proj g =	2.42017D+03
At iterate	111	f=	1.39748D+07	proj g =	1.47430D+03
At iterate	112	f=	1.39748D+07	proj g =	2.87657D+02
At iterate	113	f=	1.39748D+07	proj g =	2.09573D+02
At iterate	114	f=	1.39748D+07	proj g =	3.47348D+02
At iterate	115	f=	1.39748D+07	proj g =	2.03382D+02
At iterate	116	f=	1.39748D+07	proj g =	1.18666D+03
At iterate	117	f=	1.39748D+07	proj g =	1.44230D+02
At iterate	118	f=	1.39748D+07	proj g =	8.96923D+02
At iterate	119	f=	1.39748D+07	proj g =	1.94717D+03
At iterate	120	f=	1.39748D+07	proj g =	2.70422D+03
At iterate	121	f=	1.39748D+07	proj g =	4.43730D+02
At iterate	122	f=	1.39748D+07	proj g =	2.09764D+03



At iterate	123	f=	1.39748D+07	proj g =	5.12952D+02
At iterate	124	f=	1.39747D+07	proj g =	3.35042D+03
At iterate	125	f=	1.39747D+07	proj g =	6.08683D+03
At iterate	126	f=	1.39747D+07	proj g =	8.63608D+03
At iterate	127	f=	1.39747D+07	proj g =	7.50762D+03
At iterate	128	f=	1.39746D+07	proj g =	1.20281D+04
At iterate	129	f=	1.39746D+07	proj g =	5.11284D+03
At iterate	130	f=	1.39746D+07	proj g =	3.95162D+03
At iterate	131	f=	1.39746D+07	proj g =	4.83074D+03
At iterate	132	f=	1.39745D+07	proj g =	6.53167D+03
At iterate	133	f=	1.39745D+07	proj g =	9.08618D+03
At iterate	134	f=	1.39744D+07	proj g =	5.64972D+03
At iterate	135	f=	1.39744D+07	proj g =	8.91274D+02
At iterate	136	f=	1.39744D+07	proj g =	5.92423D+02
At iterate	137	f=	1.39744D+07	proj g =	5.47119D+02
At iterate	138	f=	1.39744D+07	proj g =	1.40906D+03
At iterate	139	f=	1.39744D+07	proj g =	1.33229D+03
At iterate	140	f=	1.39744D+07	proj g =	5.06371D+03
At iterate	141	f=	1.39744D+07	proj g =	2.54319D+03
At iterate	142	f=	1.39743D+07	proj g =	8.15274D+02
At iterate	143	f=	1.39743D+07	proj g =	4.94438D+02
At iterate	144	f=	1.39743D+07	proj g =	1.65354D+03
At iterate	145	f=	1.39743D+07	proj g =	2.53727D+03
At iterate	146	f=	1.39742D+07	proj g =	3.40668D+03

At iterate	147	f=	1.39742D+07	proj g =	1.53758D+03
At iterate	148	f=	1.39742D+07	proj g =	6.01826D+02
At iterate	149	f=	1.39742D+07	proj g =	1.20241D+03
At iterate	150	f=	1.39742D+07	proj g =	2.30846D+03
At iterate	151	f=	1.39742D+07	proj g =	2.25952D+03
At iterate	152	f=	1.39742D+07	proj g =	3.88104D+03
At iterate	153	f=	1.39742D+07	proj g =	2.72270D+03
At iterate	154	f=	1.39742D+07	proj g =	5.72316D+02
At iterate	155	f=	1.39742D+07	proj g =	2.90330D+02
At iterate	156	f=	1.39741D+07	proj g =	4.43363D+02
At iterate	157	f=	1.39741D+07	proj g =	1.41573D+03
At iterate	158	f=	1.39741D+07	proj g =	3.90497D+02
At iterate	159	f=	1.39741D+07	proj g =	2.54630D+02
At iterate	160	f=	1.39741D+07	proj g =	2.70651D+02
At iterate	161	f=	1.39741D+07	proj g =	3.25158D+02
At iterate	162	f=	1.39741D+07	proj g =	2.76340D+02
At iterate	163	f=	1.39741D+07	proj g =	7.29032D+02
At iterate	164	f=	1.39741D+07	proj g =	5.31836D+02
At iterate	165	f=	1.39741D+07	proj g =	6.81144D+02
At iterate	166	f=	1.39741D+07	proj g =	4.69521D+02
At iterate	167	f=	1.39740D+07	proj g =	8.86433D+02
At iterate	168	f=	1.39740D+07	proj g =	1.16345D+03
At iterate	169	f=	1.39740D+07	proj g =	4.49874D+03
At iterate	170	f=	1.39739D+07	proj g =	1.86085D+03

At iterate	171	f=	1.39739D+07	proj g =	2.69304D+02
At iterate	172	f=	1.39739D+07	proj g =	2.45918D+02
At iterate	173	f=	1.39739D+07	proj g =	1.44613D+02
At iterate	174	f=	1.39739D+07	proj g =	1.35208D+03
At iterate	175	f=	1.39739D+07	proj g =	5.75314D+02
At iterate	176	f=	1.39739D+07	proj g =	1.57958D+02
At iterate	177	f=	1.39739D+07	proj g =	1.84474D+02
At iterate	178	f=	1.39739D+07	proj g =	2.29464D+03
At iterate	179	f=	1.39739D+07	proj g =	1.01490D+03
At iterate	180	f=	1.39739D+07	proj g =	4.00078D+02
At iterate	181	f=	1.39739D+07	proj g =	6.08759D+02
At iterate	182	f=	1.39738D+07	proj g =	3.87800D+02
At iterate	183	f=	1.39738D+07	proj g =	1.49623D+03
At iterate	184	f=	1.39738D+07	proj g =	2.44856D+02
At iterate	185	f=	1.39738D+07	proj g =	3.81254D+02
At iterate	186	f=	1.39738D+07	proj g =	5.06946D+02
At iterate	187	f=	1.39738D+07	proj g =	1.43592D+03
At iterate	188	f=	1.39738D+07	proj g =	5.91052D+03
At iterate	189	f=	1.39738D+07	proj g =	2.48049D+03
At iterate	190	f=	1.39738D+07	proj g =	8.99968D+02
At iterate	191	f=	1.39738D+07	proj g =	2.20572D+02
At iterate	192	f=	1.39738D+07	proj g =	1.23233D+03
At iterate	193	f=	1.39738D+07	proj g =	1.53251D+03
At iterate	194	f=	1.39738D+07	proj g =	3.13069D+02

At iterate	195	f=	1.39738D+07	proj g =	3.37263D+02
At iterate	196	f=	1.39738D+07	proj g =	2.33286D+02
At iterate	197	f=	1.39737D+07	proj g =	3.49914D+02
At iterate	198	f=	1.39737D+07	proj g =	1.02836D+03
At iterate	199	f=	1.39737D+07	proj g =	5.06325D+03
At iterate	200	f=	1.39737D+07	proj g =	4.72207D+02
At iterate	201	f=	1.39737D+07	proj g =	7.36377D+02
At iterate	202	f=	1.39737D+07	proj g =	1.80872D+03
At iterate	203	f=	1.39737D+07	proj g =	5.21362D+02
At iterate	204	f=	1.39737D+07	proj g =	1.85462D+02
At iterate	205	f=	1.39737D+07	proj g =	2.16391D+02
At iterate	206	f=	1.39736D+07	proj g =	5.95842D+02
At iterate	207	f=	1.39736D+07	proj g =	8.52818D+02
At iterate	208	f=	1.39736D+07	proj g =	9.64290D+02
At iterate	209	f=	1.39736D+07	proj g =	7.46297D+02
At iterate	210	f=	1.39736D+07	proj g =	2.79579D+02
At iterate	211	f=	1.39736D+07	proj g =	2.72208D+02
At iterate	212	f=	1.39736D+07	proj g =	7.31132D+02
At iterate	213	f=	1.39736D+07	proj g =	9.87690D+01
At iterate	214	f=	1.39736D+07	proj g =	9.20078D+01
At iterate	215	f=	1.39736D+07	proj g =	5.16510D+01
At iterate	216	f=	1.39736D+07	proj g =	2.60677D+02
At iterate	217	f=	1.39736D+07	proj g =	5.27427D+01
At iterate	218	f=	1.39736D+07	proj g =	1.35506D+02

At iterate	219	f=	1.39736D+07	proj g =	2.17493D+02
At iterate	220	f=	1.39736D+07	proj g =	2.55139D+02
At iterate	221	f=	1.39736D+07	proj g =	3.36197D+02
At iterate	222	f=	1.39736D+07	proj g =	3.10502D+02
At iterate	223	f=	1.39736D+07	proj g =	1.03567D+02
At iterate	224	f=	1.39736D+07	proj g =	1.99294D+03
At iterate	225	f=	1.39736D+07	proj g =	6.48322D+02
At iterate	226	f=	1.39736D+07	proj g =	1.99149D+02
At iterate	227	f=	1.39736D+07	proj g =	2.30749D+02
At iterate	228	f=	1.39736D+07	proj g =	2.89736D+02
At iterate	229	f=	1.39736D+07	proj g =	3.78458D+02
At iterate	230	f=	1.39736D+07	proj g =	5.90151D+01
At iterate	231	f=	1.39736D+07	proj g =	2.14991D+03
At iterate	232	f=	1.39736D+07	proj g =	2.92686D+03
At iterate	233	f=	1.39736D+07	proj g =	2.46431D+03
At iterate	234	f=	1.39736D+07	proj g =	2.85196D+03
At iterate	235	f=	1.39736D+07	proj g =	1.11177D+03
At iterate	236	f=	1.39736D+07	proj g =	5.95109D+02
At iterate	237	f=	1.39736D+07	proj g =	1.74997D+02
At iterate	238	f=	1.39736D+07	proj g =	1.86624D+02
At iterate	239	f=	1.39736D+07	proj g =	5.89086D+02
At iterate	240	f=	1.39736D+07	proj g =	9.61322D+02
At iterate	241	f=	1.39736D+07	proj g =	1.27790D+03
At iterate	242	f=	1.39736D+07	proj g =	1.03793D+03

At iterate	243	f=	1.39736D+07	proj g =	1.46186D+03
At iterate	244	f=	1.39736D+07	proj g =	4.36586D+02
At iterate	245	f=	1.39736D+07	proj g =	2.04990D+02
At iterate	246	f=	1.39736D+07	proj g =	5.10972D+02
At iterate	247	f=	1.39736D+07	proj g =	6.07870D+02
At iterate	248	f=	1.39736D+07	proj g =	1.69263D+03
At iterate	249	f=	1.39736D+07	proj g =	1.44278D+03
At iterate	250	f=	1.39736D+07	proj g =	6.16132D+02
At iterate	251	f=	1.39736D+07	proj g =	3.90941D+02
At iterate	252	f=	1.39736D+07	proj g =	8.98011D+02
At iterate	253	f=	1.39736D+07	proj g =	8.99928D+02
At iterate	254	f=	1.39736D+07	proj g =	1.68616D+02
At iterate	255	f=	1.39736D+07	proj g =	1.32250D+02
At iterate	256	f=	1.39736D+07	proj g =	4.67034D+02
At iterate	257	f=	1.39736D+07	proj g =	2.11713D+02
At iterate	258	f=	1.39736D+07	proj g =	1.37127D+02
At iterate	259	f=	1.39736D+07	proj g =	2.45728D+03
At iterate	260	f=	1.39736D+07	proj g =	8.73304D+02
At iterate	261	f=	1.39736D+07	proj g =	2.78425D+02
At iterate	262	f=	1.39736D+07	proj g =	2.11079D+02
At iterate	263	f=	1.39736D+07	proj g =	9.36830D+02
At iterate	264	f=	1.39736D+07	proj g =	4.74201D+02
At iterate	265	f=	1.39736D+07	proj g =	2.69689D+02
At iterate	266	f=	1.39735D+07	proj g =	1.60361D+03

At iterate	267	f=	1.39735D+07	proj g =	1.54456D+03
At iterate	268	f=	1.39735D+07	proj g =	2.59843D+03
At iterate	269	f=	1.39735D+07	proj g =	1.38033D+03
At iterate	270	f=	1.39735D+07	proj g =	2.38934D+02
At iterate	271	f=	1.39735D+07	proj g =	4.00253D+02
At iterate	272	f=	1.39735D+07	proj g =	1.60827D+03
At iterate	273	f=	1.39735D+07	proj g =	1.15280D+03
At iterate	274	f=	1.39735D+07	proj g =	4.61883D+02
At iterate	275	f=	1.39735D+07	proj g =	3.58647D+02
At iterate	276	f=	1.39735D+07	proj g =	7.43771D+02
At iterate	277	f=	1.39735D+07	proj g =	9.45373D+02
At iterate	278	f=	1.39735D+07	proj g =	6.67161D+02
At iterate	279	f=	1.39735D+07	proj g =	1.82382D+02
At iterate	280	f=	1.39735D+07	proj g =	6.22770D+02
At iterate	281	f=	1.39735D+07	proj g =	1.06559D+03
At iterate	282	f=	1.39735D+07	proj g =	1.30463D+03
At iterate	283	f=	1.39735D+07	proj g =	1.53416D+02
At iterate	284	f=	1.39735D+07	proj g =	3.52265D+02
At iterate	285	f=	1.39735D+07	proj g =	1.34088D+03
At iterate	286	f=	1.39735D+07	proj g =	7.63291D+02
At iterate	287	f=	1.39735D+07	proj g =	2.53773D+02
At iterate	288	f=	1.39735D+07	proj g =	3.06450D+02
At iterate	289	f=	1.39735D+07	proj g =	4.87798D+02
At iterate	290	f=	1.39735D+07	proj g =	2.25505D+02

At iterate	291	f=	1.39735D+07	proj g =	1.61808D+03
At iterate	292	f=	1.39735D+07	proj g =	7.02697D+02
At iterate	293	f=	1.39735D+07	proj g =	1.59566D+02
At iterate	294	f=	1.39735D+07	proj g =	1.69347D+02
At iterate	295	f=	1.39735D+07	proj g =	1.64140D+02
At iterate	296	f=	1.39735D+07	proj g =	4.95313D+01
At iterate	297	f=	1.39735D+07	proj g =	7.03934D+01
At iterate	298	f=	1.39735D+07	proj g =	1.99829D+02
At iterate	299	f=	1.39735D+07	proj g =	2.77472D+02
At iterate	300	f=	1.39735D+07	proj g =	2.43284D+02
At iterate	301	f=	1.39735D+07	proj g =	9.15967D+01
At iterate	302	f=	1.39735D+07	proj g =	6.56687D+02
At iterate	303	f=	1.39735D+07	proj g =	1.07372D+02
At iterate	304	f=	1.39735D+07	proj g =	1.75404D+02
At iterate	305	f=	1.39735D+07	proj g =	2.10598D+02
At iterate	306	f=	1.39735D+07	proj g =	5.37473D+02
At iterate	307	f=	1.39735D+07	proj g =	2.69938D+02
At iterate	308	f=	1.39735D+07	proj g =	8.01516D+01
At iterate	309	f=	1.39735D+07	proj g =	3.13258D+02
At iterate	310	f=	1.39735D+07	proj g =	3.64530D+02
At iterate	311	f=	1.39735D+07	proj g =	2.36362D+02
At iterate	312	f=	1.39735D+07	proj g =	1.98290D+02
At iterate	313	f=	1.39735D+07	proj g =	2.22383D+02
At iterate	314	f=	1.39735D+07	proj g =	4.73294D+02



```

At iterate 315    f=  1.39735D+07    |proj g|=  4.13256D+02
At iterate 316    f=  1.39735D+07    |proj g|=  2.91543D+02
At iterate 317    f=  1.39735D+07    |proj g|=  1.02642D+02
At iterate 318    f=  1.39735D+07    |proj g|=  4.60511D+02
At iterate 319    f=  1.39735D+07    |proj g|=  2.05780D+02
At iterate 320    f=  1.39735D+07    |proj g|=  8.58002D+01
At iterate 321    f=  1.39735D+07    |proj g|=  1.51972D+02
At iterate 322    f=  1.39735D+07    |proj g|=  3.07175D+02
At iterate 323    f=  1.39735D+07    |proj g|=  1.40598D+02
At iterate 324    f=  1.39735D+07    |proj g|=  3.55955D+02
At iterate 325    f=  1.39735D+07    |proj g|=  1.44093D+02
At iterate 326    f=  1.39735D+07    |proj g|=  1.28648D+02
At iterate 327    f=  1.39735D+07    |proj g|=  2.09789D+02
At iterate 328    f=  1.39735D+07    |proj g|=  1.67837D+02
At iterate 329    f=  1.39735D+07    |proj g|=  4.10544D+02

```

\* \* \*

```

Tit   = total number of iterations
Tnf   = total number of function evaluations
Tnint = total number of segments explored during Cauchy searches
Skip  = number of BFGS updates skipped
Nact  = number of active bounds at final generalized Cauchy point
Projg = norm of the final projected gradient
F     = final function value

```

\* \* \*

N	Tit	Tnf	Tnint	Skip	Nact	Projg	F
18	329	391	1	0	0	4.105D+02	1.397D+07

F = 13973466.473627981

CONVERGENCE: REL\_REDUCTION\_OF\_F\_<=\_FACTR\*EPSMCH

```
[9]: message: CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
      success: True
      status: 0
      fun: 13973466.47362798
      x: [ 4.516e+00 -1.593e-01 ...  2.646e-01  2.487e-02]
      nit: 329
      jac: [-1.162e+01  4.614e+01 ... -4.105e+02  4.527e+01]
      nfev: 391
      njev: 391
      hess_inv: <18x18 LbfgsInvHessProduct with dtype=float64>
```

### 0.0.3 make prediction

```
[10]: # predict
      standard_posterior_mean, standard_posterior_var = model.predict_y(Xtest.values)
      ↪# predicted mean of GP, predicted variance of GP
      posterior_mean = standard_posterior_mean * train_pr90_std + train_pr90_mean #
      ↪transform mean prediction to original scale
      posterior_stddev = np.sqrt(standard_posterior_var) * train_pr90_std # transform
      ↪variance prediction to original scale standard deviation
```

```
[11]: # put output back into xarray format for calculating RMSE/plotting
      posterior_pr90 = np.reshape(posterior_mean, [86, 96, 144])
      posterior_pr90_stddev = np.reshape(posterior_stddev, [86, 96, 144])

      posterior_pr90_data = xr.DataArray(posterior_pr90, dims=pr90_truth.dims,
      ↪coords=pr90_truth.coords)
      posterior_pr90_std_data = xr.DataArray(posterior_pr90_stddev, dims=pr90_truth.
      ↪dims, coords=pr90_truth.coords)
```

```
[12]: # Compute RMSEs
      print(f"RMSE at 2050: {get_rmse(pr90_truth[35], posterior_pr90_data[35])}")
      print(f"RMSE at 2100: {get_rmse(pr90_truth[85], posterior_pr90_data[85])}")
      print(f"RMSE 2045-2055: {get_rmse(pr90_truth[30:41], posterior_pr90_data[30:
      ↪41]).mean()}")
      print(f"RMSE 2090-2100: {get_rmse(pr90_truth[75:], posterior_pr90_data[75:]).
      ↪mean()}")
      print(f"RMSE 2050-2100: {get_rmse(pr90_truth[35:], posterior_pr90_data[35:]).
      ↪mean()}")

      # RMSE for average field over last 20 years
      print(f"RMSE average last 20y: {get_rmse(pr90_truth[-20:].mean(dim='time'),
      ↪posterior_pr90_data[-20:].mean(dim='time'))}")
```

```
RMSE at 2050: 1.2130858727783458
RMSE at 2100: 1.6382599959380366
RMSE 2045-2055: 1.4754150986257621
```

RMSE 2090-2100: 1.5208997748470332  
 RMSE 2050-2100: 1.5492741600620086  
 RMSE average last 20y: 0.48368783868907234

```
[13]: from matplotlib import colors
# plotting predictions
divnorm = colors.TwoSlopeNorm(vmin=-2., vcenter=0., vmax=5)
diffnorm = colors.TwoSlopeNorm(vmin=-2., vcenter=0., vmax=2)

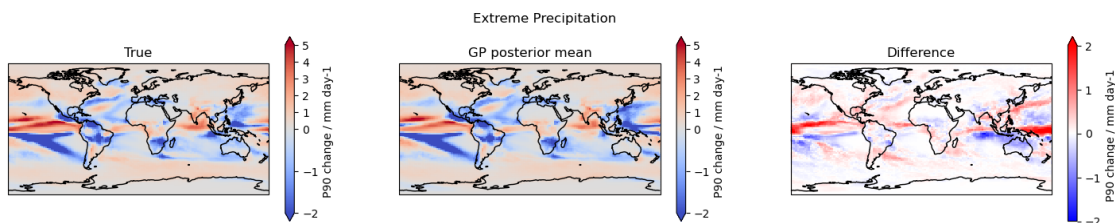
## Temperature
proj = ccrs.PlateCarree()
fig = plt.figure(figsize=(18, 3))
fig.suptitle('Extreme Precipitation')

# Test
plt.subplot(131, projection=proj)
pr90_truth.sel(time=slice(2050, None)).mean('time').plot(cmap="coolwarm",
    ↪ norm=divnorm,
                                cbar_kwargs={"label": "P90 change / mm day-1"})
plt.gca().coastlines()
plt.setp(plt.gca(), title='True')

# Emulator
plt.subplot(132, projection=proj)
posterior_pr90_data.sel(time=slice(2050, None)).mean('time').
    ↪ plot(cmap="coolwarm", norm=divnorm,
                                cbar_kwargs={"label": "P90 change / mm day-1"})
plt.gca().coastlines()
plt.setp(plt.gca(), title='GP posterior mean')

# Difference
difference = pr90_truth - posterior_pr90_data
plt.subplot(133, projection=proj)
difference.sel(time=slice(2050, None)).mean('time').plot(cmap="bwr",
    ↪ norm=diffnorm,
                                cbar_kwargs={"label": "P90 change / mm day-1"})
plt.gca().coastlines()
plt.setp(plt.gca(), title='Difference')
```

[13]: [Text(0.5, 1.0, 'Difference')]



```
[14]: model
```

```
[14]: <gpflow.models.gpr.GPR object at 0x145e1a91cca0>
```

name	shape	dtype	class	transform	prior
GPR.mean_function.c			Parameter	Identity	
True	(1,)	float64	[0.02486868]		
GPR.kernel.kernels[0].variance			Parameter	Softplus	
True	()	float64	0.6166829861364803		
GPR.kernel.kernels[0].lengthscales			Parameter	Softplus	
True	()	float64	4.52729		
GPR.kernel.kernels[1].variance			Parameter	Softplus	
True	()	float64	1.83991697104977e-52		
GPR.kernel.kernels[1].lengthscales			Parameter	Softplus	
True	()	float64	96.65484		
GPR.kernel.kernels[2].variance			Parameter	Softplus	
True	()	float64	0.016885402763358048		
GPR.kernel.kernels[2].lengthscales			Parameter	Softplus	
True	(5,)	float64	[1.73173, 125.32431, 79.52744...		
GPR.kernel.kernels[3].variance			Parameter	Softplus	
True	()	float64	0.25427230196286643		
GPR.kernel.kernels[3].lengthscales			Parameter	Softplus	
True	(5,)	float64	[6.60297, 7.39649, 13.06545...		

	GPR.likelihood.variance	Parameter	Softplus + Shift
True	( )	float64	0.834159241121816

[ ]: