# GP_dtr

November 28, 2024

[1]:
```
# imports
import numpy as np
import xarray as xr
import pandas as pd
import matplotlib.pyplot as plt
import cartopy.crs as ccrs
from esem import gp_model
from eofs.xarray import Eof
from utils import *
import gpflow
```

```
2024-11-22 05:30:38.445843: E
external/local_xla/xla/stream_executor/cuda/cuda_fft.cc:477] Unable to register
cuFFT factory: Attempting to register factory for plugin cuFFT when one has
already been registered
WARNING: All log messages before absl::InitializeLog() is called are written to
STDERR
E0000 00:00:1732278638.457860  251654 cuda_dnn.cc:8310] Unable to register cuDNN
factory: Attempting to register factory for plugin cuDNN when one has already
been registered
E0000 00:00:1732278638.461740  251654 cuda_blas.cc:1418] Unable to register
cuBLAS factory: Attempting to register factory for plugin cuBLAS when one has
already been registered
2024-11-22 05:30:38.476337: I tensorflow/core/platform/cpu_feature_guard.cc:210]
This TensorFlow binary is optimized to use available CPU instructions in
performance-critical operations.
To enable the following instructions: AVX2 FMA, in other operations, rebuild
TensorFlow with the appropriate compiler flags.
```

[2]:
```
# list of experiment data used for training

train_files= ['ssp126', 'ssp370', 'ssp585', 'historical', 'hist-GHG',␣
 ↪'hist-aer']
```

### 0.0.1 prepare data

[3]:
```
# get data
Xtrain, eof_solvers = get_Xtrain(train_files)
Ytrain_dtr = get_Ytrain(train_files)['diurnal_temperature_range'].values.
 ↪reshape(-1, 96*144)

Xtest = get_Xtest('ssp245', eof_solvers)
Ytest = xr.open_dataset('../test/outputs_ssp245.nc').compute()
dtr_truth = Ytest['diurnal_temperature_range'].mean('member')
```

[4]:
```
# drop rows including nans
train_nan_mask = Xtrain.isna().any(axis=1).values
Xtrain = Xtrain.dropna(axis=0, how='any')
Ytrain_dtr = Ytrain_dtr[~train_nan_mask]
assert Xtrain.shape[0]==Ytrain_dtr.shape[0]

test_nan_mask = Xtest.isna().any(axis=1).values
Xtest = Xtest.dropna(axis=0, how='any')
dtr_truth = dtr_truth[~test_nan_mask]
```

[5]:
```
# Standardize predictor fields requiring standardization (non-EOFs)
train_CO2_mean, train_CO2_std = Xtrain['CO2'].mean(), Xtrain['CO2'].std()
train_CH4_mean, train_CH4_std = Xtrain['CH4'].mean(), Xtrain['CH4'].std()

Xtrain['CO2'] = (Xtrain['CO2'] - train_CO2_mean) / train_CO2_std
Xtrain['CH4'] = (Xtrain['CH4'] - train_CH4_mean) / train_CH4_std

Xtest['CO2'] = (Xtest['CO2'] - train_CO2_mean) / train_CO2_std
Xtest['CH4'] = (Xtest['CH4'] - train_CH4_mean) / train_CH4_std
```

[6]:
```
# Standardize predictand fields
train_dtr_mean, train_dtr_std = Ytrain_dtr.mean(), Ytrain_dtr.std()
Ytrain_dtr = (Ytrain_dtr - train_dtr_mean) / train_dtr_std
```

### 0.0.2 Model

[7]:
```
kernel_CO2 = gpflow.kernels.Matern32(active_dims=[0]) # active_dims specifies␣
 ↪which dimension the kernel is applied to
kernel_CH4 = gpflow.kernels.Matern32(active_dims=[1])

kernel_BC = gpflow.kernels.Matern32(lengthscales=5 * [1.], active_dims=[2, 3,␣
 ↪4, 5, 6])
kernel_SO2 = gpflow.kernels.Matern32(lengthscales=5 * [1.], active_dims=[7, 8,␣
 ↪9, 10, 11])

kernel = kernel_CO2 + kernel_CH4 + kernel_BC + kernel_SO2
```

```
I0000 00:00:1732278647.319994  251654 gpu_device.cc:2022] Created device
/job:localhost/replica:0/task:0/device:GPU:0 with 79379 MB memory:  -> device:
0, name: NVIDIA A100-SXM4-80GB, pci bus id: 0000:01:00.0, compute capability:
8.0
```

[8]:
```
np.random.seed(5)

"""
In Gaussian Processes, a mean function represents the "prior mean" or the␣
 ↪expected value
of the function at any input point before observing any data.
"""
mean = gpflow.mean_functions.Constant()

model = gpflow.models.GPR(data=(Xtrain.astype(np.float64), # cast to float64␣
 ↪because gpflow requires numerical stability
                                Ytrain_dtr.astype(np.float64)),
                          kernel = kernel,
                          mean_function = mean)
```

[9]:
```
# define optimizer
optimizer = gpflow.optimizers.Scipy()

# train
optimizer.minimize(model.training_loss,
                   variables=model.trainable_variables,
                   options=dict(disp=True, maxiter=1000))
```

```
WARNING: All log messages before absl::InitializeLog() is called are written to
STDERR
I0000 00:00:1732278651.883347  251719 cuda_solvers.cc:178] Creating GpuSolver
handles for stream 0x55b88c9a98e0
 This problem is unconstrained.

RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =           18     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.26854D+07    |proj g|=  1.84502D+06

At iterate    1    f=  1.11613D+07    |proj g|=  8.94905D+05

At iterate    2    f=  1.09653D+07    |proj g|=  1.25952D+05
```

```
At iterate    3    f=  1.09480D+07    |proj g|=  1.22959D+05

At iterate    4    f=  1.08528D+07    |proj g|=  4.55159D+05

At iterate    5    f=  1.06939D+07    |proj g|=  7.28017D+05

At iterate    6    f=  1.05146D+07    |proj g|=  4.80741D+05

At iterate    7    f=  1.04623D+07    |proj g|=  1.05185D+05

At iterate    8    f=  1.04601D+07    |proj g|=  1.46344D+04

At iterate    9    f=  1.04599D+07    |proj g|=  1.13620D+04

At iterate   10    f=  1.04596D+07    |proj g|=  3.15654D+04

At iterate   11    f=  1.04590D+07    |proj g|=  5.30216D+04

At iterate   12    f=  1.04575D+07    |proj g|=  8.47461D+04

At iterate   13    f=  1.04547D+07    |proj g|=  1.11495D+05

At iterate   14    f=  1.04501D+07    |proj g|=  1.15381D+05

At iterate   15    f=  1.04479D+07    |proj g|=  3.37232D+04

At iterate   16    f=  1.04439D+07    |proj g|=  5.47281D+03

At iterate   17    f=  1.04427D+07    |proj g|=  2.76426D+04

At iterate   18    f=  1.04415D+07    |proj g|=  5.44443D+04

At iterate   19    f=  1.04400D+07    |proj g|=  6.81293D+04

At iterate   20    f=  1.04366D+07    |proj g|=  5.37858D+04

At iterate   21    f=  1.04327D+07    |proj g|=  9.64600D+03

At iterate   22    f=  1.04308D+07    |proj g|=  2.23776D+04

At iterate   23    f=  1.04300D+07    |proj g|=  1.67113D+04

At iterate   24    f=  1.04295D+07    |proj g|=  5.33135D+04

At iterate   25    f=  1.04288D+07    |proj g|=  2.90241D+04

At iterate   26    f=  1.04280D+07    |proj g|=  5.56681D+03
```

```
At iterate   27    f=  1.04274D+07    |proj g|=  2.40637D+04

At iterate   28    f=  1.04262D+07    |proj g|=  3.71249D+04

At iterate   29    f=  1.04247D+07    |proj g|=  5.35860D+04

At iterate   30    f=  1.04223D+07    |proj g|=  6.05104D+04

At iterate   31    f=  1.04213D+07    |proj g|=  8.24593D+04

At iterate   32    f=  1.04193D+07    |proj g|=  5.07177D+04

At iterate   33    f=  1.04166D+07    |proj g|=  3.36494D+04

At iterate   34    f=  1.04157D+07    |proj g|=  2.77938D+04

At iterate   35    f=  1.04144D+07    |proj g|=  1.10073D+04

At iterate   36    f=  1.04129D+07    |proj g|=  1.31858D+04

At iterate   37    f=  1.04083D+07    |proj g|=  5.92658D+03

At iterate   38    f=  1.04078D+07    |proj g|=  6.92012D+04

At iterate   39    f=  1.04063D+07    |proj g|=  8.39978D+03

At iterate   40    f=  1.04062D+07    |proj g|=  6.05119D+03

At iterate   41    f=  1.04060D+07    |proj g|=  6.40410D+03

At iterate   42    f=  1.04048D+07    |proj g|=  2.75013D+04

At iterate   43    f=  1.04034D+07    |proj g|=  3.52516D+04

At iterate   44    f=  1.03995D+07    |proj g|=  3.22421D+04

At iterate   45    f=  1.03983D+07    |proj g|=  4.24177D+04

At iterate   46    f=  1.03960D+07    |proj g|=  2.05409D+04

At iterate   47    f=  1.03899D+07    |proj g|=  6.01982D+04

At iterate   48    f=  1.03873D+07    |proj g|=  1.55718D+04

At iterate   49    f=  1.03839D+07    |proj g|=  3.08663D+04

At iterate   50    f=  1.03832D+07    |proj g|=  2.67121D+04
```

```
At iterate   51    f=  1.03818D+07    |proj g|=  2.13139D+04

At iterate   52    f=  1.03807D+07    |proj g|=  1.98444D+04

At iterate   53    f=  1.03797D+07    |proj g|=  1.53322D+04

At iterate   54    f=  1.03777D+07    |proj g|=  8.76341D+03

At iterate   55    f=  1.03769D+07    |proj g|=  9.29697D+03

At iterate   56    f=  1.03758D+07    |proj g|=  6.76058D+03

At iterate   57    f=  1.03751D+07    |proj g|=  1.83345D+04

At iterate   58    f=  1.03744D+07    |proj g|=  4.67615D+03

At iterate   59    f=  1.03739D+07    |proj g|=  3.82116D+03

At iterate   60    f=  1.03735D+07    |proj g|=  3.71910D+03

At iterate   61    f=  1.03725D+07    |proj g|=  1.95338D+04

At iterate   62    f=  1.03714D+07    |proj g|=  3.60730D+04

At iterate   63    f=  1.03694D+07    |proj g|=  6.69388D+03

At iterate   64    f=  1.03644D+07    |proj g|=  7.31974D+04

At iterate   65    f=  1.03639D+07    |proj g|=  8.13454D+04

At iterate   66    f=  1.03610D+07    |proj g|=  1.09156D+05

At iterate   67    f=  1.03588D+07    |proj g|=  9.30331D+04

At iterate   68    f=  1.03561D+07    |proj g|=  1.66522D+04

At iterate   69    f=  1.03552D+07    |proj g|=  1.13884D+04

At iterate   70    f=  1.03540D+07    |proj g|=  3.22527D+04

At iterate   71    f=  1.03524D+07    |proj g|=  3.47198D+04

At iterate   72    f=  1.03517D+07    |proj g|=  2.80599D+04

At iterate   73    f=  1.03502D+07    |proj g|=  2.29380D+04

At iterate   74    f=  1.03500D+07    |proj g|=  1.69141D+04
```

```
At iterate   75    f=  1.03498D+07    |proj g|=  8.29900D+03

At iterate   76    f=  1.03493D+07    |proj g|=  3.66840D+03

At iterate   77    f=  1.03490D+07    |proj g|=  2.56674D+03

At iterate   78    f=  1.03488D+07    |proj g|=  3.48748D+03

At iterate   79    f=  1.03488D+07    |proj g|=  1.73975D+03

At iterate   80    f=  1.03487D+07    |proj g|=  6.40853D+03

At iterate   81    f=  1.03487D+07    |proj g|=  2.61891D+03

At iterate   82    f=  1.03486D+07    |proj g|=  1.33148D+03

At iterate   83    f=  1.03486D+07    |proj g|=  1.29198D+03

At iterate   84    f=  1.03486D+07    |proj g|=  3.07908D+03

At iterate   85    f=  1.03486D+07    |proj g|=  1.49260D+03

At iterate   86    f=  1.03486D+07    |proj g|=  4.41981D+03

At iterate   87    f=  1.03485D+07    |proj g|=  2.07577D+03

At iterate   88    f=  1.03485D+07    |proj g|=  5.09375D+02

At iterate   89    f=  1.03485D+07    |proj g|=  4.58305D+02

At iterate   90    f=  1.03485D+07    |proj g|=  8.32008D+02

At iterate   91    f=  1.03485D+07    |proj g|=  1.12796D+03

At iterate   92    f=  1.03484D+07    |proj g|=  4.16021D+03

At iterate   93    f=  1.03484D+07    |proj g|=  2.89583D+03

At iterate   94    f=  1.03483D+07    |proj g|=  2.53247D+03

At iterate   95    f=  1.03482D+07    |proj g|=  1.89343D+03

At iterate   96    f=  1.03482D+07    |proj g|=  3.31398D+03

At iterate   97    f=  1.03482D+07    |proj g|=  1.00479D+04

At iterate   98    f=  1.03481D+07    |proj g|=  3.00346D+03
```

```
At iterate   99    f=  1.03481D+07    |proj g|=  1.92294D+03

At iterate  100    f=  1.03481D+07    |proj g|=  2.09850D+03

At iterate  101    f=  1.03481D+07    |proj g|=  2.10522D+03

At iterate  102    f=  1.03480D+07    |proj g|=  6.93798D+02

At iterate  103    f=  1.03480D+07    |proj g|=  1.62361D+03

At iterate  104    f=  1.03480D+07    |proj g|=  2.27719D+03

At iterate  105    f=  1.03479D+07    |proj g|=  8.56475D+02

At iterate  106    f=  1.03479D+07    |proj g|=  1.82710D+03

At iterate  107    f=  1.03479D+07    |proj g|=  2.57559D+03

At iterate  108    f=  1.03479D+07    |proj g|=  3.80839D+03

At iterate  109    f=  1.03479D+07    |proj g|=  5.00673D+03

At iterate  110    f=  1.03479D+07    |proj g|=  9.64851D+02

At iterate  111    f=  1.03478D+07    |proj g|=  1.19687D+03

At iterate  112    f=  1.03478D+07    |proj g|=  1.31978D+03

At iterate  113    f=  1.03478D+07    |proj g|=  1.78404D+03

At iterate  114    f=  1.03478D+07    |proj g|=  3.04632D+03

At iterate  115    f=  1.03478D+07    |proj g|=  6.72522D+02

At iterate  116    f=  1.03478D+07    |proj g|=  3.05818D+02

At iterate  117    f=  1.03478D+07    |proj g|=  2.69693D+03

At iterate  118    f=  1.03478D+07    |proj g|=  2.74265D+03

At iterate  119    f=  1.03477D+07    |proj g|=  2.62800D+03

At iterate  120    f=  1.03477D+07    |proj g|=  2.19291D+03

At iterate  121    f=  1.03477D+07    |proj g|=  3.87145D+02

At iterate  122    f=  1.03477D+07    |proj g|=  3.50317D+02
```

```
At iterate  123    f=  1.03477D+07    |proj g|=  4.27726D+02

At iterate  124    f=  1.03477D+07    |proj g|=  5.01082D+02

At iterate  125    f=  1.03476D+07    |proj g|=  7.37694D+02

At iterate  126    f=  1.03476D+07    |proj g|=  6.16626D+02

At iterate  127    f=  1.03476D+07    |proj g|=  5.65172D+02

At iterate  128    f=  1.03476D+07    |proj g|=  2.84644D+03

At iterate  129    f=  1.03476D+07    |proj g|=  6.74804D+02

At iterate  130    f=  1.03476D+07    |proj g|=  2.96964D+03

At iterate  131    f=  1.03476D+07    |proj g|=  1.08857D+03

At iterate  132    f=  1.03475D+07    |proj g|=  4.16544D+03

At iterate  133    f=  1.03475D+07    |proj g|=  4.71100D+02

At iterate  134    f=  1.03475D+07    |proj g|=  8.14550D+02

At iterate  135    f=  1.03475D+07    |proj g|=  5.28811D+02

At iterate  136    f=  1.03475D+07    |proj g|=  4.03918D+02

At iterate  137    f=  1.03475D+07    |proj g|=  9.17063D+02

At iterate  138    f=  1.03475D+07    |proj g|=  9.88925D+02

At iterate  139    f=  1.03475D+07    |proj g|=  1.61393D+03

At iterate  140    f=  1.03475D+07    |proj g|=  5.36518D+02

At iterate  141    f=  1.03475D+07    |proj g|=  7.94666D+02

At iterate  142    f=  1.03475D+07    |proj g|=  2.57772D+03

At iterate  143    f=  1.03475D+07    |proj g|=  3.45133D+02

At iterate  144    f=  1.03475D+07    |proj g|=  1.25835D+03

At iterate  145    f=  1.03475D+07    |proj g|=  2.03504D+03

At iterate  146    f=  1.03474D+07    |proj g|=  4.88059D+03
```

```
At iterate  147    f=  1.03474D+07    |proj g|=  1.60849D+03

At iterate  148    f=  1.03474D+07    |proj g|=  6.35520D+02

At iterate  149    f=  1.03474D+07    |proj g|=  1.30362D+03

At iterate  150    f=  1.03474D+07    |proj g|=  1.29466D+03

At iterate  151    f=  1.03474D+07    |proj g|=  8.59381D+02

At iterate  152    f=  1.03474D+07    |proj g|=  1.20615D+03

At iterate  153    f=  1.03474D+07    |proj g|=  5.52597D+02

At iterate  154    f=  1.03474D+07    |proj g|=  1.99711D+03

At iterate  155    f=  1.03473D+07    |proj g|=  8.17683D+02

At iterate  156    f=  1.03473D+07    |proj g|=  3.04962D+02

At iterate  157    f=  1.03473D+07    |proj g|=  4.75927D+02

At iterate  158    f=  1.03473D+07    |proj g|=  2.15532D+03

At iterate  159    f=  1.03473D+07    |proj g|=  1.05800D+03

At iterate  160    f=  1.03473D+07    |proj g|=  3.47748D+02

At iterate  161    f=  1.03473D+07    |proj g|=  2.68512D+02

At iterate  162    f=  1.03473D+07    |proj g|=  2.96462D+02

At iterate  163    f=  1.03473D+07    |proj g|=  3.81046D+02

At iterate  164    f=  1.03473D+07    |proj g|=  6.95497D+02

At iterate  165    f=  1.03473D+07    |proj g|=  4.63898D+03

At iterate  166    f=  1.03473D+07    |proj g|=  1.41923D+03

At iterate  167    f=  1.03473D+07    |proj g|=  4.11441D+02

At iterate  168    f=  1.03473D+07    |proj g|=  1.02047D+03

At iterate  169    f=  1.03473D+07    |proj g|=  2.05018D+03

At iterate  170    f=  1.03473D+07    |proj g|=  1.42617D+03
```

```
At iterate  171    f=  1.03473D+07    |proj g|=  5.80907D+02

At iterate  172    f=  1.03473D+07    |proj g|=  3.51029D+02

At iterate  173    f=  1.03473D+07    |proj g|=  2.85162D+02

At iterate  174    f=  1.03472D+07    |proj g|=  2.10325D+03

At iterate  175    f=  1.03472D+07    |proj g|=  1.10254D+03

At iterate  176    f=  1.03472D+07    |proj g|=  4.57910D+02

At iterate  177    f=  1.03472D+07    |proj g|=  7.39917D+02

At iterate  178    f=  1.03472D+07    |proj g|=  3.77456D+02

At iterate  179    f=  1.03472D+07    |proj g|=  1.98023D+03

At iterate  180    f=  1.03472D+07    |proj g|=  2.51866D+02

At iterate  181    f=  1.03472D+07    |proj g|=  3.11045D+02

At iterate  182    f=  1.03472D+07    |proj g|=  2.14288D+02

At iterate  183    f=  1.03472D+07    |proj g|=  1.03872D+03

At iterate  184    f=  1.03472D+07    |proj g|=  6.17596D+02

At iterate  185    f=  1.03472D+07    |proj g|=  2.74651D+02

At iterate  186    f=  1.03472D+07    |proj g|=  9.17424D+02

At iterate  187    f=  1.03472D+07    |proj g|=  1.16767D+03

At iterate  188    f=  1.03472D+07    |proj g|=  9.92276D+02

At iterate  189    f=  1.03472D+07    |proj g|=  7.65896D+02

At iterate  190    f=  1.03472D+07    |proj g|=  9.12647D+02

At iterate  191    f=  1.03472D+07    |proj g|=  1.07503D+03

At iterate  192    f=  1.03471D+07    |proj g|=  1.39835D+03

At iterate  193    f=  1.03471D+07    |proj g|=  1.00443D+03

At iterate  194    f=  1.03471D+07    |proj g|=  2.44304D+03
```

```
At iterate  195    f=  1.03471D+07    |proj g|=  1.30298D+03

At iterate  196    f=  1.03471D+07    |proj g|=  2.40127D+02

At iterate  197    f=  1.03471D+07    |proj g|=  2.17252D+02

At iterate  198    f=  1.03471D+07    |proj g|=  3.56259D+02

At iterate  199    f=  1.03471D+07    |proj g|=  1.03541D+03

At iterate  200    f=  1.03471D+07    |proj g|=  6.49419D+02

At iterate  201    f=  1.03471D+07    |proj g|=  1.46840D+03

At iterate  202    f=  1.03471D+07    |proj g|=  1.20079D+03

At iterate  203    f=  1.03471D+07    |proj g|=  2.36735D+03

At iterate  204    f=  1.03471D+07    |proj g|=  1.29942D+03

At iterate  205    f=  1.03470D+07    |proj g|=  1.69728D+03

At iterate  206    f=  1.03470D+07    |proj g|=  1.61092D+03

At iterate  207    f=  1.03470D+07    |proj g|=  9.68995D+02

At iterate  208    f=  1.03470D+07    |proj g|=  4.81133D+02

At iterate  209    f=  1.03470D+07    |proj g|=  1.24552D+03

At iterate  210    f=  1.03470D+07    |proj g|=  1.14091D+03

At iterate  211    f=  1.03470D+07    |proj g|=  1.92305D+03

At iterate  212    f=  1.03470D+07    |proj g|=  5.04897D+02

At iterate  213    f=  1.03470D+07    |proj g|=  4.46837D+02

At iterate  214    f=  1.03470D+07    |proj g|=  5.14008D+02

At iterate  215    f=  1.03470D+07    |proj g|=  7.55067D+02

At iterate  216    f=  1.03470D+07    |proj g|=  7.90439D+02

At iterate  217    f=  1.03470D+07    |proj g|=  1.56898D+02

At iterate  218    f=  1.03470D+07    |proj g|=  1.85927D+02
```

```
At iterate  219    f=  1.03470D+07    |proj g|=  8.12908D+02

At iterate  220    f=  1.03470D+07    |proj g|=  6.26000D+02

At iterate  221    f=  1.03470D+07    |proj g|=  2.49520D+02

At iterate  222    f=  1.03470D+07    |proj g|=  2.03354D+02

At iterate  223    f=  1.03470D+07    |proj g|=  2.05760D+03

At iterate  224    f=  1.03470D+07    |proj g|=  2.23880D+02

At iterate  225    f=  1.03470D+07    |proj g|=  1.70724D+02

At iterate  226    f=  1.03470D+07    |proj g|=  3.96058D+02

At iterate  227    f=  1.03470D+07    |proj g|=  4.19476D+02

At iterate  228    f=  1.03470D+07    |proj g|=  9.57200D+02

At iterate  229    f=  1.03470D+07    |proj g|=  4.46048D+02

At iterate  230    f=  1.03470D+07    |proj g|=  7.94540D+02

At iterate  231    f=  1.03470D+07    |proj g|=  2.12640D+02

At iterate  232    f=  1.03470D+07    |proj g|=  5.62001D+02

At iterate  233    f=  1.03470D+07    |proj g|=  2.99160D+02

At iterate  234    f=  1.03470D+07    |proj g|=  2.17185D+02

At iterate  235    f=  1.03470D+07    |proj g|=  4.62677D+02

At iterate  236    f=  1.03470D+07    |proj g|=  1.76351D+03

At iterate  237    f=  1.03470D+07    |proj g|=  3.84357D+02

At iterate  238    f=  1.03470D+07    |proj g|=  3.68961D+02

At iterate  239    f=  1.03469D+07    |proj g|=  3.45238D+02

At iterate  240    f=  1.03469D+07    |proj g|=  8.58798D+01

At iterate  241    f=  1.03469D+07    |proj g|=  5.21348D+02

At iterate  242    f=  1.03469D+07    |proj g|=  1.18459D+02
```

```
At iterate  243    f=  1.03469D+07    |proj g|=  3.47642D+02

At iterate  244    f=  1.03469D+07    |proj g|=  5.64775D+02

At iterate  245    f=  1.03469D+07    |proj g|=  7.82058D+02

At iterate  246    f=  1.03469D+07    |proj g|=  4.99224D+02

At iterate  247    f=  1.03469D+07    |proj g|=  1.30283D+02

At iterate  248    f=  1.03469D+07    |proj g|=  2.68365D+02

At iterate  249    f=  1.03469D+07    |proj g|=  5.88778D+02

At iterate  250    f=  1.03469D+07    |proj g|=  5.27638D+02

At iterate  251    f=  1.03469D+07    |proj g|=  1.32844D+03

At iterate  252    f=  1.03469D+07    |proj g|=  8.07888D+02

At iterate  253    f=  1.03469D+07    |proj g|=  2.52180D+02

At iterate  254    f=  1.03469D+07    |proj g|=  9.64157D+01

At iterate  255    f=  1.03469D+07    |proj g|=  1.76032D+02

At iterate  256    f=  1.03469D+07    |proj g|=  2.37315D+02

At iterate  257    f=  1.03469D+07    |proj g|=  1.29021D+02

At iterate  258    f=  1.03469D+07    |proj g|=  6.03964D+01

At iterate  259    f=  1.03469D+07    |proj g|=  3.79869D+02

At iterate  260    f=  1.03469D+07    |proj g|=  1.75018D+02

At iterate  261    f=  1.03469D+07    |proj g|=  1.14473D+02

At iterate  262    f=  1.03469D+07    |proj g|=  3.75148D+02

At iterate  263    f=  1.03469D+07    |proj g|=  1.16328D+02

At iterate  264    f=  1.03469D+07    |proj g|=  1.73356D+02

At iterate  265    f=  1.03469D+07    |proj g|=  1.98534D+02

At iterate  266    f=  1.03469D+07    |proj g|=  1.17401D+03
```

```
At iterate  267    f=  1.03469D+07    |proj g|=  5.63791D+02

At iterate  268    f=  1.03469D+07    |proj g|=  2.08435D+02

At iterate  269    f=  1.03469D+07    |proj g|=  2.07147D+02

At iterate  270    f=  1.03469D+07    |proj g|=  1.80477D+02

At iterate  271    f=  1.03469D+07    |proj g|=  5.88465D+02

At iterate  272    f=  1.03469D+07    |proj g|=  2.05732D+02

At iterate  273    f=  1.03469D+07    |proj g|=  1.02214D+02

At iterate  274    f=  1.03469D+07    |proj g|=  2.88076D+02

At iterate  275    f=  1.03469D+07    |proj g|=  1.10654D+02

At iterate  276    f=  1.03469D+07    |proj g|=  7.29175D+01

At iterate  277    f=  1.03469D+07    |proj g|=  1.11522D+02

At iterate  278    f=  1.03469D+07    |proj g|=  4.34784D+02

At iterate  279    f=  1.03469D+07    |proj g|=  2.15129D+02

At iterate  280    f=  1.03469D+07    |proj g|=  9.14786D+01

At iterate  281    f=  1.03469D+07    |proj g|=  7.93543D+01

At iterate  282    f=  1.03469D+07    |proj g|=  8.82007D+01

At iterate  283    f=  1.03469D+07    |proj g|=  7.18989D+02

At iterate  284    f=  1.03469D+07    |proj g|=  4.34482D+02

At iterate  285    f=  1.03469D+07    |proj g|=  2.17542D+02

At iterate  286    f=  1.03469D+07    |proj g|=  2.66384D+02

At iterate  287    f=  1.03469D+07    |proj g|=  2.78059D+02

At iterate  288    f=  1.03469D+07    |proj g|=  1.11593D+02

At iterate  289    f=  1.03469D+07    |proj g|=  5.02882D+02

At iterate  290    f=  1.03469D+07    |proj g|=  3.00157D+02
```

```
At iterate  291    f=  1.03469D+07    |proj g|=  9.13633D+01

At iterate  292    f=  1.03469D+07    |proj g|=  2.90096D+02

At iterate  293    f=  1.03469D+07    |proj g|=  4.72420D+02

At iterate  294    f=  1.03469D+07    |proj g|=  3.29505D+02

At iterate  295    f=  1.03469D+07    |proj g|=  5.86484D+02

At iterate  296    f=  1.03469D+07    |proj g|=  2.30897D+02

At iterate  297    f=  1.03469D+07    |proj g|=  2.87279D+02

At iterate  298    f=  1.03469D+07    |proj g|=  1.38474D+02

At iterate  299    f=  1.03469D+07    |proj g|=  2.73617D+02

At iterate  300    f=  1.03469D+07    |proj g|=  1.15892D+02

At iterate  301    f=  1.03469D+07    |proj g|=  2.01316D+02

At iterate  302    f=  1.03469D+07    |proj g|=  2.05316D+02

At iterate  303    f=  1.03469D+07    |proj g|=  2.05808D+02

At iterate  304    f=  1.03469D+07    |proj g|=  1.11330D+02

At iterate  305    f=  1.03469D+07    |proj g|=  8.98731D+01

At iterate  306    f=  1.03469D+07    |proj g|=  9.16556D+01

At iterate  307    f=  1.03469D+07    |proj g|=  3.91288D+02

At iterate  308    f=  1.03469D+07    |proj g|=  3.11762D+02

At iterate  309    f=  1.03469D+07    |proj g|=  1.41009D+02

At iterate  310    f=  1.03469D+07    |proj g|=  1.57657D+02

At iterate  311    f=  1.03469D+07    |proj g|=  4.19729D+03

At iterate  312    f=  1.03469D+07    |proj g|=  1.59147D+03

At iterate  313    f=  1.03469D+07    |proj g|=  1.65065D+03

At iterate  314    f=  1.03468D+07    |proj g|=  7.64496D+03
```

```
At iterate  315    f=  1.03468D+07    |proj g|=  6.63996D+03

At iterate  316    f=  1.03467D+07    |proj g|=  2.75496D+03

At iterate  317    f=  1.03466D+07    |proj g|=  3.35268D+03

At iterate  318    f=  1.03465D+07    |proj g|=  2.24038D+03

At iterate  319    f=  1.03464D+07    |proj g|=  5.12545D+03

At iterate  320    f=  1.03464D+07    |proj g|=  5.49716D+02

At iterate  321    f=  1.03464D+07    |proj g|=  9.13928D+02

At iterate  322    f=  1.03463D+07    |proj g|=  2.04075D+03

At iterate  323    f=  1.03463D+07    |proj g|=  5.23643D+03

At iterate  324    f=  1.03463D+07    |proj g|=  4.59974D+02

At iterate  325    f=  1.03463D+07    |proj g|=  4.56844D+02

At iterate  326    f=  1.03463D+07    |proj g|=  4.57385D+02

At iterate  327    f=  1.03463D+07    |proj g|=  2.00744D+03

At iterate  328    f=  1.03463D+07    |proj g|=  1.11238D+03

At iterate  329    f=  1.03463D+07    |proj g|=  4.46328D+02

At iterate  330    f=  1.03463D+07    |proj g|=  7.44142D+02

At iterate  331    f=  1.03462D+07    |proj g|=  3.10824D+03

At iterate  332    f=  1.03462D+07    |proj g|=  3.17574D+03

At iterate  333    f=  1.03462D+07    |proj g|=  1.08649D+04

At iterate  334    f=  1.03461D+07    |proj g|=  5.39609D+03

At iterate  335    f=  1.03461D+07    |proj g|=  7.20108D+02

At iterate  336    f=  1.03461D+07    |proj g|=  8.17555D+02

At iterate  337    f=  1.03461D+07    |proj g|=  1.75163D+03

At iterate  338    f=  1.03461D+07    |proj g|=  6.59164D+02
```

```
At iterate  339    f=  1.03461D+07    |proj g|=  6.18424D+02

At iterate  340    f=  1.03461D+07    |proj g|=  5.48852D+02

At iterate  341    f=  1.03461D+07    |proj g|=  2.82747D+02

At iterate  342    f=  1.03461D+07    |proj g|=  8.54410D+02

At iterate  343    f=  1.03460D+07    |proj g|=  7.84504D+02

At iterate  344    f=  1.03460D+07    |proj g|=  2.85255D+03

At iterate  345    f=  1.03460D+07    |proj g|=  6.01297D+02

At iterate  346    f=  1.03460D+07    |proj g|=  5.79514D+02

At iterate  347    f=  1.03460D+07    |proj g|=  2.77243D+03

At iterate  348    f=  1.03460D+07    |proj g|=  1.31154D+03

At iterate  349    f=  1.03460D+07    |proj g|=  1.24877D+03

At iterate  350    f=  1.03460D+07    |proj g|=  2.69203D+03

At iterate  351    f=  1.03459D+07    |proj g|=  2.45652D+03

At iterate  352    f=  1.03459D+07    |proj g|=  1.47760D+03

At iterate  353    f=  1.03459D+07    |proj g|=  4.07979D+02

At iterate  354    f=  1.03459D+07    |proj g|=  1.83886D+02

At iterate  355    f=  1.03459D+07    |proj g|=  7.34948D+01

At iterate  356    f=  1.03459D+07    |proj g|=  2.40268D+02

At iterate  357    f=  1.03459D+07    |proj g|=  9.28147D+01

At iterate  358    f=  1.03459D+07    |proj g|=  7.19566D+01

At iterate  359    f=  1.03459D+07    |proj g|=  1.02916D+02

At iterate  360    f=  1.03459D+07    |proj g|=  1.80016D+02

At iterate  361    f=  1.03459D+07    |proj g|=  1.07557D+02

At iterate  362    f=  1.03459D+07    |proj g|=  6.92102D+01
```

```
At iterate  363    f=  1.03459D+07    |proj g|=  7.07694D+01

At iterate  364    f=  1.03459D+07    |proj g|=  1.96821D+02

At iterate  365    f=  1.03459D+07    |proj g|=  2.26844D+02

At iterate  366    f=  1.03459D+07    |proj g|=  1.24633D+02

At iterate  367    f=  1.03459D+07    |proj g|=  5.33581D+01

At iterate  368    f=  1.03459D+07    |proj g|=  5.31464D+01

At iterate  369    f=  1.03459D+07    |proj g|=  3.10142D+02

At iterate  370    f=  1.03459D+07    |proj g|=  5.71075D+01

At iterate  371    f=  1.03459D+07    |proj g|=  7.23378D+01

At iterate  372    f=  1.03459D+07    |proj g|=  3.64692D+02

At iterate  373    f=  1.03459D+07    |proj g|=  4.13898D+02

At iterate  374    f=  1.03459D+07    |proj g|=  4.52940D+01

At iterate  375    f=  1.03459D+07    |proj g|=  4.32300D+01

At iterate  376    f=  1.03459D+07    |proj g|=  6.25902D+01

At iterate  377    f=  1.03459D+07    |proj g|=  5.37824D+02

At iterate  378    f=  1.03459D+07    |proj g|=  7.23179D+01

At iterate  379    f=  1.03459D+07    |proj g|=  1.29437D+02

At iterate  380    f=  1.03459D+07    |proj g|=  3.65073D+02

At iterate  381    f=  1.03459D+07    |proj g|=  4.08716D+02

At iterate  382    f=  1.03459D+07    |proj g|=  4.63567D+02

At iterate  383    f=  1.03459D+07    |proj g|=  4.41812D+02

At iterate  384    f=  1.03459D+07    |proj g|=  9.75782D+02

At iterate  385    f=  1.03459D+07    |proj g|=  1.50906D+02

At iterate  386    f=  1.03459D+07    |proj g|=  1.19798D+02
```

```
At iterate  387    f=  1.03459D+07    |proj g|=  4.72648D+02

At iterate  388    f=  1.03459D+07    |proj g|=  1.19357D+02

At iterate  389    f=  1.03459D+07    |proj g|=  2.64512D+02

At iterate  390    f=  1.03459D+07    |proj g|=  2.94743D+02

At iterate  391    f=  1.03459D+07    |proj g|=  1.21209D+02

At iterate  392    f=  1.03459D+07    |proj g|=  2.77889D+02

At iterate  393    f=  1.03459D+07    |proj g|=  4.65789D+02

At iterate  394    f=  1.03459D+07    |proj g|=  2.07941D+02

At iterate  395    f=  1.03459D+07    |proj g|=  5.36380D+02

At iterate  396    f=  1.03459D+07    |proj g|=  2.02173D+02

At iterate  397    f=  1.03459D+07    |proj g|=  3.27836D+02

At iterate  398    f=  1.03459D+07    |proj g|=  3.94217D+02

At iterate  399    f=  1.03459D+07    |proj g|=  1.07840D+02

At iterate  400    f=  1.03459D+07    |proj g|=  9.06739D+01

At iterate  401    f=  1.03459D+07    |proj g|=  3.68340D+02

At iterate  402    f=  1.03459D+07    |proj g|=  2.96298D+02

At iterate  403    f=  1.03459D+07    |proj g|=  1.30417D+02

At iterate  404    f=  1.03459D+07    |proj g|=  2.25668D+02

At iterate  405    f=  1.03459D+07    |proj g|=  5.53918D+01

At iterate  406    f=  1.03459D+07    |proj g|=  2.95115D+02

At iterate  407    f=  1.03459D+07    |proj g|=  2.23054D+02

At iterate  408    f=  1.03459D+07    |proj g|=  1.22273D+02

At iterate  409    f=  1.03459D+07    |proj g|=  3.08582D+02

At iterate  410    f=  1.03459D+07    |proj g|=  3.01649D+02
```

```
At iterate  411    f=  1.03459D+07    |proj g|=  8.88207D+02

At iterate  412    f=  1.03459D+07    |proj g|=  2.79645D+02

At iterate  413    f=  1.03459D+07    |proj g|=  1.12356D+02

At iterate  414    f=  1.03459D+07    |proj g|=  9.05979D+01

At iterate  415    f=  1.03459D+07    |proj g|=  8.53610D+01

At iterate  416    f=  1.03459D+07    |proj g|=  3.00598D+02

At iterate  417    f=  1.03459D+07    |proj g|=  2.52090D+02

At iterate  418    f=  1.03459D+07    |proj g|=  1.52094D+02

At iterate  419    f=  1.03459D+07    |proj g|=  8.69672D+01

At iterate  420    f=  1.03459D+07    |proj g|=  9.46733D+01

At iterate  421    f=  1.03459D+07    |proj g|=  1.41066D+02

At iterate  422    f=  1.03459D+07    |proj g|=  3.57837D+02

At iterate  423    f=  1.03459D+07    |proj g|=  5.27399D+02

At iterate  424    f=  1.03459D+07    |proj g|=  1.68805D+02

At iterate  425    f=  1.03459D+07    |proj g|=  1.29094D+02

At iterate  426    f=  1.03459D+07    |proj g|=  1.20631D+02

At iterate  427    f=  1.03459D+07    |proj g|=  3.69820D+02

At iterate  428    f=  1.03459D+07    |proj g|=  3.70843D+02

At iterate  429    f=  1.03459D+07    |proj g|=  9.28251D+02

At iterate  430    f=  1.03459D+07    |proj g|=  4.32611D+02

At iterate  431    f=  1.03459D+07    |proj g|=  1.13835D+02

At iterate  432    f=  1.03459D+07    |proj g|=  6.27791D+01

At iterate  433    f=  1.03459D+07    |proj g|=  1.00380D+02

At iterate  434    f=  1.03459D+07    |proj g|=  8.74563D+01
```

```
At iterate  435    f=  1.03459D+07    |proj g|=  2.49334D+02

At iterate  436    f=  1.03459D+07    |proj g|=  1.17325D+02

At iterate  437    f=  1.03459D+07    |proj g|=  8.81792D+01

At iterate  438    f=  1.03459D+07    |proj g|=  9.23796D+01

At iterate  439    f=  1.03459D+07    |proj g|=  1.58326D+02

At iterate  440    f=  1.03459D+07    |proj g|=  2.73369D+02

At iterate  441    f=  1.03459D+07    |proj g|=  1.05290D+02

At iterate  442    f=  1.03459D+07    |proj g|=  6.47078D+01

At iterate  443    f=  1.03459D+07    |proj g|=  7.25941D+01

At iterate  444    f=  1.03459D+07    |proj g|=  1.51523D+02

At iterate  445    f=  1.03459D+07    |proj g|=  1.10572D+02

At iterate  446    f=  1.03459D+07    |proj g|=  3.05787D+02

At iterate  447    f=  1.03459D+07    |proj g|=  2.33992D+02

At iterate  448    f=  1.03459D+07    |proj g|=  8.42328D+01

At iterate  449    f=  1.03459D+07    |proj g|=  1.27549D+02

At iterate  450    f=  1.03459D+07    |proj g|=  7.13471D+01

At iterate  451    f=  1.03459D+07    |proj g|=  1.26618D+02

At iterate  452    f=  1.03459D+07    |proj g|=  2.48143D+02

At iterate  453    f=  1.03459D+07    |proj g|=  2.33990D+02

At iterate  454    f=  1.03459D+07    |proj g|=  1.58559D+02

At iterate  455    f=  1.03459D+07    |proj g|=  8.09982D+01

At iterate  456    f=  1.03459D+07    |proj g|=  6.68271D+01

At iterate  457    f=  1.03459D+07    |proj g|=  1.91094D+02

At iterate  458    f=  1.03459D+07    |proj g|=  2.15009D+02
```

```
At iterate  459    f=  1.03459D+07    |proj g|=  6.36669D+01

At iterate  460    f=  1.03459D+07    |proj g|=  8.14919D+01

At iterate  461    f=  1.03459D+07    |proj g|=  8.64553D+01

At iterate  462    f=  1.03459D+07    |proj g|=  2.44694D+02

At iterate  463    f=  1.03459D+07    |proj g|=  5.58165D+02

At iterate  464    f=  1.03459D+07    |proj g|=  2.12776D+02

At iterate  465    f=  1.03459D+07    |proj g|=  1.67081D+02

At iterate  466    f=  1.03459D+07    |proj g|=  1.58476D+02

At iterate  467    f=  1.03459D+07    |proj g|=  2.04874D+02

At iterate  468    f=  1.03459D+07    |proj g|=  7.43555D+01

At iterate  469    f=  1.03459D+07    |proj g|=  4.51390D+02

At iterate  470    f=  1.03459D+07    |proj g|=  4.66492D+01

At iterate  471    f=  1.03459D+07    |proj g|=  5.12924D+01

At iterate  472    f=  1.03459D+07    |proj g|=  1.31161D+02

At iterate  473    f=  1.03459D+07    |proj g|=  1.45782D+02

At iterate  474    f=  1.03459D+07    |proj g|=  1.65552D+02

At iterate  475    f=  1.03459D+07    |proj g|=  1.12767D+02

At iterate  476    f=  1.03459D+07    |proj g|=  6.24726D+01

At iterate  477    f=  1.03459D+07    |proj g|=  1.54294D+02

At iterate  478    f=  1.03459D+07    |proj g|=  2.26654D+02

At iterate  479    f=  1.03459D+07    |proj g|=  2.58181D+02

At iterate  480    f=  1.03459D+07    |proj g|=  1.53836D+02

At iterate  481    f=  1.03459D+07    |proj g|=  5.44824D+01

At iterate  482    f=  1.03459D+07    |proj g|=  9.00981D+01
```

```
 At iterate  483    f=  1.03459D+07    |proj g|=  1.02409D+02

 At iterate  484    f=  1.03459D+07    |proj g|=  2.11929D+02

 At iterate  485    f=  1.03459D+07    |proj g|=  6.16088D+01

 At iterate  486    f=  1.03459D+07    |proj g|=  1.33202D+02

 At iterate  487    f=  1.03459D+07    |proj g|=  7.20604D+01

 At iterate  488    f=  1.03459D+07    |proj g|=  1.59058D+02

 At iterate  489    f=  1.03459D+07    |proj g|=  9.91841D+01

 At iterate  490    f=  1.03459D+07    |proj g|=  5.97197D+01

           * * *

 Tit   = total number of iterations
 Tnf   = total number of function evaluations
 Tnint = total number of segments explored during Cauchy searches
 Skip  = number of BFGS updates skipped
 Nact  = number of active bounds at final generalized Cauchy point
 Projg = norm of the final projected gradient
 F     = final function value

           * * *

    N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
   18    490     579      1     0     0   5.972D+01   1.035D+07
  F =   10345900.307780962

 CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
```

[9]:
```
   message: CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
   success: True
    status: 0
       fun: 10345900.307780962
         x: [ 5.214e+00  2.459e+00 … -9.517e-01 -2.079e-01]
       nit: 490
       jac: [ 4.886e+00  5.148e+00 …  5.972e+01 -1.294e+01]
      nfev: 579
      njev: 579
  hess_inv: <18x18 LbfgsInvHessProduct with dtype=float64>
```

### 0.0.3 make prediction

[10]:
```
# predict
standard_posterior_mean, standard_posterior_var = model.predict_y(Xtest.values)␣
 ↪# predicted mean of GP, predicted variance of GP
posterior_mean = standard_posterior_mean * train_dtr_std + train_dtr_mean #␣
 ↪transform mean prediction to original scale
posterior_stddev = np.sqrt(standard_posterior_var) * train_dtr_std # transform␣
 ↪variance prediction to original scale standard deviation
```

[11]:
```
# put output back into xarray format for calculating RMSE/plotting
posterior_dtr = np.reshape(posterior_mean, [86, 96, 144])
posterior_dtr_stddev = np.reshape(posterior_stddev, [86, 96, 144])

posterior_dtr_data = xr.DataArray(posterior_dtr, dims=dtr_truth.dims,␣
 ↪coords=dtr_truth.coords)
posterior_dtr_std_data = xr.DataArray(posterior_dtr_stddev, dims=dtr_truth.
 ↪dims, coords=dtr_truth.coords)
```

[12]:
```
# Compute RMSEs
print(f"RMSE at 2050: {get_rmse(dtr_truth[35], posterior_dtr_data[35])}")
print(f"RMSE at 2100: {get_rmse(dtr_truth[85], posterior_dtr_data[85])}")
print(f"RMSE 2045-2055: {get_rmse(dtr_truth[30:41], posterior_dtr_data[30:41]).
 ↪mean()}")
print(f"RMSE 2090-2100: {get_rmse(dtr_truth[75:], posterior_dtr_data[75:]).
 ↪mean()}")
print(f"RMSE 2050-2100: {get_rmse(dtr_truth[35:], posterior_dtr_data[35:]).
 ↪mean()}")

# RMSE for average field over last 20 years
print(f"RMSE average last 20y: {get_rmse(dtr_truth[-20:].mean(dim='time'),␣
 ↪posterior_dtr_data[-20:].mean(dim='time'))}")
```

```
RMSE at 2050: 0.13227912819669815
RMSE at 2100: 0.14590536779500873
RMSE 2045-2055: 0.13907161029513718
RMSE 2090-2100: 0.1625766423416543
RMSE 2050-2100: 0.15430780029689709
RMSE average last 20y: 0.10414020027323277
```

[13]:
```
from matplotlib import colors
# plotting predictions
divnorm = colors.TwoSlopeNorm(vmin=-2., vcenter=0., vmax=5)
diffnorm = colors.TwoSlopeNorm(vmin=-2., vcenter=0., vmax=2)

## Temperature
proj = ccrs.PlateCarree()
```

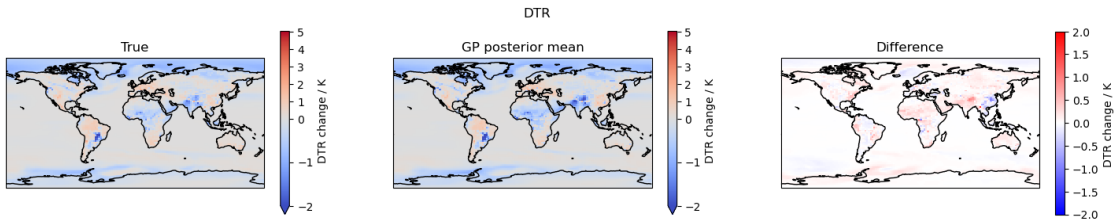
```
fig = plt.figure(figsize=(18, 3))
fig.suptitle('DTR')

# Test
plt.subplot(131, projection=proj)
dtr_truth.sel(time=slice(2050,None)).mean('time').plot(cmap="coolwarm",␣
 ↪norm=divnorm,
                                cbar_kwargs={"label":"DTR change / K "})
plt.gca().coastlines()
plt.setp(plt.gca(), title='True')

# Emulator
plt.subplot(132, projection=proj)
posterior_dtr_data.sel(time=slice(2050,None)).mean('time').
 ↪plot(cmap="coolwarm", norm=divnorm,
                        cbar_kwargs={"label":"DTR change / K "})
plt.gca().coastlines()
plt.setp(plt.gca(), title='GP posterior mean')

# Difference
difference = dtr_truth - posterior_dtr_data
plt.subplot(133, projection=proj)
difference.sel(time=slice(2050,None)).mean('time').plot(cmap="bwr",␣
 ↪norm=diffnorm,
                cbar_kwargs={"label":"DTR change / K "})
plt.gca().coastlines()
plt.setp(plt.gca(), title='Difference')
```

[13]: [Text(0.5, 1.0, 'Difference')]




[14]:
```
model
```

[14]: <gpflow.models.gpr.GPR object at 0x153b2fdf5000>

| name | class | transform | prior | trainable | shape | dtype | value |
|---|---|---|---|---|---|---|---|

```
  GPR.mean_function.c                  Parameter   Identity
True          (1,)      float64   [-0.20785411]


  GPR.kernel.kernels[0].variance       Parameter   Softplus
True          ()        float64   2.54138


  GPR.kernel.kernels[0].lengthscales   Parameter   Softplus
True          ()        float64   5.2194


  GPR.kernel.kernels[1].variance       Parameter   Softplus
True          ()        float64   0.06593512409548839


  GPR.kernel.kernels[1].lengthscales   Parameter   Softplus
True          ()        float64   0.48763960832641695


  GPR.kernel.kernels[2].variance       Parameter   Softplus
True          ()        float64   0.14007659238349016


  GPR.kernel.kernels[2].lengthscales   Parameter   Softplus
True          (5,)      float64   [0.02441118, 4.50587, 1.35779…


  GPR.kernel.kernels[3].variance       Parameter   Softplus
True          ()        float64   0.3117402754966986


  GPR.kernel.kernels[3].lengthscales   Parameter   Softplus
True          (5,)      float64   [5.80576, 4.24049, 8.60169…


  GPR.likelihood.variance              Parameter   Softplus + Shift
True          ()        float64   0.3264734783071516
```

[ ]: