GP_pr

November 28, 2024

[1]:
```
# imports
import numpy as np
import xarray as xr
import pandas as pd
import matplotlib.pyplot as plt
import cartopy.crs as ccrs
from esem import gp_model
from eofs.xarray import Eof
from utils import *
import gpflow
```

```
2024-11-22 05:27:31.246030: E
external/local_xla/xla/stream_executor/cuda/cuda_fft.cc:477] Unable to register
cuFFT factory: Attempting to register factory for plugin cuFFT when one has
already been registered
WARNING: All log messages before absl::InitializeLog() is called are written to
STDERR
E0000 00:00:1732278451.258811  250630 cuda_dnn.cc:8310] Unable to register cuDNN
factory: Attempting to register factory for plugin cuDNN when one has already
been registered
E0000 00:00:1732278451.262722  250630 cuda_blas.cc:1418] Unable to register
cuBLAS factory: Attempting to register factory for plugin cuBLAS when one has
already been registered
2024-11-22 05:27:31.277619: I tensorflow/core/platform/cpu_feature_guard.cc:210]
This TensorFlow binary is optimized to use available CPU instructions in
performance-critical operations.
To enable the following instructions: AVX2 FMA, in other operations, rebuild
TensorFlow with the appropriate compiler flags.
```

[2]:
```
# list of experiment data used for training

train_files= ['ssp126', 'ssp370', 'ssp585', 'historical', 'hist-GHG',␣
 ↪'hist-aer']
```

0.0.1 prepare data

[3]:
```
# get data
Xtrain, eof_solvers = get_Xtrain(train_files)
Ytrain_pr = get_Ytrain(train_files)['pr'].values.reshape(-1, 96*144)

Xtest = get_Xtest('ssp245', eof_solvers)
Ytest = xr.open_dataset('../test/outputs_ssp245.nc').compute()
pr_truth = 86400 * Ytest['pr'].mean('member') # convert pr to mm/day, default␣
 ↪unit is kg/m²/s
```

[4]:
```
# drop rows including nans
train_nan_mask = Xtrain.isna().any(axis=1).values
Xtrain = Xtrain.dropna(axis=0, how='any')
Ytrain_pr = Ytrain_pr[~train_nan_mask]
assert Xtrain.shape[0]==Ytrain_pr.shape[0]

test_nan_mask = Xtest.isna().any(axis=1).values
Xtest = Xtest.dropna(axis=0, how='any')
pr_truth = pr_truth[~test_nan_mask]
```

[5]:
```
# Standardize predictor fields requiring standardization (non-EOFs)
train_CO2_mean, train_CO2_std = Xtrain['CO2'].mean(), Xtrain['CO2'].std()
train_CH4_mean, train_CH4_std = Xtrain['CH4'].mean(), Xtrain['CH4'].std()

Xtrain['CO2'] = (Xtrain['CO2'] - train_CO2_mean) / train_CO2_std
Xtrain['CH4'] = (Xtrain['CH4'] - train_CH4_mean) / train_CH4_std

Xtest['CO2'] = (Xtest['CO2'] - train_CO2_mean) / train_CO2_std
Xtest['CH4'] = (Xtest['CH4'] - train_CH4_mean) / train_CH4_std
```

[6]:
```
# Standardize predictand fields
train_pr_mean, train_pr_std = Ytrain_pr.mean(), Ytrain_pr.std()
Ytrain_pr = (Ytrain_pr - train_pr_mean) / train_pr_std
```

0.0.2 Model

[7]:
```
kernel_CO2 = gpflow.kernels.Matern32(active_dims=[0]) # active_dims specifies␣
 ↪which dimension the kernel is applied to
kernel_CH4 = gpflow.kernels.Matern32(active_dims=[1])

kernel_BC = gpflow.kernels.Matern32(lengthscales=5 * [1.], active_dims=[2, 3,␣
 ↪4, 5, 6])
kernel_SO2 = gpflow.kernels.Matern32(lengthscales=5 * [1.], active_dims=[7, 8,␣
 ↪9, 10, 11])

kernel = kernel_CO2 + kernel_CH4 + kernel_BC + kernel_SO2
```

```
I0000 00:00:1732278460.080766  250630 gpu_device.cc:2022] Created device
/job:localhost/replica:0/task:0/device:GPU:0 with 79379 MB memory:  -> device:
0, name: NVIDIA A100-SXM4-80GB, pci bus id: 0000:01:00.0, compute capability:
8.0
```

[8]:
```
np.random.seed(5)

"""
In Gaussian Processes, a mean function represents the "prior mean" or the␣
 ↪expected value
of the function at any input point before observing any data.
"""
mean = gpflow.mean_functions.Constant()

model = gpflow.models.GPR(data=(Xtrain.astype(np.float64), # cast to float64␣
 ↪because gpflow requires numerical stability
                                Ytrain_pr.astype(np.float64)),
                          kernel = kernel,
                          mean_function = mean)
```

[9]:
```
# define optimizer
optimizer = gpflow.optimizers.Scipy()

# train
optimizer.minimize(model.training_loss,
                   variables=model.trainable_variables,
                   options=dict(disp=True, maxiter=1000))
```

```
WARNING: All log messages before absl::InitializeLog() is called are written to
STDERR
I0000 00:00:1732278464.500881  250688 cuda_solvers.cc:178] Creating GpuSolver
handles for stream 0x558c7ebca720
 This problem is unconstrained.
RUNNING THE L-BFGS-B CODE

           * * *

Machine precision = 2.220D-16
 N =           18     M =           10

At X0         0 variables are exactly at the bounds

At iterate    0    f=  1.46144D+07    |proj g|=  6.59595D+05

At iterate    1    f=  1.44612D+07    |proj g|=  1.03765D+05

At iterate    2    f=  1.44451D+07    |proj g|=  1.01717D+05
```

```
At iterate    3    f=  1.43598D+07    |proj g|=  3.61807D+05

At iterate    4    f=  1.42008D+07    |proj g|=  6.35110D+05

At iterate    5    f=  1.40093D+07    |proj g|=  4.53002D+05

At iterate    6    f=  1.39421D+07    |proj g|=  1.07261D+05

At iterate    7    f=  1.39380D+07    |proj g|=  1.25869D+04

At iterate    8    f=  1.39374D+07    |proj g|=  1.47148D+04

At iterate    9    f=  1.39363D+07    |proj g|=  5.03315D+04

At iterate   10    f=  1.39340D+07    |proj g|=  9.61485D+04

At iterate   11    f=  1.39291D+07    |proj g|=  1.52290D+05

At iterate   12    f=  1.39207D+07    |proj g|=  1.90517D+05

At iterate   13    f=  1.39107D+07    |proj g|=  1.59457D+05

At iterate   14    f=  1.39063D+07    |proj g|=  5.31489D+04

At iterate   15    f=  1.39041D+07    |proj g|=  8.22843D+03

At iterate   16    f=  1.39038D+07    |proj g|=  6.19192D+03

At iterate   17    f=  1.39022D+07    |proj g|=  5.44523D+03

At iterate   18    f=  1.38971D+07    |proj g|=  6.56393D+03

At iterate   19    f=  1.38946D+07    |proj g|=  1.18746D+04

At iterate   20    f=  1.38939D+07    |proj g|=  2.35818D+03

At iterate   21    f=  1.38929D+07    |proj g|=  2.35574D+03

At iterate   22    f=  1.38928D+07    |proj g|=  5.22936D+04

At iterate   23    f=  1.38922D+07    |proj g|=  1.68163D+04

At iterate   24    f=  1.38920D+07    |proj g|=  3.95429D+03

At iterate   25    f=  1.38917D+07    |proj g|=  5.38691D+03

At iterate   26    f=  1.38912D+07    |proj g|=  1.32978D+04
```

```
At iterate   27    f=  1.38903D+07    |proj g|=  2.06653D+04

At iterate   28    f=  1.38891D+07    |proj g|=  1.79990D+04

At iterate   29    f=  1.38889D+07    |proj g|=  1.20462D+04

At iterate   30    f=  1.38888D+07    |proj g|=  5.19825D+03

At iterate   31    f=  1.38886D+07    |proj g|=  1.98195D+03

At iterate   32    f=  1.38885D+07    |proj g|=  1.19767D+04

At iterate   33    f=  1.38885D+07    |proj g|=  7.77967D+03

At iterate   34    f=  1.38884D+07    |proj g|=  1.23403D+03

At iterate   35    f=  1.38884D+07    |proj g|=  4.15653D+03

At iterate   36    f=  1.38882D+07    |proj g|=  1.21737D+04

At iterate   37    f=  1.38880D+07    |proj g|=  1.91959D+04

At iterate   38    f=  1.38876D+07    |proj g|=  2.06356D+04

At iterate   39    f=  1.38874D+07    |proj g|=  6.08329D+04

At iterate   40    f=  1.38867D+07    |proj g|=  3.13328D+04

At iterate   41    f=  1.38861D+07    |proj g|=  4.53158D+03

At iterate   42    f=  1.38859D+07    |proj g|=  6.11316D+03

At iterate   43    f=  1.38858D+07    |proj g|=  1.04670D+04

At iterate   44    f=  1.38857D+07    |proj g|=  8.60749D+03

At iterate   45    f=  1.38855D+07    |proj g|=  1.25795D+04

At iterate   46    f=  1.38851D+07    |proj g|=  1.30907D+04

At iterate   47    f=  1.38846D+07    |proj g|=  2.17656D+04

At iterate   48    f=  1.38840D+07    |proj g|=  2.24199D+04

At iterate   49    f=  1.38830D+07    |proj g|=  2.41501D+04

At iterate   50    f=  1.38826D+07    |proj g|=  9.31404D+03
```

```
At iterate   51    f=  1.38824D+07    |proj g|=  2.28015D+03

At iterate   52    f=  1.38824D+07    |proj g|=  1.17493D+03

At iterate   53    f=  1.38822D+07    |proj g|=  2.11887D+03

At iterate   54    f=  1.38818D+07    |proj g|=  6.93841D+03

At iterate   55    f=  1.38811D+07    |proj g|=  7.73984D+03

At iterate   56    f=  1.38809D+07    |proj g|=  9.22402D+03

At iterate   57    f=  1.38806D+07    |proj g|=  6.21923D+03

At iterate   58    f=  1.38806D+07    |proj g|=  2.40178D+04

At iterate   59    f=  1.38804D+07    |proj g|=  1.80779D+03

At iterate   60    f=  1.38804D+07    |proj g|=  4.89360D+02

At iterate   61    f=  1.38804D+07    |proj g|=  1.94055D+03

At iterate   62    f=  1.38804D+07    |proj g|=  2.94741D+02

At iterate   63    f=  1.38804D+07    |proj g|=  2.00487D+02

At iterate   64    f=  1.38804D+07    |proj g|=  4.30380D+02

At iterate   65    f=  1.38804D+07    |proj g|=  6.57906D+02

At iterate   66    f=  1.38804D+07    |proj g|=  8.18184D+02

At iterate   67    f=  1.38804D+07    |proj g|=  5.40235D+02

At iterate   68    f=  1.38803D+07    |proj g|=  5.27257D+02

At iterate   69    f=  1.38803D+07    |proj g|=  1.08323D+03

At iterate   70    f=  1.38803D+07    |proj g|=  3.67446D+03

At iterate   71    f=  1.38802D+07    |proj g|=  1.83421D+03

At iterate   72    f=  1.38802D+07    |proj g|=  1.10948D+03

At iterate   73    f=  1.38802D+07    |proj g|=  3.35544D+03

At iterate   74    f=  1.38802D+07    |proj g|=  2.40202D+03
```

```
At iterate   75    f=  1.38801D+07    |proj g|=  8.49208D+02

At iterate   76    f=  1.38801D+07    |proj g|=  6.23777D+02

At iterate   77    f=  1.38801D+07    |proj g|=  1.26973D+03

At iterate   78    f=  1.38800D+07    |proj g|=  1.60965D+03

At iterate   79    f=  1.38800D+07    |proj g|=  1.15734D+03

At iterate   80    f=  1.38800D+07    |proj g|=  5.51579D+02

At iterate   81    f=  1.38800D+07    |proj g|=  2.26081D+03

At iterate   82    f=  1.38800D+07    |proj g|=  6.04739D+02

At iterate   83    f=  1.38800D+07    |proj g|=  5.59483D+02

At iterate   84    f=  1.38800D+07    |proj g|=  5.28812D+02

At iterate   85    f=  1.38800D+07    |proj g|=  1.07899D+03

At iterate   86    f=  1.38800D+07    |proj g|=  5.57077D+02

At iterate   87    f=  1.38800D+07    |proj g|=  1.22276D+02

At iterate   88    f=  1.38800D+07    |proj g|=  2.96914D+02

At iterate   89    f=  1.38800D+07    |proj g|=  5.90939D+02

At iterate   90    f=  1.38800D+07    |proj g|=  5.71992D+02

At iterate   91    f=  1.38800D+07    |proj g|=  1.18993D+02

At iterate   92    f=  1.38800D+07    |proj g|=  5.84000D+02

At iterate   93    f=  1.38800D+07    |proj g|=  2.15788D+02

At iterate   94    f=  1.38800D+07    |proj g|=  1.02405D+02

At iterate   95    f=  1.38800D+07    |proj g|=  4.00186D+02

At iterate   96    f=  1.38800D+07    |proj g|=  7.60258D+02

At iterate   97    f=  1.38800D+07    |proj g|=  1.37134D+03

At iterate   98    f=  1.38800D+07    |proj g|=  1.75573D+03
```

```
At iterate   99    f=  1.38800D+07    |proj g|=  3.66014D+03

At iterate  100    f=  1.38799D+07    |proj g|=  2.57752D+03

At iterate  101    f=  1.38799D+07    |proj g|=  8.12744D+02

At iterate  102    f=  1.38799D+07    |proj g|=  3.34610D+02

At iterate  103    f=  1.38799D+07    |proj g|=  9.54951D+02

At iterate  104    f=  1.38799D+07    |proj g|=  7.80272D+02

At iterate  105    f=  1.38799D+07    |proj g|=  2.10644D+03

At iterate  106    f=  1.38799D+07    |proj g|=  1.59084D+03

At iterate  107    f=  1.38799D+07    |proj g|=  2.55669D+02

At iterate  108    f=  1.38799D+07    |proj g|=  2.76138D+02

At iterate  109    f=  1.38799D+07    |proj g|=  7.27118D+02

At iterate  110    f=  1.38799D+07    |proj g|=  1.18463D+03

At iterate  111    f=  1.38799D+07    |proj g|=  5.48404D+02

At iterate  112    f=  1.38799D+07    |proj g|=  3.04844D+03

At iterate  113    f=  1.38799D+07    |proj g|=  1.35453D+03

At iterate  114    f=  1.38799D+07    |proj g|=  4.05294D+02

At iterate  115    f=  1.38799D+07    |proj g|=  4.20632D+02

At iterate  116    f=  1.38799D+07    |proj g|=  6.79926D+02

At iterate  117    f=  1.38799D+07    |proj g|=  1.44369D+03

At iterate  118    f=  1.38799D+07    |proj g|=  7.38268D+02

At iterate  119    f=  1.38799D+07    |proj g|=  8.97231D+02

At iterate  120    f=  1.38799D+07    |proj g|=  1.33957D+03

At iterate  121    f=  1.38799D+07    |proj g|=  1.44994D+03

At iterate  122    f=  1.38799D+07    |proj g|=  2.12252D+03
```

```
At iterate  123    f=  1.38799D+07    |proj g|=  2.75525D+02

At iterate  124    f=  1.38799D+07    |proj g|=  6.54309D+02

At iterate  125    f=  1.38798D+07    |proj g|=  4.09143D+02

At iterate  126    f=  1.38798D+07    |proj g|=  2.35760D+03

At iterate  127    f=  1.38798D+07    |proj g|=  1.45718D+03

At iterate  128    f=  1.38798D+07    |proj g|=  8.39462D+02

At iterate  129    f=  1.38798D+07    |proj g|=  4.89188D+02

At iterate  130    f=  1.38798D+07    |proj g|=  1.86925D+03

At iterate  131    f=  1.38798D+07    |proj g|=  6.87362D+02

At iterate  132    f=  1.38798D+07    |proj g|=  2.35855D+03

At iterate  133    f=  1.38798D+07    |proj g|=  4.16407D+03

At iterate  134    f=  1.38797D+07    |proj g|=  3.45393D+03

At iterate  135    f=  1.38797D+07    |proj g|=  2.07590D+03

At iterate  136    f=  1.38796D+07    |proj g|=  2.55605D+03

At iterate  137    f=  1.38796D+07    |proj g|=  3.05356D+03

At iterate  138    f=  1.38795D+07    |proj g|=  3.25831D+03

At iterate  139    f=  1.38795D+07    |proj g|=  2.76758D+03

At iterate  140    f=  1.38794D+07    |proj g|=  2.21910D+03

At iterate  141    f=  1.38794D+07    |proj g|=  1.30251D+03

At iterate  142    f=  1.38793D+07    |proj g|=  1.05344D+03

At iterate  143    f=  1.38793D+07    |proj g|=  3.84649D+03

At iterate  144    f=  1.38793D+07    |proj g|=  3.07274D+03

At iterate  145    f=  1.38791D+07    |proj g|=  2.06812D+03

At iterate  146    f=  1.38791D+07    |proj g|=  2.74917D+02
```

```
At iterate  147    f=  1.38790D+07    |proj g|=  7.86086D+02

At iterate  148    f=  1.38790D+07    |proj g|=  1.88741D+03

At iterate  149    f=  1.38790D+07    |proj g|=  2.30236D+02

At iterate  150    f=  1.38790D+07    |proj g|=  1.78193D+02

At iterate  151    f=  1.38790D+07    |proj g|=  7.81475D+02

At iterate  152    f=  1.38790D+07    |proj g|=  5.39023D+02

At iterate  153    f=  1.38790D+07    |proj g|=  4.71607D+02

At iterate  154    f=  1.38790D+07    |proj g|=  7.20800D+02

At iterate  155    f=  1.38790D+07    |proj g|=  3.08601D+02

At iterate  156    f=  1.38790D+07    |proj g|=  2.20553D+02

At iterate  157    f=  1.38790D+07    |proj g|=  2.16488D+02

At iterate  158    f=  1.38790D+07    |proj g|=  3.24533D+02

At iterate  159    f=  1.38790D+07    |proj g|=  1.34061D+03

At iterate  160    f=  1.38790D+07    |proj g|=  3.73513D+02

At iterate  161    f=  1.38790D+07    |proj g|=  3.50374D+03

At iterate  162    f=  1.38789D+07    |proj g|=  3.62707D+03

At iterate  163    f=  1.38789D+07    |proj g|=  3.70265D+03

At iterate  164    f=  1.38788D+07    |proj g|=  1.00598D+03

At iterate  165    f=  1.38788D+07    |proj g|=  6.64945D+02

At iterate  166    f=  1.38788D+07    |proj g|=  1.92741D+03

At iterate  167    f=  1.38788D+07    |proj g|=  5.83066D+02

At iterate  168    f=  1.38788D+07    |proj g|=  7.16416D+02

At iterate  169    f=  1.38787D+07    |proj g|=  5.79695D+02

At iterate  170    f=  1.38787D+07    |proj g|=  1.93046D+03
```

```
At iterate  171    f=  1.38787D+07    |proj g|=  1.13749D+03

At iterate  172    f=  1.38787D+07    |proj g|=  1.80680D+02

At iterate  173    f=  1.38787D+07    |proj g|=  3.84356D+02

At iterate  174    f=  1.38787D+07    |proj g|=  2.76636D+03

At iterate  175    f=  1.38787D+07    |proj g|=  9.76633D+02

At iterate  176    f=  1.38787D+07    |proj g|=  3.30204D+02

At iterate  177    f=  1.38787D+07    |proj g|=  1.34253D+03

At iterate  178    f=  1.38787D+07    |proj g|=  1.95179D+03

At iterate  179    f=  1.38787D+07    |proj g|=  1.55222D+03

At iterate  180    f=  1.38787D+07    |proj g|=  1.55836D+03

At iterate  181    f=  1.38787D+07    |proj g|=  5.76959D+02

At iterate  182    f=  1.38787D+07    |proj g|=  2.07444D+02

At iterate  183    f=  1.38787D+07    |proj g|=  1.22993D+03

At iterate  184    f=  1.38787D+07    |proj g|=  1.83989D+03

At iterate  185    f=  1.38787D+07    |proj g|=  2.24251D+03

At iterate  186    f=  1.38787D+07    |proj g|=  1.31254D+03

At iterate  187    f=  1.38787D+07    |proj g|=  4.95930D+02

At iterate  188    f=  1.38787D+07    |proj g|=  1.05696D+03

At iterate  189    f=  1.38787D+07    |proj g|=  9.40357D+02

At iterate  190    f=  1.38787D+07    |proj g|=  1.46320D+03

At iterate  191    f=  1.38787D+07    |proj g|=  7.36171D+02

At iterate  192    f=  1.38787D+07    |proj g|=  4.01782D+02

At iterate  193    f=  1.38787D+07    |proj g|=  9.80970D+02

At iterate  194    f=  1.38787D+07    |proj g|=  6.09178D+02
```

```
At iterate  195    f=  1.38787D+07    |proj g|=  2.88021D+02

At iterate  196    f=  1.38787D+07    |proj g|=  7.87463D+02

At iterate  197    f=  1.38787D+07    |proj g|=  8.96087D+02

At iterate  198    f=  1.38787D+07    |proj g|=  4.52942D+02

At iterate  199    f=  1.38787D+07    |proj g|=  1.72723D+02

At iterate  200    f=  1.38787D+07    |proj g|=  2.63938D+02

At iterate  201    f=  1.38787D+07    |proj g|=  3.30887D+02

At iterate  202    f=  1.38787D+07    |proj g|=  1.17976D+02

At iterate  203    f=  1.38787D+07    |proj g|=  1.46297D+02

At iterate  204    f=  1.38787D+07    |proj g|=  8.12010D+02

At iterate  205    f=  1.38787D+07    |proj g|=  4.59343D+02

At iterate  206    f=  1.38787D+07    |proj g|=  6.98475D+02

At iterate  207    f=  1.38787D+07    |proj g|=  6.77466D+02

At iterate  208    f=  1.38786D+07    |proj g|=  1.25189D+03

At iterate  209    f=  1.38786D+07    |proj g|=  3.24916D+02

At iterate  210    f=  1.38786D+07    |proj g|=  2.19146D+02

At iterate  211    f=  1.38786D+07    |proj g|=  5.95641D+02

At iterate  212    f=  1.38786D+07    |proj g|=  8.65185D+01

At iterate  213    f=  1.38786D+07    |proj g|=  5.62174D+02

At iterate  214    f=  1.38786D+07    |proj g|=  9.62439D+02

At iterate  215    f=  1.38786D+07    |proj g|=  1.18508D+03

At iterate  216    f=  1.38786D+07    |proj g|=  3.61523D+02

At iterate  217    f=  1.38786D+07    |proj g|=  4.63325D+02

At iterate  218    f=  1.38786D+07    |proj g|=  1.37587D+03
```

```
At iterate  219    f=  1.38786D+07    |proj g|=  3.54754D+02

At iterate  220    f=  1.38786D+07    |proj g|=  9.53945D+01

At iterate  221    f=  1.38786D+07    |proj g|=  9.60815D+02

At iterate  222    f=  1.38786D+07    |proj g|=  5.07174D+02

At iterate  223    f=  1.38786D+07    |proj g|=  1.51651D+02

At iterate  224    f=  1.38786D+07    |proj g|=  1.69177D+02

At iterate  225    f=  1.38786D+07    |proj g|=  1.16628D+02

At iterate  226    f=  1.38786D+07    |proj g|=  1.49536D+02

At iterate  227    f=  1.38786D+07    |proj g|=  3.15258D+02

At iterate  228    f=  1.38786D+07    |proj g|=  2.34330D+02

At iterate  229    f=  1.38786D+07    |proj g|=  1.38766D+03

At iterate  230    f=  1.38786D+07    |proj g|=  7.57636D+02

At iterate  231    f=  1.38786D+07    |proj g|=  5.88174D+02

At iterate  232    f=  1.38786D+07    |proj g|=  1.24168D+03

At iterate  233    f=  1.38786D+07    |proj g|=  1.48574D+02

At iterate  234    f=  1.38786D+07    |proj g|=  9.27341D+02

At iterate  235    f=  1.38786D+07    |proj g|=  1.42042D+03

At iterate  236    f=  1.38786D+07    |proj g|=  4.61751D+02

At iterate  237    f=  1.38786D+07    |proj g|=  2.91573D+02

At iterate  238    f=  1.38786D+07    |proj g|=  2.18298D+02

At iterate  239    f=  1.38786D+07    |proj g|=  2.43141D+03

At iterate  240    f=  1.38786D+07    |proj g|=  4.33345D+02

At iterate  241    f=  1.38786D+07    |proj g|=  1.84476D+02

At iterate  242    f=  1.38786D+07    |proj g|=  2.61532D+02
```

```
At iterate  243    f=  1.38786D+07    |proj g|=  7.15925D+02

At iterate  244    f=  1.38786D+07    |proj g|=  1.17276D+03

At iterate  245    f=  1.38786D+07    |proj g|=  1.42723D+03

At iterate  246    f=  1.38786D+07    |proj g|=  1.72870D+03

At iterate  247    f=  1.38786D+07    |proj g|=  4.88020D+02

At iterate  248    f=  1.38786D+07    |proj g|=  2.82709D+02

At iterate  249    f=  1.38786D+07    |proj g|=  1.11907D+02

At iterate  250    f=  1.38786D+07    |proj g|=  2.46166D+02

At iterate  251    f=  1.38786D+07    |proj g|=  1.85512D+02

At iterate  252    f=  1.38786D+07    |proj g|=  5.83439D+02

At iterate  253    f=  1.38786D+07    |proj g|=  4.19015D+02

At iterate  254    f=  1.38786D+07    |proj g|=  1.86069D+02

At iterate  255    f=  1.38786D+07    |proj g|=  1.27028D+02

At iterate  256    f=  1.38786D+07    |proj g|=  2.37299D+02

At iterate  257    f=  1.38786D+07    |proj g|=  3.01346D+02

At iterate  258    f=  1.38786D+07    |proj g|=  2.87895D+03

At iterate  259    f=  1.38786D+07    |proj g|=  4.58686D+02

At iterate  260    f=  1.38785D+07    |proj g|=  1.17042D+02

At iterate  261    f=  1.38785D+07    |proj g|=  4.29038D+02

At iterate  262    f=  1.38785D+07    |proj g|=  4.35273D+02

At iterate  263    f=  1.38785D+07    |proj g|=  2.62256D+02

At iterate  264    f=  1.38785D+07    |proj g|=  1.42082D+02

At iterate  265    f=  1.38785D+07    |proj g|=  4.18270D+02

At iterate  266    f=  1.38785D+07    |proj g|=  1.54288D+02
```

```
At iterate  267    f=  1.38785D+07    |proj g|=  1.71061D+02

At iterate  268    f=  1.38785D+07    |proj g|=  9.68739D+02

At iterate  269    f=  1.38785D+07    |proj g|=  4.69593D+02

At iterate  270    f=  1.38785D+07    |proj g|=  2.31919D+03

At iterate  271    f=  1.38785D+07    |proj g|=  1.26157D+03

At iterate  272    f=  1.38785D+07    |proj g|=  2.38203D+03

At iterate  273    f=  1.38785D+07    |proj g|=  3.71330D+03

At iterate  274    f=  1.38785D+07    |proj g|=  3.96270D+03

At iterate  275    f=  1.38785D+07    |proj g|=  3.14434D+03

At iterate  276    f=  1.38785D+07    |proj g|=  1.39650D+03

At iterate  277    f=  1.38785D+07    |proj g|=  1.23414D+02

At iterate  278    f=  1.38785D+07    |proj g|=  5.82708D+02

At iterate  279    f=  1.38785D+07    |proj g|=  6.23982D+02

At iterate  280    f=  1.38784D+07    |proj g|=  1.70875D+02

At iterate  281    f=  1.38784D+07    |proj g|=  4.98491D+02

At iterate  282    f=  1.38784D+07    |proj g|=  5.50697D+02

At iterate  283    f=  1.38784D+07    |proj g|=  1.05800D+02

At iterate  284    f=  1.38784D+07    |proj g|=  4.46848D+02

At iterate  285    f=  1.38784D+07    |proj g|=  8.01548D+02

At iterate  286    f=  1.38784D+07    |proj g|=  1.02176D+03

At iterate  287    f=  1.38784D+07    |proj g|=  1.21392D+03

At iterate  288    f=  1.38784D+07    |proj g|=  1.95055D+02

At iterate  289    f=  1.38784D+07    |proj g|=  3.47219D+02

At iterate  290    f=  1.38784D+07    |proj g|=  2.99065D+02
```

```
 At iterate  291    f=  1.38784D+07    |proj g|=  8.48272D+02

 At iterate  292    f=  1.38784D+07    |proj g|=  4.52123D+02

 At iterate  293    f=  1.38784D+07    |proj g|=  5.41263D+02

 At iterate  294    f=  1.38784D+07    |proj g|=  2.07413D+02

 At iterate  295    f=  1.38784D+07    |proj g|=  7.58341D+01

 At iterate  296    f=  1.38784D+07    |proj g|=  1.27791D+02

 At iterate  297    f=  1.38784D+07    |proj g|=  1.48761D+02

 At iterate  298    f=  1.38784D+07    |proj g|=  1.31808D+02

 At iterate  299    f=  1.38784D+07    |proj g|=  6.78409D+01

           * * *

 Tit   = total number of iterations
 Tnf   = total number of function evaluations
 Tnint = total number of segments explored during Cauchy searches
 Skip  = number of BFGS updates skipped
 Nact  = number of active bounds at final generalized Cauchy point
 Projg = norm of the final projected gradient
 F     = final function value

           * * *

    N    Tit     Tnf  Tnint  Skip  Nact     Projg        F
   18    299     343      1     0     0   6.784D+01   1.388D+07
  F =   13878416.465636913

 CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
```

```
[9]:   message: CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH
       success: True
        status: 0
           fun: 13878416.465636913
             x: [ 4.890e+00  2.082e-02 …  2.367e-01  2.222e-02]
           nit: 299
           jac: [ 8.350e+00 -5.607e+01 … -6.784e+01 -2.699e+01]
          nfev: 343
          njev: 343
      hess_inv: <18x18 LbfgsInvHessProduct with dtype=float64>
```

0.0.3 make prediction

[10]:
```
# predict
standard_posterior_mean, standard_posterior_var = model.predict_y(Xtest.values)␣
 ↪# predicted mean of GP, predicted variance of GP
posterior_mean = standard_posterior_mean * train_pr_std + train_pr_mean #␣
 ↪transform mean prediction to original scale
posterior_stddev = np.sqrt(standard_posterior_var) * train_pr_std # transform␣
 ↪variance prediction to original scale standard deviation
```

[11]:
```
# put output back into xarray format for calculating RMSE/plotting
posterior_pr = np.reshape(posterior_mean, [86, 96, 144])
posterior_pr_stddev = np.reshape(posterior_stddev, [86, 96, 144])

posterior_pr_data = xr.DataArray(posterior_pr, dims=pr_truth.dims,␣
 ↪coords=pr_truth.coords)
posterior_pr_std_data = xr.DataArray(posterior_pr_stddev, dims=pr_truth.dims,␣
 ↪coords=pr_truth.coords)
```

[12]:
```
# Compute RMSEs
print(f"RMSE at 2050: {get_rmse(pr_truth[35], posterior_pr_data[35])}")
print(f"RMSE at 2100: {get_rmse(pr_truth[85], posterior_pr_data[85])}")
print(f"RMSE 2045-2055: {get_rmse(pr_truth[30:41], posterior_pr_data[30:41]).
 ↪mean()}")
print(f"RMSE 2090-2100: {get_rmse(pr_truth[75:], posterior_pr_data[75:]).
 ↪mean()}")
print(f"RMSE 2050-2100: {get_rmse(pr_truth[35:], posterior_pr_data[35:]).
 ↪mean()}")

# RMSE for average field over last 20 years
print(f"RMSE average last 20y: {get_rmse(pr_truth[-20:].mean(dim='time'),␣
 ↪posterior_pr_data[-20:].mean(dim='time'))}")
```

```
RMSE at 2050: 0.38358933179796506
RMSE at 2100: 0.5677319656198956
RMSE 2045-2055: 0.49878060177806066
RMSE 2090-2100: 0.5261891933905046
RMSE 2050-2100: 0.5337930762172088
RMSE average last 20y: 0.17560346542905994
```

[13]:
```
from matplotlib import colors
# plotting predictions
divnorm = colors.TwoSlopeNorm(vmin=-2., vcenter=0., vmax=5)
diffnorm = colors.TwoSlopeNorm(vmin=-2., vcenter=0., vmax=2)

## Temperature
proj = ccrs.PlateCarree()
```

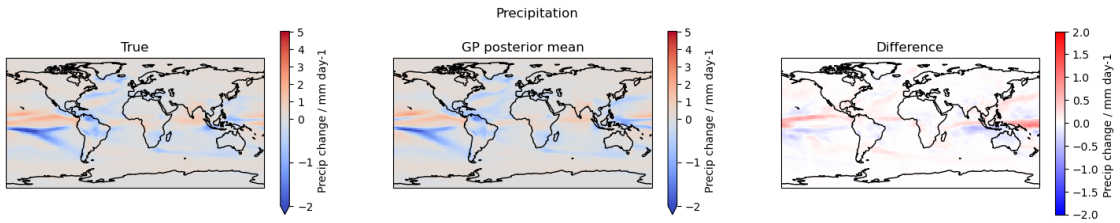
```
fig = plt.figure(figsize=(18, 3))
fig.suptitle('Precipitation')

# Test
plt.subplot(131, projection=proj)
pr_truth.sel(time=slice(2050,None)).mean('time').plot(cmap="coolwarm",␣
 ↪norm=divnorm,
                                cbar_kwargs={"label":"Precip change / mm day-1 "})
plt.gca().coastlines()
plt.setp(plt.gca(), title='True')

# Emulator
plt.subplot(132, projection=proj)
posterior_pr_data.sel(time=slice(2050,None)).mean('time').plot(cmap="coolwarm",␣
 ↪norm=divnorm,
                        cbar_kwargs={"label":"Precip change / mm day-1 "})
plt.gca().coastlines()
plt.setp(plt.gca(), title='GP posterior mean')

# Difference
difference = pr_truth - posterior_pr_data
plt.subplot(133, projection=proj)
difference.sel(time=slice(2050,None)).mean('time').plot(cmap="bwr",␣
 ↪norm=diffnorm,
                cbar_kwargs={"label":"Precip change / mm day-1 "})
plt.gca().coastlines()
plt.setp(plt.gca(), title='Difference')
```

[13]:
```
[Text(0.5, 1.0, 'Difference')]
```



[14]:
```
model
```

[14]:
```
<gpflow.models.gpr.GPR object at 0x1544c42f6590>
```

name	class	transform	prior	trainable	shape	dtype	value

```
  GPR.mean_function.c                  Parameter   Identity
True          (1,)      float64   [0.02222345]


  GPR.kernel.kernels[0].variance       Parameter   Softplus
True          ()        float64   0.7036111603622266


  GPR.kernel.kernels[0].lengthscales   Parameter   Softplus
True          ()        float64   4.89768


  GPR.kernel.kernels[1].variance       Parameter   Softplus
True          ()        float64   5.782079471538937e-23


  GPR.kernel.kernels[1].lengthscales   Parameter   Softplus
True          ()        float64   45.4764


  GPR.kernel.kernels[2].variance       Parameter   Softplus
True          ()        float64   0.019000624069268043


  GPR.kernel.kernels[2].lengthscales   Parameter   Softplus
True          (5,)      float64   [1.91525, 74.69821, 52.75336…


  GPR.kernel.kernels[3].variance       Parameter   Softplus
True          ()        float64   0.27657674195095316


  GPR.kernel.kernels[3].lengthscales   Parameter   Softplus
True          (5,)      float64   [6.52178, 7.57278, 12.40975…


  GPR.likelihood.variance              Parameter   Softplus + Shift
True          ()        float64   0.8184576881089053
```

[ ]: