

Course Project Part 1: Investigation of the Exponential Distribution

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Overview

In this project we are going to investigate the Exponential Distribution ($X \sim \text{Exp}(\lambda)$) and compare it with the Central Limit Theorem (CLT). The mean of the exponential distribution and standard deviation are $1/\lambda$.

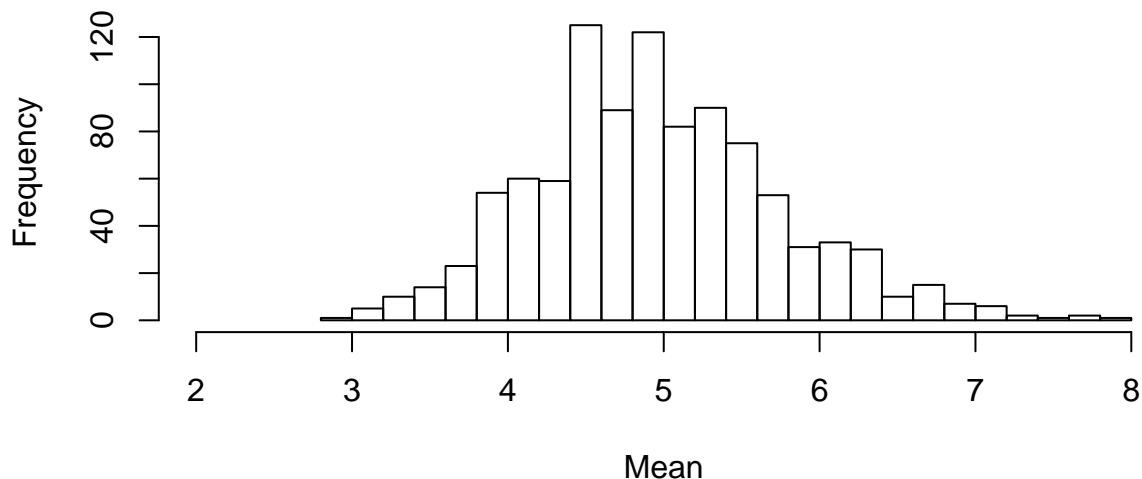
Simulation parameters:

- rate, $\lambda = 0.2$ (for all of the simulations),
- number of observation, $n = 40$,
- number of simulations, $\text{sims} = 1000$.

Simulations

```
# set parameters for simulation
lambda <- 0.2
n <- 40
sims <- 1000
# run simulations
exp <- replicate(sims, rexp(n = n, rate = lambda))
# calculate mean of every simulation
exp_means <- apply(x = exp, MARGIN = 2, FUN = mean)
# plot histogram of means
hist(exp_means, breaks = 30, xlim = c(2,8), main = "Distribution of Means", xlab = "Mean")
```

Distribution of Means

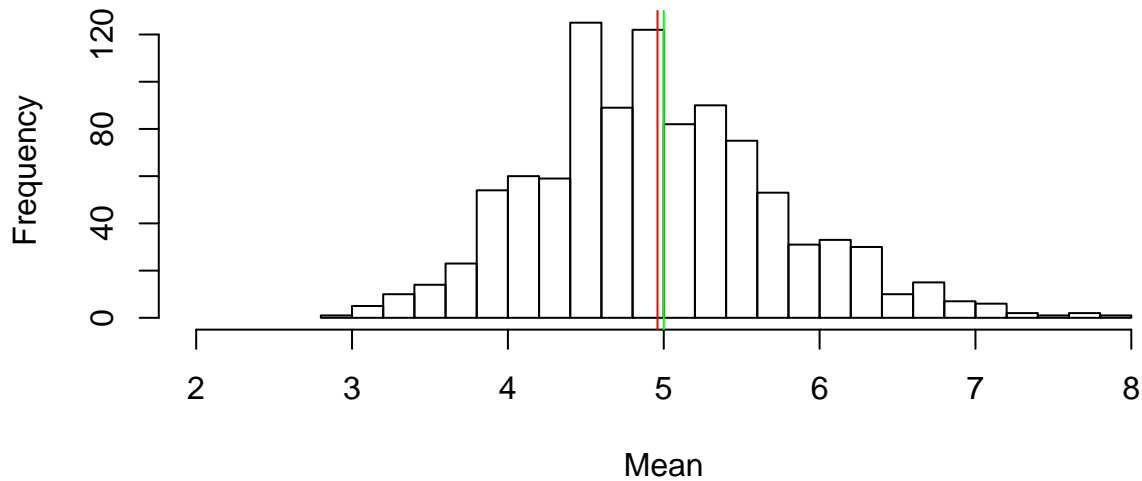


Results

1. Sample Mean versus Theoretical Mean

Lets calculate the mean of our simulations means distribution and plot it and on histogram.

Distribution of Means



The sample mean 4.9598071 (red line) is very close to the theoretical mean 5 (green line)

2. Sample Variance versus Theoretical Variance

```
exp_var <- var(exp_means)
exp_sd <- sd(exp_means)
t_var <- (1/lambda)^2 / n
t_sd <- 1/(lambda * sqrt(n))
```

Variances and Standart Deviations are very close too:

standart deviation sample/ theoretical: 0.7921655/ 0.7905694

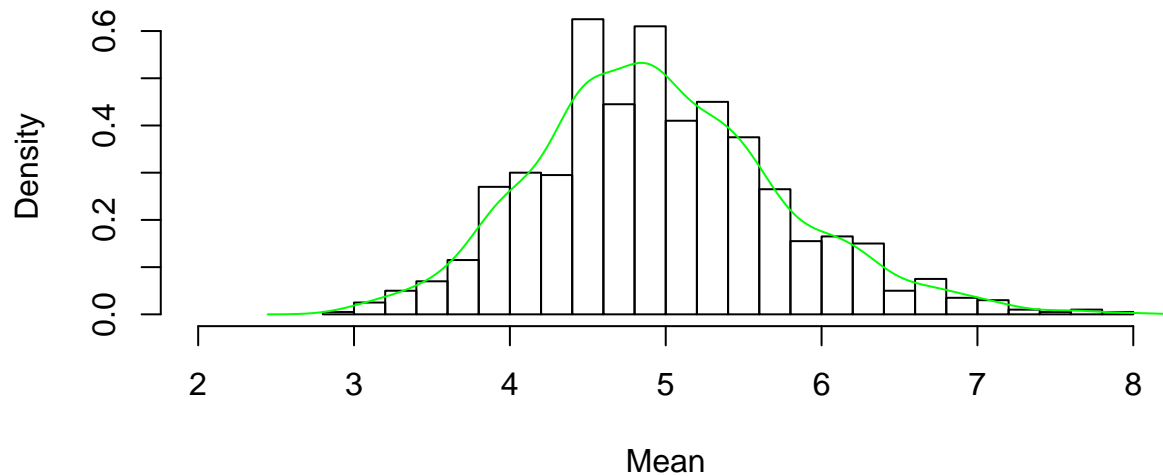
variance sample/ theoretical: 0.6275263 / 0.625

3. Distribution

The distribution of means looks like normal destribution curve:

```
hist(exp_means, probability = TRUE, breaks = 30, xlim = c(2,8),
     main = "Distribution of Means", xlab = "Mean")
lines(density(exp_means), col = "green")
```

Distribution of Means



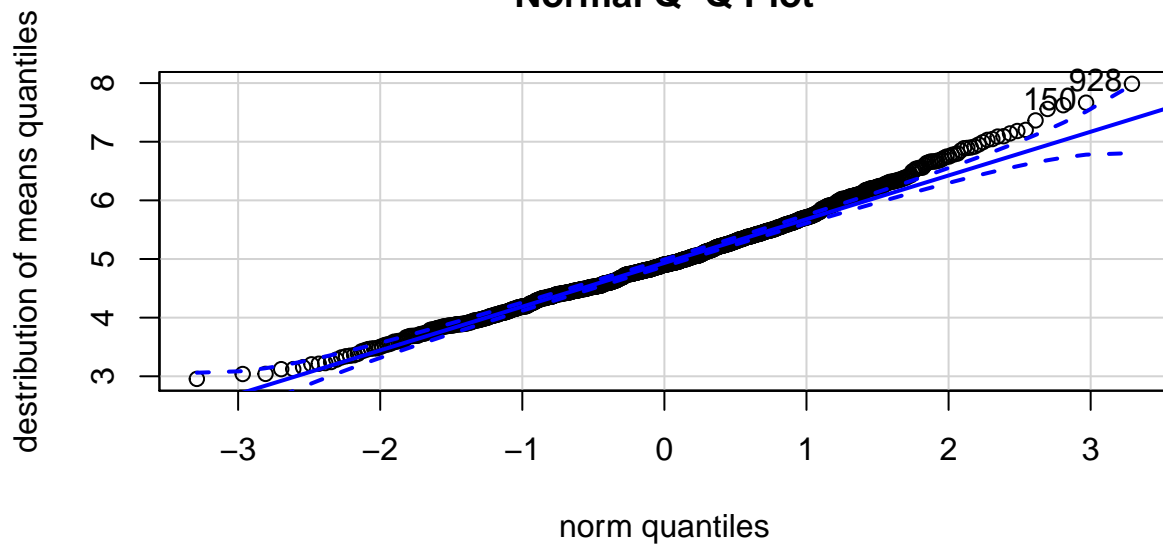
Lets look at quantile-quantile plot:

```
library(car)
```

```
## Loading required package: carData
```

```
qqPlot(exp_means, ylab = "distribution of means quantiles", main = "Normal Q-Q Plot")
```

Normal Q-Q Plot



```
## [1] 928 150
```

Most of the points lie very close to a stright line, whith shows normal quantiles.

And finally we can count the 95% confidence interval of mean:

```
mean(exp_means)+ c(-1,1) * qnorm(0.975) * sd(exp_means)/sqrt(n)
```

```
## [1] 4.714317 5.205297
```