Chapter 1

Numerical Methods

1.1 Thomas Method: Tridiagonal Systems

Consider a single row in the tri-diagonal system,

$$\beta_j \phi_{j-1} + D_j \phi_j + \alpha_j \phi_{j+1} = C_j$$

Suppose the perceding row in the tri-diagonal system,

$$D_{j-1} \phi_{j-1} + \alpha_{j-1} \phi_j = C_{j-1}$$

The forward sweep of the thomas algorithm seeks to eliminate the sub-diagonal terms of the tri-diagonal system. For the two rows of the tri-diagonal system shown above, the sub-diagonal term is ϕ_{j-1}. Manpulating the perceding row,

$$D_{j-1}\beta_j \phi_{j-1} + \alpha_{j-1}\beta_j \phi_j = \beta_j C_{j-1}$$

Manipulating the following row,

$$\beta_j D_{j-1} \phi_{j-1} + D_j D_{j-1} \phi_j + \alpha_j D_{j-1} \phi_{j+1} = D_{j-1} C_j$$

Subtracting the following row by the perceding row

$$\beta_j D_{j-1} \phi_{j-1} + D_j D_{j-1} \phi_j + \alpha_j D_{j-1} \phi_{j+1} - D_{j-1}\beta_j \phi_{j-1} - \alpha_{j-1}\beta_j \phi_j = D_{j-1} C_j - \beta_j C_{j-1}$$

$$D_j D_{j-1} \phi_j - \alpha_{j-1}\beta_j \phi_j + \alpha_j D_{j-1} \phi_{j+1} = D_{j-1} C_j - \beta_j C_{j-1}$$

$$[D_j D_{j-1} - \alpha_{j-1}\beta_j]\phi_j + \alpha_j D_{j-1} \phi_{j+1} = D_{j-1} C_j - \beta_j C_{j-1}$$

$$\phi_j + \left[\frac{\alpha_j D_{j-1}}{D_j D_{j-1} - \alpha_{j-1}\beta_j}\right] \phi_{j+1} = \frac{D_{j-1} C_j - \beta_j C_{j-1}}{D_j D_{j-1} - \alpha_{j-1}\beta_j}$$

The results above would Complete the forward sweep of the thomas algorithm for the first row until the sec ond last row. The last row is simply a more speCific case of the expression above wherein $a_j = 0$. Substituting for only the last row,

$$\phi_j = \frac{D_{j-1}C_j - \beta_j C_{j-1}}{D_j D_{j-1} - \alpha_{j-1}\beta_j}$$

The last row in the tri-diagonal system is solved after the forward sweep of the thomas algorithm. After t he forward sweep of the thomas algorithm, the perceding row,

$$\phi_j + \alpha_j \phi_{j+1} = C_j$$

The following row,

$$\phi_{j+1} = C_{j+1}$$

Substituting the following row to the perceding row,

$$\phi_j + \alpha_j C_{j+1} = C_j$$

$$\phi_j = C_j - \alpha_j C_{j+1}$$

This would be true because the main diagonal after the forward sweep of the thomas algorithm would all be just 1. The Thomas algorithm implemented in fortran is shown below,