# 0.1

### 0.1.1

## 0.2 Numerical Formulation of Navier Stokes

### 0.2.1 Conservation Equations

Conservation of mass is shown below,

$$\frac{\partial}{\partial t}[\rho] + \nabla \cdot (\rho \bar{v_f}) = 0$$

Conservation of linear momentum,

$$\frac{\partial}{\partial t}(\rho \bar{v_f}) + \nabla \cdot (\rho \bar{v_f} \bar{v_f}) = -\nabla P_r + \nabla \cdot \bar{\bar{\tau}} + \bar{f}$$

Conservation of energy,

$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{1}{2}|v_f|^2\right)\right] + \nabla \cdot \left[\rho \bar{v_f}\left(e + \frac{1}{2}|\bar{v_f}|^2\right)\right] = -\nabla \cdot (\rho \bar{v_f}) + \nabla \cdot (\bar{\bar{\tau}} \cdot \bar{v_f}) - \nabla \cdot \bar{q} + \bar{f} \cdot \bar{v_f} + Q$$

### 0.2.2 Viscous Stress Relations

The viscous stress tensor is symmetric. Using the Stokes' hypothesis, the viscous stress tensor,

$$\tau_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial v_k}{\partial x_k}\delta_{ij}$$

wherein dynamic viscosity $\mu$ is largely a function of temperature,

$$\mu = \mu(T)$$

### 0.2.3 Empirical Assumptions

Fourier's heat conduction law,

$$q_i = -k_f\frac{\partial T}{\partial x_i}$$

wherein the conductivity of the fluid $k_f$ may be a function of temperature,

$$k_f = k_f(T)$$

The last relation needed is the ideal gas law,

$$P_r = (\gamma - 1)\rho e \quad , \quad T = (\gamma - 1)e/R$$

### 0.2.4 Vector form

In 3-dimensions, there are 5 equations for the Navier-stokes, 1 continuity, 3 for momentum in each direction $x_1$, $x_2$ and $x_3$ and the last 1 for conservation of energy. The Navier-Stokes equation in vector form can be rewritten as follows,

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} = \frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y} + \frac{\partial G_v}{\partial z} + S$$

wherein the solution vector $U$ is shown below,

$$U = \begin{bmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho v_3 \\ E_t \end{bmatrix}$$

wherein $E_t$ is defined as,

$$E_t = \rho e + \frac{1}{2}\rho(v_1^2 + v_2^2 + v_3^2)$$

To obtain the velocities in 3 dimensions,

$$v_1 = \frac{\rho v_1}{\rho} \quad , \quad v_2 = \frac{\rho v_2}{\rho} \quad , \quad v_3 = \frac{\rho v_3}{\rho}$$

Susbtituting in terms of the elements of the solution vector,

$$v_1 = \frac{U_2}{U_1} \quad , \quad v_2 = \frac{U_3}{U_1} \quad , \quad v_3 = \frac{U_4}{U_1}$$

The vectors $E$, $F$ and $G$ are shown below,

$$E = \begin{bmatrix} \rho v_1 \\ \rho v_1^2 + P_r \\ \rho v_1 v_2 \\ \rho v_1 v_3 \\ (E_t + P_r)v_1 \end{bmatrix} \quad , \quad F = \begin{bmatrix} \rho v_2 \\ \rho v_1 v_2 \\ \rho v_2^2 + P_r \\ \rho v_2 v_3 \\ (E_t + P_r)v_2 \end{bmatrix} \quad , \quad G = \begin{bmatrix} \rho v_3 \\ \rho v_1 v_3 \\ \rho v_2 v_3 \\ \rho v_3^2 + P_r \\ (E_t + P_r)v_3 \end{bmatrix}$$

The vectors $E_v$, $F_v$ and $G_v$ are defined below,

$$E_v = \begin{bmatrix} 0 \\ \tau_{11} \\ \tau_{21} \\ \tau_{31} \\ v_1\tau_{11} + v_2\tau_{12} + v_3\tau_{13} + g_1 \end{bmatrix} \quad , \quad F_v = \begin{bmatrix} 0 \\ \tau_{12} \\ \tau_{22} \\ \tau_{32} \\ v_1\tau_{21} + v_2\tau_{22} + v_3\tau_{23} + g_2 \end{bmatrix}$$

$$G_v = \begin{bmatrix} 0 \\ \tau_{13} \\ \tau_{23} \\ \tau_{33} \\ v_1\tau_{31} + v_2\tau_{32} + v_3\tau_{33} + g_3 \end{bmatrix}$$

$$S = 0$$

### 0.2.5