Control Archives

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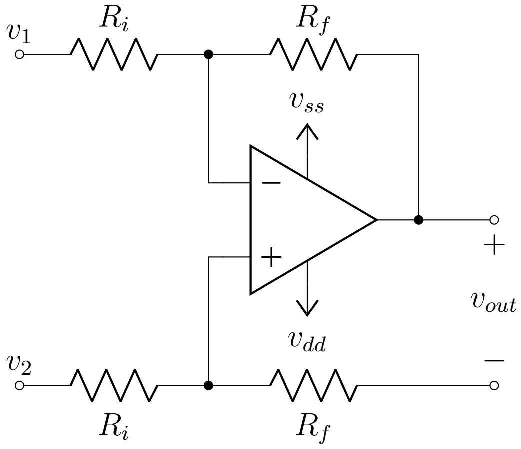
Practically, the operational amplifier is a 5-terminal device: source voltage, drain voltage, positive terminal, negative terminal, and output. Both the source voltage and drain voltage are used to power the various nmosfets and pmosfets inside the op amplifier. The ideal operational amplifier is a device with infinite gain. If the voltage at the positive terminal is greater than the voltage at the negative terminal, then the output will be positive infinity. However, practically, the highest output the operational amplifier can output is the source voltage. Therefore, if the voltage at the positive terminal is greater than the negative terminal, then the output will be the source voltage. Likewise, following the same reasoning, if the voltage at the negative terminal is greater than the positive terminal, then the output will be the drain volage. An operational amplifier could act as a comparator, but it is not recommended.

Chapter 3

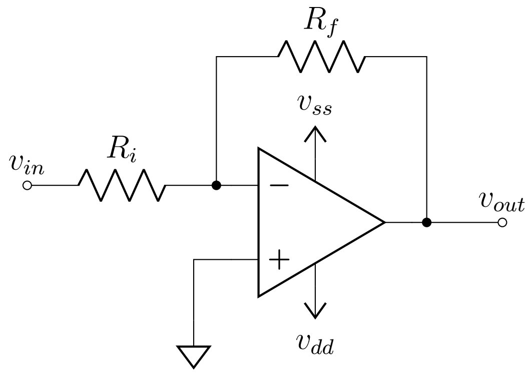
Op-Amplifier Arrangements

3.1 Differential Amplifier



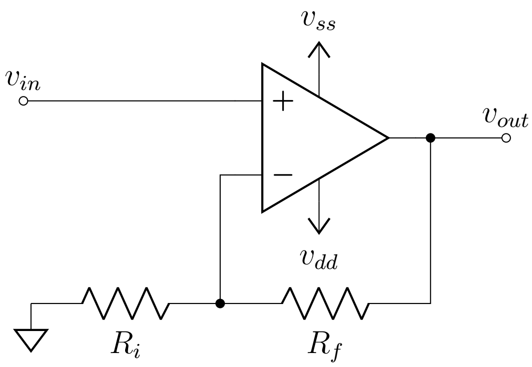
$$v_{out} = \frac{R_f}{R_i}(v_2 - v_1)$$

3.2 Inverting Amplifier



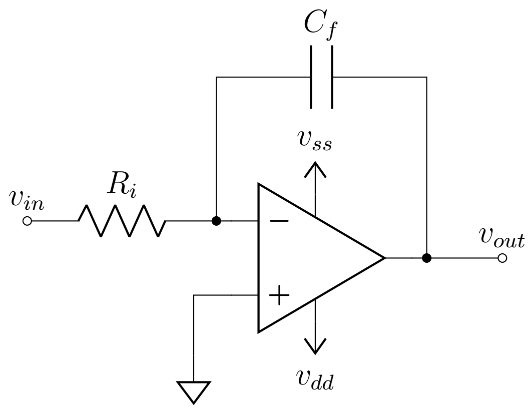
$$v_{out} = -\frac{R_f}{R_i} v_{in}$$

3.3 Non-Inverting Amplifier



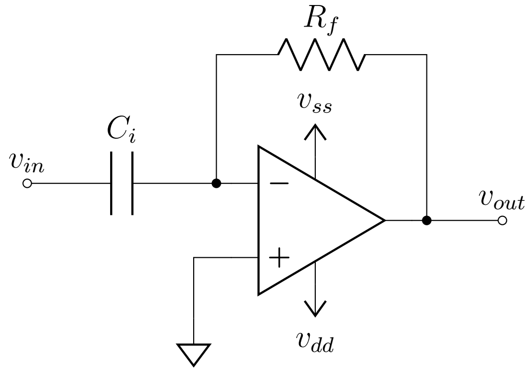
$$v_{out} = \left(1 + \frac{R_f}{R_i}\right) v_{in}$$

3.4 Integrating Amplifier



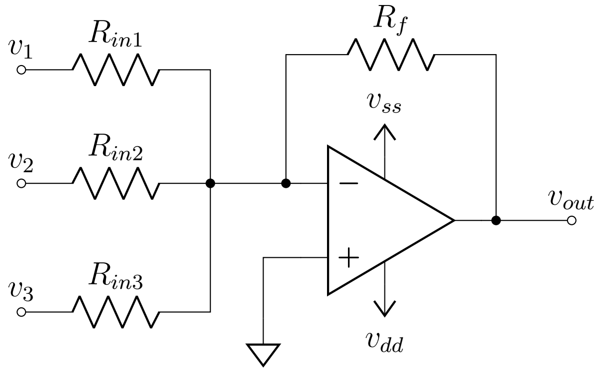
$$\frac{d}{dt}[v_{out}] = -\left(\frac{1}{R_iC_f}\right)v_{in}$$

3.5 Differentiating Amplifier



$$v_{out} = -R_f C_i \frac{d}{dt}[v_{in}]$$

3.6 Summing Amplifier



$$v_{out} = -R_f \left[\frac{v_1}{R_{in1}} + \frac{v_2}{R_{in2}} + \frac{v_3}{R_{in3}} \right]$$

Chapter 4

State-Space Implementations in Circuits

Chapter 5

Long-Term Behaviour of Circuits

5.1 Final Value Theorem

The final value theorem for an arbitrary function $f(t)$ is written as,

$$\lim_{t\to\infty}[f(t)] = \lim_{s\to 0}[sF(s)]$$

wherein $F(s)$ represents the laplace transform of $f(t)$. The proof of this particular form of the final value theorem is shown below,

$$\lim_{s\to 0}\left\{\mathcal{L}\left[f'(t)\right]\right\} = \lim_{s\to 0}\left\{\int_0^\infty e^{-st}f'(t)\,dt\right\} = \int_0^\infty f'(t)\,dt$$

By fundamental theorem of calculus, and change of variable,

$$\lim_{s\to 0}\left\{\mathcal{L}\left[f'(t)\right]\right\} = \lim_{r\to\infty}\left\{f(r) - f(0)\right\} = \lim_{t\to\infty}\left\{f(t) - f(0)\right\}$$

Taking the approach of the Laplace Transform of derivatives,

$$\mathcal{L}\left[f'(t)\right] = s\mathcal{L}f(t) - f(0) = sF(s) - f(0)$$

Therefore, taking the limits as before,

$$\lim_{s\to 0}\left\{\mathcal{L}\left[f'(t)\right]\right\} = \lim_{s\to 0}\left\{sF(s)\right\}\} - f(0)$$

By equating the $\lim_{s\to 0}\left\{\mathcal{L}\left[f'(t)\right]\right\}$ to each other,

$$\lim_{t\to\infty}\left\{f(t)\right\} - f(0) = \lim_{s\to 0}\left\{sF(s)\right\}\} - f(0)$$

This completes the proof,

$$\lim_{t\to\infty}\left\{f(t)\right\} = \lim_{s\to 0}\left\{sF(s)\right\}\}$$

5.2 Impulse Response

The transfer function $G(s)$ is defined as

$$G(s) = \frac{Y(s)}{U(s)}$$

wherein the $Y(s)$ is the output and $U(s)$ is the input, both in the laplace domain. In the laplace domain, the delta-dirac impulse function with arbitrary magnitude μ_0,

$$\mathcal{L}[\mu_0\, \delta(t-c)] = \mu_0 e^{-cs}$$

If the system described by the transfer function $G(s)$ has an input of the dirac delta function of arbitrary magnitude,

$$G(s) = \mu_0 e^{-cs}\, Y(s)$$

Manipulating for $Y(s)$ to be the subject of the equation,

$$Y(s) = \left(\frac{e^{cs}}{\mu_0}\right) G(s)$$

To find the long term behaviour of the output of the system $\lim\limits_{t\to\infty} \big[y(t)\big]$, we should consult the final value theorem. The final value theorem for an arbitrary function $f(t)$,

$$\lim_{t\to\infty} [f(t)] = \lim_{s\to 0} [sF(s)]$$

By performing the substitution $f(t) = y(t)$ and $F(s) = Y(s)$, wherein $y(t)$ represents the output function in the time domain and $Y(s)$ represents the output function in the laplace domain accordingly,

$$\lim_{t\to\infty} [y(t)] = \lim_{s\to 0} [sY(s)]$$

If the input function is the dirac-delta function as shown previously, then $Y(s) = \left(\frac{e^{cs}}{\mu_0}\right) G(s)$. Therefore,

$$\lim_{t\to\infty} [y(t)] = \lim_{s\to 0} \left[s \left(\frac{e^{cs}}{\mu_0}\right) G(s) \right]$$

A famous identity for limits of two arbitrary functions $q(t)$ and $p(t)$,

$$\lim_{t\to t_0} \big[p(t) \times q(t)\big] = \lim_{t\to t_0} \big[p(t)\big] \times \lim_{t\to t_0} \big[q(t)\big]$$

By the identity above,

$$\lim_{t\to\infty} [y(t)] = \lim_{s\to 0} \big[sG(s)\big] \times \lim_{s\to 0} \left[\frac{e^{cs}}{\mu_0}\right] = \frac{1}{\mu_0} \lim_{s\to 0} \big[sG(s)\big]$$

5.3 Step Response

Let $G(s)$ represent the transfer function of some system in the laplace domain, $Y(s)$ represent the output of some system in the laplace domain, and $U(s)$ represent the input in the laplace domain.

$$Y(s) = G(s)U(s)$$

Consider the heaviside function $u(t-c)$ wherein c is some arbitrary time the heaviside function "turns on". The laplace transform of the arbitray heaviside function,

$$\mathcal{L}[u(t-c)] = \frac{e^{-}cs}{s}$$

If the input $u(t) = \mu_0$ wherein μ_0 is a constant for all t, then

$$\mu_0 G(0) = \lim_{t\to\infty}[y(t)]$$

5.4 Pure Sinusoid Response

The convolution of two arbitrary functions $f(t)$ and $g(t)$ are defined as

$$f * g(t) = g * f(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau = \int_0^t g(\tau)f(t-\tau)\,d\tau$$

Suppose a system has a transfer function $G(s)$ and the output and input in the laplace domain respectively is $Y(s)$ and $U(s)$.

$$G(s) = \frac{Y(s)}{U(s)}$$

Therefore,

$$y(t) = \mathcal{L}^{-1}\left[G(s)U(s)\right] = \mathcal{L}^{-1}\left[G(s)\right] * \mathcal{L}^{-1}\left[U(s)\right] = g(t) * u(t) = \int_0^t g(\tau)u(t-\tau)\,d\tau$$

wherein $\mathcal{L}^{-1}G(s) = g(t)$, and $\mathcal{L}^{-1}U(s) = u(t)$. Here, the function $u(t)$ does not represent the heaviside unit function. Assuming that the input function is reasonably well-behaved and without loss of generality,

$$u(t) = \sum_{k=-\infty}^{\infty}\left[a_k e^{-ik\omega_0 t}\right] \quad , \quad a_k = \frac{1}{\tau}\int_0^{\tau} e^{ik\omega_0 t} f(t)dt \quad , \quad \omega_0 = 2\pi/\tau$$

Substituting the Fourier representation of the input function,

$$y(t) = \int_0^t g(\tau)\sum_{k=-\infty}^{\infty}\left[a_k e^{-ik\omega_0(t-\tau)}\right]\,d\tau = \int_0^t g(\tau)\sum_{k=-\infty}^{\infty}\left[a_k e^{ik\omega_0\tau}e^{-ik\omega_0 t}\right]\,d\tau$$

$$y(t) = \sum_{k=-\infty}^{\infty}\left[\int_0^t g(\tau)e^{ik\omega_0\tau}\,d\tau a_k e^{-ik\omega_0 t}\right]$$

Observing the long term-behaviour of the output, $t = \infty$. Therefore,

$$y(t) = \sum_{k=-\infty}^{\infty}\left[\int_0^{\infty} g(\tau)e^{ik\omega_0\tau}\,d\tau a_k e^{-ik\omega_0 t}\right]$$

The term $\int_0^{\infty} g(\tau)e^{ik\omega_0\tau}\,d\tau$ represents a laplace transform with $s = -ik\omega_0$. Therefore,

$$G(-ik\omega_0) = \int_0^{\infty} g(\tau)e^{ik\omega_0\tau}\,d\tau$$

By substitution,

$$y(t) = \sum_{k=-\infty}^{\infty}\left[G(-ik\omega_0)a_k e^{-ik\omega_0 t}\right]$$

For real sinusoidal inputs $u(t) = k\sin(\omega t)$,

$$\lim_{t\to\infty}[y(t)] = k|G(i\omega)|\sin\{\omega t + arg[G(i\omega)]\}$$

Chapter 6

State-Space