Chapter 1

Potential Flows

Potential and inviscid flows are flows wherein the effects of viscosity is neglected. The degeneracy from the general Navier Stokes equation is shown below,
Consider the continuity differential governing equation,

$$0 = \frac{\partial}{\partial t}[\rho] + \nabla \cdot (\rho \bar{v_f})$$

For a steady-state flow, $\frac{\partial}{\partial t}[\rho] = 0$. Substituting to the continuity differential governing equation,

$$0 = \nabla \cdot (\rho \bar{v_f})$$

1.1 Compressible Potential Flow

For the compressible potential flow, let the potential function ψ_c be defined as,

$$\rho \bar{v_f} = \nabla \psi_c$$

Substituting the definition of the potential function into the steady state continuity differential governing equation,

$$0 = \nabla \cdot (\nabla \psi_c) = \nabla^2 \psi_c$$

In cartesian coordinates, this yields,

$$0 = \frac{\partial^2}{\partial x^2}[\psi_c] + \frac{\partial^2}{\partial y^2}[\psi_c] + \frac{\partial^2}{\partial z^2}[\psi_c]$$

1.2 Incompressible Potential Flow

Due to the nature of the fluid, the density could be considered a scalar constant. The $\frac{\partial}{\partial t}[\rho]$ term in this case is evaluates to zero under two conditions: steady state means that the gradient is unchanging, but also since density is non-changing, this particular term also evaluates to zero. Therefore, for for incompressible flows, the potential flow function would still be applicable for non-steady fluid states.

$$0 = \nabla \cdot (\rho \bar{v_f}) = \rho \nabla \cdot (\bar{v_f})$$

Division of both sides by ρ,

$$0 = \nabla \cdot (\bar{v_f})$$

For the incompressible potential flow, let the potential function ψ_i be defined as,

$$\bar{v_f} = \nabla \psi_i$$

By substituting the fluid velocity field into the continuity degenerate differential form,

$$0 = \nabla \cdot (\nabla \psi_i) = \nabla^2 \psi_i = \frac{\partial^2}{\partial x^2}[\psi_i] + \frac{\partial^2}{\partial y^2}[\psi_i] + \frac{\partial^2}{\partial z^2}[\psi_i]$$