Aircraft Sizing Algorithm Documentation

Team 7:
Cooper LeComp, John A. Papas Dennerline,
Ian Greene, Christos Levy, Daniel Qi, Hans Suganda

27th August 2022

Contents

1 Rationale

We need to determine 2 very important things:

1. The thrust to weight ratio affects:
   - acceleration:
   - climb rate:
   - maximum speed:
   - turn rate:
   - Aircraft Endurance:
2. The stall speed is directly determined by the wing loading and the maximum lift coefficient. Wing Loading will affect:
   - stall speed
   - induced drag
   - take-off distance

2 Sizing Methodology

1. First Method:
   (a) Firstly use the stall speed to determine wing loading Stall speed should be independent of engine size
   (b) After figuring out wing loading, determine the thrust to weight ratio based on takeoff distance Rate of climb
2. Second Alternative Method:
   (a) Determine the Thrust to Weight Ratio
   (b) Determine the wing loading by direct computation

3 Common Definitions

q is the dynamic pressure of an aircraft flying. It is defined as

$$q = \frac{1}{2}\rho v^2$$

wherein ρ represents the density of the air, v represents the velocity the aircraft is flying at. The `Matlab` script below defines the global variables that is needed for this entire simulation,

```
%%Global variable list
global g; %This is local gravitational acceleration constant (m/s^2)
global L_wi; %This is a wing loading (N/m^2)
global T_W; %This is the thrust to weight ratio (dimensionless)
global rho; %This is the air density (kg/m^3)
global v_stall; %This is the stall speed of our aircraft (m/s)
global c_Lmax; %This is an estimation of how good is our biggest possible lift coefff
global q; %This is the dynamic pressure
```

```
global c_D0; %This is the drag coefficient near stall speed
global A_r; %Aspect ratio of the main wing
global e; %efficiency of the wing due to deviating from elliptic lift distribution
global G; %Gradient of ascent = rise of flight/run of flight
global endurance; %This is how long the aircraft should be flying in terms of seconds
global Motor_Power; %This is how much power the motor consumes (J/s)
global Batt_Specific_Energy; %This has units of (J/kg)
global Batt_Mass; %This has units of kg, this is the mass of the battery
global Engine_Thrust; %This is the engine thrust of our aircraft (Newtons)
```

The script below handles the global variable initializations,

```
%%Give Definition of Global variable
Global_Variables;

%%Variables which are known/guessed (independent)

%Variables Needed to Compute the Wing Loading and Thrust to Weight Ratio
g = 9.81; %This is the local gravitational accel (m/s^2)
rho = 1.225; %This is the density of the air
v_stall = 8; %This is the estimation for stall speed
c_Lmax = 1.2; %the maximum lift coefficient we can get
climb_angle = 25; %This is from the RFP
c_D0 = 0.03; %This is the drag coefficient near stall speed
A_r = 9; %Aspect ratio of the main wing, typical values
e = 0.8; %efficiency of the wing due to deviating from elliptic lift distribution

%Variables that are needed to compute the weight of the aircraft
Engine_Thrust = ; %This is force produced by engine, units are in Newtons (N)

%Variables that are needed to compute the weight of the powerplant system
endurance = 900; %seconds of flight. This corresponds to 15 minutes of flight
Motor_Power = ; %This is the power of the motor (W)
Batt_Specific_Energy = ; %This is Energy/Mass, so its unit has to be (J/kg)

%%Variables that are dependent on other variables (dependent)
q = 1/2*rho*v_stall^2; %This is the dynamic pressure
G = tan(deg2rad(climb_angle)); %Gradient of ascent = rise of flight/run of flight
```

4 Stall Speed Constraint

Wing loading is basically the weight of the aircraft divided by the total wing area.

$$L_{wi} = \frac{W}{S} = \frac{1}{2}\rho v_{stall}^2 c_{L,max}$$

Our typical would be around $c_{L,max} \approx 1.2 \rightarrow 1.5$, Sweep only reduces your maximum coefficient of lift. The `Matlab` implementation is shown below,

```
%Give Definition of Global variable
Global_Variables;

%This Matlab script is used to compute the wing loading using the stall speed of the aircraft.
%We are getting the stall speed from the Requirements for the aircraft.
L_wi = 1/2*rho*(v_stall^2)*c_Lmax;
```

5 Climb Rate Constraint

The cimb rate G is defined as the ratio between vertical and horizontal distance travelled when the aircraft is climbing D is going to represent drag.

$$\frac{D}{W} = \frac{T}{W} - G \tag{1}$$

There is also another relation for drag to weight ratio,

$$\frac{D}{W} = \frac{qc_{D,0}}{L_{wi}} + L_{wi}\frac{1}{q\pi A_r e} \tag{2}$$

wherein A_r is the aspect ratio of the wings, and e represents the efficiency factor due to deviating from the elliptic lift distribution. Substituting equation 1 into equation 2 and solving for $\frac{T}{W}$,

$$\frac{D}{W} = \frac{T}{W} - G = \frac{qc_{D,0}}{L_{wi}} + L_{wi}\frac{1}{q\pi A_r e}$$

$$\frac{T}{W} = \frac{qc_{D,0}}{L_{wi}} + L_{wi}\frac{1}{q\pi A_r e} + G$$

wherein $c_{D,0}$ represents the zero-lift drag coefficient of an aircraft. For a clean propeller aircraft, $c_{D,0} = 0.02$. For a dirty propeller aircraft, then $c_{D,0} = 0.03$. The expression for thrust to weight ratio computations are implemented below,

```
%Give Definition of Global variable
Global_Variables;

%This computes the thrust to weight ratio based on the climb rate
T_W  = (q*c_D0)/(L_wi) + L_wi/(q*pi*A_r*e) + G;
```

6 Estimation of Motor Power Rating

The equation below was found in page 118,

$$\frac{T}{W} = \left(\frac{\eta_p}{V}\right)\left(\frac{P}{W}\right)$$

wherein η_p represents the propeller efficiency, P represents power of the engines, W represents the weight of the aircraft, V represents the true air speed of the aircraft.

7 Sizing Method

1. From the RFP, compute 2 of the most arguably important parameters: Wing Loading and Thrust to Weight Ratio
2. Choose a motor-propeller combination that exists within the market, and figure out:
   (a) The thrust that the setup can produce.
   (b) The power that the motor will consume.
3. Based on the Thrust to Weight Ratio and the Thrust that the motor-propeller setup produces, figure out the loaded weight of the entire aircraft.

4. Based on the power consumption of the motor and the required endurance, as well as the specific energy of the batteries, determine the weight of the batteries.

5. Based on the Wing Loading, and the loaded weight of the aircraft, figure out the weight of the wings.

   (a) Based on the wing loading and weight of the loaded aircraft, figure out the area of the aircraft's main wings.

   (b) Based on the area of the aircraft's main wings and the material of choice, estimate the weight of the wings.

6. We already know a few things: Weight of the propulsion system (batteries, motor, propeller), Weight of the Lift Devices (Weight of the Wing), Weight of the Payload, Weight of the Aircraft. Subtract the weight of all the known components from the total weight of the aircraft. This is essentially the weight for the fuselage and tail and excess margin mass. We seek to figure out a reasonable number for this fuselage weight.

7. If the fuselage and excess weight quota is too low, choose a propeller motor combination that gives a larger thrust. Re-iterate all the steps above using the `Matlab` script.

8 Textual References

1. Page 117 has table for thrust to weight of typical aircrafts.

2. Page 119 has a table for power to weight ratio of typical aircrafts.

3. Page 124 has a tabel for typical wing loadings.

4. Page 126 has description on typical $c_{L,max}$ values.

5. Page 135 has descriptions of what $c_{D,0}$ should be.