

# lecture2&3

Vincent

2023-02-06

## Causality

### Selection Bias in ATT with Observational Data

$$E[Y_i^1|D_1 = 1] - E[Y_i^0|D_1 = 1] + E[Y_i^0|D_1 = 1] - E[Y_i^0|D_1 = 0]$$

$$\text{ATT: } E[Y_i^1|D_1 = 1] - E[Y_i^0|D_1 = 1]$$

$$\text{Selection Bias: } E[Y_i^0|D_1 = 1] - E[Y_i^0|D_1 = 0]$$

### Selection Bias in ATE with Observational Data

$$E[Y_i^1] - E[Y_i^0] + Pr(D_i = 1)(E[Y_i^0|D_1 = 1] - E[Y_i^0|D_1 = 0]) + Pr(D_i = 0)(E[Y_i^1|D_1 = 1] - E[Y_i^1|D_1 = 0])$$

$$\text{ATT: } E[Y_i^1] - E[Y_i^0]$$

$$\text{Selection Bias: } Pr(D_i = 1)(E[Y_i^0|D_1 = 1] - E[Y_i^0|D_1 = 0]) + Pr(D_i = 0)(E[Y_i^1|D_1 = 1] - E[Y_i^1|D_1 = 0])$$

## Regression with a Single Regressor: Hypothesis Tests and Confidence Intervals

### The Least Squares Assumptions:

1.  $E(u|X = x) = 0$  (or we can say that there is no correlation between the focus and other factors, i.e. causality are 0) 2.  $(X_i, Y_i), i = 1, \dots, n$ , are i.i.d. 3. Large outliers are rare  $E(X^4) < \infty, E(Y^4) < \infty$ .

$\hat{\beta} \sim N(\beta_1, \frac{\sigma_v^2}{n\sigma_X^2})$ , where  $v_i = (X_i - \mu_X)u_i$  In other word, the model is normally distributed

$$\text{TestScore} = \beta_0 + \beta_1 \text{STR}$$

$\beta_1$  is the slope of population regression line or the change in test score for a unit change in STR

### t-testing

$$\beta_{1,0} = 0$$

$$\frac{\hat{\beta}_1 - \beta_{1,0}}{SE(\hat{\beta}_1)}$$

### confidence interval

$$\beta_1 = \{\hat{\beta}_1 \pm C \times SE(\hat{\beta}_1)\}$$

$C$  is the critical value

The G-M theorem says that among all possible choices of  $\{w_i\}$ , the OLS weights yield the smallest  $var(\hat{\beta}_1)$