# Math115A 2/10 notes

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2023-02-10

The matrix representation of the zero linear transformation $0 : V \to W$ with respect to any ordered basis $\beta, \gamma$ is $[0]_\beta^\gamma = (0)_{ij} \in M_{m\times n}(F)$ so the matrix that has all entries $= 0$

If $V = W$ and $\beta$ is an ordered basis for $V$ then the identity linear transformation $I_V : V \to V$ defined by $I_V(x) = x, \forall x \in V$ has matrix rep $[I_V]_\beta^\beta = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} \in M_{m\times n}$

### 14.1 Definition

Let $T, U : V \to W$ be two functions between vector spaces $V, W$ over some field F. We define $T + U : V \to W$ by $(T + U)(v) = T(v) + U(v), \forall v \in V$. Also, if $c \in F$, we define $cT : V \to W$ by $(cT)(v) = cT(v), \forall v \in V$

### 14.2 Theorem

Let $V, W$ be vector spaces and let $T, U : V \to W$ linear.
(a) $T + U$ and $cT$ are linear, $\forall c \in F$
(b) The set $\mathbb{L}(V, W)$ of linear transformations from V to W, with the operation of addition $T + U$ and scalar multiplication $cT$ defined in 14.1, is a vector space over the field F.
**Proof:**
(a). if $x, y \in V$ Then $(T+U)(x+y) = T(x+y)+U(x+y) = T(x)+T(y)+U(x)+U(y) = (T+U)(x)+(T+U)(y)$ Thus showing that $T + U$ preservers addition similarly $(T + U)(cx) = T(cx) + U(cx) = cT(x) + cU(x) = c(T + U)(x) \forall c \in F$ So $T + U : V \to W$ is a linear. Similarly one shows that $cT : V \to W$ is linear

(b) With the operations of addition and scalar multip. in (a) in defining 0 to be the zero transformation, that takes any $v \in V$ to $0_w$, its very easy to check all VS1-8 axioms.

### 14.3 Notation

As already mentioned in 14.2 proof (b), we denote by $\mathbb{Z}(V, W)$. The set of linear transformations from V to W endowed with vector space structure defined in Theom 14.2
When V=W, denote $\mathbb{L} : (V, V) = \mathbb{L}(V)$

### 14.4 Theorem

Let V, W be linear dim vector spaces with ordered basis $\beta$, resp $\gamma$.
If $T, U : V \to W$ are linear, Then:
(a) $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$

(b) $[cT]_\beta^\gamma = c[T]_\beta^\gamma, \forall c \in F$

## 14.6 Corollary

Let V, W be finite dim vector and $\beta, \gamma$ ordered basis for V, resp W.
The transformation from $\mathbb{L}(V, W)$ to $M_{m\times n}(F)$ defined by
$T \to [T]_\beta^\gamma$ is linear and one to one
**Proof:**
We already established linearity of this transformation and showed that $[T]_\beta^\gamma = [U]_\beta^\gamma$ then $T + U$, so one to one.