Math115A 11/1 notes

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A set is a collection of mathmatical object. The members of a set are called elements of the set: we write a∈A to mean: “a is an element of the set A”

We often use curly brackets for sets whose elements can be “enumerated”

Example:/newline

{-1,2,3,7} is the set consisting of the numbers(elements) -1,2,3, and 7.

More often we describe a set as “elements with same properties”, like A = {x: x has property P} No such cases are often mentions that x is from a larger set (more often set of numbers) that defined and has an established notation.

Important fact element sets: one can not have at the same time: x∈A and x!∈A

Example: (x ∈ R: x >= -2} means “the set of real numbers larger than or equal to -2”

Suppose A, B are sets, We say A is a subset of B and write A<B if for any a∈A we have a∈B (every elemet of A is an element of B)

For any set A we have 0 ∈ A and A ∈ A

Two sets A, B are equal if A∈B and B∈A, we write A=B

**Operations with sets

Suppose A, B are sets, We write A∪B for the set {x:x∈A or x∈B} we read A∪B

We write A∩B for the set {x: x∈A and x∈B} we read A∩

We write $A-B$ is $\{x : x\in A$ and $x!\in B\}$

If $A\in X$ the we call $X-A$ the complement of A in X

let $A,B \in x$ Then $A \in B$ if and only if $x-B \in x-A$ proof:
The statement asks us to move the things
1. if $A\in B$ then $X-B\in X-A$
2. if $X-B\in X-A$ then $A\in B$(“<=”) meet of “1.”
proof of 1:
Assume $A \in B$, let $x\in X-B$ we want to show that $x\in X-A$ i.e. that x does not belong to A. indeed, for if $x\in A$ then we would have $x\in B$ (because $A\in B$) giving us $x\in A$ and $x!\in A$ at the same time which is contradiction.

proof of 2: (i.e. f “<=”)
Assume $X-B \in X-A$. We want to move that $A\in B$, Let $a\in A$ if we assume by contradiction that $x\in B$, then $x\in X-B$. But this implies $x\in X-A$, in other words $x !\in A$. again we get $a !\in A$, $a\in A$, contradiction.

if and only if = “<=>” or “iff”

if $A \in X$ Then $X-(X-A) = A$ (The complement of the complement of a set A is the set A itself)

For arbitary sets A,B,x $A\in B$ implies $X-B \in X-A$ proof Need to show 1. $X-(X-A)\in A$ and 2. $A\in X-(X-A)$
proof of 1: Let $x\in X-(X-A)$. This means $x!\in X-A$. We want to show that $x\in$. indeed, for if we assume that $x!\in A$, then $x\in X-A$, to we get $x\in X-A$ and $x!\in X-A$, a contradiction.

proof of 2:
If A,B,X are arbitrary sets then $A\in B$ implies $X-B\in X-A$
Proof:

De morgan’s law
$X-(A\cup B) = (X-A)\cap(X-B)$
$X-(A\cap B) = (X-A)\cup(X-B)$

theorem 1 proof:
1. if $x\in X-(A\cup B)$ Then $x \in X-A$ (because $A \in A\cup B$ or $X-(A\cup B)\in X-A$) and $x\in X-B$ (because $B\in A\cup B$). Thus $x \in(X-A)\cap(X-B)$

2. Assume $\in(X-A)\cap(X-B)$. This means $x!\in A$ and $x!\in B$. We want to move that $x\in X-(A\cup B)$. Assume by contradiction that this is not true. Thus $x!\in X-(A\cup B)$ which means $x\in A\cup B$. So we have at the same time $x\in A\cup B$ and $x!\in A\cup B$ which is a contradiction.

theorem 2 proof:
1. if $x\in X-(A\cap B)$. Then $(X-A) \in x$ and $(X-B) \in x$. Thus $x\in(X-A)\cup(X-B)$.

2 Assume $x\in(X-A)\cup(X-B)$, which means $x\in(X-A)$ or $x\in(X-B)$.($x!\in A\cap B$) Base on the other side of the equation, $x\in X-(A\cap B)$, we can assume by contradiction that this is not true.(where $X-(A\cap B) \neq (X-A)\cup(X-B)$). Thus, $x!\in X-(A\cap B)$ which means $x\in A\cap B$. So we have at the same time $x\in A\cap B$ and $x!\in A\cap B$ which is a contradiction.

2.1 Definition: A function is a triple considering of: a set X called the domain of the function
a set Y called the codomain of the function
a rule of assigning to each element x∈X a unique element y∈Y (often this “rule” or “assignment” is given by a formula)
We write such a triple f:X->Y with the y assigned x denoted f(x) or read it to x maps to f(x) (we often use the notation x↦f(x) to emphesize that the function f assigns f(x) to x read it “x maps to f(x)”)

2.2 Definition: (2.2.1)let f:X-> Y be a function
we say that f is injective if whenever x1,x2 ∈ X are be that f(x1) = f(x2), it implies x1=x2 (non_equal elements in X map to non_equal elemetns in Y, under f i.e. $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$)

we say that f is surjective if for any y∈Y there exist x∈X such that f(x)=y (Any y∈Y is the image of x∈X, under f)

we say that bijective if f is both injective and surjective (this is some as saying that any y∈Y is the image of exactly any x∈X, under f)

suppose f:X->Y on g:Y -> Z are functions. The composition of f and g is the function g of f: X_>Z defined by g of f(x)= g(f(x))

let f:X->Y be a function and $X_0 < X_a$ subset the restriction of f to X0, denoted $f|_X$, X_0->Y is the function with domain X_0, codomain Y and arraignment given by : for x∈X_0 $f|_x(x)$ = f(x)

Exercise:
Show that if f is surjective then n>=m
proof:
We say that a function is surjective when any y∈Y have a exist x∈X, Thus, the number of x∈X will always greater or equal to the number of y∈Y in the function. Which means n>=m

Show that if f is injective, then n<=m
proof:
We say that a function is injective when any x∈X have a unique exist y∈Y, Thus the number of y∈Y will always greater or equal to the number of x∈X in the function. Which means n<=m

Show that if f in bijective, the n=m
proof:
We say that a function is bijective when any x∈X have a unique exist y∈Y and any y∈Y have a unique exist x∈X. Thus the number of the x∈X must be equal to the number of x∈X in the function. which means n=m

Show that if n>m Then there must exist y∈Y be that f(x_1)=y, f(x_2)=y for some x_1, x_2∈X, x_1≠x_2
proof:
We say that if $x_1 \neq x_2$, where both x_1 and x_2 have a solution in the codomain. Also, we can tell that n>m, where the number of x∈X is greater than y∈Y. Thus, f(x_1)=y, f(x_2)=y, where two x will "point toward" a single y.

Let f:X->Y, g:Y -Z be functions
a) if g of f is injective then f is injective the f is injective
b) if g of f is surjective then g is surjective.
c) if f, g are injective then g of f is injective.
d) if f, g are surjective the g of f is surjective