Math115 1/23 notes

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2023-01-24

6.1 Definition

If S_1, S_2 are nonempty subsets of a vector space V then the sum of S_1 and S_2, denoted $S_1 + S_2$ is the set $\{x + y : x \in S_1, y \in S_2\}$

6.2 Definition

Let W_1, W_2 be subspaces of the vector space V. We say that V is the direct sum of W_1 and W_2 if $W_1 \cap W_2 =$ {0} and $W_1 + W_2 = V$, and we then write $V = W_1 + W_2$

6.3 Exercise

Show that $V = \mathbb{R}^2$ is the direct sum of $W_1 =$ {$(x, x) : x \in \mathbb{R}$} and $W_2 =$ {$(y, -y) : y] \in \mathbb{R}$}
Solution
First note that W_1, W_2 are indeed vector subspace of $\mathbb{R}^2$ indeed, if we take two elements $(x, x) \in W_1, (Z, Z) \in W_2$, then $(x, x) + (z, z) = (x + z, x + z) \in W_1$. Also, if $c \in \mathbb{R}$ is a scalar, then $c(x, x) = (cx, cx) \in W_1$
Similarly for W_2
Also, we see that $0 = (0, 0) \in W_1$ and $0 = (0, -0) \in W_2$. So W_1, W_2 satisfy the conditions in Theom 5.2 so they are subspace of $V = \mathbb{R}^2$