MATH115A 2/08 notes

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Matrix Representaion of a Linear Transformation(2.2)

up to now, we considered basis of a finite dim vector space V as a set of elements $S = \{v_1, ..., v_n\}$, where the order in which we take the vectors $v_1, ..., v_n$ didn't matter. But we now want to take into consideration the order

13.1 Definition

Let V be a finite dim vector space. An ordered basis for V is a basis of V with a specified order of its elements, i.e. it is a finite sequence $v_1, ..., v_n$ of linearly independent vectors in V that span V.
We will still write it $\{v_1, , , v_n\}$ but we will specify

13.2 Example

In $V = \mathbb{R}^n$ we usually take basis $e_1 = (1, 0, ..., 0)$, $e_2 = (0, 1, 0, ...0), ..., e_n = (0, 0, ...0, 1)$. We then say $\{e_1, ..., e_n\}$ is a basis for $\mathbb{R}^n$ as a set, we can take it to be $\{e_2, e_1, e_4, e_3, e_5, ..., e_n\}$ as well, or any other re-ordering. It is still no some set, so some basis

13.3 Definition

Let $\beta = \{v_1, ..., v_n\}$ be an ordered basis for a finite dim vector space V let $v \in V$ and $c_1, ..., c_n \in F$ be the unique sca;ars such that $v = \Sigma_{i=1}^n c_i v_i$. we define the coordinate vector of V relative to the ordered basis β by $[v]_\beta = \begin{pmatrix} c_1 \\ | \\ c_n \end{pmatrix} \in F^n$ so it is a column m-tuple

13.4 Definition

Let V, W be finite dim vector spaces with ordered basis $\beta = \{v_1, ..., v_n\}$ for V $\gamma = \{w_1, ...w_m\}$ for W. Let $T : V \to W$ be linear. Notice that for each $j = 1, 2, ..., n$ there exist unique scalars $a_{ij} \in F, i = 1, ..., m$ such that $T(v_j) = \Sigma_{i=1}^m a_{ij} w_i$, $j = 1, ..., n$

We call the matrix $A \in M_{m\times n}(F)$ defined by $(A)_{ij} = a_{ij}$ the matrix representation of T with respect to the ordered basis β and γ we write $A = [T]_\beta^\gamma$