Midterm 1 Review content

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De Morgan's law

X-(A∪B) = (X-A)∩(X-B)
X-(A∩B) = (X-A)∪(X-B)

A function is a triple considering of: a set X called the domain of the function
a set Y called the codomain of the function
a rule of assigning to each element x∈X a unique element y∈Y (often this "rule" or "assignment" is given by a formula)
We write such a triple f:X->Y with the y assigned x denoted f(x) or read it to x maps to f(x) (we often use the notation x↦f(x) to emphesize that the function f assigns f(x) to x read it "x maps to f(x)")

We say that a function is surjective when any y∈Y have a exist x∈X
We say that a function is injective when any x∈X have a unique exist y∈Y
We say that a function is bijective when any x∈X have a unique exist y∈Y and any y∈Y have a unique exist x∈X.

1.1 Field

A field F is a set on which one has two operations +, ∗, called addition and multiplication, are defined so that for each x,y ∈ F corresponds a unique element in F denoted x+y and a unique element denoted $x * y$ such that the following properties are satisfied for all elements a, b, c ∈F:
(F1) $a + b = b + a$, $a * b = b * a$

(F2) $(a + b) + c = a + (b + c); (a * b) * c = a * (b * c)$

(F3) There exist distinct elements 0 and 1 in F such that 0+a=a and 1∗a = a, ∀a ∈F

(F4) For each a∈F and each b∈F, b!=0 there exist elements c∈F, d∈F such that a+c=0, b*d=1

(F5) $a * (b + c) = a * b + a * c$ distributivity of multiplication.

The element x+y called the sum of x&y x∗y called the product of x&y
The element 0 is called the identity element for addition
The element 1 called the identity element for multiplication
The element c in (F4) with property a + c = 0 called the addition inverse of a.
The element d in (F4) with property a∗d = 1 called the multiplication inverse of c

let(F,+,$*$) be a field. For any a,b,c $\in$ F we have:
(1) if a+b =c+b Then a=c
(2) if ab=cb and b!=0, then a=c.

The element 0 and 1 in a field are unique. Also the additive inverse of an element and the multiplicative inverse of a != 0 element are unique

1.2 Vector Space

Definition: A vector space V over a field F consists of a set V on which two operations(called addition and scalar multiplication) are defined, so that for each x,y $\in$V, we have a unique element x+y in V and for each x$\in$V and a$\in$F we have a unique element ax$\in$V(scalar) such that the following conditions hold:
(VS1) x+y=y+x, $\forall$ x,y$\in$V
(VS2) (x+y)+z=x+(y+z), $\forall$ x,y,z$\in$V
(VS3) There exists on element in V denoted 0 such that x+0=x, $\forall$x$\in$V
(VS4) For each x$\in$V have exist y$\in$V such that x+y=0
(VS5) For each x$\in$V we have 1x=x
(VS6) For each x$\in$V, a,b$\in$F we have (ab)x=a(bx)
(VS7) For each x,y$\in$V, a$\in$F we have a(x+y) =ax+ay
(VS8) For each x$\in$V, a,b$\in$F we have (a+b)x=ay+bx

1.3 subspace

A subset W of a vector space V over a field F is called a subspace of V if W is a vector space over F with the operations of addition and scalar multiplication defined on V.

To check if a subset is under a Field:
(1) $0 \in S$
(2) closure under addition, $x + y \in S, x, y \in S$ (3) closure under multiplication, $cx \in S, x \in S, c \in F$

If S_1, S_2 are nonempty subsets of a vector space V then the sum of S_1 and S_2, denoted $S_1 + S_2$ is the set $\{x + y : x \in S_1, y \in S_2\}$

Let W_1, W_2 be subspace of the vector space V. We say that V is the direct sum of W_1 and W_2 if $W_1 \cap W_2$ =$\{0\}$ and $W_1 + W_2 = V$, and we then write $V = W_1 + W_2$

1.4 linear combination

Let V be a vector space over a field F and $S \in V$ a nonempty subset of V. A vector $v \in V$ is called a linear combination of vectors in S. If there exist a finite number of vectors $u_1, ..., u_n \in S$ and scalars $c_1, ..., c_n \in F$ such that $v = c_1u_1 + c_2u_2... + c_nu_n$

If V is a vector space and $S \neq 0 \in V$, Then the span of S, denoted $span(S)$, is the set of all linear combinations of vectors in S.

The span of any subset S of a vector space V is a subspace of V. Any subspace of V that contains S must contain $span(S)$
The span of a subspace is always a subspace itself

1.5 linear independent

A subset of vectors since vector space V is linearly dependent if there exist finitely many distinct vectors $v_1, .., v_n \in S$ and scalars $c_1, ..., c_n \in F$ not all of there 0, such that $c_1 v_1 + c_2 v_2 + ... + c_n v_n = 0$
In other word: if one can express the vector 0 as a linear combination of distinct vectors in S with non-zero coefficients.