Note 3

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2023-10-05

Random Numbers Generate

TRNG true Random number generator PRNG

Math formulas random # table

Random number → u → $Unif(0,1)$ 'runif(n)"

Inverse Transformation method

Wish to generate $x \sim F_X(x)$

$$u = F_X(x) \sim Unif(0,1)$$

$$Y = F_X^{-1}(u) = inf\{y : F_X(y) = u\}$$

$$y \sim F_Y(y)$$

show $F_x(x) \sim Unif(0,1)$

$$F_y(x) = P_Y(Y < x) = P_Y(F^{-1}(u) < x) = P_U(U < F_X(x)) = \int_0^{F_X(x)} 1dt = F_X(x)$$

$$f(u) = \frac{1}{1-0}$$

$$F(u) = \frac{u-a}{b-a}$$

Ex:

$$x \sim Unif(a,b)$$

$$u = F_X(x) = \frac{x-a}{b-a}$$

$$a < x < b$$

$$x = u(b-a) + a = F^{-1}(u)$$
$$u_n \sim Unif(0,1)$$

$$x_i = u_i(b-a) + a$$

Exponential Prob Function

$$f(x) = \lambda e^{-\lambda x}$$
$$F(x) = 1 - e^{-\lambda x}$$

$$u = 1 - e^{-\lambda x}$$
$$x = \frac{ln(1-u)}{\lambda} = \frac{ln(u)}{\lambda}$$

Since u and 1-u belong to the same distribution then we can have only u inside the natrual log.

$$1 - u \sim Unif(0,1)$$

Polynomial Density

$$f(x) = k * x^{k-1}$$
$$F_X(x) = \int_0^x kt^{k-1}dt = x^k$$

$$x = u^{\frac{1}{k}}$$

Discrete Case

1. Sort Sample in order
2. Draw the CDF of the discrete data

$$F^{-1}(u) = inv\{x : F_X(x) = u\}$$
$$F^{-1}(u) = inv\{x_{(i)} : F_X(x_{i-1}) < u \leq F(x_i)\}$$

$$P(F_X(x_{i-1}) < U \leq F_X(x_i))$$

$$= F_X(x_i) - F_X(x_{i-1}) = P(x_1) + P(x_2) + ... + P(x_i) - P(x_1) - P(x_2) - ... - P(x_{i-1}) = P(x_i)$$

Ex:

$x : 0; 1; 2$ $P(x) : 1/4, 2/4, 1/4$

Generate $u \sim Unif(0,1)$

If $u \leq \frac{1}{4} \to 0,\ \frac{1}{4} < u \leq \frac{3}{4} \to 1,\ \frac{3}{4} < u \to 2$

Geometric

$$P(x) = q^x p, q = 1 - p$$

$$F_X(x) = \Sigma_{t=0}^{x} q^t p = 1 - q^{x+1}$$

Generate $u \sim Unif(0, 1)$
if $F_x(x-1) < u \leq F_X(x)$

$$1 - q^x < u \leq 1 - q^{x-1}$$

$$q^x > 1 - u \geq q^{x+1}$$
$$xlog(q) > log(1-u) \geq (x+1)log(q)$$

$$x < \frac{log(1-u)}{log(q)} \leq x + 1$$

$$\frac{log(1-u)}{log(q)} = x + 1$$