Note 1

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Conditional prob function

$$P(Y|X = x) = \frac{P_{xy}(x, y)}{P_X(x)}$$

$$P(X|Y = y) = \frac{P_{xy}(x, y)}{P_Y(y)}$$

$$P_{xy}(x, y) = P(Y|X = x) * P_X(x) = P(X|Y = y) * P_Y(y)$$

With all of these cases above, we state that Y and X are independent

$$P(Y|X = x) = P(y)$$
$$P(X|Y = y) = P(x)$$
$$\rightarrow P(x, y) = P(x)P(y)$$

$$F_{xy}(x, y) = F_X(x)F_Y(y)$$

$$E(X) = \Sigma X P_X(x)$$

$$Var(X) = E[(X - \mu)^2] = E(X^2) - E(X)^2$$

$$E(X^3) : skewness$$
$$E(X^4) : kurtosis$$

Smaller $E(X^4)$ will have a flatter shape of the distribution

$$g(x|r) = x^r$$
$$E[g(x|r)]$$

$$g(x) = e^x$$
$$g(x, y) = log(x) - log(y)$$

Bernulli

$$x = \{0, 1\}$$

$$P(x) = p^x(1-p)^{1-x}$$
$$X_1 \sim Ber(p)$$
$$X_2 \sim Ber(p)$$
$$X_n \sim Ber(p)$$

$$\Sigma X_i \sim Ber(p)$$

as $p \to 0$ and $n \to \infty$

$$Y \sim Pos(\lambda)$$
$$\lambda = np$$
$$E(Y) = Var(Y) = \lambda$$

$$F_x(x) = P(X \leq x) = 1 - (1-p)^x$$

memoryless

$$P(X > a + b | X > a) = P(X > b)$$
$$= \frac{P(X > a + b)}{P(X > a)} = \frac{1 - P(X \leq a + b)}{1 - P(X \leq a)}$$

$$= \frac{(1-p)^{a+b}}{(1-p)^a} = (1-p)^b = P(X > b)$$

negative binomial

$$\Sigma_1^r X_i \sim NB(r, p)$$

uniform

$$f(x) = \frac{1}{b-a}$$
$$a \leq x \leq b$$

I(statement)

$$f(x) = \frac{I(a \leq x \leq b)}{b-a}$$

Exponential

X: the wait time (discrete) between succssive events from a possion process

Example:

$$\lambda = 10 calls/hour$$
$$\frac{1}{\lambda} = calls/hour = 6 min/call$$

Let λ be the expected # of class during a one-minute interval. If $\lambda = 2$, the expected # of calls in 1 minute is 2 calls will be $2 \times 2 = 4$

N_x the # of calls in x-minute interval $N_x \sim Possion(\lambda x)$