Note 4

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From last time

$$F_X(x), u \sim unif(0,1)$$

$$F^{-1}(u) = inf\{x : F_X(x) = u\}$$

$$F^{-1}(u) = inf\{x_i : F_X(x_{i-1}) < u \leq F_X(x_i)\}$$

Geometric distribution $x = 0, 1, ...$

$$F^{-1}(u) = inf\{x : F_X(x) < u \leq F_X(x+1)\}$$

$$\frac{log(u)}{log(q)} \leq x + 1$$

ceiling function

$$\lceil \frac{log(u)}{log(q)} \rceil - 1 = x$$

Log distribution function

$$P(x) = \frac{\theta^x}{-log_{10}(1-\theta)x}; x = 1, 2, ...; 0 < \theta < 1$$

number of items purchased by a customer in a unit of time.

$$F(x) = P(1) + P(2) + ... + P(x)$$

$$F_X = \begin{pmatrix} F(1) \\ F(2) \\ \vdots \\ F(x) \\ F(x+1) \\ \vdots \end{pmatrix} = \begin{pmatrix} I(u \geq F(1)) \\ I(u \geq F(2)) \\ \vdots \\ I(u \geq F(x)) \\ I(u \geq F(x)) \\ \vdots \end{pmatrix} = I_F$$

Generate $u \sim Unif(0,1)$ cheich if $u < F_X(x)$ by order $x = 1, 2, ...$ until $u \geq F_X(x) \rightarrow$ return x

Ex:

if $u < F(1)$

$$I_F = \begin{pmatrix} F \\ F \\ F \\ \vdots \\ F \\ F \end{pmatrix}$$

$$sum(I_F) = 0 + 1$$

if $u < F(4)$

$$I_F = \begin{pmatrix} T \\ T \\ T \\ \vdots \\ F \\ F \end{pmatrix}$$

$$sum(I_F) = 3 + 1$$

$$F_X(x) = \Sigma_{k=1}^{x} P(x) = F(x-1) + P(x)$$
$$u \sim unif(0,1)$$

$$while(u > F_X(x))\{x = x + 1 Fx = Fx[x-1] + px\} return x$$

```
logdist <- function(u, Fx){

  while(u>Fx){
    x <- x+1
    newFx <- Fx + px
  }
  return(x)
}
```

Recursive equation

$$P(x) = rec(P(x-1))$$

$$P(x=1) = \frac{\theta}{-log(1-\theta)}$$
$$P(x=2) = \frac{\theta^2}{-log_{10}(1-\theta)2} = \frac{\theta}{-log_{10}(1-\theta)}\frac{\theta}{2} = P(x=1)\frac{\theta}{2}$$
$$P(x=3) = \frac{\theta^3}{-log_{10}(1-\theta)3} = \frac{\theta}{-log_{10}(1-\theta)}\frac{\theta}{2}\frac{2\theta}{3} = P(x=2)\frac{2\theta}{3}$$
$$P(x) = P(x-1)\frac{(x-1)\theta}{x}$$

$$rlog(\theta)\{x = 1p(x) = \frac{\theta}{-log_{10}(1-\theta)} = F(x)u \sim unif(0,1)while(u > F_X(x))\{x = x+1P(x) = P(x-1)\frac{(x-1)\theta}{x}F(x) = F(x-1$$

```r
logdist <- function(theta,n){
  x <- 1
  px <- theta/-log((1-theta),10)
  Fx <- px
  u <- runif(n)
  while(u>Fx){
    x <- x+1
    newpx <- px*((x-1)*theta)/x
    newFx <- Fx + px
  }
  return(x)
}
```

Some of the function that I can't be able to use the inverse CDF method, since it does not have a close form.

$$f(x) = \frac{1}{(2\pi)^2}e^{-\frac{x^2}{2}}$$

This is a example

Acceptance-Rejection Method (indirect method)

We will find a g(x) to generate samples

$$g(x) \to x_1^g...x_n^g$$

Then will create a mechanism to determine if x_i^g is accepted $x \sim f(x)$

Constraints:

$$1.f(x) > 0, g(x) > 0$$
$$2.\frac{f(x)}{g(x)} \leq M_{const} \to f(x) \leq g(x)M_{const}; \forall x$$

$$x_g \sim g(x); x_f \sim f(x)$$

if $\frac{f(x_g)}{Mg(x_g)}$ is small it will be less likely from $f(x)$ if $\frac{f(x_g)}{Mg(x_g)}$ is large it will be more likely from $f(x)$