Quiz #7

Started: 7 Dec at 12:19

Quiz instructions

General Quiz Information

(https://bruinlearn.ucla.edu/courses/173226/files/14494000?wrap=1)

You will answer the quiz questions with sensible answers to gain full credit. Any blank, non-sense, unfinished answers will not be counted. You may upload pdf or png files to answer the questions.

Here is some general information that may be helpful in using Canvas Quizzes.

- You must complete and submit your answers for each quiz by the due date
- For a timed quiz, **you can't stop the clock once you begin**. If time runs out, your quiz will close.
- When you are done answering the questions and are ready to submit your answers for grading, click **Submit Quiz**.
- If you experience a technical problem that interferes with your ability to complete a quiz during the specified time, contact your instructor as soon as possible—you don't have to wait until the quiz has closed.

Question 1 — 2 pts

Consider two urns A and B, where we place n black balls in urn A and n white balls in urn B. At each step, we randomly choose a ball from each urn and interchange the two balls. Let X_t, $t \geq 0$, be the number of black balls in urn A at step t. Please find the transition probabilities.

$$P(X_{\{t+1\}} = k+1 \mid Xt = k) = \frac{m-(n-k)}{m} \times \frac{k}{n}$$

$$P(X_{\{t+1\}} = k-1 \mid Xt = k) = \frac{(n-k)}{m} \times \frac{m-(n-k)}{n}$$

$$P(X_{t+1} = k \mid X_t = k) = 1 - P(X_{t+1} = k+1 \mid X_t = k) - P(X_{t+1} = k-1 \mid X_t = k)$$

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Question 2 3 pts

The weather on any given day on a tropical island could be rainy, sunny, or cloudy, and the probability of tomorrow's weather only depends on today's weather and not any other previous days. Suppose that we obtained the transition probabilities as below.

- P(Rainy tomorrow | Rainy today) = 0.5
- P(Rainy tomorrow | Cloudy today) = 0.5
- P(Sunny tomorrow | Cloudy today) = 0.5
- P(Rainy tomorrow | Sunny today) = 0.25
- P(Cloudy tomorrow | Rain today) = 0.25
- P(Cloudy tomorrow | Sunny today) = 0.25

(a) Please find the transition matrix with the state space {1: Rainy, 2: Cloudy, 3: Sunny}.

(b) Suppose today is sunny. What is the expected weather two days from now?

(c) Is this Markov chain irreducible? Explain. Can you find a stationary distribution?

b)

If today is sunny, the

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0.421875 to be rainy 0.328125 to be cloudy 0.250000 sunny
```

c)

This Markov chain is irreducible because it is possible to transition from any state to any other state in two steps, as indicated by all entries being positive in the two-day transition matrix.

Stationary distribution:

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[0.4444444 0.3333333 0.2222222]
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Question 3

4 pts

Consider the transition matrix for the state space {0, 1, 2} shown below.

$$\begin{pmatrix} 1 & 0 & 0 \\ 2/3 & 0 & 1/3 \\ 0 & 1/2 & 1/2 \end{pmatrix}.$$

(a) Is this Markov chain aperiodic?

(b) What is the probability of getting stuck in state 0 given that the initial state = i for i = 0, 1, 2.

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state 0, we can see that it always stays in state 0, meaning it has a period of 1. Therefore, without looking further, we can say the Markov chain is aperiodic because at least one state (state 0) has a self-loop, indicating a period of 1.

b)

- If the initial state is 0 (i.e., i=0), the probability of getting stuck in state 0 is 1, since state 0 is an absorbing state in the transition matrix.
- If the initial state is 1 (i.e., i=1), the probability of moving to state 0 in the next step is $\frac{2}{3}$.
- If the initial state is 2 (i.e., i=2), the probability of moving to state 0 in the next step is 0, as there is no direct transition to state 0

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Question 4

3 pts

Consider the transition matrix for the state space {0, 1} below.

$\begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$. Can we find a stationary distribution $\pi = [\pi_0, \pi_1]$ with this transition matrix? If so, please find the equations for computing π_0 and π_1.

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12pt Paragraph B I U A T²

1. $\pi_0(1-a)+\pi_1 b=\pi_0$
2. $\pi_0 a + \pi_1(1-b) = \pi_1$
3. $\pi_0 + \pi_1 = 1$

transition matrix:

$\pi_0 = \frac{b}{a+b}$

$\pi_1 = \frac{a}{a+b}$

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Saved at 12:42 Submit quiz