Quiz #5

Started: 15 Nov at 11:33

Quiz instructions

🗒️General Quiz Information

[(https://bruinlearn.ucla.edu/courses/173225/files/14446578?wrap=1)](https://bruinlearn.ucla.edu/courses/173225/files/14446578?wrap=1)

You will answer the quiz questions with sensible answers to gain full credit. Any blank, non-sense, unfinished answers will not be counted. You may upload PDF or PNG files to answer the questions.

Here is some general information that may help use Canvas Quizzes.

- You must complete and submit your answers for each quiz by the due date
- For a timed quiz, **you can't stop the clock once you begin**. If time runs out, your quiz will close.
- When you are done answering the questions and are ready to submit your answers for grading, click **Submit Quiz**.
- If you experience a technical problem that interferes with your ability to complete a quiz during the specified time, contact your instructor as soon as possible—you don't have to wait until the quiz has closed.

Question 1	**8 pts**

Consider the problem of estimating $\theta = \int_0^1 e^x\, dx$. Please write R code with the following methods to estimate θ.

(a) Use the simple Monte Carlo method and denote the estimator as $\hat{\theta}_{mc}$.

(b) Use the antithetic variate approach and denote the estimator as $\hat{\theta}_{av}$.

(c) Use the control variate approach and denote the estimator as $\hat{\theta}_c$.

(d) Under what conditions can we expect $\hat{\theta}_c$ to be more efficient than $\hat{\theta}_{mc}$?

c)

```
control_variate <- function(n) {
  u <- runif(n)
  f <- function(x) exp(x^2)
  g <- function(x) exp(x)
  c <- cov(f(u), g(u)) / var(g(u))
  theta_hat_c <- mean(f(u)) - c * (mean(g(u)) - (exp(1) - 1))
  return(theta_hat_c)
}
```

d)

The control variate approach is more efficient when the control variate is highly correlated with the function being integrated, and its integral is known or can

p

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Question 2

Suppose $\hat{\theta}_1$ and $\hat{\theta}_2$ are both unbiased estimators of θ. Then for any constant C.

(a) Please show $\hat{\theta} = (1 - C)\hat{\theta}_1 + C\hat{\theta}_2$ is an unbiased estimator of θ

(b) Assume the correlation of $\hat{\theta}_1$ and $\hat{\theta}_2$ is -1. Please find C such that $Var(\hat{\theta})$ achieves the minimum.

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$$E\left(\hat{\theta}\right) = (1 - C)\theta + C\theta = \theta$$

b)

$$Var(\theta) = Var((1-C)\hat{\theta}_1 + C\hat{\theta}_2)$$

$$Var(\hat{\theta}) = (1-C)2Var(\hat{\theta}_1) + C2Var(\hat{\theta}_2) - 2C(1-C)Cov(\hat{\theta}_1, \hat{\theta}_2)$$

$$\frac{d}{dC} = 2CVar(\hat{\theta}_2) - 2(1-C)Var(\hat{\theta}_1) + 2(1-2C)(-\sigma\hat{\theta}_1\sigma\hat{\theta}_2)$$

$$C = \frac{\sigma(\hat{\theta}_1)\sigma(\hat{\theta}_2)}{2Var(\hat{\theta}_1) - Var(\hat{\theta}_2)}$$

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Question 3   2 pts

Consider the problem of estimating $\theta = \int_0^2 \frac{e^{-x}}{1+x} dx$. Please write an algorithm (or R code) to obtain the stratified estimator $\hat{\theta}_k$ with k strata.

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```
stratified_estimator <- function(k, n_per_stratum) {
  theta_hat <- 0
  for (i in 1:k) {
    a <- (i - 1) * 2 / k
    b <- i * 2 / k

    u <- runif(n_per_stratum, min = a, max = b)

    f_u <- exp(-u) / (1 + u^2)

    theta_hat <- theta_hat + (b - a) * mean(f_u)
  }
```

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Saved at 11:51   Submit quiz