

Note 8

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2023-10-24

From last time:

$$\begin{aligned}\Theta &= \int_x g(x)f(x)dx \\ x_1, \dots, x_n &\sim f(x) \\ \hat{\theta} &= \frac{\sum g(x_i)}{n} \\ \text{Var}(\hat{\theta}) &= \frac{\sigma^2}{n} \\ \hat{\sigma}^2 &= \frac{\sum (g(x_i) - \bar{g})^2}{n}\end{aligned}$$

Estimate $\text{Var}(\hat{\theta}_{MC})$ by bootstrap

Antithetic variable

$$\begin{aligned}x_1, \dots, x_m &\sim f(x) \\ u_1, \dots, u_m &\sim \text{Unif}(0, 1) \\ Y &= g(F^{-1}(u)) = h(u) \\ Y' &= g(F^{-1}(1-u)) = h(1-u) \\ \text{corr}(Y, Y') &\leq 0 \\ u_1, \dots, u_m &\sim \text{Unif}(0, 1)\end{aligned}$$