Quiz #6

Started: 29 Nov at 23:34

Quiz instructions

General Quiz Information (https://bruinlearn.ucla.edu/courses/173226/files/14494000?wrap=1)

You will answer the quiz questions with sensible answers to gain full credit. Any blank, non-sense, unfinished answers will not be counted. You may upload pdf or png files to answer the questions.

Here is some general information that may be helpful in using Canvas Quizzes.

- You must complete and submit your answers for each quiz by the due date
- For a timed quiz, **you can't stop the clock once you begin**. If time runs out, your quiz will close.
- When you are done answering the questions and are ready to submit your answers for grading, click **Submit Quiz**.
- If you experience a technical problem that interferes with your ability to complete a quiz during the specified time, contact your instructor as soon as possible—you don't have to wait until the quiz has closed.

Question 1 3 pts

We showed that the variance of importance sampling estimator $\mathrm{Var}_f(\hat{\theta}) = \frac{1}{m}\mathrm{Var}_\phi(\frac{g(X)f(X)}{\phi(X)})$, where $\hat{\theta} = \frac{1}{m}\sum_{i=1}^m \frac{g(x_i)f(x_i)}{\phi(x_i)}$ and $x_i \sim \phi(x)$. equal to $\mathrm{E}_\phi[\frac{(g(X)f(X)-\theta\phi(X))^2}{\phi^2(X)}]$. Here $f(x$

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importance sampling function.

$\hat{\theta} = \frac{1}{m}\sum_{i=1}^{m}\frac{g(x_i)f(x_i)}{\phi(x_i)}$

$\mathbf{Var}_\phi(\hat{\theta}) = \frac{1}{m}\mathbf{Var}_\phi\left(\frac{g(X)f(X)}{\phi(X)}\right)$

$\mathrm{E}_\phi\left[\left(\frac{g(X)f(X)}{\phi(X)} - \hat{\theta}\right)^2\right]$

$\mathbf{Var}_\phi(\hat{\theta}) = \mathrm{E}_\phi\left[\left(\frac{g(X)f(X)}{\phi(X)} - \mathrm{E}_\phi\left[\frac{g(X)f(X)}{\phi(X)}\right]\right)^2\right]$

$\mathbf{Var}_\phi(\hat{\theta}) = \mathrm{E}_\phi\left[\left(\frac{g(X)f(X)}{\phi(X)} - \hat{\theta}\right)^2\right]$

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Question 2

2 pts

Suppose we want to estimate $\theta = \int_{-1}^{1} 5e^{-2|x|}dx$. Please write an algorithm (or R code) using Importance sampling to estimate θ. You may use a normal density as the importance sampling function for your algorithm.

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```
f <- function(x) {
5 * exp(-2 * abs(x))
} phi <- function(x) {
dnorm(x, mean = 0, sd = 0.5)
} n <- 10000
samples <- rnorm(n, mean = 0, sd = 0.5)
theta_hat <- mean(f(samples) / phi(samples))
```

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Question 3

3 pts

Please find the normalizing constants for the following unnormalized densities.

(a) $q_a(x) = 3e^{-x^2/2}$ for $x > 0$

(b) $q_a(x) = e^{-5x}$ for $x > 0$

(c) $q_a(x) = x^3(1-x)^2$ for $x \in [0, 1]$

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$C_a = \frac{\sqrt{2}}{3\sqrt{\pi}}$

$C_b = 5$

$C_c = 60$

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Question 4

2 pts

Suppose we sample $X_1, \ldots, X_m$ from an importance sampling function $\phi(x)$. Please show that $E(\sum_i^m w_i) = E(f(x_i)/\phi(x_i)) = m$, where f is target density.

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$$\mathbf{E}\left[\sum_{i=1}^{m} w_i\right] = \mathbf{E}\left[\sum_{i=1}^{m} \frac{f(x_i)}{\phi(x_i)}\right]$$

$$\mathbf{E}\left[\frac{f(x_i)}{\phi(x_i)}\right] = \int \frac{f(x)}{\phi(x)}\phi(x)dx$$

$$\mathbf{E}\left[\frac{f(x_i)}{\phi(x_i)}\right] = \int f(x)dx$$

$$\mathbf{E}\left[\frac{f(x_i)}{\phi(x_i)}\right] = 1$$

$$\sum_{i=1}^{m} \mathbf{E}\left[\frac{f(x_i)}{\phi(x_i)}\right] = m \times 1 = m$$

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