Note 5

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From last time:

$$\frac{f(x)}{g(x)} \leq M \to f(x) \leq Mg(x); \forall x$$

if $u < \frac{f(x_g)}{Mg(x_g)}$ accept x_g as x_f; if $u \geq \frac{f(x_g)}{Mg(x_g)}$ reject x_g as x_f

$$f(x) = \frac{P(X \; accepted \; in (x, x + \triangle x))}{\triangle x} = \frac{P(x \in (x, x + \triangle x))}{\triangle x}$$

$$P(x \in (x, x + \triangle x)) = \triangle f(x)$$

$$P(x \in (x, x + \triangle x)) = \frac{number \; of \; points \; survived \; in (x, x + \triangle x)}{accross \; the \; all \; bins, \; the \; number \; of \; points \; survived}$$

Total # of points in $(x, x + \triangle x)$:

Assume that we have N as the total number of sample,

$$N * g(x) * \triangle x$$

$$P(acceptance \; for \; x \in (x, x + \triangle x)) = \frac{f(x)}{Mg(x)}$$

$$N * g(x) * \triangle x \frac{f(x)}{Mg(x)} = \frac{N}{M} \triangle x f(x)$$

Total \$ of points survived:

$$\Sigma_{all \; bins} \frac{N}{M} \triangle x f(x) = \frac{N}{M} \Sigma_{all \; bins} \triangle x f(x) = \frac{N}{M}$$

Finally:

$$P(x \in (x, x + \triangle x)) = \frac{\frac{N}{M} \triangle x f(x)}{\frac{N}{M}} = \triangle x f(x)$$

$$0 < \frac{f(x)}{Mg(x)} \leq 1$$

$$\frac{f(x)}{g(x)} \leq M$$

$$M \geq \frac{f(x)}{g(x)} \rightarrow M \geq Max(\frac{f(x)}{g(x)})$$

Ex:

$$x \sim f(x)$$

$$f(x) = sin(x); 0 < x < \frac{\pi}{x}$$

$$g(x) = \frac{1}{\frac{\pi}{2} - 0} = \frac{2}{\pi}$$

$$u_2 \sim unif(0,1)$$

$$M = max(\frac{f(x)}{g(x)})$$

$$x^{(g)} = u(b-a) + u = u_1 \frac{\pi}{2}$$

$$if\ u_2 < \frac{f(x^{(g)})}{Mg(x^{(g)})}; accept\ X_g$$

To generate random normal distribution:

$$f(z) = \frac{1}{(2\pi)^{\frac{1}{2}}} e^{-\frac{z^2}{2}}$$

$$x = |z|$$

$$f(x) = 2 * \frac{1}{(2\pi)^{\frac{1}{2}}} e^{-\frac{x^2}{2}} = (\frac{2}{\pi})^{\frac{1}{2}} e^{\frac{x^2}{2}}; x \geq 0$$

$$g(x) = e^{-x}; X \sim exp(\lambda = 1)$$

$$M = max(\frac{f(x)}{g(x)})$$

$$\frac{f(x)}{g(x)} = \frac{(\frac{2}{\pi})^{\frac{1}{2}} e^{\frac{x^2}{2}}}{e^{-x}} = (\frac{2}{\pi})^{\frac{1}{2}} e^{\frac{x^2}{2} - x}$$

$$Max_x((\frac{2}{\pi})^{\frac{1}{2}} e^{\frac{x^2}{2} - x}) \rightarrow Min_x(\frac{x^2}{2} - x)$$

$$x \rightarrow x^* = 1$$

$$M = max(\frac{f(x)}{g(x)}) = (\frac{2}{\pi})^{\frac{1}{2}} e^{\frac{1}{2}} = 1.315$$