Quiz #4

Started: 3 Nov at 10:47

Quiz instructions

General Quiz Information

(https://bruinlearn.ucla.edu/courses/173226/files/14494000?wrap=1)

You will answer the quiz questions with sensible answers to gain full credit. Any blank, non-sense, unfinished answers will not be counted. You may upload pdf or png files to answer the questions.

Here is some general information that may be helpful in using Canvas Quizzes.

- You must complete and submit your answers for each quiz by the due date
- For a timed quiz, **you can't stop the clock once you begin**. If time runs out, your quiz will close.
- When you are done answering the questions and are ready to submit your answers for grading, click **Submit Quiz**.
- If you experience a technical problem that interferes with your ability to complete a quiz during the specified time, contact your instructor as soon as possible—you don't have to wait until the quiz has closed.

Question 1 — 4 pts

Consider the problem of estimating $\theta = \int_0^1 e^x dx$.

(a) Please write R code using the Monte Carlo method to estimate θ.

(b) If we modify the interval in the previous problem to (0, 1/2), denoting it as $\theta_{1/2} = \int_0^{1/2} e^x dx$, do you need to rewrite your algorithm from (a) to estimate $\theta_{1/2}$? If so, please write R code to estimate $\theta_{1/2}$.

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12pt Paragraph

a)

df <- function(n){

u <- runif(n)

return(mean(exp(u)))

}

b)

df <- function(n){

u <- runif(n, 0, 1/2)

p

28 words

Question 2

4 pts

We want to use the "hit-or-miss" approach to estimate $\theta = \int_0^{0.5} e^{-x} dx$.

(a) Please write the algorithm (or R code) to estimate θ.

(b) Please calculate the exact variance of the estimator with a sample size of 1,000.

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12pt Paragraph

}

return (result)

}

(b)

$$Var[\hat{\theta}] = \frac{Var[g(X)]}{m} = (E(g(X)^2) - (E(g(X)))^2)/1000^2$$

$$= (2[-0.5e^{-1} + 0.5] - (2[-e^{-0.5} + 1])^2)/1000^2$$

p 51 words </>

Question 3 2 pts

Suppose $X = (X_1, X_2)^T$ follows the bivariate standard normal distribution. We are interested in $E(|X_1 - X_2|)$ and $Var(|X_1 - X_2|)$. Please write R code using the bootstrap method to estimate both quantities.

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12pt Paragraph

```
bootstrap_estimate <- function(n = 1000, B = 1000) {
  X1 <- rnorm(n)
  X2 <- rnorm(n)
  differences <- abs(X1 - X2)
  bootstrap_means <- numeric(B)
  bootstrap_vars <- numeric(B)

  for (i in 1:B) {
    bootstrap_sample <- sample(differences, n, replace = TRUE)
    bootstrap_means[i] <- mean(bootstrap_sample)
    bootstrap_vars[i] <- var(bootstrap_sample)
  }
```

p 69 words </>

Saved at 11:15 Submit quiz