Quiz #3

Started: 28 Oct at 18:21

Quiz instructions

General Quiz Information

(https://bruinlearn.ucla.edu/courses/173226/files/14494000?wrap=1)

You will answer the quiz questions with sensible answers to gain full credit. Any blank, non-sense, unfinished answers will not be counted. You may upload pdf or png files to answer the questions.

Here is some general information that may be helpful in using Canvas Quizzes.

- You must complete and submit your answers for each quiz by the due date
- For a timed quiz, **you can't stop the clock once you begin**. If time runs out, your quiz will close.
- When you are done answering the questions and are ready to submit your answers for grading, click **Submit Quiz**.
- If you experience a technical problem that interferes with your ability to complete a quiz during the specified time, contact your instructor as soon as possible—you don't have to wait until the quiz has closed.

Question 1 2 pts

Suppose we can only generate random samples from Unif(0, 1). Please write an algorithm (or R code) using the composition methods to generate the random samples from t-distribution with k degrees of freedom.

```
rt_composition <- function(n, k) {

  u1 <- runif(n)
  u2 <- runif(n)
  z <- sqrt(-2 * log(u1)) * cos(2 * pi * u2)


  chi_sq <- rep(0, n)
  for (i in 1:k) {
    exp_rv <- -log(runif(n))
    chi_sq <- chi_sq + exp_rv
  }
```

p 51 words </>

Question 2

4 pts

The density function of x is given by $f_X(x) = 3(1-x)^2$, $0 < x < 1$. Set $Y = (1-x)^3$.

(a) Please find $f_Y(y)$

(b) Please write an algorithm (or R code) to generate random samples from $f_X(x)$ via $f_Y(y)$.

a)

$$x = 1 - y^{\frac{1}{3}}$$

$$f_Y(y) = 3(1-(1-y^{\frac{1}{3}}))^2 \times 3(1-y^{\frac{1}{3}})^2 = 9(1-y^{\frac{1}{3}})^4$$

b)

```
generate_samples <- function(n) {

  y_samples <- runif(n, 0, 1)  # Since the domain of y is also [0,1]

  x_samples <- 1 - y_samples^(1/3)
```

p 28 words </>

Question 3 2 pts

Please specify how to generate the random samples from $N(\mu_p, \Sigma_{pxp})$ using the spectral decomposition method.

```
eigen_decomp <- eigen(Sigma)
Q <- eigen_decomp$vectors
Lambda <- diag(eigen_decomp$values)

L <- chol(Lambda)

p <- length(mu)
samples <- matrix(0, n, p)
for (i in 1:n) {
  Z <- rnorm(p)
  X_star <- Q %*% L %*% Z
  samples[i,] <- mu + t(X_star)
}
```

p 58 words </>

Question 4 2 pts

Suppose we define $f(x) = \sum_{i=1}^{n} w_i f_i(x|p_i)$, where $w_i = 1/n$ and $f_i(x|p_i)$ =Bernulli(p_i) function for $x \in \{0, 1\}$. Is $f(x)$ a valid probability function? Explain.

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12pt Paragraph

To determine if f(x) is a valid probability function, it must satisfy the following criteria:

1. For all x in the domain, $(0 \leq f(x) \leq 1)$
2. The sum of f(x) over all values of x in the domain must be equal to 1.

Given:
$f(x) = \sum_{i=1}^{n} w_i f_i(x|p_i)$
Where:
$w_i = \frac{1}{n}$
$f_i(x|p_i)$ is a Bernoulli function defined as:

p 3 231 words </>

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