Quiz week 4 key  4

Question 4

| $y$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(Y=y)$ | 0,01 | 0,1 | 0,4 | 0,3 | 0,19 |

A = there is less than 2 attacks per week.

$P(A) = P(Y < 2) = P(Y=1) + P(Y=0)$
$= 0.1 + 0.01 = 0.11$ //

Let $P(A) = p = 0.11$

Define $X$ = # of weeks out of 10 in which A happens.

$P(X \geq 2)$ is what is being asked.

Assumptions: ⓞ Attacks one week independent of attacks other weeks.
ⓞ Each week $P(A) = 0.11$.
ⓞ Independence, then can be assumed, weeks are independent.

$X \sim Bin(n=10, p=0.11)$. because the population of weeks is very large. So 10 weeks out of many weeks is probably less than 10% of total # of weeks. If you used hypergeometric, you would have to know the Population size, which is not given

$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[\binom{10}{0}(0.11)^0(1-0.11)^{10} + \binom{10}{1}(0.11)(1-0.11)^9\right]$

$= 1 - 0.6972092$

$= 0.3027908$

# Quiz Week 4

Key.





Question 1. Stadium experiment.

(a) $S = \{C, AC, BC, ABC, BAC, AAC, BBC, ABBC, BABC, AABC, \ldots\ldots, \}$.

where C = event that individual enters via C door
A = event that individual enters via A door
B = event that individual enters via B door

and, for example, ABC means that the first individual observed entered via door A, the second via door B and the third via door C, where I stop observing.

This is a <u>countably infinite</u> sample space. The experiment could go on forever!

(b) The values of random variable X, where.
X = # of people observed,
can be any of the natural numbers $N^+$ (nonnegative):
$X: \{1, 2, 3, 4, \ldots\}$.

For the sample space I gave, I write x below the outcome

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $S=\{$ | C, | AC, | BC, | ABC, | BAC, | AAC, | BBC, | ABBC, | BABC, | AABC, $\ldots\ldots\}$ |
| X: | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 |

(c) $P(A) = 0.3$, $P(B) = 0.5$, $P(C) = 0.2$.

| x | P(X=x) | event | work for P(X=x) |
|---|---|---|---|
| 1 | 0.2 | {C} | P(C) is given as 0.2 |
| 2 | 0.16 | {AC, BC} | $P(AC)+P(BC) = (0.3)(0.2)+(0.5)(0.2)$ |
| 3 | 0.128 | {ABC, BAC, AAC, BBC} | $P(ABC)+P(BAC)+P(AAC)+P(BBC)$ $=(0.3)(0.5)(0.2)+(0.5)(0.3)(0.2)$ $+(0.3^2)(0.2)+(0.5)^2(0.2)$ |
| --- | | | |

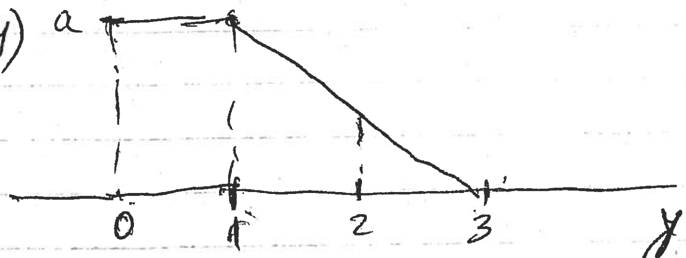
(d) (1) Independence of observations ✓, (2) not binary outcomes at each trial, (3) Not measuring # of successes - # entering C. <u>Not Binomial</u>

Quiz Week 4 Key (2)



Question 2

(a) $f(y)$




The area under the $f(y)$ must be one.

$$\underbrace{(1\times a)}_{\text{for rectangle}} + \underbrace{\frac{a\times 2}{2}}_{\text{for triangle}} = 1 \Rightarrow 2a = 1 \Rightarrow \boxed{a=\frac{1}{2}}$$

(b). The median or $50^{th}$ percentile is the value of y such that 50% of the area is to the left of this value and 50% above.

The area to the left of 1 is $1\times\frac{1}{2} = \frac{1}{2}$.

Thus, $\boxed{\text{median} = 1.}$

(c) (c.1) Let's calculate first $P(Y>2)$.

(i) calculate height of $f(y)$ at $Y=2$.
For that find equation of line in the rectangle.

$$\left.\begin{array}{l} f(y) = 0 = c - 3a \\ f(y) = \frac{1}{2} = c - 1a \end{array}\right\} \Rightarrow c = \frac{3}{4},\ a = \frac{1}{4}$$

For interval $1-2$, $f(y) = \frac{3}{4} - \frac{1}{4}y$.

So $f(2) = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$.

(ii). $P(Y>2) = \frac{1}{2}\left(1\times\frac{1}{4}\right) = \frac{1}{8} = 0.125$ //

(c,2) $P(\text{both individuals have } Y>2)$.
$= [P(Y>2)]^2$ by product rule.
But $[P(Y>2)]^2 = \left(\frac{1}{8}\right)^2 = 0.015625$

Quiz week 4 key    

Question 3

The density should have been
$f(y) = (\frac{1}{2})(ay+1), \quad -1 < y < 1$
for it to integrate to 1.

still, area does not depend on a. but a must be $a \in [-1,1]$

Note: the one posted in the exam was not a density since it was not integrating to 1.
But let's calculate the expected value with what was given.

(a) $f(y) = \frac{3}{2}(ay+1), \quad -1 < y < 1.$

$$E(Y) = \int_{-1}^{1} y\,\frac{3}{2}(ay+1)\,dy = \int_{-1}^{1} \frac{3}{2}ay^2\,dy + \int_{-1}^{1} \frac{3}{2}y\,dy$$

$$= \frac{3}{2}a\frac{y^3}{3}\Big|_{-1}^{1} + \frac{3}{2}\frac{y^2}{2}\Big|_{-1}^{1}$$

$$= \left(\frac{\cancel{3}}{2}\frac{a}{\cancel{3}} + \frac{\cancel{3}}{2}\frac{a}{\cancel{3}}\right) + \left(\frac{3}{4} - \frac{3}{4}\right)$$

$$= 2\frac{a}{2} = a$$

$$E(Y^2) = \int_{-1}^{1} y^2\,\frac{3}{2}(ay+1)\,dy = \int_{-1}^{1}\frac{3}{2}ay^3\,dy + \int_{-1}^{1}\frac{3}{2}y^2\,dy$$

$$= \frac{3}{2}a\frac{y^4}{4}\Big|_{-1}^{1} + \frac{\cancel{3}}{2}\frac{y^3}{\cancel{3}}\Big|_{-1}^{1} = \left(\frac{3}{8}a - \frac{3}{8}a\right) + \left(\frac{1}{2} + \frac{1}{2}\right) = 1$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 1 - a^2$$

$$\sigma_Y = \sqrt{1-a^2}$$

(b). Continuing with wrong $f(y)$.    $IQR = c - b$.

$\int_{-1}^{c} \frac{3}{2}(ay+1)\,dy = 0.75$    solve for c.

$\int_{-1}^{b} \frac{3}{2}(ay+1)\,dy = 0.25$    solve for b.

$$\frac{3}{2}a\frac{y^2}{2} + \frac{3}{2}y\Big|_{-1}^{c} = \frac{3}{4}ac^2 + \frac{3}{2}c - \left[\frac{3}{4}a + \frac{3}{2}\right] = 0.75$$

$$\frac{3}{2}a\frac{y^2}{2} + \frac{3}{2}y\Big|_{-1}^{b} = \frac{3}{4}ab^2 + \frac{3}{2}b - \left[\frac{3}{4}a + \frac{3}{2}\right] = 0.25.$$