**Stat 100A(2) Essay questions where you had to show work. KEY. Copyright Juana Sanchez. Distribution outside Canvas course web page for Stat 100A(2) Fall 2021 prohibited.**

**Question Z generation (1 point)** A Z generation student has 3000 songs in their playlist. 15% of these songs are classic rock. The student selects their songs at random. What is the probability that in the next 100 songs chosen, the number of classic rock songs is larger than 70? Show work and justify your answer.

Define random variable (0.1 points): X = number of classic rock songs in the sample

Check assumptions for Binomial (0.1 points): (i) Songs chosen at random, so each song choice is an independent trial. (ii) small sample relative to 3000 so p constant and p=0.15= "probability that a son is classic rock."

Conclusion (0.1): Can assume that X is Binomial(n=100, p=0.15)

Check conditions for normal approximation to binomial (0.1 points): (i) np= 100(0.15) = 15>10; (ii) 100(0.85) =85 > 10. Conditions hold, so can use the Normal approximation.

Calculate expected value and standard deviation for binomial (0.2 pts):

$$\mu_X = np = 15; \qquad \sigma_X = \sqrt{np(1-p)} = \sqrt{100(0.15)(0.85)} = 3.570714$$

Work set up for answer: P(X> 70) (0.2pts) = "approximately 0" (0.2 pts) by looking at the app for calculating normal probabilities posted in the course web site.

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**Question one die (1 pt).** A friend tells me that a six-sided die that we are using for a game is fair.You roll the die 100 times and find 75 numbers that are even. Is the die fair or not?

Define random variable (0.1 points): X = number of even numbers in the roll of fair 6-sided

Check assumptions for Binomial (0.1 points): (i) Each roll (or trial) is an independent trial. (ii) rolling a die is like sampling with replacement, so p constant and p=0.5= "probability that a roll is an even number"

Conclusion (0.1): Can assume that X is Binomial(n=100, p=0.5)

Check conditions for normal approximation to binomial (0.1 points): (i) np= 100(0.5) = 50>10; (ii) 100(0.5) =50 > 10. Conditions hold, so can use the Normal approximation.

Calculate expected value and standard deviation for binomial (0.2 pts):

$$\mu_X = np = 50; \qquad \sigma_X = \sqrt{np(1-p)} = \sqrt{100(0.5)(0.5)} = 5$$

Work set up for answer: P(X> 75) (0.1pts) = "approximately 0" (0.1 pts) by looking at the app for calculating normal probabilities posted in the course web site. So we conclude: die is not fair (0.2pts)

**Question types of r.v.(1 pt).** For each of the following types of random variables, provide the expected value of the following function:

$$\boldsymbol{g(x)} = 20 + 30\boldsymbol{X^2} - 5\boldsymbol{X}$$

(a) (0.2 pts, 0.1 for work, 0.1 for final number) X ~ Binomial (n=20, p=0.3)

$$E\big(g(X)\big) = 20 + 30E(\boldsymbol{X^2}) - 5E(X) = 20 + 30(np(1-p) + (np)^2) - 5np$$
$$= 20 + 30(4.2 + 36) - 5(6) = 1196$$

(b) (0.2pts, 0.1 for work, 0.1 for final number) X~ Poisson ($\lambda$=5)

$$E\big(g(X)\big) = 20 + 30E(\boldsymbol{X^2}) - 5E(X) = 20 + 30(\lambda + (\lambda)^2) - 5\lambda = 20 + 30(5 + 25) - 5(5)$$
$$= 895$$

(c) (0.2pts, 0.1 for work, 0.1 for final number) X~ Negative Binomial (p=0.2, r=5)

$$E\big(g(X)\big) = 20 + 30E(\boldsymbol{X^2}) - 5E(X) = 20 + 30\left(\frac{r(1-p)}{p^2} + \left(\frac{r}{p}\right)^2\right) - 5\frac{r}{p}$$
$$= 20 + 30(100 + 625) - 5(25) = 21645$$

(d) (0.2pts, 0.1 for work, 0.1 for final number) X~ Exponential($\lambda$=5)

$$E\big(g(X)\big) = 20 + 30E(\boldsymbol{X^2}) - 5E(X) = 20 + 30\left(\frac{1}{\lambda^2} + \left(\frac{1}{\lambda}\right)^2\right) - 5\frac{1}{\lambda}$$
$$= 20 + 30(0.04 + 0.04) - 5(0.2) = 21.4$$

(e) (0.2pts, 0.1 for work, 0.1 for final number) X~ Gaussian($\mu$=100, $\sigma$=5)

$$E\big(g(X)\big) = 20 + 30E(\boldsymbol{X^2}) - 5E(X) = 20 + 30(\sigma^2 + (\mu)^2) - 5\mu$$
$$= 20 + 30(25 + 10000) - 5(100) = 300270$$

**Question factories(1 pt).** Two factories are used to produce the microchips sold by the Microchipsellers company. Factory A produces 60% of the microchips and its microchips have, on average, 2 defects per microchip. Factory B produces the remaining 40% of the microchips and its microchips have, on average, 1 defect per microchip.

If we buy a microchip sold by Microchipsellers, what is the probability that it has 1 error? Show work and provide a final answer.

Defines random variable (0.2pts): Let Y="number of errors per microchip."

Justifies Poisson (0.2 pts):Assuming errors are independently and equally likely to happen in any of the microchips, we can assume that Y is Poisson distributed. However, there are two Poisson distributions, one for Factory A and one for factory B.

$$
\begin{aligned}
P(Y=1)(0.1pts) &= P(Y=1|A)P(A)+P(Y=1|B)P(B) \\
&= \left(\frac{2^1e^{-2}}{1!}\right)0.6+\left(\frac{1^1e^{-1}}{1!}\right)0.4(work\ 0.4\ pts) \\
&= 0.3095541(final\ answer\ 0.1pts)
\end{aligned}
$$

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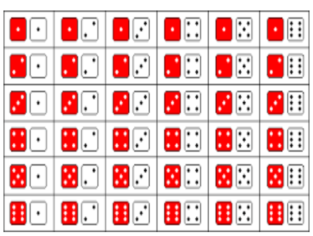
**Question two dice (two dice) (1pt) :** Suppose we roll two fair six-sided dice, one red and one blue. Showing work, answer the following question. You may submit a file for this question with your hand-written or typed answer.

(a) the



(0.6 pts for the pmf) Construct the probability mass function of random variable

Y=(a-b),

where "a" is the value obtained with the red one and "b" the value obtained with the blue one.

| | | | | | |
|---|---|---|---|---|---|
| 5 | 4 | 3 | 2 | 1 | 0 |
| 4 | 3 | 2 | 1 | 0 | -1 |
| 3 | 2 | 1 | 0 | -1 | -2 |
| 2 | 1 | 0 | -1 | -2 | -3 |
| 1 | 0 | -1 | -2 | -3 | -4 |
| 0 | -1 | -2 | -3 | -4 | -5 |

| y=(a-b) | P(y) |
|---|---|
| -5 | 1/36 |
| -4 | 2/36 |
| -3 | 3/36 |
| -2 | 4/36 |
| -1 | 5/36 |
| 0 | 6/36 |
| 1 | 5/36 |
| 2 | 4/36 |
| 3 | 3/36 |
| 4 | 2/36 |
| 5 | 1/36 |

(b) (0.2 pts for mean and standard deviation) Calculate the probability that Y is less than or equal to 1. Calculate the expected value and standard deviation of Y.

$$P(Y \leq 1) = \frac{26}{36} = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36}$$

$$E(Y) = \sum_y yP(Y = y) = -5 \times \frac{1}{36} + \cdots \ldots \ldots + 5 \times \frac{1}{36} = 0$$

$$E(Y^2) = \sum_y y^2P(Y = y) = 25 \times \frac{1}{36} + \cdots \ldots \ldots + 25 \times \frac{1}{36} = 5.83333$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 5.83333$$

$$\sigma_Y = \sqrt{5.8333} = 2.415229$$

(b) Characterize the distribution of Y as skewed right, skewed left or symmetric.

It is symmetric. (0.2pts)

**Question grocery. (1 pt)** The length of time that it takes to process the grocery shopping at a checkout counter in a store with only one checkout counter is exponentially distributed with parameter $\lambda$=1/7. If someone arrives immediately ahead of you to the checkout counter, what is the probability that you will have to wait 3 minutes?

Let Y ="time it takes to process grocery shopping" (0.2pts defines r.v.)

You will have to wait 3 minutes if the processing of the grocery shopping of the person ahead of you takes 3 minutes or more

$P(Y \geq 3) = 1 - P(Y < 3) = 1 - (1 - e^{-3/7})$ = 0.6514391. ( 0.6, 0.2pts )

Note: Y is a continuous random variable. Consequently f(3) =0, because there is no area in a line. You can not calculate the probability of Y=3, but you can calculate the probability that Y is larger or equal to 3.

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**Question with image of train station.** List three random variables, X, Y, T that apply to this scene and indicate whether they are discrete or continuous and what could be a possible model for them.

1/3 point for each. Examples will vary.

**X=** The daily number of people arriving at this station from wherever the train that is parked there originated from. This would be a Poisson random variable.

**Y=**The weight of the people arriving in the train that is now parked in the station. This is a continuous random variable and it could not be normal, because there could be children and adults, and therefore there will be more than one peak in the distribution (what we call a bimodal distribution). It could be a mixture of normal distributions.

**T=** The time it takes a passenger arriving in this train to go from the train to the top of the stairs. This would be a continuous random variable, exponential.

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**Question of system with components (1pt) :** This system has 8 components. The reliability of each component is given as indicated. We have, for convenience, split the system into 5 subcomponents that we label as $C1\ to\ C5$. Showing work calculate the reliability of the whole system.

Reliability= P(system works) (0.2pt)= $0.95 \times 0.95 \times (1 - 0.3^3) \times (1 - 0.25^2) \times 0.9\ (0.6\ pt) = 0.7409243 (0.2pt)$

**Question on taxicab hit and run. (1pt)**

Let

Labeling events and identifying probabilities (0.2pt)

G=green cab; B=blue cab ; C =witness identified a cab as blue; $C^c$ =witness identified the cab as green.

P(G) =0.85 ; P(B) = 0.15 ; P(C|B) = 0.8; P(L|G) = 0.8

Want (work: 0.1, 0.6, 0.1 pts )

$$P(B|C) = \frac{P(C|B)P(B)}{P(C|B)P(B) + P(C|G)P(G)} = \frac{0.8 \times 0.15}{0.8 \times 0.15 + 0.2 \times 0.85} = 0.4137931$$

So the cab is more likely to be green than blue.