Started on	Sunday, 16 October 2022, 11:29 PM
State	Finished
Completed on	Friday, 21 October 2022, 1:32 PM
Time taken	4 days 14 hours
Grade	**17.75** out of 20.00 (**88.75**%)

Question **1**
Correct
Mark 1.00 out of 1.00

Version of Section 5.3.7, exercise 1

A resident of Boston spends the Summers in the Grand Tetons, Wyoming. Every day there is expectation that a moose may pass in front of the house. Moose are wild animals that live around that area. The daily sighting (number of times seen) of moose has the following probability mass function, where X is the daily number of sightings:

x	0	1	2
P(X=x)	0.1	0.5	0.4

If this Boston resident spends 5 randomly chosen days in the Grand Tetons, what is the expected number of days in which there will be at least one moose sighting? There is uncertainty, of course, the true number of days could be different from the expected value. By how much will the total number of days depart from the expected value, on average?

Guide for you to find the solution- An exercise in modeling:

(1) Explore conditions, information and assumptions you can make. Sounds like we are sampling five days from the many days in the life of this resident. In each of these days the person could or could not sight a moose. The probability of sighting at least one moose is 0.9, let's call that p=0.9, and the probability of not sighting a moose is q=1-p=0.1. Notice that we are defining a success here as "sighting at least one moose." We see the probabilities in the table given. Can we approximate this with a Binomial? We check. 5/365 days is only 1% of the days of a year, so much less if this person lives beyond one year.

(2) Decide which random variable among the ones you are familiar with based on (1). So although sampling without replacement (can't live the same day twice), we have the condition to approximate a Hypergeometric with the Binomial. So we can define the random variable

X=number of days out of n=5 in which there was at least one moose sighting.

Given our discussion, X follows a Binomial model. Identify the binomial model and then you will use it to calculate the requested quantities.

(3) Use this model to answer the question.

Select one:

- a. 6.5 and

  $\sqrt{2.04992}$

  , respectively

- b. 4.01411 and 0.8 respectively
- c. 4.5 and 0.6708 respectively ✔
- d. 1.71293 and

  $\sqrt{0.3176261}$

  , respectively

Your answer is correct.

For a Binomial random variable X, E(X)= np, Var(X) = np(1-p) for the model and the standard deviation, which is what you must use, is the square root of the variance for all random variables.

X is Bin(n=5, p=0.9).

The correct answer is: 4.5 and 0.6708 respectively

Question **2**
Correct
Mark 1.00 out of 1.00

Let the random variable X represent the difference between the number of heads and the number of tails obtained when a fair coin is tossed 3 times. That is,

X = number of heads - number of tails.

The possible values this random variable can take, and the sample space on which the random variable is defined are:

Hint to find the solution: Remember the definition of a random variable as a function whose origin is events in the sample space and its image is the Real numbers.

Select one:

a. X can be equal to either of the folllowing: -3, -1, 1, 0, 3. And S={0,1,2,3}

b. X can be equal to either of the folllowing: -3, -1, 1, 3. And S={0,1,2,3}

c. X can be equal to either of the folllowing: -1, -2, 1, 3. And S={HHH,HHT, HTH, HTT, THH, THT, TTH, TTT}

d. X can be equal to either of the folllowing: -3, -1, 1, 3. And S={HHH,HHT, HTH, HTT, THH, THT, TTH, TTT} ✔ The sample space has hhh, hht, hth, htt, thh, tht, tth, ttt. For example, the difference between the number of heads and number of tails in hhh is 3 -0 = 3...

Your answer is correct.

It always helps to think of the sample space and remembering that a random variable is just a function assigned to events in a sample space.

S={HHH,HHT, HTH, HTT, THH, THT, TTH, TTT}

For each outcome, deduct the number of tails from the number of heads. For example, the outcome HHH has 3 heads and 0 tails. So the random variable

X have value 3.

The values of X corresponding to each of the outcomes in S, respectively, are

3,1,1,-1, 1, -1, -1, -3.

Each of those values has probability 1/8. With all this information, you can put together the probability mass function and supplement it with the information about the event for which the random variable takes those values. Remember that a pmf has unique values of X.

x	**-3**	**-1**	**1**	**3**
P(X=x)	1/8	3/8	3/8	1/8
Event	{TTT}	{(HTT, THT, TTH}	{HHT, HTH, THH}	{HHH}

$$\mu_x = E(X) = 0$$

$$\sigma_X^2 = Var(X) = 3$$

$$\sigma_X = \sqrt{3} = 1.732$$

The correct answer is: X can be equal to either of the folllowing: -3, -1, 1, 3. And S={HHH,HHT, HTH, HTT, THH, THT, TTH, TTT}

Question **3**

Incorrect

Mark 0.00 out of 1.00

Select one:

- a. 0.0799468
- b. 0.07529536
- c. 2
- d. 1
- e. 0.1542013 ✖

Your answer is incorrect.

We define a random variable Y as the number of defectives found in the quality control inspection. The expected value of the random variable Y is, approximately,

Y	P(Y)
0	0.9223
1	**4(0.0188) =0.07529 = P(A) = 4 (0.02)(0.98^3)**
2	0.0004+0.000392+4(0.000384)=0.0023284

E(Y) = 0.07529+2*0.0023284=0.0799468

The correct answer is: 0.07529536

Question 4
Correct
Mark 1.00 out of 1.00

An unfair die looks exactly like an ordinary six-sided die. The probability of a face landing up on this die is proportional to the number of dots on the face. Let Y denote the number of dots on the up face. Give the probability mass function of Y

Select one:

a.

Y	1	2	3	4	5	6
P(Y)	1/6	2/6	3/6	4/6	5/6	6/6

b.

Y	1	2	3	4	5	6
P(Y)	1/21	2/21	3/21	4/21	5/21	6/21

c.

Y	1	2	3	4	5	6
P(Y)	1/6	1/6	1/6	1/6	1/6	1/6

d. None of the other options

Your answer is correct.

We know that the sum of the probabilities must be 1. So
1k + 2k + 3k + 4k + 5k + 6k = 21 k = 1
So k= constant of proportionality = 1/21

Y	1	2	3	4	5	6
P(Y)	1/21	2/21	3/21	4/21	5/21	6/21

The correct answer is:

Y	1	2	3	4	5	6
P(Y)	1/21	2/21	3/21	4/21	5/21	6/21

Question 5
Correct
Mark 1.00 out of 1.00

The number of hours spent watching TV (X) per day by middle aged school teachers is as follows

X	0	1	2	3	4
P(X)	0.3	0.3	0.1	0.2	0.1

The Expected number of hours spent watching TV per day is

Select one:
- a. 1.5 ✔
- b. 1.36
- c. 2
- d. 0.5

Your answer is correct.

The correct answer is: 1.5

Question 6
Correct
Mark 1.00 out of 1.00

Let X be the max(a,b) of the roll of two fair six-sided dice, where a is the number in the first die and b the number in the second. For example, (3,4) means that the first die was a 3 and the second a 4.

On average, we should expect X to be closest to

Select one:
- a. 3
- b. 6
- c. 5
- d. 4 ✔

Your answer is correct.

The max can only take values 1, 2,3,4,5,6. Each of those values is associated with an event. For example,

1 is the value of the random variable for {(1,1)}. Then 2 is the value of the random variable for the event {(2,1),(1,2)}, etc... By calculating the probabilities of the event associated with the value of the random variable, you will get the probability mass function of X, and that pmf allows you to calculate the expected value of X using the definition of expected value of a discrete random variable.

The correct answer is: 4

Question 7
Partially correct
Mark 0.75 out of 1.00

Mini quiz Chapter 5, question 5.

Students were asked to give an example of probability mass functions. They are given below. Then other students were asked to judge whether the students did indeed give a pmf. Decide which of those below are not pmf.

There could be more than one answer.

Select one or more:
- a. $P(x) = x/3, \qquad x = -2, -1, +2$ ✔

b. $P(x) = x/4, \qquad x = 1, 2, 3$

c. $P(x) = x/3, \qquad x = -1, +1, +3$

d. $P(x) = x^2/8, \qquad x = 1, 2, 3$

Your answer is partially correct.

You have correctly selected 3.
A probability mass function is summarizing the whole sample space of the experiment. Each value of X is based on an event in S.

Therefore, the pmf must satisfy the axioms.

(a) P(S) = 1; so the sum of the probabilities of X over all X=x must be one.

(b) Probability is nonnegative and no larger than 1. There can not be negative probabilities or probabilities larger than 1.

(c) The probability of mutually exclusive events is the sum of their probabilities.

The correct answers are:

$$P(x) = x/4, \qquad x = 1, 2, 3$$

,

$$P(x) = x^2/8, \qquad x = 1, 2, 3$$

,

$$P(x) = x/3, \qquad x = -2, -1, +2$$

,

$$P(x) = x/3, \qquad x = -1, +1, +3$$

Question **8**
Correct
Mark 1.00 out of 1.00

Chapter 7, Mini quiz question 2

What is the constant k that makes the following function a valid density?

$$f(x) = kx^9(1-x)^2, \qquad 0 \le x \le 1$$

Hint: review properties of density functions.

Select one:

- a. 0.00151
- b. 210
- c. 660 ✔
- d. 2

Your answer is correct.

Take the integral and equal it to 1 and solve for k.

The correct answer is: 660

Question **9**
Correct
Mark 1.00 out of 1.00

Chapter 7, mini quiz question 3.

Let X be the time that it takes to drive between point A and point B during the afternoon rush hour period in highway 4005. The density function of X is

$$f(x) = \frac{1}{2}x, \quad 0 \le x \le 2$$

Find the median.

Hint: Think that the median is the 50th percentile and when computing percentiles the cumulative distribution is set as follow: F(c) = P(X $\le$ c)= 0.5 for this case. You must find c.

Select one:

- a. 0.034
- b. 1
- c. 5/16
- d. 1.414 ✔

Your answer is correct.

The correct answer is: 1.414

Question **10**
Correct
Mark 1.00 out of 1.00

Chapter 7, section 7.2.1 Exercise 4

If X is a continuous random variable with cumulative distribution function

$$F(x) = 1 - e^{-x/4}, \qquad x \geq 0$$

calculate $P(2 \leq X \leq 5)$

Select one:

- a. approximately 0
- b. 0.491801
- c. 0.6799741
- d. 0.3200259 ✔

Your answer is correct.

$$P(2 \leq X \leq 5) = F(5) - F(2) = (1 - e^{-\frac{5}{4}}) - (1 - e^{\frac{-2}{4}}) = e^{-2/4} - e^{-5/4} = 0.3200259$$

The correct answer is: 0.3200259

Question **11**
Correct
Mark 1.00 out of 1.00

Chapter 7, section 7.2.1 Exercise 8 textbook

(This problem is based on Scheaffer (1995,150).) Daily total solar radiation for a certain location in Florida during the month of October has the following density function:

$$f(x) = \frac{3}{32}(x-2)(6-x), \qquad 2 \leq x \leq 6$$

where X is the solar radiation in hundreds of calories. Find the expected daily solar radiation for October in that location.

Select one:

- a. 3 hundred calories
- b. 1.5 hundred calories
- c. 4.5 hundred calories
- d. 4 hundred calories ✔

Your answer is correct.

$$f(x) = \frac{3}{32}(x-2)(6-x), \qquad 2 \leq x \leq 6$$

$$E(x) = \int_2^6 x\frac{3}{32}(x-2)(6-x)dx = \int_2^6 \frac{3}{32}x(6x - x^2 - 12 + 2x)dx$$

$$= \int_2^6 \frac{3}{32}x(8x - x^2 - 12)dx = \int_2^6 \frac{3}{32}(8x^2 - x^3 - 12x)dx$$

$$= \frac{3}{32}\left[\frac{8x^3}{3} - \frac{x^4}{4} - \frac{12x^2}{2}\right]_2^6 = \frac{3}{32}[36 - (-6.6667)] = 4$$

The correct answer is: 4 hundred calories

Question 12
Correct
Mark 1.00 out of 1.00

The following expression

$$\int_x \mu_x f(x)dx$$

, where f(x) is a density function and the integration is over all the domain of the random variable X,

equals

Select one:

- a. 1
- b. μ_x
- c. σ_x^2
- d. $2\mu_x$

Your answer is correct.

The correct answer is:

μ_x

Question 13
Correct
Mark 1.00 out of 1.00

A student of probability told us that they had finally come up with the correct density function for volume of water (in liters) that a typical student drinks during a half hour workout.

The student gave us

$$f(x) = 8x, \quad ; \qquad 0 \le x \le ???$$

Unfortunately, the student did not give us the complete domain. So we do not really have the density well specified yet. What would be the missing piece of the domain of this proposed density function?

Select one:

- a. 3
- b. 1
- c. 2
- d. 1/2

Your answer is correct.

The correct answer is:

1/2

Question **14**

Incorrect

Mark 0.00 out of 1.00

Los Angeles County had 10.4 million people in 2019, and possibly much more in 2022. That is a very large population. And yet, some individuals are called for Jury Duty pretty often, almost every 2 or 3 years. More specifically, they are called to be in a panel from which jurors are chosen. So sometimes do not get selected to be in the jury. But they have to go and be in the panel for a day nonetheless. The article in our course web side titled: "The Binomial and Hypergeometric Probability Distributions in Jury Selection" by Jude T. Sommerfeld (a copy of which is available for view for your convenience and only for this exam) talks about panels and jury selection. The article considers that it is appropriate to use the hypergeometric distribution in which of the following scenarios?

Hint: read the article in the supplementary materials in this week's module, page with textbook reading and lectures.

- [ ] a. Calculating the probability of having 9 women in the choice of 100 potential jurors out of a jury panel of 350 people consisting of 102 women from a district's population which was 53% female.
- [ ] b. Calculating the probability of having 8 black persons in a jury pool of 100 people drawn from a total population of 16000 men.
- [x] c. Both of the cases presented in the other two choices given in this question. ✖

Your answer is incorrect.

The correct answer is:
Calculating the probability of having 9 women in the choice of 100 potential jurors out of a jury panel of 350 people consisting of 102 women from a district's population which was 53% female.

Question **15**

Correct

Mark 1.00 out of 1.00

Chapter 5, Section 5.2.4, exercise 1

Let Y be a random variable denoting the sum of the roll of two fair six-sided dice. Posted below is a table for the pmf. The table is incomplete. Before doing the table, remember that if the die is fair, then each outcome in the roll of the two dice is [equally likely] ✔ and therefore it is appropriate to use [the classical definition of probability] ✔ to calculate the probability of each outcome (Review Lecture 1 and chapter 1 if you do not remember that). Also, based on Lecture 7, each roll of two fair six sided dice is a [random sample of size 2 from an urn containing numbers 1 to 6 with replacement] ✔ . And after studying independence, we can calculate the probability of rolling, for example, (2,3) using the [product rule for independent events] ✔ . Now that we have reviewed that, here is the table. You will complete what is missing. A couple of examples are included.

y	P(Y=y)	Outcomes in the event mapping to that y
2	1/36	{(1,1)}
3	2/36	{(1,2), (2,1)}
4	[3/36] ✔	
5	[4/36] ✔	
6	[5/36] ✔	
7	6/36	{(1,6), (6,1),(2,5),(5,2), (3,4), (4,3)
8	5	[{(2,6), (6,2), (3,5), (5,3), (4,4)}] ✔
9	4/36	[{(3,6),(6,3), (4,5),(5,4)}]

10 3/36	{(4,6), (6,4), (5,5)}
11 2/36	{(6,5), (5,6) }
12 1/36	{(6,6) }

Notice that if you revisit the app studied together with Lecture 1
https://www.randomservices.org/random/apps/DiceExperiment.html

you will be able to construct other probability mass functions for the sum of 2 dice when the dice are not fair. Practice on your own those other distributions, because in those cases you will not be able to use the classical definition, but you can use the product rule for independent events to calculate the probability of each outcome.

Your answer is correct.

The correct answer is:
Chapter 5, Section 5.2.4, exercise 1

Let Y be a random variable denoting the sum of the roll of two fair six-sided dice. Posted below is a table for the pmf. The table is incomplete. Before doing the table, remember that if the die is fair, then each outcome in the roll of the two dice is [equally likely] and therefore it is appropriate to use [the classical definition of probability] to calculate the probability of each outcome (Review Lecture 1 and chapter 1 if you do not remember that). Also, based on Lecture 7, each roll of two fair six sided dice is a [random sample of size 2 from an urn containing numbers 1 to 6 with replacement]. And after studying independence, we can calculate the probability of rolling, for example, (2,3) using the [product rule for independent events]. Now that we have reviewed that, here is the table. You will complete what is missing. A couple of examples are included.

y	P(Y=y)	Outcomes in the event mapping to that y
2	1/36	{(1,1)}
3	2/36	{(1,2), (2,1)}
4	[3/36]	
5	[4/36]	
6	[5/36]	
7	6/36	{(1,6), (6,1),(2,5),(5,2), (3,4), (4,3)
8	5	[{(2,6), (6,2), (3,5), (5,3), (4,4)}]
9	4/36	[{(3,6),(6,3), (4,5),(5,4)}]
10	3/36	{(4,6), (6,4), (5,5)}
11	2/36	[{(6,5), (5,6) }]
12	1/36	{(6,6) }

Notice that if you revisit the app studied together with Lecture 1
https://www.randomservices.org/random/apps/DiceExperiment.html

you will be able to construct other probability mass functions for the sum of 2 dice when the dice are not fair. Practice on your own those other distributions, because in those cases you will not be able to use the classical definition, but you can use the product rule for independent events to calculate the probability of each outcome.

Question **16**
Correct
Mark 1.00 out of 1.00

Chapter 5 Section 5.2.4, exercise 3

Daily tooth brushing by residents in a remote country was found to follow the probability mass function given below, where the random variable Y represents the number of times brushing teeth per day

y	0	1	2	3
P(Y=y)	0.325	0.474	0.15	0.051

The value 0.474 means that 47.4% of the residents brush their teeth once per day.

The statement "F(2) equals 1-0.051 " is

Select one:

- True ✔
- False

The correct answer is 'True'.

Question **17**
Correct
Mark 1.00 out of 1.00

Survival time in years (X) after lung transplant has the following pdf:

$$f(x) = 5e^{-5x}, \qquad x \geq 0$$

We are interested in the median survival time.

Select one:

- a. 0.1386294 ✔
- b. 5
- c. 1.289341
- d. 0.89189

Your answer is correct.

The correct answer is: 0.1386294

Question **18**
Correct
Mark 1.00 out of 1.00

The Old Faithful is a famous geyser in Yellowstone
(https://www.yellowstonepark.com/things-to-do/geysers-hot-springs/about-old-faithful/)

The geyser varies in the time it makes visitors wait for its next eruption. Using past data we calculated that it takes an average of 35 minute waiting time from the moment the last eruption occurred to a new eruption, when the last eruption was short. You arrive at the site where you can watch Old Faithful, but you arrive right at the end of a short eruption. What is the probability that you will have to wait less than 30 minutes for its next eruption?

- a. 0.42437
- b. 0.57563 ✔
- c. 1
- d. $1 - e^{-35}$

Your answer is correct.

$f(x) = \frac{1}{35} e^{-\frac{1}{35}x}, x \geq 0$

The correct answer is:
0.57563

Question **19**
Correct
Mark 1.00 out of 1.00

Watch the video posted in Module on the Maxwell-Boltzman pdf. Each major has its distributions, because distributions are associated with types of experiments.

Question

The Maxwell-Boltzman probability density function for the speed of air particles in a room does the following when the room temperature increases. Choose all that applies

- a. It shifts to the right of the speed horizontal axis ✔
- b. Is such that the speed has a higher expected value ✔
- c. it shifts to the left of the speed horizontal axis
- d. Is such that the speed has a lower expected value

Your answer is correct.
The correct answers are:
It shifts to the right of the speed horizontal axis ,

Is such that the speed has a higher expected value

Question **20**

Correct

Mark 1.00 out of 1.00

Two different manufacturers supply a component with an exponentially distributed lifetime, that is, the length of service the component gives until if fails is an exponentially distributed random variable. Manufacturer A's device has expected lifetime 4 months and manufacturer B's has 10 months. A particular user has a batch of devices of which 40% came from manufacturer A and 60% from manufacturer B. If a randomly selected device from this batch is used, what is the probability that the lifetime of this device is more than 2 months?

Hint: Think law of total probability. First define the random variable X as the length of service of a randomly chosen component. You are given that it is exponential. But you have two exponentials, one for manufacturer A and one for manufacturer B.

P(X>2) = P(X>2|A)P(A) + P(X>2|B) P(B)

Now you would have to use the exonential pdf to calculate the P(X>2|A) and the P(X>2|B).

- a. 0.73385 ✔
- b. 0.6
- c. 0.4
- d. 0.18127

Your answer is correct.

X= is the lifetime of a random component.

\(P(X>2) =P(X>2|\lambda = 1/4) 0.4 + P(X> 2 |\lambda=1/10)0.6 =

0.73385

The correct answer is:
0.73385