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Grade	**12.90** out of 13.00 (**99.23**%)

Question 1

Correct

Mark 3.00 out of 3.00

In chapter 4, we focus on probability sampling. The urn model helps us visualize this process of sampling and helps with the calculations. Imagine an urn with 4 balls in it, numbered 1, 2, 3, 4. We are going to draw 3 balls. If there are 24 possible samples, this means that we are drawing

without replacement

, and if the number of possible samples we get is 64 we are drawing

with replacement

. The key difference between the two methods of sampling is that

in sampling without replacement, to calculate probabilities, it does not matter that we assume order matters or not. However, in sampling with replacement, it mak

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1/4

or the 3-tuple calculation

6/24

when drawing without replacement, but that equivalence disappears when sampling with replacement. Under the latter, the number of unordered outcomes (or sets) is not 4, but rather, it is

14

and the number of ordered samples or 3-tuples is

64

. Thus the probability of getting the numbers 2,3,4 must be calculated with the 3-tuples, and that would be

6/64

.

without replacement

with replacement

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1/4

6/24

14

64

6/64

Your answer is correct.

Chapter 4 draws a distinction between sets and samples. A sample of size n is an n-tuple. The order matters for the number of samples to count. When sampling without replacement, calculating probability using the number of tuples (with order -permutations- accounted for), or with the sets (without counting the permutations). But when sampling with replacement, only the tuples give you the right probability.

The correct answer is:
In chapter 4, we focus on probability sampling. The urn model helps us visualize this process of sampling and helps with the calculations. Imagine an urn with 4 balls in it, numbered 1, 2, 3, 4. We are going to draw 3 balls. If there are 24 possible samples, this means that we are drawing [without replacement], and if the number of possible samples we get is 64 we are drawing [with replacement]. The key difference between the two methods of sampling is that [in sampling without replacement, to calculate probabilities, it does not matter that we assume order matters or not. However, in sampling with replacement, it makes a difference in the calculus of probabilities that we assume order matters or not.]. For example, the probability that the numbers 2,4,3 come out when we draw 3 balls is equal to the set expression [1/4] or the 3-tuple calculation [6/24] when drawing without replacement, but that equivalence disappears when sampling with replacement. Under the latter, the number of unordered outcomes (or sets) is not 4, but rather, it is [14] and the number of ordered samples or 3-tuples is [64]. Thus the probability of getting the numbers 2,3,4 must be calculated with the 3-tuples, and that would be [6/64].

Question **2**

Correct

Mark 1.00 out of 1.00

In Chapter 4, we distinguish between sampling with and sampling without replacement. In games of chance, like rolling a fair six sided dice, the rolling 3 times can be seen as taking a random sample from an urn with 6 numbers, 1 to 6, with replacement. According to what we learned in the chapter, the number of samples (samples mean that order matters when counting) is 216 ✔ . The number of sets is tedious to calculate, we can instead calculate the number of unordered 3-tuples when drawing with replacement is $\binom{6+3-1}{3} = 56$. The probability of obtaining the numbers 1,2,3 in the sample would be 0.027778 ✔ if using samples to calculate the probability (order matters), but it would be 0.01785714 ✔ if we just use the 56 3-tuples (order does not matter). We notice that the probability with samples (order matters) does not equal ✔ the probability with the unordered 3-tuples.

Your answer is correct.

With the samples, the probability is 6/216 but with the 3-tuples without ordering mattering, it will be 1/56.

The correct answer is:
In Chapter 4, we distinguish between sampling with and sampling without replacement. In games of chance, like rolling a fair six sided dice, the rolling 3 times can be seen as taking a random sample from an urn with 6 numbers, 1 to 6, with replacement. According to what we learned in the chapter, the number of samples (samples mean that order matters when counting) is [216]. The number of sets is tedious to calculate, we can instead calculate the number of unordered 3-tuples when drawing with replacement is $\binom{6+3-1}{3} = 56$. The probability of obtaining the numbers 1,2,3 in the sample would be [0.027778] if using samples to calculate the probability (order matters), but it would be [0.01785714] if we just use the 56 3-tuples (order does not matter). We notice that the probability with samples (order matters) [does not equal] the probability with the unordered 3-tuples.

Question 3
Correct
Mark 1.00 out of 1.00

In Chapter 4, we look also at urns that contain two types of balls. Like a box of 20 chocolates containing 4 defective and 16 non-defective chocolates. And we looked at the way to calculate a solution for probability of finding k number of defectives in a sample of size n when drawing without replacement. Separately we gave the formula for calculating the probability of finding k defectives when drawing with replacement, but we did not apply the latter to the chocolate example. Suppose we draw a random sample of 3 chocolates. Match the following:

Probability of 2 defectives in the sample of 3 chocolates if drawing without replacement is	0.0842 ✔
Probability of 2 defectives in the sample of 3 chocolates if drawing wit replacement is	0.096 ✔
For the without replacement case, I used the	hypergeometric formula ✔
For the with replacement case, I used the	binomial formula ✔

Your answer is correct.

The correct answer is:
Probability of 2 defectives in the sample of 3 chocolates if drawing without replacement is → 0.0842,

Probability of 2 defectives in the sample of 3 chocolates if drawing wit replacement is → 0.096,

For the without replacement case, I used the → hypergeometric formula,

For the with replacement case, I used the → binomial formula

Question **4**

Correct

Mark 1.00 out of 1.00

A warehouse contains ten printing machines, four of which are defective. A company randomly selects five of the machines for purchase. What is the probability that all of the machines are not defective?

When doing sampling in real life, when we say we draw a random sample, it does not make sense to think it is with replacement. Why would we sample the same machine twice? So in most applications, a random sample means without replacement. Rolling a dice, however, is sampling with replacement because the numbers of the dice remain in the dice. The fact that we roll a six, does not preclude that we get another six.

Select one:

- a. 0.9762
- b. 0.0238 ✔
- c. 0
- d. 0.4567
- e. 0.2194

Your answer is correct.

Using the general product rule:

let i=1,2,..., 10 denote the order in which machines are drawn.

Let D denote the event that a machine is defective. There is only one possible outcome where the 5 machines are nondefective.

$$P(D^c \cap D^c \cap D^c \cap D^c \cap D^c) = P(D_1^c)P(D_2^c|D_1^c)P(D_3^c|D_1^c D_2^c)P(D_4^c|D_1^c D_2^c D_3^c)P(D_5^c|D_1^c D_2^c D_3^c D_4^c) = (6/10) * (5/9) * (4/8) * (3/7) * (2/6) = 0.0238.$$

You can also use the hypergeometric formula. If A denotes the event "5 nondefectives" in a sample of n=5,

$$P(A) = \frac{\binom{6}{5}\binom{4}{0}}{\binom{10}{5}} = 0.0238$$

The correct answer is: 0.0238

Question 5
Correct
Mark 1.00 out of 1.00

There are two types of workers in an office: 10 administrative assistants and 5 fund managers. Two workers will be chosen randomly to represent the office on the board of directors and at the town's city hall, one worker to each place. Let A be the event that two fund managers are chosen. What is the probability of A?

Select one:

- a. 0.3156
- b. 0.095238 ✔
- c. 0.6954
- d. 0.0910

Your answer is correct.

When sampling in real life, we do not sample the same item twice, so we are sampling without replacement. The hypergeometric formula must be used.

The correct answer is:
0.095238

Question **6**

Complete

Mark 1.00 out of 1.00

Chapter 4, version of Exercise 7 at end of chapter exercises.

Suppose there are 40 alumni signed up to travel to Egypt with the university alumni association during the summer. In this group of alumni, 25 received a bachelor of science (BS) degree from the university and 15 received a master of science (MS) degree. We must select a random sample of 7 people. (i) What is the probability that the sample contains 4 BS recipients and 3 MS recipients? (ii) What is the probability that there is at least one BS in the sample?

Type your answer in the space provided.
To enter math directly in the space provided, remember to include all the math between. the following. $maths ymbols latex$
A binomial coefficient in latex is as follows, for example, 6 choose 5 is written as {6 \choose 5}
A fraction is written as, for example, \frac{10}{2}
Do not attempt to copy paste. Type the text. It is part of the exercise to do that.

(i) The probability that the sample contains 4 BS recipients and 3 MS recipients when we select a random sample of 7 people. is equal to $\frac{{25 \choose 4}{15 \choose 3}}{{40 \choose 7}} = 0.30873$. Where there is a 0.30873 probability that there are 4 BS and 3 MS recipients when we randomly choose 7 people from the population.

(ii) The probability that the sample contains least 1 BS recipients when we select a random sample of 7 people is equal to $\Sigma_{x=1}^{7} \frac{{25 \choose x}{15 \choose 7-x}}{{40 \choose 7}} = 0.9997$. Which means that there is a 0.9997 probability that there is at least one BS in the sample.

Solution

(i)

$$P(\{"\ 4BS\}) = \frac{{25 \choose 4}{15 \choose 3}}{{40 \choose 7}} = 0.3087259$$

(ii)

$$P(\{"\ at least 1 BS\ "\}) = 1 - P(\{"\ no BS\ "\}) = 1 - \frac{{25 \choose 0}{15 \choose 7}}{{40 \choose 7}} = 0.999$$

Comment:

Question 7
Complete
Mark 0.90 out of 1.00

Version of Chapter 4, exercise 13 end of chapter problems

During one year (365 days) 23 earthquakes occurred over an area of interest.

(a) We want to calculate the probability that two or more earthquakes occurred on the same day of the year. Describe the experiment that we would do before you calculate the probability and what an example of a tuple outcome in the sample space would look like).

Show your work. See the other work questions in this review assignment for hints about using Latex to type notation that we use in Chapter 4.

(b) Relate this problem to the birthday problem discussed during the TA session. Write a question in that context that would be the same question as in the earthquake problem but for that birthday context.

(c) How would you do a simulation to find out the probabilities in either case (a) or (b) ? Give the probability model used, what a trial consists of, what you would calculate at each trial, and the summary calculation you would do with your trials at the end to obtain an estimate of the probabilities. Also indicate how many trials you would do.

a) Consider that two or more earthquakes will occurred in a same day, it will be the number of 1 minus the probability that earthquake will occur less than twice in a same day. Therefore, it will equal to $1-\prod_{n=0}^{22}(1-n/(365)) = 1-0.4927 = 0.5073.$

b) There are 23 people inside the room, what is the probability that two people will have the same birthday.

c)

n = the number of trial

prod(1 — (0:n-1)/365)

factorial(n)*choose(365, n)/365^n

Solution

The experiment is observing the earthquakes in the

(a)

$$P(\{"\ atleast1thesameday\ "\}) = 1-\frac{365!/342!}{365^{23}} = 0.4927$$

(b) This is the same question as the birthday problem. We would look at the sample of earthquakes that occur in a year and would observe the day of the year in which they happened. For example, if there are n=5 earthquakes, a sample of this experiment could be {16, 90, 90, 310, 310}. This would be like sampling with replacement from an urn with 365 balls.

(c) Probability model: a discrete uniform distribution with P(X)=1/365, x= 1, 2,....., 365

Trial: draw a sample of size n=5 from the numbers 1 to 365

Repeat: 10000 (at least 1000 or more)

Record at each trial: there are matches (0) ? There are no matches? 1.

Compute at the end: add the number of 1's and divide by 10000. That would be the estimate of the probability of no matches. Subtract from 1 to obtain the probability that 2 or more earthquakes occurred in the same day of the year.

Comment:
c needs more details and need to mention repeats

Question **8**
Correct
Mark 1.00 out of 1.00

A box with 15 VLSI (Very Large Scale Integrated) chips contains five defective ones. If a random sample of three chips is drawn, what is the probability that more than 1 is defective?

Remember, in real life it would not make sense to draw the same chip over and over...

Select one:

a. 0.8148

0.8148

b. 0.2417582 ✔

c. 0.0952381

d. 0.156108

Your answer is correct.

Can use hypergeometric or do it from first principles. From first principles, let D be defective and N nondefective

Let X equal the number of defectives. D denotes defective and N nondefective.

S ={ DDD, DDN, DND, DNN, NDD, NDN, NND, NNN}

X : 3 2 2 1 2 1 1 0

P : (5/15)(4/14)(3/13), (5/15)(4/14)(10/13) (5/15)(10/14)(4/13) (5/15)(10/14)(9/13)

$$P(X > 1) = P(X = 2) + P(X = 3) = 3(5/15)(4/14)(10/13) + (5/15)(4/14)(3/13) = 0.2417582$$

The correct answer is: 0.2417582

Question **9**
Correct
Mark 1.00 out of 1.00

Suppose that a large university contains 10 percent mountain climbers. If four students are randomly sampled from the university, what is the probability that exactly one student is a mountain climber?

Select one:
- a. 0.057
- b. 0.1
- c. 0.2916✔
- d. 0.001
- e. 0.3439

Your answer is correct.

Use the binomial formula. You can assume that the number of students you are sampling from is very large so it is equivalent to drawing with replacement.

$$P(X=1)=\binom{10}{5}(1/10)(9/10)^3=4(1/10)(9/10)^3=0.2916$$

Alternatively, to be literal, since in real life when we sample we do not sample with replacement in fact, the hypergeometric formula would be appropriate.

The correct answer is: 0.2916

Question **10**
Correct
Mark 1.00 out of 1.00

A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and to accept the lot only if all 3 are nondefective. If 30 percent of the lots have 4 defective components and 70 percent have only 1, what proportion of lots does the purchaser reject?

Notice that you must use the law of total probability in this problem. The conditional probabilities are calculated with the hypergeometric, since you are drawing from a rather small lot, 10, so you can not make the assumption that the lot is large.

Calculate

P(accept) = P(accept|lot 1)P(lot1) + P(accept|lot 2)P(lot 2) by law of total probability

Then calculate, by complement rule,

P(reject=1-P(accept)

Select one:
- a. 54%
- b. 46%✔
- c. 33%
- d. 90%

Calculate

P(accept) = P(accept|lot 1)P(lot1) + P(accept|lot 2)P(lot 2) by law of total probability

Then calculate, by complement rule,

P(reject=1-P(accept)

The correct answer is: 46%

Question **11**
Complete
Mark 1.00 out of 1.00

A version of CHAPTER 4-Selected Section 4.3.2, Ex 3

Factories produce millions of items. Thus, when we sample them to observe quality, we can pretend we are sampling with replacement, although in reality we are sampling at random without replacement. That is the assumption in industrial reliability and other contexts where populations from which we are sampling (The urns) are very large. We are going to select three randomly chosen silicon wafers from a factory producing silicon wafers. The defective rate in this factory is 20% (maybe you wonder why they are not yet out of business, but disregard that).

(a) List the sample space of this experiment completely as tuples. Make sure to label your notation.

(b) Indicate the probability of each outcome of the sample space. Show how you calculate it.

(c) Use the binomial formula to calculate the probability that there are two defectives. Show work.

(d) Use only the third axiom of probability to calculate the probability that there are two defectives.

(e) If we can simply use the third axiom to calculate probabilities of an event in this instance, why use the binomial formula? Give one circumstance when having a formula would be the only practical and feasible way of doing the calculation. Be specific, give an example in the same context as that in this problem.

Notice: two defectives is not the same event as "at least two defectives".

Type your answer in the space provided. See one of the other work questions in this review, to see how to type latex notation used in this chapter.

$p(A_k) = \binom{n}{k}(p)^k(1-p)^{n-k}$ (a) A = Not Defective, B = Defective

S = {AAA, AAB, ABA, BAA, ABB, BAB, BBA, BBB}

(b)

Event X = All defective = {BBB},

Event Y = All not defective = {AAA},

Event Z = Exactly two defective = {ABB, BAB, BBA},

Event M = Exactly one defective = {AAB, ABA, BAA}

P(X) = $(0.2)^3 * (0.8)^0 = 0.008$,

P(Y) = $(0.8)^3 * (0.2)^0 = 0.512$,

P(Z) = $\binom{3}{2}(0.2)^2 * (0.8)^1 = 0.096$,

P(M) = $\binom{3}{1}(0.2)^1 * (0.8)^2 = 0.384$

(c)

According to the binomial formula, $p(A_k) = \binom{n}{k}p^k(1-p)^{n-k}$, where the probability of exactly two samples are defective is equal to $\binom{3}{2}(0.2)^2 * (0.8)^1 = 0.096$.

(d)

The third axiom states that $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$, where these three events are independents to each other. The probability of each components are defective are independents to each other, therefore, three events that happen together are equal to 3(0.2)(0.2)(0.8) = 0.096

(e)

The difference between the third axiom and binomial formula is that the third axiom is build for calculating probability of union of disjoint events, however binomial formula can calculate events that has only two outcomes. For example, There are 25 percent of people who likes soda, and 75 percent of people don't like it. If I want collect sample of 1000 from population and asking for the probability that exactly 700 people from the sample I collect likes soda. This example will be practical and feasible when using binomial formula.

Let 1= denote defective and 0=nondefective.

Then

(a) S= {111, 110, 101, 100, 011, 010, 001, 000}

(b) P(in the order of the outcomes) :
$0.2^3, 0.2^2(0.8), 0.2^2(0.8), 0.2(0.8^2), 0.2^2(0.8), 0.8(0.2^2), 0.2(0.8^2), 0.8^3$

(c) P("2 defectives") = $\binom{3}{2}0.2^3(0.8) = 0.096$

(d) By the third axiom, we consider the outcomes in the event "2 defectives" = {110,101,011}. Since the outcomes are mutually exclusive, we just add the probabilities of each outcome to obtain the probability of the event two defectives.

P("2 defectives"} = P(110) +P(101) + P(011) =
$0.8(0.2^2) + 0.8 * (0.2^2) + 0.8 * 0.2^2 = 0.096$

(e) If we had sampled for example, 1000 wafers, there is no way we could write the whole sample space and figure out all the outcomes with 3 defective. We would have to resort to the formula to calculate it. Notice that it is not the size of the big population that matters but rather the sample size from that large population.

Comment: