Long Run Macroeconomics

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Lecture 5

(note: this lecture will be recorded)

Econ 110A - Housekeeping

- No Office Hours Today, Make Up on Wed April 19, 12 pm to 1 pm

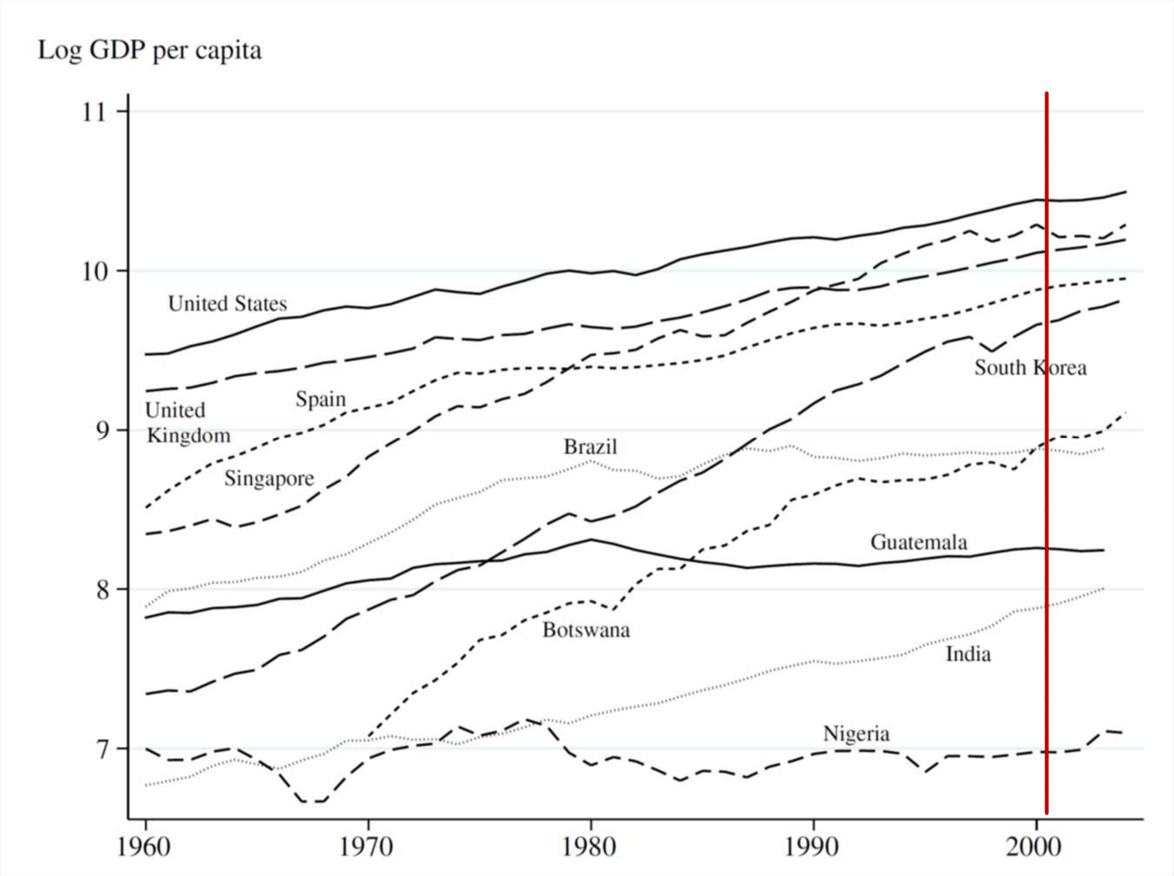
- Practice Problem Set 2 posted

Plan for Lecture 5

- The Production Function
  - Marginal Products
  - Returns to Scale

- A Production Model
  - Profits Maximization and Factors Demand
  - Equilibrium
  - Implications

Why do countries have persistent differences in GDP per capita?



Reproduced from Ch. 1 of Daron Acemoglu, “Introduction to Modern Economic Growth,” 2014

Our Approach

- growth theories of the aggregate supply (production)
- investigate explanations of
  - differences in levels of GDP per capita (facts 3 and 6)
  - differences in growth experiences (fact 3)
  - sustained growth at the frontier (fact 2)

"Say's Law"

The Production Function

- Example: Ice-cream Factory

Milk

Sugar

Salt

Chocolate

Strawberry

Freezer

Machine

Workers

Transportation

Cones / Cups / Containers

Recipe

Factory / Building

Money ?

License

The Production Function

- we are interested in modeling the production of value added
- from the income approach to GDP, we know what factors are ultimately responsible for creation of value
  - labor, management, capital, government
- this suggests a "factor-based" representation of the production function:

$$Y = F(A, K, L)$$

F: production function

A: Ideas

K: capital

L: Labor

Y: Output "Value Added"

Cobb-Douglas:

$$Y = A K^{\alpha} L^{1-\alpha}$$

$$\alpha \in (0,1)$$

Production Function

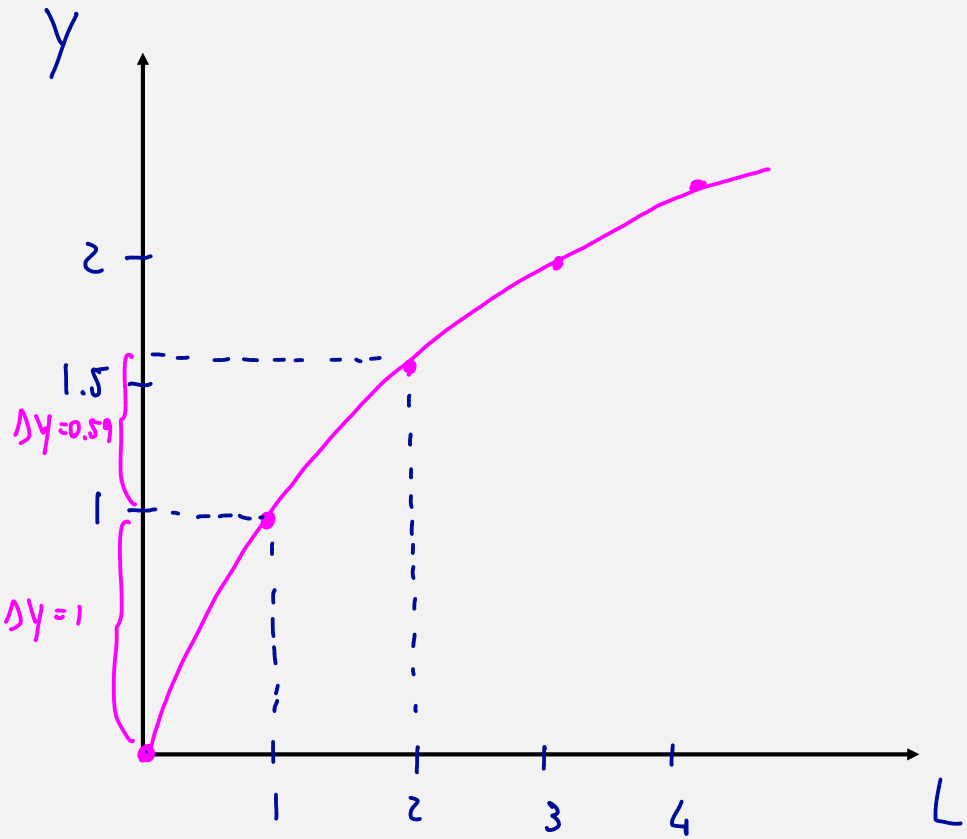
Marginal Product: extra output produced by increasing one factor while keeping all the other factors fixed.

recall: economic agents make decision by ``reasoning at the margin''

Example: Labor

$$Y = \bar{A}K^{1/3}L^{2/3}$$

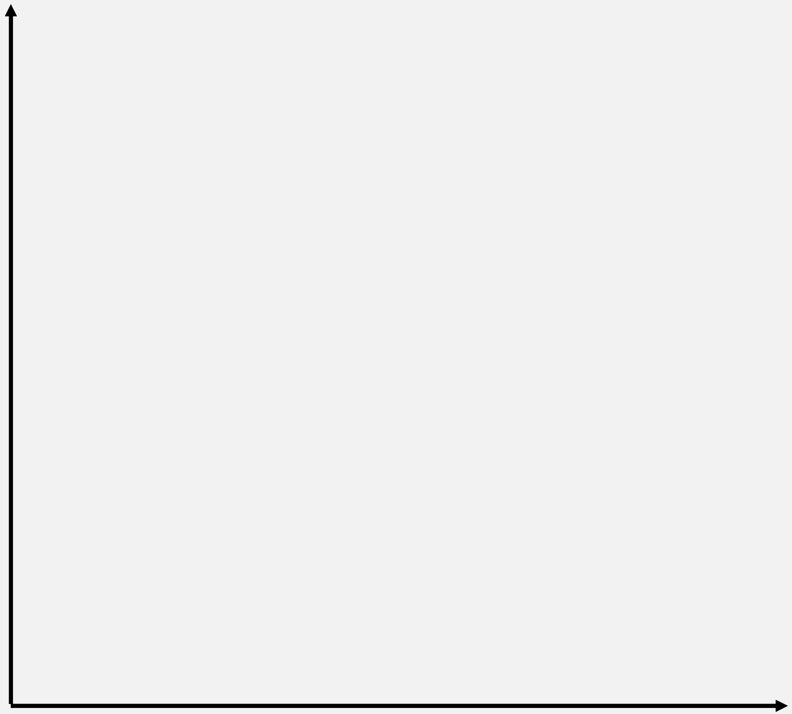
L and *Y*, when *K*=1,$\bar{A}$=1	
L	**Y**
1	1
2	1.59
3	2.08
4	2.52



Example: Capital

$$Y = \bar{A}K^{1/3}L^{2/3}$$

***K* and *Y*, when *L*=1,$\bar{A}$=1**	
K	**Y**
1	1
2	1.26
3	1.44
4	1.59



Production Function

Diminishing Marginal Product: Extra output produced by increasing one factor while keeping all the other fixed is decreasing in the one increasing factor.

$$MPL \equiv \frac{\partial F(K,L)}{\partial L} \qquad MPK \equiv \frac{\partial F(K,L)}{\partial K}$$

CD Production Function: $F(K,L) = \bar{A} K^{\alpha} L^{1-\alpha}$

$$MPL = (1-\alpha)\bar{A} K^{\alpha} L^{-\alpha} = (1-\alpha)\bar{A}\left(\frac{K}{L}\right)^{\alpha}$$

$$MPK = \alpha \bar{A} K^{\alpha-1} L^{1-\alpha} = \alpha \bar{A}\left(\frac{L}{K}\right)^{1-\alpha}$$

Production Function

Returns to Scale (RS): change in output when *all* factors are changed by the same proportion.

$$F(\lambda K, \lambda L) = \lambda^{s} Y \qquad \lambda > 0$$

$s = 1$: constant returns to scale

$s > 1$: increasing returns to scale $\qquad \lambda = 1.1 \quad , \quad \lambda^{2} = 1.1^{2} > 1.1$

$s < 1$: decreasing returns to scale $\qquad \lambda = 1.1 \quad , \quad \lambda^{1/2} = 1.1^{1/2} < 1.1$

Ex: $F(K, L) = \bar{A} K^{\alpha} L^{1-\alpha}$, what is s ?

$$\bar{A} (\lambda K)^{\alpha} (\lambda L)^{1-\alpha} = \lambda^{\overbrace{\alpha + 1 - \alpha}^{s}} \underbrace{\bar{A} K^{\alpha} L^{1-\alpha}}_{Y} = \lambda^{1} Y$$

Production Function

Assumption: Constant Returns to Scale (CRS).
When *all* the production factors are scaled by λ, output is also scaled by λ.

Intuition: absent any restrictions on availability of factors, one can always exactly double the current production by building an exact replica of the existing production arrangement. This is known as the "standard replication argument."

Production Model

- production function
- profit maximization
- factors demand
- aggregation
- factors supply
- equilibrium

Profits Maximization

Assumption: Firms choose K and L such that profits are maximized.

$$\Pi = P \times F(K, L) - r \times K - w \times L$$

w: wage rate

r: rental rate of capital

P: price of a unit of output $= 1$

(numéraire)

First order conditions

$$K: \frac{\partial \Pi}{\partial K} = 0 \Rightarrow \frac{\partial F(K,L)}{\partial K} - r = 0$$

$$L: \frac{\partial \Pi}{\partial L} = 0 \Rightarrow \frac{\partial F(K,L)}{\partial L} - w = 0$$

so $MPL = w$, $MPK = r$

Intuition:

Suppose $MPK > r$

$MPK < r$

Production Model

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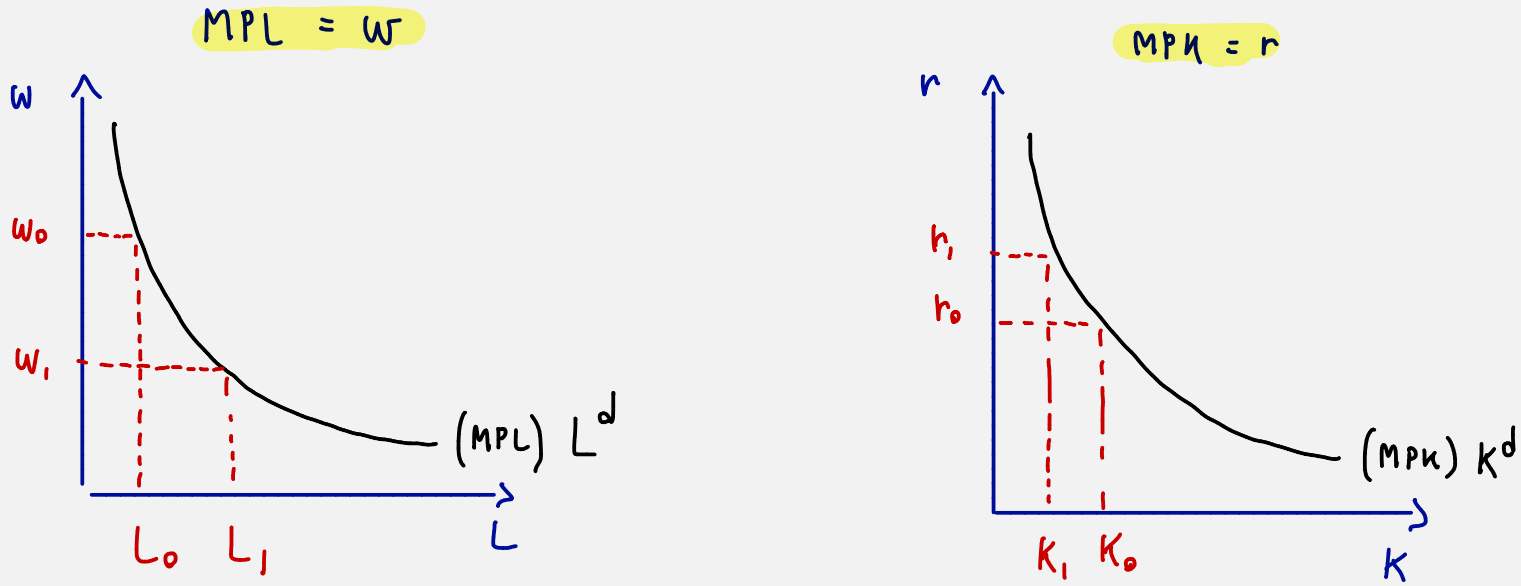
Profits Maximization and Factors Demand

First order conditions for K and L from the firm's profit maximization problem constitute a system of factor demands.



Production Model

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Aggregation

Assumption: the total value added in the economy can be represented by the production function of a single "representative" firm where K is aggregate capital used in the economy, and L is total labor employed.

$$Y_A = F(K_A, L_A) \qquad Y_B = F(K_B, L_B)$$

$$Y = P_A Y_A + P_B Y_B$$

Aggregation: $Y = F(K, L)$

where $L = L_A + L_B$, $K = P_A^K K_A + P_B^K K_B$

Production Model

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Factors Supply

Assumption: the total supply of capital and the total supply of labor are assumed to be inelastic with respect to their prices.