Long Run Macroeconomics

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Lecture 6

(note: this lecture will be recorded)

Plan for Lecture 6

- A Production Model
  - Equilibrium
  - Implications

- Development Accounting: the role of TFP

Econ 110A - Housekeeping

- Office Hours today: 5:30 pm to 6:30 pm

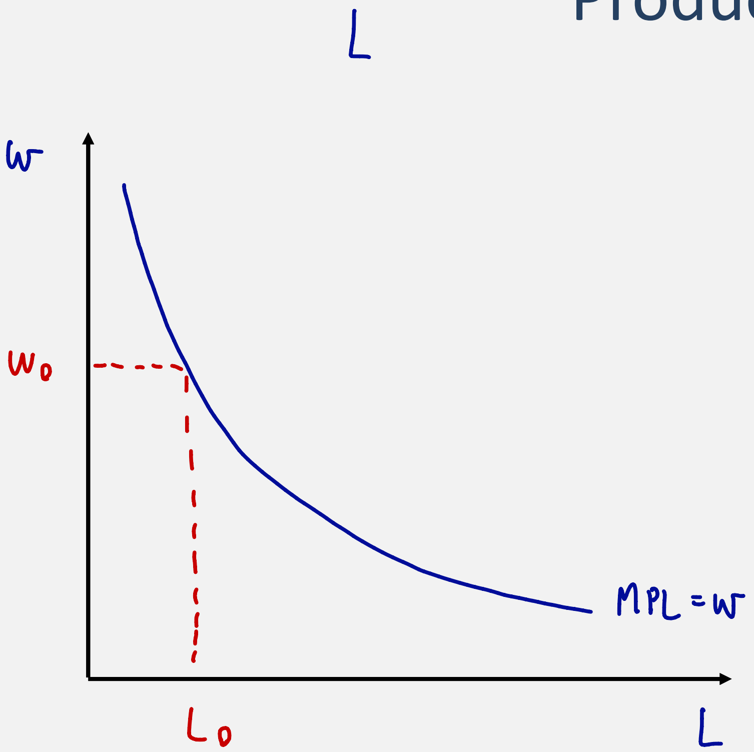
Production Model: recap

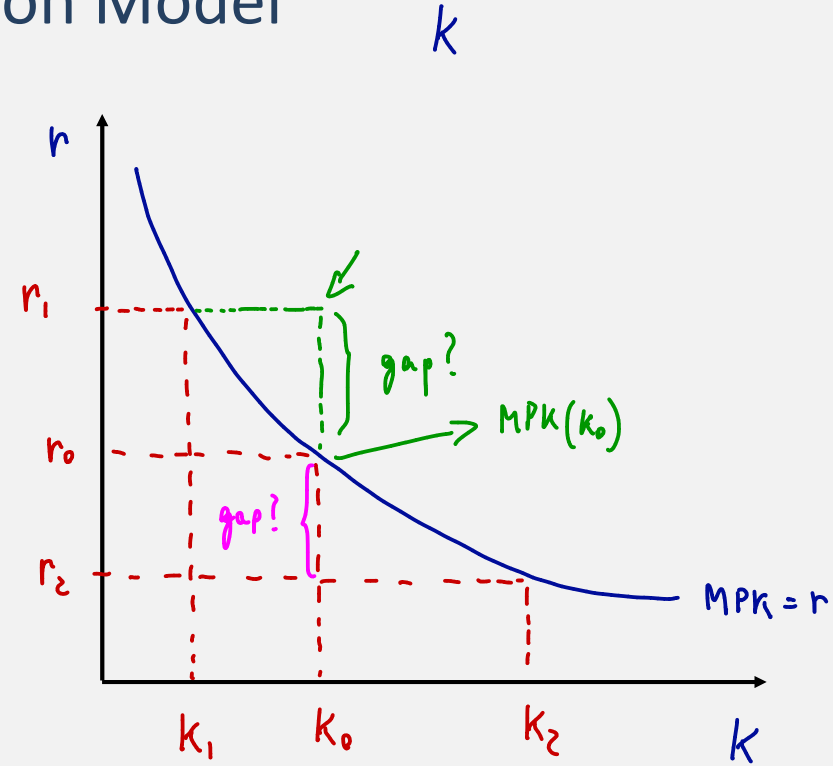
- production function
- profit maximization
- factors demand
- factors supply
- equilibrium

Aggregation

Assumption: the total value added in the economy can be represented by the production function of a single "representative" firm where K is aggregate capital used in the economy, and L is total labor employed.

Production Model





Production Model

- equilibrium

Assumption: equilibrium prices are such that the total demand for capital is equal to the supply of capital, and the total demand for labor is equal to the supply of labor.

Equilibrium: Equations

$$L^d = L^s \qquad K^d = K^s$$

unknowns : Y, K, L, w, r

solution : Y^*, K^*, L^*, w^*, r^*

$$Y = \bar{A} K^{\alpha} L^{1-\alpha}$$

$$r = \alpha \bar{A} \left(\frac{L}{K}\right)^{1-\alpha} \quad (= MPK)$$

$$w = (1-\alpha) \bar{A} \left(\frac{K}{L}\right)^{\alpha} \quad (= MPL)$$

$$L = \bar{L} \qquad K = \bar{K}$$

$$L^* = \bar{L}, \quad K^* = \bar{K}$$

$$r^* = \alpha \bar{A} \left(\frac{\bar{L}}{\bar{K}}\right)^{1-\alpha}$$

$$w^* = (1-\alpha) \bar{A} \left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}$$

$$Y^* = \bar{A} \bar{K}^{\alpha} \bar{L}^{1-\alpha}$$

Equilibrium: Numerical Example

$Y = K^{1/3} L^{2/3}$, $\bar{K} = 20$, $\bar{L} = 160$

$K^* = 20$, $L^* = 160$, $Y^* = 20^{1/3}\, 160^{2/3} = 80$

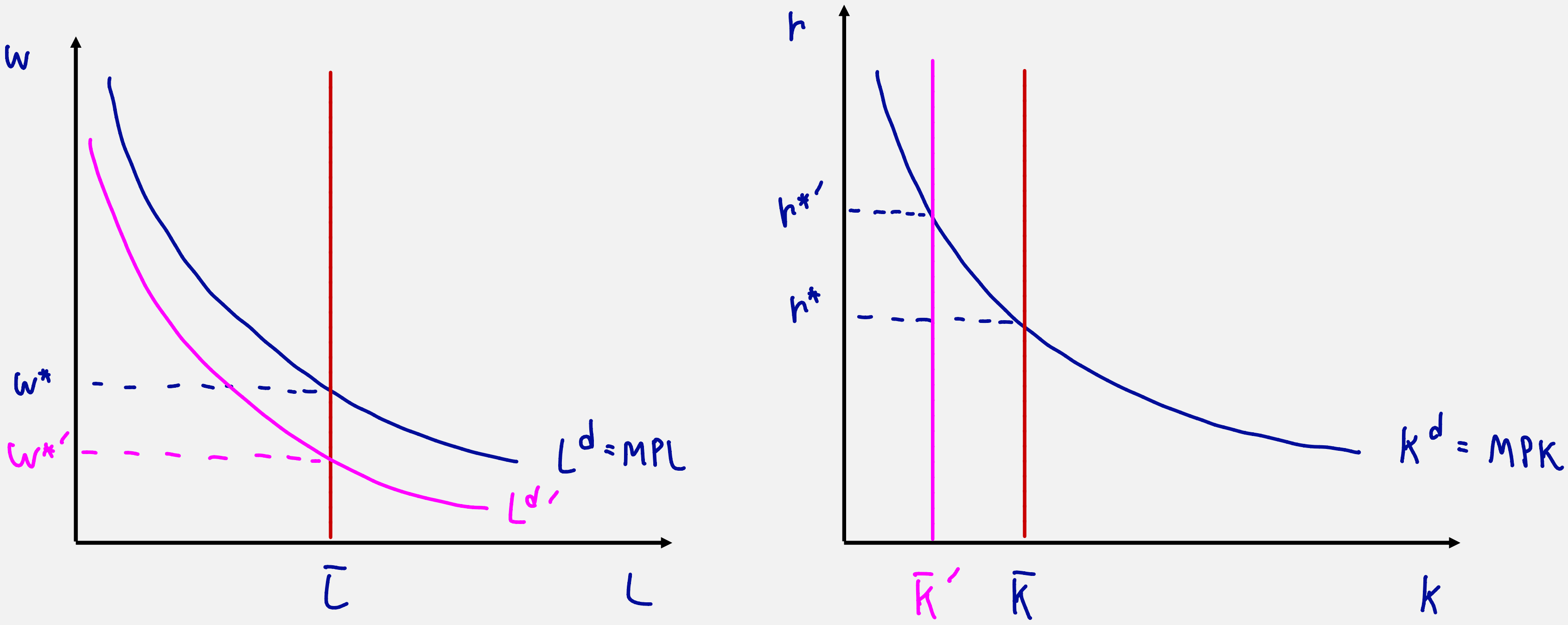
$r^* = \frac{1}{3}\left(\frac{160}{20}\right)^{2/3} = \frac{4}{3}$, $w^* = \frac{2}{3}\left(\frac{20}{160}\right)^{1/3} = 1/3$

Suppose $\bar{K}' = 10$, what happens K^*, L^*, Y^*, r^*, w^* ?

$K^{*\prime} = 10$, $L^{*\prime} = 160$, $Y^{*\prime} = 10^{1/3}\, 160^{2/3} = 63.5$

$r^{*\prime} = \frac{1}{3}\left(\frac{160}{10}\right)^{2/3} = 2.11$, $w^{*\prime} = \frac{2}{3}\left(\frac{10}{160}\right)^{1/3} = 0.26$

Equilibrium: Graphical Representation of Comparative Static



Production Model: Implications

Production Model: 4 Implications

1. All available factors are utilized in equilibrium, so production depends on endowments of factors

$$Y = \bar{A}\,\bar{K}^{\alpha}\,\bar{L}^{1-\alpha}$$

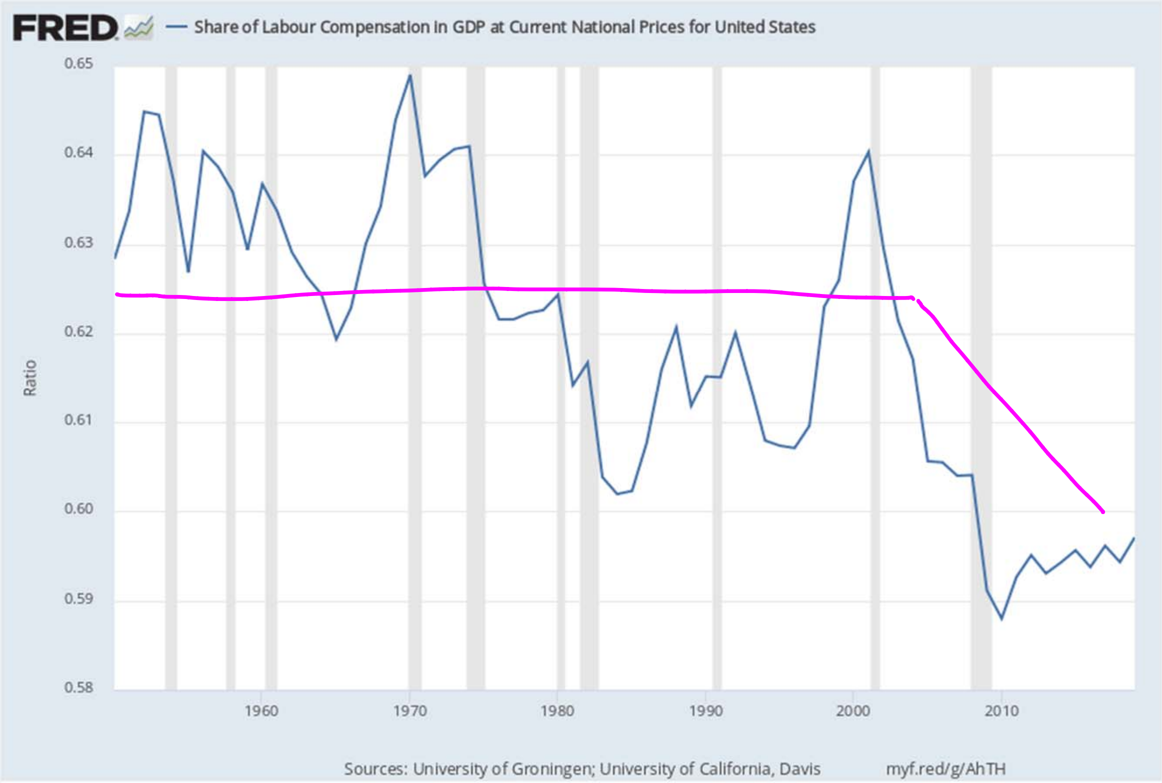
2. Total payments to factors as share of output (factor shares) are determined by the production function

Labor share: $\dfrac{w^* \times L^*}{Y^*} = \dfrac{(1-\alpha)\,\cancel{\bar{A}}\left(\dfrac{\cancel{\bar{K}}}{\cancel{\bar{L}}}\right)^{\alpha} \times \cancel{\bar{L}}}{\cancel{\bar{A}}\,\cancel{\bar{K}^{\alpha}}\,\cancel{\bar{L}^{1-\alpha}}} = 1-\alpha$

Capital share: $\dfrac{r^* \times K^*}{Y^*} = \alpha$

Production Model: 4 Implications

2. (Corollary) Under Cobb-Douglas, factor shares are constant.



$1-\alpha \approx 2/3$

so $\alpha \approx 1/3$

Production Model: 4 Implications

3. Production (value added) is equal to Income

$$Y^* = \bar{K}^{\alpha} \bar{L}^{1-\alpha} = r^* \times K^* + w^* \times L^*$$

4. The profits of the representative firm are zero

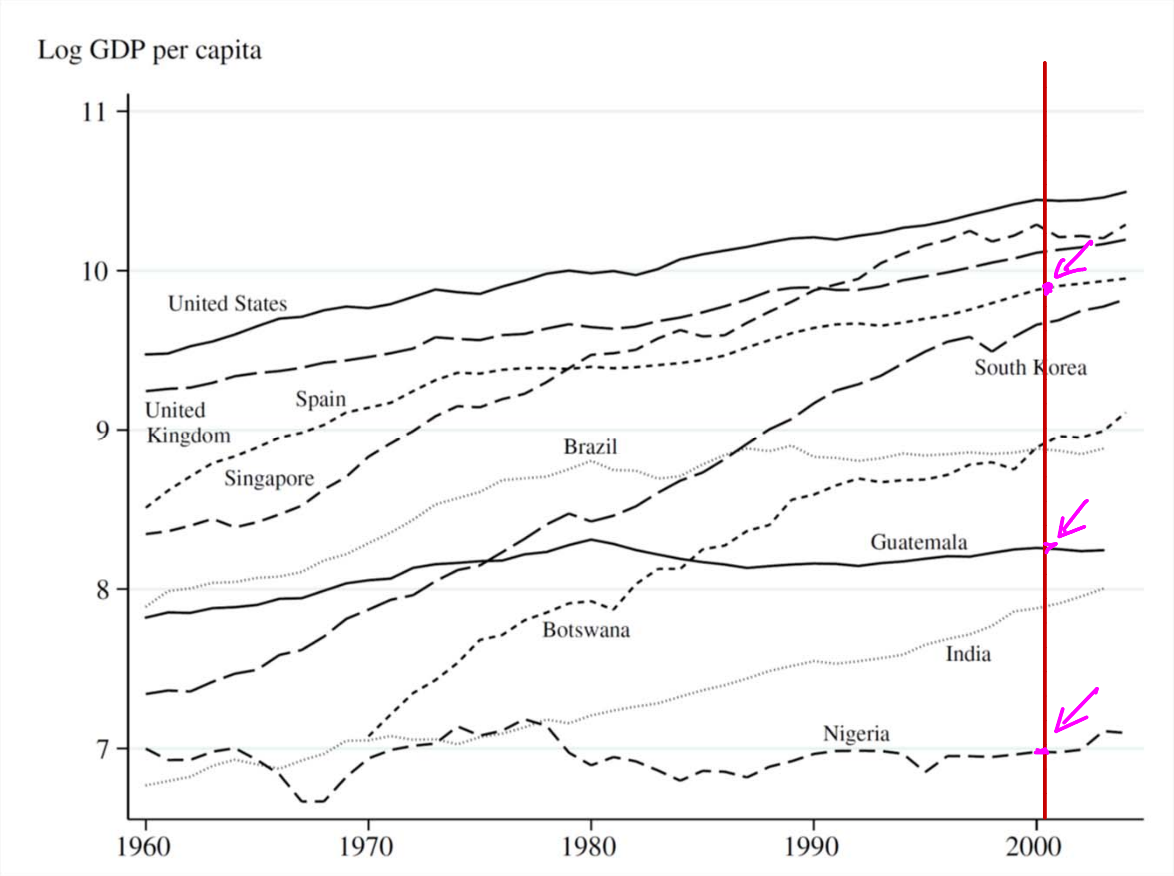
$$\Pi^* = Y^* - r^* \times K^* - w^* \times L^* = 0$$

Production Model: Summary of Assumptions

1. Firms maximize profits
2. Factor markets are competitive: prices are taken as given by firms
3. There exists a representative firm (aggregation)
4. Factors supply are inelastic to changes in prices
5. Prices are determined in equilibrium when supply equals demand

How can we use the model?

Why do countries have persistent differences in GDP per capita?



Reproduced from Ch. 1 of Daron Acemoglu, "Introduction to Modern Economic Growth," 2014

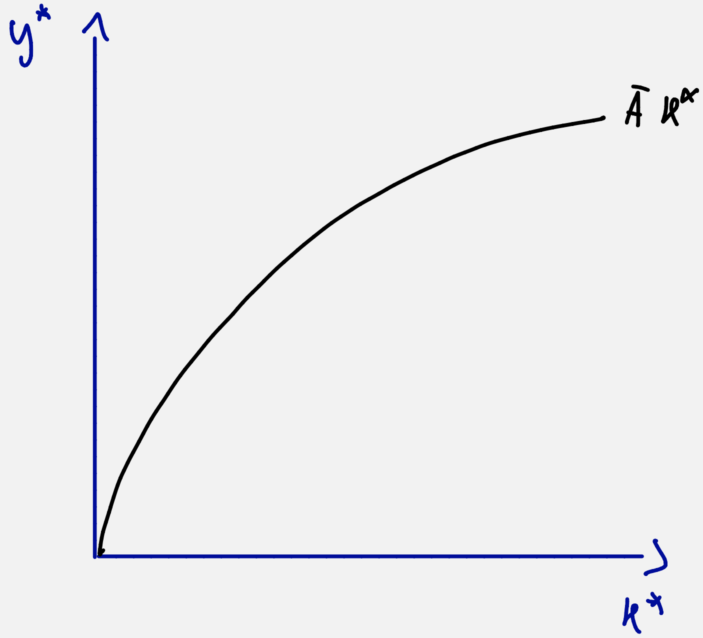
GDP per Capita

$$y \equiv \frac{Y}{L} \qquad k = \frac{K}{L}$$

$$y^* = \frac{Y^*}{L^*} = \frac{\bar{A}\,\bar{K}^{\alpha}\,\bar{L}^{1-\alpha}}{\bar{L}} = \bar{A}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}$$

$$k^* = \frac{K^*}{L^*} = \frac{\bar{K}}{\bar{L}}$$

so $$y^* = \bar{A}\,(k^*)^{\alpha}$$

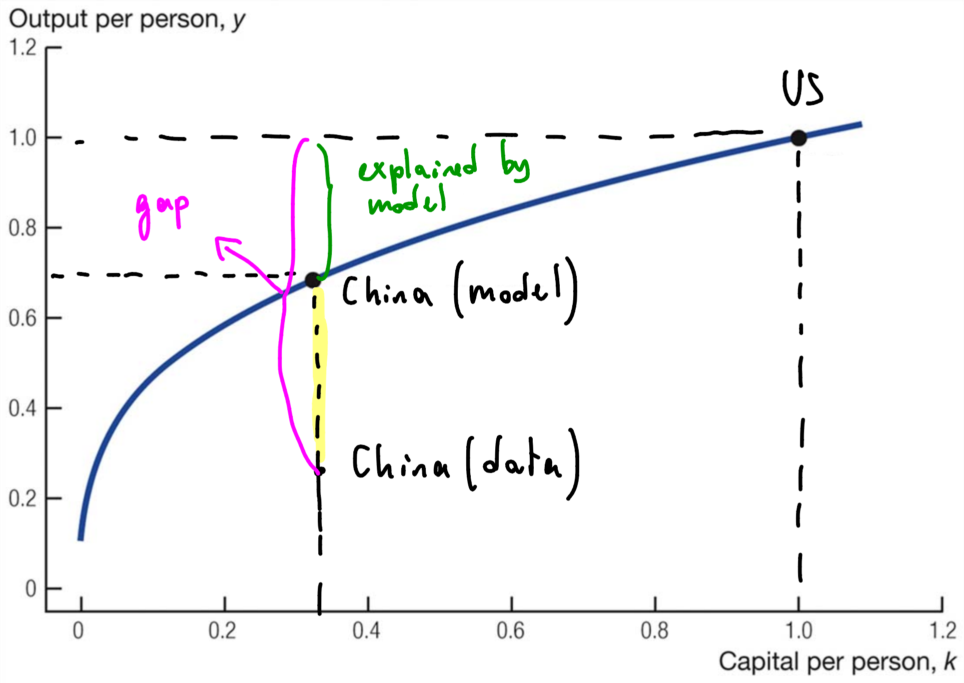


Development Accounting

Experiment 1:
Assuming all countries have the same Total Factor of Productivity (TFP), $\bar{A}$, can differences in capital-per-worker, k, explain differences in GDP per-capita, y?

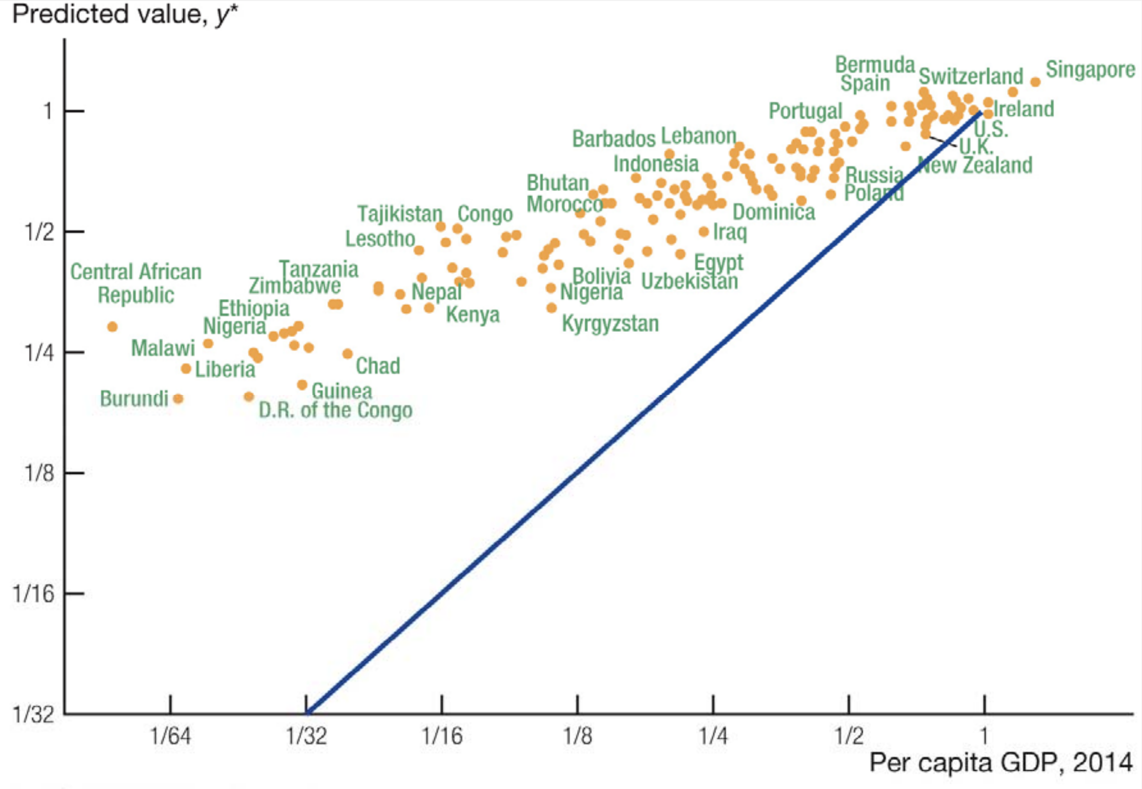
Experiment 1: set $\bar{A} = 1$, measure $\bar{k}$ in data, compute $y = \bar{k}^{\frac{1}{3}}$, compare with GDP data.

Country	$\bar{k}$ in data	$y = \bar{k}^{1/3}$	y in data
USA	1.000	1.000	1.000
China	0.323	0.686	0.241



Experiment 1: set $\bar{A} = 1$, measure $\bar{k}$ in data, compute $y = \bar{k}^{\frac{1}{3}}$, compare with GDP data.

Country	Observed capital per person, $\bar{k}$	Predicted per capita GDP $y = \bar{k}^{1/3}$	Observed per capita GDP
United States	1.000	1.000	1.000
Switzerland	1.416	1.123	1.147
Japan	1.021	1.007	0.685
Italy	1.124	1.040	0.671
Spain	1.128	1.041	0.615
United Kingdom	0.832	0.941	0.733
Brazil	0.458	0.771	0.336
China	0.323	0.686	0.241
South Africa	0.218	0.602	0.232
India	0.084	0.437	0.105
Burundi	0.007	0.192	0.016



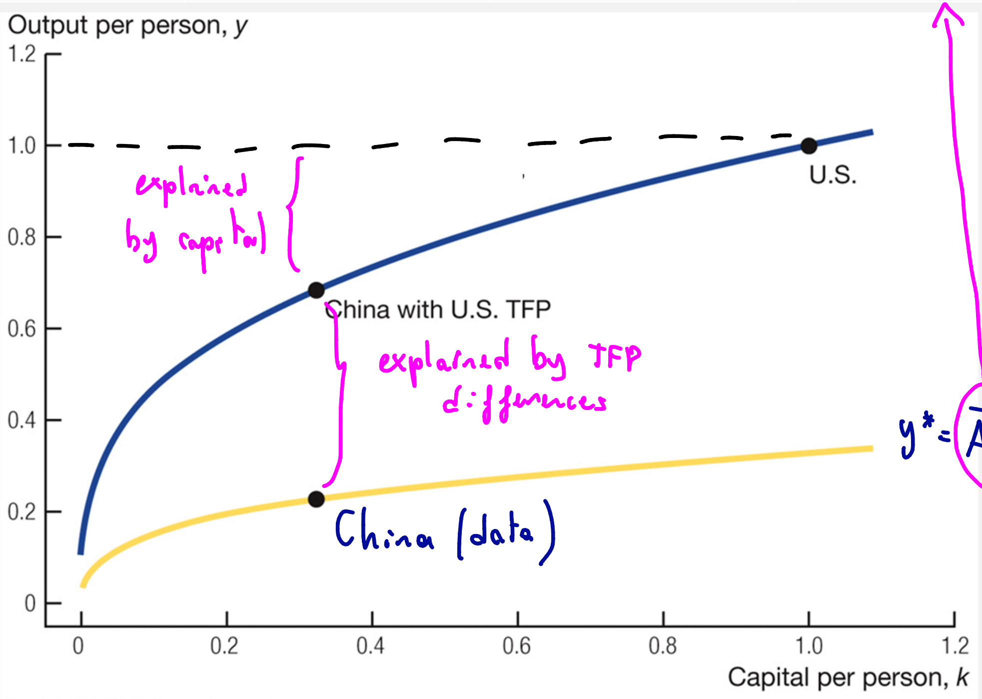
Experiment 1: Assuming all countries have the same Total Factor of Productivity (TFP), $\bar{A}$, can differences in capital-per-worker, k, explain differences in GDP per-capita, y?

No, the Production Model predicts smaller differences in GDP per capita. Why?

Experiment 2: assuming the Production Model fits perfectly for each country, what are the implied differences in Total Factor of Productivity across countries?

Experiment 2: measure y with GDP data, compute $\bar{A} = y/\bar{k}^{\frac{1}{3}}$.

Country	y in data	$\bar{k}^{1/3}$	$\bar{A} = y/\bar{k}^{1/3}$
USA	1.000	1.000	1.000
China	0.241	0.686	0.351



$y^* = \bar{A}_{CH} k^{*1/3}$

Experiment 2: measure y with actual GDP data, compute $\bar{A} = y/\bar{k}^{\frac{1}{3}}$.

Country	Per capita GDP (y)	$\bar{k}^{1/3}$	Implied TFP ($\bar{A}$)
United States	1.000	1.000	1.000
Switzerland	1.147	1.123	1.022
United Kingdom	0.733	0.941	0.779
Japan	0.685	1.007	0.680
Italy	0.671	1.040	0.646
Spain	0.615	1.041	0.590
Brazil	0.336	0.771	0.436
South Africa	0.232	0.602	0.386
China	0.241	0.686	0.351
India	0.105	0.437	0.240
Burundi	0.016	0.192	0.085

Insight from the Model: TFP vs Capital-per-Worker?

$$\underbrace{\frac{y^*_{\text{rich}}}{y^*_{\text{poor}}}}_{70} = \underbrace{\frac{\bar{A}_{\text{rich}}}{\bar{A}_{\text{poor}}}}_{14} \cdot \underbrace{\left(\frac{\bar{k}_{\text{rich}}}{\bar{k}_{\text{poor}}}\right)^{1/3}}_{5}$$

$$70 = 14 \times 5$$

- Per Capita GDP of 5 richest countries is 70 times that of 5 poorest
- Capital per person explains a factor of about 5 of this difference
- The rest, a factor of 14, is “explained” by differences in TFP