Long Run Macroeconomics

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Lecture 7
(note: this lecture will be recorded)

Looking ahead to the Midterm

- April 25, 27: The Solow Model
- May 2: How firms make investment decisions
- May 4: Review ahead of Midterm
- **Monday May 8:**

  Out-of-Class Midterm, 7 pm – 9 pm, Solis 107

Econ 110A - Housekeeping

- Problem Set 3 is posted
- No Office Hours today, make-up tomorrow noon-1pm

Plan for Lecture 7

- What did we learn from the Production Model?

- The Solow Model
  - Capital Accumulation
  - The Real Interest Rate
  - The Solow Diagram

The Production Model: what did we learn?

$y = \bar{A} k^{1/3}$

$$\underbrace{\frac{y^*_{\text{rich}}}{y^*_{\text{poor}}}}_{70} = \underbrace{\frac{\bar{A}_{\text{rich}}}{\bar{A}_{\text{poor}}}}_{14} \times \underbrace{\left(\frac{\bar{k}_{\text{rich}}}{\bar{k}_{\text{poor}}}\right)^{1/3}}_{5}$$

- Differences in per-capita GDP largely due to Total Factor of Productivity (TFP)

Why are some countries more efficient at using capital and labor than others?

Institutions

Human Capital

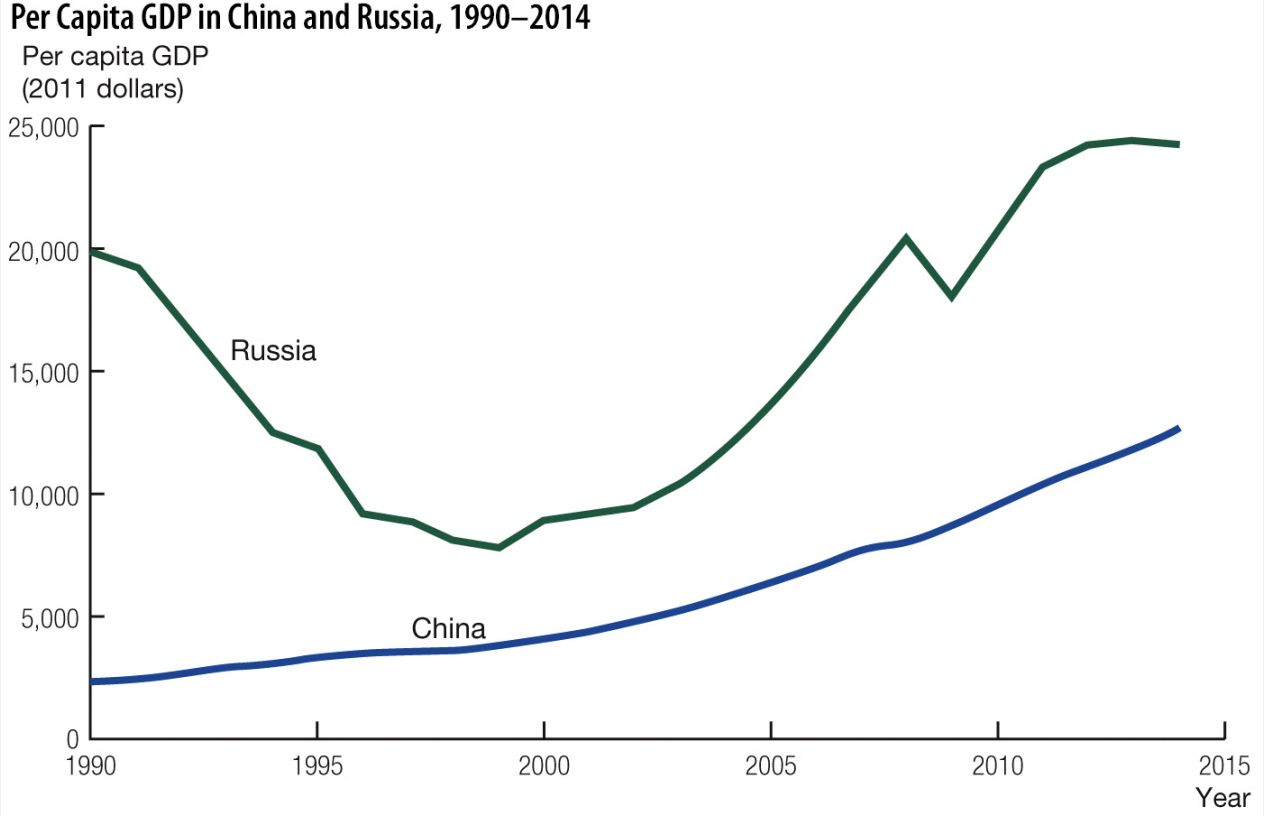
Climate

Innovation / Technology

Misallocation

Importance of Institutions/Government is Clear…

…but Role of Institutions is Complicated



Taking Stock and Next Step

- **Why some countries are so much richer than others?**
  Capital matters but only partially. TFP plays a much bigger role.

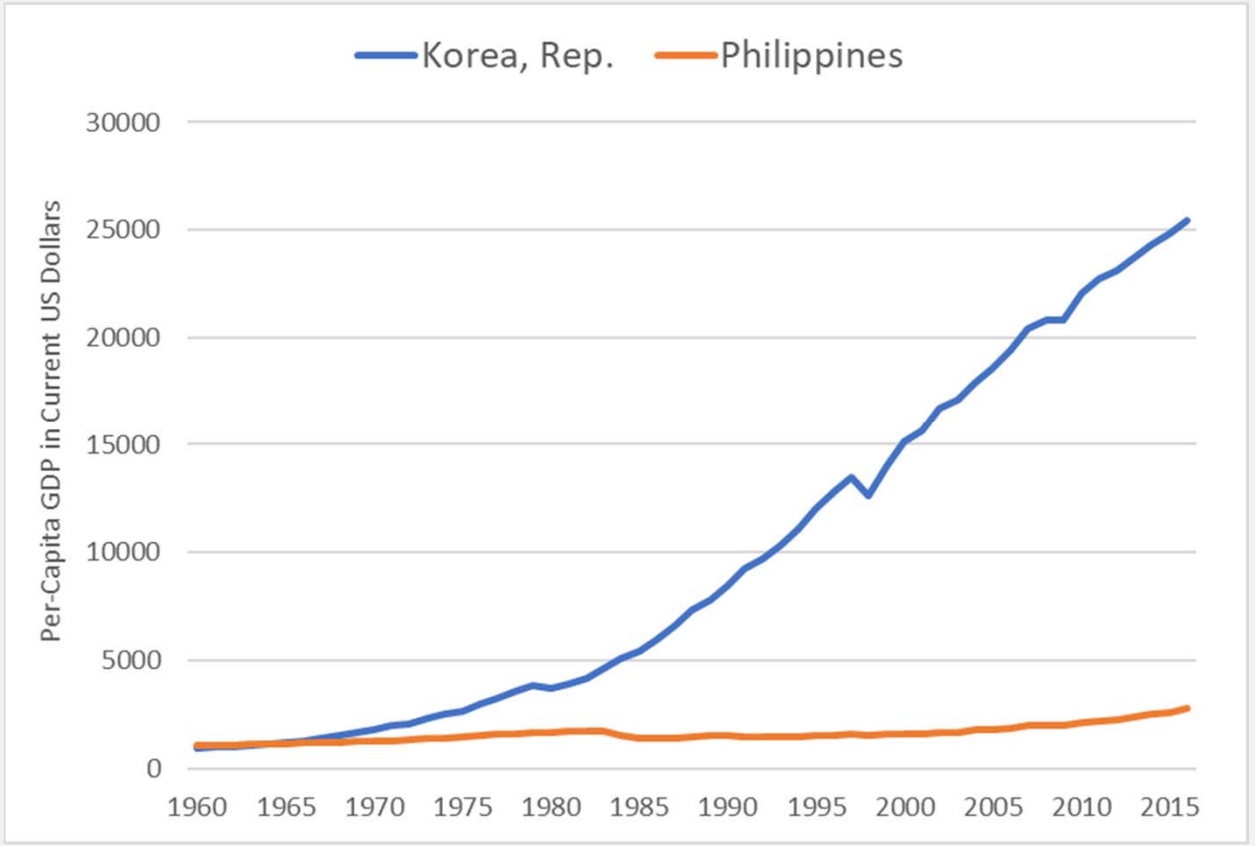
- **Why do some countries grow faster than others?**
  Can the answer to this question help understand the role of TFP?

Two Pictures from 1960

Two Pictures from 1960

	South Korea	Philippines
Per Capita GDP	$1,500	$1,500
Population	25M	25M
Working Age Population	50%	50%
Attending College at 20	5%	13%

Two Pictures from 1960



More Specific Questions :

1. Can differences in **capital accumulation** explain differences in *growth* of GDP per capita across countries?

2. Is **capital accumulation** the *ultimate* source of sustained growth in GDP per capita?

The Solow Growth Model

Introducing *dynamics* into the production model.

Robert Solow

The Solow Growth Model

1.Production

$$Y_t = \bar{A} K_t^{\alpha} L_t^{1-\alpha} \qquad t = 0, 1, 2, 3, \ldots$$

t → Time

2.Resource Constraint

$$Y_t = C_t + S_t$$

S_t: Saving

The Solow Growth Model

3.Capital Accumulation

$\bar{d}$: depreciation rate

I_t : Investment

$$K_{t+1} = K_t + I_t - \bar{d} K_t$$

define $\Delta K_{t+1} = K_{t+1} - K_t$

Then

$$\Delta K_{t+1} = I_t - \bar{d} K_t$$

for U.S. in 2019:

$\bar{d} K_t$ = \$3 Trillion

I_t = \$3.3 Trillion

K_t = \$20 Trillion

$\frac{\bar{d} K_t}{K_t} = \bar{d} = \frac{\$3T}{\$20T} \approx 15\%$

Capital Accumulation: Example

Time t	Capital K_t	Investment I_t	Depreciation $\bar{d}K_t$	Change in Capital ΔK_{t+1}
0	1000	200	100	100
1	1100	200	110	90
2	1190	200	119	81
3	1271	200		
4		200		

$$\Delta K_{t+1} = I_t - \bar{d}K_t \qquad \bar{d} = 0.10$$

The Solow Growth Model

4.Labor

$$L_t = \bar{L}$$

5.Investment

$$I_t = S_t$$

$$S_t = \bar{s} Y_t$$

$\bar{s}$: saving rate (investment rate)

The Solow Growth Model

Summary: 5 Equations in 5 Unknowns

$$Y_t = \bar{A} K_t^{\alpha} L_t^{1-\alpha}$$

$$Y_t = C_t + I_t$$

$$\Delta K_{t+1} = \bar{I}_t - \bar{d} K_t$$

$$L_t = \bar{L}$$

$$I_t = \bar{s} Y_t$$

$$t = 0, 1, 2, 3, \ldots$$

unknowns:

$$Y_t, K_{t+1}, L_t, C_t, I_t$$

$$t \in \{0, 1, 2, 3, \ldots\}$$

parameters: $\bar{d}, \bar{s}, \bar{A}$

exogeneous: $\bar{L}, K_0$

The Solow Growth Model

Quick check:

1. Where are the markets and prices of capital and labor?

$r = MPK$ $\qquad$ $W = MPL$

2. Where is the consumption equation?

$Y_t = C_t + S_t = C_t + I_t = C_t + \bar{s} Y_t \Rightarrow C_t = (1 - \bar{s}) Y_t$

3. What is a stock, what is a flow?

stock: K_t, $\qquad$ flow: Y_t, I_t, S_t, C_t

Solving The Solow Model

Solving fully the model with equations is not possible.

Our strategy:

1. Reduce equations to strictly necessary
2. Show solution on a diagram (Solow Diagram)
3. Solve for the “Long Run” of the model (Steady State)

Solve Equations

start $I_t = \bar{s} Y_t$ and sub into ΔK_{t+1}:

$$\Delta K_{t+1} = \bar{s} Y_t - \bar{d} K_t$$

note $Y_t = \bar{A} K_t^{\alpha} L_t^{1-\alpha}$ and $L_t = \bar{L}$ so

$$\Delta K_{t+1} = \bar{s} \bar{A} K_t^{\alpha} \bar{L}^{1-\alpha} - \bar{d} K_t$$

Solow Diagram: Capital Dynamics