ECON 110A notes

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Two-Period Neoclassical Consumption Model

1. The economy consists of a representative consumer who only lives for two periods: today (period 1), and the future (period 2).

2. The consumer earns income in both periods; can save (or borrow) and receives (or pays) some interest.

Y_1: income in period 1
Y_2: income in period 2
S: savings(borrowing)

C_1: consumptions in period 1
C_2: consumptions in period 2
$1 + R$: gross interest rate

$$Y_1 = C_1 + S$$

$$Y_2 + S(1 + R) = C_2$$

so

$$Y_1 = C_1 + \frac{C_2 - Y_2}{1 + R}$$

$$C_1 + \frac{C_2}{1 + R} = Y_1 + \frac{Y_2}{1 + R}$$

$\frac{1}{1+R}$: price of consumption in the future in terms of consumption today
$1 + R$: Price of consumption today in terms of consumption in the future

$$U(C_1) + \beta U(C_2), \beta \in [0, 1]$$

$\triangle_1$: ice cream now
$\beta\triangle_1$: ice cream future

3. The consumer maximizes lifetime utility subject to the intertemporal budget constraint

$$max_{C_1,C_2} U(C_1) + \beta U(C_2) such that C_1 + \frac{C_2}{1 + R} = Y_1 + \frac{Y_2}{1 + R}$$

recall that

$$S = Y_1 - C_1$$

$$solution : U'(C_1) = \beta(1 + R)U'(C_2)$$
$$U'(C) = \frac{dU(C)}{d(C)}$$

Suppose:

$$U'(C_1) > \beta(1 + R)U'(C_2)$$

$$U'(C_1) < \beta(1 + R)U'(C_2)$$

Gross Domestic Product

Market value of the final goods and services produced in an economy over a certain period of time.

1. Market value – allows to add things up
2. Final – avoids double counting
3. Goods and Services – tangibles and intangibles
4. Produced – not all sales are GDP: used cars/houses, assets…
5. In an Economy – within certain boundaries (physical, political)
6. Certain Period of Time – GDP is a flow, not a stock

Production = Income = Expenditure

Production: value added produced

Income: remuneration to factors of production

Expenditure: end-use of value added produced

GDP by Value Added (= Sales – Cost of Inputs)

GDP by Income (= Wages + Profits)

The Expenditure Approach

$$GDP = C + I + G + X - IM$$

C: Private Consumption Expenditure

I: Private Investment Expenditure

G: Government Expenditure

X: Exports (Foreign Expenditure in Domestic Goods/Services)

IM: Imports (Domestic Expenditure in Foreign Goods/Services)

GDP by Income (= Wages + Profits) = Wages and Benefits to Employees + Taxes (less subsidies) on Businesses + Profits + Depreciation of Fixed Capital

- Wages: remuneration to labor as factor of production
- Taxes: remuneration to government as factor of production
- Profits: remuneration to owners/managers as factor of production
- Depreciation of Fixed Capital: remuneration to capital as factor of production

Nominal and Real GDP

$$GDP_t = \Sigma P_y^i \times Q_t^i$$

where i = food, rent, cars; t = time

$$RGDP_t = \Sigma P_X^i \times Q_t^i$$

- Initial Price method(Laspeyres):P_X^i areearliestdateprices
- Final Price method (Paasche): P_X^i are latest date prices
- Chained-Weighted method: P_X^i are "weighted" averages across dates

The Role of Prices: Comparing GDP Across Countries

$$GDP_{t,P^{US}}^{CH} = GDP_t^{CH} \times E_t \times \frac{P_t^{US}}{P_t^{CH}}$$

E_t (\$/¥): Exchange Rate(Dollars per Yuan)

$\frac{P_t^{US}}{P_t^{CH}}$: Price Level Ratio GDP Conversion Factor (World Bank Data)

Growth Rate and Compounding

Growth rate g of variable y between t and $t+1$

$$g_{t+1} = \frac{y_{t+1} - y_t}{y_t}$$

In percentage: $100 \times g$

Annualized growth rate $\bar{g}$ of variable y between year 0 and t assuming that previous growth is "reinvested".

$$y_t = (1+\bar{g})^t y_0$$

GDP per Capita

$$y = \frac{Y}{L}, k = \frac{K}{L}$$

$$y^* = \frac{Y^*}{L^*} = \frac{\bar{A}\bar{K}^\alpha \bar{L}^{1-\alpha}}{\bar{L}} = \bar{A}(\frac{\bar{K}}{\bar{L}})^\alpha$$

$$K^* = \frac{K^*}{L^*} = \frac{\bar{K}}{\bar{L}}$$

So

$$y^* = \bar{A}(k^*)^\alpha$$

Production Model

The Production Function

$$Y = F(A, K, L)$$

Cobb-Douglas:

$$Y = AK^{\alpha}L^{1-\alpha}$$

- F = production function
- A = Ideas
- K = capital
- L = labor
- Y = output "value added"

Marginal Product: extra output produced by increasing one factor while keeping all the other factors fixed.

Diminishing Marginal Product: Extra output produced by increasing one factor while keeping all the other fixed is decreasing in the one increasing factor.

$$MPL = \frac{dF(K, L)}{dL}$$
$$MPK = \frac{dF(K, L)}{dK}$$

- Production Function: $F(K, L) = \bar{A}K^{\alpha}L^{1-\alpha}$
- $MPL = (1-\alpha)\bar{A}K^{\alpha}L^{-\alpha} = (1-\alpha)\bar{A}(\frac{K}{L})^{\alpha}$
- $MPK = \alpha\bar{A}K^{\alpha-1}L^{1-\alpha} = \alpha\bar{A}(\frac{L}{K})^{1-\alpha}$

Returns to Scale (RS): change in output when all factors are changed by the same proportion.

$$F(\lambda K, \lambda L) = \lambda^{s}Y, \lambda > 0$$

- S=1: constant return to scale
- s>1: increasing returns to scale, $\lambda = 1.1$, $\lambda^2 = 1.1^2 > 1.1$
- s<1: decreasing returns to scale, $\lambda = 1.1$, $\lambda^{\frac{1}{2}} = 1.1^{\frac{1}{2}} < 1.1$

Assumption: Constant Returns to Scale (CRS): When all the production factors are scaled by λ, output is also scaled by λ.

Profit Maximization

Assumption: Firms choose K and L such that profits are maximized.

$$\pi = P \times F(K, L) - r \times K - w \times L$$

- w: wage rate
- r: rental rate of capital
- P: price of a unit of output = 1

First order conditions

$$K : \frac{d\pi}{dK} = 0 \rightarrow \frac{dF(K, L)}{dK} - r = 0$$
$$K : \frac{d\pi}{dL} = 0 \rightarrow \frac{dF(K, L)}{dL} - w = 0$$

so $MPL = w$, $MPK = r$

Aggregation

Assumption: the total value added in the economy can be represented by the production function of a single "representative" firm where K is aggregate capital used in the economy, and L is total labor employed.

$$Y_A = F(K_A, L_A), Y_B = F(K_B, L_B)$$
$$Y = P_A Y_A + P_B Y_B$$

Aggregation:

$$Y = F(K, L)$$

where:

$$L = L_A + L_B, K = P_A^K K_A + P_B^K K_B$$

Equilibrium

$$L^d = L^s, K^d = K^s$$

unknowns: Y, K, L, w, r

$$Y = \bar{A} K^\alpha L^{1-\alpha}$$
$$r = \alpha \bar{A} (\frac{L}{K})^{1-\alpha} (= MPK)$$
$$w = (1 - \alpha) \bar{A} (\frac{K}{L})^\alpha (= MPL)$$

$$L = \bar{L}, K = \bar{K}$$

solution:Y^*, K^*, L^*, W^*, r^*

$$L^* = \bar{L}, K^* = \bar{K}$$
$$Y = \bar{A}K^{\alpha}L^{1-\alpha}$$

$$r^* = \alpha\bar{A}(\frac{\bar{L}}{\bar{K}})^{1-\alpha}$$

$$w^* = (1-\alpha)\bar{A}(\frac{\bar{K}}{\bar{L}})^{\alpha}$$

$$Y^* = \bar{A}\bar{K}^{\alpha}\bar{L}^{1-\alpha}$$

Production Model: 4 Implications

1. All available factors are utilized in equilibrium, so production depends on endowments of factors

$$Y = \bar{A}\bar{K}^{\alpha}\bar{L}^{1-\alpha}$$

2. Total payments to factors as share of output (factor shares) are determined by the production function

Labor share:

$$\frac{w^* \times L^*}{Y^*} = \frac{(1-\alpha)\bar{A}(\frac{\bar{K}}{\bar{L}})^{\alpha} \times \bar{L}}{\bar{A}\bar{K}^{\alpha}\bar{L}^{1-\alpha}} = 1 - \alpha$$

Capital share:

$$\frac{w^* \times L^*}{Y^*} = \frac{\alpha\bar{A}(\frac{\bar{L}}{\bar{K}})^{1-\alpha}}{\bar{A}\bar{K}^{\alpha}\bar{L}^{1-\alpha}} = \alpha$$

3. Production (value added) is equal to Income

$$Y^* = \bar{K}^{\alpha}\bar{L}^{1-\alpha} = r^* \times K^* + w^* \times L^*$$

4. The profits of the representative firm are zero

$$\pi^* = Y^* - r^* \times K^* - w^* \times L^* = 0$$

Solow Model

Production

$$Y_t = \bar{A} K_t^{\alpha} L_t^{1-\alpha}, t \in [1, \inf]$$

Resource Constraint

$$Y_t = C_t + S_t$$

Capital Accumulation

$$K_{t+1} = K_t + I_t - \bar{d} K_t$$

define

$$\triangle K_{t+1} = K_{t+1} - K_t$$

then

$$\triangle K_{t+1} = I_t - \bar{d} K_t$$

$\bar{d}$: depreciation rate

I_t: Investment

Labor

$$L_t = \bar{L}$$

Investment

$$I_t = S_t$$
$$S_t = \bar{s} Y_t$$

$\bar{s}$: saving rate (investment rate)

Solve Equations

Start

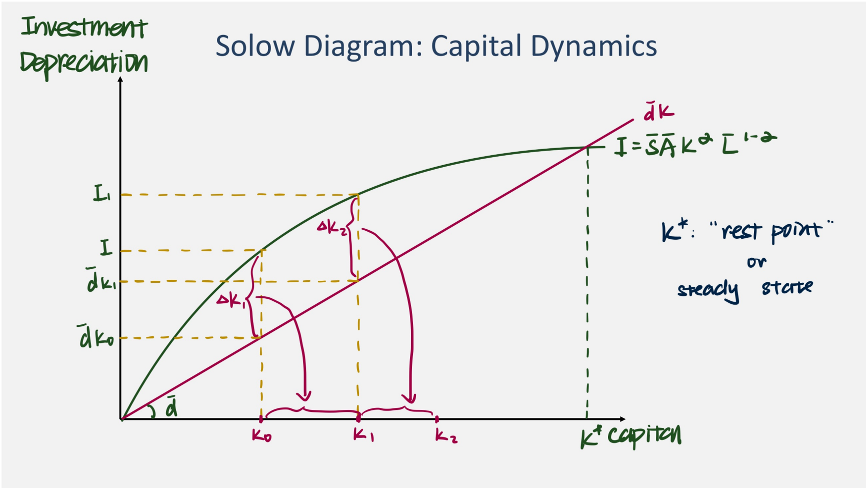
$$I^t = \bar{s} Y_t$$

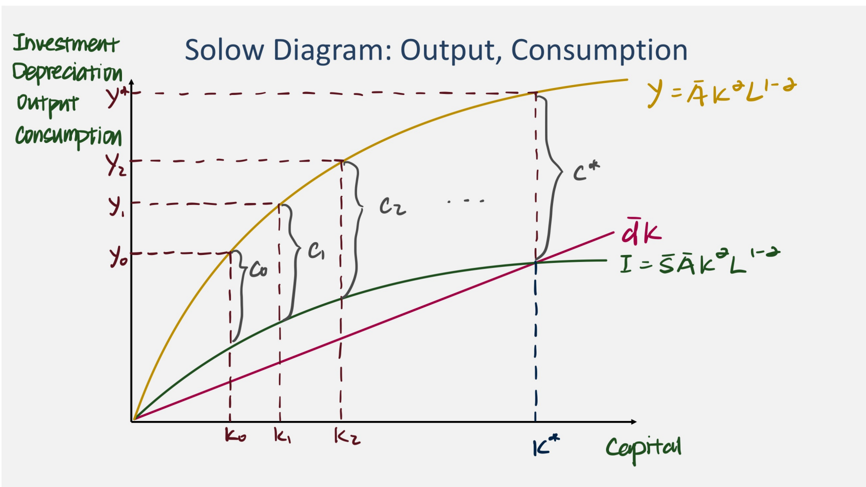
and sub into $\triangle K_{t+1}$:

$$\triangle K_{t+1} = \bar{s} Y_t - \bar{d} K_t$$

note

$$Y_t = \bar{A} K_t^{\alpha} L_t^{1-\alpha} - d K_t$$





Long Run: Steady State Capital

$$K^* : \triangle K_{t+1} = 0 \rightarrow \bar{s}\bar{A}K^{*\alpha}\bar{L}^{1-\alpha} = \bar{d}K^*$$

solve for K^* :

$$\frac{\bar{s}}{\bar{d}}\bar{A}\bar{L}^{1-\alpha} = \frac{K^*}{K^{*\alpha}} = K^{*1-\alpha}$$

$$(K^{*1-\alpha})^{\frac{1}{1-\alpha}} = (\frac{\bar{s}}{\bar{d}}\bar{A}\bar{L}^{1-\alpha})^{\frac{1}{1-\alpha}} \rightarrow K^* = (\frac{\bar{s}}{\bar{d}}\bar{A})^{\frac{1}{1-\alpha}}\bar{L}$$

capital per-worker in steady state:

$$\frac{K^*}{\bar{L}} = (\frac{\bar{s}}{\bar{d}}\bar{A})^{\frac{1}{1-\alpha}}$$

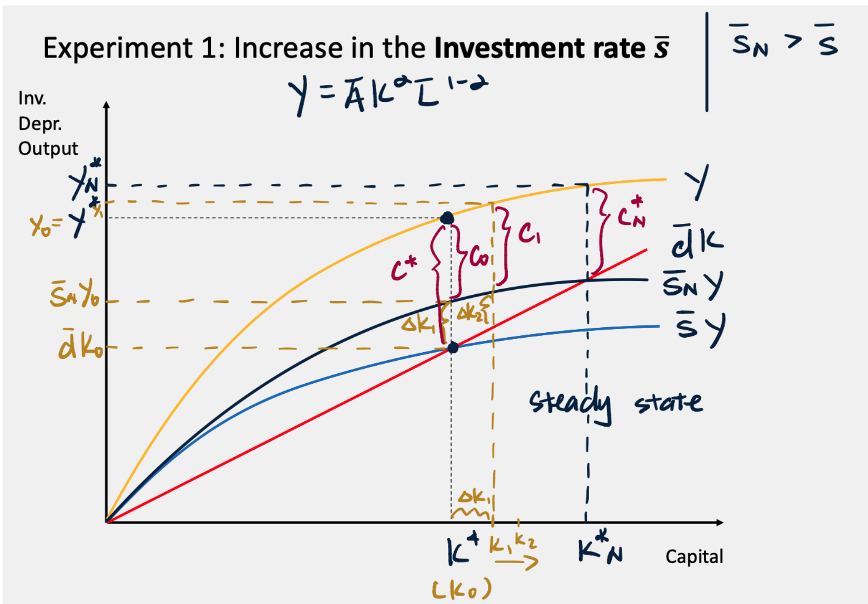
$$\begin{aligned} Y^* &= \bar{A}K^{*\alpha}\bar{L}^{1-\alpha} \\ &= \bar{A}[(\frac{\bar{s}}{\bar{d}}\bar{A})^{\frac{1}{1-\alpha}}\bar{L}]^{\alpha}\bar{L}^{1-\alpha} \\ &= \bar{A}^{1+\frac{\alpha}{1+\alpha}}(\frac{\bar{s}}{\bar{d}})^{\frac{\alpha}{1-\alpha}}\bar{L} \end{aligned}$$

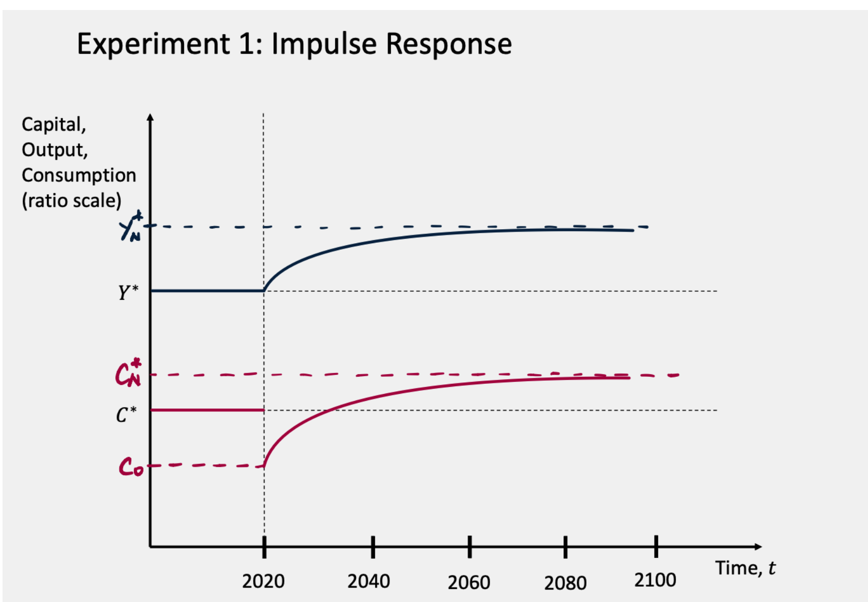
$$\bar{A}^{\frac{1}{1-\alpha}}\left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}\bar{L}$$

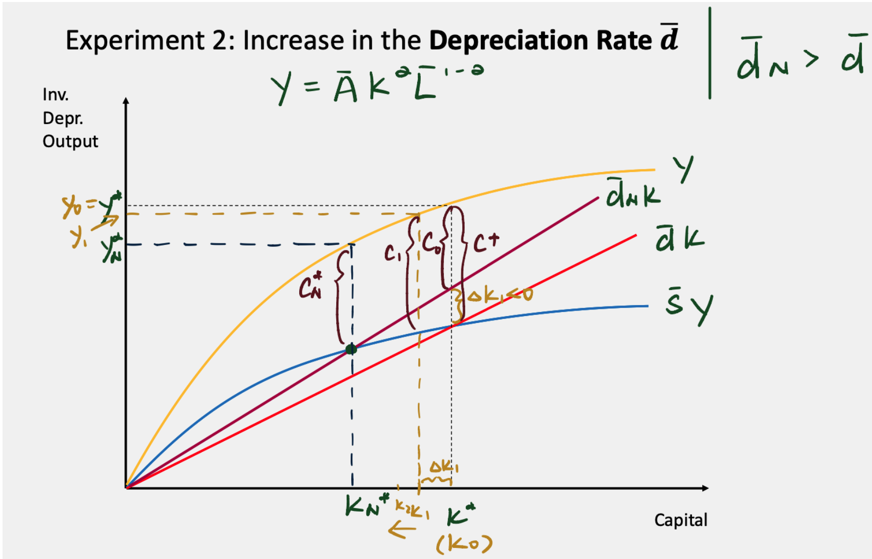
$$y^* = \frac{Y^*}{\bar{L}} = \bar{A}^{\frac{1}{1-\alpha}}\left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}$$

set $\alpha = \frac{1}{3}$

$$y^* = \bar{A}^{\frac{3}{2}}\left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{1}{2}}$$







The Real Interest Rate

Amount of output a person can earn by saving one unit of output for a year, or amount of output a person must pay to borrow one unit of output for a year.

Financial View

Save 1 unit of time t $\boxed{1} \longrightarrow \boxed{1+R}$ Receive $1+R$ units of $t+1$

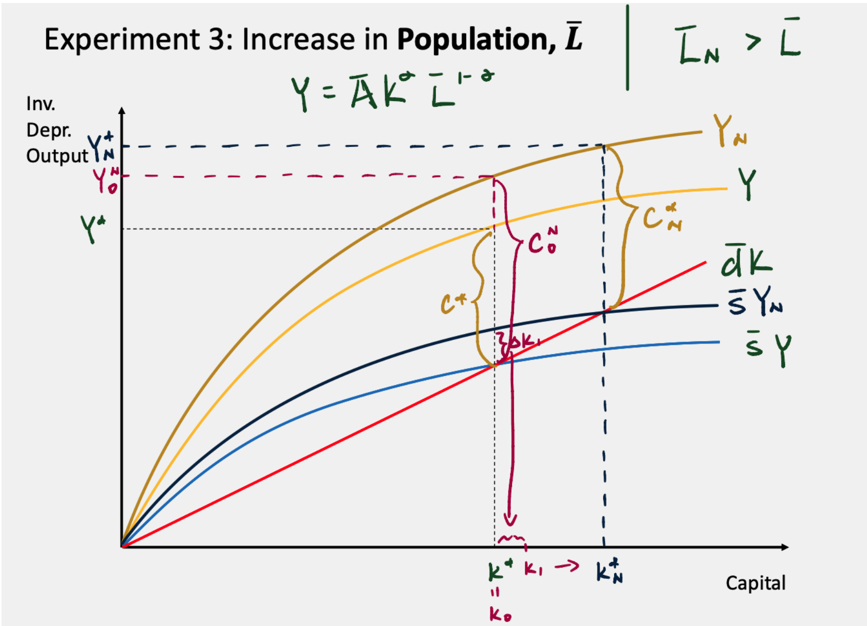
Production View

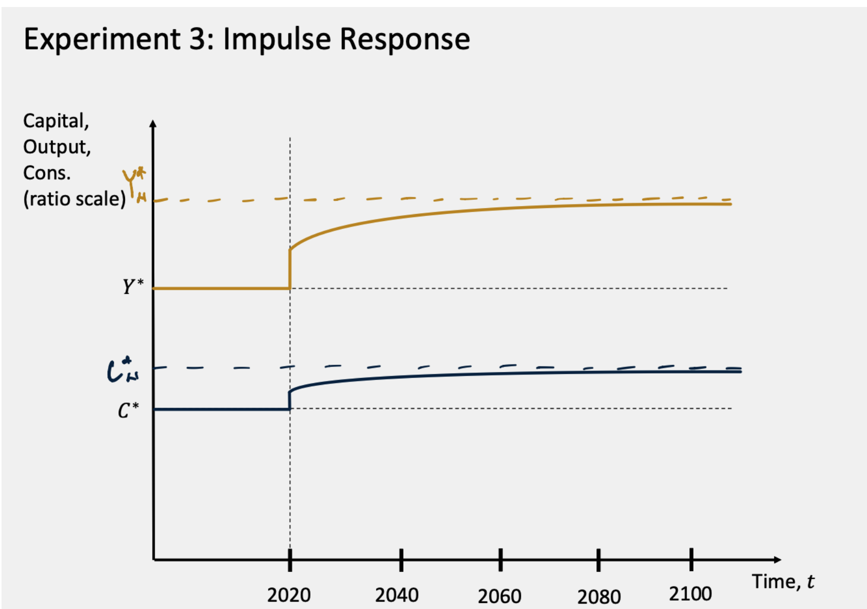
save 1 unit of time t $\rightarrow$ invest 1 unit, I_t $\rightarrow$ get 1 unit of K_{t+1} $\begin{cases} \text{rented of } r = MPK \\ \text{get back } 1-\bar{d} \text{ units} \end{cases}$

$\boxed{1} \longrightarrow \boxed{r + 1 - \bar{d}}$

by no-arbitrage : $1 + R = r + 1 - \bar{d}$

$R = r - \bar{d}$ $\quad (r = R + \bar{d})$





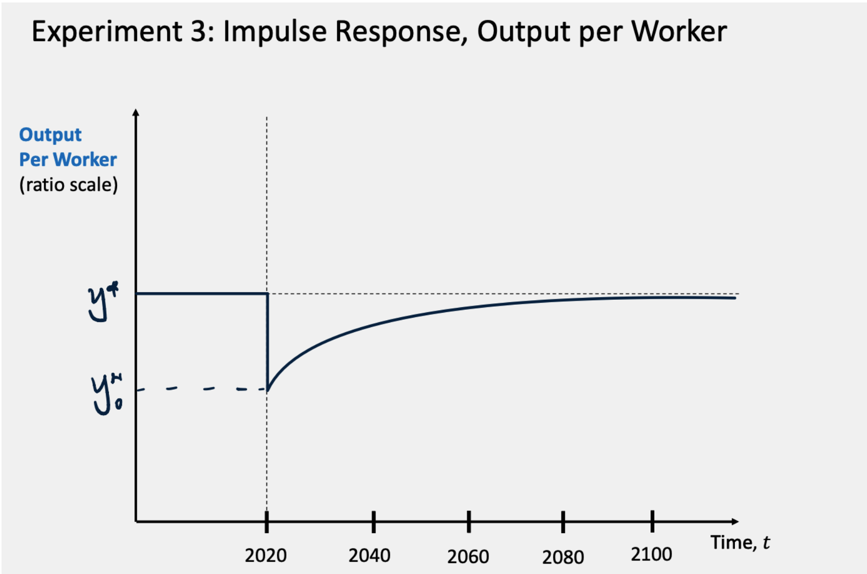
Experiment 3: Increase in Population, $\bar{L}$

What is the effect :on output per worker (GDP per capita)?

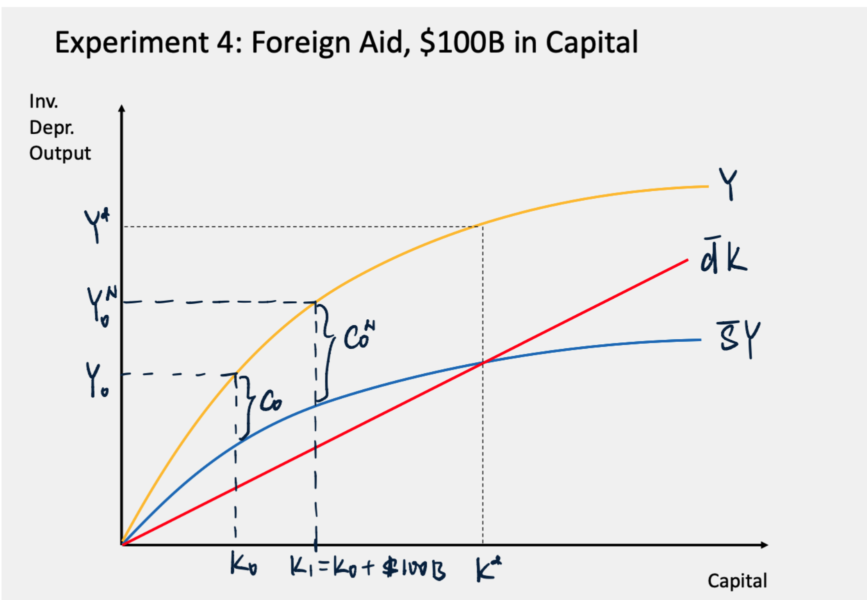
$$Y^* = \bar{A}^{\frac{1}{1-\alpha}}(\frac{\bar{s}}{\bar{d}})^{\frac{\alpha}{1-\alpha}}$$

No impact in steady state

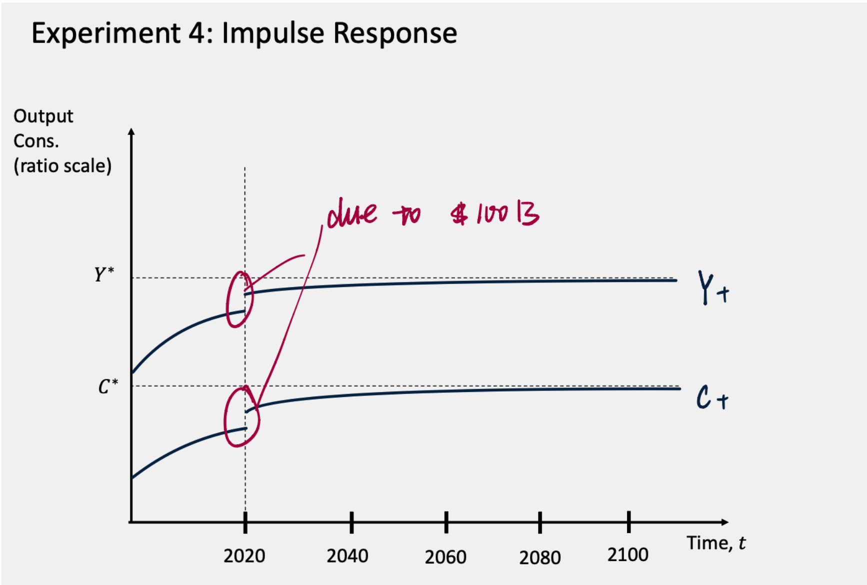
$$Y_0^N = \frac{\bar{A}K^{*\alpha}\bar{L}_N^{1-\alpha}}{\bar{L}_N} = \bar{A}(\frac{K^*}{\bar{L}_N}) < \bar{A}(\frac{K^*}{\bar{L}})^{\alpha} = Y^*$$



Experiment 3: Impulse Response, Output per Worker
Output
Per Worker
(ratio scale)
y*
y₀ᴺ
2020
2040
2060
2080
2100
Time, t

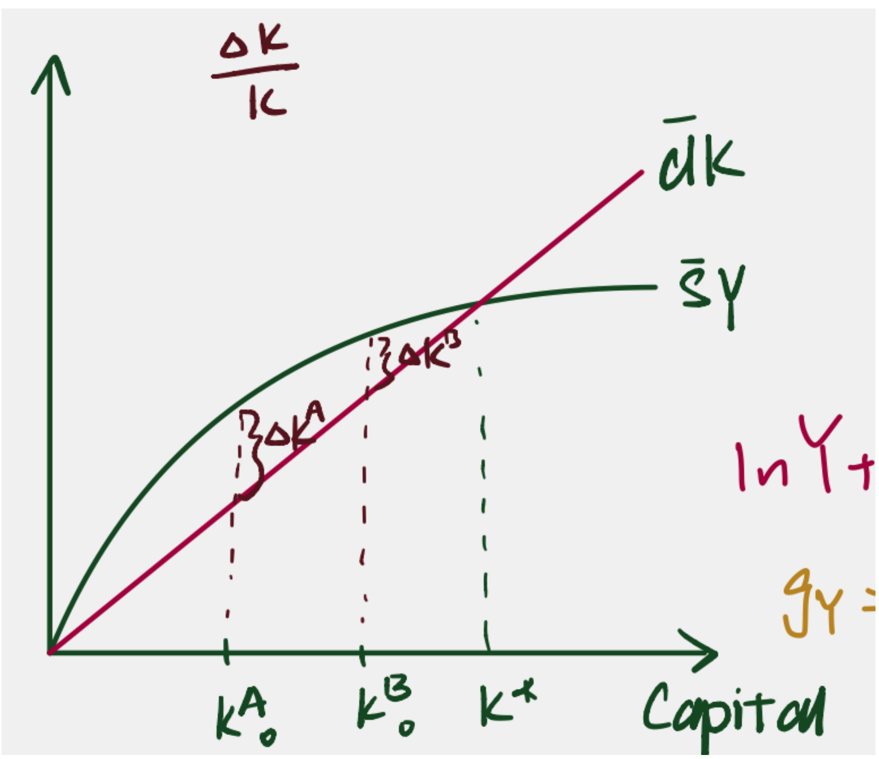


Experiment 4: Foreign Aid, $100B in Capital
Inv.
Depr.
Output
Y*
Y₀ᴺ
Y₀
Y
d̄K
s̄Y
C₀ᴺ
C₀
K₀
K₁=K₀+$100B
K*
Capital



The Principle of Transition Dynamics

The farther below the steady state capital an economy is, the faster the economy will grow.



$$\frac{\triangle K_{t+1}}{K_t} = \bar{s}\frac{Y^*}{K^*}((\frac{K^*}{K_t})^{1-\alpha} - 1)$$

$$Y = \bar{A}K^{\alpha}\bar{L}^{1-\alpha}$$

$$ln(Y_t) = ln(\bar{A}) + \alpha ln(K) + (1-\alpha)ln(\bar{L})$$

$$g_Y = \frac{\triangle Y_t}{Y_t} = \alpha \frac{\triangle K_t}{K_t} = g_K$$

No-Arbitrage Equation for Investment

Consider a firm thinking of investing $\$P_K$. The firm has two options: bank deposit or physical capital

Bank Deposit "No-Arbitrage" Physical Capital(rent out)

$$\frac{P_K(1+R)}{P_K} - \frac{P_K}{P_K} \qquad = \qquad \frac{r \times P_K + P_K^{used}}{P_K} - \frac{P_K}{P_K}$$

$$R = r + \frac{P_K^u - P_K}{P_K} \to \frac{P_K' - P_K}{P_K} - d \to \frac{\triangle P_K}{P_K} - \bar{d}$$

$$R = r + \frac{\triangle P_K}{P_K} - \bar{d}$$

The User Cost of Capital

$$MPK = R + \bar{d} - \frac{\triangle P_K}{P_K} = uc$$

- R: opportunity cost of funds
- $\bar{d}$: depreciation cost
- $\frac{\triangle P_K}{P_K}$: capital gain (if +)/ loss (if -)

Ideas are Nonrivalrous

Important: distinguish nonrivalry from scarcity and excludability

- nonrivalry: once they are created, it is feasible for ideas to be used by anybody
- scarcity: new ideas are scarce, always better to have more
- excludability: use of ideas can be restricted by property rights

Production and Cost

- Production: $Y = L^\alpha$
- Fixed Cost: F
- Total Function: $C = F + w \times L, L = Y^{\frac{1}{2}}$
- Cost Function : $C(Y) = F + w \times Y^{\frac{1}{2}}$
- Marginal Cost: $MC = \frac{dC(Y)}{dY} = \frac{1}{2} + w \times Y^{\frac{1-\alpha}{\alpha}}$
- Average Cost: $AC = \frac{C(Y)}{Y} = \frac{F}{Y} + \frac{wY^{\frac{1}{2}}}{Y} = \frac{F}{Y} + wY^{\frac{1-\alpha}{\alpha}}$
- Profits per unit: $\frac{\pi}{Y} = \frac{P \times Y - C(Y)}{Y} = P - AC$

Returns to Scale and Average Cost

Average Cost:

$$AC(Y) = \frac{C(Y)}{Y}$$
$$AC(Y) increasing in Y : decreasing RS$$
$$AC(Y) constant in Y : constant RS$$
$$AC(Y) decreasing in Y : increasing RS$$

Fixed Cost, Average Cost, and Increasing Returns to Scale

Production: $Y = L$
Fixed Cost: F

- $F = 0, C(Y) = wY, AC(Y) = w$
- $F > 0$

$$Y$$
$$= 0 when C(Y) \leq F \rightarrow AC(Y) = \frac{wL + F}{Y} = \frac{wY + F}{Y} = w + \frac{F}{Y} = \infty$$
$$= L when C(Y) > F \rightarrow AC(Y) = \frac{F}{Y} + \frac{wL}{Y} = \frac{F}{Y} + w$$

decreasing in Y, so increasing returns!