SNESL formalization

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0 Level-0

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0.1 Source language syntax

(Ignore empty sequence for now)
Expressions:

$$e ::= x \mid \textbf{let } x = e_1 \textbf{ in } e_2 \mid \varphi(x_1, ..., x_k) \mid \{e : x \textbf{ in } y \textbf{ using } \cdot\}$$

$$\varphi = \textbf{const}_n \mid \textbf{iota} \mid \textbf{plus}$$

Values:

$$n \in \mathbf{Z}$$

$$v ::= n \mid \{v_1, ..., v_k\}$$

0.2 Type system

$$\tau ::= \textbf{int} | \{\tau_1\}$$

Type environment $\Gamma = [x_1 \mapsto \tau_1, ..., x_i \mapsto \tau_i]$.

- Judgment $\boxed{\Gamma \ \vdash \ e : \tau}$

$$\frac{}{\Gamma \ \vdash \ x : \tau}\ (\Gamma(x) = \tau) \qquad \frac{\Gamma \ \vdash \ e_1 : \tau_1 \qquad \Gamma[x \mapsto \tau_1] \ \vdash \ e_2 : \tau}{\Gamma \ \vdash \ \textbf{let } x = e_1 \textbf{ in } e_2 : \tau}$$

$$\frac{\vdash \ Typ_\varphi(\tau_1, ..., \tau_k) : \tau}{\Gamma \ \vdash \ \varphi(x_1, ..., x_k) : \tau}\ ((\Gamma(x_i) = \tau_i)_{i=1}^k) \qquad \frac{\Gamma[x \mapsto \tau_1] \ \vdash \ e : \tau}{\Gamma \ \vdash \ \{e : x \textbf{ in } y \textbf{ using } \cdot\} : \{\tau\}}\ (\Gamma(y) = \{\tau_1\})$$

- Auxiliary Judgment $\boxed{\vdash \ Typ_\varphi(\tau_1, ..., \tau_k) : \tau}$

$$\frac{}{\vdash \ \textbf{const}_n() : \textbf{int}} \qquad \frac{}{\vdash \ \textbf{iota}(\textbf{int}) : \{\textbf{int}\}} \qquad \frac{}{\vdash \ \textbf{plus}(\textbf{int}, \textbf{int}) : \textbf{int}}$$

0.3 Source language semantics

- Judgment $\boxed{\rho \ \vdash \ e \downarrow v}$

$$\frac{}{\rho \ \vdash \ x \downarrow v}\ (\rho(x) = v) \qquad \frac{\rho \ \vdash \ e_1 \downarrow v_1 \qquad \rho[x \mapsto v_1] \ \vdash \ e_2 \downarrow v}{\rho \ \vdash \ \textbf{let } e_1 = x \textbf{ in } e_2 \downarrow v}$$

$$\frac{\vdash \ Eva_\varphi(v_1, ..., v_k) \downarrow v}{\rho \ \vdash \ \varphi(x_1, ..., x_k) \downarrow v}\ ((\rho(x_i) = v_i)_{i=1}^k) \qquad \frac{([x \mapsto v_i] \ \vdash \ e \downarrow v_i')_{i=1}^k}{\rho \ \vdash \ \{e : x \textbf{ in } y \textbf{ using } \cdot\} \downarrow \{v_1', ..., v_k'\}}\ (\rho(y) = \{v_1, ..., v_k\})$$

- Auxiliary Judgment $\boxed{\vdash \ Eva_{\varphi}(v_1, ..., v_k) \downarrow v}$

$$\frac{}{\vdash \ \mathbf{const}_n() \downarrow n} \qquad \frac{}{\vdash \ \mathbf{iota}(n) \downarrow \{0, 1, ..., n-1\}} \qquad \frac{}{\vdash \ \mathbf{plus}(n_1, n_2) \downarrow n_3} \ (n_3 = n_1 + n_2)$$

0.4 SVCODE syntax

Stream id:

$$s \in \mathbf{N} = \{0, 1, 2...\}$$

Stream tree:

$$st ::= s \mid (st_1, s)$$

SVCODE expressions:

$$\varphi ::= \texttt{Ctrl} \mid \texttt{Const}_\texttt{a} \mid \texttt{ToFlags} \mid \texttt{Usum} \mid \texttt{MapTwo} \mid \texttt{ScanPlus}$$

SVCODE program:

$$\begin{aligned} p ::= & \ \epsilon \\ & \mid s := \psi(s_1, ..., s_i) \\ & \mid st := \texttt{WithCtrl}(s, p) \\ & \mid p_1; p_2 \end{aligned}$$

Target language values:

$$\begin{gathered} b \in \{\texttt{T}, \texttt{F}\} \\ a ::= n \mid b \mid () \\ \vec{b} = \langle b_1, ..., b_i \rangle \\ \vec{a} = \langle a_1, ..., a_i \rangle \\ \vec{v} ::= \vec{a} \mid (\vec{v}, \vec{b}) \end{gathered}$$

Define some operations for convenience:

- $++ : \vec{v} \to \vec{v} \to \vec{v}$
  $\langle a_1, ..., a_i \rangle ++ \langle a_2, ..., a_j \rangle = \langle \vec{a}_1, ..., a_i, a_2, ..., a_j \rangle$
  $(\vec{v}_1, \vec{b}_1) ++ (\vec{v}_2, \vec{b}_2) = (\vec{v}_1 ++ \vec{v}_2, \vec{b}_1 ++ \vec{b}_2)$
- $\texttt{tail} : \vec{a} \to \vec{a}$
  $\texttt{tail}(\langle a_1, a_2, ..., a_i \rangle) = \langle a_2, ..., a_i \rangle$

0.5 SVCODE semantics

$\sigma = [s_1 \mapsto \vec{a}_1, ..., s_i \mapsto \vec{a}_i]$
$\vec{a}_c$ is the control stream.

- Judgment $\boxed{\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'}$

$$\frac{}{\langle \epsilon, \sigma \rangle \downarrow^{\vec{a}_c} \sigma} \qquad \frac{SEva_{\varphi}(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\vec{a}_c} \vec{a}}{\langle s := \varphi(s_1, ..., s_k), \sigma \rangle \downarrow^{\vec{a}_c} \sigma[s \mapsto \vec{a}]} \ ((\sigma(s_i) = \vec{a}_i)_{i=1}^k)$$

$$\frac{}{\langle st := \texttt{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{a}_c} \sigma[st \rightarrowtail \langle \rangle]} \ (\sigma(s) = \langle \rangle)$$

$$\frac{\langle p, \sigma \rangle \downarrow^{\vec{a}_s} \sigma'}{\langle st := \texttt{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{a}_c} \sigma'} \ (\sigma(s) = \vec{a}_s = \langle a_1, ..., a_i \rangle)$$

$$\frac{\langle p_1, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'' \qquad \langle p_2, \sigma'' \rangle \downarrow^{\vec{a}_c} \sigma'}{\langle p_1; p_2, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'}$$

- Auxiliary Judgment $\boxed{SEva_\varphi(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\vec{a}_c} \vec{a}}$

$$\frac{Block_\varphi(\vec{a}_{11}, ..., \vec{a}_{k1}) \Downarrow \vec{a}_1 \qquad SEva_\varphi(\vec{a}_{12}, ..., \vec{a}_{k2}) \downarrow^{\texttt{tail}(\vec{a}_c)} \vec{a}_2}{SEva_\varphi(\vec{a}_{11} +\!\!+ \vec{a}_{12}, ..., \vec{a}_{k1} +\!\!+ \vec{a}_{k2}) \downarrow^{\vec{a}_c} \vec{a}} \; (\vec{a} = \vec{a}_1 +\!\!+ \vec{a}_2)$$

$$\frac{}{SEva_\varphi(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\langle\rangle} \langle\rangle}$$

- Auxiliary Judgment $\boxed{Block_\varphi(\vec{a}_1, ..., \vec{a}_k) \Downarrow \vec{a}}$

$$\frac{}{\texttt{Const}_\texttt{a} \Downarrow \langle a\rangle} \qquad \frac{}{\texttt{ToFlags}(\langle n\rangle) \Downarrow \langle \texttt{F}_1, ..., \texttt{F}_n, \texttt{T}\rangle} \qquad \frac{}{\texttt{MapTwo}(\langle n_1\rangle, \langle n_2\rangle) \Downarrow \langle n_3\rangle} \; (n_3 = n_1 + n_2)$$

$$\frac{Unary_\varphi(\langle \texttt{F}\rangle, ..., \vec{a}_{k1}) \Downarrow \vec{a}_1 \qquad Block_\varphi(\vec{a}_{12}, ..., \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_\varphi(\langle \texttt{F}\rangle +\!\!+ \vec{a}_{12}, ..., \vec{a}_{k1} +\!\!+ \vec{a}_{k2}) \Downarrow \vec{a}} \; (\vec{a} = \vec{a}_1 +\!\!+ \vec{a}_2)$$

$$\frac{Unary_\varphi(\langle \texttt{T}\rangle, ..., \vec{a}_k) \Downarrow \vec{a}}{Block_\varphi(\langle \texttt{T}\rangle, ..., \vec{a}_k) \Downarrow \vec{a}}$$

$$\frac{Unary_{\varphi,n_0}(\langle \texttt{F}\rangle, ..., \vec{a}_{k1}) \Downarrow^{n_0'} \langle n_1\rangle \qquad Block_{\varphi,n_0'}(\vec{a}_{12}, ..., \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_{\varphi,n_0}(\langle \texttt{F}\rangle +\!\!+ \vec{a}_{12}, ..., \vec{a}_{k1} +\!\!+ \vec{a}_{k2}) \Downarrow \langle n_1\rangle +\!\!+ \vec{a}_2}$$

$$\frac{Unary_{\varphi,n_0}(\langle \texttt{T}\rangle, ..., \vec{a}_k) \Downarrow \langle n_1\rangle}{Block_{\varphi,n_0}(\langle \texttt{T}\rangle, ..., \vec{a}_k) \Downarrow \langle n_1\rangle}$$

- Auxiliary Judgment $\boxed{Unary_\varphi(\langle b\rangle, ..., \vec{a}_k) \Downarrow \vec{a}}$

$$\frac{}{\texttt{Usum}(\langle \texttt{F}\rangle) \Downarrow \langle()\rangle} \qquad \frac{}{\texttt{Usum}(\langle \texttt{T}\rangle) \Downarrow \langle\rangle}$$

$$\frac{}{\texttt{ScanPlus}_{n_0}(\langle \texttt{F}\rangle, \langle n\rangle) \Downarrow^{n_0+n} \langle n_0\rangle} \qquad \frac{}{\texttt{ScanPlus}_{n_0}(\langle \texttt{T}\rangle, \langle\rangle) \Downarrow \langle n_0\rangle}$$

0.6 Translation

$\delta = [x_1 \mapsto st_1, ..., x_i \mapsto st_i]$

- (??not necessary)

$$\frac{\delta \vdash e \xRightarrow[s_1]{s_0+1} (p, st)}{\delta \vdash e \xRightarrow[s_1]{s_0} \texttt{let } s_0 := \texttt{Ctrl}; p \texttt{ in } st}$$

- Judgment $\boxed{\delta \vdash e \xRightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{\delta \vdash x \xRightarrow[s_0]{s_0} (\epsilon, st)} \; (\delta(x) = st) \qquad \frac{\delta \vdash e_1 \xRightarrow[s_0']{s_0} (p_1, st_1) \qquad \delta[x \mapsto st_1] \vdash e_2 \xRightarrow[s_1]{s_0'} (p_2, st)}{\delta \vdash \textbf{let } x = e_1 \textbf{ in } e_2 \xRightarrow[s_1]{s_0} (p_1; p_2, st)}$$

$$\frac{\vdash Trans_\varphi(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)}{\delta \vdash \varphi(x_1, ..., x_k) \xRightarrow[s_1]{s_0} (p, st)} \; ((\delta(x_i) = st_i)_{i=1}^k)$$

$$\frac{[x \mapsto st_1] \ \vdash \ e \xRightarrow[s_1]{s_0+1} (p, st)}{\delta \ \vdash \ \{e : x \ \textbf{in} \ y \ \textbf{using} \ \cdot\} \xRightarrow[s_1]{s_0} (s_0 := \texttt{Usum}(s_2); st := \texttt{WithCtrl}(s_0, p), (st, s_2))} \ (\delta(y) = (st_1, s_2))$$

- Auxiliary Judgment $\boxed{\vdash \ Trans_\varphi(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{\textbf{const}_a() \xRightarrow[s_0+1]{s_0} (s_0 := \texttt{Const}_\texttt{a}, s_0)}$$

$$\frac{}{\textbf{iota}(s) \xRightarrow[s_4]{s_0} (p, (s_3, s_0))} \left(\begin{array}{l} s_{i+1} = s_i + 1 \\ p = s_0 := \texttt{ToFlags}(s); \\ \quad s_1 := \texttt{Usum}(s_0); \\ \quad s_2 := \texttt{WithCtrl}(s_1, s_2 := \texttt{Const}_1); \\ \quad s_3 := \texttt{ScanPlus}(s_0, s_2) \end{array} \right)$$

$$\frac{}{\textbf{plus}(s_1, s_2) \xRightarrow[s_0+1]{s_0} (s_0 := \texttt{MapTwo}(s_1, s_2), s_0)}$$

0.7 Value representation

- Judgment $\boxed{v \rhd_\tau \vec{v}}$

$$\frac{}{n \rhd_{\textbf{int}} \langle n \rangle} \qquad \frac{(v_i \rhd_\tau \vec{v}_i)_{i=1}^k}{\{v_1, ..., v_k\} \rhd_{\{\tau\}} (\vec{v}, \langle \texttt{F}_1, ..., \texttt{F}_k, \texttt{T} \rangle)} \ (\vec{v} = \vec{v}_1 +\!\!+ \vec{v}_2 +\!\!+ ... +\!\!+ \vec{v}_k)$$

0.8 Correctness proof

Lemma 1. *If*

(i) $\vdash \ Typ_\varphi(\tau_1, ..., \tau_k) : \tau$

(ii) $\vdash \ Eva_\varphi(v_1, ..., v_k) \downarrow v$

(iii) $\vdash \ Trans_\varphi(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)$

then

(i) $\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'$ *(by* $\mathcal{P}$*)*

(ii) $v \rhd_\tau \sigma(st)$ *(by* $\mathcal{V}$*)*

(iii) $\sigma' \overset{<s_0}{=\!=} \sigma$

(iv) $sids(st) < s_1$

(v) $s_0 \leq s_1$

Theorem 2. *If*

(i) $\Gamma \ \vdash \ e : \tau$ *(by some derivation* $\mathcal{T}$*)*

(ii) $\rho \ \vdash \ e \downarrow v$ *(by* $\mathcal{E}$*)*

(iii) $\delta \ \vdash \ e \xRightarrow[s_1]{s_0} (p, st)$ *(by* $\mathcal{C}$*)*

(iv) $\forall x \in dom(\Gamma).\rho(x) : \Gamma(x) \wedge sids(\delta(x)) < s_0 \wedge \rho(x) \rhd_{\Gamma(x)} \sigma(\delta(x))$

then

(i) $\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'$ *(by* $\mathcal{P}$*)*

(ii) $v \triangleright_\tau \sigma(st)$ *(by $\mathcal{V}$)*

(iii) $\sigma' \xlongequal{<s_0} \sigma$

(iv) $sids(st) \lessdot s_1$

(v) $s_0 \leq s_1$