# SNESL formalization

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## 0  Level-0

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### 0.1  Source language syntax

(Ignore empty sequence for now)
Expressions:

$$e ::= x \mid \textbf{let } x = e_1 \textbf{ in } e_2 \mid \varphi(x_1, ..., x_k) \mid \{e : x \textbf{ in } y \textbf{ using } \cdot\}$$

$$\varphi = \textbf{const}_n \mid \textbf{iota} \mid \textbf{plus}$$

Values:

$$n \in \mathbf{Z}$$

$$v ::= n \mid \{v_1, ..., v_k\}$$

### 0.2  Type system

$$\tau ::= \textbf{int} \mid \{\tau_1\}$$

Type environment $\Gamma = [x_1 \mapsto \tau_1, ..., x_i \mapsto \tau_i]$.

- Judgment $\boxed{\Gamma \ \vdash \ e : \tau}$

$$\frac{}{\Gamma \ \vdash \ x : \tau} \ (\Gamma(x) = \tau) \qquad \frac{\Gamma \ \vdash \ e_1 : \tau_1 \qquad \Gamma[x \mapsto \tau_1] \ \vdash \ e_2 : \tau}{\Gamma \ \vdash \ \textbf{let } x = e_1 \textbf{ in } e_2 : \tau}$$

$$\frac{\vdash \ Typ_\varphi(\tau_1, ..., \tau_k) : \tau}{\Gamma \ \vdash \ \varphi(x_1, ..., x_k) : \tau} \ ((\Gamma(x_i) = \tau_i)_{i=1}^k) \qquad \frac{\Gamma[x \mapsto \tau_1] \ \vdash \ e : \tau}{\Gamma \ \vdash \ \{e : x \textbf{ in } y \textbf{ using } \cdot\} : \{\tau\}} \ (\Gamma(y) = \{\tau_1\})$$

- Auxiliary Judgment $\boxed{\vdash \ Typ_\varphi(\tau_1, ..., \tau_k) : \tau}$

$$\frac{}{\vdash \ \textbf{const}_n() : \textbf{int}} \qquad \frac{}{\vdash \ \textbf{iota}(\textbf{int}) : \{\textbf{int}\}} \qquad \frac{}{\vdash \ \textbf{plus}(\textbf{int}, \textbf{int}) : \textbf{int}}$$

### 0.3  Source language semantics

- Judgment $\boxed{\rho \ \vdash \ e \downarrow v}$

$$\frac{}{\rho \ \vdash \ x \downarrow v} \ (\rho(x) = v) \qquad \frac{\rho \ \vdash \ e_1 \downarrow v_1 \qquad \rho[x \mapsto v_1] \ \vdash \ e_2 \downarrow v}{\rho \ \vdash \ \textbf{let } e_1 = x \textbf{ in } e_2 \downarrow v}$$

$$\frac{\vdash \ Eva_\varphi(v_1, ..., v_k) \downarrow v}{\rho \ \vdash \ \varphi(x_1, ..., x_k) \downarrow v} \ ((\rho(x_i) = v_i)_{i=1}^k) \qquad \frac{([x \mapsto v_i] \ \vdash \ e \downarrow v_i')_{i=1}^k}{\rho \ \vdash \ \{e : x \textbf{ in } y \textbf{ using } \cdot\} \downarrow \{v_1', ..., v_k'\}} \ (\rho(y) = \{v_1, ..., v_k\})$$

- Auxiliary Judgment $\boxed{\vdash\ Eva_\varphi(v_1,...,v_k)\downarrow v}$

$$\frac{}{\vdash\ \mathbf{const}_n()\downarrow n}\qquad\frac{}{\vdash\ \mathbf{iota}(n)\downarrow\{0,1,...,n-1\}}\qquad\frac{}{\vdash\ \mathbf{plus}(n_1,n_2)\downarrow n_3}\ (n_3=n_1+n_2)$$

## 0.4 SVCODE syntax

Stream id:
$$s\in\mathbf{N}=\{0,1,2...\}$$

Stream tree:
$$st::=s\mid(st_1,s)$$

SVCODE expressions:
$$\varphi::=\mathtt{Ctrl}\mid\mathtt{Const_a}\mid\mathtt{ToFlags}\mid\mathtt{Usum}\mid\mathtt{MapTwo}\mid\mathtt{ScanPlus}$$

SVCODE program:
$$\begin{aligned}p::=&\ \epsilon\\ \mid&\ s:=\psi(s_1,...,s_i)\\ \mid&\ st:=\mathtt{WithCtrl}(s,p)\\ \mid&\ p_1;p_2\end{aligned}$$

Target language values:
$$\begin{aligned}b&\in\{\mathtt{T},\mathtt{F}\}\\ a&::=n\mid b\mid()\\ \vec{b}&=\langle b_1,...,b_i\rangle\\ \vec{a}&=\langle a_1,...,a_i\rangle\\ \vec{v}&::=\vec{a}\mid(\vec{v},\vec{b})\end{aligned}$$

Define some operations for convenience:

- $++:\vec{v}\to\vec{v}\to\vec{v}$
$\langle a_1,...,a_i\rangle++\langle a_2,...,a_j\rangle=\langle\vec{a}_1,...,a_i,a_2,...,a_j\rangle$
$(\vec{v}_1,\vec{b}_1)++(\vec{v}_2,\vec{b}_2)=(\vec{v}_1++\vec{v}_2,\vec{b}_1++\vec{b}_2)$

- $\mathtt{tail}:\vec{a}\to\vec{a}$
$\mathtt{tail}(\langle a_1,a_2,...,a_i\rangle)=\langle a_2,...,a_i\rangle$

## 0.5 SVCODE semantics

$\sigma=[s_1\mapsto\vec{a}_1,...,s_i\mapsto\vec{a}_i]$
$\vec{a}_c$ is the control stream.

- Judgment $\boxed{\langle p,\sigma\rangle\downarrow^{\vec{a}_c}\sigma'}$

$$\frac{}{\langle\epsilon,\sigma\rangle\downarrow^{\vec{a}_c}\sigma}\qquad\frac{SEva_\varphi(\vec{a}_1,...,\vec{a}_k)\downarrow^{\vec{a}_c}\vec{a}}{\langle s:=\varphi(s_1,...,s_k),\sigma\rangle\downarrow^{\vec{a}_c}\sigma[s\mapsto\vec{a}]}\ ((\sigma(s_i)=\vec{a}_i)_{i=1}^k)$$

$$\frac{}{\langle st:=\mathtt{WithCtrl}(s,p),\sigma\rangle\downarrow^{\vec{a}_c}\sigma[st\rightarrowtail\langle\rangle]}\ (\sigma(s)=\langle\rangle)$$

$$\frac{\langle p,\sigma\rangle\downarrow^{\vec{a}_s}\sigma'}{\langle st:=\mathtt{WithCtrl}(s,p),\sigma\rangle\downarrow^{\vec{a}_c}\sigma'}\ (\sigma(s)=\vec{a}_s=\langle a_1,...,a_i\rangle)$$

$$\frac{\langle p_1,\sigma\rangle\downarrow^{\vec{a}_c}\sigma''\qquad\langle p_2,\sigma''\rangle\downarrow^{\vec{a}_c}\sigma'}{\langle p_1;p_2,\sigma\rangle\downarrow^{\vec{a}_c}\sigma'}$$

- Auxiliary Judgment $\boxed{SEva_\varphi(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\vec{a}_c} \vec{a}}$

$$\frac{Block_\varphi(\vec{a}_{11}, ..., \vec{a}_{k1}) \Downarrow \vec{a}_1 \qquad SEva_\varphi(\vec{a}_{12}, ..., \vec{a}_{k2}) \downarrow^{\mathtt{tail}(\vec{a}_c)} \vec{a}_2}{SEva_\varphi(\vec{a}_{11} ++ \vec{a}_{12}, ..., \vec{a}_{k1} ++ \vec{a}_{k2}) \downarrow^{\vec{a}_c} \vec{a}} \; (\vec{a} = \vec{a}_1 ++ \vec{a}_2)$$

$$\frac{}{SEva_\varphi(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\langle\rangle} \langle\rangle}$$

- Auxiliary Judgment $\boxed{Block_\varphi(\vec{a}_1, ..., \vec{a}_k) \Downarrow \vec{a}}$

$$\frac{}{\mathtt{Const_a} \Downarrow \langle a\rangle} \qquad \frac{}{\mathtt{ToFlags}(\langle n\rangle) \Downarrow \langle \mathtt{F}_1, ..., \mathtt{F}_n, \mathtt{T}\rangle} \qquad \frac{}{\mathtt{MapTwo}(\langle n_1\rangle, \langle n_2\rangle) \Downarrow \langle n_3\rangle} \; (n_3 = n_1 + n_2)$$

$$\frac{Unary_\varphi(\langle\mathtt{F}\rangle, ..., \vec{a}_{k1}) \Downarrow \vec{a}_1 \qquad Block_\varphi(\vec{a}_{12}, ..., \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_\varphi(\langle\mathtt{F}\rangle ++ \vec{a}_{12}, ..., \vec{a}_{k1} ++ \vec{a}_{k2}) \Downarrow \vec{a}} \; (\vec{a} = \vec{a}_1 ++ \vec{a}_2)$$

$$\frac{Unary_\varphi(\langle\mathtt{T}\rangle, ..., \vec{a}_k) \Downarrow \vec{a}}{Block_\varphi(\langle\mathtt{T}\rangle, ..., \vec{a}_k) \Downarrow \vec{a}}$$

$$\frac{Unary_{\varphi,n_0}(\langle\mathtt{F}\rangle, ..., \vec{a}_{k1}) \Downarrow^{n_0'} \langle n_1\rangle \qquad Block_{\varphi,n_0'}(\vec{a}_{12}, ..., \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_{\varphi,n_0}(\langle\mathtt{F}\rangle ++ \vec{a}_{12}, ..., \vec{a}_{k1} ++ \vec{a}_{k2}) \Downarrow \langle n_1\rangle ++ \vec{a}_2}$$

$$\frac{Unary_{\varphi,n_0}(\langle\mathtt{T}\rangle, ..., \vec{a}_k) \Downarrow \langle n_1\rangle}{Block_{\varphi,n_0}(\langle\mathtt{T}\rangle, ..., \vec{a}_k) \Downarrow \langle n_1\rangle}$$

- Auxiliary Judgment $\boxed{Unary_\varphi(\langle b\rangle, ..., \vec{a}_k) \Downarrow \vec{a}}$

$$\frac{}{\mathtt{Usum}(\langle\mathtt{F}\rangle) \Downarrow \langle()\rangle} \qquad \frac{}{\mathtt{Usum}(\langle\mathtt{T}\rangle) \Downarrow \langle\rangle}$$

$$\frac{}{\mathtt{ScanPlus}_{n_0}(\langle\mathtt{F}\rangle, \langle n\rangle) \Downarrow^{n_0+n} \langle n_0\rangle} \qquad \frac{}{\mathtt{ScanPlus}_{n_0}(\langle\mathtt{T}\rangle, \langle\rangle) \Downarrow \langle n_0\rangle}$$

## 0.6 Translation

$\delta = [x_1 \mapsto st_1, ..., x_i \mapsto st_i]$

- (??not necessary)

$$\frac{\delta \;\vdash\; e \xRightarrow[s_1]{s_0+1} (p, st)}{\delta \;\vdash\; e \xRightarrow[s_1]{s_0} \mathtt{let}\; s_0 := \mathtt{Ctrl}; p \;\mathtt{in}\; st}$$

- Judgment $\boxed{\delta \;\vdash\; e \xRightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{\delta \;\vdash\; x \xRightarrow[s_0]{s_0} (\epsilon, st)} \; (\delta(x) = st) \qquad\qquad \frac{\delta \;\vdash\; e_1 \xRightarrow[s_0']{s_0} (p_1, st_1) \qquad \delta[x \mapsto st_1] \;\vdash\; e_2 \xRightarrow[s_1]{s_0'} (p_2, st)}{\delta \;\vdash\; \mathbf{let}\; x = e_1 \;\mathbf{in}\; e_2 \xRightarrow[s_1]{s_0} (p_1; p_2, st)}$$

$$\frac{\vdash Trans_\varphi(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)}{\delta \;\vdash\; \varphi(x_1, ..., x_k) \xRightarrow[s_1]{s_0} (p, st)} \; ((\delta(x_i) = st_i)_{i=1}^k)$$

$$\frac{[x \mapsto st_1] \;\vdash\; e \xRightarrow[s_1]{s_0+1} (p, st)}{\delta \;\vdash\; \{e : x \textbf{ in } y \textbf{ using } \cdot\} \xRightarrow[s_1]{s_0} (s_0 := \texttt{Usum}(s_2); st := \texttt{WithCtrl}(s_0, p), (st, s_2))} \; (\delta(y) = (st_1, s_2))$$

- Auxiliary Judgment $\boxed{\;\vdash\; Trans_\varphi(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)\;}$

$$\frac{}{\textbf{const}_a() \xRightarrow[s_0+1]{s_0} (s_0 := \texttt{Const}_\texttt{a}, s_0)}$$

$$\frac{}{\textbf{iota}(s) \xRightarrow[s_4]{s_0} (p, (s_3, s_0))} \begin{pmatrix} s_{i+1} = s_i + 1 \\ p = s_0 := \texttt{ToFlags}(s); \\ \quad s_1 := \texttt{Usum}(s_0); \\ \quad s_2 := \texttt{WithCtrl}(s_1, s_2 := \texttt{Const}_\texttt{1}); \\ \quad s_3 := \texttt{ScanPlus}(s_0, s_2) \end{pmatrix}$$

$$\frac{}{\textbf{plus}(s_1, s_2) \xRightarrow[s_0+1]{s_0} (s_0 := \texttt{MapTwo}(s_1, s_2), s_0)}$$

## 0.7 Value representation

- Judgment $\boxed{v \rhd_\tau \vec{v}}$

$$\frac{}{n \rhd_{\textbf{int}} \langle n \rangle} \qquad\qquad \frac{(v_i \rhd_\tau \vec{v}_i)_{i=1}^k}{\{v_1, ..., v_k\} \rhd_{\{\tau\}} (\vec{v}, \langle \texttt{F}_1, ..., \texttt{F}_k, \texttt{T} \rangle)} \; (\vec{v} = \vec{v}_1 + \!\!+ \, \vec{v}_2 + \!\!+ \, ... + \!\!+ \, \vec{v}_k)$$

## 0.8 Correctness proof