# SNESL formalization

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## 0 Level-0

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### 0.1 Source language syntax

(Ignore empty sequence for now)
Expressions:

$$e ::= x \mid \textbf{let } x = e_1 \textbf{ in } e_2 \mid \phi(x_1, ..., x_k) \mid \{e : x \textbf{ in } y \textbf{ using } \cdot\}$$

$$\phi = \textbf{const}_n \mid \textbf{iota} \mid \textbf{plus}$$

Values:

$$n \in \mathbf{Z}$$

$$v ::= n \mid \{v_1, ..., v_k\}$$

### 0.2 Type system

$$\tau ::= \textbf{int} | \{\tau_1\}$$

Type environment $\Gamma = [x_1 \mapsto \tau_1, ..., x_i \mapsto \tau_i]$.

- Judgment $\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma \vdash x : \tau} (\Gamma(x) = \tau) \qquad \frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma[x \mapsto \tau_1] \vdash e_2 : \tau}{\Gamma \vdash \textbf{let } x = e_1 \textbf{ in } e_2 : \tau}$$

$$\frac{\vdash Typ_\phi(\tau_1, ..., \tau_k) : \tau}{\Gamma \vdash \phi(x_1, ..., x_k) : \tau} ((\Gamma(x_i) = \tau_i)_{i=1}^k) \qquad \frac{[x \mapsto \tau_1] \vdash e : \tau}{\Gamma \vdash \{e : x \textbf{ in } y \textbf{ using } \cdot\} : \{\tau\}} (\Gamma(y) = \{\tau_1\})$$

- Auxiliary Judgment $\boxed{\vdash Typ_\phi(\tau_1, ..., \tau_k) : \tau}$

$$\frac{}{\vdash \textbf{const}_n() : \textbf{int}} \qquad \frac{}{\vdash \textbf{iota}(\textbf{int}) : \{\textbf{int}\}} \qquad \frac{}{\vdash \textbf{plus}(\textbf{int}, \textbf{int}) : \textbf{int}}$$

### 0.3 Source language semantics

- Judgment $\boxed{\rho \vdash e \downarrow v}$

$$\frac{}{\rho \vdash x \downarrow v} (\rho(x) = v) \qquad \frac{\rho \vdash e_1 \downarrow v_1 \qquad \rho[x \mapsto v_1] \vdash e_2 \downarrow v}{\rho \vdash \textbf{let } e_1 = x \textbf{ in } e_2 \downarrow v}$$

$$\frac{\vdash Eva_\phi(v_1, ..., v_k) \downarrow v}{\rho \vdash \phi(x_1, ..., x_k) \downarrow v} ((\rho(x_i) = v_i)_{i=1}^k) \qquad \frac{([x \mapsto v_i] \vdash e \downarrow v_i')_{i=1}^k}{\rho \vdash \{e : x \textbf{ in } y \textbf{ using } \cdot\} \downarrow \{v_1', ..., v_k'\}} (\rho(y) = \{v_1, ..., v_k\})$$

- Auxiliary Judgment $\boxed{\vdash \; Eva_{\phi}(v_1, ..., v_k) \downarrow v}$

$$\frac{}{\vdash \; \mathbf{const}_n() \downarrow n} \qquad \frac{}{\vdash \; \mathbf{iota}(n) \downarrow \{0, 1, ..., n-1\}} \qquad \frac{}{\vdash \; \mathbf{plus}(n_1, n_2) \downarrow n_3} \; (n_3 = n_1 + n_2)$$

## 0.4 SVCODE syntax

Stream id:

$$s \in \mathbf{N} = \{0, 1, 2...\}$$

Stream tree:

$$st ::= s \mid (st_1, s)$$

SVCODE expressions:

$$\psi ::= \texttt{Ctrl} \mid \texttt{Const}_\texttt{a} \mid \texttt{ToFlags} \mid \texttt{Usum} \mid \texttt{MapTwo} \mid \texttt{ScanPlus}$$

SVCODE program:

$$\begin{aligned} p ::= \; & \epsilon \\ & \mid s := \psi(s_1, ..., s_i) \\ & \mid st := \texttt{WithCtrl}(s, p) \\ & \mid p_1; p_2 \end{aligned}$$

Target language values:

$$\begin{gathered} b \in \{\texttt{T}, \texttt{F}\} \\ a ::= n \mid b \mid () \\ \vec{b} = \langle b_1, ..., b_i \rangle \\ \vec{a} = \langle a_1, ..., a_i \rangle \\ \vec{v} ::= \vec{a} \mid (\vec{v}, \vec{b}) \end{gathered}$$

Define some operations for convenience:

- $++ : \vec{v} \to \vec{v} \to \vec{v}$
  $\langle a_1, ..., a_i \rangle ++ \langle a_2, ..., a_j \rangle = \langle \vec{a}_1, ..., a_i, a_2, ..., a_j \rangle$
  $(\vec{v}_1, \vec{b}_1) ++ (\vec{v}_2, \vec{b}_2) = (\vec{v}_1 ++ \vec{v}_2, \vec{b}_1 ++ \vec{b}_2)$
- $\texttt{tail} : \vec{a} \to \vec{a}$
  $\texttt{tail}(\langle a_1, a_2, ..., a_i \rangle) = \langle a_2, ..., a_i \rangle$

## 0.5 SVCODE semantics

$\sigma = [s_1 \mapsto \vec{a}_1, ..., s_i \mapsto \vec{a}_i]$
$\vec{a}_c$ is the control stream.

- Judgment $\boxed{\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'}$

$$\frac{}{\langle \epsilon, \sigma \rangle \downarrow^{\vec{a}_c} \sigma} \qquad \frac{SEva_{\psi}(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\vec{a}_c} \vec{a}}{\langle s := \psi(s_1, ..., s_k), \sigma \rangle \downarrow^{\vec{a}_c} \sigma[s \mapsto \vec{a}]} \; ((\sigma(s_i) = \vec{a}_i)_{i=1}^k)$$

$$\frac{}{\langle st := \texttt{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{a}_c} \sigma[st \rightarrowtail \langle \rangle]} \; (\sigma(s) = \langle \rangle)$$

$$\frac{\langle p, \sigma \rangle \downarrow^{\vec{a}_s} \sigma'}{\langle st := \texttt{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{a}_c} \sigma'} \; (\sigma(s) = \vec{a}_s = \langle a_1, ..., a_i \rangle)$$

$$\frac{\langle p_1, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'' \qquad \langle p_2, \sigma'' \rangle \downarrow^{\vec{a}_c} \sigma'}{\langle p_1; p_2, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'}$$

- Auxiliary Judgment $\boxed{SEva_{\psi}(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\vec{a}_c} \vec{a}}$

$$\frac{Block_{\psi}(\vec{a}_{11}, ..., \vec{a}_{k1}) \Downarrow \vec{a}_1 \qquad SEva_{\psi}(\vec{a}_{12}, ..., \vec{a}_{k2}) \downarrow^{\mathtt{tail}(\vec{a}_c)} \vec{a}_2}{SEva_{\psi}(\vec{a}_{11} +\!\!+ \vec{a}_{12}, ..., \vec{a}_{k1} +\!\!+ \vec{a}_{k2}) \downarrow^{\vec{a}_c} \vec{a}} \; (\vec{a} = \vec{a}_1 +\!\!+ \vec{a}_2)$$

$$\frac{}{SEva_{\psi}(\vec{a}_1, ..., \vec{a}_k) \downarrow^{\langle\rangle} \langle\rangle}$$

- Auxiliary Judgment $\boxed{Block_{\psi}(\vec{a}_1, ..., \vec{a}_k) \Downarrow \vec{a}}$

$$\frac{}{\mathtt{Const_a} \Downarrow \langle a \rangle} \qquad \frac{}{\mathtt{ToFlags}(\langle n \rangle) \Downarrow \langle \mathtt{F}_1, ..., \mathtt{F}_n, \mathtt{T} \rangle} \qquad \frac{}{\mathtt{MapTwo}(\langle n_1 \rangle, \langle n_2 \rangle) \Downarrow \langle n_3 \rangle} \; (n_3 = n_1 + n_2)$$

$$\frac{Unary_{\psi}(\langle \mathtt{F} \rangle, ..., \vec{a}_{k1}) \Downarrow \vec{a}_1 \qquad Block_{\psi}(\vec{a}_{12}, ..., \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_{\psi}(\langle \mathtt{F} \rangle +\!\!+ \vec{a}_{12}, ..., \vec{a}_{k1} +\!\!+ \vec{a}_{k2}) \Downarrow \vec{a}} \; (\vec{a} = \vec{a}_1 +\!\!+ \vec{a}_2)$$

$$\frac{Unary_{\psi}(\langle \mathtt{T} \rangle, ..., \vec{a}_k) \Downarrow \vec{a}}{Block_{\psi}(\langle \mathtt{T} \rangle, ..., \vec{a}_k) \Downarrow \vec{a}}$$

$$\frac{Unary_{\psi,n_0}(\langle \mathtt{F} \rangle, ..., \vec{a}_{k1}) \Downarrow^{n_0'} \langle n_1 \rangle \qquad Block_{\psi,n_0'}(\vec{a}_{12}, ..., \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_{\psi,n_0}(\langle \mathtt{F} \rangle +\!\!+ \vec{a}_{12}, ..., \vec{a}_{k1} +\!\!+ \vec{a}_{k2}) \Downarrow \langle n_1 \rangle +\!\!+ \vec{a}_2}$$

$$\frac{Unary_{\psi,n_0}(\langle \mathtt{T} \rangle, ..., \vec{a}_k) \Downarrow \langle n_1 \rangle}{Block_{\psi,n_0}(\langle \mathtt{T} \rangle, ..., \vec{a}_k) \Downarrow \langle n_1 \rangle}$$

- Auxiliary Judgment $\boxed{Unary_{\psi}(\langle b \rangle, ..., \vec{a}_k) \Downarrow \vec{a}}$

$$\frac{}{\mathtt{Usum}(\langle \mathtt{F} \rangle) \Downarrow \langle () \rangle} \qquad \frac{}{\mathtt{Usum}(\langle \mathtt{T} \rangle) \Downarrow \langle\rangle}$$

$$\frac{}{\mathtt{ScanPlus}_{n_0}(\langle \mathtt{F} \rangle, \langle n \rangle) \Downarrow^{n_0+n} \langle n_0 \rangle} \qquad \frac{}{\mathtt{ScanPlus}_{n_0}(\langle \mathtt{T} \rangle, \langle\rangle) \Downarrow \langle n_0 \rangle}$$

## 0.6 Translation

$\delta = [x_1 \mapsto st_1, ..., x_i \mapsto st_i]$

- (??not necessary)

$$\frac{\delta \vdash e \xRightarrow[s_1]{s_0+1} (p, st)}{\delta \vdash e \xRightarrow[s_1]{s_0} \mathtt{let}\ s_0 := \mathtt{Ctrl}; p\ \mathtt{in}\ st}$$

- Judgment $\boxed{\delta \vdash e \xRightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{\delta \vdash x \xRightarrow[s_0]{s_0} (\epsilon, st)} \; (\delta(x) = st) \qquad \frac{\delta \vdash e_1 \xRightarrow[s_0']{s_0} (p_1, st_1) \qquad \delta[x \mapsto st_1] \vdash e_2 \xRightarrow[s_1]{s_0'} (p_2, st)}{\delta \vdash \textbf{let}\ x = e_1\ \textbf{in}\ e_2 \xRightarrow[s_1]{s_0} (p_1; p_2, st)}$$

$$\frac{\vdash Trans_{\phi}(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)}{\delta \vdash \phi(x_1, ..., x_k) \xRightarrow[s_1]{s_0} (p, st)} \; ((\delta(x_i) = st_i)_{i=1}^{k})$$

$$\frac{[x \mapsto st_1] \ \vdash \ e \xrightarrow[s_1]{s_0+1} (p, st)}{\delta \ \vdash \ \{e : x \ \textbf{in} \ y \ \textbf{using} \ \cdot\} \xRightarrow[s_1]{s_0} (s_0 := \texttt{Usum}(s_2); st := \texttt{WithCtrl}(s_0, p), (st, s_2))} \ (\delta(y) = (st_1, s_2))$$

- Auxiliary Judgment $\boxed{\vdash \ Trans_\phi(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{\textbf{const}_a() \xrightarrow[s_0+1]{s_0} (s_0 := \texttt{Const}_\texttt{a}, s_0)}$$

$$\frac{}{\textbf{iota}(s) \xRightarrow[s_4]{s_0} (p, (s_3, s_0))} \left( \begin{array}{l} s_{i+1} = s_i + 1 \\ p = s_0 := \texttt{ToFlags}(s); \\ \quad s_1 := \texttt{Usum}(s_0); \\ \quad s_2 := \texttt{WithCtrl}(s_1, s_2 := \texttt{Const}_1); \\ \quad s_3 := \texttt{ScanPlus}(s_0, s_2) \end{array} \right)$$

$$\frac{}{\textbf{plus}(s_1, s_2) \xrightarrow[s_0+1]{s_0} (s_0 := \texttt{MapTwo}(s_1, s_2), s_0)}$$

## 0.7 Value representation

- Judgment $\boxed{v \rhd_\tau \vec{v}}$

$$\frac{}{n \rhd_{\textbf{int}} \langle n \rangle} \qquad \frac{(v_i \rhd_\tau \vec{v}_i)_{i=1}^k}{\{v_1, ..., v_k\} \rhd_{\{\tau\}} (\vec{v}, \langle \texttt{F}_1, ..., \texttt{F}_k, \texttt{T} \rangle)} \ (\vec{v} = \vec{v}_1 +\!\!+ \vec{v}_2 +\!\!+ ... +\!\!+ \vec{v}_k)$$

## 0.8 Correctness proof

**Lemma 1.** *If*

*(i)* $\vdash \ Typ_\phi(\tau_1, ..., \tau_k) : \tau$

*(ii)* $\vdash \ Eva_\phi(v_1, ..., v_k) \downarrow v$

*(iii)* $\vdash \ Trans_\phi(st_1, ..., st_k) \xRightarrow[s_1]{s_0} (p, st)$

*(iv)* $(v_i \rhd_{\tau_i} st_i)_{i=1}^k$

*(v)* $\texttt{sids}((st_1, ..., st_k)) \lessdot s_0$

*then*

*(i)* $\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'$ *(by $\mathcal{P}$)*

*(ii)* $v \rhd_\tau \sigma'(st)$ *(by $\mathcal{V}$)*

*(iii)* $\sigma' \xlongequal{\lessdot s_0} \sigma$

*(iv)* $\texttt{sids}(st) \lessdot s_1$

*(v)* $s_0 \leq s_1$

**Theorem 2.** *If*

*(i)* $\Gamma \ \vdash \ e : \tau$ *(by some derivation $\mathcal{T}$)*

*(ii)* $\rho \ \vdash \ e \downarrow v$ *(by $\mathcal{E}$)*

*(iii)* $\delta \ \vdash \ e \xRightarrow[s_1]{s_0} (p, st)$ *(by $\mathcal{C}$)*

*(iv)* $\forall x \in dom(\Gamma).\rho(x) : \Gamma(x) \land \texttt{sids}(\delta(x)) \lessdot s_0 \land \rho(x) \rhd_{\Gamma(x)} \sigma(\delta(x))$

*then*

*(i)* $\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'$ *(by $\mathcal{P}$)*

*(ii)* $v \triangleright_\tau \sigma'(st)$ *(by $\mathcal{V}$)*

*(iii)* $\sigma' \xlongequal{< s_0} \sigma$

*(iv)* $\mathtt{sids}(st) \lessdot s_1$

*(v)* $s_0 \leq s_1$