

SNESL formalization

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September 3, 2017

0 Level-0

Draft version 0.0.1

0.1 Source language syntax

(Ignore empty sequence for now)

Expressions:

$$e ::= x \mid \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 \mid \varphi(x_1, \dots, x_k) \mid \{e : x \ \mathbf{in} \ y \ \mathbf{using} \ \cdot\} \\ \varphi = \mathbf{const}_n \mid \mathbf{iota} \mid \mathbf{plus}$$

Values:

$$n \in \mathbf{Z} \\ v ::= n \mid \{v_1, \dots, v_k\}$$

0.2 Type system

$$\tau ::= \mathbf{int} \mid \{\tau_1\}$$

Type environment $\Gamma = [x_1 \mapsto \tau_1, \dots, x_i \mapsto \tau_i]$.

- Judgment $\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma \vdash x : \tau} (\Gamma(x) = \tau) \quad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma[x \mapsto \tau_1] \vdash e_2 : \tau}{\Gamma \vdash \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 : \tau} \\ \frac{\vdash \mathit{Typ}_\varphi(\tau_1, \dots, \tau_k) : \tau}{\Gamma \vdash \varphi(x_1, \dots, x_k) : \tau} ((\Gamma(x_i) = \tau_i)_{i=1}^k) \quad \frac{\Gamma[x \mapsto \tau_1] \vdash e : \tau}{\Gamma \vdash \{e : x \ \mathbf{in} \ y \ \mathbf{using} \ \cdot\} : \{\tau\}} (\Gamma(y) = \{\tau_1\})$$

- Auxiliary Judgment $\boxed{\vdash \mathit{Typ}_\varphi(\tau_1, \dots, \tau_k) : \tau}$

$$\frac{}{\vdash \mathbf{const}_n() : \mathbf{int}} \quad \frac{}{\vdash \mathbf{iota}(\mathbf{int}) : \{\mathbf{int}\}} \quad \frac{}{\vdash \mathbf{plus}(\mathbf{int}, \mathbf{int}) : \mathbf{int}}$$

0.3 Source language semantics

- Judgment $\boxed{\rho \vdash e \downarrow v}$

$$\frac{}{\rho \vdash x \downarrow v} (\rho(x) = v) \quad \frac{\rho \vdash e_1 \downarrow v_1 \quad \rho[x \mapsto v_1] \vdash e_2 \downarrow v}{\rho \vdash \mathbf{let} \ e_1 = x \ \mathbf{in} \ e_2 \downarrow v} \\ \frac{\vdash \mathit{Eva}_\varphi(v_1, \dots, v_k) \downarrow v}{\rho \vdash \varphi(x_1, \dots, x_k) \downarrow v} ((\rho(x_i) = v_i)_{i=1}^k) \quad \frac{([x_i \mapsto v_i] \vdash e \downarrow v'_i)_{i=1}^k}{\rho \vdash \{e : x \ \mathbf{in} \ y \ \mathbf{using} \ \cdot\} \downarrow \{v'_1, \dots, v'_k\}} (\rho(y) = \{v_1, \dots, v_k\})$$

- Auxiliary Judgment $\boxed{\vdash \text{Eva}_\varphi(v_1, \dots, v_k) \downarrow v}$

$$\frac{}{\vdash \mathbf{const}_n() \downarrow n} \quad \frac{}{\vdash \mathbf{iota}(n) \downarrow \{0, 1, \dots, n-1\}} \quad \frac{}{\vdash \mathbf{plus}(n_1, n_2) \downarrow n_3} (n_3 = n_1 + n_2)$$

0.4 SVCODE syntax

Stream id:

$$s \in \mathbf{N} = \{0, 1, 2, \dots\}$$

Stream tree:

$$st ::= s \mid (st_1, s)$$

SVCODE expressions:

$$\varphi ::= \mathbf{Ctrl} \mid \mathbf{Const}_a \mid \mathbf{ToFlags} \mid \mathbf{Usum} \mid \mathbf{MapTwo} \mid \mathbf{ScanPlus}$$

SVCODE program:

$$\begin{aligned} p ::= & \epsilon \\ & \mid s := \psi(s_1, \dots, s_i) \\ & \mid st := \mathbf{WithCtrl}(s, p) \\ & \mid p_1; p_2 \end{aligned}$$

Target language values:

$$\begin{aligned} b & \in \{\mathbf{T}, \mathbf{F}\} \\ a & ::= n \mid b \mid () \\ \vec{v} & ::= \langle a_1, \dots, a_i \rangle \end{aligned}$$

Define some operations on \vec{v} for convenience:

- $\vec{v}_1 ++ \vec{v}_2$: append \vec{v}_2 to \vec{v}_1
- $\mathbf{tail}(\langle a_1, a_2, \dots, a_i \rangle) = \langle a_2, \dots, a_i \rangle$

0.5 SVCODE semantics

- Judgment $\boxed{\langle p, \sigma \rangle \downarrow^{\vec{v}_c} \sigma'}$

$$\frac{}{\langle \epsilon, \sigma \rangle \downarrow^{\vec{v}_c} \sigma} \quad \frac{\text{SEva}_\varphi(\vec{v}_1, \dots, \vec{v}_k) \downarrow^{\vec{v}_c} \vec{v}}{\langle s := \varphi(s_1, \dots, s_k), \sigma \rangle \downarrow^{\vec{v}_c} \sigma[s \mapsto \vec{v}]} ((\sigma(s_i) = \vec{v}_i)_{i=1}^k)$$

$$\frac{}{\langle st := \mathbf{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{v}_c} \sigma[st \mapsto \langle \rangle]} (\sigma(s) = \langle \rangle)$$

$$\frac{\langle p, \sigma \rangle \downarrow^{\vec{v}_s} \sigma'}{\langle st := \mathbf{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{v}_c} \sigma'} (\sigma(s) = \vec{v}_s = \langle a_1, \dots, a_i \rangle)$$

$$\frac{\langle p_1, \sigma \rangle \downarrow^{\vec{v}_c} \sigma'' \quad \langle p_2, \sigma'' \rangle \downarrow^{\vec{v}_c} \sigma'}{\langle p_1; p_2, \sigma \rangle \downarrow^{\vec{v}_c} \sigma'}$$

- Auxiliary Judgment $\boxed{\text{SEva}_\varphi(\vec{v}_1, \dots, \vec{v}_k) \downarrow^{\vec{v}_c} \vec{v}}$

$$\frac{\text{Block}_\varphi(\vec{v}_{11}, \dots, \vec{v}_{k1}) \downarrow \vec{v}_1 \quad \text{SEva}_\varphi(\vec{v}_{12}, \dots, \vec{v}_{k2}) \downarrow^{\mathbf{tail}(\vec{v}_c)} \vec{v}_2}{\text{SEva}_\varphi(\vec{v}_{11} ++ \vec{v}_{12}, \dots, \vec{v}_{k1} ++ \vec{v}_{k2}) \downarrow^{\vec{v}_c} \vec{v}} (\vec{v} = \vec{v}_1 ++ \vec{v}_2)$$

$$\frac{}{\text{SEva}_\varphi(\vec{v}_1, \dots, \vec{v}_k) \downarrow^{\langle \rangle} \langle \rangle}$$

- Auxiliary Judgment $\boxed{Block_\varphi(\vec{v}_1, \dots, \vec{v}_k) \Downarrow \vec{v}}$

$$\frac{}{Const_a \Downarrow \langle a \rangle} \quad \frac{}{ToFlags(\langle n \rangle) \Downarrow \langle F_1, \dots, F_n, T \rangle} \quad \frac{}{MapTwo(\langle n_1 \rangle, \langle n_2 \rangle) \Downarrow \langle n_3 \rangle} (n_3 = n_1 + n_2)$$

$$\frac{Unary_\varphi(\langle F \rangle, \dots, \vec{v}_{k1}) \Downarrow \vec{v}_1 \quad Block_\varphi(\vec{v}_{12}, \dots, \vec{v}_{k2}) \Downarrow \vec{v}_2}{Block_\varphi(\langle F \rangle ++ \vec{v}_{12}, \dots, \vec{v}_{k1} ++ \vec{v}_{k2}) \Downarrow \vec{v}} (\vec{v} = \vec{v}_1 ++ \vec{v}_2)$$

$$\frac{Unary_\varphi(\langle T \rangle, \dots, \vec{v}_k) \Downarrow \vec{v}}{Block_\varphi(\langle T \rangle, \dots, \vec{v}_k) \Downarrow \vec{v}}$$

$$\frac{Unary_{\varphi, n_0}(\langle F \rangle, \dots, \vec{v}_{k1}) \Downarrow^{n'_0} \langle n_1 \rangle \quad Block_{\varphi, n'_0}(\vec{v}_{12}, \dots, \vec{v}_{k2}) \Downarrow \vec{v}_2}{Block_{\varphi, n_0}(\langle F \rangle ++ \vec{v}_{12}, \dots, \vec{v}_{k1} ++ \vec{v}_{k2}) \Downarrow \langle n_1 \rangle ++ \vec{v}_2}$$

$$\frac{Unary_{\varphi, n_0}(\langle T \rangle, \dots, \vec{v}_k) \Downarrow \langle n_1 \rangle}{Block_{\varphi, n_0}(\langle T \rangle, \dots, \vec{v}_k) \Downarrow \langle n_1 \rangle}$$

- Auxiliary Judgment $\boxed{Unary_\varphi(\langle b \rangle, \dots, \vec{v}_k) \Downarrow \vec{v}}$

$$\frac{}{Usum(\langle F \rangle) \Downarrow \langle () \rangle} \quad \frac{}{Usum(\langle T \rangle) \Downarrow \langle \rangle}$$

$$\frac{}{ScanPlus_{n_0}(\langle F \rangle, \langle n \rangle) \Downarrow^{n_0+n} \langle n_0 \rangle} \quad \frac{}{ScanPlus_{n_0}(\langle T \rangle, \langle \rangle) \Downarrow \langle n_0 \rangle}$$

0.6 Translation

- $$\frac{\delta \vdash e \xrightarrow[s_1]{s_0+1} (p, st)}{\delta \vdash e \xrightarrow[s_1]{s_0} \text{let } s_0 := \text{Ctrl}; p \text{ in } st}$$

- Judgment $\boxed{\delta \vdash e \xrightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{\delta \vdash x \xrightarrow[s_0]{s_0} (p, st)} (\delta(x) = st) \quad \frac{\delta \vdash e_1 \xrightarrow[s'_0]{s_0} (p_1, st_1) \quad \delta[x \mapsto st_1] \vdash e_2 \xrightarrow[s_1]{s'_0} (p_2, st)}{\delta \vdash \text{let } x = e_1 \text{ in } e_2 \xrightarrow[s_1]{s_0} (p_1; p_2, st)}$$

$$\frac{Trans_\varphi(st_1, \dots, st_k) \xrightarrow[s_1]{s_0} (p, st)}{\delta \vdash \varphi(x_1, \dots, x_k) \xrightarrow[s_1]{s_0} (p, st)} ((\delta(x_i) = st_i)_{i=1}^k)$$

$$\frac{[x \mapsto st_1] \vdash e \xrightarrow[s_1]{s_0+1} (p, st)}{\delta \vdash \{e : x \text{ in } y \text{ using } \cdot\} \xrightarrow[s_1]{s_0} (s_0 := Usum(s_2); st := WithCtrl(s_0, p), st)} (\delta(y) = (st_1, s_2))$$

- Auxiliary Judgment $\boxed{Trans_\varphi(st_1, \dots, st_k) \xrightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{const_a() \xrightarrow[s_0+1]{s_0} (s_0 := Const_a, s_0)}$$

$$\frac{\text{iota}(s) \xrightarrow[s_4]{s_0} (p, (s_3, s_0))}{\text{plus}(s_1, s_2) \xrightarrow[s_0+1]{s_0} (s_0 := \text{MapTwo}(s_1, s_2), s_0)} \left(\begin{array}{l} s_{i+1} = s_i + 1 \\ p = s_0 := \text{ToFlags}(s); \\ s_1 := \text{Usum}(s_0); \\ s_2 := \text{WithCtrl}(s_1, s_2 := \text{Const}_1); \\ s_3 := \text{ScanPlus}(s_0, s_2) \end{array} \right)$$

0.7 Value representation

TODO: define $\sigma(st)$

Judgment $\boxed{\sigma \vdash v \triangleright_{\tau} st}$

$$\frac{}{\sigma \vdash n \triangleright_{\text{int}} s} (\sigma(s) = \langle n \rangle) \quad \frac{(\sigma \vdash n_i \triangleright_{\text{int}} s_i)_{i=1}^k}{\sigma \vdash \{n_1, \dots, n_k\} \triangleright_{\{\text{int}\}} (s, s')} \left(\begin{array}{l} \sigma(s) = \sigma(s_1) ++ \sigma(s_2) ++ \dots ++ \sigma(s_k) \\ \sigma(s') = \langle F_1, \dots, F_k, T \rangle \end{array} \right)$$

$$\frac{(\sigma \vdash v_i \triangleright_{\tau} (st_i, s_i))_{i=1}^k}{\sigma \vdash \{v_1, \dots, v_k\} \triangleright_{\{\tau\}} ((st, s), s')} \left(\begin{array}{l} \tau \neq \text{int} \\ \sigma(st) = \sigma(st_1) ++ \sigma(st_2) ++ \dots ++ \sigma(st_k) \\ \sigma(s) = \sigma(s_1) ++ \sigma(s_2) ++ \dots ++ \sigma(s_k) \\ \sigma(s') = \langle F_1, \dots, F_k, T \rangle \end{array} \right)$$

0.8 Correctness proof