

# SNESL formalization

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September 4, 2017

## 0 Level-0

Draft version 0.0.2

### 0.1 Source language syntax

(Ignore empty sequence for now)

Expressions:

$$e ::= x \mid \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 \mid \varphi(x_1, \dots, x_k) \mid \{e : x \ \mathbf{in} \ y \ \mathbf{using} \ \cdot\} \\ \varphi = \mathbf{const}_n \mid \mathbf{iota} \mid \mathbf{plus}$$

Values:

$$n \in \mathbf{Z} \\ v ::= n \mid \{v_1, \dots, v_k\}$$

### 0.2 Type system

$$\tau ::= \mathbf{int} \mid \{\tau_1\}$$

Type environment  $\Gamma = [x_1 \mapsto \tau_1, \dots, x_i \mapsto \tau_i]$ .

- Judgment  $\boxed{\Gamma \vdash e : \tau}$

$$\frac{}{\Gamma \vdash x : \tau} (\Gamma(x) = \tau) \quad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma[x \mapsto \tau_1] \vdash e_2 : \tau}{\Gamma \vdash \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 : \tau} \\ \frac{\vdash \mathit{Typ}_\varphi(\tau_1, \dots, \tau_k) : \tau}{\Gamma \vdash \varphi(x_1, \dots, x_k) : \tau} ((\Gamma(x_i) = \tau_i)_{i=1}^k) \quad \frac{\Gamma[x \mapsto \tau_1] \vdash e : \tau}{\Gamma \vdash \{e : x \ \mathbf{in} \ y \ \mathbf{using} \ \cdot\} : \{\tau\}} (\Gamma(y) = \{\tau_1\})$$

- Auxiliary Judgment  $\boxed{\vdash \mathit{Typ}_\varphi(\tau_1, \dots, \tau_k) : \tau}$

$$\frac{}{\vdash \mathbf{const}_n() : \mathbf{int}} \quad \frac{}{\vdash \mathbf{iota}(\mathbf{int}) : \{\mathbf{int}\}} \quad \frac{}{\vdash \mathbf{plus}(\mathbf{int}, \mathbf{int}) : \mathbf{int}}$$

### 0.3 Source language semantics

- Judgment  $\boxed{\rho \vdash e \downarrow v}$

$$\frac{}{\rho \vdash x \downarrow v} (\rho(x) = v) \quad \frac{\rho \vdash e_1 \downarrow v_1 \quad \rho[x \mapsto v_1] \vdash e_2 \downarrow v}{\rho \vdash \mathbf{let} \ e_1 = x \ \mathbf{in} \ e_2 \downarrow v} \\ \frac{\vdash \mathit{Eva}_\varphi(v_1, \dots, v_k) \downarrow v}{\rho \vdash \varphi(x_1, \dots, x_k) \downarrow v} ((\rho(x_i) = v_i)_{i=1}^k) \quad \frac{([x \mapsto v_i] \vdash e \downarrow v'_i)_{i=1}^k}{\rho \vdash \{e : x \ \mathbf{in} \ y \ \mathbf{using} \ \cdot\} \downarrow \{v'_1, \dots, v'_k\}} (\rho(y) = \{v_1, \dots, v_k\})$$

- Auxiliary Judgment  $\boxed{\vdash \text{Eva}_\varphi(v_1, \dots, v_k) \downarrow v}$

$$\frac{}{\vdash \mathbf{const}_n() \downarrow n} \quad \frac{}{\vdash \mathbf{iota}(n) \downarrow \{0, 1, \dots, n-1\}} \quad \frac{}{\vdash \mathbf{plus}(n_1, n_2) \downarrow n_3} (n_3 = n_1 + n_2)$$

## 0.4 SVCODE syntax

Stream id:

$$s \in \mathbf{N} = \{0, 1, 2, \dots\}$$

Stream tree:

$$st ::= s \mid (st_1, s)$$

SVCODE expressions:

$$\varphi ::= \mathbf{Ctrl} \mid \mathbf{Const}_a \mid \mathbf{ToFlags} \mid \mathbf{Usum} \mid \mathbf{MapTwo} \mid \mathbf{ScanPlus}$$

SVCODE program:

$$\begin{aligned} p ::= & \epsilon \\ & \mid s := \psi(s_1, \dots, s_i) \\ & \mid st := \mathbf{WithCtrl}(s, p) \\ & \mid p_1; p_2 \end{aligned}$$

Target language values:

$$\begin{aligned} b & \in \{\mathbf{T}, \mathbf{F}\} \\ a & ::= n \mid b \mid () \\ \vec{b} & = \langle b_1, \dots, b_i \rangle \\ \vec{a} & = \langle a_1, \dots, a_i \rangle \\ \vec{v} & ::= \vec{a} \mid (\vec{v}, \vec{b}) \end{aligned}$$

Define some operations for convenience:

- $++ : \vec{v} \rightarrow \vec{v} \rightarrow \vec{v}$   
 $\langle a_1, \dots, a_i \rangle ++ \langle a_2, \dots, a_j \rangle = \langle \vec{a}_1, \dots, a_i, a_2, \dots, a_j \rangle$   
 $(\vec{v}_1, \vec{b}_1) ++ (\vec{v}_2, \vec{b}_2) = (\vec{v}_1 ++ \vec{v}_2, \vec{b}_1 ++ \vec{b}_2)$
- $\mathbf{tail} : \vec{a} \rightarrow \vec{a}$   
 $\mathbf{tail}(\langle a_1, a_2, \dots, a_i \rangle) = \langle a_2, \dots, a_i \rangle$

## 0.5 SVCODE semantics

$$\sigma = [s_1 \mapsto \vec{a}_1, \dots, s_i \mapsto \vec{a}_i]$$

$\vec{a}_c$  is the control stream.

- Judgment  $\boxed{\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'}$

$$\frac{}{\langle \epsilon, \sigma \rangle \downarrow^{\vec{a}_c} \sigma} \quad \frac{SEva_\varphi(\vec{a}_1, \dots, \vec{a}_k) \downarrow^{\vec{a}_c} \vec{a}}{\langle s := \varphi(s_1, \dots, s_k), \sigma \rangle \downarrow^{\vec{a}_c} \sigma[s \mapsto \vec{a}]} ((\sigma(s_i) = \vec{a}_i)_{i=1}^k)$$

$$\frac{}{\langle st := \mathbf{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{a}_c} \sigma[st \mapsto \langle \rangle]} (\sigma(s) = \langle \rangle)$$

$$\frac{\langle p, \sigma \rangle \downarrow^{\vec{a}_s} \sigma'}{\langle st := \mathbf{WithCtrl}(s, p), \sigma \rangle \downarrow^{\vec{a}_c} \sigma'} (\sigma(s) = \vec{a}_s = \langle a_1, \dots, a_i \rangle)$$

$$\frac{\langle p_1, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'' \quad \langle p_2, \sigma'' \rangle \downarrow^{\vec{a}_c} \sigma'}{\langle p_1; p_2, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'}$$

- Auxiliary Judgment  $\boxed{SEva_\varphi(\vec{a}_1, \dots, \vec{a}_k) \Downarrow^{\vec{a}_c} \vec{a}}$

$$\frac{Block_\varphi(\vec{a}_{11}, \dots, \vec{a}_{k1}) \Downarrow \vec{a}_1 \quad SEva_\varphi(\vec{a}_{12}, \dots, \vec{a}_{k2}) \Downarrow^{\text{tail}(\vec{a}_c)} \vec{a}_2}{SEva_\varphi(\vec{a}_{11} ++ \vec{a}_{12}, \dots, \vec{a}_{k1} ++ \vec{a}_{k2}) \Downarrow^{\vec{a}_c} \vec{a}} \quad (\vec{a} = \vec{a}_1 ++ \vec{a}_2)$$

$$\overline{SEva_\varphi(\vec{a}_1, \dots, \vec{a}_k) \Downarrow^{\langle \rangle} \langle \rangle}$$

- Auxiliary Judgment  $\boxed{Block_\varphi(\vec{a}_1, \dots, \vec{a}_k) \Downarrow \vec{a}}$

$$\overline{\text{Const}_a \Downarrow \langle a \rangle} \quad \overline{\text{ToFlags}(\langle n \rangle) \Downarrow \langle F_1, \dots, F_n, T \rangle} \quad \overline{\text{MapTwo}(\langle n_1 \rangle, \langle n_2 \rangle) \Downarrow \langle n_3 \rangle} \quad (n_3 = n_1 + n_2)$$

$$\frac{Unary_\varphi(\langle F \rangle, \dots, \vec{a}_{k1}) \Downarrow \vec{a}_1 \quad Block_\varphi(\vec{a}_{12}, \dots, \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_\varphi(\langle F \rangle ++ \vec{a}_{12}, \dots, \vec{a}_{k1} ++ \vec{a}_{k2}) \Downarrow \vec{a}} \quad (\vec{a} = \vec{a}_1 ++ \vec{a}_2)$$

$$\frac{Unary_\varphi(\langle T \rangle, \dots, \vec{a}_k) \Downarrow \vec{a}}{Block_\varphi(\langle T \rangle, \dots, \vec{a}_k) \Downarrow \vec{a}}$$

$$\frac{Unary_{\varphi, n_0}(\langle F \rangle, \dots, \vec{a}_{k1}) \Downarrow^{n'_0} \langle n_1 \rangle \quad Block_{\varphi, n'_0}(\vec{a}_{12}, \dots, \vec{a}_{k2}) \Downarrow \vec{a}_2}{Block_{\varphi, n_0}(\langle F \rangle ++ \vec{a}_{12}, \dots, \vec{a}_{k1} ++ \vec{a}_{k2}) \Downarrow \langle n_1 \rangle ++ \vec{a}_2}$$

$$\frac{Unary_{\varphi, n_0}(\langle T \rangle, \dots, \vec{a}_k) \Downarrow \langle n_1 \rangle}{Block_{\varphi, n_0}(\langle T \rangle, \dots, \vec{a}_k) \Downarrow \langle n_1 \rangle}$$

- Auxiliary Judgment  $\boxed{Unary_\varphi(\langle b \rangle, \dots, \vec{a}_k) \Downarrow \vec{a}}$

$$\overline{\text{Usum}(\langle F \rangle) \Downarrow \langle () \rangle} \quad \overline{\text{Usum}(\langle T \rangle) \Downarrow \langle \rangle}$$

$$\overline{\text{ScanPlus}_{n_0}(\langle F \rangle, \langle n \rangle) \Downarrow^{n_0+n} \langle n_0 \rangle} \quad \overline{\text{ScanPlus}_{n_0}(\langle T \rangle, \langle \rangle) \Downarrow \langle n_0 \rangle}$$

## 0.6 Translation

$$\delta = [x_1 \mapsto st_1, \dots, x_i \mapsto st_i]$$

- (??not necessary)

$$\frac{\delta \vdash e \xrightarrow[s_1]{s_0+1} (p, st)}{\delta \vdash e \xrightarrow[s_1]{s_0} \text{let } s_0 := \text{Ctrl}; p \text{ in } st}$$

- Judgment  $\boxed{\delta \vdash e \xrightarrow[s_1]{s_0} (p, st)}$

$$\frac{}{\delta \vdash x \xrightarrow[s_0]{s_0} (\epsilon, st)} \quad (\delta(x) = st) \quad \frac{\delta \vdash e_1 \xrightarrow[s'_0]{s_0} (p_1, st_1) \quad \delta[x \mapsto st_1] \vdash e_2 \xrightarrow[s_1]{s'_0} (p_2, st)}{\delta \vdash \text{let } x = e_1 \text{ in } e_2 \xrightarrow[s_1]{s_0} (p_1; p_2, st)}$$

$$\frac{\vdash Trans_\varphi(st_1, \dots, st_k) \xrightarrow[s_1]{s_0} (p, st)}{\delta \vdash \varphi(x_1, \dots, x_k) \xrightarrow[s_1]{s_0} (p, st)} \quad ((\delta(x_i) = st_i)_{i=1}^k)$$

$$\frac{[x \mapsto st_1] \vdash e \xrightarrow[s_1]{s_0+1} (p, st)}{\delta \vdash \{e : x \text{ in } y \text{ using } \cdot\} \xrightarrow[s_1]{s_0} (s_0 := \text{Usum}(s_2); st := \text{WithCtrl}(s_0, p), (st, s_2))} (\delta(y) = (st_1, s_2))$$

- Auxiliary Judgment  $\boxed{\vdash \text{Trans}_\varphi(st_1, \dots, st_k) \xrightarrow[s_1]{s_0} (p, st)}$

$$\frac{\text{const}_a() \xrightarrow[s_0+1]{s_0} (s_0 := \text{Const}_a, s_0)}{\text{iota}(s) \xrightarrow[s_4]{s_0} (p, (s_3, s_0))} \left( \begin{array}{l} s_{i+1} = s_i + 1 \\ p = s_0 := \text{ToFlags}(s); \\ s_1 := \text{Usum}(s_0); \\ s_2 := \text{WithCtrl}(s_1, s_2 := \text{Const}_1); \\ s_3 := \text{ScanPlus}(s_0, s_2) \end{array} \right)$$

$$\frac{}{\text{plus}(s_1, s_2) \xrightarrow[s_0+1]{s_0} (s_0 := \text{MapTwo}(s_1, s_2), s_0)}$$

## 0.7 Value representation

- Judgment  $\boxed{v \triangleright_\tau \vec{v}}$

$$\frac{}{n \triangleright_{\text{int}} \langle n \rangle} \quad \frac{(v_i \triangleright_\tau \vec{v}_i)_{i=1}^k}{\{v_1, \dots, v_k\} \triangleright_{\{\tau\}} (\vec{v}, \langle \mathbf{F}_1, \dots, \mathbf{F}_k, \mathbf{T} \rangle)} (\vec{v} = \vec{v}_1 ++ \vec{v}_2 ++ \dots ++ \vec{v}_k)$$

## 0.8 Correctness proof

**Lemma 1.** *If*

- (i)  $\vdash \text{Typ}_\varphi(\tau_1, \dots, \tau_k) : \tau$
- (ii)  $\vdash \text{Eva}_\varphi(v_1, \dots, v_k) \downarrow v$
- (iii)  $\vdash \text{Trans}_\varphi(st_1, \dots, st_k) \xrightarrow[s_1]{s_0} (p, st)$
- (iv)  $(v_i \triangleright_{\tau_i} st_i)_{i=1}^k$
- (v)  $\text{sids}((st_1, \dots, st_k)) \leq s_0$

then

- (i)  $\langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma'$  (by  $\mathcal{P}$ )
- (ii)  $v \triangleright_\tau \sigma'(st)$  (by  $\mathcal{V}$ )
- (iii)  $\sigma' \xrightarrow{\leq s_0} \sigma$
- (iv)  $\text{sids}(st) \leq s_1$
- (v)  $s_0 \leq s_1$

**Theorem 2.** *If*

- (i)  $\Gamma \vdash e : \tau$  (by some derivation  $\mathcal{T}$ )
- (ii)  $\rho \vdash e \downarrow v$  (by  $\mathcal{E}$ )
- (iii)  $\delta \vdash e \xrightarrow[s_1]{s_0} (p, st)$  (by  $\mathcal{C}$ )
- (iv)  $\forall x \in \text{dom}(\Gamma). \rho(x) : \Gamma(x) \wedge \text{sids}((\delta(x)) \leq s_0 \wedge \rho(x) \triangleright_{\Gamma(x)} \sigma(\delta(x))$

then

$$(i) \quad \langle p, \sigma \rangle \downarrow^{\vec{a}_c} \sigma' \text{ (by } \mathcal{P})$$

$$(ii) \quad v \triangleright_{\tau} \sigma(st) \text{ (by } \mathcal{V})$$

$$(iii) \quad \sigma' \stackrel{< s_0}{=} \sigma$$

$$(iv) \quad \mathbf{sids}(st) \leq s_1$$

$$(v) \quad s_0 \leq s_1$$