HW7

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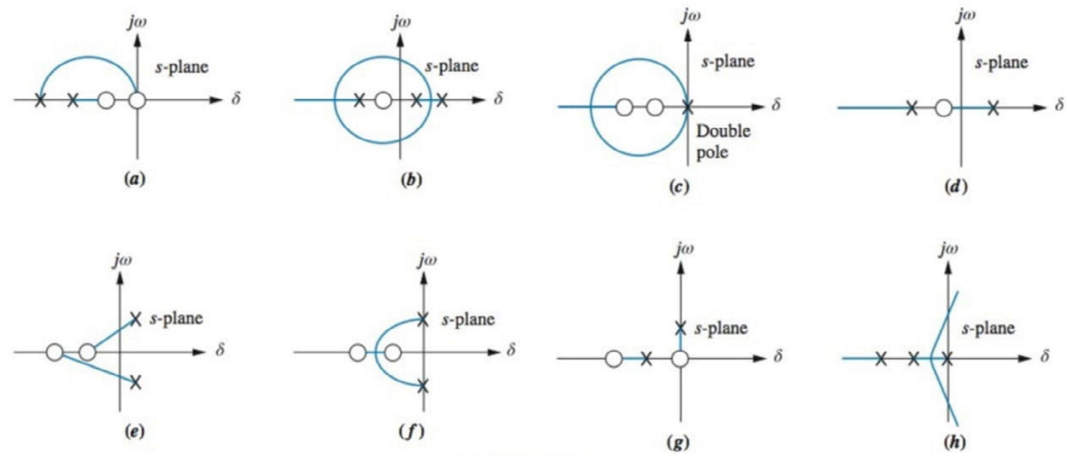
ENME462
Section 0105

1. For each of the root loci shown in the figure below, tell whether or not the sketch can be a root locus. If the sketch cannot be a root locus, explain why. Give all reasons.



(a) $\boxed{\text{Not a root locus}}$

- Not symmetric around real axis
- Missing segment between zeros and poles
- No real axis segment between pole and zero

(b) $\boxed{\text{Not a root locus}}$

- Only two branches but three finite poles
- No real-axis root to

- No real-axis root to the far left of zero
- There should not be real-axis locus to left of third pole
- One branch needs to end at the finite zero

(c) $\boxed{\text{Not a root locus}}$

- Each branch should finish at finite zero but only one does
- Should be real-axis root locus segment between two zeros
- Shouldn't be real axis root locus to the left of leftmost zero

(d) $\boxed{\text{Root locus}}$

(e) $\boxed{\text{Not a root locus}}$

(e) $\boxed{\text{Not a root locus}}$

- Not symmetric around real axis
- Missing real-axis segment between zeros

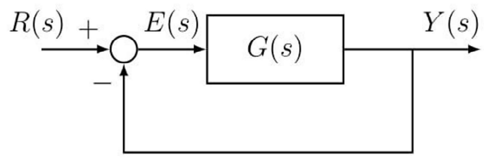
(f) $\boxed{\text{Root locus}}$

(g) $\boxed{\text{Not a root locus}}$

- Not symmetric around real axis

(h) $\boxed{\text{Root locus}}$

2. For the unity feedback system



Sketch the root locus for the closed-loop systems with forward-path transfer function as given below. Just do a quick sketch, no need to calculate break-in/break-out point(s) or exact imaginary axis crossing point.

(a) $G(s) = \dfrac{K(s+2)(s+6)}{s^2+8s+25}$ (b) $G(s) = \dfrac{K(s^2+4)}{s^2+1}$

(c) $G(s) = \dfrac{K(s^2+1)}{s^2}$ (d) $G(s) = \dfrac{K}{(s+1)^3(s+4)}$

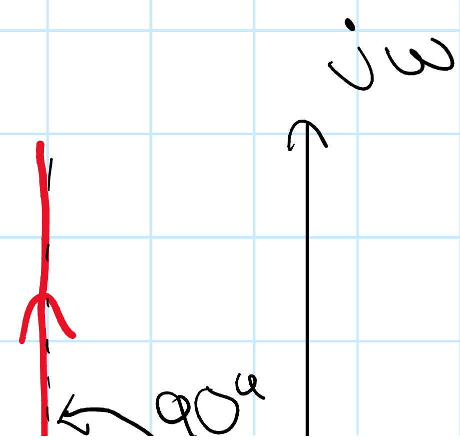
Example: To help you with the sketches, determine for each system: the number and locations of the finite poles and zeros, the real-axis root locus set, number of poles and zeros at infinity (if any), and the asymptote intercept and angles (if any). For example, consider the forward-path transfer function

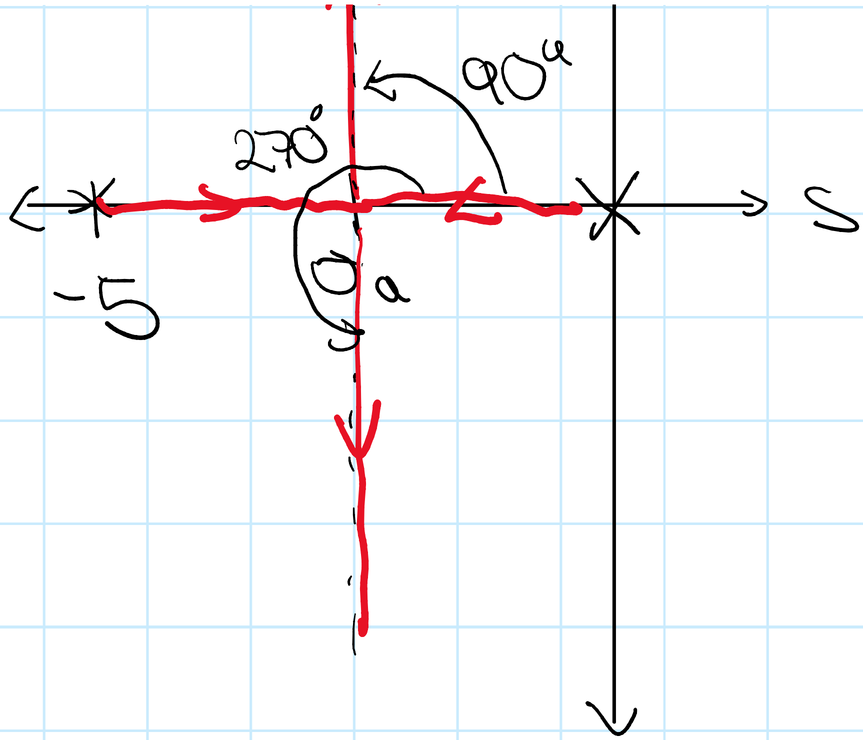
$$G(s) = \frac{K(b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0)}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = \frac{K}{s^2+5s} = \frac{K}{s(s+5)}$$

for which we determine

- Finite poles: $s = \{-5, 0\}$. No finite zeros.
- Scanning the s-plane from right to left and keeping track of whether there is an odd or even number of finite poles or zeros to the right, we find that there is real-axis root locus for $s \in [-5, -0]$.
- Number of poles at infinity: $\max(m,n) - n = 2 - 2 = 0$. Number of zeros at infinity: $\max(m,n) - m = 2 - 0 = 2$.
- Two asymptotes needed (number of poles and zeros at infinity):
  - The intercept is found at: $\sigma_a = \frac{\sum(\text{finite poles}) - \sum(\text{finite zeros})}{\#\text{ finite poles} - \#\text{ finite zeros}} = \dfrac{(-5+0)-0}{2-0} = -2.5$
  - The asymptote angles from the real-axis are:

$$\theta_a = \frac{(2\ell+1)180^\circ}{\#\text{ finite poles} - \#\text{ finite zeros}} = \{90^\circ, 270^\circ\}, \qquad \ell = 0, 1$$





(a) $G(s) = \dfrac{K(s+2)(s+6)}{s^2+8s+25}$

Finite Poles

$s = \dfrac{-8 \pm \sqrt{8^2 - 4(25)}}{2}$

$= -4+3j,\ -4-3j$

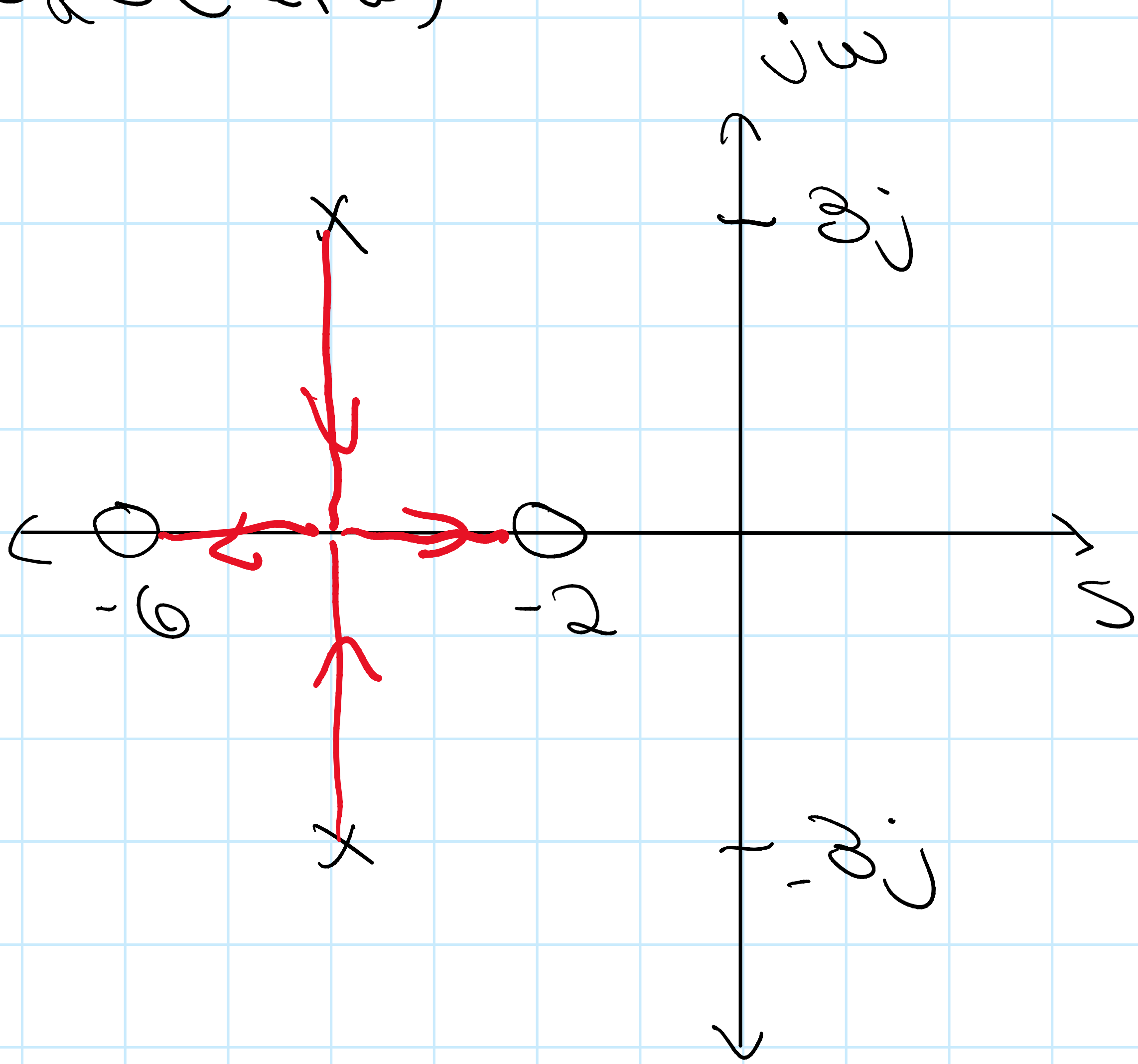
Finite Zeros

$s = -2, -6$

$s \in [-6, -4]$

Poles at infinity = 0

poles at infinity = 0
zeros at infinity = 0
No asymtotes
$\sigma_a \in (-6, 2)$



(b) $G(s) = \dfrac{K(s^2+4)}{s^2+1}$

Finite Poles
$s = 1j, -1j$

$S = 1j, -1j$

Finite Zeros

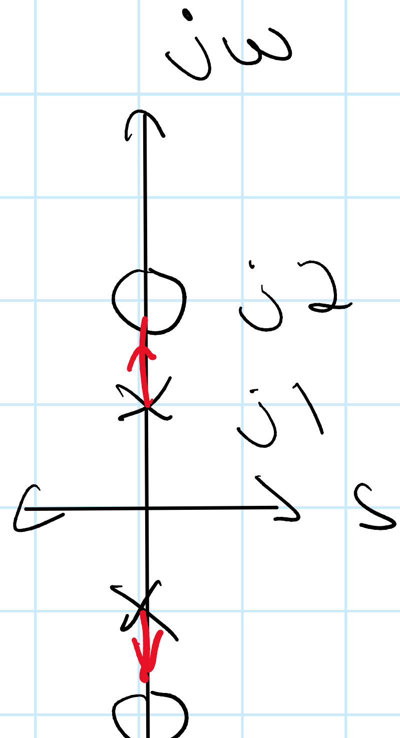
$S = 2j, -2j$

No real-axis root locus

poles at infinity = 0

zeros at infinity = 0

No asymtotes



(c) $G(s) = \frac{K(s^2+1)}{s^2}$

Finite Poles

Finite Poles

$s = 0, 0$

Finite Zeros

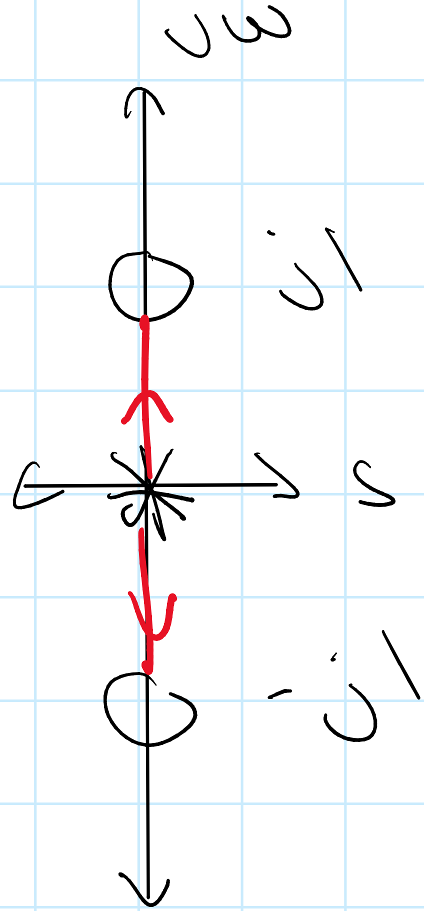
$s = j1, -j1$

No real-axis root locus

poles at infinity = 0

zeros at infinity = 0

No asymtotes.



(d) $G(s) = K$

(d) $G(s) = \dfrac{K}{(s+1)^3(s+4)}$

Finite Poles

$s = -1, -1, -1, -4$

Finite Zeros

none

poles at infinity = 0

zeros at infinity = 4

4 asymtotes

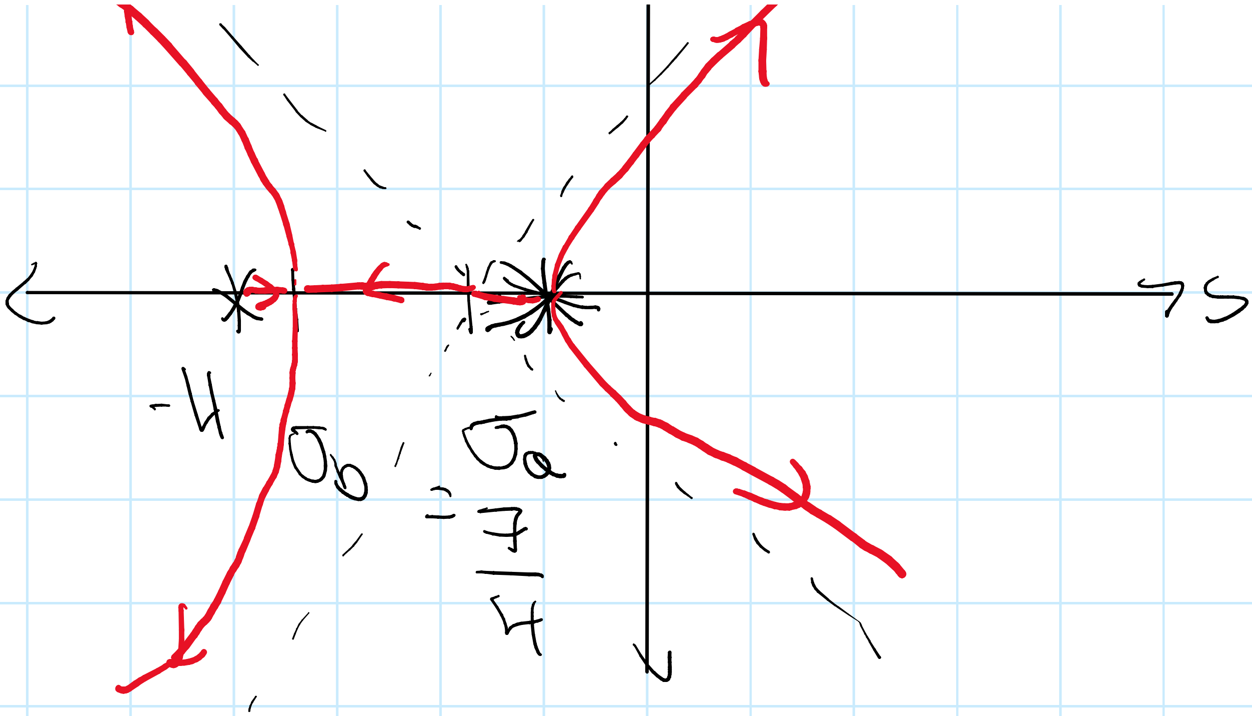
$\sigma_a = \dfrac{-7-0}{4} = -\dfrac{7}{4}$

$\theta_a = \dfrac{(2k+1)180°}{4} = 45°, 135°, 225°, 315°$

$\sigma_b \in (-4,-1)$

More poles a $s=-1$, ∴

breakaway point at $s=-4$ closer



3. For a unity feedback system with forward-path transfer function:

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)(s+5)}$$

(a) Sketch the root locus.

i. Draw the finite open-loop poles and zeros.

ii. Draw the real-axis root locus.

iii. Draw the asymptotes and root locus branches.

(b) Find the value of gain that will make the system marginally stable.

(c) Find the value of gain for which the closed-loop transfer function will have a pole on the real axis at $s = -0.5$.

3. $G(s) = \dfrac{K(s+2)}{s(s+1)(s+3)(s+5)}$

(a) Finite poles

$s = 0, -1, -3, -5$

Finite zeros

$s = -2$

Real-axis root locus

$s \in (-\infty, -5] \cup [-3, -2] \cup [-1, 0]$

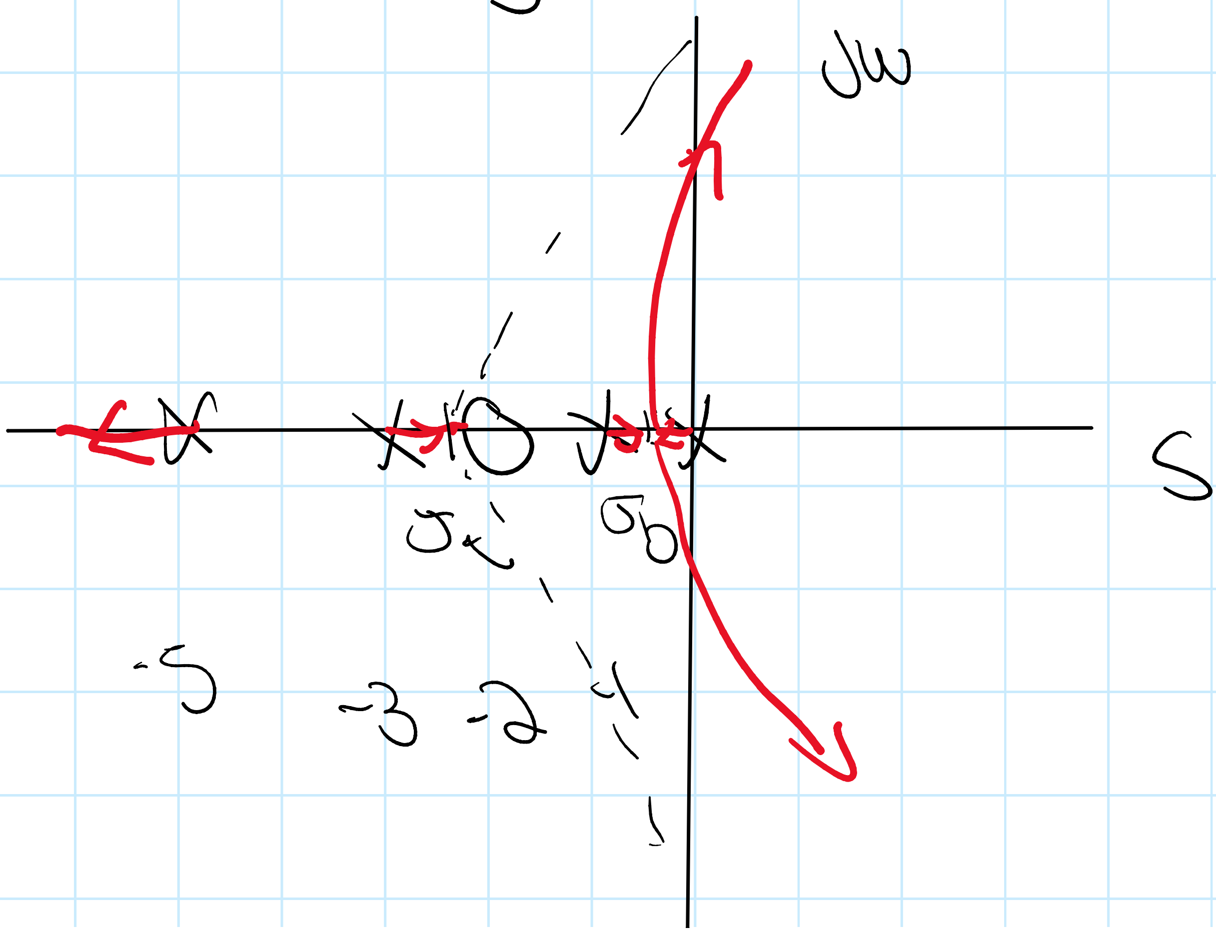
Root locus depart real axis $\sigma_b \in (-1, 0)$

Poles at infinity = 0

zeros at infinity = 3

$$\sigma_a = \frac{-9 + 2}{4 - 1} = -\frac{7}{3}$$

$$\theta_a = \frac{(2k+1)180^\circ}{3} = 60^\circ, 180^\circ, 300^\circ$$



(b) $P(s) = s^4 + 9s^3 + 23s^2 + (K+15)s + 2K$

s^4	1	23	$2K$	
s^3	9	$K+15$	0	
s^2	$\dfrac{-\begin{vmatrix} 1 & 23 \\ 9 & K+15 \end{vmatrix}}{9} = 192-K$	$18K$	0	
s^1	$\dfrac{-\begin{vmatrix} 9 & K+15 \\ 192-K & 18K \end{vmatrix}}{192-K} = \dfrac{-K^2+15K+2880}{192-K}$	0	0	0
s^0	$18K$	0	0	0

$-K^2 + 15K + 2880$

$K = 61.6872$

$D(s) = s^2 + 8.5208$

$s = \pm 2.919j \quad K = 61.6872$

(c) $s = -5$

$K = \dfrac{1}{|G(-5)|} = \dfrac{(0.5)(0.5)(25)(48)}{1.8}$

$= \boxed{1.575}$

4. Consider the unity feedback system with forward-path transfer function:

$$G(s) = \frac{8}{(s+4)(s^2+2s+2)}$$

Use the definition of the root locus to determine if the following points are closed-loop poles of the system. If they are, determine the gain K at those points.

(a) $s = -5$

(b) $s = -2 \pm j$

(c) $s = \pm j3.165$

4. $G(s) = \dfrac{8}{(s+4)(s^2+2s+2)}$

(a) $\angle G(s)\Big|_{s=-5}$

$= (2k+1)180°$

$$= \angle\left(\frac{8}{(s+4)(s^2+2s+2)}\right)\Bigg|_{s=-5}$$

$$= \angle\left(\frac{8}{-17}\right)$$

$$\tan^{-1}\left(\frac{0}{-8}\right) = 180^\circ$$

$$-\frac{8}{17} \qquad \boxed{\text{On the root locus}}$$

$$K = \frac{1}{|G(s)|_{s=-5}} = \frac{17}{8} = \boxed{2.125}$$

(b) $s = -2 \pm j$

$$\angle G(s)\Big|_{s=-2+j} = (2k+1)180^\circ$$

$$\angle\left(\frac{8}{(s+4)(s^2+2s+2)}\right)\Bigg|_{s=-2+j}$$

$$= \angle\left(\frac{8}{(2+j)((4-1-4j)+(-4+2j)+2)}\right)$$

$$= \angle\left(\frac{8}{(2+j)(1-2j)}\right) = \angle\left(\frac{8}{4-3j}\right)$$

$$= \angle(8) - \angle(4-3j)$$

$$= \tan^{-1}\left(\frac{0}{8}\right) - \tan^{-1}\left(\frac{-3}{4}\right)$$

$$= 0 - 36.9° = 36.9 \neq (2k+1)180°$$

$$\boxed{\text{Not on root locus}}$$

(c) $s = \pm 3.165j$

$$\angle G(s)\Big|_{s=3.165j} = (2k+1)180°$$

$$= \angle\left(\frac{8}{(s+4)(s^2+2s+2)}\right)\Bigg|_{s=3.165j}$$

$$= \angle\left(\frac{8}{(4+3.165j)(-8.017+6.33j)}\right)$$

$$= \angle(8) - \angle(4-3.165j)$$

$$- \angle(-8.017+6.33j)$$

$$- \angle(-8.017 + 6.23j))$$

$$= \tan^{-1}\left(\frac{0}{9}\right) - \tan^{-1}\left(\frac{3.163}{4}\right)$$

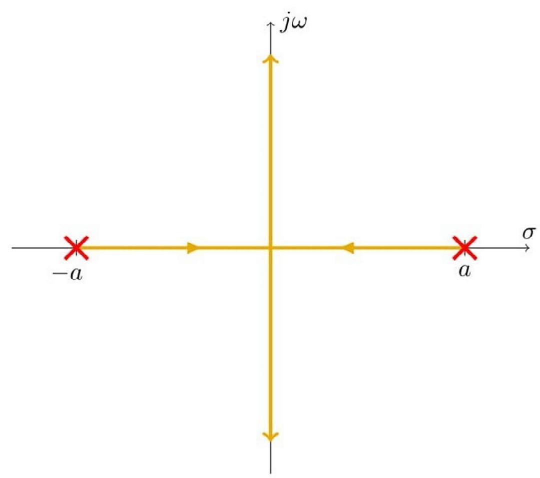
$$- \tan^{-1}\left(\frac{6.23}{-8.017}\right)$$

$$= 0 - 38.3^\circ - 141.7^\circ = -180^\circ$$

On the root locus

$$K = \left.\frac{1}{|G(s)|}\right|_{s=3.165j} = \boxed{6.51}$$

5. Considering the root locus diagram below for $a > 0$, is the corresponding closed loop system ever BIBO stable for gains $K > 0$? Motivate your answer.



$\boxed{\text{No the system will not be banded input banded output stable for any value of K}}$

- If gain is small closed loop poles are real and distinct however one pole in RHP makes it unbounded
- For gain, poles become zero which means imaginary axis with multiplicity 2 and becomes unbounded
- For large gain, poles are imagnary complex and system is marginally stable, but input of

resonance make output unbounded

6. Consider the unity feedback system with forward-path transfer function:

$$G(s) = \frac{K}{s(s+3)(s+4)(s+8)}$$

Use MATLAB to write a simple program to:

(a) Display the root locus. (**Hint:** `rlocus`)

(b) Do a close-up of the root locus with axes going from $[-2, 0]$ on the real axis and from $[-2, 2]$ on the imaginary axis. (**Hint:** `axis`)

(c) Overlay a 10% overshoot line on the close-up root locus. (**Hint:** `sgrid`)

(d) Select interactively the point where the root locus crosses the 10% overshoot line, and provide the gain at that point as well as the closed-loop poles for that gain. (**Hint:** `rlocfind`)

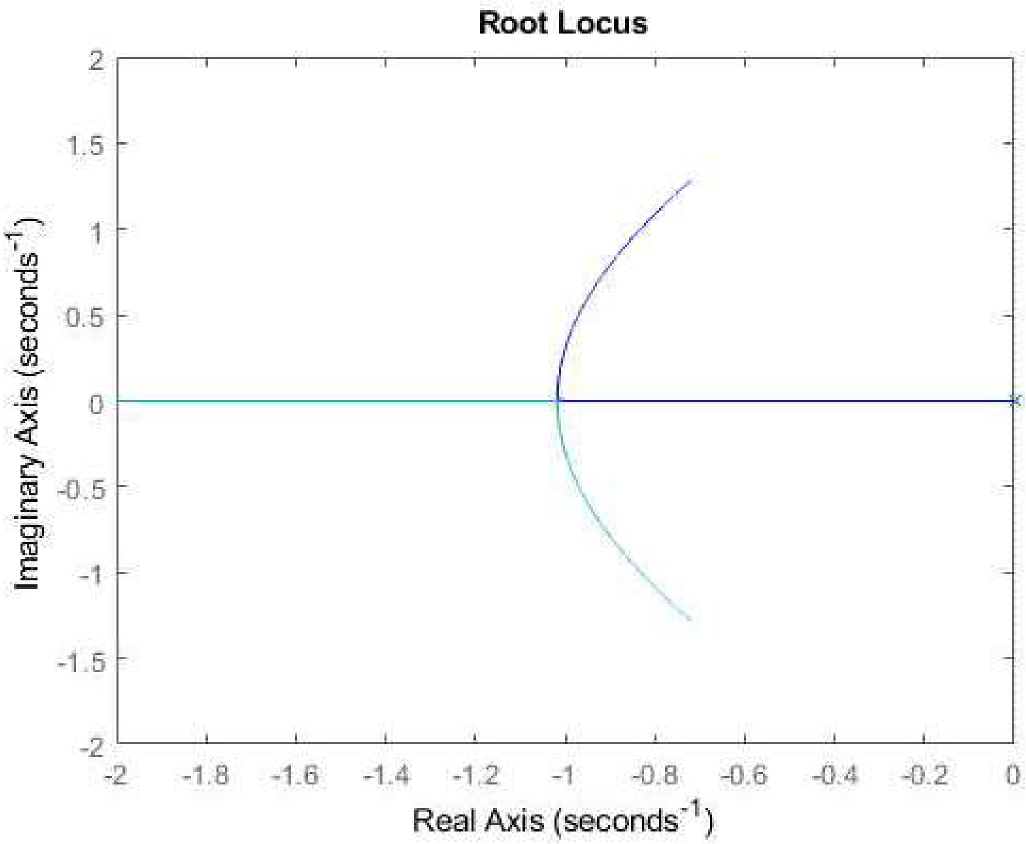
(e) Generate the step response for the closed-loop system at the gain selected for a 10% overshoot. (**Hint:** `step`)

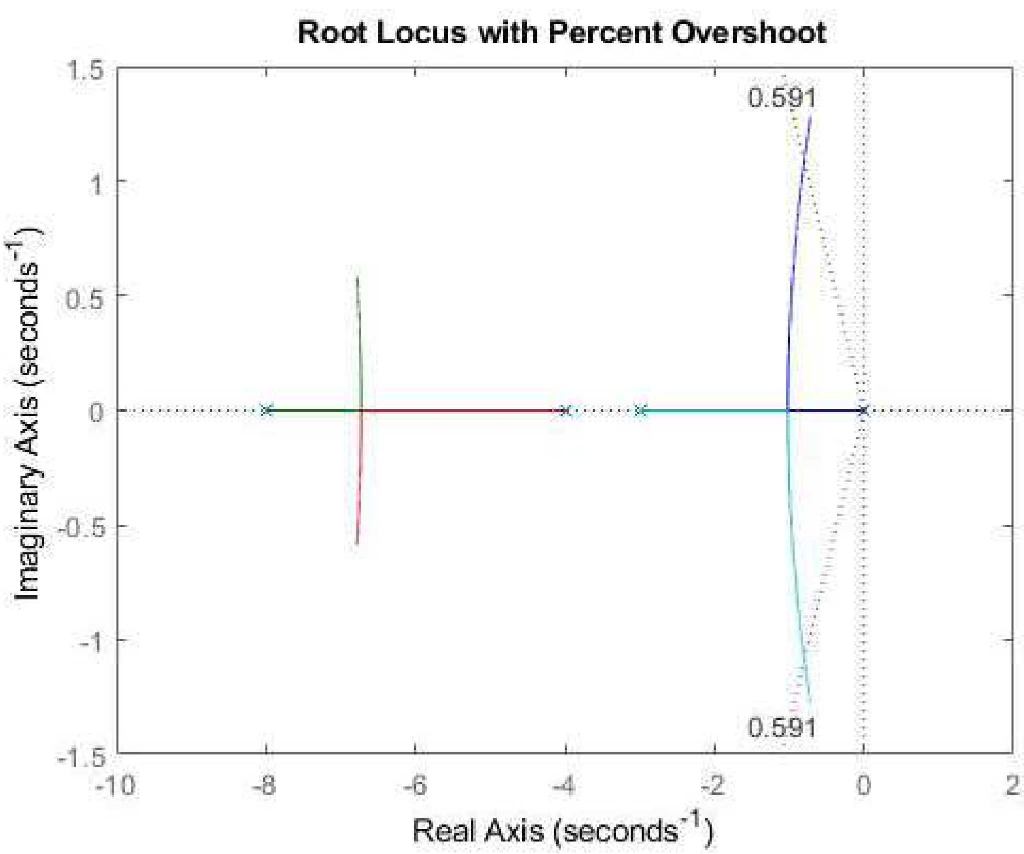
Include the script (m-file) and resulting figures for each part in your solution.

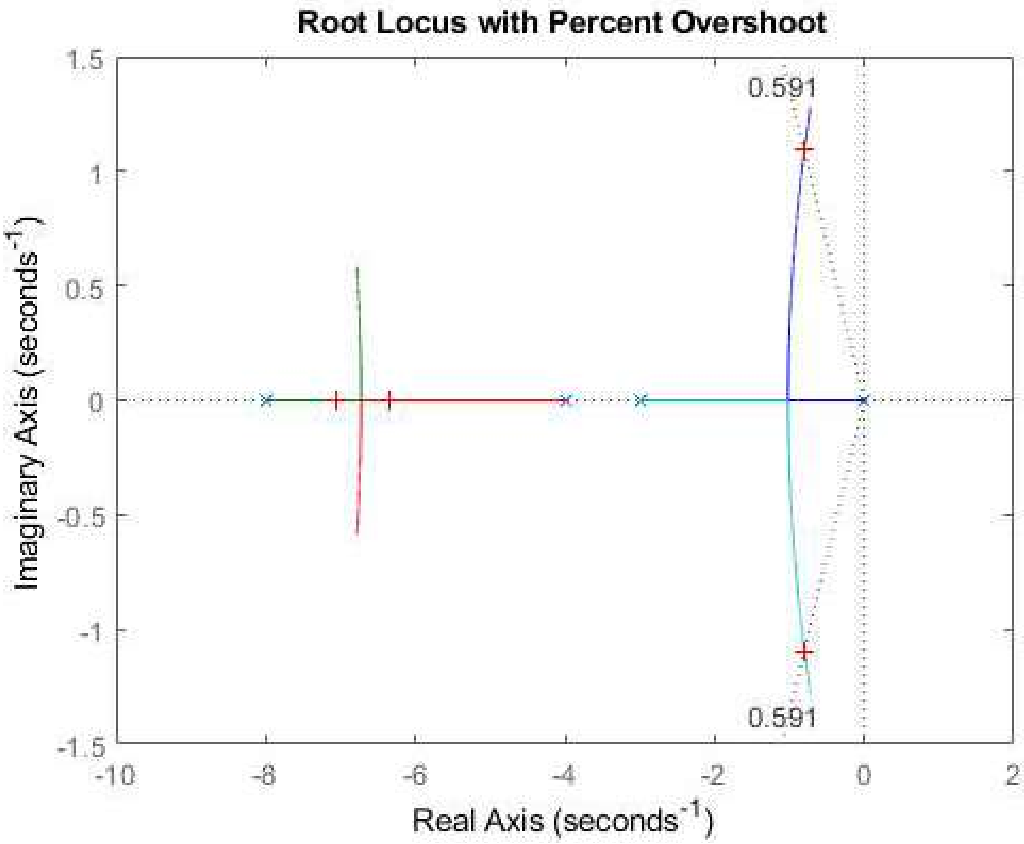
```
%(a)
G = tf([1],poly([0 -3 -4 -8]));
%Gain vector
K =[0:0.01:100];
figure(1)
rlocus(G,K)
```

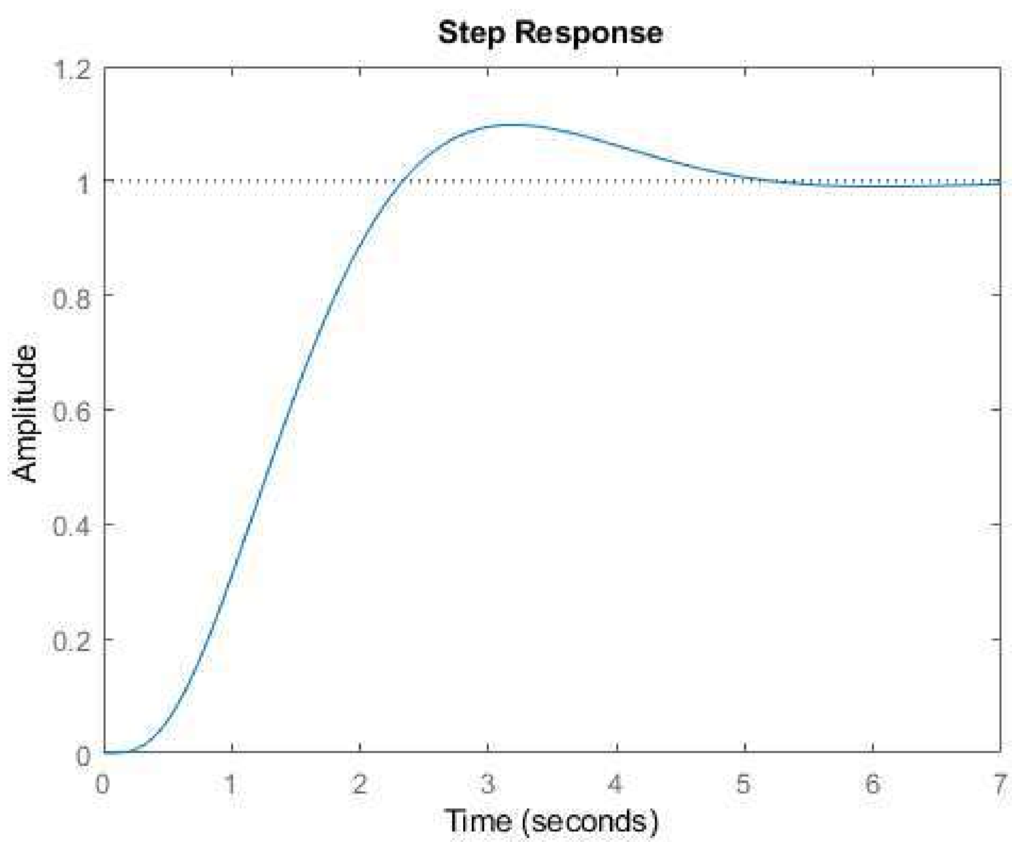
```
%(b)
G = tf([1],poly([0 -3 -4 -8]));
%Gain vector
K =[0:0.01:100];
figure(2)
rlocus(G,K)
axis([-2 0 -2 2])
%(c)
%OS=.1
figure(3)
rlocus(G,K)
z = -log(.1)/sqrt(pi^2+log(0.1)^2) ;
sgrid(z,0)
title ('Root Locus with Percent Overshoot')
figure(4)
rlocus(G,K)
z = -log(.1)/sqrt(pi^2+log(0.1)^2) ;
sgrid(z,0)
title ('Root Locus with Percent Overshoot')
%(d)
[K,p]= rlocfind(G)
%(e)
figure(5)
T = feedback (K*G,1);
step(T);
```

```
Select a point in the graphics window

selected_point =

  -0.8152 + 1.1064i


K =

   82.4893


p =

  -7.0586 + 0.0000i
  -6.3502 + 0.0000i
  -0.7956 + 1.0988i
  -0.7956 - 1.0988i
```


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1.8
-1.6
-1.4
-1.2
-0.8
-0.6
-0.4
-0.2


Root Locus with Percent Overshoot
0.591
0.591
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
1.5
1
0.5
0
-0.5
-1
-1.5
-10
-8
-6
-4
-2
2


Root Locus with Percent Overshoot
0.591
0.591
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
1.5
1
0.5
0
-0.5
-1
-1.5
-10
-8
-6
-4
-2
0
2


Step Response
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7