HW4

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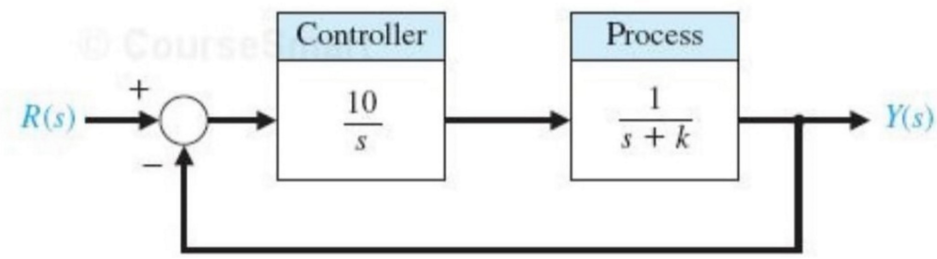
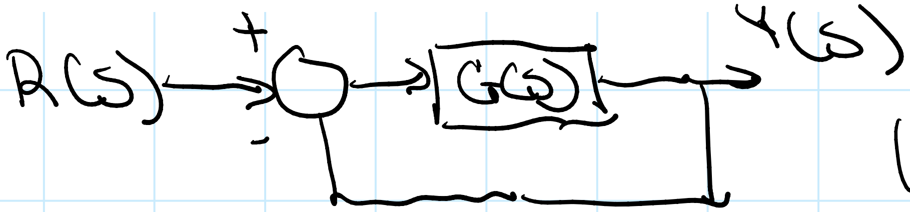
ENME462
Section 0105

1. Consider the closed-loop system shown below.



1) Determine the value(s) of k such that the percent overshoot due to a unit step input is between 1% and 10%.
2) Determine the steady state value of y(t). Does the steady state value depend on k? Justify your answer.



$$G(s) = \frac{10}{s(s+k)}$$

Unity Feedback

1) $Y(s) = G(s)R(s)$

$= G(s)(R(s) - Y(s))$

$= G(s)R(s) - G(s)Y(s)$

$Y(s) + G(s)Y(s) = G(s)R(s)$

$Y(s)(1+G(s)) = G(s)R(s)$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$ ← Form

$$\frac{Y(s)}{R(s)} = \frac{\frac{10}{s(s+k)}}{\quad}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{10}{s(s+k)}}{1+\frac{10}{s(s+k)}}$$

$$= \frac{10}{s(s+k)} \cdot \frac{1}{\left(1+\frac{10}{s(s+k)}\right)}$$

$$= \frac{10}{s(s+k)+10}$$

$$Y(s) = \frac{1}{s} \cdot \frac{10}{s(s+k)+10}$$

$$= \frac{10}{s(s^2+ks+10)} = \frac{\omega_n^2 k}{(s^2+2\zeta\omega_n s+\omega_n^2}$$

$\omega_n^2 = 10 \quad \omega_n = \sqrt{10} = 3.16$ rad/s

$0.01 < \%OS < 0.1$

$$\zeta = \frac{-\ln(\%OS)}{\sqrt{\pi^2+\ln(\%OS)^2}}$$

$$\zeta < \frac{-\ln(0.01)}{\sqrt{\pi^2+\ln(0.01)^2}} = 0.826$$

$$\zeta > \frac{-\ln(0.1)}{\sqrt{\pi^2+\ln(0.1)^2}} = 0.591$$

$$\sqrt{\pi^2+\ln(0.1)^2}$$

$0.591 < \zeta < 0.826$

$K = 2\zeta\omega_n$

$K < 2(0.826)(3.16) = 5.22$

$K > 2(0.591)(3.16) = 3.73$

$\boxed{3.74 < K < 5.22}$

2)

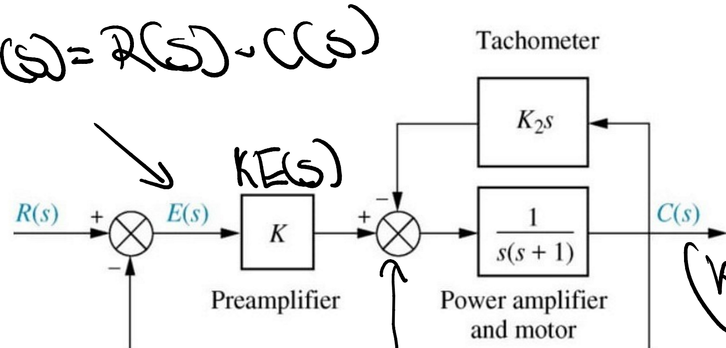
$$y(\infty) = \lim_{s\to 0} sY(s) = \cancel{s}\cdot\frac{1}{\cancel{s}}\cdot\frac{10}{s(s+K)+10}$$

$= 1$

∴ The steady state value does not depend on K

2. Design K_1 and K_2 in the following block diagram to yield a damping ratio of 0.69. The natural frequency of the system before the addition of the tachometer is 10 rad/s.

$E(s) = R(s) - C(s)$



$KE(s)$

$C(s) = \left(KE(s) - K_2 s\, C(s)\right)\left(\frac{1}{s(s+1)}\right)$

$KE(s) - K_2 s\, C(s)$

$KE(s) - K_2 s C(s)$

$$C(s) = \left[K(R(s) - C(s)) - K_2 s C(s)\right]\left(\frac{1}{s(s+1)}\right)$$

$$= \frac{KR(s)}{s(s+1)} - \frac{KC(s)}{s(s+1)} - \frac{K_2 s\, C(s)}{s(s+1)}$$

$$C(s) + \frac{KC(s)}{s(s+1)} + \frac{K_2 s}{s(s+1)} C(s) = \frac{KR(s)}{s(s+1)}$$

$$C(s)\left(1 + \frac{K}{s(s+1)} + \frac{K_2 s}{s(s+1)}\right) = R(s)\left(\frac{K}{s(s+1)}\right)$$

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+1)}}{1 + \frac{K}{s(s+1)} + \frac{K_2 s}{s(s+1)}}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+1)} \cdot \frac{1}{1 + \frac{K}{s(s+1)} + \frac{K_2 s}{s(s+1)}}$$

$$= \frac{K}{s(s+1) + K_2 s + K}$$

Before Tachometer, $\omega_n = 10 \frac{rad}{s}$, $K_2 = 0$

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+1)+K} = \frac{K}{s^2+s+K} = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$K=\omega_n^2=10^2=\boxed{100}$$

$$\frac{C(s)}{R(s)}=\frac{K}{s(s+1)+K_2 s+K}=\frac{K}{s^2+(K_2+1)s+K}$$

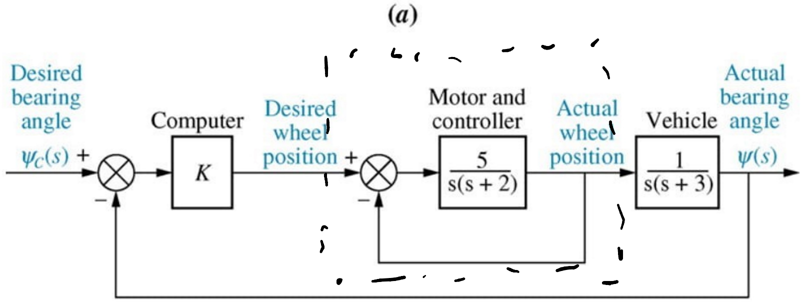
$$=\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\qquad K_2+1=2\zeta\omega_n$$

$$K_2=2\zeta\omega_n-1$$

$$K_2=2(0.69)(10)-1=\boxed{12.8}$$

3. Find the closed-loop transfer function of the HelpMate transport robot shown below, by reducing its block diagram.

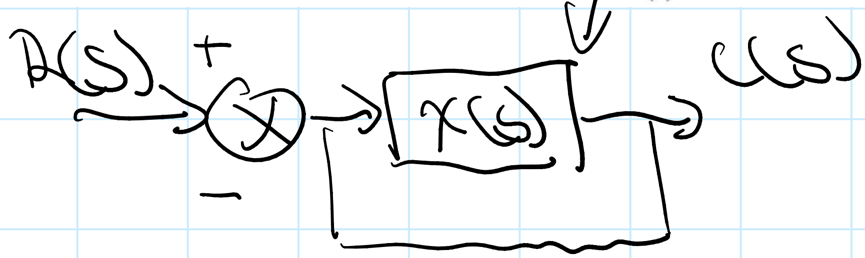
(a)



(b)

R(s) + C(s)

(b)



$$X(s) = \frac{5}{s(s+2)}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{X(s)}{1+X(s)} = \frac{5}{s(s+2)+5}$$

$$G(s) = K(T(s))\left(\frac{1}{s(s+3)}\right)$$

$$\frac{\psi(s)}{\psi_c(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{K\left(\frac{5}{s(s+2)+5}\right)\left(\frac{1}{s(s+3)}\right)}{1+K\left(\frac{5}{s(s+2)+5}\right)\left(\frac{1}{s(s+3)}\right)}$$

$$= \frac{5K}{\quad} \cdot \frac{1}{\quad}$$

$$(s(s+2)+5)(s(s+3)) \quad 1+\frac{5K}{(s(s+2)+5)(s(s+3))}$$

$$\boxed{= \frac{5K}{s(s+3)(s^2+2s+5)+5K}}$$