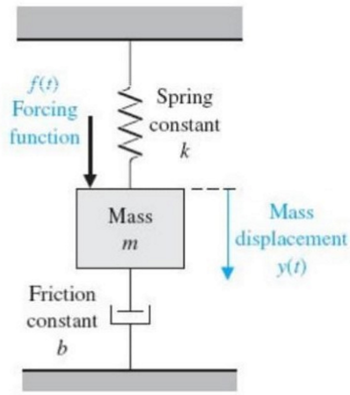
HW3

Gino Rospigliosi
UID: 115443037

ENME462
Section 0105

1. Consider the mechanical system depicted below. The input is given by *f(t)*, and the output is *y(t)*.

(1) Let m = 10, k=1, and b=0.5. Derive the transfer function from *f(t)* to *y(t)*. Plot the system response to a unit step input. Show that the peak amplitude of the output is about 1.8.
(2) Determine the damping ratio and the natural frequency for the system.
(3) Is this system over-damped, critically damped or underdamped? Justify your answer.
(4) For a Mass m=10, choose values for b and k to yield the same natural frequency as original system but is critically damped.



(1) $L = T - U$

$$= \frac{1}{2} m \dot{y}^2 - \frac{1}{2} k y^2 \qquad D = \frac{1}{2} b \dot{y}^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = F \cdot \frac{\partial r}{\partial y} - \frac{\partial D}{\partial \dot{y}}$$

$$\frac{d}{dt}(m\dot{y}) + ky = F - b\dot{y}$$

$$m\ddot{y} + b\dot{y} + ky = F$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F(t)$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F(t)$$

$$Y(s)(ms^2 + bs + K) = F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}} = \frac{\frac{1}{10}}{s^2 + \frac{0.5}{10}s + \frac{1}{10}}$$

$$= \boxed{\frac{0.1}{s^2 + 0.05s + 0.1}}$$

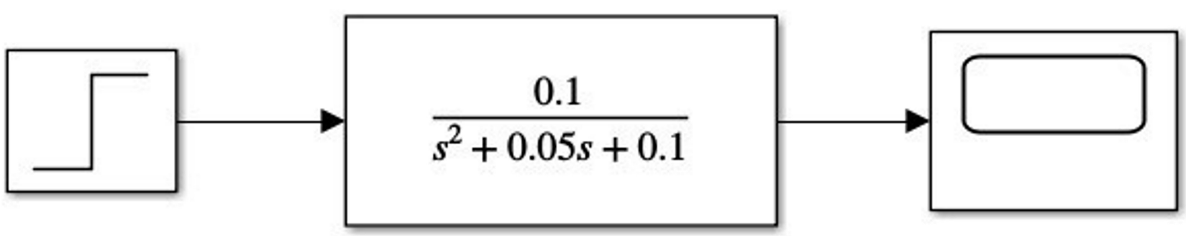
$F(t) = u(t)$

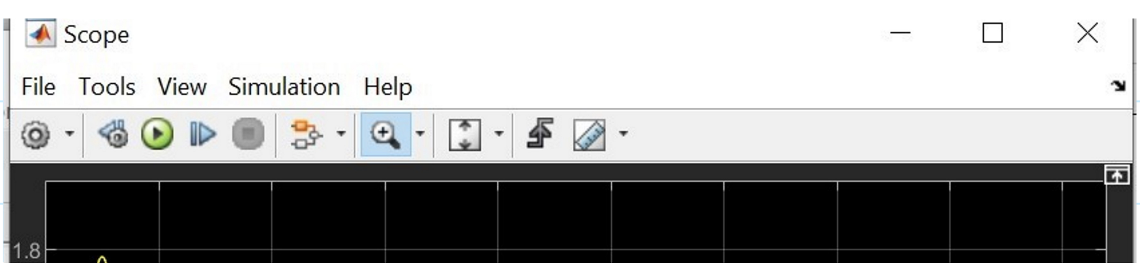
$F(s) = \frac{1}{s}$

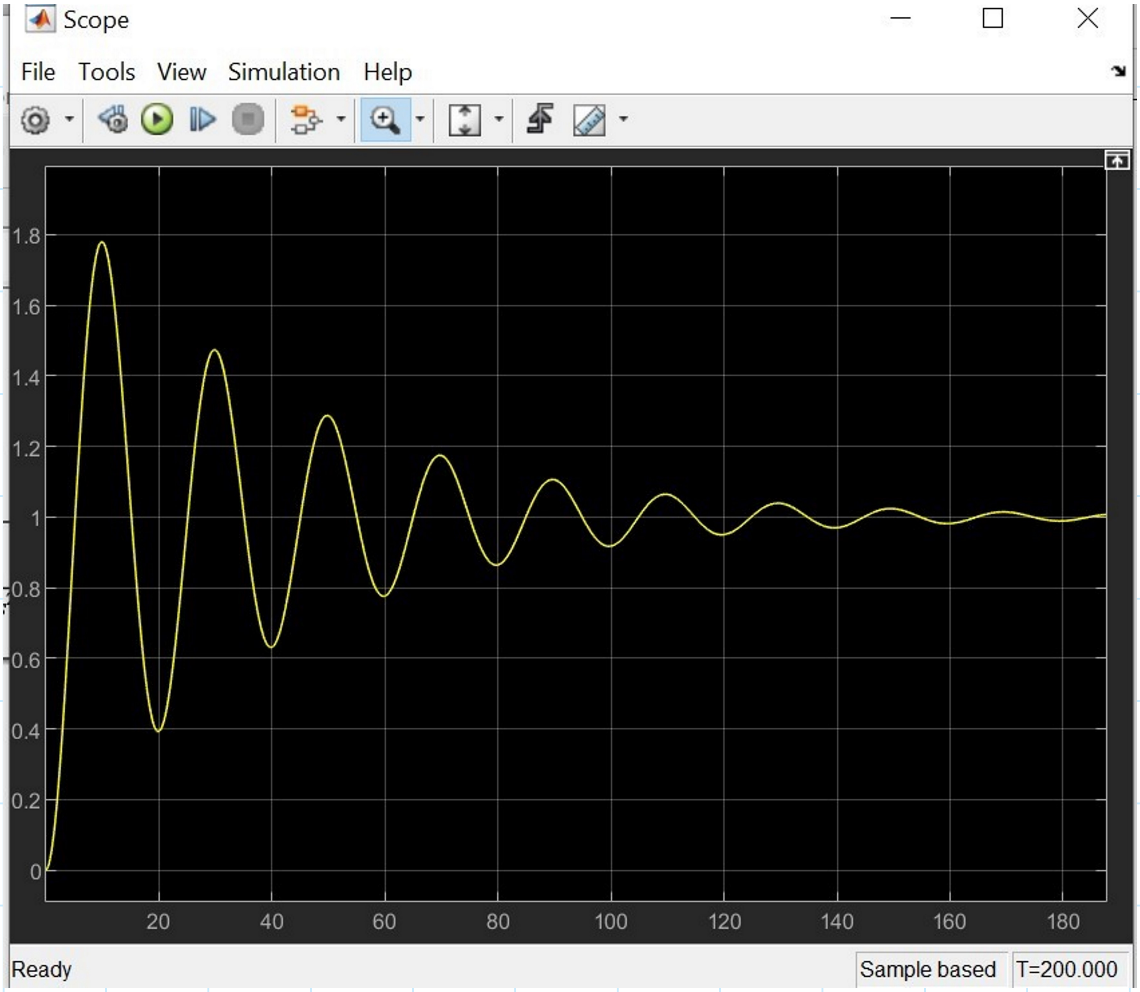
$$Y(s) = F(s)\,G(s)$$

$$= \frac{1}{s}\,\frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}}$$

$$Y(t) = \mathcal{L}^{-1}\left(\frac{1}{s}\,\frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}}\right)$$







(2) $\dfrac{Y(s)}{F(s)} = \dfrac{0.1}{s^2 + 0.05s + 0.1}$

$= \dfrac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\omega_n = \sqrt{k/m} = \sqrt{0.1} = \boxed{0.316 \text{ rad/s}}$

$\zeta = \dfrac{b}{2\sqrt{km}} = \dfrac{\frac{b}{m}}{2\sqrt{k/m}} = \dfrac{0.05}{2\sqrt{0.1}} = \boxed{0.79}$

(3) Underdamped since $\zeta < 1$

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(4) $m = 10$ $\quad \omega_n = 0.316$ rad/s

$\sqrt{1/10} = \sqrt{k/m}$ $\quad k = 1$ N/m

$\zeta = 1 = \dfrac{b}{2\sqrt{km}}$

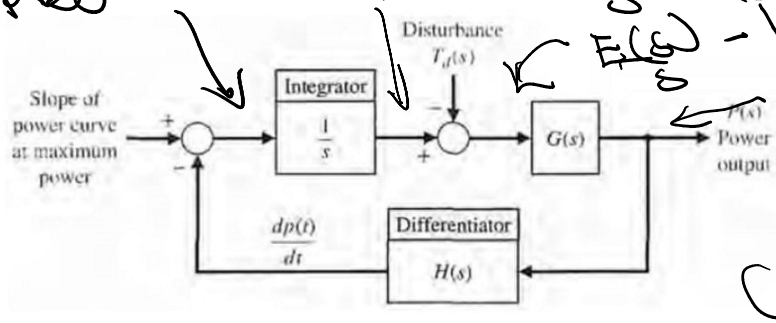
$b = 2\sqrt{km} = 2\sqrt{10} = \boxed{6.32 \text{ Ns/m}}$

2. Solar cells generate a DC voltage that can be used to drive DC motors or that can be converted to AC power and added to the distribution network. It is desirable to maintain the power out of the array at its maximum available as the solar incidence changes during the day. One such closed-loop system is shown below. The transfer function for the process is:

$E(s) = R(s) - P(s)\cdot H(s)$ $\quad G(s) = \dfrac{20}{s+20}$ $\quad \dfrac{E(s)}{s}$

$\dfrac{E(s)}{s} - T_d(s)$

$R(s)$



$P(s) = G(s)\left(\dfrac{E(s)}{s} - T_d(s)\right)$

$= s$

(a) Find the time constant of the closed-loop system.

(b) Find the settling time to within 2% of the final value of the system to a unit step disturbance.

$P(s) = G(s)\left(\dfrac{E(s)}{s} - T_d(s)\right)$

$= G(s)\left(\dfrac{R(s)}{s} - \dfrac{P(s)H(s)}{s} - \cancel{T_d(s)}\right)$

$= \left(\dfrac{G(s)}{s}R(s) - \dfrac{G(s)}{s}P(s)H(s)\right)$

$$P(s) + P(s)\frac{G(s)}{s}s = R(s)\frac{G(s)}{s}$$

$$P(s)\left(1 + \frac{G(s)}{s}\cdot s\right) = R(s)\left(\frac{G(s)}{s}\right)$$

$$\frac{P(s)}{R(s)} = \frac{G(s)/s}{1 + \frac{G(s)}{s}\cdot s} = \frac{1}{s}\cdot\frac{G(s)}{1+G(s)}$$

$$= \frac{1}{s}\cdot\frac{\frac{20}{s+20}}{1+\frac{20}{s+20}} = \frac{1}{s}\cdot\frac{20}{s+20}\cdot\frac{1}{1+\frac{20}{s+20}}$$

$$= \frac{1}{s}\cdot\frac{20}{(s+20)\left(1+\frac{20}{s+20}\right)}$$

$$= \frac{1}{s}\cdot\frac{20}{\left(s+20+\frac{20\cancel{(s+20)}}{\cancel{(s+20)}}\right)} = \frac{20}{s(s+40)}$$

$$= \frac{A}{s} + \frac{B}{s+40}$$

$$20 = A(s+40) + B(s)$$

$$20 = As + 40A + Bs$$

$$0 = A + B \qquad A = -B \qquad B = -1$$

$20 = A(s+40) + Bs$

$0 = A + B \qquad A = -B \qquad B = -\frac{1}{2}$

$20 = 40A \qquad A = \frac{1}{2}$

$$= \frac{1}{2}\left(\frac{1}{s}\right) + \frac{1}{2}\left(\frac{1}{s+40}\right)$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s+40}\right)$$

$$P(t) = \frac{1}{2} - \frac{1}{2}e^{-40t} = \frac{1}{2}\left(1 - e^{-40t}\right)$$

$$\tau = \frac{1}{a} = \boxed{\frac{1}{40}\ s} \qquad \frac{K}{\tau s + 1} = \frac{\frac{1}{2}}{\frac{1}{40}s + 1}$$

$$T_s = \frac{4}{a} = 0.1\ s$$

(b)

$T_d(s) \rightarrow \boxed{G(s)} \rightarrow P(s)$ $\qquad T(s) = 1/s$

Open-Loop

$$P(s) = T_d(s)\,G(s)$$

$$= \frac{1}{s}\left(\frac{20}{s+40}\right)$$

$$= \frac{1}{s}\left(\frac{20}{s+20}\right)$$

<u>or</u>

$$P(s) = G(s)\left(E(s) - T_d(s)\right)$$

$$= G(s)\left(\frac{R(s)}{s} - \frac{P(s)s}{s} - \frac{1}{s}\right)$$

$$= \left(\frac{G(s)}{s}\cancel{R(s)} - \frac{G(s)}{\cancel{s}}P(s)\,\cancel{s} - G(s)T_d(s)\right)$$

$$\frac{P(s)}{T_d(s)} = -G(s)$$

$$P(s) = \frac{1}{s}\left(-G(s)\right) = \frac{1}{s}\left(\frac{-20}{s+20}\right)$$

$$= \frac{1}{s}\left(\frac{-1}{\frac{1}{20}s+1}\right) \qquad \frac{K}{\tau s+1}$$

$$\tau = \frac{1}{a} = \frac{1}{20}$$

$$T_s = \frac{4}{a} = \frac{4}{20} = \boxed{0.2\ s}$$

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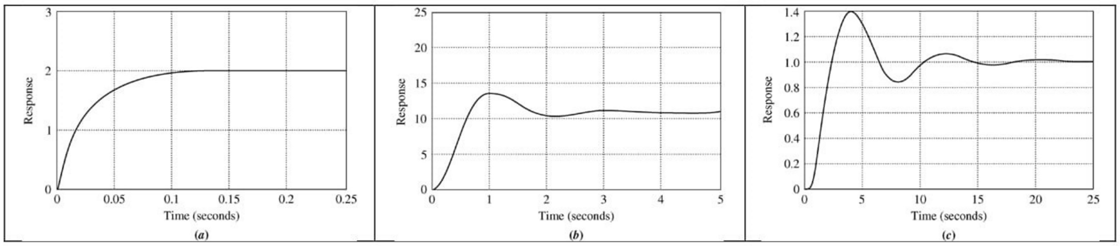
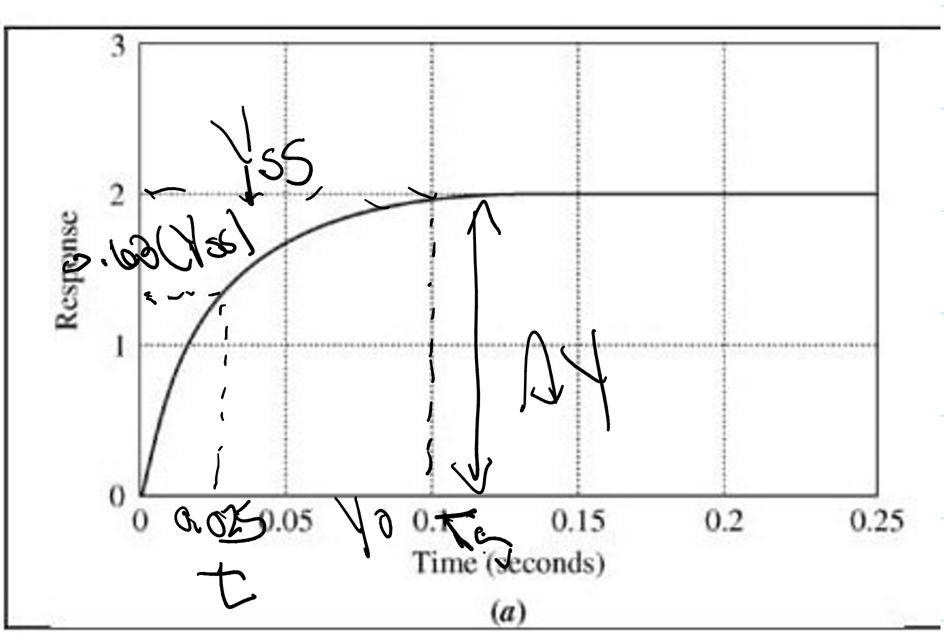
3. For each of the unit step responses shown below, find the transfer function of the system.





First-Order

a) $K = \frac{\Delta Y}{\Delta U} = \frac{2}{1}$

$Y(\tau) = 0.6325(2) = 1.265$

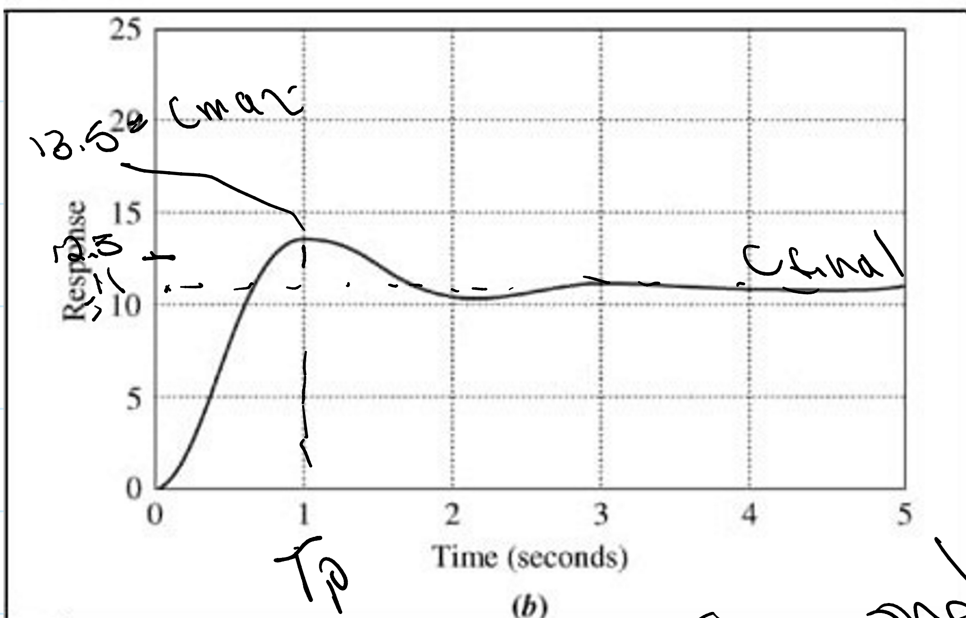
$T_s = \frac{4}{a} = 0.1 \quad a = \frac{4}{0.1}$

$\tau = \frac{1}{\ } = \frac{1}{\ } = 0.025\,s$

$$\tau = \frac{1}{a} = \frac{1}{40} = 0.025\,s$$

$$\boxed{T(s) = \frac{2}{0.025s+1}}$$

b)



Second Order

$$\%OS = \frac{C_{max} - C_{final}}{C_{final}} = \frac{13.5 - 11}{11}$$

$$= 0.227 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$-1.48 = -\zeta\pi(1-\zeta^2)^{-1/2}$$

$$0 = -\zeta\pi(1-\zeta^2)^{-1/2} + 1.48$$

Solve Computationally

Solve Computationally

$\zeta = 0.427$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1$$

$$\omega_n = \frac{\pi}{\sqrt{1-\zeta^2}} = \frac{\pi}{\sqrt{1-0.427^2}} = 3.47$$

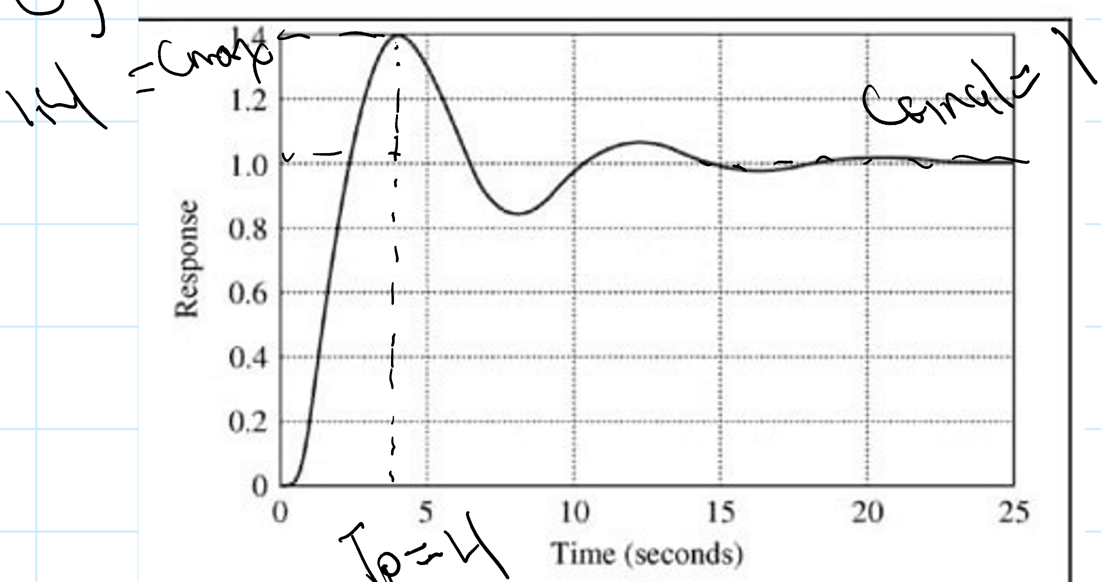
$$T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{3.47^2 K}{s^2+2(0.427)(3.47s)+3.47^2}$$

$$= \boxed{\frac{132.77}{s^2+2.97s+12.07}}$$

$K = 11$

c)



(c)

Second Order

$T_p = 4$ Time (seconds)

(c)

$$\%OS = \frac{C_{max} - C_{final}}{C_{final}} = \frac{1.4 - 1}{1}$$

$$= 0.4 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$-0.916 = -\zeta\pi(1-\zeta^2)^{-1/2}$$

$$0 = -\zeta\pi(1-\zeta^2)^{-1/2} + 0.916$$

Solve computationally

$$\zeta = 0.28$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 4$$

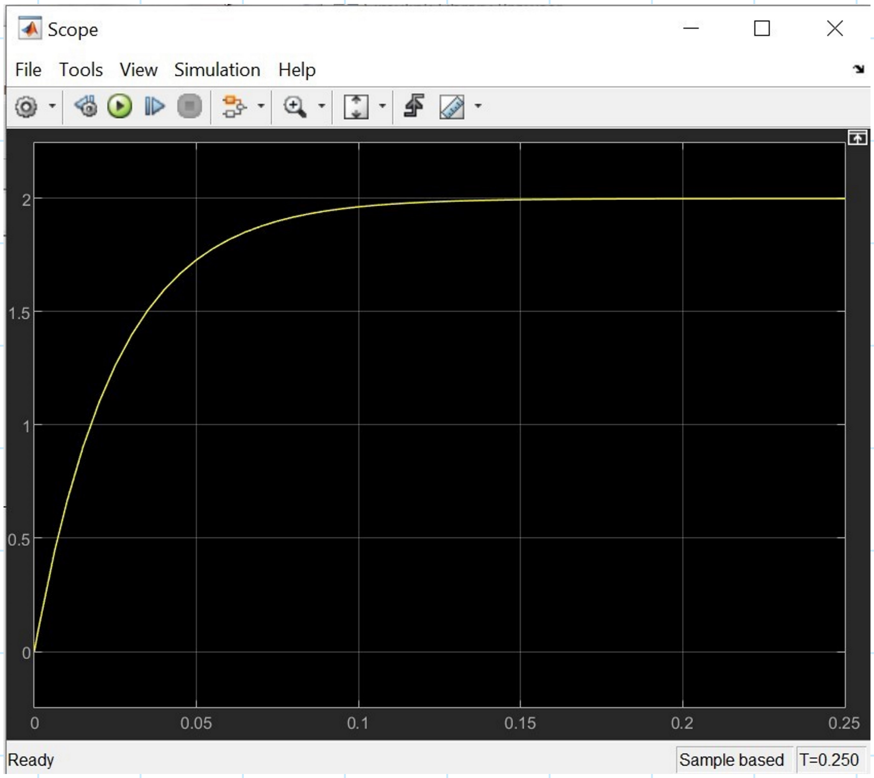
$$\omega_n = \frac{\pi}{\sqrt{1-\zeta^2}} = \frac{\pi}{4\sqrt{1-0.28^2}} = 0.818$$

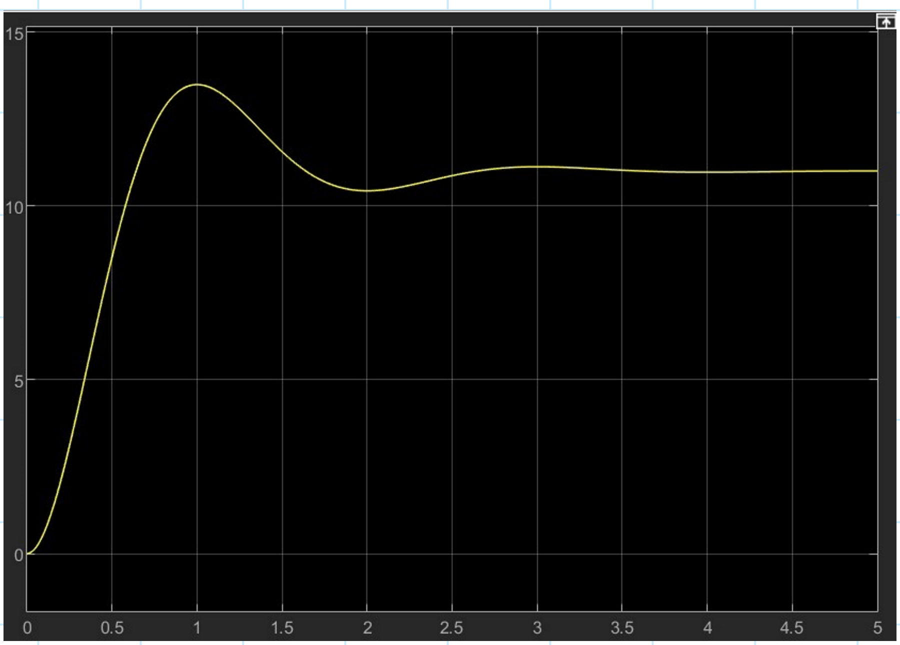
$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

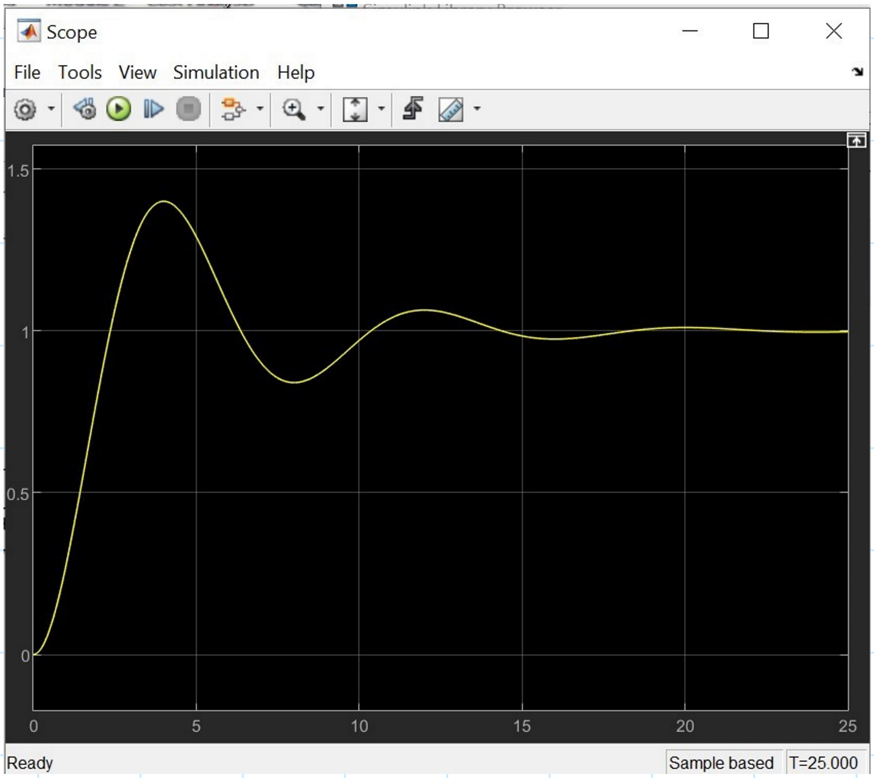
$$= \frac{0.818^2 \, K}{s^2 + 2(0.28)(0.818)s + 0.818^2} \qquad K = 1$$

$$= \boxed{\frac{0.669}{s^2 + 0.458 s + 0.669}}$$

$$= \frac{\quad}{s^2 + 0.4583s + 0.6664}$$







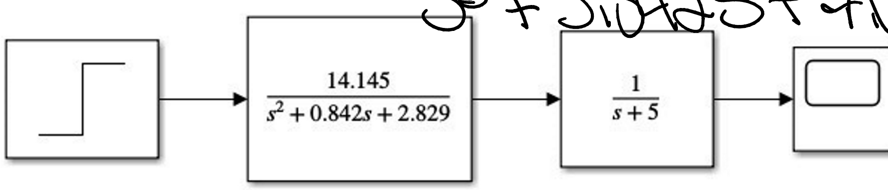
4. For the transfer function given by

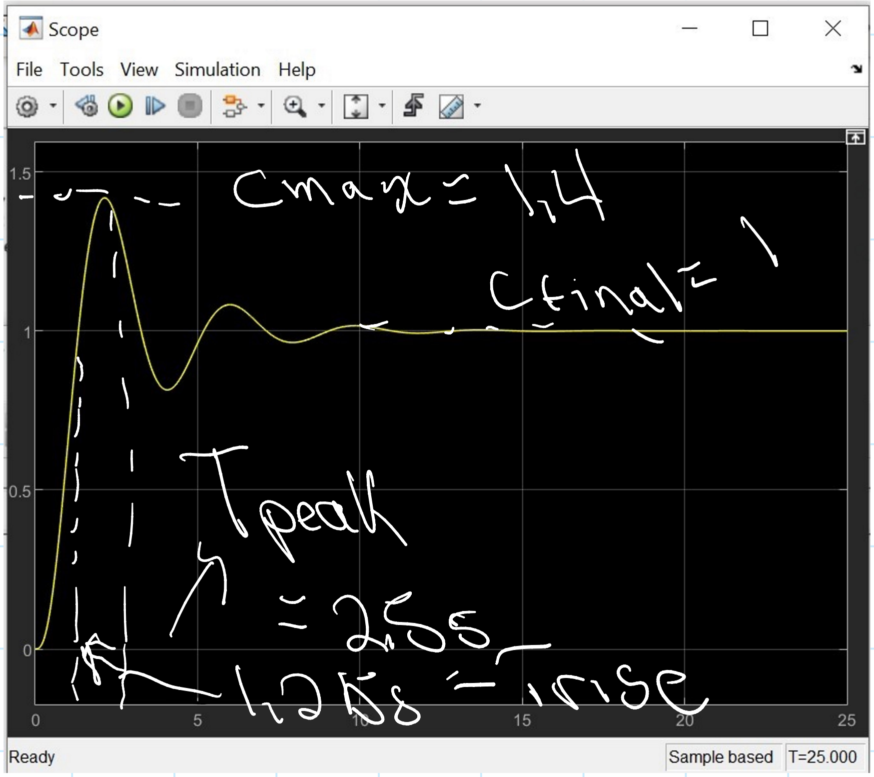
$$T(s) = \frac{14.145}{(s^2 + 0.842s + 2.829)(s + 5)}$$

(a) Find the percent overshoot, settling time, rise time and peak time.

(b) Verify the answers you obtained in part (a) numerically using Matlab (Hint: Use STEP command to simulate the step response of T(s), and then extract the specifications using STEPINFO command).

$$= \frac{14.145}{s^3 + 5.842s^2 + 7.039s + 14.145}$$





(a) $\%OS = \dfrac{C_{max} - C_{final}}{C_{final}} = \boxed{40\%}$

$T_{peak} = \boxed{2.1\ s}$

$T_{rise} \approx \boxed{0.8\ s}$

Graphical Approximations

(b)

Editor - C:\Users\GRosp\Desktop\Senior S2\ENME462\HW3.m

HW3.m

```
sys = tf([14.145],[1 5.842 7.039 14.145]);
stepinfo(sys)
```

Editor - C:\Users\GRosp\Desktop\Senior S2\ENME462\HW3.m

HW3.m

```
1 -   sys = tf([14.145],[1 5.842 7.039 14.145]);
2 -   stepinfo(sys)
```

Command Window

```
  struct with fields:

        RiseTime: 0.8099
    SettlingTime: 8.5743
     SettlingMin: 0.8144
     SettlingMax: 1.4181
       Overshoot: 41.8056
      Undershoot: 0
            Peak: 1.4181
        PeakTime: 2.1368

fx >>
```