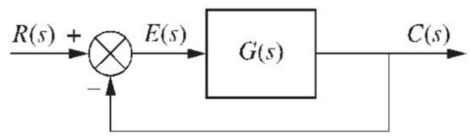
HW5

Gino Rospigliosi
UID: 115443037
ENME462
Section 0105

1. For the unity feedback system shown below where $G(s) = \frac{500}{(s+28)(s^2+8s+12)}$, find the steady-state error for inputs of 20u(t), 60tu(t), and 81t²u(t).



$$E(s) = \frac{R(s)}{1+G(s)}$$

$$R(s)_1 = 20u(t) = \frac{20}{s}$$

$$e(\infty) = \lim_{s\to0} \frac{s\left(\frac{20}{s}\right)}{1+\frac{500}{(s+28)(s^2+8s+12)}}$$

$$= \lim_{s\to0} \frac{20}{1+\frac{500}{(s+28)(s^2+8s+12)}} = \lim_{s\to0} \frac{20}{1+\frac{500}{28(12)}} = \frac{20}{\left(\frac{209}{84}\right)}$$

$$= \frac{1680}{209} = \boxed{8.0386}$$

$$R(s)_2 = 60tu(t) = \frac{60}{s(s)} = \frac{60}{s^2}$$

$$R(s)_2 = 60t\,u(t) = \frac{60}{s(s)} = \frac{60}{s^2}$$

$$e(\infty) = \lim_{s \to 0} \frac{s\left(\frac{60}{s^2}\right)}{1 + \frac{500}{(s+25)(s^2+8s+12)}}$$

$$= \lim_{s \to 0} \frac{60}{s\left(1 + \frac{500}{(s+25)(s^2+8s+12)}\right)}$$

$$= \lim_{s \to 0} \frac{60}{s + \frac{500s}{(s+25)(s^2+8s+12)}} = \frac{60}{0} = \boxed{\infty}$$

$$R(s)_3 = 81t^2 u(t) = \frac{81}{(s^2)s} = \frac{81}{s^3}$$

$$e(\infty) = \lim_{s \to 0} \frac{s\left(\frac{81}{s^3}\right)}{1 + \frac{500}{(s+25)(s^2+8s+12)}}$$

$$= \lim \frac{81}{}$$

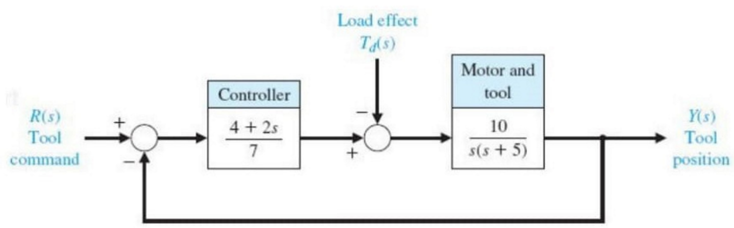
$$s \to 0 \quad s^2\left(1 + \frac{500}{(s+25)(s^2+8s+12)}\right)$$

$$= \lim_{s \to 0} \frac{81}{s^2 + \frac{500 s^2}{(s+25)(s^2+8s+12)}} = \frac{81}{0} = \boxed{\infty}$$

2. A machine tool is designed to follow a desired path so that

$$r(t) = (1-t)u(t)$$

where u(t) is the unit step function. The machine tool control system is shown below.



1) Determine the steady-state error if r(t) is the desired path as given and $T_d(s)=0$.

2) Plot the error for Part 1) for $0 \le t \le 10$ s using MATLAB/SIMULINK and verify your answer to 1).

3) If r(t)=0, find the steady-state error if $T_d(s)$ is a unit step input.

4) Plot the error for Part 3) for $0 \le t \le 10$ s using MATLAB/SIMULINK and verify your answer to 3).

1) $r(t) = (1-t)u(t) = \left(1 - \frac{1}{s}\right)\frac{1}{s}$

$$= \frac{1}{s} - \frac{1}{s^2}$$

When $T_d(s) = 0$

$$G(s) = \left(\frac{4+2s}{7}\right)\left(\frac{10}{s(s+5)}\right)$$

$$= \frac{10(4+2s)}{7s(s+5)}$$

$$E(s) = \frac{R(s)}{1+G(s)}$$

$$e(\infty) = \lim_{s\to 0} \frac{s\left(\frac{1}{s}-\frac{1}{s^2}\right)}{1+\frac{10(4+2s)}{7s(s+5)}}$$

$$= \lim_{s\to 0} \frac{1-\frac{1}{s}}{1+\frac{10(4+2s)}{7s(s+5)}} = \lim_{s\to 0} \frac{1-\frac{1}{s}}{\frac{7s(s+5)+10(4+2s)}{7s(s+5)}}$$

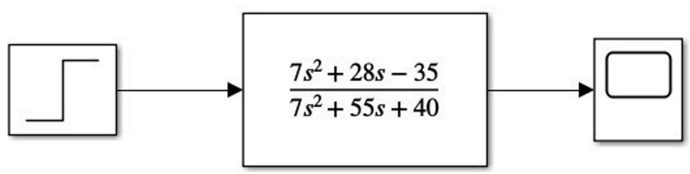
$$= \lim_{s\to 0} \frac{7s(s+5)\left(1-\frac{1}{s}\right)}{7s(s+5)+10(4+2s)} = \lim_{s\to 0} \frac{7(s(s+5)-1(s+5))}{7s(s+5)+10(4+2s)}$$

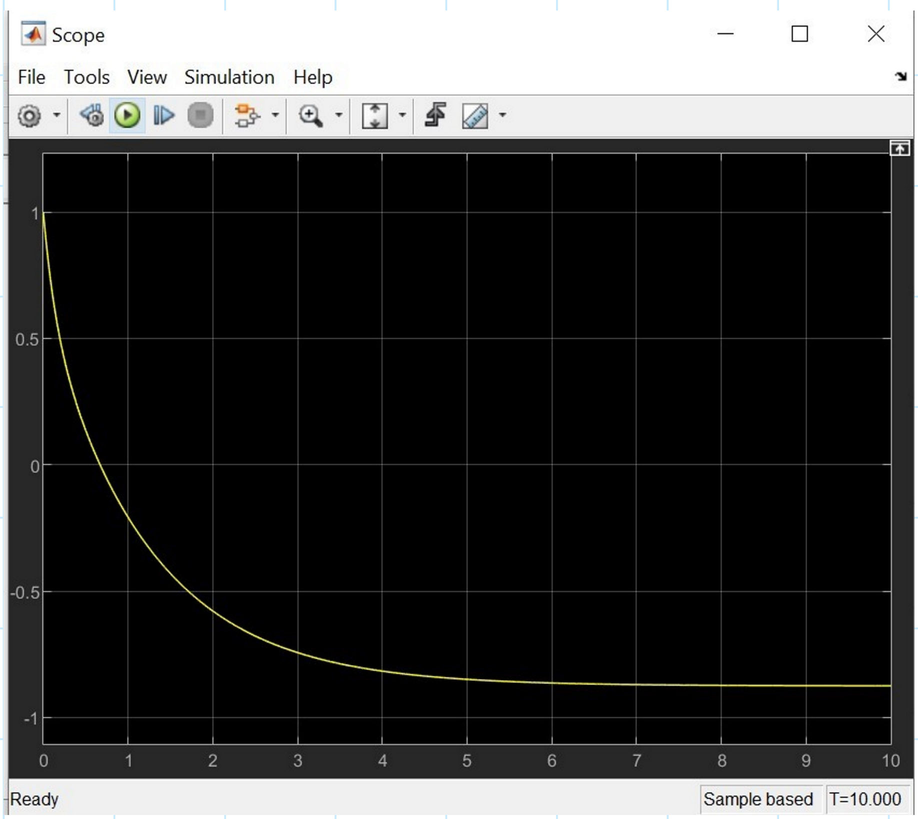
$$= \lim_{s\to 0} \frac{7(s-1)(s+5)}{7s(s+5)+10(4+2s)} = \frac{-35}{40} = \frac{-7}{8}$$

$$= \boxed{-0.875}$$

2) $$E(s) = \frac{\frac{1}{s} - \frac{1}{s^2}}{\frac{7s(s+5)+10(4+2s)}{7s(s+5)}} = \frac{\frac{s-1}{s^2}}{\frac{7s(s+5)+10(4+2s)}{7s(s+5)}}$$

$$= \frac{7(s^2+4s-5)}{s(7s^2+55s+40)} = \frac{7s^2+28s-35}{7s^3+55s^2+40s}$$





3) If $R(s)=0$

$$T_d(s)=\frac{1}{s}$$

$$\frac{Y(s)}{-T_d(s)}=\frac{G(s)}{1+G(s)H(s)}$$

$$=\frac{\frac{10}{s(s+5)}}{1+\left(\frac{10}{s(s+5)}\right)\left(\frac{4+2s}{7}\right)}$$

$$=\frac{70}{7(s+5)+10(4+2s)}$$

$$=\left(\frac{70}{7s^2+55s+40}\right)$$

$$Y(s)=E(s)=\left(\frac{70}{7s^2+55s+40}\right)-T_d(s)$$

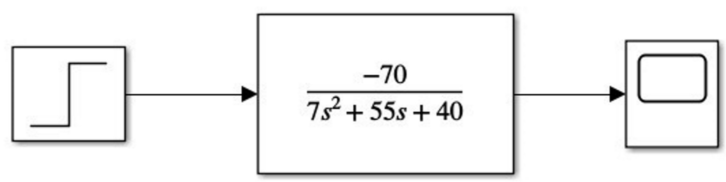
$$=\frac{-1}{s}\left(\frac{70}{7s^2+55s+40}\right)$$

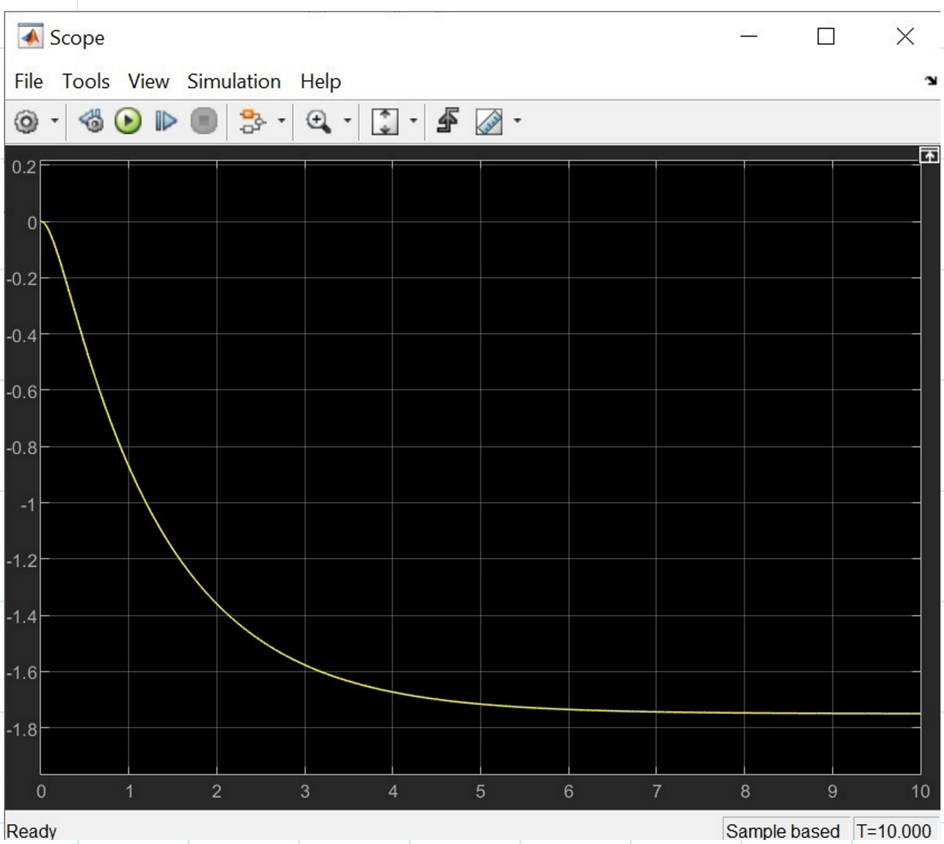
$$e(\infty) = \lim_{s \to 0} sE(s)$$

$$= \frac{-70}{7s^2+55s+40} = \frac{-70}{40}$$

$$= \boxed{-1.75}$$

4)



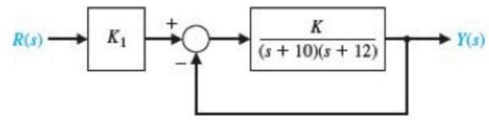


3. Consider the system shown below:

(a) Determine the steady- state value of the error $E(s) = R(s) - Y(s)$ for a unit step input in terms of K and K_1.

(b) Select K_1 (if any) so that the steady-state error is zero.

(c) What is the system type number? Justify your answer.



$$R(s) = K_1 R(s)$$

$$E(s) = \frac{R(s)}{1+G(s)} = \frac{K_1 R(s)}{1+\frac{K}{(s+10)(s+12)}}$$

$$e(\infty) = \lim_{s \to 0} \frac{s K_1 \left(\frac{1}{s}\right)}{1+\frac{K}{(s+10)(s+12)}}$$

$$= \frac{K_1}{1+\frac{K}{120}} = \frac{K_1}{\frac{120+K}{120}}$$

$$= \boxed{\frac{120 K_1}{120+K}}$$

$$\boxed{120 + K}$$

(b) $0 = \dfrac{120 K_1}{120 + K}$

$K_1 = 0$

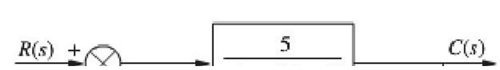
$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{K_1 \dfrac{K}{(s+10)(s+12)}}{1 + \dfrac{K}{(s+10)(s+12)}}$$

$$= \frac{K_1 K}{(s+10)(s+12) + K}$$

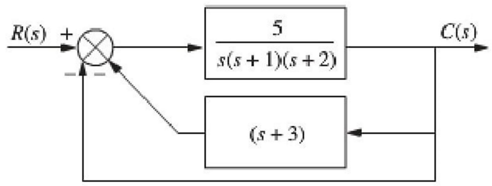
No poles at zero

$\therefore$ $\boxed{\text{System Type} = 0}$

4. For the system shown below:



4. For the system shown below:



(a) Determine and draw an equivalent unity feedback system.

(b) Find K_p, K_v, and K_a (position error constant, velocity error constant and acceleration error constant, respectively).

(c) Find the steady-state error for an input of $50u(t)$, $50tu(t)$, and $50t^2u(t)$.

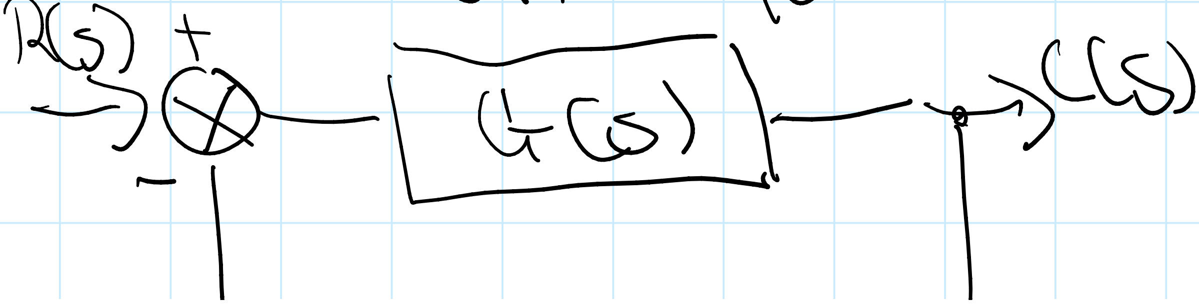
(d) What is the system type number?

a) $$G(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{\frac{5}{s(s+1)(s+2)}}{1 + \frac{5}{s(s+1)(s+2)}(s+3)}$$

$$= \frac{5}{s(s+1)(s+2) + 5(s+3)}$$

Equivalent System



b)

$$T(s) = \frac{G(s)}{1+G(s)}$$

$$= \frac{\dfrac{5}{s(s+1)(s+2)+5(s+3)}}{1+\dfrac{5}{s(s+1)(s+2)+5(s+3)}}$$

$$= \frac{5}{s(s+1)(s+2)+5(s+3)+5}$$

$$K_p = \lim_{s \to 0} T(s)$$

$$= \lim_{s \to 0} \frac{5}{s(s+1)(s+2)+5(s+3)+5}$$

$$= \frac{5}{\,} = \boxed{0.25}$$

20

$K_v = \lim_{s \to 0} sT(s)$

$= \lim_{s \to 0} \dfrac{5s}{s^3+3s^2+2s+5s+20}$

$= \lim_{s \to 0} \dfrac{5s}{s^3+3s^2+7s+20}$

$= \dfrac{0}{20} = \boxed{0}$

$K_a = \lim_{s \to 0} s^2T(s)$

$= \lim_{s \to 0} \dfrac{5s^2}{s^3+3s^2+7s+20}$

$= \dfrac{0}{20} = \boxed{0}$

c) $R(s) = 50u(t) = \dfrac{50}{s}$ step

$$e_{step}(\infty) = 50\left(\frac{1}{1+K_p}\right)$$

$$= \frac{50}{1+0.125} = \boxed{40}$$

$$R(s) = 50t\,u(t) = \frac{50}{s^2} \text{ ramp}$$

$$e_{ramp}(\infty) = 50\left(\frac{1}{K_v}\right) = \frac{50}{0}$$

$$= \boxed{\infty}$$

$$R(s) = 50t^2 u(t) = \frac{50}{s^3} \text{ parabolic}$$

$$e_{parabolic}(\infty) = 50\left(\frac{1}{K_a}\right) = \frac{50}{0}$$

$$= \boxed{\infty}$$

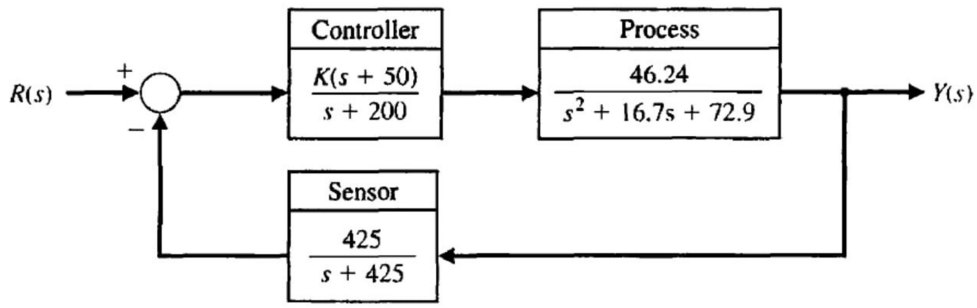
d) $K_p =$ constant

$K_v = 0$

$K_a = 0$

$\therefore$ $\boxed{\text{System Type } 0}$

5. Consider the feedback control system shown below.



(a) Determine the steady-state error for a unit step input in terms of K.

(b) Determine the %OS for the step response for 40 ≤ K ≤ 400.
(For this problem, use MATLAB to simulate the closed-loop system response and find the %OS.)

(c) Plot the %OS and steady-state error with respect to K.

$$G(s) = \frac{K(s+50)(46.24)}{(s+200)(s^2+16.7s+72.9)}$$

$$T(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\dfrac{K(s+50)(46.24)}{(s+200)(s^2+16.7s+72.9)}}{1 + \dfrac{K(s+50)(46.24)}{(s+200)(s^2+16.7s+72.9)(s+425)}(425)}$$

$$\overline{(s+200)(s^2+10.7s+72.9)(s+425)}$$

$$\boxed{= \frac{K(s+50)(46.24)}{(s+200)(s^2+10.7s+72.9) + \frac{425K(s+50)(46.24)}{s+425}}}$$

$$K_p = \lim_{s \to 0} G(s)$$

$$= \frac{K(50)(46.24)}{(200)(72.9) + \frac{425(K)(46.24)}{20}}$$

$$= \boxed{\frac{2312K}{14580 + 2312K}}$$

$$E(s) = \frac{s}{1+G(s)}$$

$$e(\infty) = \lim_{s \to 0} \frac{1}{1+G(s)} = \frac{1}{1+K_p}$$

$$= \frac{1}{\quad} = \frac{1}{\quad}$$

$$= \frac{1}{1+\frac{2312K}{14580+2312K}} = \frac{1}{\frac{14580+4624K}{14580+2312K}}$$

$$= \boxed{\frac{14580+2312K}{14580+4624K}}$$

b) $\%OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$

$T(s) =$

$$= \frac{K(s+50)(46.24)}{(s+200)(s^2+16.7s+72.9) + \frac{425K(s+50)(46.24)}{s+425}}$$

$$= \frac{K(s+50)(46.24)(s+425)}{(s+200)(s^2+16.7s+72.9)(s+425) + 425K(s+50)(46.24)}$$

$$s_{1,2} = \frac{-16.7 \pm \sqrt{16.7^2 - 4(72.9)}}{2}$$

$$2$$

$$= -8.35 \pm 3.56\,i$$

Poles >> $5\sigma_d$

$\therefore$ Approximate by 2nd order system

$$T(s) \approx \frac{46.24K}{s^2 + 16.7s + (72.9 + 19652K)}$$

$$\omega_n = \sqrt{72.9 + 19652K}$$

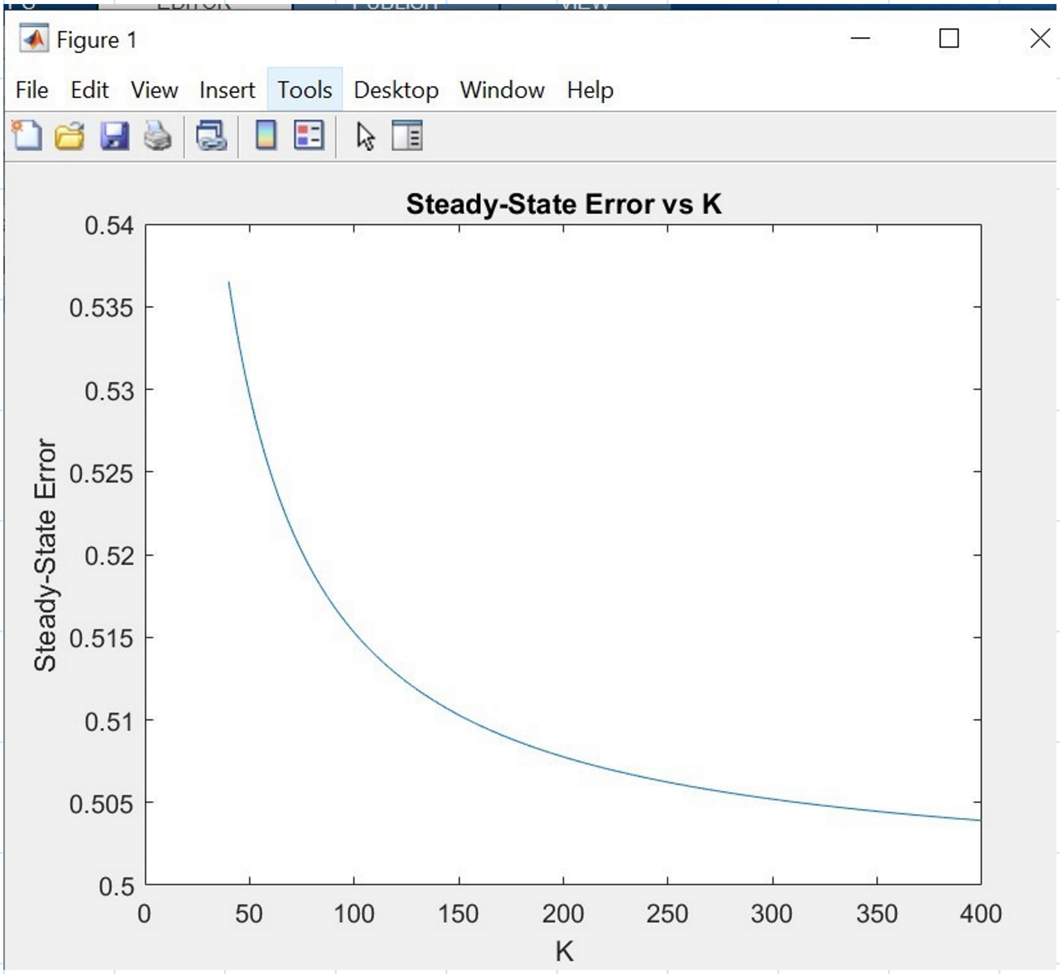
$$16.7 = 2\zeta\omega_n$$

$$\zeta = \frac{16.7}{2\sqrt{72.9 + 19652K}}$$

Editor - C:\Users\GRosp\Desktop\Senior S2\ENME462\HW5.m

HW5.m

```
K=[40:0.1:400];
e=(14580+2312.*K)./(14580+4624.*K);
Z=(16.7)./(2.*sqrt((72.9+19652.*K)));
OS=100.*exp((-pi.*Z)./((1-Z.^2).^(1/2)));
figure(1)
plot(K,e)
title('Steady-State Error vs K')
xlabel('K')
ylabel('Steady-State Error')
figure(2)
plot(K,OS)
title('Percent Overshoot vs K')
xlabel('K')
ylabel('Percent Overshoot')
```



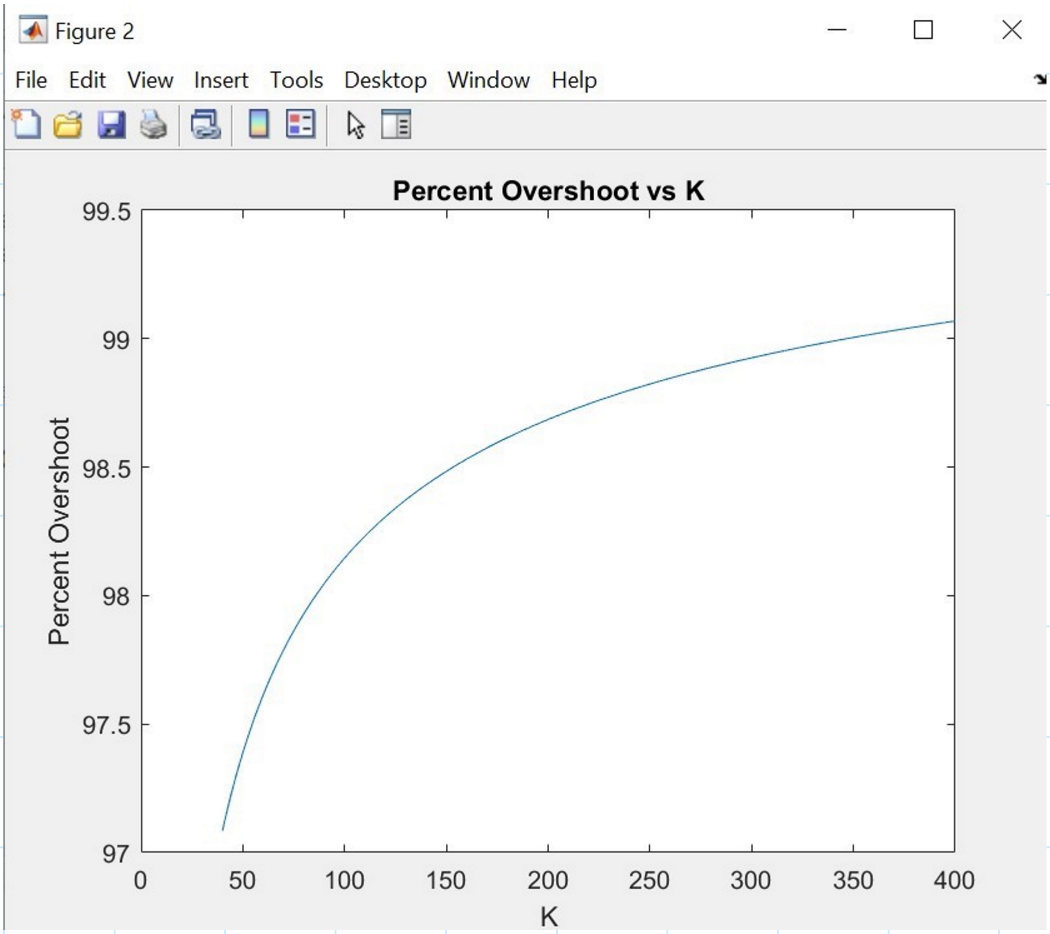


Figure 2
File Edit View Insert Tools Desktop Window Help
Percent Overshoot vs K
Percent Overshoot
99.5
99
98.5
98
97.5
97
0
50
100
150
200
250
300
350
400
K