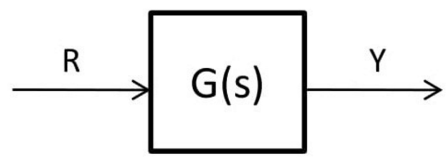
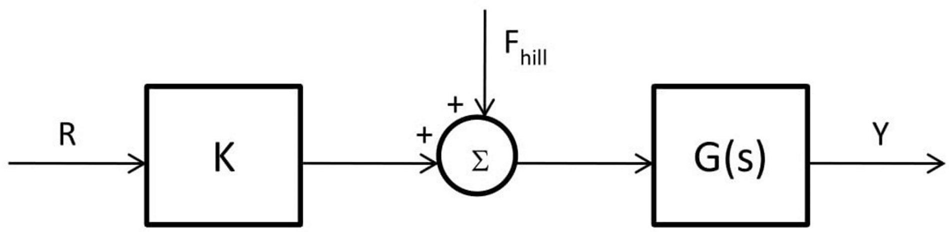
Gino Rospigliosi ENME462
UID: 115443037 Section 0105

1. Transfer functions are useful because they algebraically describe the behavior of a system. A signal input is multiplied by the transfer function to provide the signal output as seen in the simple block diagram below (also described in lecture). In this case Y(s) = G(s)R(s).

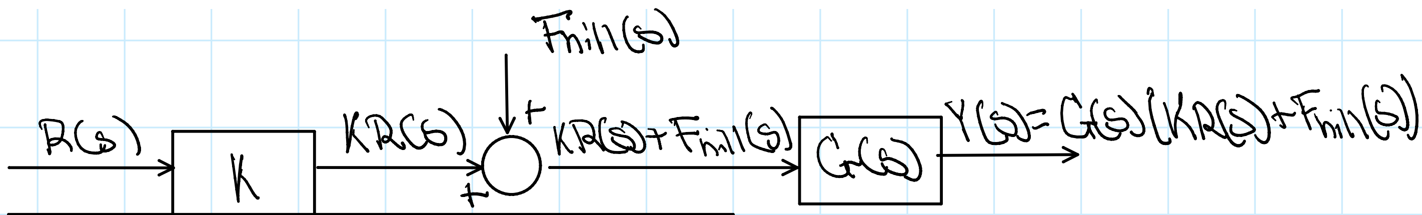


A more complex block diagram describing an open loop system of a cruise controller with a system transfer function G(s), reference input R(s), gain K, disturbance $F_{hill}(s)$, and output Y(s) is shown below.



a. Show that the output Y(s) = G(s)*(KR(s)+F_{hill}(s)).
b. Calculate the transfer function Y(s)/R(s) assuming that F_{hill}(s) = 0 and Y(s)/F_{hill}(s) assuming that R(s) = 0.
c. A closed loop cruise control system is pictured below. Find an expression for the output Y(s).

a)



$$\boxed{Y(s) = G(s)(KR(s) + F_{hill}(s))}$$

b) $\frac{Y(s)}{R(s)}$, $F_{hill}(s) = 0$ $\qquad$ $\frac{Y(s)}{F_{hill}(s)}$, $R(s) = 0$

$$Y(s) = G(s)KR(s) + G(s)F_{hill}(s) = R(s)\left(G(s)K + \frac{G(s)F_{hill}(s)}{R(s)}\right)$$

$$\frac{Y(s)}{R(s)} = G(s)K + \frac{G(s)F_{hill}(s)}{R(s)}$$

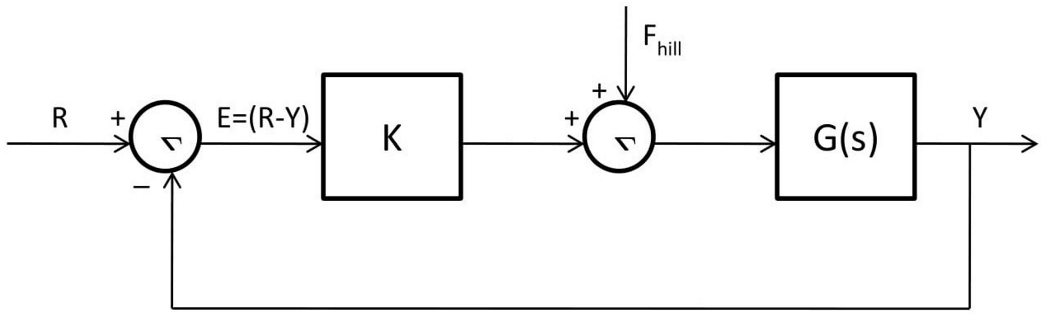
$$\left.\frac{Y(s)}{R(s)}\right|_{F_{hill}(s)=0} = G(s)K + \frac{G(s)(0)}{R(s)} \qquad \boxed{\frac{Y(s)}{R(s)} = G(s)K}$$

$$Y(s) = G(s)KR(s) + G(s)F_{hill}(s) = F_{hill}(s)\left(\frac{G(s)KR(s)}{F_{hill}(s)} + G(s)\right)$$

$$\frac{Y(s)}{F_{hill}(s)} = \frac{G(s)KR(s)}{F_{hill}(s)} + G(s)$$

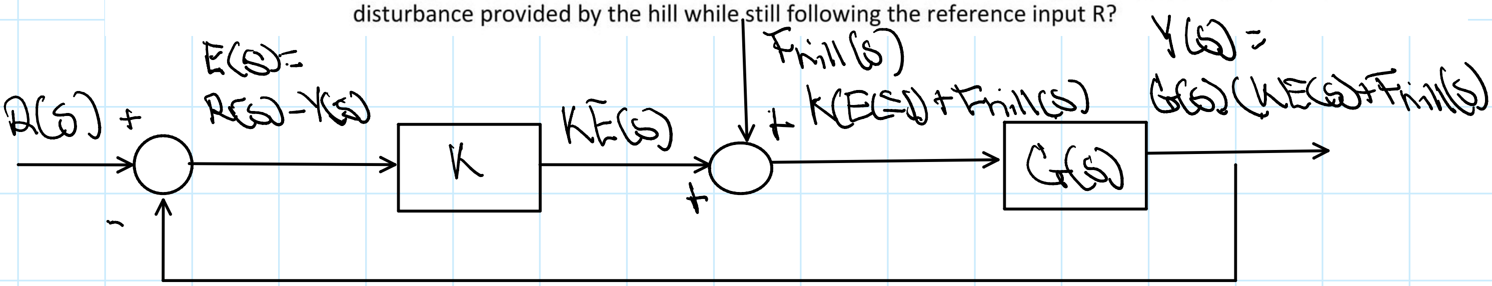
$$\left.\frac{Y(s)}{F_{hill}(s)}\right|_{R(s)=0} = \frac{G(s)K(0)}{F_{hill}(s)} + G(s) \qquad \boxed{\frac{Y(s)}{F_{hill}(s)} = G(s)}$$

C.)



d. Repeat part (b) for the closed loop system.

e. Looking at the transfer functions you found in (d), how would you minimize the effect of disturbance provided by the hill while still following the reference input R?



Drop s dependence

$$Y(s) = G(s)(KE(s) + F_{hill}(s))$$

$$= G(s)(K(R(s) - Y(s)) + F_{hill}(s))$$

$$= G(s)KR(s) - G(s)KY(s) + G(s)F_{hill}(s)$$

$$Y(s) + G(s)KY(s) = G(s)KR(s) + G(s)F_{hill}(s)$$

$$Y(s)(1 + G(s)K) = G(s)KR(s) + G(s)F_{hill}(s)$$

$$\boxed{Y(s) = \frac{G(s)KR(s) + G(s)F_{hill}(s)}{1 + G(s)K}}$$

d) $\frac{Y(s)}{R(s)}$, $F_{hill}(s) = 0$ $\frac{Y(s)}{F_{hill}(s)}$, $R(s) = 0$

$$Y(s)\Big|_{F_{hill}(s)=0} = \frac{G(s)KR(s) + G(s)(0)}{1 + G(s)K}$$

$$= \frac{G(s)KR(s)}{1 + G(s)K} \qquad \boxed{\frac{Y(s)}{R(s)} = \frac{G(s)K}{1 + G(s)K}}$$

$$Y(s)\Big|_{R(s)=0} = \frac{G(s)K(0) + G(s)F_{hill}(s)}{1 + G(s)K}$$

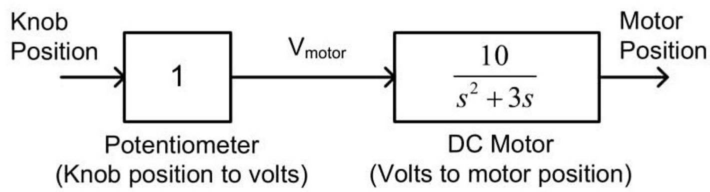
$$= \frac{G(s)F_{hill}(s)}{1 + G(s)K} \qquad \boxed{\frac{Y(s)}{F_{hill}(s)} = \frac{G(s)}{1 + G(s)K}}$$

e) Minimize disturbance $F_{hill}(s)$, maximize input $R(s)$

If $K \to \infty$, $\frac{Y(s)}{F_{hill}(s)} = 0$, $\frac{Y(s)}{R(s)} = 1$

$\therefore$ Large K closely matches the input and rejects disturbance

2. You would like to control the position (shaft angle) of a DC motor with a knob connected to a potentiometer. The open loop system is given as follows:



(1) Plot the poles of this open loop transfer function on the s-plane.

(2) Use MATLAB/SIMULINK to plot the motor position response to a step input (i.e. turning the knob to a particular position). Is it possible to use the knob to specify a particular motor position?

(3) Now try using negative feedback to change this system response. Draw a block diagram with the DC motor, a controller with gain K in the forward path, and a gain of 1 in the feedback path (i.e. assume that the sensor you are using to measure motor position has a transfer function = 1).

(4) Calculate the closed loop transfer function (Motor Position/Knob Position).

(5) Use MATLAB/SIMULINK to plot the motor position response to a step input if K=1. Now is it possible to use the knob to specify a particular motor angle?

(6) What are the poles of the closed loop transfer function as a function of the controller gain K?

(7) At what value of K does the closed loop system become critically damped?

(8) If you increase K above this critically damped value, how will this affect the settling time, damped frequency, and percent overshoot (i.e. do they increase, decrease or stay the same)?

(9) Shade in the allowable regions in the s-plane for second order system poles to achieve a %OS less than 5%.

(10) What value of K will result in a 5% overshoot? Confirm your answer by plotting the motor position response to a step input in MATLAB/SIMULINK.

$$G(s) = (1)\left(\frac{10}{s^2+3s}\right) = \frac{10}{s(s+3)}$$

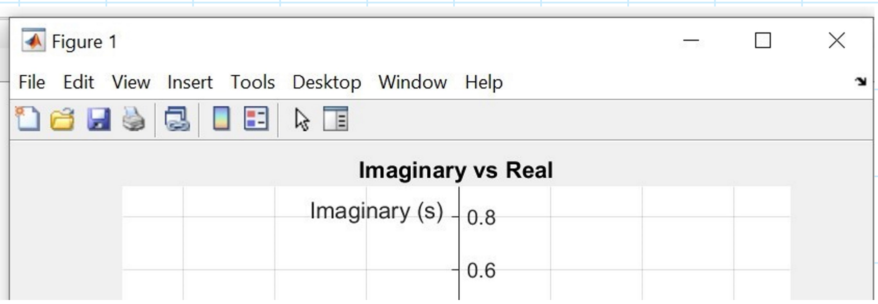
Poles: $0 = s(s+3)$ $\boxed{s=0 \quad s=-3}$

(1)

Editor - C:\Users\GRosp\Desktop\Senior S2\ENME462\HW2.m

HW2.m | SimulinkHW2.m

```
%(1)
s1=[-3 0];
s2=[0,0];
clf;
figure(1);
scatter(s1,s2, 100, 'r','x')
xlim([-4 4]);
title("Imaginary vs Real")
```



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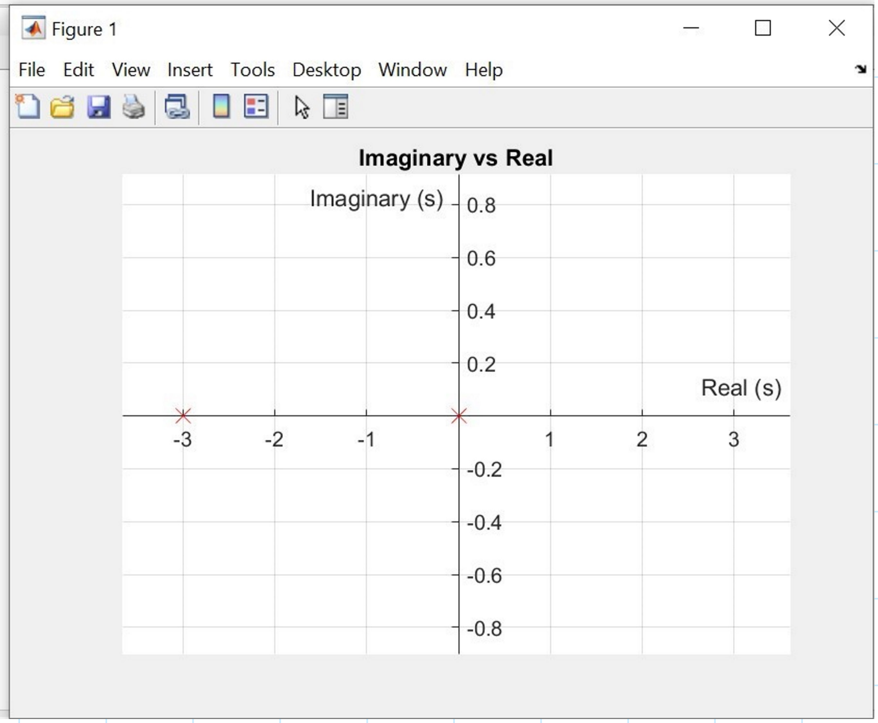
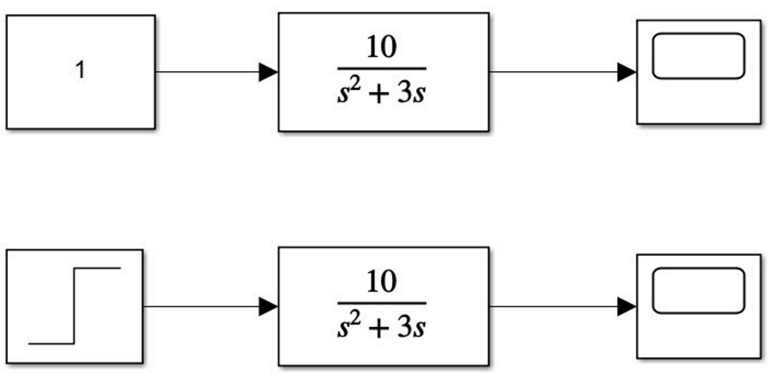
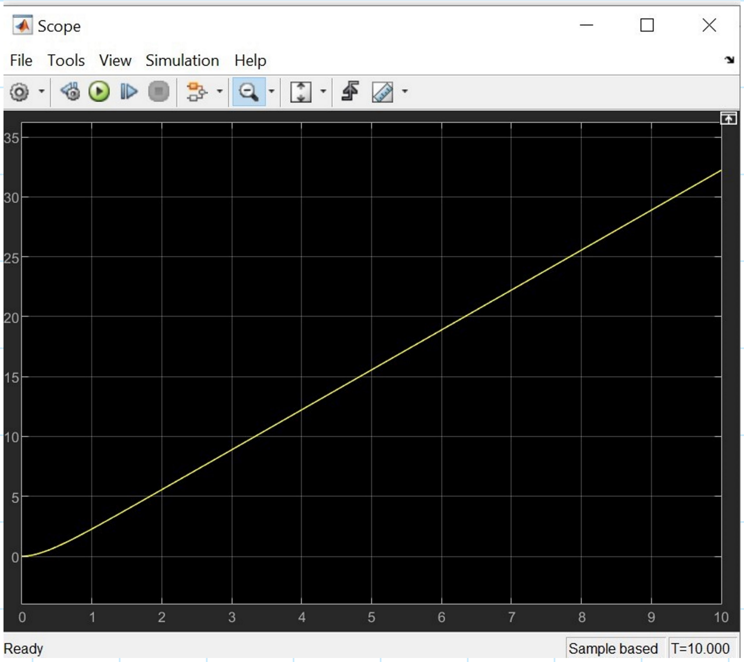
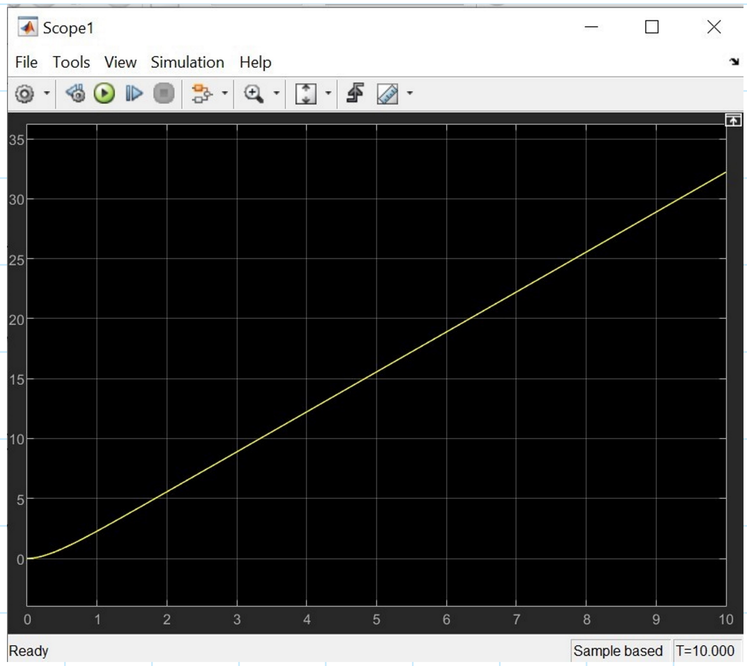
title("Imaginary vs Real")
ylabel("Imaginary (s)")
xlabel("Real (s)")
ax = gca;
ax.XAxisLocation = 'origin';
ax.YAxisLocation = 'origin';
grid on
```



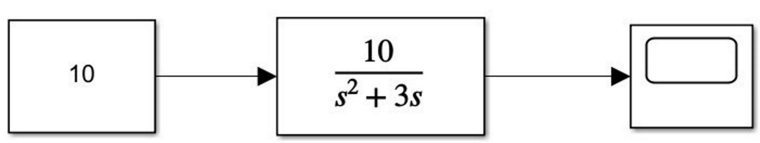
(2)

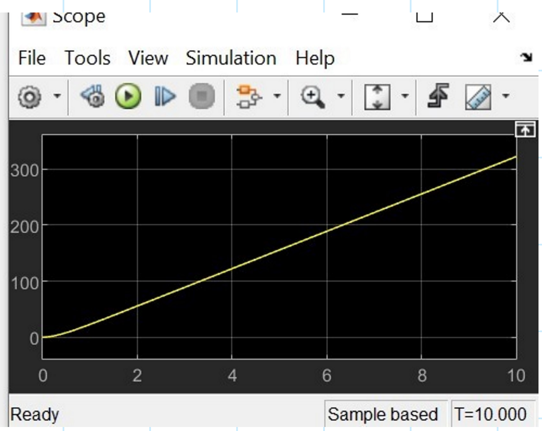




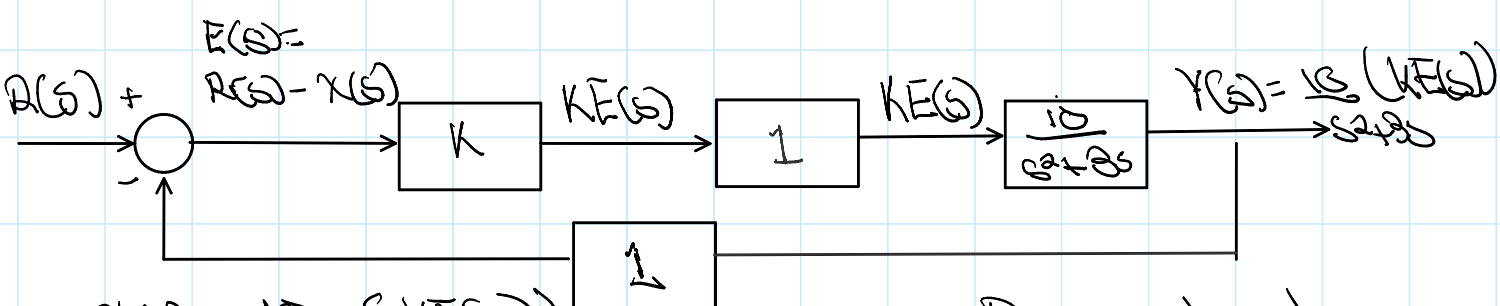


You can control the knob to change step input constant / final value to control motor position.





(3)



$X(s) = \frac{10}{s^2+3s}(KE(s))$

Drop s dependence

$X(s) = \frac{10}{s^2+3s}(KE(s))$ Drop s dependence

$= Y(s)$

(4) $Y(s) = \frac{10}{s^2+3s}(KE(s)) = \frac{10}{s^2+3s}(K(R(s) - Y(s)))$

$$Y(s) = \frac{10}{s^2+3s}(KR(s) - KY(s))$$

$$= \frac{10KR(s)}{s^2+3s} - \frac{10K\,Y(s)}{s^2+3s}$$

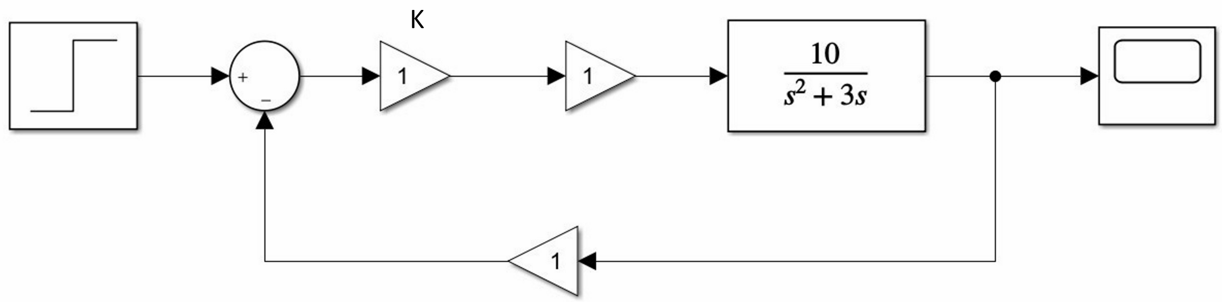
$$Y(s) + \frac{10KY(s)}{s^2+3s} = \frac{10KR(s)}{s^2+3s}$$

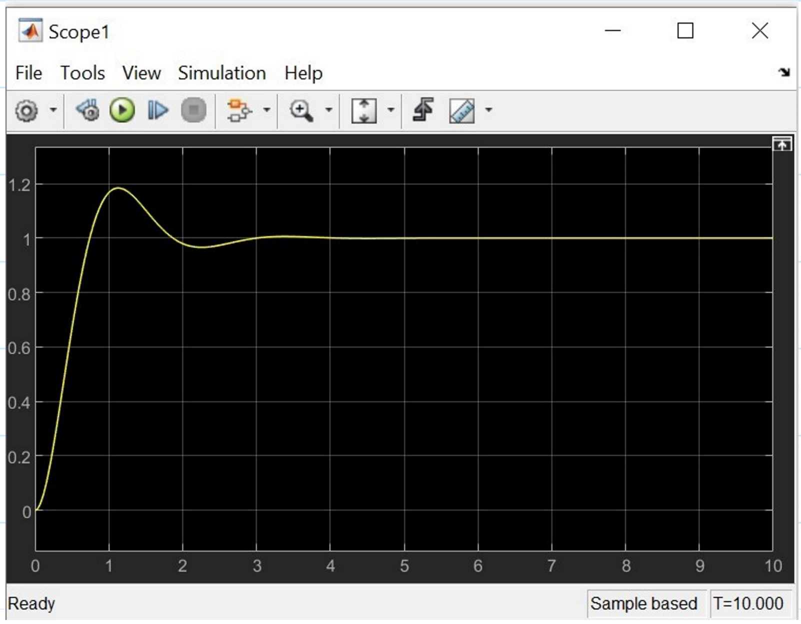
$$Y(s)\left(1 + \frac{10K}{s^2+3s}\right) = R(s)\left(\frac{10K}{s^2+3s}\right)$$

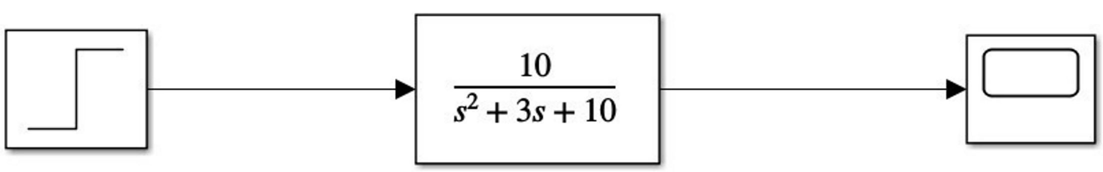
$$\frac{Y(s)}{R(s)} = \frac{\left(\frac{10K}{s^2+3s}\right)}{\left(1 + \frac{10K}{s^2+3s}\right)} = \frac{10K}{s^2+\cancel{3s}} \cdot \frac{s^2\cancel{+3s}}{s^2+3s+10K}$$

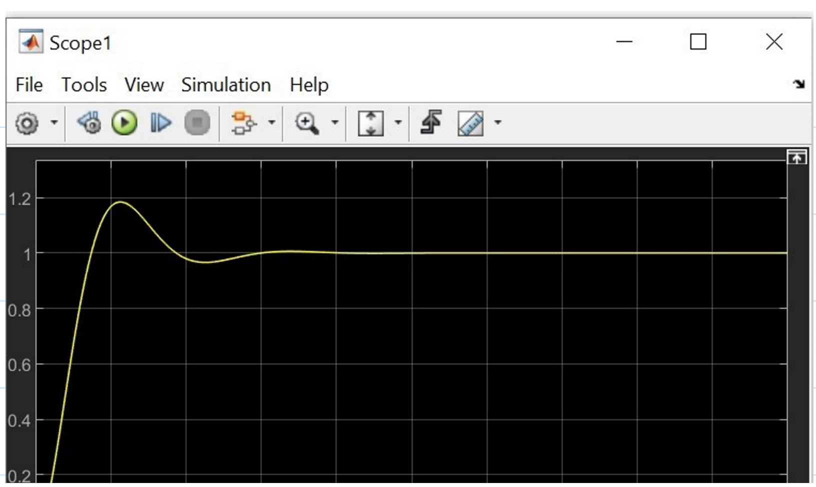
$$\boxed{\frac{Y(s)}{R(s)} = \frac{10K}{s^2+3s+10K}}$$

(5)


K
+
−
1
1
10
s^2 + 3s
1


Scope1
File Tools View Simulation Help
1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
Ready
Sample based
T=10.000


10
s^2 + 3s + 10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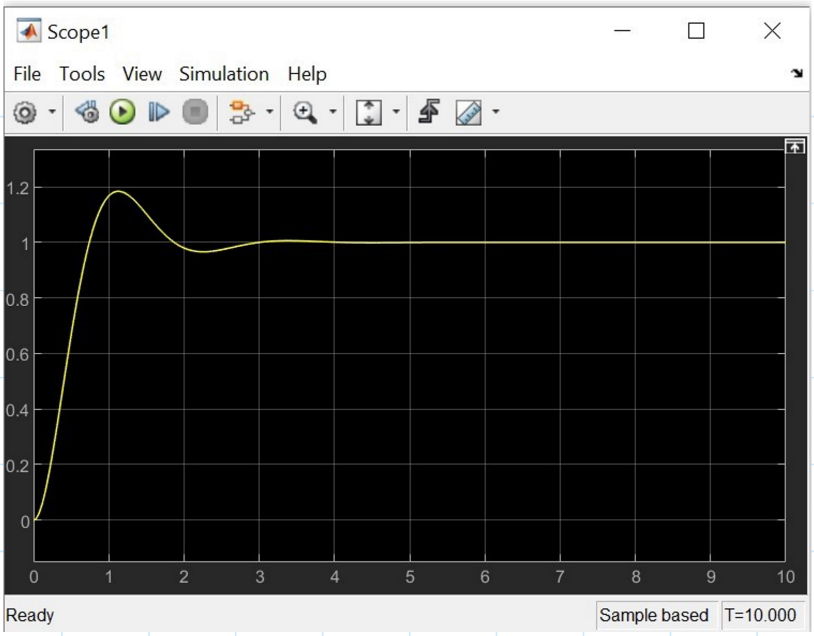
0.2



You can control the knob to change step input constant / final value to control motor position.

(6) $$\frac{Y(s)}{R(s)} = \frac{10K}{s^2+3s+10K}$$

Poles: $0 = s^2+3s+10K$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{or} \quad s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

$$= \frac{-3 \pm \sqrt{9-40K}}{2}$$

$$\boxed{s = \frac{-3+\sqrt{9-40K}}{2} \qquad s = \frac{-3-\sqrt{9-40K}}{2}}$$

(7) $$\frac{Y(s)}{R(s)} = \frac{10K}{s^2+3s+10K} = \frac{K}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Critically Damped $\zeta = 1$ ☆

$R(s) \quad s^2+3s+10K \quad s^2+2\zeta\omega_n s+\omega_n^2$

Critically Damped $\zeta=1$ ✰

$\omega_n^2 = 10K \qquad \omega_n=\sqrt{10K}$

$2\zeta\omega_n=3 \qquad 2\sqrt{10K}=3 \qquad K=\frac{1}{10}\left(\frac{9}{4}\right)=\boxed{\frac{9}{40}}$

(8) If $K > \frac{9}{40}$, Let $K=\frac{1}{2}$

$\omega_n=\sqrt{5} \qquad \zeta=\frac{3}{2\omega_n}=\frac{3}{2(\sqrt{5})}=0.67$

∴ Becomes underdamped

$T_s=\frac{4}{\zeta\omega_n}=\frac{4}{\sqrt{9/4}}=2.66$
$\zeta=1$

$T_s=\frac{4}{\zeta\omega_n}=\frac{4}{(0.67)(\sqrt{5})}=2.66$
$\zeta=0.67$

$\boxed{\text{Settling time does not change}}$

$\omega_d=\omega_n\sqrt{1-\zeta^2}$

$\omega_d\big|_{\zeta=1}=\sqrt{9/4}(0)=0$

$\omega_d\big|_{\zeta=0.67}=\sqrt{5}\left(\sqrt{1-(0.67)^2}\right)=1.66$

$\boxed{\text{Damped Frequency increases}}$

$\%OS=100 \times e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

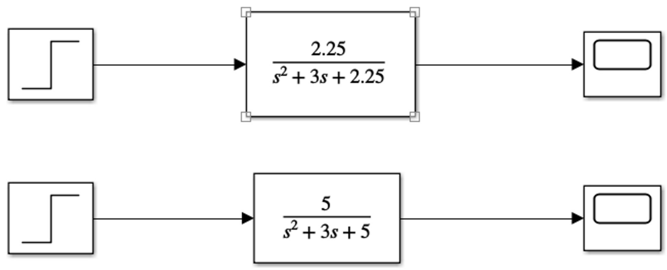
$\%OS|=100\, e^{0}=100 \Rightarrow$ No Overshoot

$$\%OS = 100 \times e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\%OS\Big|_{\zeta=1} = 100\,e^{0} = 100 \Rightarrow \text{No Overshoot}$$

$$\%OS\Big|_{\zeta=0.67} = 100\,e^{-\frac{0.67\pi}{\sqrt{1-(0.67)^2}}} = 5.86$$

$$\boxed{\text{Percent overshoot increases}}$$

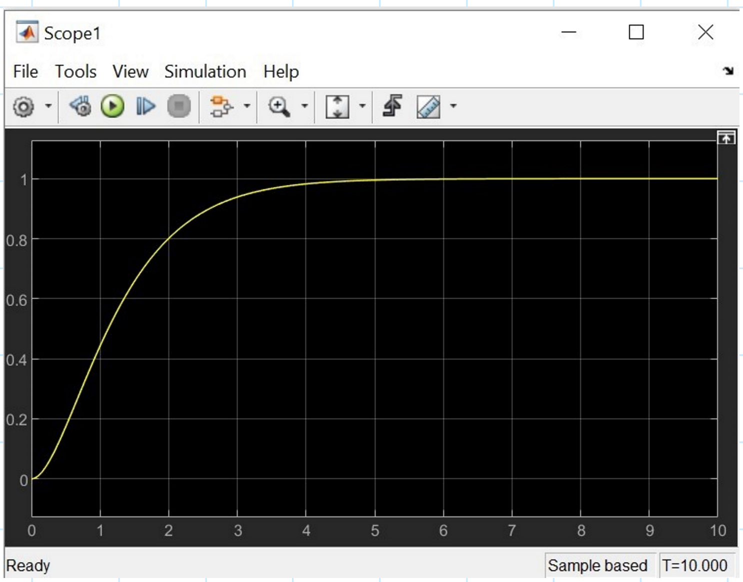


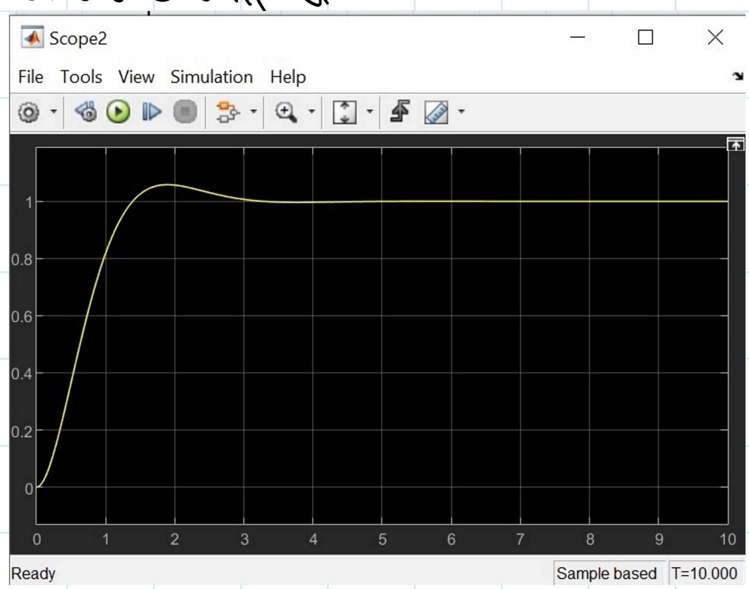
Critically Damped

Underdamped

Critically Damped

Underdamped





(4) $\%OS = 100 \times e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \geq 5$

$$0.05 \geq e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$-\frac{\ln(0.05)}{\pi} \leq \frac{\zeta}{\sqrt{1-\zeta^2}} \qquad 0 \leq \frac{\zeta}{\sqrt{1-\zeta^2}} - 0.954$$

$\zeta \geq 0.69$

$2\zeta\omega_n = 3 \qquad \omega_n = \dfrac{3}{2\zeta} < \dfrac{3}{2(0.69)} = 2.17$

$\omega_n^2 = 10k \qquad k = \dfrac{\omega_n^2}{10} = \dfrac{(2.17)^2}{10} = 0.472$

$s = \dfrac{-3 + \sqrt{9 - 40(0.472)}}{2} = -1.5 + 1.572i$

$s = \dfrac{-3 - \sqrt{9 - 40(0.472)}}{2} = -1.5 - 1.572i$

NAVIGATE EDIT BREAKPOINTS

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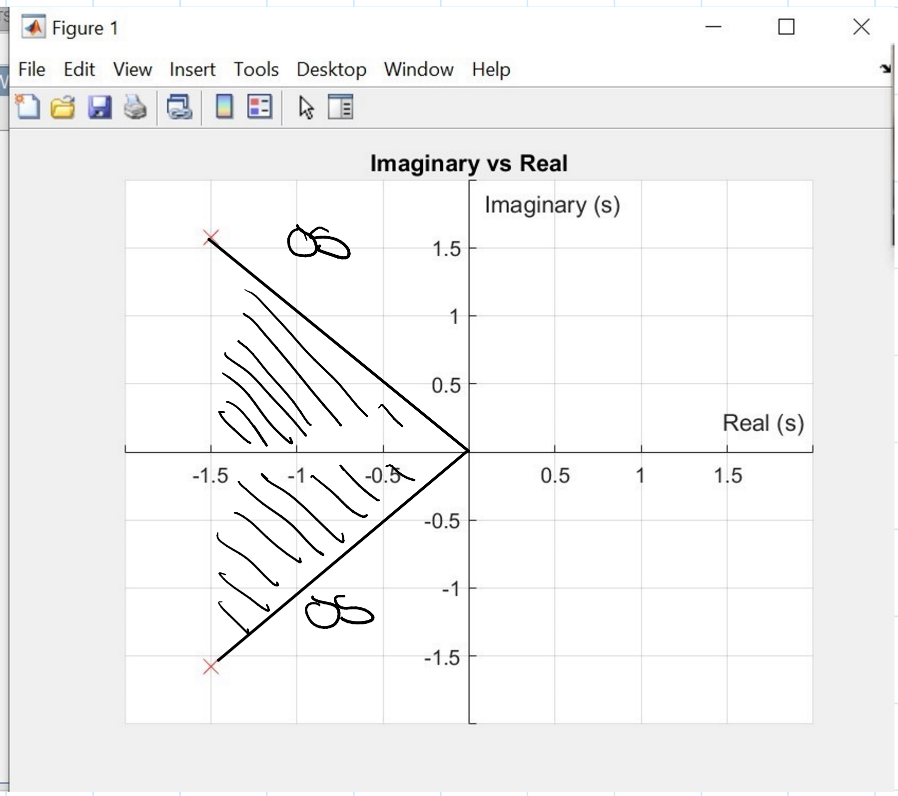
Editor - C:\Users\GRosp\Desktop\Senior S2\ENME462\HW

HW2.m × SimulinkHW2.m × +

```
%(9)
s1=[-1.5 -1.5];
s2=[1.572 -1.572];
clf;
figure(1);
scatter(s1,s2, 100, 'r','x')
xlim([-2 2]);
ylim([-2 2]);
title("Imaginary vs Real")
ylabel("Imaginary (s)")
xlabel("Real (s)")
ax = gca;
ax.XAxisLocation = 'origin';
ax.YAxisLocation = 'origin';
grid on
```



(10) $\zeta \geq 0.69$

$2\zeta\omega_n = 3 \qquad \omega_n = \dfrac{3}{2\zeta} < \dfrac{3}{2(0.69)} = 2.17$

$\omega_n^2 = 10k \qquad k = \dfrac{\omega_n^2}{} = \dfrac{(2.17)^2}{} = \boxed{0.472}$

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