Studio 07

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ENME462
Section 0105

1. (Example 6.2, Nise) Determine the stability of the closed-loop transfer function below:

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

There are several methods to ascertain stability of the aforementioned system. For example, one can use Matlab commands such as `pole()`, `roots()`. Alternatively, one can use methods such as root locus (to be covered later in the course), Routh-Hurwitz criterion, etc. The Studio Instructor will

a) Construct the Routh table for the closed-loop transfer function above and study the stability of the closed-loop system.
b) Use the Matlab commands `pole()` and `roots()` to verify the answer obtained in (a).
c) It is also possible to obtain the various cells of the Routh array using symbolic computation in Matlab. The Studio Instructor will now demonstrate to the students the Matlab file *ch6sp1.m*, and its applicability in obtaining the array. Please note that this file can also be found in the Studio 7 Assignment page on Canvas.

a)

s^5	1	3	5	+
s^4	2	6	3	+
s^3	$\frac{\begin{vmatrix}1&3\\2&6\end{vmatrix}}{-6}=0 \to \varepsilon$	$\frac{\begin{vmatrix}1&5\\2&3\end{vmatrix}}{-2}=3.5$	0	+
s^2	$\frac{\begin{vmatrix}2&6\\\varepsilon&3.5\end{vmatrix}}{-\varepsilon}=\frac{6\varepsilon-7}{\varepsilon}$	$\frac{\begin{vmatrix}2&3\\\varepsilon&0\end{vmatrix}}{-\varepsilon}=3$	0	−
s^1	$\frac{\begin{vmatrix}\varepsilon&3.5\\\frac{6\varepsilon-7}{\varepsilon}&3\end{vmatrix}}{\frac{6\varepsilon-7}{\varepsilon}}=\frac{-6\varepsilon^2+42\varepsilon-49}{12\varepsilon-14}$	0	0	+
s^0	3	0	0	+

$\boxed{\therefore \text{ Not stable}}$

b)

```
%Problem 1
G = tf([10],[1 2 3 6 5 3])
s_d= pole(G)
disp("Unstable since poles in the RHP")
```

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G = tf([10],[1 2 3 6 5 3])
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```

```
G =

                10
  ------------------------------------
  s^5 + 2 s^4 + 3 s^3 + 6 s^2 + 5 s + 3

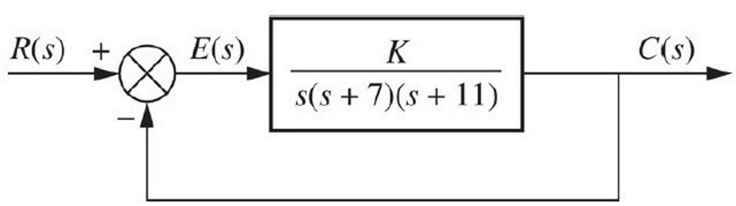
Continuous-time transfer function.


s_d =

   0.3429 + 1.5083i
   0.3429 - 1.5083i
  -1.6681 + 0.0000i
  -0.5088 + 0.7020i
  -0.5088 - 0.7020i

Unstable since poles in the RHP
```

2. (Example 6.9, Nise) It is also possible to utilize the Routh array to design parameters or controller gains in the system, from the stability perspective. In the system shown in the Figure below, the Studio Instructor will first obtain a range of the gain K that will lead to a closed-loop system that is stable, unstable, or marginally stable. (Which transfer function should be used for obtaining the range?)



It is also possible to obtain this range using Matlab. The Studio Instructor will now demonstrate this by going through the Matlab script given in the file `ch6p2.m`, which can be obtained from the Studio 7 Assignment page on Canvas.

$$G(s) = \frac{T(s)}{1+T(s)} = \frac{K}{s(s+7)(s+11)+K}$$

$$G(s) = \frac{[illegible]}{1+T(s)} = \frac{K}{s(s+7)(s+11)+K}$$

$$= \boxed{\frac{K}{s^3+18s^2+77s+K}}$$

s^3	1	77	$+$
s^2	18	0	$+$
s^1	$\dfrac{\begin{vmatrix} 1 & 77 \\ 18 & K \end{vmatrix}}{-18} = \dfrac{1386-K}{18}$	0	$+$
s^0	$\dfrac{\begin{vmatrix} 18 & K \\ \frac{1386-K}{K} & 0 \end{vmatrix}}{-\frac{1386-K}{18}} = K$	0	$+$

$0 < K < 1386$ Stable

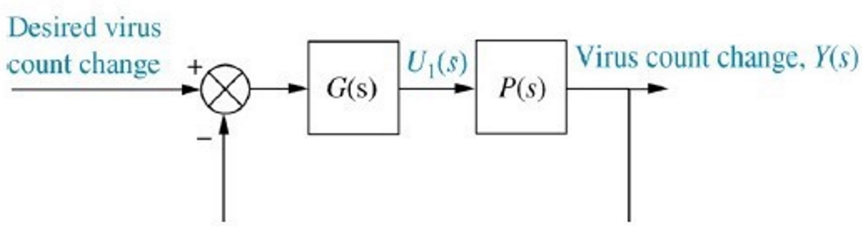
$K > 1386$ Unstable

$K = 1386$ Marginally Stable

1. Control of HIV/AIDS: The linearized HIV infection model has the following transfer function:

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126}$$

It is desired to develop a policy for drug delivery to maintain the virus count at prescribed levels. For the purpose of obtaining an appropriate $u_1(t)$, a feedback will be used as shown below:



As a first approach, consider G(s)=K, where K is a constant to be selected.

(a) Use the Routh-Hurwitz criterion to find the range of K for which the closed-loop system is stable.
(b) Following the Matlab file `ch6p2.m`, write a Matlab code to obtain this range using Matlab.
(c) Select a value of K for which the system is marginally stable and plot the unit step response for t=0 till t=1000s.
(d) Select a value of K for which the system is stable and plot the unit step response for t=0 till t=1000s.
(e) Select a value of K for which the system is unstable and plot the unit step response for t=0 till t=10s.

(a) $T(s) = KP(s)$

$$G(s) = \frac{T(s)}{1+T(s)}$$

$$= \frac{K(-520s - 10.3844)}{s^3 + 2.6817s^2 + 0.11s + 0.0126 + (-520s - 10.3844)K}$$

$$= \frac{(-520K)s - 10.3844K}{s^3 + 2.6817s^2 + (0.11 - 520K)s + 0.0126 - 10.3844K}$$

s^3	1	$0.11-520K$	$+$
s^2	2.6817	$0.0126-10.3844K$	$+$
s^1	$0.105-516.127K$	0	$+$
s^0	$0.0126-10.3844K$	0	$+$

s^1:

$$\frac{\begin{vmatrix} 1 & 0.11-520K \\ 2.6817 & 0.0126-10.3844K \end{vmatrix}}{-2.6817}$$

$$=\frac{\left[(0.0126-10.3844K)-(2.6817)(0.11-520K)\right]}{-2.6817}$$

$$=\frac{-0.282+1384.09K}{-2.6817}$$

$$0.105-516.127K \qquad 0 \qquad +$$

s^0:

$$\frac{\begin{vmatrix} 2.6817 & 0.0126-10.3844K \\ 0.105-516.127K & 0 \end{vmatrix}}{0.105-516.127K}$$

$$=0.0126-10.3844K \qquad 0 \qquad +$$

$0.105 - 516.12K > 0$

$K < 2.04 \times 10^{-4}$

$0.0126 - 10.38444K > 0$

$K < 1.21 \times 10^{-3}$

$\therefore$ Stable when

$$\boxed{K < 2.04 \times 10^{-4}}$$

b)

```
K=logspace(-4,-1,10000);              % Define range of K from 1 to 2000 in
                                      % steps of 1.
for n=1:length(K);                    % Set up length of DO LOOP to equal
                                      % number of K values to be tested.
       dent=[1 2.6817 (0.11-520*K(n)) (0.0126-10.3844*K(n))];
                                      % Define the denominator of T(s) for
                                      % the nth value of K.
       poles=roots(dent);             % Find the poles for the nth value of
                                      % K.
       r=real(poles);                 % Form a vector containing the real
                                      % parts of the poles for K(n).
             if max(r)>=0,            % Test poles found for the nth value
                                      % of K for a real value > or = 0.
                  poles               % Display first pole values where
                                      % there is a real part > or = 0.
                  K=K(n)              % Display corresponding value of K.
                  break               % Stop loop if rhp poles are found.
       end                            % End if.
end                                   % End for.
```

```
poles =

  -2.6817 + 0.0000i
   0.0000 + 0.0625i
   0.0000 - 0.0625i


K =

   2.0414e-04
```

Marginally Stable

c) $K = 2.04 \times 10^{-4}$

d) Stable

$K < 2.04 \times 10^{-4}$ $K = .1 \times 10^{-4}$

e) Unstable

$K > 2.04 \times 10^{-4}$ $K = 3 \times 10^{-4}$

```
%1c)
%K=2.0414e-4
K1= 2.0414e-4
G1=tf(K1*[-520 -10.3844],[1 2.6817 0.11-520*K1 0.0126-10.3844*K1])
figure(1)
step(G1,1000)
title("Marginally Stable")
%1d)
%K<2.0414e-4
K2= 1e-4
G2=tf(K2*[-520 -10.3844],[1 2.6817 0.11-520*K2 0.0126-10.3844*K2])
figure(2)
step(G2,1000)
title("Stable")
%1)
%K>2.0414e-4
K3= 3e-4
G3=tf(K3*[-520 -10.3844],[1 2.6817 0.11-520*K3 0.0126-10.3844*K3])
figure(3)
step(G3,1000)
title("Unstable")
```

```
K1 =

   2.0414e-04


G1 =

           -0.1062 s - 0.00212
  --------------------------------------
  s^3 + 2.682 s^2 + 0.003847 s + 0.01048

Continuous-time transfer function.


K2 =

   1.0000e-04


G2 =

          -0.052 s - 0.001038
  -----------------------------------
  s^3 + 2.682 s^2 + 0.058 s + 0.01156

Continuous-time transfer function.

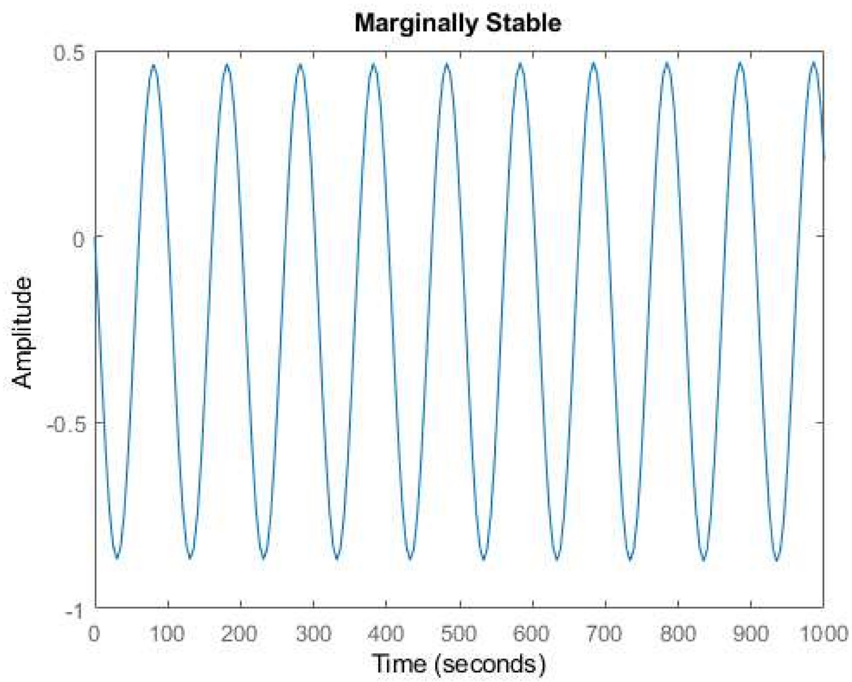

K3 =

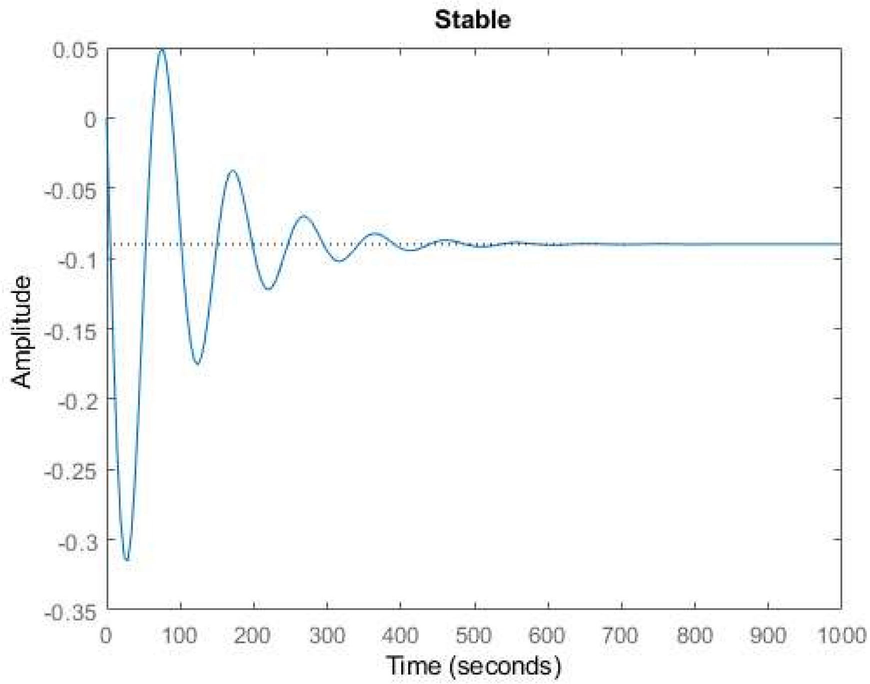
   3.0000e-04
```

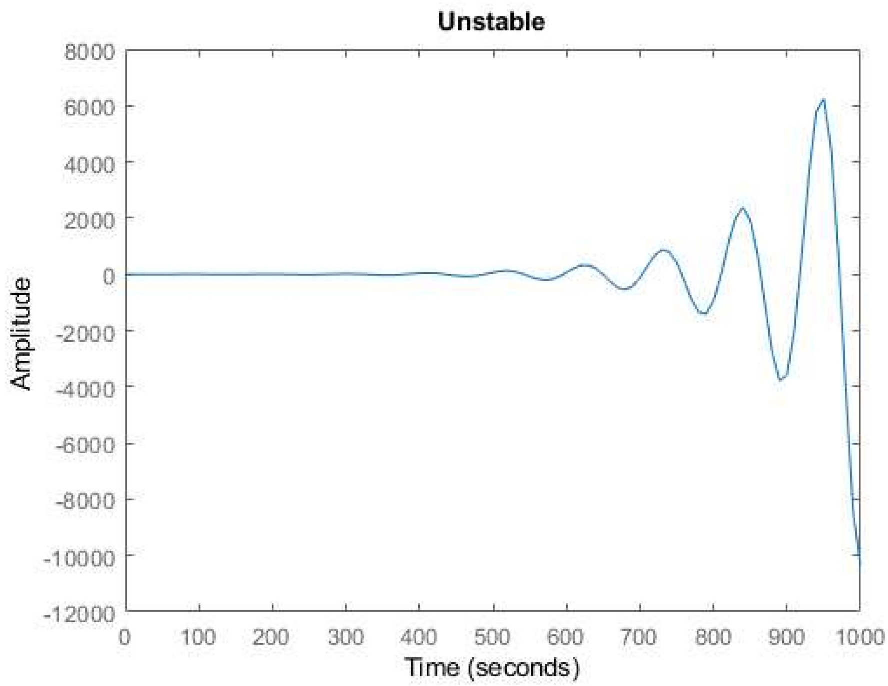
```
G3 =

          -0.156 s - 0.003115
  ------------------------------------
  s^3 + 2.682 s^2 - 0.046 s + 0.009485

Continuous-time transfer function.
```




Stable
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
Amplitude
0
100
200
300
400
500
600
700
800
900
1000
Time (seconds)


Unstable
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
-12000
Amplitude
0
100
200
300
400
500
600
700
800
900
1000
Time (seconds)