Studio 06

2.1 STEADY-STATE ERROR DUE TO STEP INPUT

1. In the simulation, set the **Input** toggle switch to **Step**, **kp** to 5, **kd** to 0.25, and **ki** to 0. Press the **Start** button. The simulation will run for a duration of 5 seconds and apply a step input with an amplitude of 1 rad to the PID position controller. Observe the behavior of the QUBE-Servo 2 and estimate the amount of steady-state error in your system.
    a. What steady-state error did you find? Select the answer that is closest to your measurement.
        i. 0.01 rad
        ii. 0.03 rad
        iii. 0.1 rad
2. Explore how different PID gains affect the system's steady-state error by adjusting the **kp**, **ki**, and **kd** sliders.
    a. How did the proportional gain affect steady-state error?
        i. Larger kp values increased steady-state error
        ii. Larger kp values reduced steady-state error but also increased overshoot
        iii. It had no effect
    b. How did the integrator gain affect the steady-state error?
        i. Larger ki values increased steady-state error
        ii. Larger ki values decreased steady-state error
        iii. It had no effect
    c. How did the derivative gain affect the steady-state error?
        i. Larger kd values increased steady-state error
        ii. Larger kd values decreased steady-state error
        iii. It had no effect

2.2 STEADY-STATE ERROR DUE TO RAMP INPUT

3. In the simulation, set the **Input** toggle switch to **Ramp**, **kp** to 5, **kd** to 0.25, and **ki** to 0. Press the **Start** button. The simulation will run for a duration of 5 seconds and apply a ramp input with a slope of 5 rad/s to the PID position controller. Observe the behavior of the QUBE-Servo 2 and estimate the amount of steady-state error in your system.
   a. What steady-state error did you find? Select the answer that is closest to your measurement.
      i. 0.025 rad
      ii. 0.35 rad
      iii. 3.0 rad
4. Explore how different PID gains affect the system's steady-state error by adjusting the **kp**, **ki**, and **kd** sliders .
   a. How did the proportional gain affect steady-state error?
      i. Larger kp values increased steady-state error
      ii. Larger kp values reduced steady-state error but also increased overshoot and caused oscillations/instability
      iii. It had no effect
   b. How did the integrator gain affect the steady-state error?
      i. Larger ki values increased steady-state error
      ii. Larger ki values decreased steady-state error
      iii. It had no effect
   c. How did the derivative gain affect the steady-state error?
      i. Larger kd values increased steady-state error
      ii. Larger kd values decreased steady-state error
      iii. It had no effect

1) Find the closed-loop transfer function $E(s)/R(s)$ for the PID controller shown in Figure 3. The transfer function will represent the dynamics between the desired position, $R(s) = \Theta_d(s)$, and the error, $E(s) = R(s) - Y(s) = \Theta_d(s) - \Theta_m(s)$.

$$\frac{E(s)}{R(s)} = \frac{R(s) - Y(s)}{R(s)}$$

$$= 1 - \frac{Y(s)}{R(s)} = 1 - G(s) = 1 - \frac{\Theta_m(s)}{\Theta_d(s)}$$

$$= 1 - \frac{KK_p s}{\tau} + \frac{KK_i}{\tau}$$

$$= 1 - \frac{\frac{KK_p s}{\tau} + \frac{KK_i}{\tau}}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s + \frac{KK_i}{\tau}}$$

$$= \boxed{\frac{s^3 + \frac{1+KK_d}{\tau}s^2}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s + \frac{KK_i}{\tau}}}$$

2) Using your answer from the previous question, find the error transfer function $E(s)$ for a step input of $R(s) = R_0/s$, where R_0 is the step magnitude. Then, evaluate the analytical steady-state error of the system due to this step input when using a PD controller (i.e. k_i = 0) and identify the system type.

$$E(s) = R(s)G(s) = \boxed{\frac{R_0}{s}\left(\frac{s^3 + \frac{1+KK_d}{\tau}s^2}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s + \frac{KK_i}{\tau}}\right)}$$

PD, $K_i = 0$

$$e(\infty) = \lim_{s \to 0} \frac{R_0 \cancel{s}}{\cancel{s}}\left(\frac{s^3 + \frac{1+KK_d}{\tau}s^2}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s}\right)$$

$$= R_0\left(\frac{3s^2 + 2\left(\frac{1+KK_d}{\tau}\right)s}{3s^2 + 2\left(\frac{1+KK_d}{\tau}\right)s + \frac{KK_p}{\tau}}\right)$$

$$= \boxed{0}$$

System Type = 1

3) Compare the expected steady-state error of a step input to a PD controller (as calculated in the previous question) with the steady-state error that you measured in this lab (section 2.1, step 1). Do they match? If not, provide one reason why there is a difference.

calculated e(s) = 0 does not match with expirimental e(s) = 0.03, there is an error because of nonlinearities such as friction

4) Using your answer from the Question 1, find the error transfer function $E(s)$ for a ramp input of $R(s) = R_0/s^2$.

5) Calculate the steady-state error of the system due to a ramp input when using a PD controller (i.e. k_i = 0) and the PID controller. Compare and comment on the results.

$$E(s) = R(s)G(s) = \frac{R_0}{s^2}\left(\frac{s^3 + \frac{1+KK_d}{\tau}s^2}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s + \frac{KK_i}{\tau}}\right)$$

PD, Ki = 0

$$e(s) = \lim_{s \to 0} \frac{R_0 s}{s^2}\left(\frac{s^3 + \frac{1+KK_d}{\tau}s^2}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s}\right)$$

$$= R_0\left(\frac{s^2 + \frac{1+KK_d}{\tau}s}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s}\right)$$

$$\left(s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s\right.$$

$$= R_0\left(\frac{2s + \frac{1+KK_d}{\tau}}{3s^2 + 2\left(\frac{1+KK_d}{\tau}\right)s + \frac{KK_p}{\tau}}\right)$$

$$= \boxed{\frac{R_0\left(\frac{1+KK_d}{\tau}\right)}{\left(\frac{KK_p}{\tau}\right)}} \quad \boxed{\text{System Type} = 1}$$

PID, $K_i \neq 0$

$$e(\infty) = \lim_{s \to 0} \frac{R_0 s}{s^2}\left(\frac{s^3 + \frac{1+KK_d}{\tau}s^2}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s + \frac{KK_i}{\tau}}\right)$$

$$= R_0\left(\frac{s^2 + \frac{1+KK_d}{\tau}s}{s^3 + \frac{1+KK_d}{\tau}s^2 + \frac{KK_p}{\tau}s + \frac{KK_i}{\tau}}\right)$$

$$= \boxed{0} \quad \boxed{\text{System Type} = 2}$$

PD $\quad e(s) = \dfrac{R_0\left(\frac{1+KK_d}{\tau}\right)}{\left(\frac{KK_p}{\tau}\right)}$

agrees with above since when $K_i = 0$, $e(\infty)$ depends when $k_i = 0$, $e(s)$ depends on k_d and k_p where larger k_p reduces $e(s)$ and k_d increases $e(s)$

PID $e(s) = 0$

agrees with above since when k_i is increased $e(s)$ decreases

6) Using your results from the previous question, calculate the expected steady-state error of the system using a PD controller when applying a ramp input with an amplitude of $R_0 = 5$ rad/s. Assume a the following system parameters: $K = 22.5$ rad/s/V and $\tau = 0.165$ s, and the PD gains $k_p = 5$ V/rad and $k_d = 0.25$ V/(rad/s).

$$e(s) = \frac{R_0\left(1 + \frac{K k_d}{\tau}\right)}{\left(\frac{K k_p}{\tau}\right)}$$

$$= \frac{5\left(1 + \frac{22.5(0.25)}{0.165}\right)}{\left(\frac{22.5(5)}{0.165}\right)} = \boxed{0.257 \text{ rad}}$$

7) Compare the expected steady-state error when applying the ramp input to the PD controller (as calculated in the previous question) with the actual steady-state error that you measured in this lab (section 2.2, step 1). Do they match? If not, provide one reason why there is a difference.

lab (section 2.2, step 1). Do they match? If not, provide one reason why there is a difference.

Calculated $e(\infty) = 0.1237$ does not match with expirement $e(S) = 0.135$ because of nonlinearities such as friction