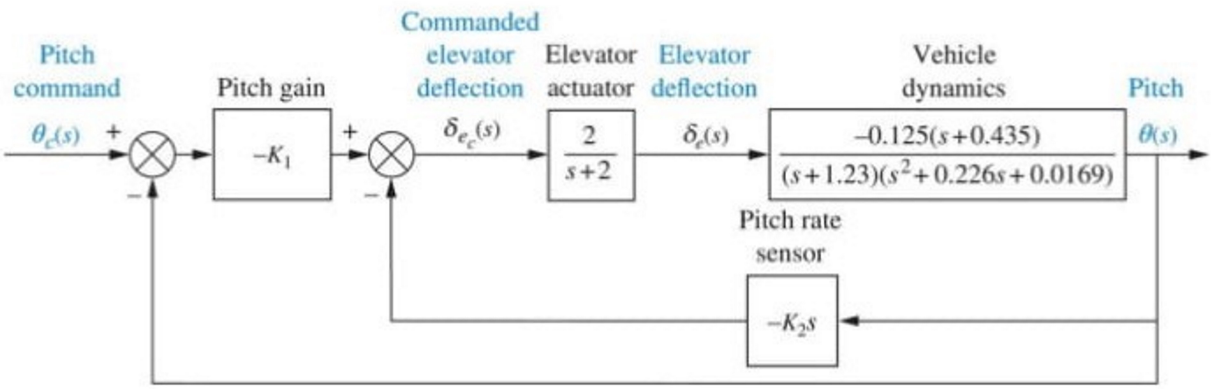
# Studio 04

Gino Rospigliosi
UID: 115443037
ENME462
Section 0105




1. For the UFSS system studied in Part 4, use Simulink to compute the unit step open-loop response of the transfer function between the elevator deflection input and the pitch angle output. Using the plot, compute the percentage overshoot, rise time, peak time, settling time and compare them with the theoretical predictions. Show these values on the plot as well.

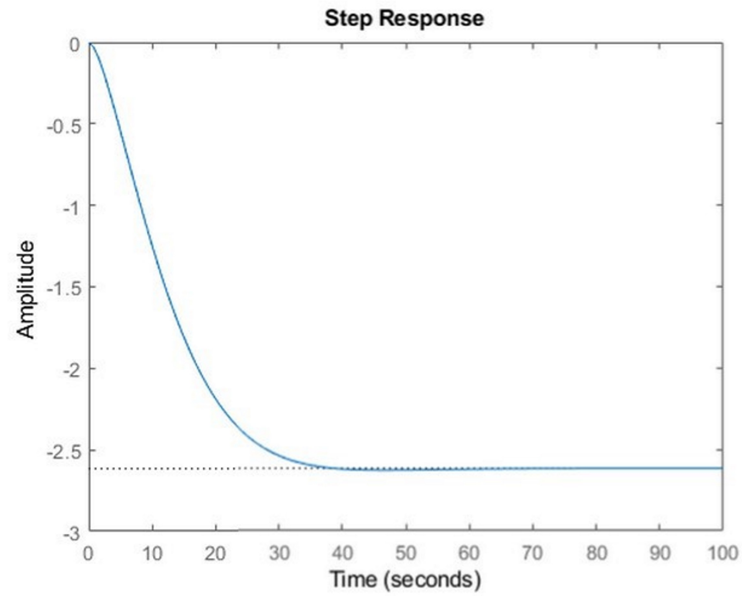
Using MATLAB

```
sys = tf(-0.125*[1 0.435],conv([1 1.23],[1 0.226 0.0169]))
step(sys,100);
stepinfo(sys)
```

```
sys =

        -0.125 s - 0.05437
  ------------------------------------
  s^3 + 1.456 s^2 + 0.2949 s + 0.02079

Continuous-time transfer function.
```

Continuous-time transfer function.

```
ans =

  struct with fields:

        RiseTime: 20.3983
    SettlingTime: 31.7417
     SettlingMin: -2.6269
     SettlingMax: -2.3660
       Overshoot: 0.4233
      Undershoot: 0
            Peak: 2.6269
        PeakTime: 46.8668
```




Using Simulink

Block Parameters: Transfer Fcn

Transfer Fcn

The numerator coefficient can be a vector or matrix expression. The denominator coefficient must be a vector. The output width equals the number of rows in the numerator coefficient. You should specify the coefficients in descending order of powers of s.

Parameters

Numerator coefficients:

-0.125*[1 0.435]

Denominator coefficients:

conv([1 1.23],[1 0.226 0.0169])

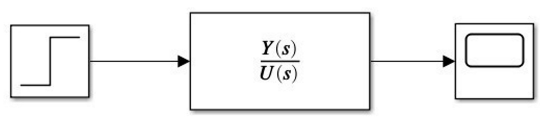
Absolute tolerance:

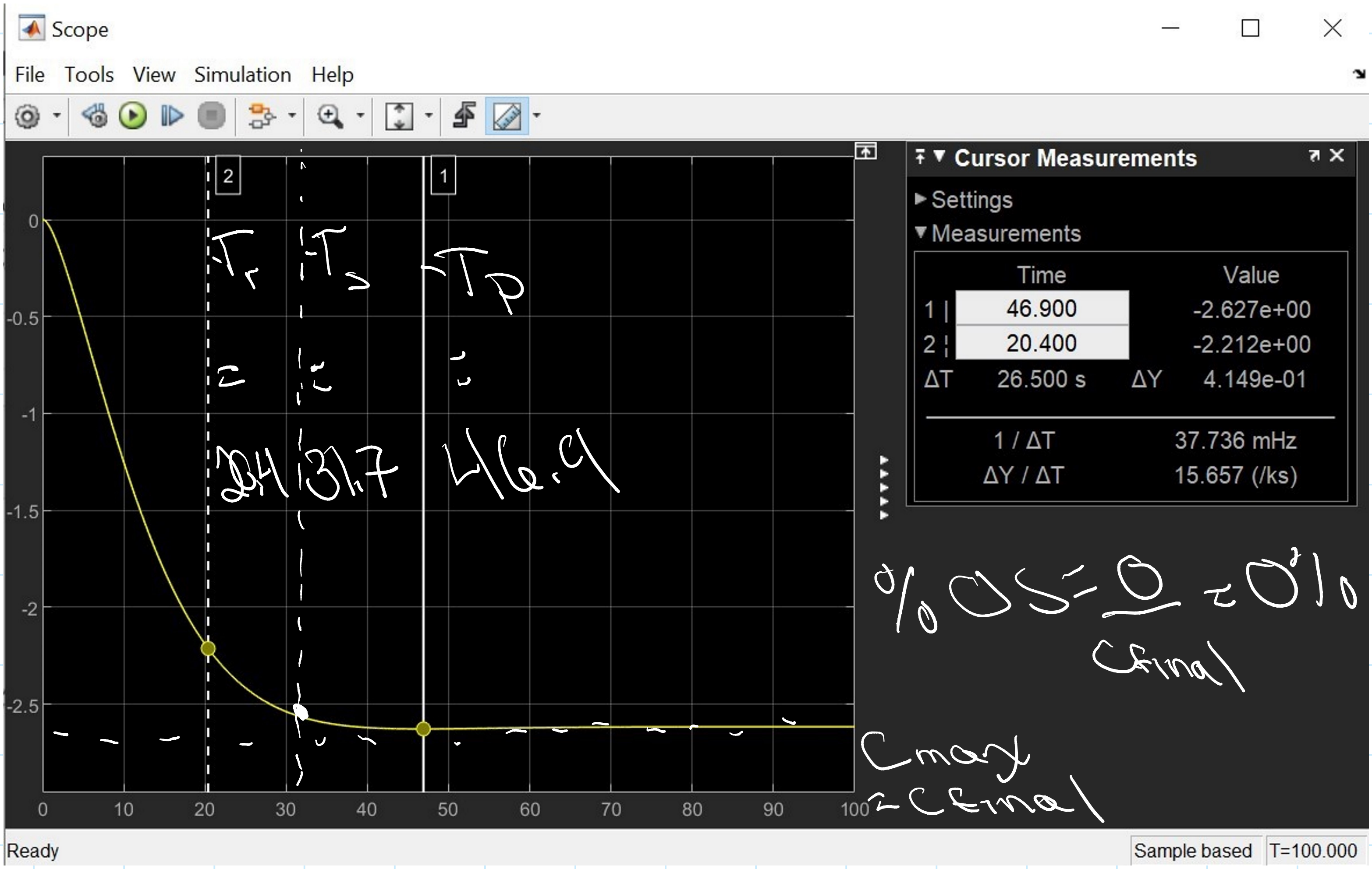
auto

State Name: (e.g., 'position')

''

OK  Cancel  Help  Apply






Theoretical

Approximate as Second Order

$$G(s) = \frac{-0.125(s+0.435)}{(s+1.23)(s^2+0.226s+0.0164)}$$

Poles $s_1 = -1.23$ Zeros $s_4 = -0.435$

$$s_{2,3} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \underline{0.226} \pm \underline{\sqrt{-.0165}}$$

$$\frac{\quad}{2} \qquad \frac{\quad}{2}$$

$= 0.113 \pm 0.129i$

Approximate as

$$G(s) = \frac{-0.125K}{(s^2+0.226s+0.0169)}$$

since $S_{1,2}$ dominates / closer to origin than $S_{2,3}$

$2\zeta\omega_n = 0.226 \qquad \omega_n^2 = 0.0169$

$\zeta = 0.869 \qquad \omega_n = 0.13$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 48.4$$

$$T_s = \frac{4}{\zeta\omega_n} = 35.4 \qquad K = \frac{Peak_1}{Peak_2}$$

$$\%OS = 100 \times e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.399\%$$

$\omega_n T_r = 2.75 \leftarrow$ From Plot $T_r = 21.2$

$$Peak_2 = f(\infty) = \lim_{s\to 0} sF(s) =$$

$$\lim_{s\to\infty} \frac{s}{s}\left(\frac{-0.125}{s^2+0.226s+0.0169}\right) = \frac{-0.125}{0.0169}$$

$5500\ S\ (S^2+0.226s+0.0169)\quad 0.0169$

$= -7.41$

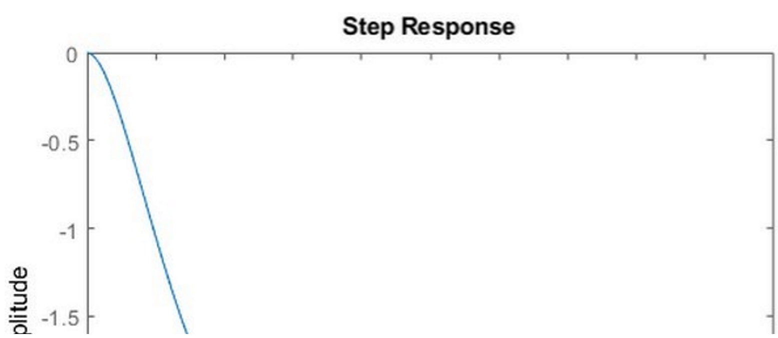
$$K = \frac{Peak_1}{Peak_2} = \frac{-2.6269}{-7.426} = 0.354$$

$$G(S) = \frac{-0.0442}{(s^2+0.226s+0.0169)}$$

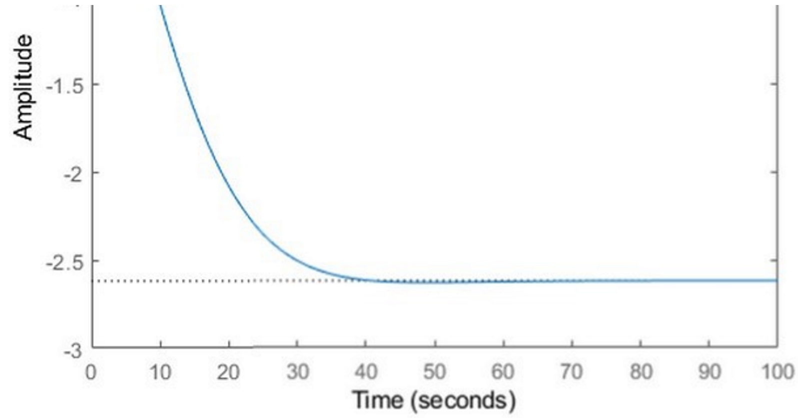
MATLAB

```
K = 0.354
sys = tf(K*[-0.125],[1 0.226 0.0169])
step(sys,100);
stepinfo(sys)
```

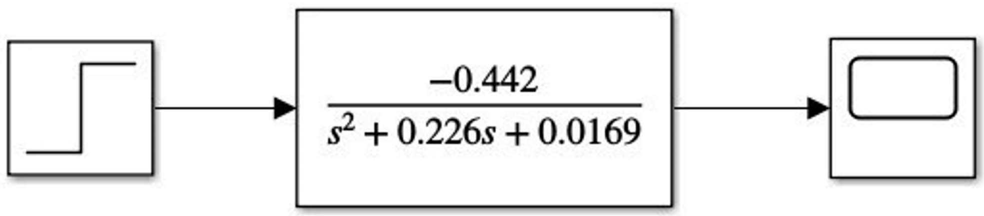
```
K =

    0.3540


sys =

          -0.04425
  ----------------------
  s^2 + 0.226 s + 0.0169

Continuous-time transfer function.


ans =

  struct with fields:

        RiseTime: 21.1397
    SettlingTime: 33.6748
     SettlingMin: -2.6288
```

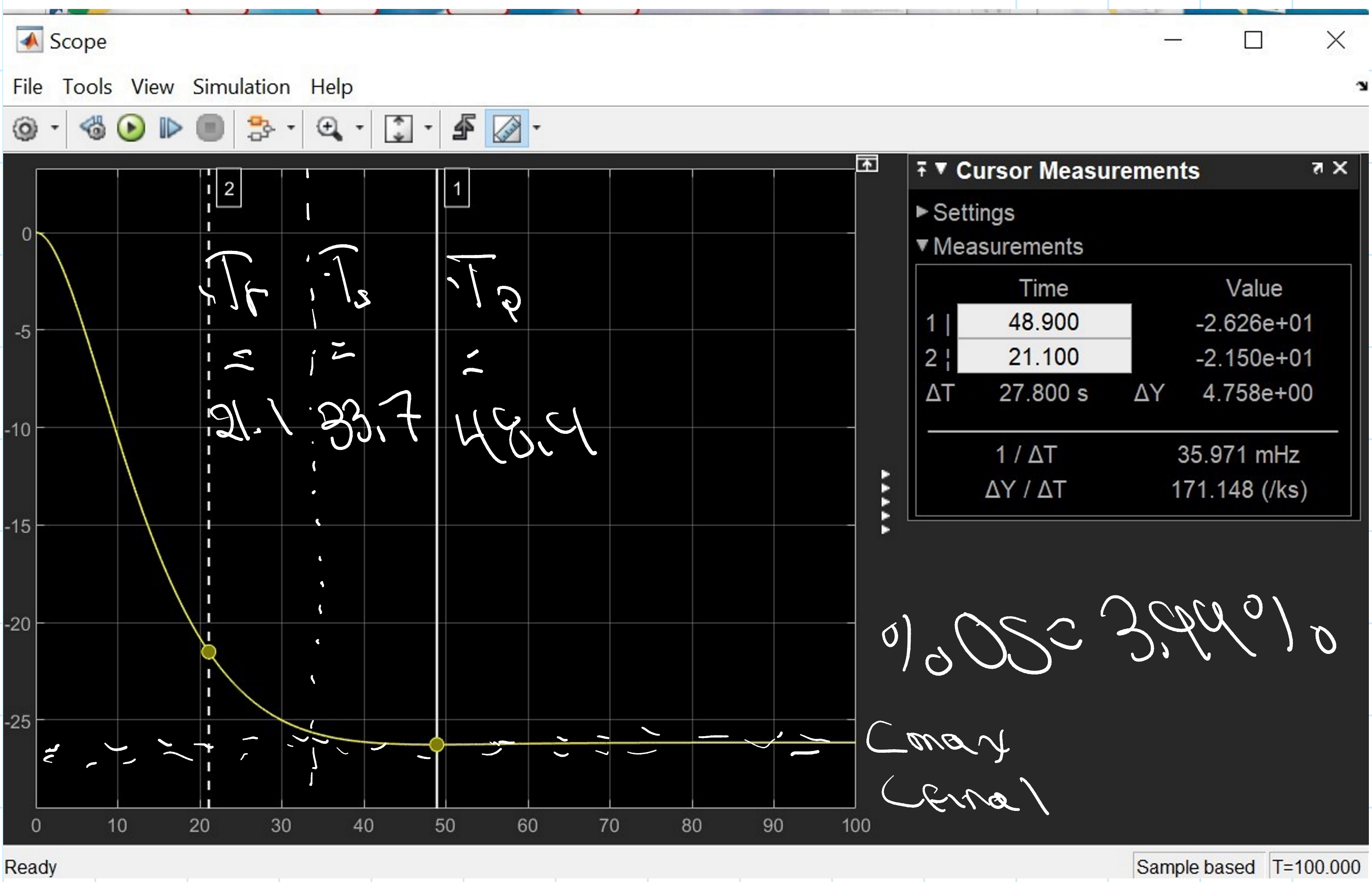


```
    RiseTime: 21.1397
SettlingTime: 33.6748
 SettlingMin: -2.6288
 SettlingMax: -2.3618
   Overshoot: 0.3993
  Undershoot: 0
        Peak: 2.6288
    PeakTime: 48.9045
```




Simulink



Comparison

| True Response | Approximate Response |
| --- | --- |
| $\%OS = 0\%$ | $\%OS \approx 3.99\%$ |
| $T_r = 20.41$ s | $T_r \approx 21.1$ s |
| $T_p = 46.4$ s | $T_p \approx 48.9$ s |
| $T_s = 31.7$ s | $T_s \approx 33.7$ s |

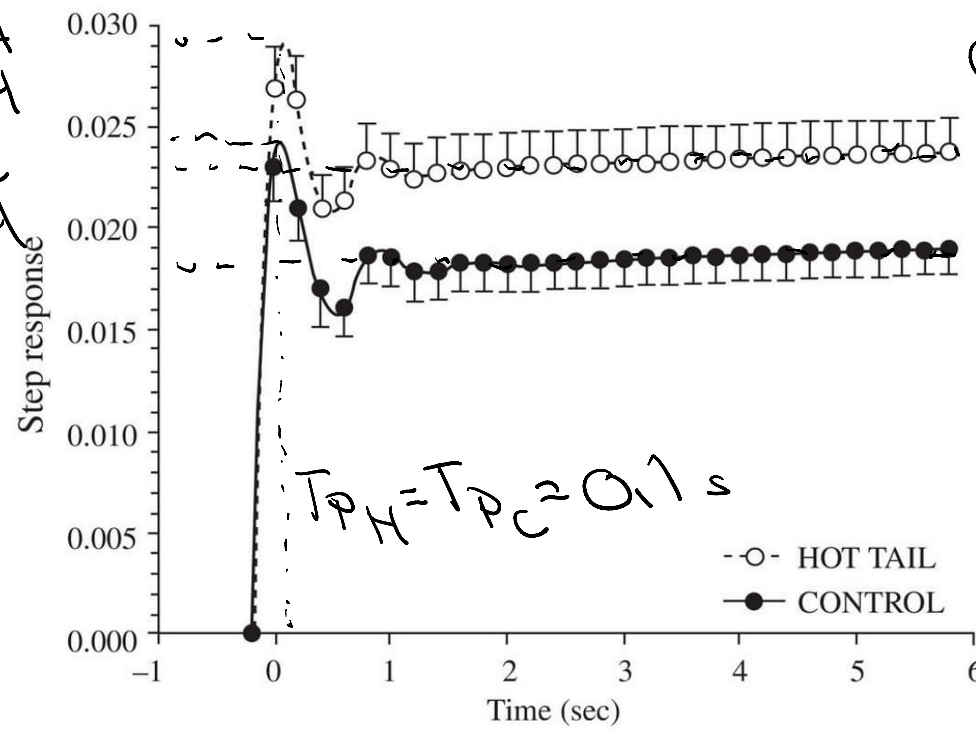
2. (Nise, 7th Edition) Several factors affect the working of the kidneys. The figure below shows how a step change in arterial flow pressure affects renal blood flow in rats. In the “hot tail” part of the experiment, peripheral thermal receptor stimulation is achieved by inserting the rat’s tail in heated water. The vertical lines indicate the variations between different test subjects. It has been argued in the literature that the “control” and “hot tail” responses are identical except their steady state values.

a) Using the figure below, obtain the normalized ($c_{final} = 1$) transfer function for both responses.

b) Use MATLAB to prove or disprove the assertion about the control and hot-tail responses.

$C_{max\,H} = 0.029$

$C_{max\,C} = 0.024$




$C_{final\,H} = 0.023$

$C_{final\,C} = 0.018$

(a)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\%OS_H = \frac{C_{max_H} - C_{final_H}}{C_{final_H}} = \frac{0.029 - 0.023}{0.023}$$

$$= 0.261$$

$$\%OS_C = \frac{C_{max_C} - C_{final_C}}{} = \frac{0.024 - 0.018}{}$$

$$\%OS_c = \frac{C_{max_c} - C_{final_c}}{C_{final_c}} = \frac{0.024 - 0.018}{0.018}$$

$$= 0.333$$

$$\zeta = \frac{-\ln(\%OS)}{\sqrt{\pi^2 + \ln(\%OS)^2}}$$

$$\zeta_H = \frac{-\ln(0.261)}{\sqrt{\pi^2 + \ln(0.261)^2}} = 0.393$$

$$\zeta_c = \frac{-\ln(0.333)}{\sqrt{\pi^2 + \ln(0.333)^2}} = 0.330$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

$$\omega_{n_H} = \frac{\pi}{(0.1)\sqrt{1-0.393^2}} = 34.2$$

$$\omega_{n_c} = \frac{\pi}{(0.1)\sqrt{1-0.330^2}} = 33.3$$

$$G_H(s) = \frac{34.2^2 \, K}{s^2 + 2(0.393)(34.2)s + 34.2^2} \qquad K = 1 \text{ (Normalized)}$$

$$\boxed{= \frac{1167.5}{s^2 + 26.9s + 1167.5}}$$ $K = C_{final\,H} = 0.023$

$$f_H(\infty) = \lim_{s \to 0} sF(s) = \lim_{s \to \infty} \frac{s}{s}\left(G_H(s)\right) = 1$$

$$G_H(s) = \frac{26.9}{s^2 + 26.9s + 1167.5}$$ (Non-normalized)

$$G_C(s) = \frac{33.3^2 K}{s^2 + 2(0.330)(33.3)s + 33.3^2}$$ $K = 1$ (Normalized)

$$= \boxed{\frac{1107.7}{s^2 + 21.97 + 1107.7}}$$ $K = C_{final\,C} = 0.018$

$$f_C(\infty) = \lim_{s \to 0} sF(s) = \lim_{s \to \infty} \frac{s}{s}\left(G_H(s)\right) = 1$$

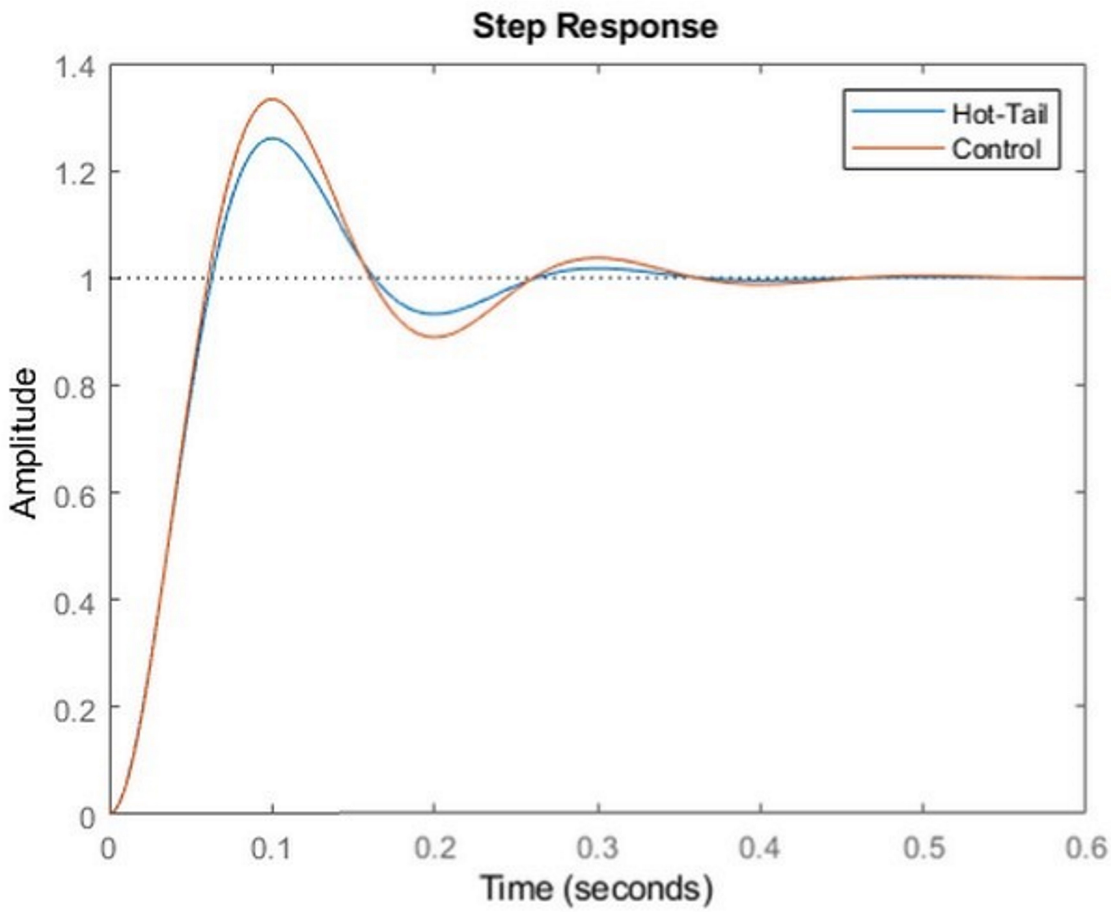
$$G_C(s) = \frac{19.9}{s^2 + 21.97 + 1107.7}$$ (Non-normalized)

(b)

```
sys_hot_tail = tf([1167.5],[1 26.9 1167.5])
sys_control = tf([1107.7],[1 21.97 1107.7])
step(sys_hot_tail,sys_control)
legend({'Hot-Tail','Control'})
hot_tail = stepinfo(sys_hot_tail)
control = stepinfo(sys_control)
```

```
sys_hot_tail =

         1168
  -------------------
  s^2 + 26.9 s + 1168

Continuous-time transfer function.


sys_control =

          1108
  --------------------
  s^2 + 21.97 s + 1108

Continuous-time transfer function.
```

```
hot_tail =

  struct with fields:

        RiseTime: 0.0426
    SettlingTime: 0.2461
     SettlingMin: 0.9271
     SettlingMax: 1.2604
       Overshoot: 26.0395
      Undershoot: 0
            Peak: 1.2604
        PeakTime: 0.0993


control =

  struct with fields:

        RiseTime: 0.0410
    SettlingTime: 0.3341
     SettlingMin: 0.8889
     SettlingMax: 1.3333
       Overshoot: 33.3314
      Undershoot: 0
            Peak: 1.3333
        PeakTime: 0.1006
```




∴ From the normalized analysis we disprove the assertion "control and hot-tail responses are 'identical' besides their steady state values". One characteristic that differs apparent from the graph is max peak. Other characteristics max peak. Other characteristics from the step response can be deducted.