Studio 10

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ENME462
Section 0105

Bode Plot Examples

A. Consider the following transfer functions:

$$G_1(s) = \frac{1}{s(s+2)(s+4)}, \qquad G_2(s) = \frac{s+5}{(s+2)(s+4)}, \qquad G_3(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$$

1. Find analytical expressions for the magnitude and phase responses associated with each of the above transfer functions.

2. Sketch the Bode asymptotic magnitude and phase plots for each of the above transfer functions.

3. Use MATLAB's 'bode' command to create the Bode magnitude and phase plots for each of the above transfer functions, and compare them with your sketches.

1.

$$G_1(s) = \frac{1}{s(s+2)(s+4)} = \frac{1}{s^3+6s^2+8s}$$

$$G_1(j\omega) = \left.\frac{1}{s(s+2)(s+4)}\right|_{s \to j\omega}$$

$$= \frac{1}{j\omega(j\omega+2)(j\omega+4)} = \frac{1}{-6\omega^2 + j\omega(8-\omega^2)}$$

$$M_1(\omega) = |G_1(j\omega)| =$$

$$\boxed{\frac{1}{\sqrt{(6\omega^2)^2 + (\omega(8-\omega^2))^2}}}$$

$$\Phi_1(\omega) = \angle G(j\omega) = \angle(1)$$

$$= \angle(-6\omega^2 + j\omega(8-\omega^2))$$

$$= \tan^{-1}(0/1) - \tan^{-1}\left(\frac{\omega(8-\omega^2)}{-6\omega^2}\right)$$

$$\boxed{= \tan^{-1}(0) - \tan^{-1}\left(\frac{(8-\omega)^2}{-6\omega}\right)}$$

2. $G_1(s) = \frac{1}{s(s+2)(s+4)} = \frac{\frac{1}{8}}{s(\frac{s}{2}+1)(\frac{s}{4}+1)}$

$s = 0.1j \qquad G_1(0.1j) = \frac{1/8}{0.1j}$

$= -1.25j$

$dB(M_0) = 20\log 1.25 = 1.94$

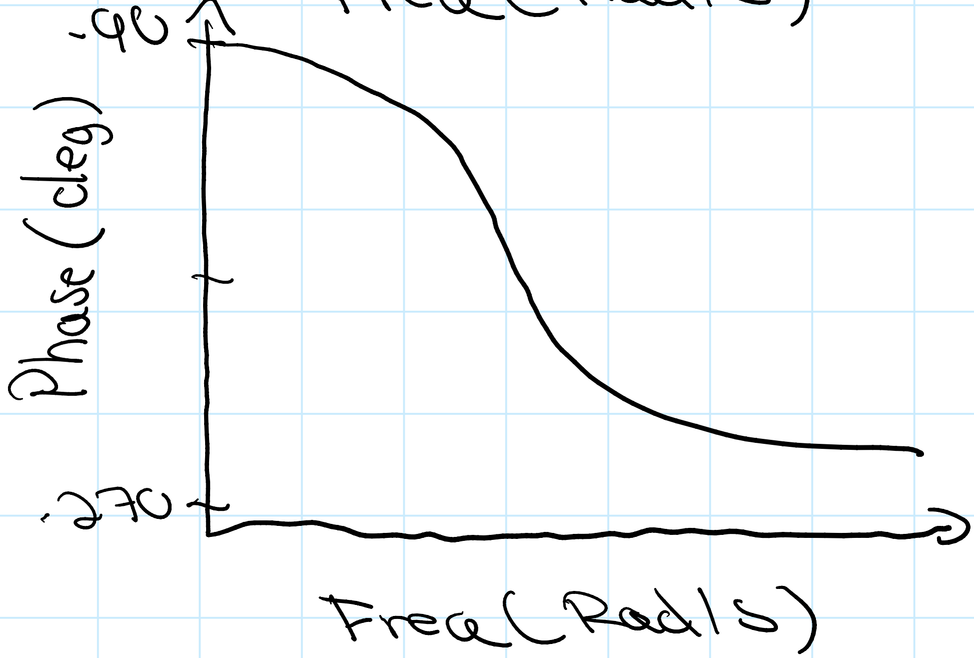
$\Phi_0 = -90°$

Magnitude Table

Description	Frequency		
	$0.1 < \omega < 2$	$2 < \omega < 4$	$\omega > 4$
$\frac{1}{s}$	-20	-20	-20
$1/\frac{s}{2}+1$	0	-20	-20
$1/\frac{s}{4}+1$	0	0	-20
Total Slope (dB/dec)	**-20**	**-40**	**-60**

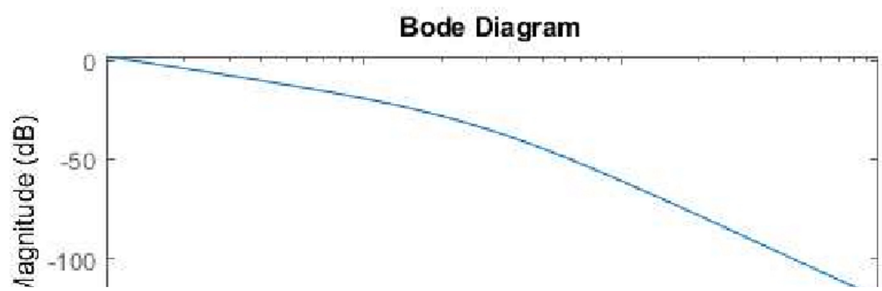
Phase Table

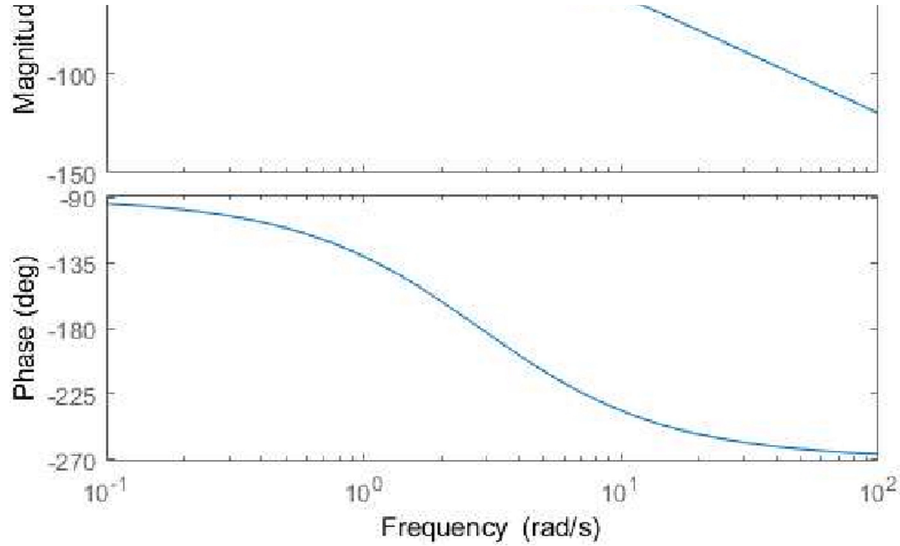
Description	Frequency			
	$0.2 < \omega < 0.4$	$0.4 < \omega < 20$	$20 < \omega < 40$	$\omega > 40$
$\frac{1}{s}$	0(-90°)	0(-90°)	0(-90°)	0(-90°)
$1/\frac{s}{2}+1$	-45	-45	0	0
$1/\frac{s}{4}+1$	0	-45	-45	0
Total Slope (dB/dec)	**-45**	**-90**	**-45**	**0**



3.

```
G1= zpk([],[0,-2,-4],1);
bode(G1);
```





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1.

$$G_2(s) = \frac{s+5}{(s+2)(s+4)} = \frac{s+5}{s^2+6s+8}$$

$$G_2(j\omega) = \left.\frac{s+5}{(s+2)(s+4)}\right|_{s\to j\omega}$$

$$= \frac{j\omega+5}{\ldots} = \frac{j\omega+5}{\ldots}$$

$$= \frac{j\omega+5}{(j\omega+2)(j\omega+4)} = \frac{j\omega+5}{-\omega^2+6j\omega+8}$$

$$= \frac{j\omega+5}{8-\omega^2+6j\omega}$$

$$M_2(\omega) = |G_2(j\omega)|$$

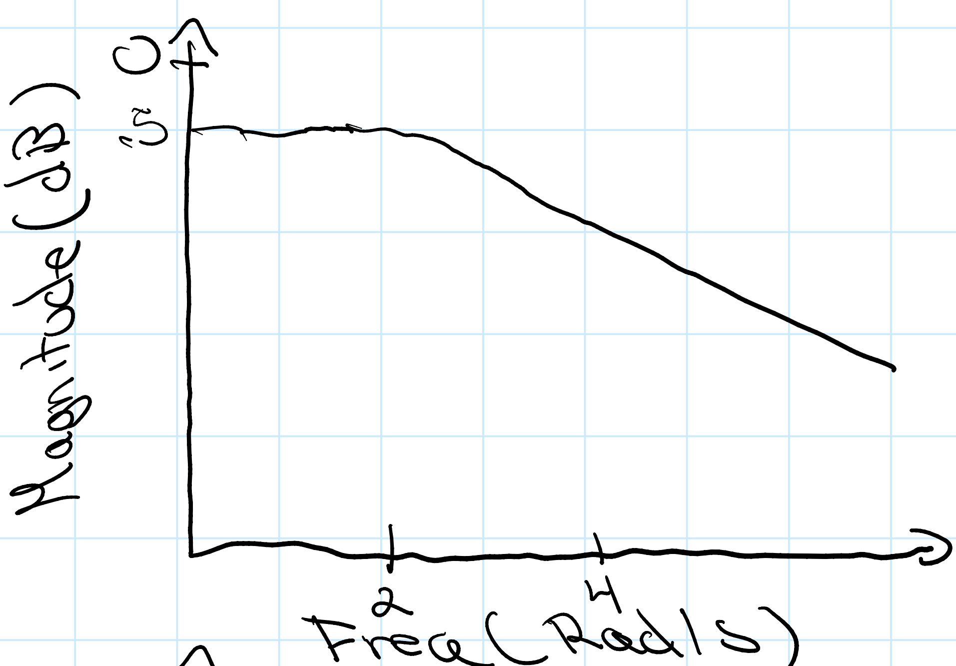
$$= \boxed{\frac{\sqrt{(5)^2+(\omega)^2}}{\sqrt{(8-\omega^2)^2+(6\omega)^2}}}$$

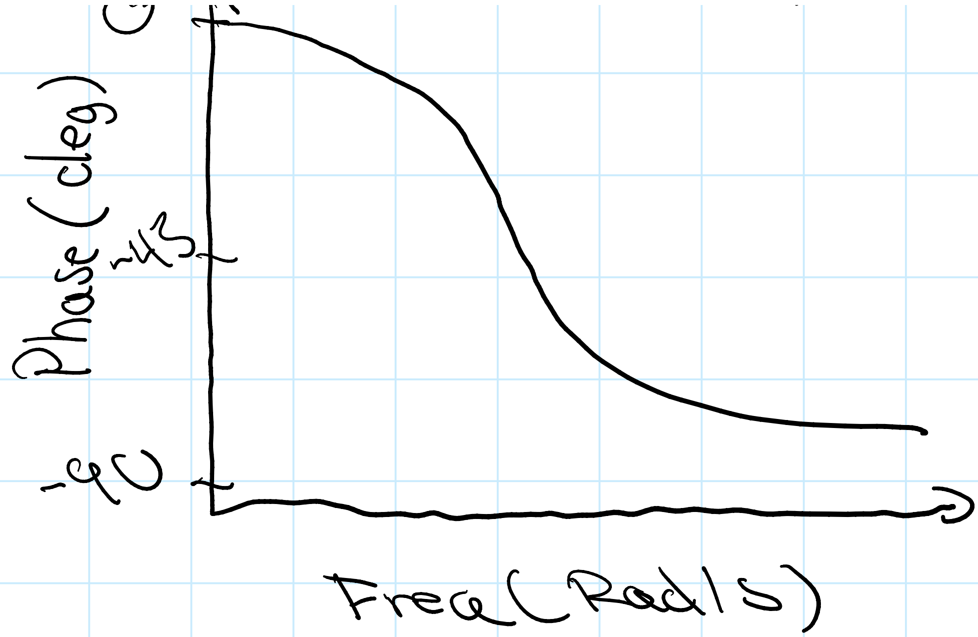
$$\phi_2(\omega) = \angle G(j\omega) = \angle(j\omega+5)$$

$$- \angle(8-\omega^2+6j\omega)$$

$$= \boxed{\tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right)}$$

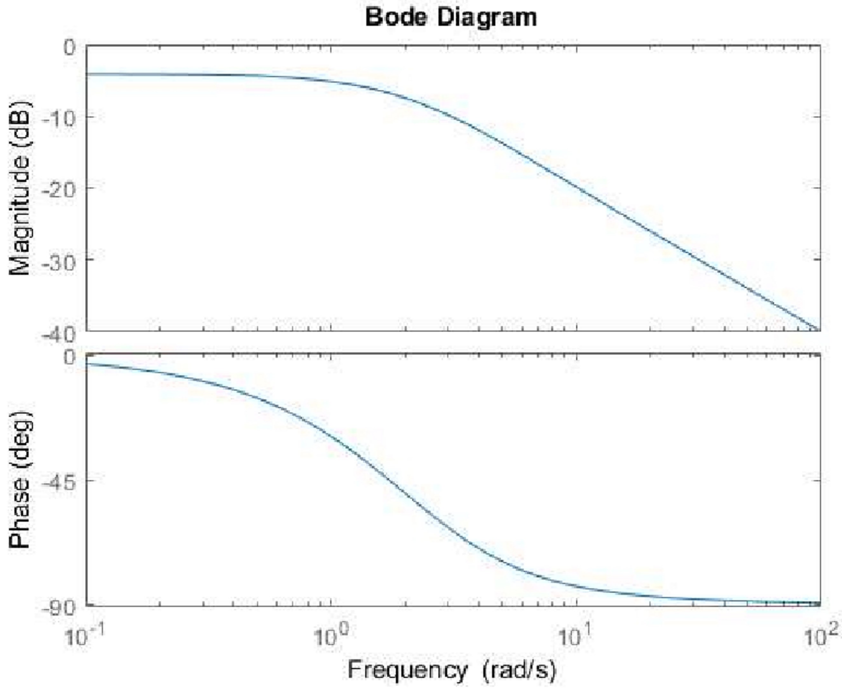
2.





3.

```
G2= zpk([-5],[-2,-4],1);
bode(G2);
```



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1. $G_3(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$

$= \frac{s^2+8s+15}{s^3+6s^2+8s}$

$G_3(j\omega) = \left.\frac{(s+3)(s+5)}{s(s+2)(s+4)}\right|_{s \to j\omega}$

$= \frac{(j\omega+3)(j\omega+5)}{j\omega(j\omega+2)(j\omega+4)} =$

$\frac{15-\omega^2+8j\omega}{-6\omega^2+j\omega(8-\omega^2)}$

$M_3(\omega) = |G_3(j\omega)|$

$= \boxed{\frac{\sqrt{(15-\omega^2)^2+(8\omega)^2}}{\sqrt{(6\omega^2)^2+(\omega(8-\omega^2))^2}}}$

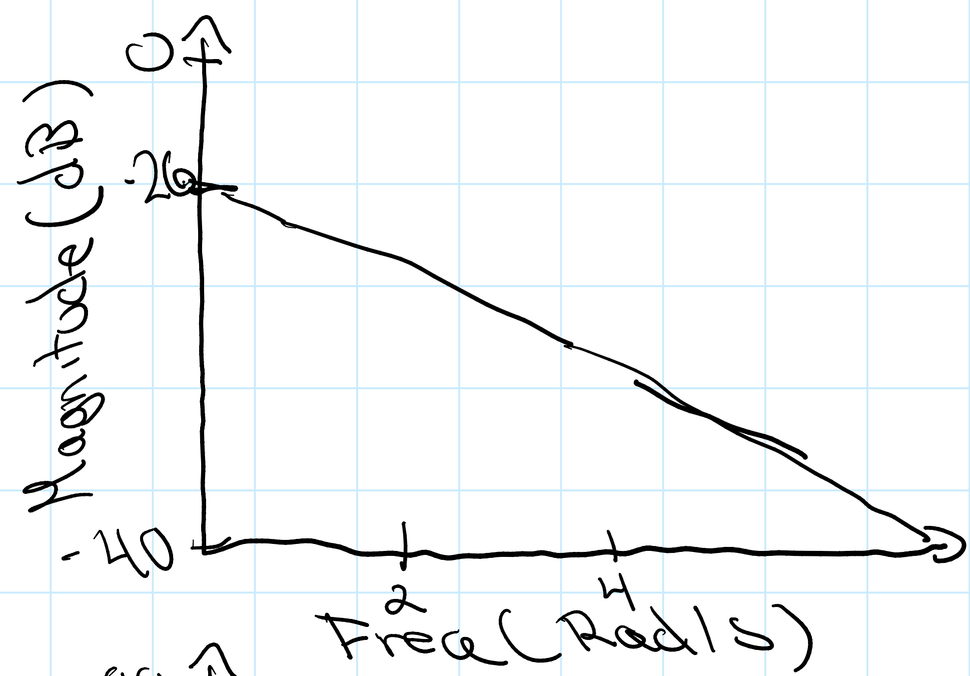
$\Phi_3(\omega) = \angle G(j\omega) = \angle(15-\omega^2+8j\omega)$

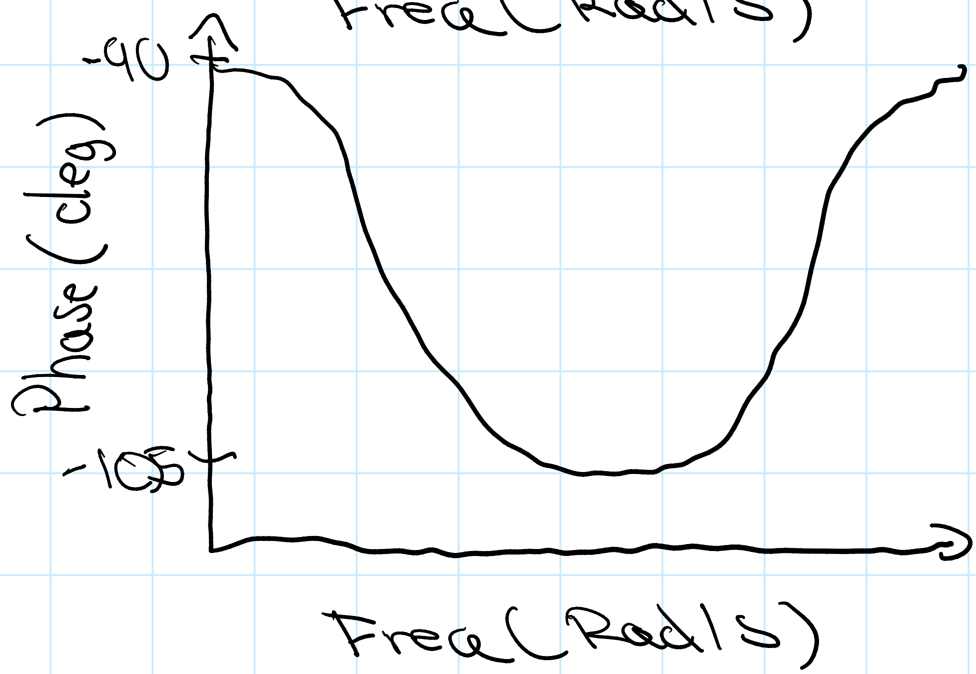
$- \angle(-6\omega^2+j\omega(8-\omega^2))$

$= \tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) - \tan^{-1}\left(\frac{\omega(8-\omega^2)}{-6\omega^2}\right)$

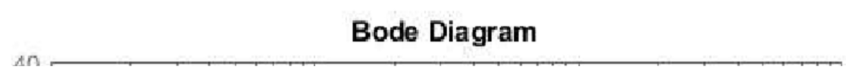
$$= \boxed{\tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) - \tan^{-1}\left(\frac{(8-\omega)^2}{-6\omega}\right)}$$

2.

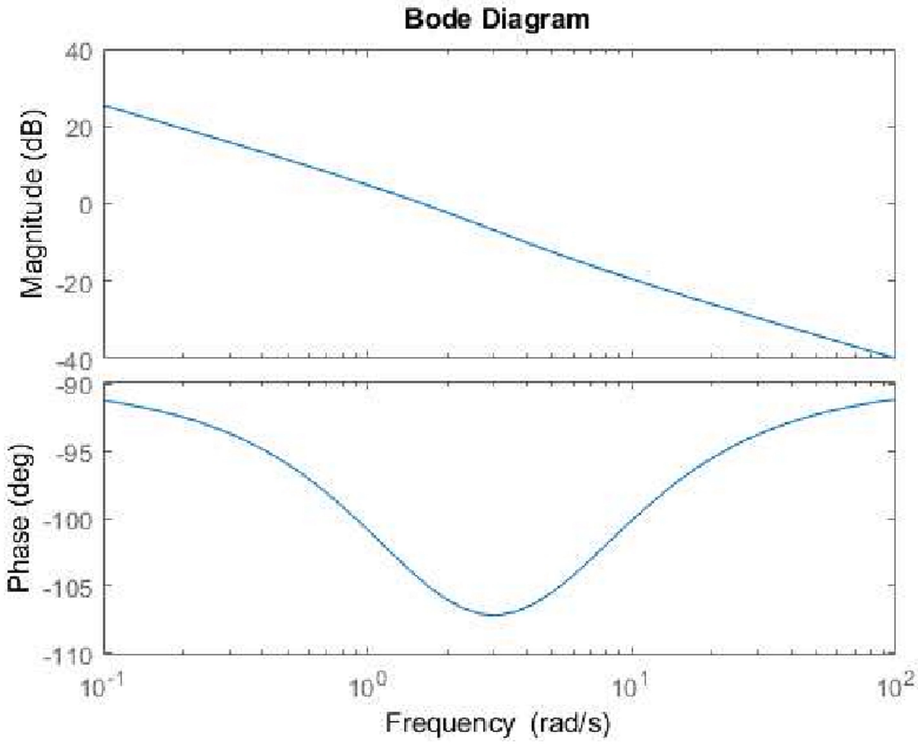





```
G3= zpk([-3,-5],[0,-2,-4],1);
bode(G3);
```



```
G3= zpk([-3,-5],[0,-2,-4],1);
bode(G3);
```



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B. The open-loop dynamics from DC voltage armature to angular position of a robotic manipulator joint is given by

$$P(s) = \frac{48500}{s^2 + 2.89s}$$

Draw by hand the Bode plot using the asymptotic approximations (both for magnitude and angle). Then use MATLAB to plot the exact Bode plot and compare with your sketch.

$$P(s) = \frac{485000}{s(s+2.89)} = \frac{485000}{s^2+2.89s}$$

$$P(j\omega) = \left.\frac{485000}{s(s+2.89)}\right|_{s \to j\omega}$$

$$= \frac{485000}{j\omega(j\omega+2.89)} = \frac{485000}{-\omega^2+2.89j\omega}$$

$$M(\omega) = |P(j\omega)|$$

$$= \boxed{\frac{\sqrt{485000}}{\sqrt{(\omega^2)^2+(2.89\omega)^2}}}$$

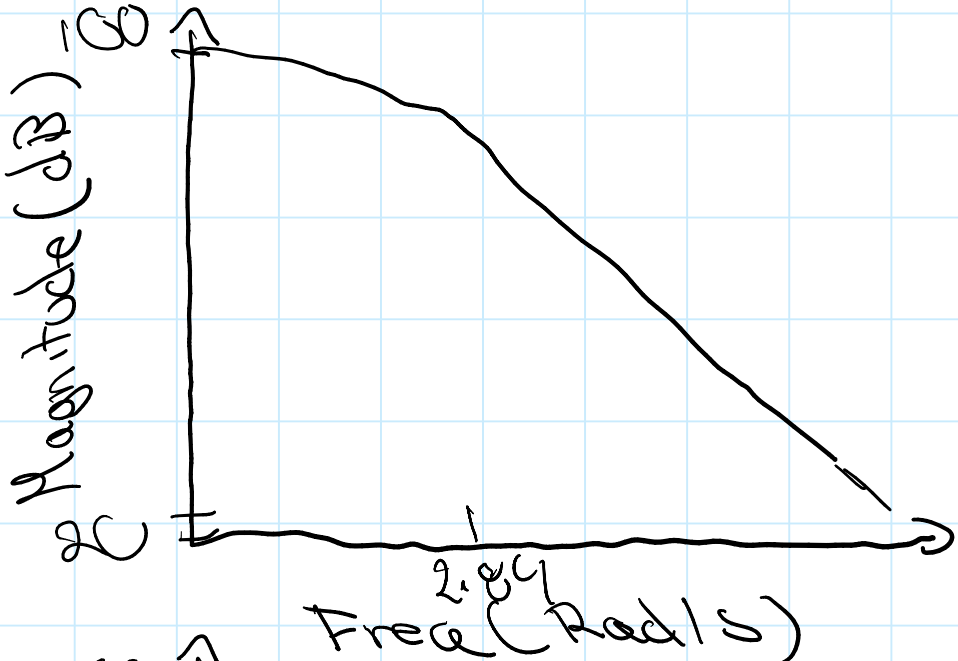
$$\phi = \angle P(j\omega)$$

$$= \angle(485000) - \angle(-\omega^2+2.89j\omega)$$

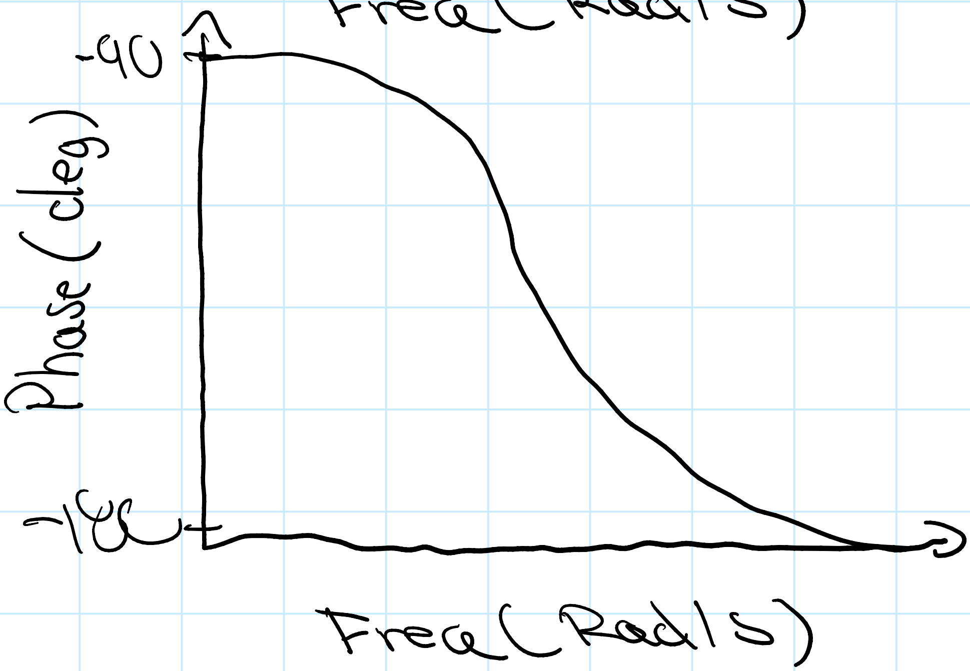
$$= \tan^{-1}\left(\frac{0}{48500}\right) - \tan^{-1}\left(\frac{2.89\omega}{-\omega^2}\right)$$

$$= \boxed{\tan^{-1}(0) - \tan^{-1}\left(\frac{2.89}{-\omega}\right)}$$

100

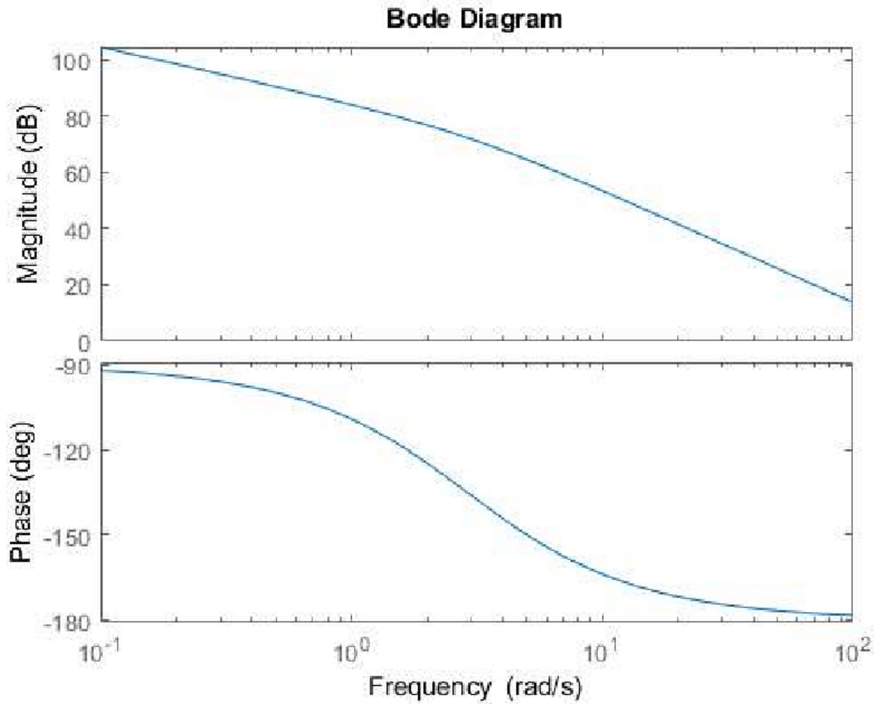


Magnitude (dB)
180
20
2.84
Freq (Rad/s)



Phase (deg)
-90
-180
Freq (Rad/s)

```
Ps= 48500*zpk([],[0,-2.89],1);
bode(Ps);
```



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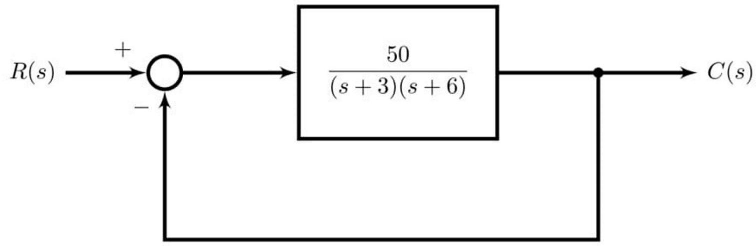
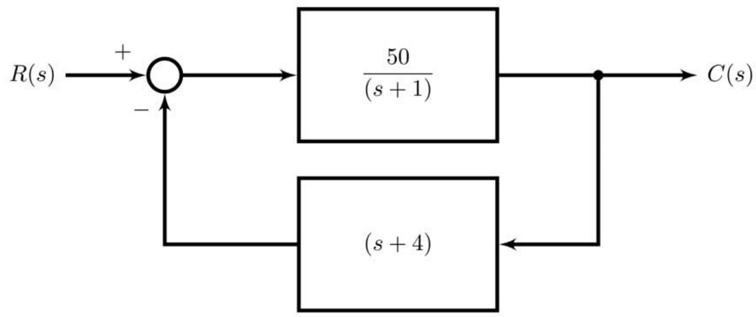
Nyquist Plot Examples

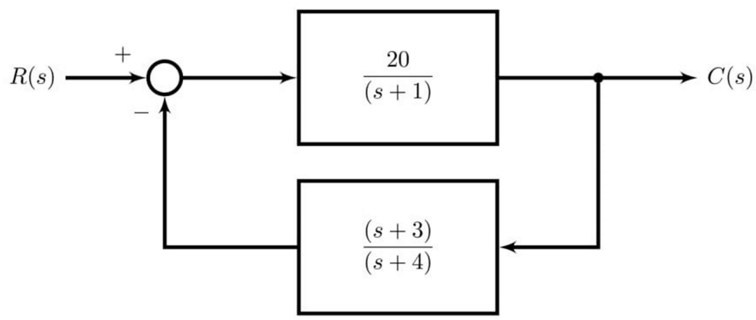
A. Sketch the Nyquist diagram for each of the systems below. To aid in your sketch, find for each system:
   1. The rationalized frequency response of the loop gain $F(j\omega) = G(j\omega)H(j\omega)$.
   2. The low-frequency response: $\lim_{\omega \to 0} F(j\omega)$
   3. The high-frequency response: $\lim_{\omega \to +\infty} F(j\omega)$
   4. The real-axis intercepts: $\{\omega \geq 0 \mid Imag(F(j\omega)) = 0\}$
   5. The imaginary-axis intercepts: $\{\omega \geq 0 \mid Real(F(j\omega)) = 0\}$



System 1



System 2



System 3

S1.

$$G(s)H(s) = \frac{50}{(s+3)(s+6)}$$

$$F(j\omega) = G(j\omega)H(j\omega)$$

$$= \frac{50}{(j\omega+3)(j\omega+6)}$$

$$= \frac{50}{(18-\omega^2) + 9j\omega} = \frac{50\left((18-\omega^2) - 9j\omega\right)}{(18-\omega^2)^2 + 81\omega^2}$$

$$\lim_{\omega \to 0} F(j\omega) = \frac{50}{18} = \frac{25}{9} = \boxed{2.78}$$

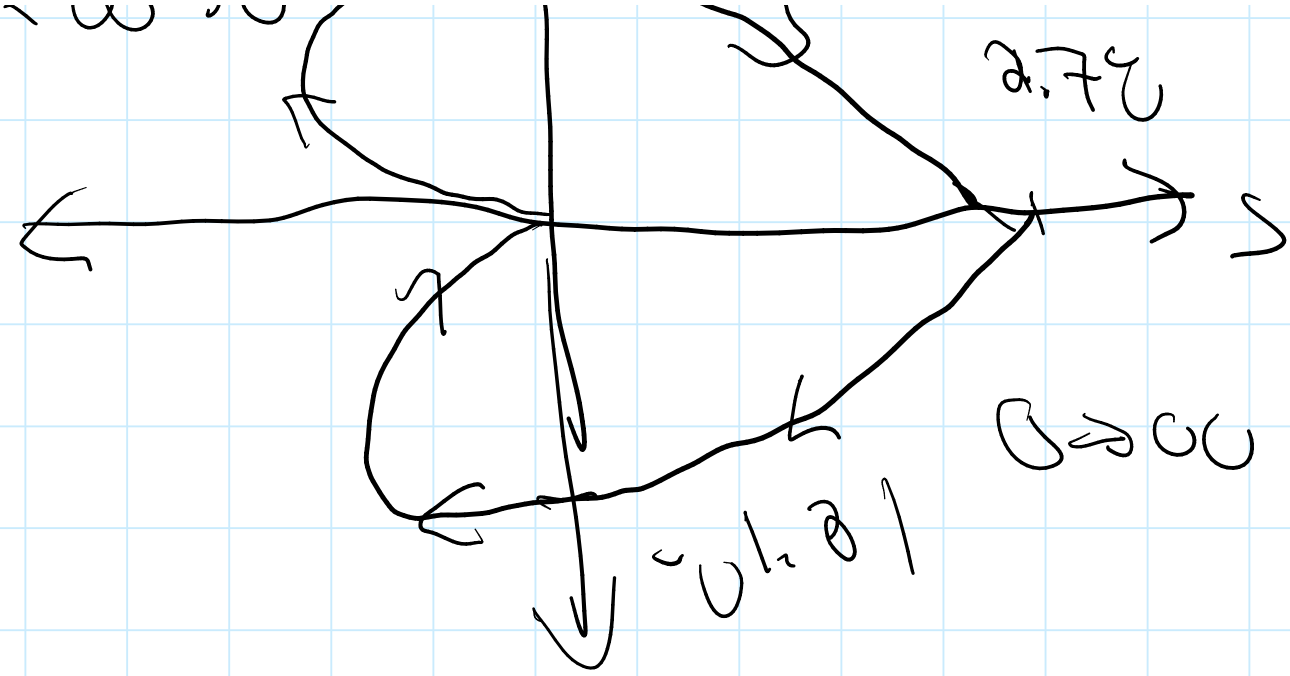
$$\lim_{\omega \to \infty} F(j\omega) = \frac{-50}{\omega^2} = \boxed{0\angle 180^\circ}$$

$$\text{Imag}(F(j\omega)) = 0$$

$$9\omega = 0 \qquad \omega = \boxed{0}$$

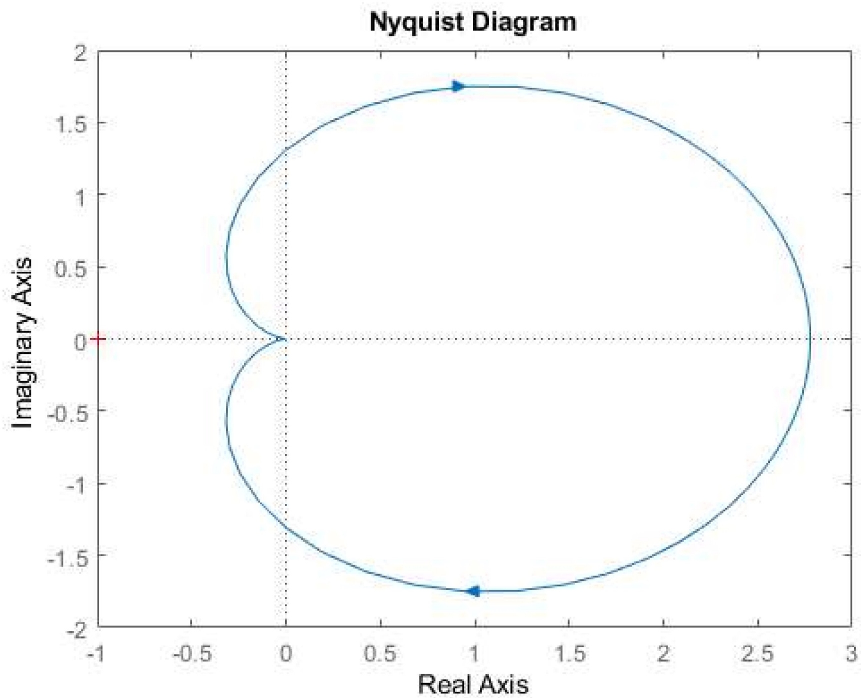
$$\text{Real}(F(j\omega)) = \boxed{0}$$

$$18-\omega^2 = 0 \qquad \boxed{\begin{aligned} \omega &= \pm 3\sqrt{2} \\ &\approx \pm 4.241 \\ F(j4.24) &= \pm j1.31 \end{aligned}}$$



2.78
0→∞
-0.481

```
S1= zpk([],[-3,-6],50);
nyquist(S1);
```



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Q2.

$G(s)H(s) = \frac{50(s+4)}{\ldots}$

$G(j\omega)H(j\omega) = \frac{50(s+4)}{(s+1)}$

$F(j\omega) = G(j\omega)H(j\omega)$

$= \frac{50(j\omega+4)}{(j\omega+1)}$

$= \frac{50(j\omega+4)}{(j\omega+1)} \cdot \frac{(j\omega-1)}{(j\omega-1)} = \frac{50((\omega^2-4)+3j\omega)}{1-\omega^2}$

$\lim_{\omega \to 0} F(j\omega) = \frac{200}{1} = \boxed{200}$

$\lim_{\omega \to \infty} F(j\omega) = \frac{-200}{\omega^2} = \boxed{0\angle 180^\circ}$

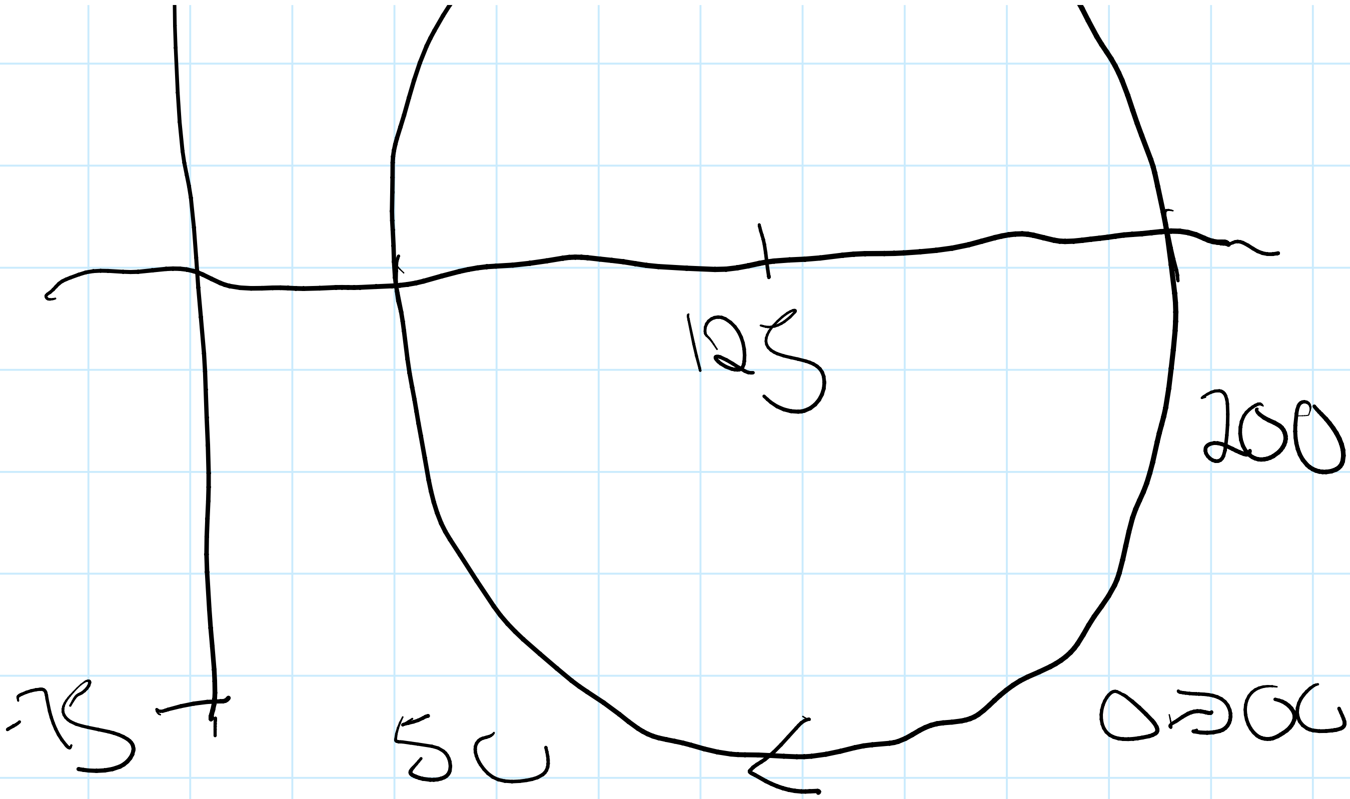
$\text{Imag}(F(j\omega)) = 0$

$3\omega = 0 \qquad \omega = \boxed{0}$

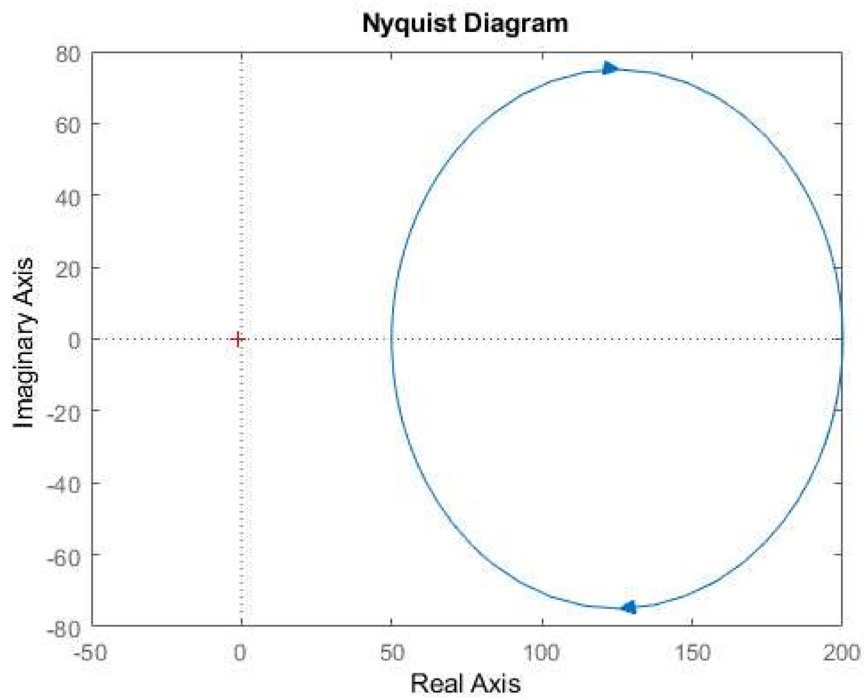
$\text{Real}(F(j\omega)) = \boxed{0}$

$-\omega^2-4 = 0 \qquad \omega = \pm 2$

$F(2j) = 75j$


125
200
-75
50
0200

```
S2= zpk([-4],[-1],50);
nyquist(S2);
```



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S3:

$G(s)H(s) = 2(s+3)$

$$G(s)H(s) = \frac{20(s+3)}{(s+1)(s+4)}$$

$$F(j\omega) = G(j\omega)H(j\omega)$$

$$= \frac{20(j\omega+3)}{(j\omega+1)(j\omega+4)}$$

$$= \frac{20(j\omega+3)}{(-\omega^2+4)+5j\omega} = \frac{20(j\omega+3)((-\omega^2+4)-5j\omega)}{(-\omega^2+4)^2-25\omega^2}$$

$$\lim_{\omega \to 0} F(j\omega) = \frac{20(3)(4)}{(16)} = \boxed{15}$$

$$\lim_{\omega \to \infty} F(j\omega) = \frac{20}{-25\omega^2} = \boxed{0\angle 180^\circ}$$

$$\text{Imag}(F(j\omega)) = 0$$

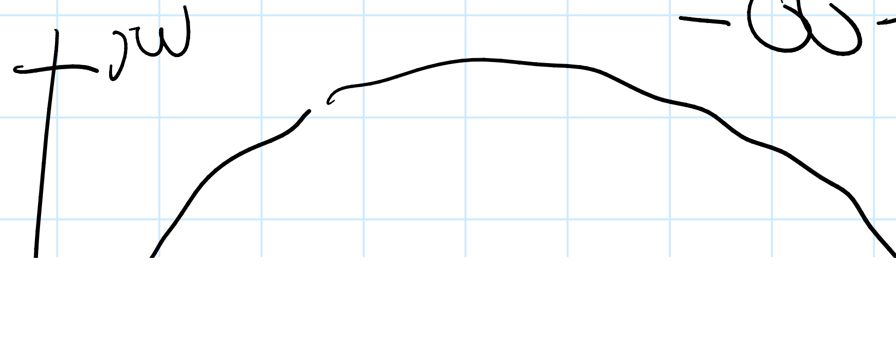
$$-\omega^2+4-15\omega = \boxed{16}$$

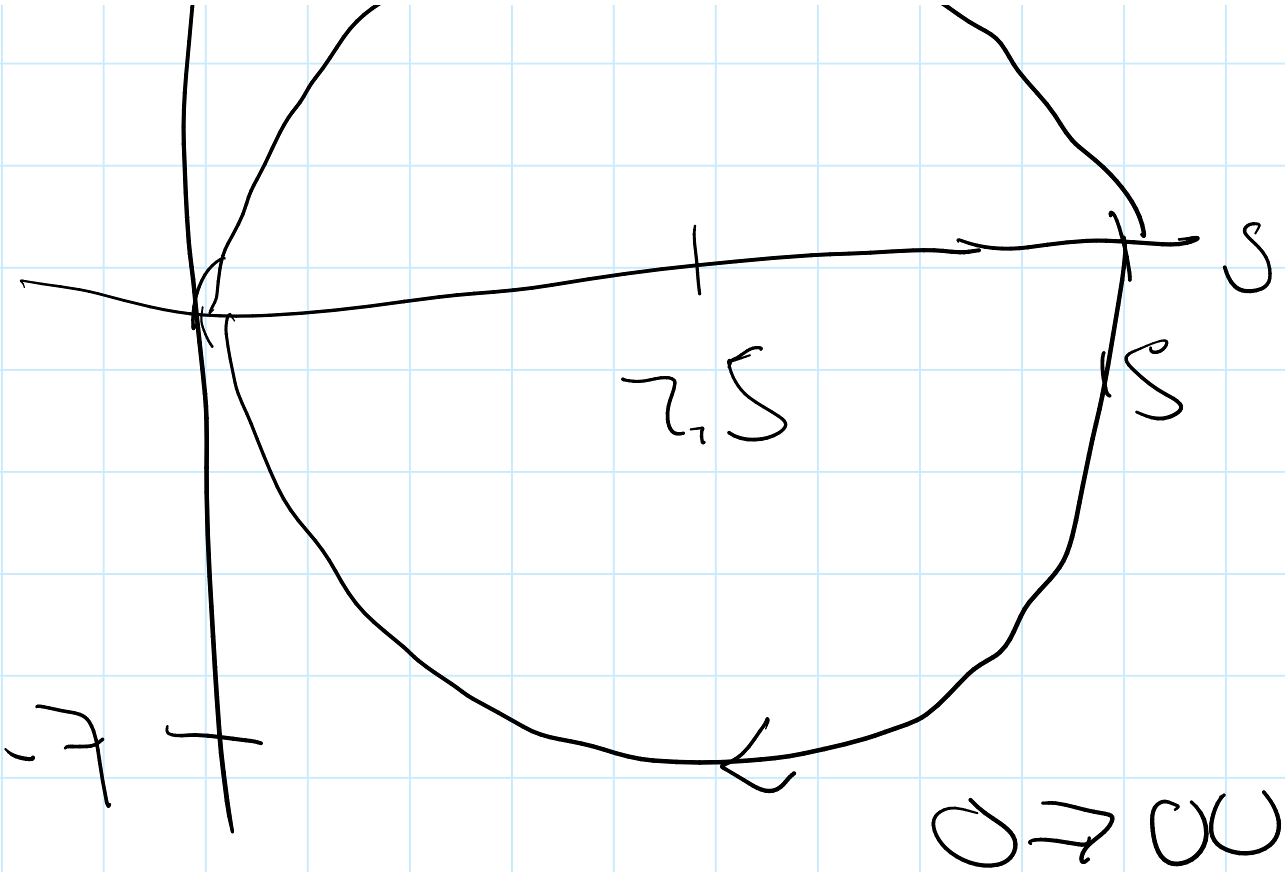
$$\text{Real}(F(j\omega)) = \boxed{0}$$

$$\omega = 0$$

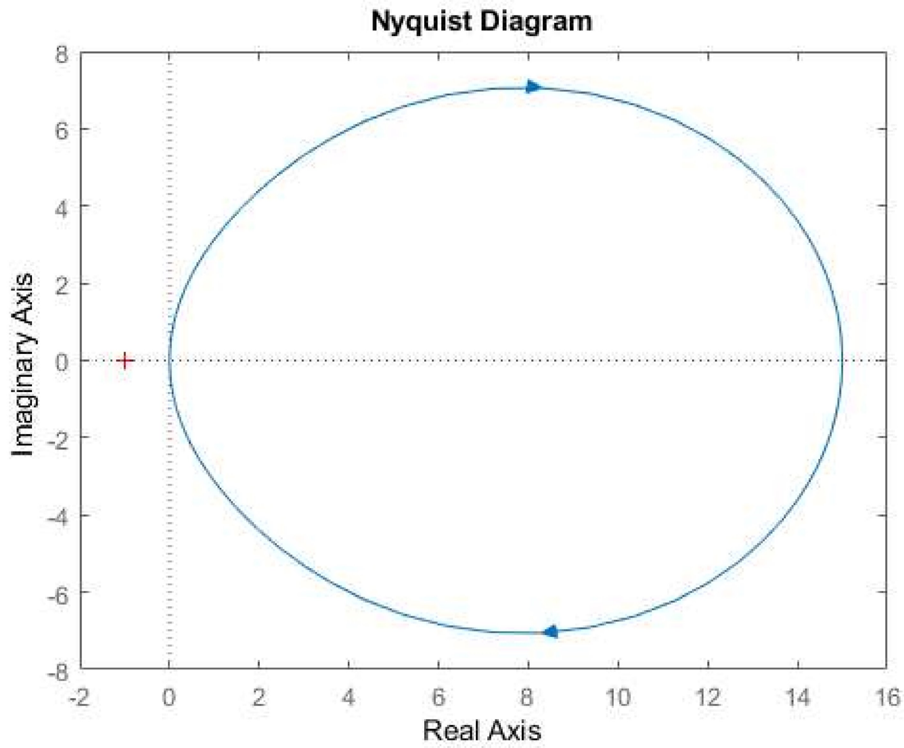
7

$-\infty \to 0$




s
7,5
15
-7
0700

```
S3= zpk([-3],[-1,-4],20);
nyquist(S3);
```



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