Studio 05

2 LAB PROCEDURE

Let us assume that in an earlier activity you have mathematically derived the voltage-to-position transfer function of the DC motor of the QUBE-Servo 2 based on its equations of motion as follows:

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{23.8}{s(0.1s+1)} \tag{10}$$

$K = 23.8$

$\tau = 0.1$

The system has a steady-state gain of K = 23.8 rad/s/V and a time constant of τ = 0.1 seconds. Let us further assume that you are required to design a PD position controller that has an overshoot of less than 5% and a peak time of no more than 0.2 seconds.

1. Using Equations 4 and 5 determine the required natural frequency (ω_n) and damping ratio (ζ) that will satisfy the overshoot and peak time requirements of the controller.
   a. What does the natural frequency of the system quantify?
      i. It is the frequency at which the system tends of oscillate when continuously subjected to an external harmonic force
      ii. It quantifies the frequency at which the system tends to oscillate in the absence of any driving force
      iii. None of the above
   b. Based on the damping ratio that you have calculated, what category best describes the system?
      i. Underdamped system
      ii. Critically damped system
      iii. Overdamped system

Q6

1. $\%OS < 5\%$ $T_p \leq 0.2$

$$\%OS = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} < 0.05$$

$$\zeta > \frac{-\ln(\%OS)}{\sqrt{\pi^2+\ln(\%OS)^2}} = \frac{-\ln(0.05)}{\sqrt{\pi^2+\ln(0.05)^2}}$$

$$\zeta > \boxed{0.69}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \leq 0.2$$

$$\omega_n > \frac{\pi}{} = \boxed{21.7 \text{ rad}}$$

$$\omega_n \geq \frac{\pi}{(0.2)(\sqrt{1-0.69^2})} = \boxed{21.7 \frac{rad}{s}}$$

a. ii

b. i

2.

2. Using Equations 8 and 9, as well as the system parameters given above, calculate the corresponding theoretical control gains k_p and k_d that will satisfy your controller requirements.
   a. What gains did you find? Select the answer that is closest to your calculations.
      i. $k_p = 0.98$ and $k_d = 0.08$
      ii. $k_p = 1.98$ and $k_d = 0.08$
      iii. $k_p = 2.98$ and $k_d = 1.32$

Q7

$$k_p = \frac{\omega_n^2 \tau}{K} = \frac{(21.7)^2(0.1)}{(23.8)} = 1.98$$

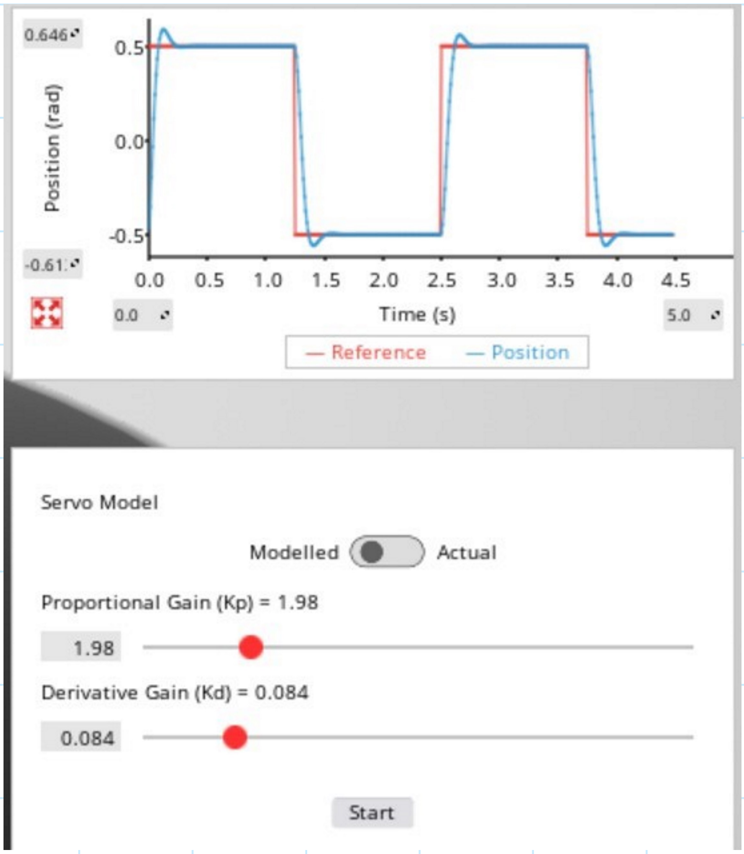
$$k_d = \frac{2\zeta\omega_n\tau - 1}{K} = \frac{2(0.69)(21.7)(0.1) - 1}{(23.8)}$$

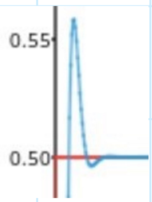
$= 0.084$

ii

3.

3. Use the simulation below to verify that your calculated gains result in the required overshoot and peak time using your modeled system as given in Equation 10. Set the toggle switch to **Modelled** and use the **gain sliders** to enter your calculated gain values. Press the **Start** button and examine the system's response to a square wave with an amplitude of +/- 0.5 rad and a frequency of 0.4 Hz.





$$\% OS = \frac{C_{max} - C_{final}}{C_{final}}$$

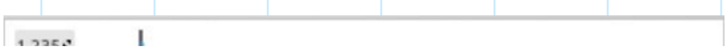
$$= \frac{0.52 - 0.5}{0.5} = 0.4$$

$$= 4\%$$

$$t_p = t_{max} - t_0 = 0.15\ s$$

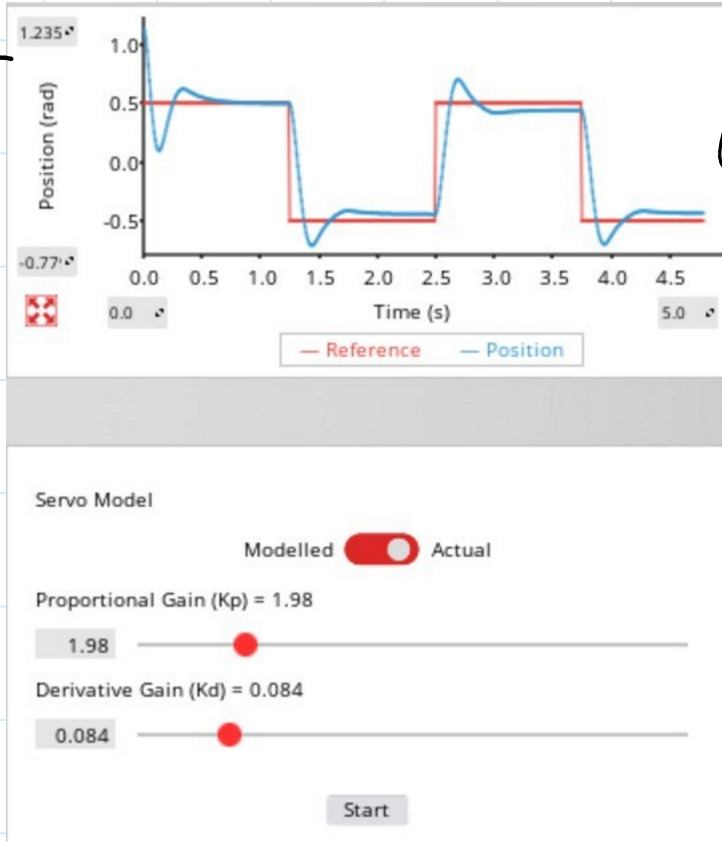
4. Once you have determined that your theoretical gains result in a satisfactory response, let's apply the same gains to a simulated version of an actual QUBE-Servo 2. While the simulation is running, set the toggle switch to **Actual** and examine the system's response.
    a. When using the theoretical gains, did you notice any differences between the modeled response and the actual response? Select all that apply.
        i. Actual response has steady-state error.
        ii. Actual response has a different peak time and overshoot.
        iii. Actual response was similar to modeled response.

Q4



iii. Actual response was similar to modeled response.

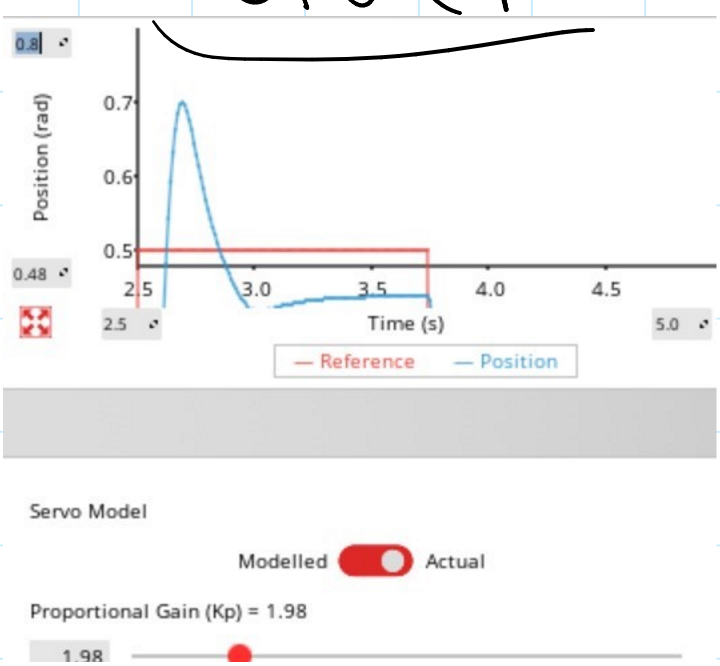
Q4



← Actual

5. Measure the percent overshoot and peak time of the your QUBE-Servo 2 response. If they do not meet the desired specifications, use the gain sliders to manually tune your controller gains in order to get as close as possible to the design requirements.

Actual



Modelled Actual

Proportional Gain (Kp) = 1.98

1.98

Derivative Gain (Kd) = 0.084

0.084

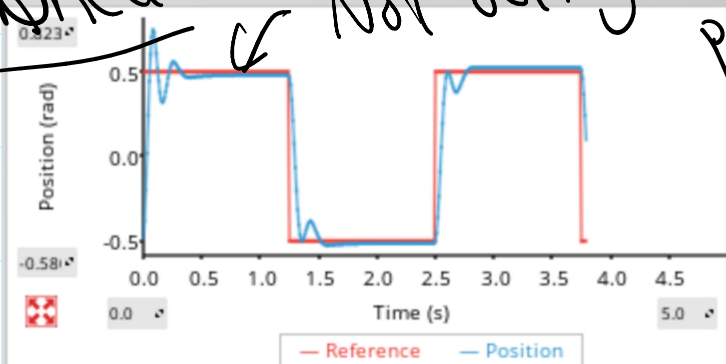
Start

$$\%OS = \frac{0.7 - 0.45}{0.45} = \boxed{55\%}$$

$$t_p = 1.43 - 1.23 = \boxed{0.2\ s}$$ ✓

Tuned

Not using this peak



Servo Model

Modelled Actual

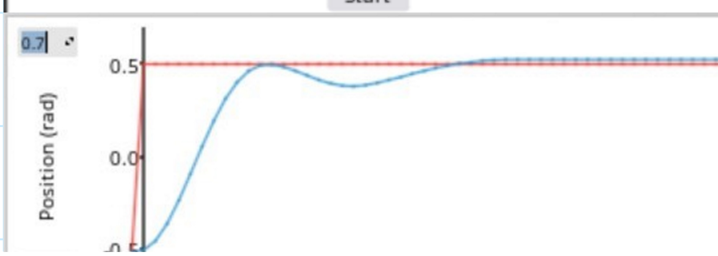
Proportional Gain (Kp) = 3.64

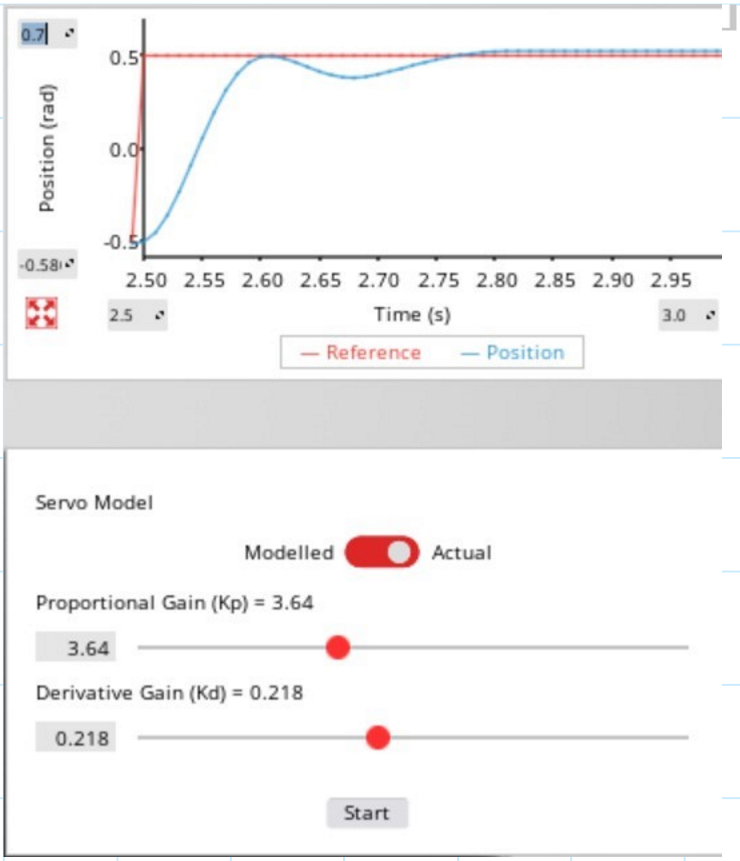
3.64

Derivative Gain (Kd) = 0.218

0.218

Start





$$\%OS = \frac{0.5 - 0.5}{0.5}$$

$$= \boxed{0\%}$$

$$T_p = 2.6 - 2.5$$

$$= \boxed{0.1\,s}$$