HW6

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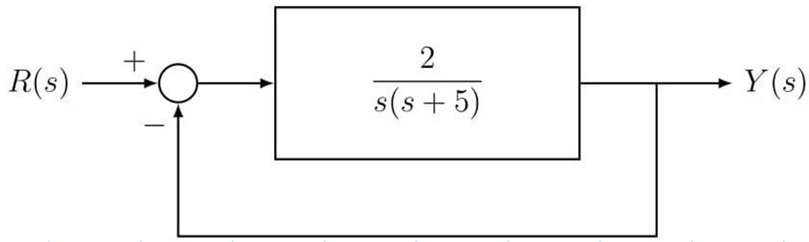
UID: 115443037 Section 0105

1. Use the Routh-Hurwitz Criterion to determine the stability of the following feedback systems. Verify your results in Matlab.

(a)



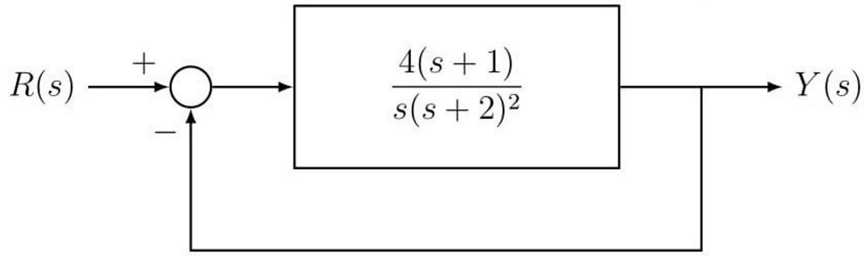
$$\frac{Y(s)}{R(s)} = T(s) = \frac{G(s)}{1+G(s)} = \frac{2}{s(s+5)+2}$$

$$= \frac{2}{s^2+5s+2} = \frac{2}{a^2s^2+a^1s+a_0}$$

s^2	1	2	0
s^1	5	0	0
s^0	$-\frac{\begin{vmatrix}1 & 2 \\ 5 & 0\end{vmatrix}}{5} = 2$	0	0

$\boxed{\text{No sign changes in 1st column} \therefore \text{System is stable}}$

(b)



$$\frac{Y(s)}{R(s)} = T(s) = \frac{G(s)}{1+G(s)} = \frac{4(s+1)}{s(s^2+2)^2 + 4(s+1)}$$

$$= \frac{4s+4}{s(s+2)(s+2)+4} \qquad \frac{4s+4}{s^3+4s^2+4s+4s+4}$$

$$= \frac{4s+4}{s^3+4s^2+8s+4}$$

s^3	1	8	0
s^2	4	4	0
s^1	7	0	0
s^0	4	0	0

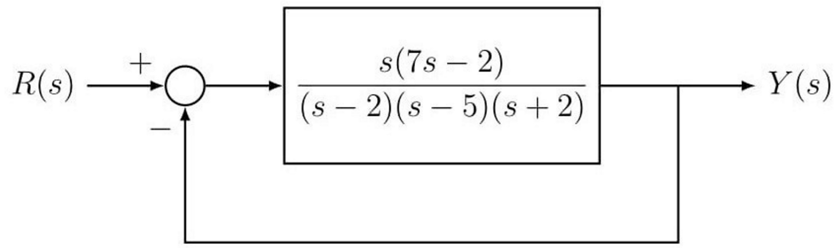
$$b_1 = \frac{-\begin{vmatrix} 1 & 8 \\ 4 & 4 \end{vmatrix}}{4} = 7$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 0 \\ 4 & 0 \end{vmatrix}}{4} = \frac{0}{4} = 0$$

$$c_1 = \frac{-\begin{vmatrix} 4 & 4 \\ 7 & 0 \end{vmatrix}}{7} = 4$$

$$\boxed{\text{No sign changes in 1st column } \therefore \text{ System is stable}}$$

(c)



$$\frac{Y(s)}{R(s)} = T(s) = \frac{G(s)}{1+G(s)} = \frac{s(7s-2)}{(s-2)(s-5)(s+2)+s(7s-2)}$$

$$= \frac{7s^2-2s}{(s^2-7s+10)(s+2)+7s^2-2s}$$

$$= \frac{7s^2-2s}{s^3-\cancel{7s^2}+10s+2s^2-14s+20+\cancel{7s^2}-2s}$$

$$= \frac{7s^2-2s}{s^3+2s^2-6s+20}$$

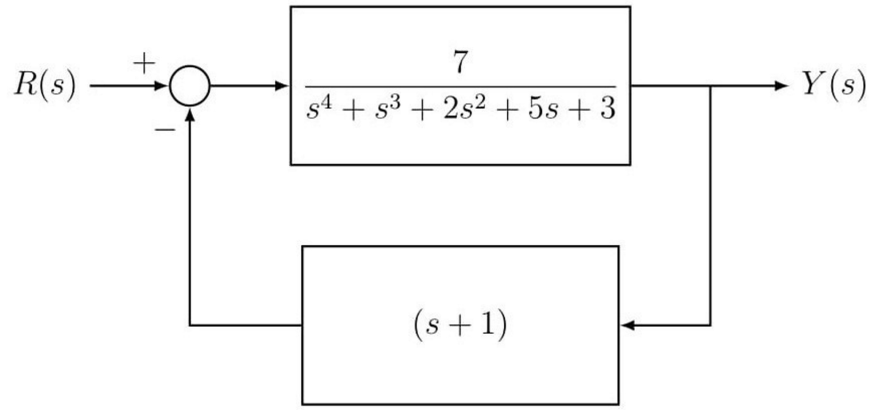
s^3	1	-6
s^2	2	20
s	-16	0
s^0	20	0

$$b_1 = \frac{-\begin{vmatrix} 1 & -6 \\ 2 & 20 \end{vmatrix}}{2} = \frac{-32}{2} = -16$$

$$c_1 = \frac{-\begin{vmatrix} 2 & 20 \\ -16 & 0 \end{vmatrix}}{-16} = \frac{-320}{-16} = 20$$

$\boxed{\text{2 sign changes in 1st column} \therefore \text{System is unstable}}$

(d)



$$\frac{Y(s)}{R(s)} = T(s) = \frac{G(s)}{1+H(s)G(s)} = \frac{7}{s^4+s^3+2s^2+5s+3+7(s+1)}$$

$$= \frac{7}{s^4+s^3+2s^2+12s+10}$$

s^4	1	2	10
s^3	1	12	0
s^2	-10	10	0
s	13	0	0
s^0	10	0	0

$$b_1 = -\frac{\begin{vmatrix} 1 & 2 \\ 1 & 12 \end{vmatrix}}{1} = -10$$

$$b_2 = -\frac{\begin{vmatrix} 1 & 10 \\ 1 & 0 \end{vmatrix}}{1} = 10$$

$$c_1 = -\frac{\begin{vmatrix} 1 & 12 \\ -10 & 10 \end{vmatrix}}{-10} = \frac{-130}{-10} = 13$$

$$c_2 = 0$$

$$d_1 = -\begin{vmatrix} -10 & 10 \end{vmatrix} = 130$$

$$d_1 = \frac{-\begin{vmatrix} -10 & 10 \\ 13 & 0 \end{vmatrix}}{13} = \frac{130}{13} = 10$$

$\boxed{\text{2 sign changes in 1st column} \therefore \text{System is unstable}}$

```
%Problem 1a
sys_a= tf(feedback(zpk([],[0 -5],2),1))
s_a= pole(sys_a)
disp("Stable since poles only in the LHP")
%Problem 1b
sys_b= tf(feedback(zpk([-1],[0 -2 -2],4),1))
s_b= pole(sys_b)
disp("Stable since poles only in the LHP")
%Problem 1c
sys_c= tf(feedback(zpk([0 2/7],[2 5 -2],7),1))
s_c= pole(sys_c)
disp("Unstable since poles in the RHP")
%Problem 1d
G = tf([7],[1 1 2 5 3]);
fb = tf([1 1], 1);
sys_d= tf(feedback(G, fb))
s_d= pole(sys_d)
disp("Unstable since poles in the RHP")
```

```
sys_a =

        2
  -------------
  s^2 + 5 s + 2

Continuous-time transfer function.


s_a =

   -4.5616
   -0.4384

Stable since poles only in the LHP

sys_b =

         4 s + 4
  ---------------------
  s^3 + 4 s^2 + 8 s + 4

Continuous-time transfer function.


s_b =

  -1.6478 + 1.7214i
  -1.6478 - 1.7214i
  -0.7044 + 0.0000i

Stable since poles only in the LHP

sys_c =
```

```
       7 s^2 - 2 s
  ----------------------
  s^3 + 2 s^2 - 6 s + 20

Continuous-time transfer function.


s_c =

  -4.3981 + 0.0000i
   1.1991 + 1.7634i
   1.1991 - 1.7634i

Unstable since poles in the RHP

sys_d =

                7
  -----------------------------
  s^4 + s^3 + 2 s^2 + 12 s + 10

Continuous-time transfer function.


s_d =

   0.9237 + 2.1353i
   0.9237 - 2.1353i
  -1.8474 + 0.0000i
  -1.0000 + 0.0000i

Unstable since poles in the RHP
```

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2. For the characteristic polynomial

$$p(s) = s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50$$

(a) Use the Routh-Hurwitz Criterion to determine the number of roots of $p(s)$ in the right-half plane, in the left-half plane, and on the $j\omega$-axis.

(b) Use Matlab to determine the roots of $p(s)$, and verify your results in part 2a.

(a)

s^5	1	24	-25
s^4	2	48	-50
s^3	$\frac{-\begin{vmatrix} 1 & 24 \\ 2 & 48 \end{vmatrix}}{2} = 0$	$\frac{-\begin{vmatrix} 1 & -25 \\ 2 & -50 \end{vmatrix}}{2} = 0$	0 ← Row of zeros
s^2			
s			
s^0			

so factor out auxillary polynomial

$P(s) = 2s^4 + 48s^2 - 50$

$\frac{dP}{ds} = 8s^3 + 96s$

$= 8(s^2 + 12)$

s^5	1	24	-25
s^4	2	48	-50
s^3	$\cancel{8}$ 1	$\cancel{96}$ 12	0
s^2	12 $\cancel{24}$	-25 $\cancel{-50}$	0
s	$\frac{169}{12}$	0	0
s^0	-25		

$c_1 = \frac{-\begin{vmatrix} 2 & 48 \\ 1 & 12 \end{vmatrix}}{1} = 24$

$c_2 = \frac{-\begin{vmatrix} 2 & -50 \\ 1 & 0 \end{vmatrix}}{1} = -50$

$d_1 = -\begin{vmatrix} 1 & 12 \\ [illegible] \end{vmatrix} = 169$

[illegible] 1st column

1 sign change in 1st colum column ∴ System is Unstable

$$d_1 = \frac{-\begin{vmatrix} 13 & 12 \\ 12 & -25 \end{vmatrix}}{12} = \frac{169}{12}$$

$$e_1 = \frac{-\begin{vmatrix} 12 & -25 \\ \frac{169}{12} & 0 \end{vmatrix}}{\frac{169}{12}} = -25$$

(b)

```
p_s = [1 2 24 48 -25 -50];
roots(p_s)
disp("Unstable since roots in the RHP")
```

```
ans =

  -0.0000 + 5.0000i
  -0.0000 - 5.0000i
   1.0000 + 0.0000i
  -2.0000 + 0.0000i
  -1.0000 + 0.0000i

Unstable since roots in the RHP
```

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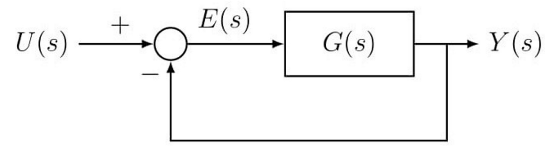
3. For the unity feedback system



with plant

$$G(s) = \frac{(2s+1)K}{s(s-2)(s+3)},$$

find the *range* of K for closed-loop stability.

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{(2s+1)K}{s(s-2)(s+3) + 2(s+1)K}$$

$$= \frac{2Ks+K}{(s^2-2s)(s+3) + 2Ks+K}$$

$$= \frac{2Ks+K}{s^3+3s^2-2s^2-6s+2Ks+K}$$

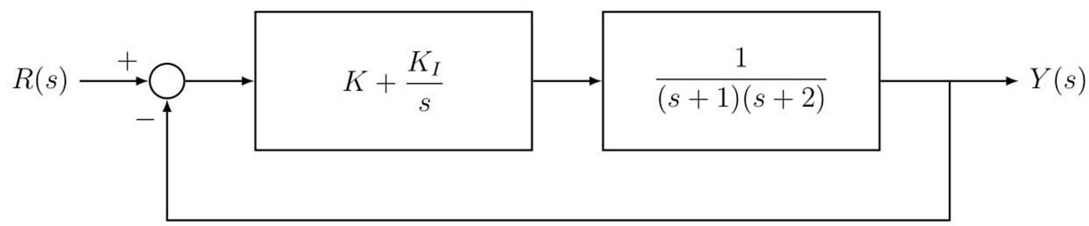
$$= \frac{2Ks+K}{s^3+s^2+(2K-6)s+K}$$

s^3	1	$(2K-6)$
s^2	1	K
s^1	$-\frac{1}{1}\begin{vmatrix}1 & 2K-6 \\ 1 & K\end{vmatrix} = K-6$	0
s^0	$-\frac{1}{K-6}\begin{vmatrix}1 & K \\ K-6 & 0\end{vmatrix} = K$	0

$$K-6$$

For stability number must stay postive, ∴ $K-6>0$, $K>0$
∴ $K>0$ for a stable system

4. Consider the following *Proportional-Integral* (PI) compensation scheme, which allows regulation proportional to the error signal as well as proportional to the accumulated (integrated) error.



(a) Assume that $K_I = 0$ and determine $e_{step}(\infty)$ as a function of the proportional gain K.

(b) Now let $K, K_I > 0$. Find the range of controller gains (K, K_I) so the feedback system is stable.

(c) Determine $e_{step}(\infty)$ for any (K, K_I) in the range found in part 4b. How does the inclusion of the integral term affect steady-state error?

(a) $K_i = 0$

$$G(s) = \frac{K}{(s+1)(s+2)}$$

$$e_{step} = \frac{1}{1+\lim_{s\to 0} \frac{K}{(s+1)(s+2)}} = \frac{1}{1+\frac{K}{2}}$$

$$= \boxed{2}$$

$$= \boxed{\frac{2}{2+K}}$$

(b) $K, k_i > 0$

$$G(s) = \frac{Ks+K_i}{s}\left(\frac{1}{(s+1)(s+2)}\right)$$

$$= \frac{Ks+K_i}{s(s+1)(s+2)} = \frac{Ks+K_i}{s^3+3s^2+2s}$$

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{Ks+K_i}{s^3+3s^2+(2+K)s+K_i}$$

s^3	1	$(K+2)$	
s^2	3	K_i	
s^1	$\dfrac{-\begin{vmatrix} 1 & K+2 \\ 3 & K_i \end{vmatrix}}{3} = K+2-\dfrac{K_i}{3}$	0	
s^0	$-\begin{vmatrix} 3 & K_i \\ K+2-\frac{K_i}{3} & 0 \end{vmatrix} = K_i = 0$		

must be positive to prevent sign change

$$K > -2 + \frac{K_i}{3}$$

$$K_i > 0$$

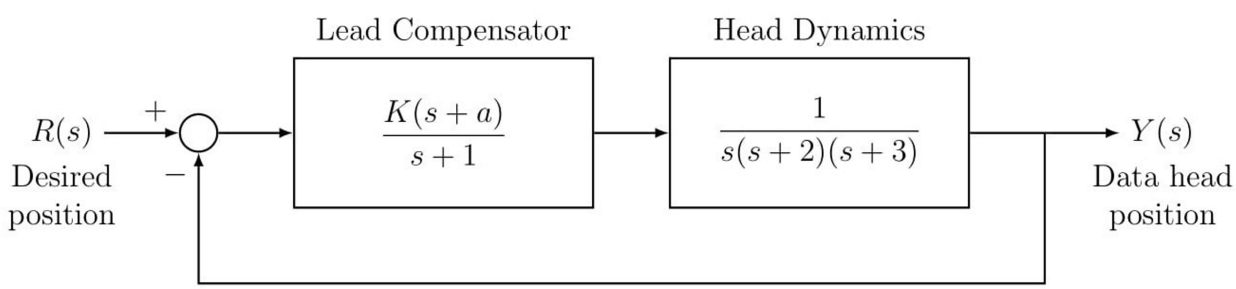
c) $k_I = 1$, $k = 1$, 4(b) is satisfied

and $G(s) = \dfrac{s+1}{s^3+3s^2+2s}$

$$e(\infty) = \frac{1}{1+\lim\limits_{s\to 0}\left(\frac{s+1}{s^3+3s^2+2s}\right)} = \frac{1}{1+\infty} = \boxed{0}$$

Make SS-Error go to zero

5. A welding head that is used in the auto industry is moved to different positions on the auto body, and a rapid, accurate response is required. The control engineers have decided to design a *lead compensator* to achieve the desired performance specifications. A block diagram of a welding head position system is shown below.



Assuming that $K = 40$, use the Routh-Hurwitz Criterion to determine the range of values of a (if any) for which the system is stable.

$G(s) = \dfrac{K(s+a)}{s(s+1)(s+2)(s+3)}$ $K = 40$

$s(s+1)(s+2)(s+3)$

$$= \frac{40s + 40a}{s(s+1)(s+2)(s+3)} = \frac{40s + 40a}{s^4 + 6s^3 + 11s^2 + 6s}$$

$$T(s) = \frac{40s + 40a}{s^4 + 6s^3 + 11s^2 + 46s + 40a}$$

$$\begin{array}{c|ccc}
s^4 & 1 & 11 & 40a \\
s^3 & 6 & 46 & 0 \\
s^2 & \frac{10}{3} & 40a & 0 \\
s^1 & 23-36a & 0 & 0 \\
s^0 & 40a & 0 & 0
\end{array}$$

$$b_1 = -\frac{\begin{vmatrix} 1 & 11 \\ 3 & 23 \end{vmatrix}}{3} = \frac{10}{3}$$

$$b_2 = -\frac{\begin{vmatrix} 1 & 40a \\ 3 & 0 \end{vmatrix}}{3} = 40a$$

For stabily first colum must stay same sign

$40a > 0$

$23 - 36a > 0$

$$c_1 = -\frac{\begin{vmatrix} 3 & 23 \\ \frac{10}{3} & 40a \end{vmatrix}}{\frac{10}{3}} = 23 - 36a$$

$$d_1 = -\frac{\begin{vmatrix} 10/3 & 40a \\ 23-36a & 0 \end{vmatrix}}{23-36a}$$

$36 - 36a$

$$\boxed{0 < a < \frac{23}{\ldots}}$$

$$\boxed{0 < a < \frac{23}{36} \quad \text{for stable stystem}}$$

$36 - 36a$

$= \frac{40a(23-36a)}{23-36a}$

$= 40a$