Studio 08

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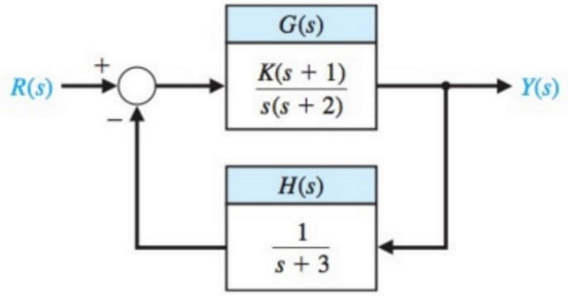
Section 0105

Exercise 1. Root Locus (RL) Analysis by hand and using Matlab

Given the feedback system shown below.



(a) Determine the characteristic equation for the closed loop system
(b) Determine the number of asymptotes and their angles
(c) Sketch the RL
(d) Determine the break-away point (if any)
(e) Determine the range of K for which the system is stable.
(f) Write a Matlab script to plot the root locus and compare your root locus sketch to the Matlab output. Use the Matlab plot to determine the break-away point and compare what you get to your answer in part (d).
Hint: use the function `rlocus(G*H)`

(a) $$T(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$T(s) = \frac{\frac{K(s+1)}{s(s+2)}}{1+\frac{K(s+1)}{s(s+2)(s+3)}} = \frac{K(s+1)}{s(s+2)+\frac{K(s+1)}{(s+3)}}$$

$$= \frac{K(s+1)(s+3)}{s(s+2)(s+3)+K(s+1)}$$

$$= \frac{K(s+1)(s+3)}{s(s+2)(s+3)+K(s+1)}$$

$$= \frac{K(s^2+4s+3)}{s^3+5s^2+(6+K)s+K}$$

$$\boxed{P(s) = s^3+5s^2+(6+K)s+K}$$

b) $G(s)H(s) = \frac{K(s+1)}{s(s+2)(s+3)}$

zeros: $s=-1$

poles: $s=0,-2,-3$

$\boxed{2 \text{ asymtotes}}$

$$\theta_a = \frac{(2k+1)180^\circ}{2} = \boxed{90^\circ, 270^\circ}$$

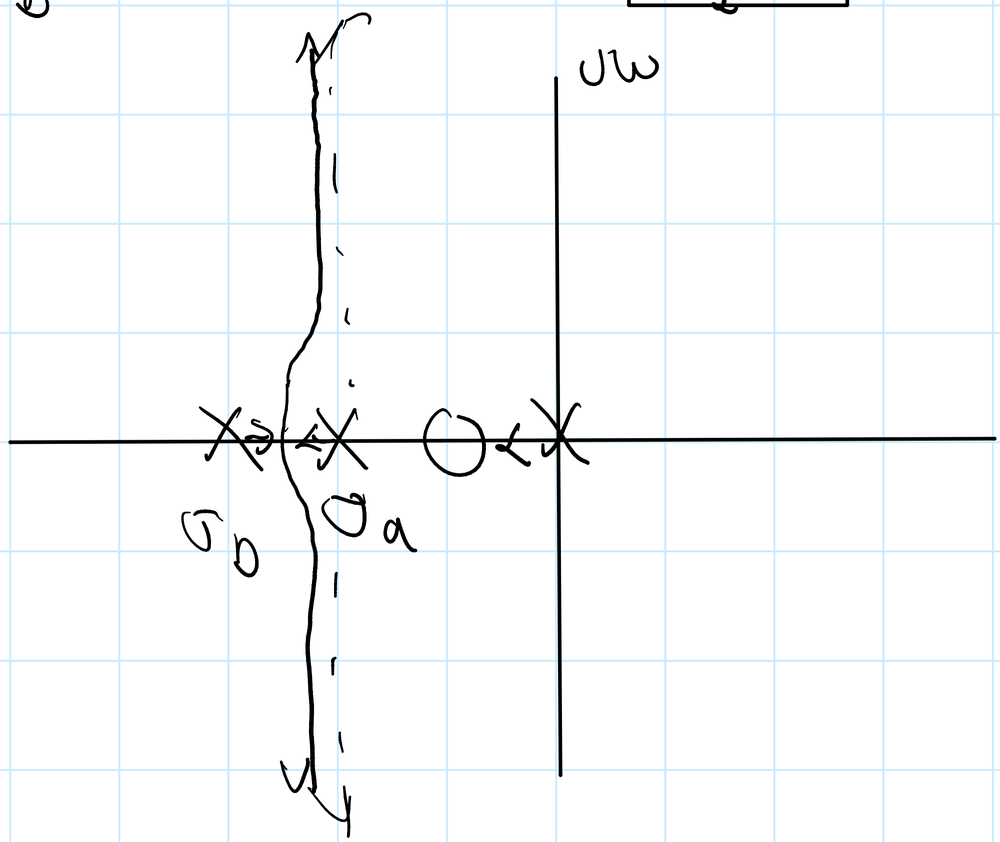
c) $\sigma_a = \frac{(-5)+1}{2} = \boxed{-2}$

d)

$$\frac{1}{\sigma+1} = \frac{1}{\sigma} + \frac{1}{\sigma+2} + \frac{1}{\sigma+3}$$

$$\sigma(\sigma+2)(\sigma+3) = (\sigma+2)(\sigma+3)(\sigma+1) + \sigma(\sigma+1)(\sigma+3) + \sigma(\sigma+1)(\sigma+2)$$

$$\sigma^3+5\sigma^2+6\sigma=(\sigma^3+6\sigma^2+11\sigma+6)$$
$$+(\sigma^3+4\sigma^2+3\sigma)+(\sigma^3+3\sigma^2+2\sigma)$$
$$0=\sigma^3+4\sigma^2+5\sigma+3$$
$$\sigma_b=-2.4656\approx\boxed{-2.5}$$



e) $P(s)=s^3+5s^2+(6+K)+K$

s^3	1	$6+K$	$+$
s^2	5	K	$+$
s^1	$\dfrac{\begin{vmatrix} 1 & 6+K \\ 5 & K \end{vmatrix}}{-5} = \dfrac{K-(30+5K)}{-5}$ $\dfrac{-4K-30}{-5} = \dfrac{4}{5}K+6$	0	
s^0	K		

$$\frac{4}{5}K+6>0$$

$$K>-6\left(\frac{5}{4}\right)$$

$$K>-7.5$$

$$K>0$$

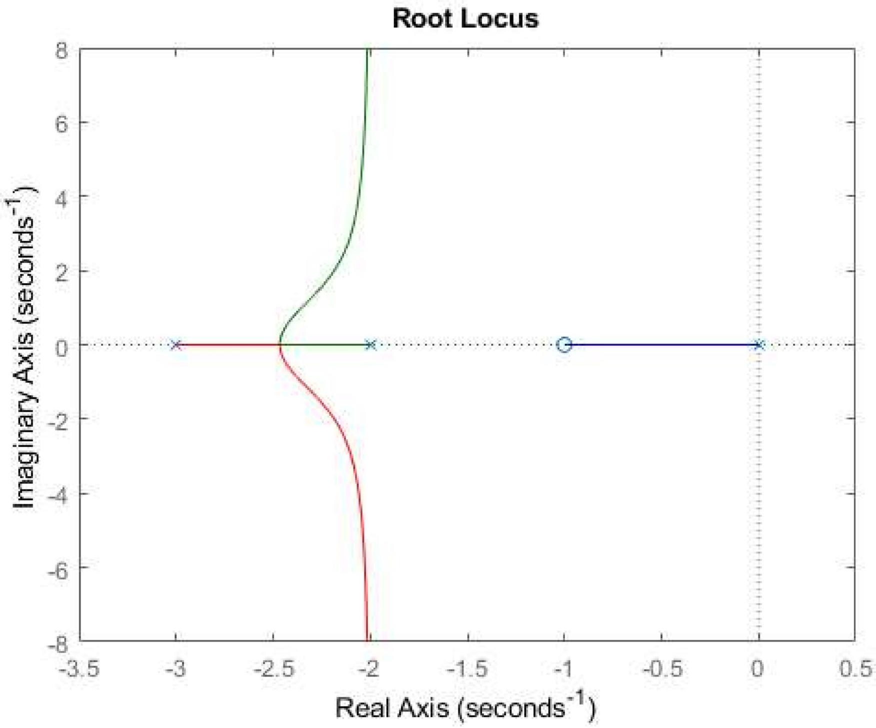
$$\boxed{\therefore\ K>0 \text{ for stable system}}$$

system

f)

```
G = tf([1 1],[1 2 0])
H = tf([1],[1 3])
rlocus(G*H)
```

```
G =

    s + 1
  ---------
  s^2 + 2 s

Continuous-time transfer function.


H =

    1
  -----
  s + 3

Continuous-time transfer function.
```



Breakaway

points match