PS4: Q1

$x_1 = 3;\ x_2 = \pi$

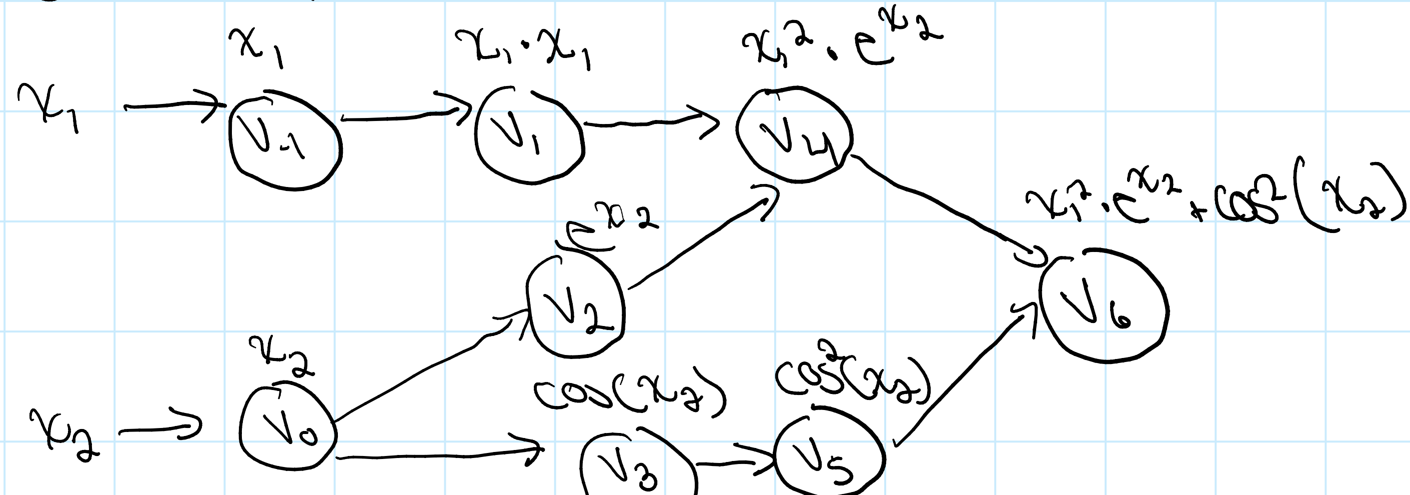
$$f(x_1, x_2) = x_1^2 \cdot e^{x_2} + \cos^2(x_2) = 209.266$$

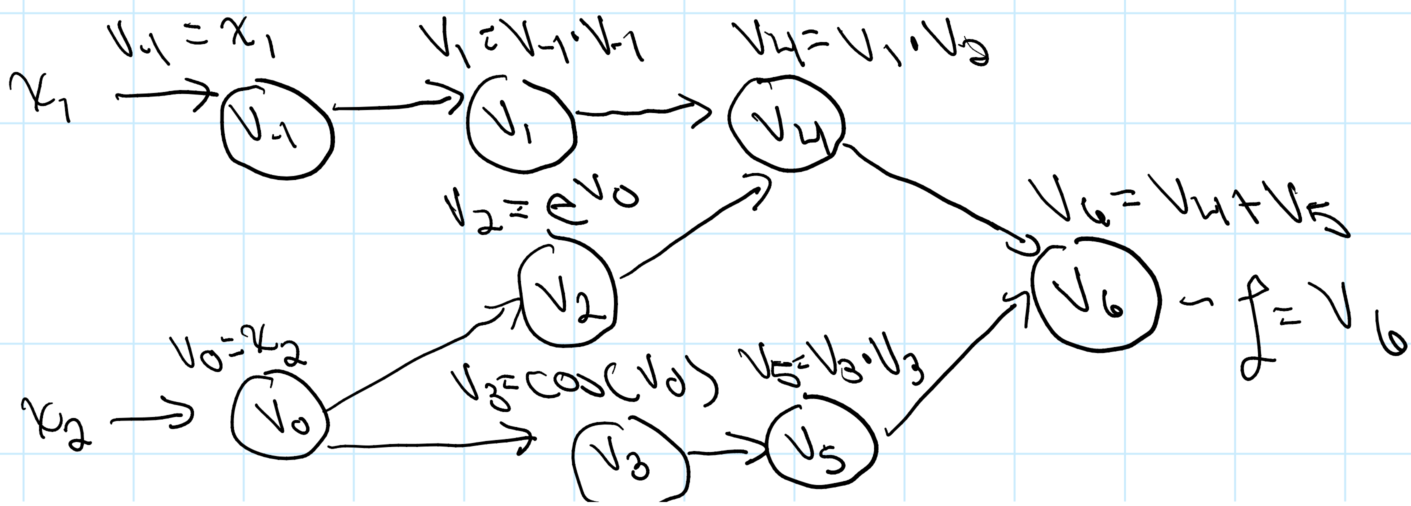
$$\frac{\partial f}{\partial x_1} = 2x_1 \cdot e^{x_2} = 6e^{\pi} = 138.844$$

$$\frac{\partial f}{\partial x_2} = x_1^2 \cdot e^{x_2} + 2\cos(x_2) \cdot -\sin(x_2)$$

$$= 9e^{\pi} + (-2)(0) = 208.266$$

1. Computational Graph





2. Foward Pass

$v_{-1} = x_1 = 3$

$v_0 = x_2 = 3.141$

$v_1 = v_{-1} \cdot v_{-1} = 9$

$v_2 = e^{v_0} = 23.141$

$v_3 = \cos(v_0) = -1$

$v_4 = v_1 \cdot v_2 = 208.266$

$v_5 = v_3 \cdot v_3 = 1$

$v_6 = v_4 + v_5 = 209.266$

$f = v_6 = 209.266$

3. Reverse-Mode Automatic Differentiation

Adjoint $= \bar{v}_i = \dfrac{\partial f}{\partial v_i}$

Forward Pass

$v_{-1} = x_1 = 3$

Backward Mode AD

$\frac{\partial f}{\ } = \bar{v}_{-1} = \frac{\partial f}{\ } \cdot \frac{\partial v_j}{\ }$

$v_{-1} = x_1 = 3$

$v_0 = x_2 = 3.141$

$v_1 = v_{-1} \cdot v_{-1} = 9$

$v_2 = e^{v_0} = 23.141$

$v_3 = \cos(v_0) = -1$

$v_4 = v_1 \cdot v_2 = 208.266$

$v_5 = v_3 \cdot v_3 = 1$

$v_6 = v_4 + v_5 = 209.266$

$f = v_6 = 209.266$

$\frac{\partial f}{\partial v_{-1}} = \bar{v}_{-1} = \frac{\partial f}{\partial v_1} \cdot \frac{\partial v_1}{\partial v_{-1}}$

$= \bar{v}_1 \cdot 2v_{-1}$

$= 138.846$

$\frac{\partial f}{\partial v_0} = \bar{v}_0 = \frac{\partial f}{\partial v_2} \cdot \frac{\partial v_2}{\partial v_0}$

$+ \frac{\partial f}{\partial v_3} \cdot \frac{\partial v_3}{\partial v_0} = \bar{v}_2 \cdot e^{v_0}$

$+ \bar{v}_3 \cdot -\sin(v_0)$

$= 208.266$

$\frac{\partial f}{\partial v_1} = \bar{v}_1 = \frac{\partial f}{\partial v_4} \cdot \frac{\partial v_4}{\partial v_1}$

$= \bar{v}_4 \cdot v_2$

$= 23.141$

$\frac{\partial f}{\partial v_2} = \bar{v}_2 = \frac{\partial f}{\partial v_4} \cdot \frac{\partial v_4}{\partial v_2}$

$= \bar{v}_4 \cdot v_1$

$= 9$

$\frac{\partial f}{\partial v_3} = \bar{v}_3 = \frac{\partial f}{\partial v_5} \cdot \frac{\partial v_5}{\partial v_3}$

$= \bar{v}_5 \cdot 2v_3$

$= -2$

$\frac{\partial f}{\partial v_4} = \bar{v}_4 = \frac{\partial f}{\partial v_6} \cdot \frac{\partial v_6}{\partial v_4}$

$= \bar{v}_6 \cdot 1 = 1$

$\frac{\partial f}{\partial v_5} = \bar{v}_5 = \frac{\partial f}{\partial v_6} \cdot \frac{\partial v_6}{\partial v_5}$

$= \bar{v}_6 \cdot 1 = 1$

$\frac{\partial f}{\partial v_6} = \bar{v}_6 = 1$

In [4]:
```
#4.
import torch
import math
import numpy as np
#Manual Derivative
def true_grad(x0,x1):
    return np.array([
        2*x0*np.exp(x1),
        x0*x0*np.exp(x1) + 2*np.cos(x1)*-np.sin(x1)
    ])
print(true_grad(3,3.141))
#Using Backward AD Pytorch
x = torch.tensor([3, 3.141], requires_grad=True)
print(x)
f= x[0]**2*torch.exp(x[1])+torch.cos(x[1])
f.backward()
x.grad
```

```
[138.76189369 208.14402584]
tensor([3.0000, 3.1410], requires_grad=True)
```

Out[4]:
```
tensor([138.7619, 208.1423])
```

Matches By-Hand