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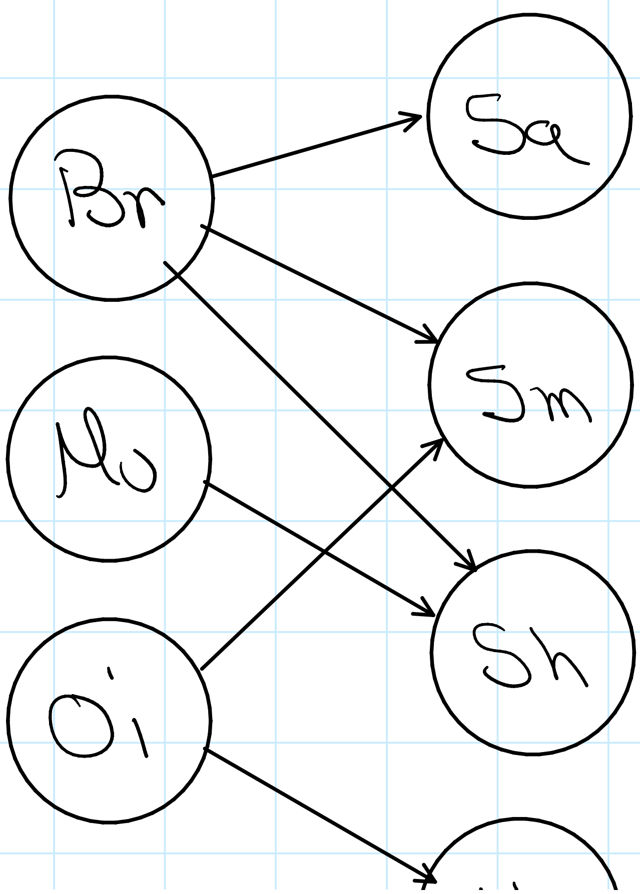
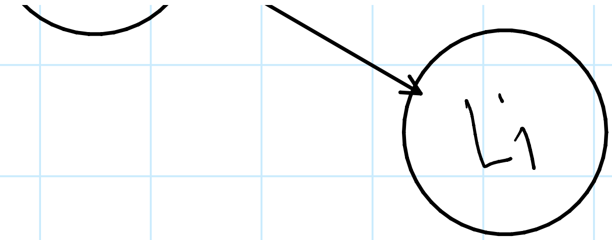
Section 0101

# 1: Alice and the Mechanic (5 pts)

Alice takes her car to a dishonest mechanic, who claims that it requires $2500 dollars in repairs. Doubting this diagnosis, Alice decides to verify it with a graphical model. Suppose that the car can have three possible problems: brake trouble (Br), muffler trouble (Mu), and low oil (Oi), all which are a priori marginally independent. The diagnosis is performed by testing for four possible symptoms: squealing noises (Sq), smoke (Sm), shaking (Sh), and engine light (Li). The conditional probabilities of these symptoms are related to the underlying causes as follows. Squealing depends only on brake problems, whereas smoke depends on brake problems and low oil. Shaking is related to brake problems and muffler problems, whereas the engine light depends only on the oil level.

1. DGM



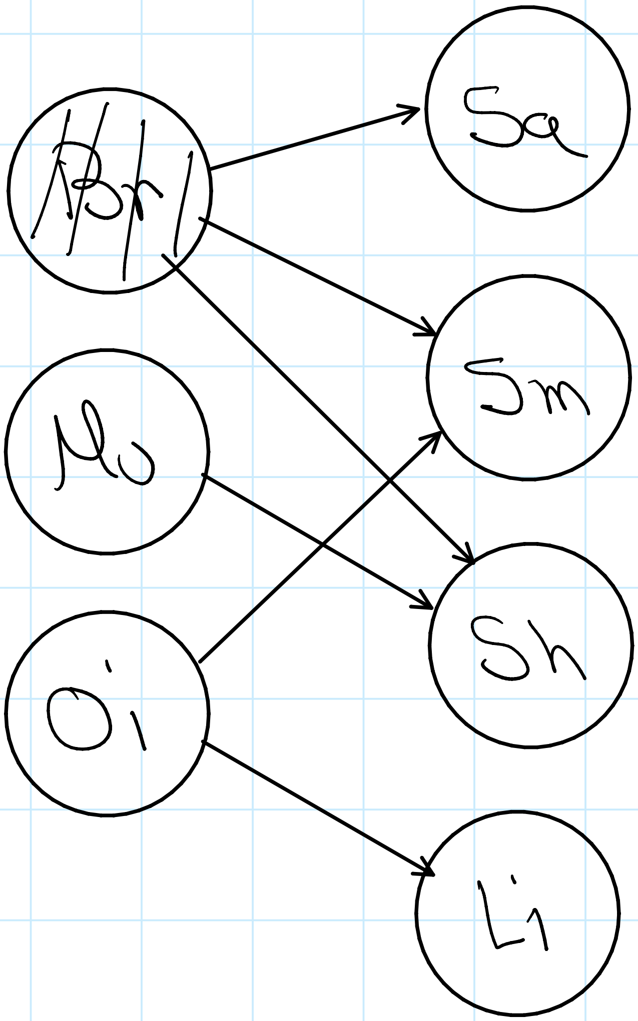



2. By observing the engine light(Li), we gain information about low oil (Oi)
3. If we see the car belching oily clouds of smoke(Sm), we know there is possible brake trouble(Br) and/or low oil (Oi). By observing the engine light (Li), this increases the probability of low oil (Oi) over brake trouble (Br)
4. If the car is shaking violently (Sh) and smoking (Sm), brake trouble (Br), muffler trouble (Mu), and low oil (Oi)

trouble (Mu), and low oil (Oi) are possible. A flickering engine light (Li) tells us there is a lesser probability of brake trouble (Br) since smoking (Sm) is explained. Also, there is a greater probability of muffler trouble (Mu) due to shaking (Sh). However, the cause of shaking (Sh) could still be brake trouble (Br) or muffler trouble (Mu).

5. If alice knows the car has a brake problem (Br), it is not useful to measure smoke level to asses a muffler problem (Mu) since

smoke (Sm) and muffler trouble (Mu) are independent




$Sm \perp Mu \mid Br$

$\rightarrow$

Smoke and muffler trouble are independent

# 2: Maximum Likelihood Estimation (10 pts)

In class, we showed how to derive the mean and variance for the Gaussian (Normal) distribution by optimizing the Log-Likelihood of multiple N i.i.d. samples:

$P(x \mid \mu, \sigma) = \prod_{i=1}^{N} N(x_i \mid \mu, \sigma)$

In the same way, show how to get the MLE parameters for the following distributions given N i.i.d. samples:

1. Find the MLE for $\mu$, where: $P(x_i \mid \mu) = \mu^{x_i}(1-\mu)^{1-x_i}$
2. Find the MLE for $\mu$, where $x_i$ take on non-negative integer values and: $P(x_i \mid \mu) = \frac{\mu^{x_i} e^{-\mu}}{x_i!}$

$P(x \mid \mu, \sigma) = \prod_{i=1}^{N} N(x_i \mid \mu, \sigma)$

1. $P(x_i \mid \mu) = \mu^{x_i}(1-\mu)^{1-x_i}$

given N i.i.d samples

$LL = \ln(P(x_i \mid \mu))$

$= \ln(\mu^{x_i}(1-\mu)^{1-x_i})$

$= \ln(\mu^{x_i}) + (1-x_i)\ln(1-\mu)$

$\frac{\partial LL}{\partial \mu} = \frac{1}{\mu^{x_i}} \cdot x_i \mu^{x_i-1} + \frac{1-x_i}{1-\mu} \cdot -1$

$= \frac{x_i \mu^{x_i} \mu^{-1}}{\mu^{x_i}} - \frac{1-x_i}{1-\mu}$

$$= \frac{x_i \mu^{x_i - 1}}{\mu^{x_i}} - \frac{1 - x_i}{1 - \mu}$$

$$= \frac{x_i}{\mu} - \frac{1 - x_i}{1 - \mu} = \frac{(x_i)(1-\mu)}{(\mu)(1-\mu)} - \frac{(1-x_i)(\mu)}{(1-\mu)(\mu)}$$

$$= \frac{x_i - x_i\mu - \mu + x_i\mu}{(1-\mu)(\mu)} = \frac{x_i - \mu}{(1-\mu)(\mu)}$$

$$= 0$$

$$\boxed{\mu = x_i}$$

2. $P(x_i|\mu) = \frac{\mu^{x_i} e^{-\mu}}{}$ given N i.i.d samples and $x_i!$ where $x_i$ takes on non-negative integer values

$LL = \ln(P(x_i|\mu))$

$$= \ln\left(\frac{\mu^{x_i} e^{-\mu}}{x_i!}\right)$$

$$= x_i \ln(\mu) + \ln(e^{-\mu})$$

$- \ln(x_i!)$

$$\frac{\partial LL}{\partial \mu} = \frac{x_i}{\mu} + \frac{1}{e^{-\mu}} \cdot -e^{-\mu}$$

$$= \frac{x_i}{\mu} - 1 = 0$$

$$\boxed{\mu = x_i}$$